Modulus and Thickness of the Pavement Surface Layer from SASW Tests

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The spectral analysis of surface waves (SASW) test can be used rapidly in the field to determine the stiffness and thickness of the pavement surface layer. The test is equally applicable to asphalt concrete and portland cement concrete pavements. One of the most important features is that testing can be performed quickly (in approximately 5 min at each location). Values of Young's modulus and thickness of the surface layer are determined using a straightforward procedure. Analytical studies are presented to substantiate this procedure and to optimize its use. Several case studies from asphalt concrete pavements and one Portland cement concrete pavement are presented. The results show that this adaptation of the SASW test provides values of Young's modulus that are sensitive to the elastic stiffness of the surface layer and also provides reasonable estimates of the thickness of the surface layer. In addition, changes in the stiffness of the surface layer with time and temperature are easily monitored in situ.

Reliable measurements of the in situ conditions of pavements are an important aspect in effectively managing pavement systems. Existing nondestructive devices for moduli measurements, such as the Dynaflect or falling weight deflectometer, cannot be used to perform an independent measure of only the surface layer. In addition, these devices can be somewhat insensitive to the modulus of the pavement surface layer, specially for the cases of a thin surface layer on the order of a few inches thick or under those conditions where bedrock is near the surface. Optimum results are also obtained with these tests when the thickness of the layers in the pavement are known a priori. On the other hand, the spectral analysis of surface waves (SASW) test is very sensitive to the value of Young's modulus of the surface layer and bedrock conditions do not affect the near-surface measurements. In addition, the thickness of surface layer is not required to evaluate the measurements but can be estimated from the field data.

As originally proposed, the SASW method (*1–3*) has been a rather complex nondestructive method involving the use of surface waves to evaluate the modulus profile of the entire pavement system. However, if only the stiffness and thickness of the surface layer are required, the SASW test can be greatly simplified so that testing can be performed rapidly and values of moduli and thickness can be determined immediately in the field. This adaptation of the SASW test, originally proposed by Sheu et al. (*4*), is based on a theoretically sound procedure that is simple, easy to implement, and does not require knowledge of any of the layer thicknesses in the pavement profile.

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In the following sections, this adaptation of the SASW test is briefly described along with an analytical study of the dispersive properties of surface waves in the pavement surface layer. Typical test results from several pavements, including one where the Portland cement concrete was curing, are then presented.

GENERAL BACKGROUND

Evolution of SASW Method

The SASW method (*1–3*) is an in situ seismic method that is used for near-surface profiling of pavement sites. The SASW method is a modification of the steady-state Rayleigh wave technique introduced in the 1950s for the measurement of elastic properties of pavements (*5,6*). The original technique involved testing with bulky equipment and analyzing the data with an empirical approach. These two shortcomings resulted in the method's never gaining wide acceptance. In fact, the empirical basis for data analysis resulted in erroneous results under certain conditions that often occur in pavement systems.

Because of the development of portable, sophisticated electronic equipment capable of performing accurate, high-frequency data acquisition and complex mathematical manipulations rapidly in the field, the bulky equipment associated with the steady-state technique is no longer required. In addition, a theoretically sound basis for data analysis has been developed (*7–10*). These two developments have resulted in the application of the SASW method to nondestructive pavement testing. One of the important areas in which the SASW method can easily be used is the rapid determination of the modulus and thickness of the pavement surface layer. This application is possible because of the simplicity of data analysis in a uniform top layer of any layered system.

Equipment and Field Testing

The general arrangement of the source, receivers (accelerometers), and recording equipment in an SASW test is shown schematically in Figure 1. No boreholes are required because both the source and receivers are placed on the pavement surface. A piezoelectric shaker is an effective source for generating a group of surface waves over frequencies ranging from about 1 to 50 kHz. These high frequencies are necessary to sample the surface layer. A digital waveform analyzer

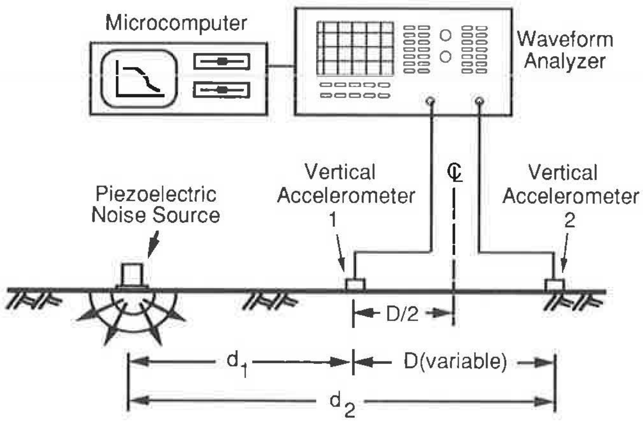

FIGURE 1 General configuration of equipment used to evaluate the surface layer.

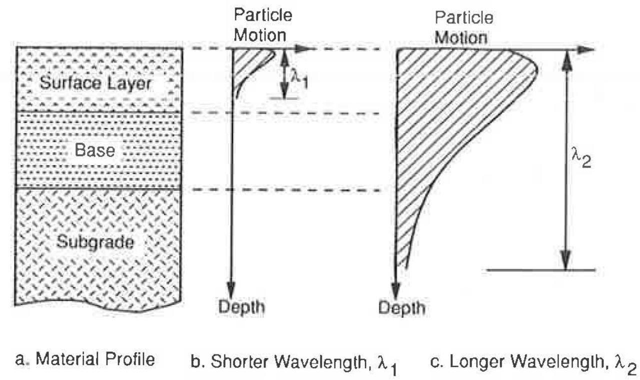

FIGURE 2 Approximate distribution of vertical particle motion with depth for two surface waves of different wavelengths.

coupled with a microcomputer is used to capture and process the outputs from the receivers.

The vertical accelerometers and source are arranged in a linear array. The distance D between receivers (see Figure 1) is usually 6 in. but may be varied by the operator to optimize the test results for a particular site. (Distances of 3 to 12 in. have been used in practice.) The distance d_1 between the source and the first receiver is usually kept equal to D but may also be increased by the operator to minimize destructive interference from body wave reflections. However, d_1/D = 1.0 is normally a good arrangement, as shown in the following analytical studies.

Surface Wave Dispersion

The dispersive property of surface waves permits use of the SASW method. Dispersion refers to the variation of surface wave phase velocity with wavelength (or frequency). Dispersion arises because surface waves of different wavelengths sample different parts of the pavement profile, as shown in Figure 2. As wavelength increases, particle motion extends to greater depths in the profile. The velocities of surface waves are representative of the material stiffness over depths for which there is significant particle motion. For example, the particle motion of a wave that has a wavelength less than the thickness of the pavement surface layer is confined to this layer (Figure 2b). Therefore, the wave velocity is affected by the stiffness of the surface layer and not by the lower layers. The velocity of a wave with a wavelength of several feet is influenced by the properties of the surface layer, base, and subgrade because a significant portion of the particle motion is in these layers (Figure 2c). Thus, by using surface waves over a wide range of wavelengths, it is possible to assess material properties over a range of depths. However, to monitor only the stiffness of a surface layer, only wavelengths less than the thickness of this surface layer need to be generated and measured.

The overall objective in SASW testing is to make field measurements of surface wave dispersion (i.e., measurements of surface wave velocity V_R at various wavelengths L_R) and then to determine the stiffnesses of the layers in the profile. For the case of a uniform surface layer, the surface wave phase velocity V_R is related to the shear wave velocity V_S of the material by Poisson's ratio ν. The ratio of V_R to V_S varies from 0.874 to 0.955 for values of Poisson's ratio ν ranging from 0 to 0.5. Therefore, once the surface wave phase velocity of the uniform surface layer has been measured, it is a simple matter to calculate the shear wave velocity and, hence, Young's modulus of the surface layer using the following relationships:

$$V_S = C \cdot V_R \quad (1)$$

$$G = (\gamma/g) \cdot V_S^2 \quad (2)$$

$$E = 2\,G\,(1 + \nu) \quad (3)$$

where

C = 1.135 − 0.182 · ν (for $\nu \geq 0.1$),
G = shear modulus,
γ = total unit weight,
g = acceleration due to gravity, and
E = Young's modulus.

Because the values of moduli calculated in Equations 1 through 3 are a result of seismic measurements, these values represent the moduli at small strain amplitudes. Moduli measured at these strain levels are maximum values of moduli. Additionally, if the material stiffness is frequency dependent (such as for asphalt concrete), then seismic tests will result in higher values of stiffness than determined by static tests because of the high frequencies used in seismic testing. As a result, the seismically determined values should be adjusted accordingly.

ANALYTICAL STUDIES

To apply the SASW test effectively to measurements of the surface layer, analytical studies of the dispersive characteristic of waves propagating in a uniform layer over a half-space were conducted. Two general cases were studied: (a) dispersion of plane Rayleigh waves, and (b) dispersion of combined Rayleigh and body waves. The theoretical solution involving plane Rayleigh wave propagation forms the basis for the simplest analysis procedure used to interpret SASW field data. However, vertical excitation at a point on the surface of a

layered system creates a group of seismic waves that are composed of body waves [compression (P) and shear (SV) waves] as well as surface (Rayleigh) waves, which propagate radially away from the source. Therefore, simulation of wave dispersion from the combined body and surface waves is important to understanding SASW testing, particularly at distances from the source that are small relative to the wavelength (near-field effect). In the following sections, the dispersion characteristics of both plane Rayleigh wave propagation and combined waves excited by a vertical dynamic load are discussed.

The mathematical model consists in both cases of a horizontally layered half-space with homogeneous properties within each layer. The solution to the differential equations of motion for each layer permits the stresses and displacements at the top of the layer to be related to the stresses and displacements at the bottom of the layer for a given frequency and wave number (or wavelength). The stresses and displacements are given by a system of equations in terms of a matrix **T** called the "transfer" or "propagator" matrix. Expressions for the elements of the matrix **T** can be found in the literature (*7,11*).

Alternatively, the stresses at the top and bottom of a layer can be expressed in terms of the displacements at the top and bottom through a dynamic stiffness matrix, as suggested by Kausel and Roesset (*9*).

Dispersion of Plane Rayleigh Waves

By imposing compatibility of displacements and equilibrium of stresses at the interfaces between two layers, a series of multiplications of the transfer matrices **T** of each layer provides a relationship between the stresses and displacements at the free surface and those at any depth. By assuming no excitation at the top and no waves propagating upward within the underlying half-space, the system of equations can be reduced to a set of homogeneous equations in terms of a 2 × 2 matrix. To obtain nontrivial solutions, this matrix must be singular. The values of the wave numbers k (for a fixed frequency) that make the determinant of this matrix equal to zero provide the wave numbers of the Rayleigh waves propagating at that frequency through the soil profile. For each value of k, one can then obtain the wavelength $2\pi/k$ and the corresponding propagation velocity V_R.

Using instead the dynamic stiffness matrices of the layers, one can assemble a stiffness matrix for the complete soil profile following the same procedures used in matrix structural analysis. Again to obtain the modes of propagation, the determinant of the global stiffness matrix is set equal to zero. The total stiffness matrix is a tridiagonal matrix in terms of 2 × 2 submatrices, and therefore the evaluation of the determinant is rather simple.

In the case of a uniform half-space, the zero determinant matrices in both approaches lead to a frequency (or wavelength) independent solution for the characteristic equation. The dispersion curve is thus a straight line as shown in Figure 3. In this special case, propagation velocity is independent of wavelength because the half-space has a uniform stiffness and only plane Rayleigh waves are being considered.

For a layer resting on a half-space with different properties, V_R will vary with frequency. At very low frequencies (long wavelengths), the velocity will tend to the velocity of the half-space. At very high frequencies (short wavelengths), the value of V_R will equal the value of Rayleigh wave velocity in the top layer. Figure 4 shows a typical dispersion curve for a softer layer overlying a stiffer half-space, with stiffness ratio $E_1/E_2 = 0.25$. This system could represent an asphalt concrete (AC) layer over a thick cemented base. Figure 5 shows a similar dispersion curve for a stiffer layer overlying a softer half-space, with $E_1/E_2 = 4$. This system could represent a rather soft surface layer over a stiff uncemented base and subgrade with similar stiffnesses or it could represent a Portland cement concrete (PCC) layer over a thick AC base. In either of the cases shown in Figures 4 and 5, the top layer in the profile appears as though it were a uniform half-space for waves with very short wavelengths (high-frequency waves). In other words, these short-wavelength surface waves sample only the stiffness of the top layer. As such, the shear wave velocity, shear modulus, and Young's modulus of the top layer may be calculated using the relationships in Equations 1 through 3. This important point forms the basis of the application of the SASW method presented herein. In addition, the thickness h of the top layer may be estimated using the critical wavelength L_C, as shown in Figure 6.

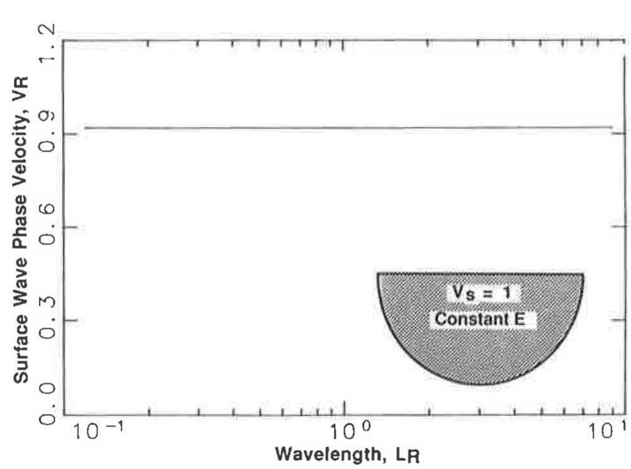

FIGURE 3 Dispersion curve for plane Rayleigh waves propagating in a uniform half-space.

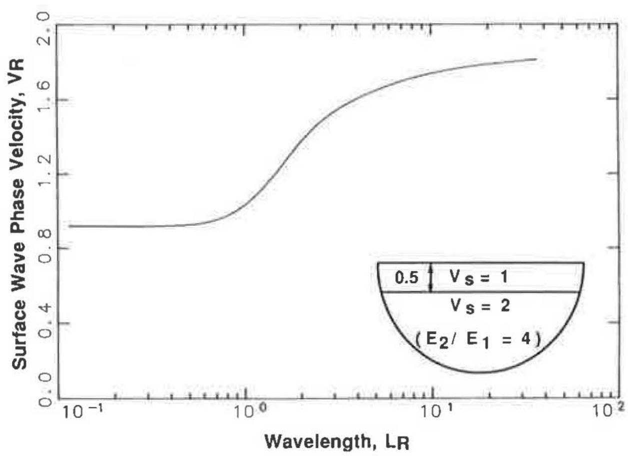

FIGURE 4 Dispersion curve for plane Rayleigh waves propagating in a softer layer over a stiffer half-space.

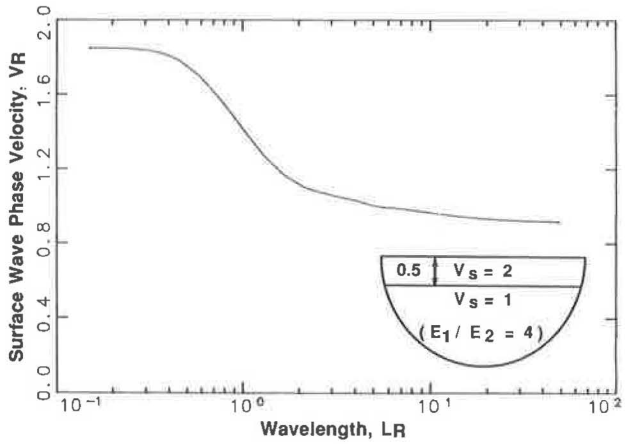

FIGURE 5 Dispersion curve for plane Rayleigh waves propagating in a stiffer layer over a softer half-space.

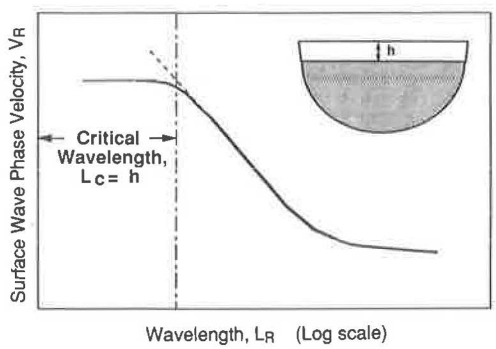

FIGURE 6 Determination of the surface layer thickness from the dispersion curve for plane Rayleigh waves.

Dispersion of Combined Body and Rayleigh Waves

The physical phenomenon is more complicated when applying a vertical impulse at a point on top of a layered system. Waves generated in this case involve both surface waves that propagate radially outward from the source along a cylindrical wave front and body waves that propagate radially outward along a hemispherical wave front. The analytical formulation requires the following processes:

1. Decomposition of the load into a series of cylindrical functions (Bessel functions) in the radial direction. Each term of the series corresponds to a wave number k.
2. Calculation of displacements and stresses for a given frequency and wave number using the global stiffness matrix of the complete layered system. The results are the Green's functions.
3. Determination of total displacements and stresses integrating the product of the Green's functions by the corresponding terms of the load decomposition.

Because the terms of the stiffness matrices of each layer are transcendental functions (complex exponentials), the integrals involved in the calculation of the Green's functions are done normally by numerical integration. Formulations along these lines have been implemented by Gazetas and Roesset (*12*) in Cartesian coordinates and Apsel and Luco (*13*) in cylindrical coordinates. This procedure is particularly convenient when dealing with a uniform half-space or a small number of layers, but expensive when a large number of layers is needed to reproduce the variation of properties with depth. An alternative to this formulation is to use the exact analytical expressions (displacements and stresses) in the two horizontal (or radial and circumferential) directions, and a simpler polynomial expansion in the vertical (z) direction if the thickness of the layers is sufficiently small. The approximation in the z direction leads to much simpler algebraic expressions for the terms of the stiffness matrices of the layers. By expressing the solution in terms of the mode shapes of the waves propagating through the layered system, Kausel (*14*) was able to obtain explicit solutions for the displacements caused by harmonic loads at any point in the system. Using Kausel's formulation with an approximate solution for a half-space at the bottom of a layered stratum (*11*) and with the rule suggested by Shao (*15*) in dividing automatically the physical layers into finer sublayers to provide an appropriate thickness for each sublayer, dispersion data for the SASW test can be evaluated.

For example, the dispersion curve for a uniform half-space with shear wave velocity of 1, Poisson's ratio of 0.25, mass density of 1, and material damping ratio of 0.02 is shown in Figure 7. By applying a vertical load at a point on the surface of the half-space, the response (amplitude and phase) at five other points spaced 1, 1.2, 1.5, 2, and 3 units from the source was computed; then the surface wave phase velocities were obtained from the phase differences and distances between adjacent receivers. This source-receiver configuration was chosen to evaluate the field SASW setup and to verify the near-field effect. The four dispersion curves are similar over the entire range. For relatively high frequencies ($f > 2$ Hz, which corresponds to $L_R < 0.5$), the curves match very well and correspond to the stiffness of the half-space. For values of d_1/L_R less than 2 ($f = 2$ Hz corresponds to $d_1/L_R = 2$, $d_1 = 1$, and $L_R = 0.5$, as calculated from $L_R = V_R/f$), surface

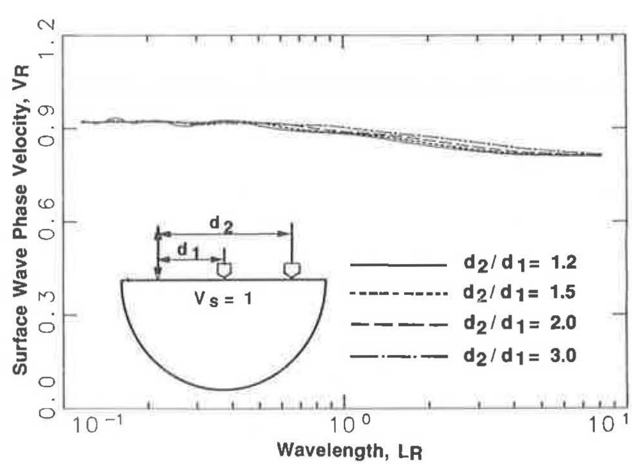

FIGURE 7 Dispersion curve for combined waves propagating in a uniform half-space.

wave phase velocities are smaller (within 10 percent difference) than the phase velocities of the plane Rayleigh wave. This difference is believed to be caused by the coupling effect of body waves and Rayleigh waves in the zone near the source (often called the near-field effect). Slight fluctuations can be seen in the curve corresponding to the smallest distance ratio d_2/d_1. This result indicates that larger values of d_2/d_1 are preferred. In SASW testing, a ratio of two is commonly used.

PARAMETRIC STUDIES

A number of studies were conducted to investigate the appropriate SASW configurations (spacings between source and receivers) to minimize the near-field effect. In considering d_1 and d_2 as the distances from the source to the two receivers, L_R as the wavelength (computed by dividing the phase velocity by the frequency), and h as the thickness of the top layer, the agreement between the dispersion curves corresponding to plane Rayleigh waves and those computed by taking into account all the propagating waves is in general a function of the ratios d_1/L_R, d_2/L_R or $(d_2 - d_1)/L_R$, d_1/h, and d_2/h. The results are also influenced to some extent by the stiffness contrast between the upper layer and the half-space.

To simulate the pavement system, where the top layer is often stiffer than the underlying layers, a set of simplified, two-layer systems consisting of a surface layer with shear wave velocities of 1.414, 2, 3, 5, and 10 overlying a half-space with a shear wave velocity of 1 was studied. These cases correspond to stiffness ratios E_1/E_2 of 2, 4, 9, 25, and 100. (Mass density and Poisson's ratio of both layers were assigned as 1 and 0.25, respectively, in all cases.) For each stiffness ratio, values of the thickness h of the top layer of 0.1, 0.25, 0.5, 1, 2, and 5 were used. Because the distance d_1 from the source to the first receiver remained 1 in all cases, the ratios d_1/h were 10, 4, 2, 1, 0.5, and 0.2. Four values of SASW spacing ratios d_2/d_1 of 1.2, 1.5, 2, and 3 were studied. Linear material damping values of 0 and 2 percent were assumed in the calculation of the plane Rayleigh wave solution and the discrete Green's solutions, respectively, to differentiate the simplified theoretical SASW interpretation from a more complete representation of the field results.

Two sets of results for stiffness ratios of 4 and 25 are shown in Figures 8 through 11. Significant fluctuations in the dispersion curves result from coupling of the body and Rayleigh waves. The parametric studies show that the best results are generally obtained when d_2/d_1 is of the order of 1.5 to 2. The main complication in this case is that, for wavelengths of the order of the layer thickness, there are reflections at the bottom of the layer that result in large oscillations in the complete solution. These oscillations are more pronounced for small values of d_2/d_1 and increase with increasing contrast in stiffnesses between the layer and the half-space. Values of d_1/L_R between 0.5 and 2 generally produce results that are very close to those of the plane Rayleigh waves. When the modulus of the top layer is much larger than that of the underlying material, as would happen with a PCC layer over an uncemented base and subgrade, determination of the dispersion curves in the range of wavelengths around the thickness of the layer is always difficult because of the large fluctuations.

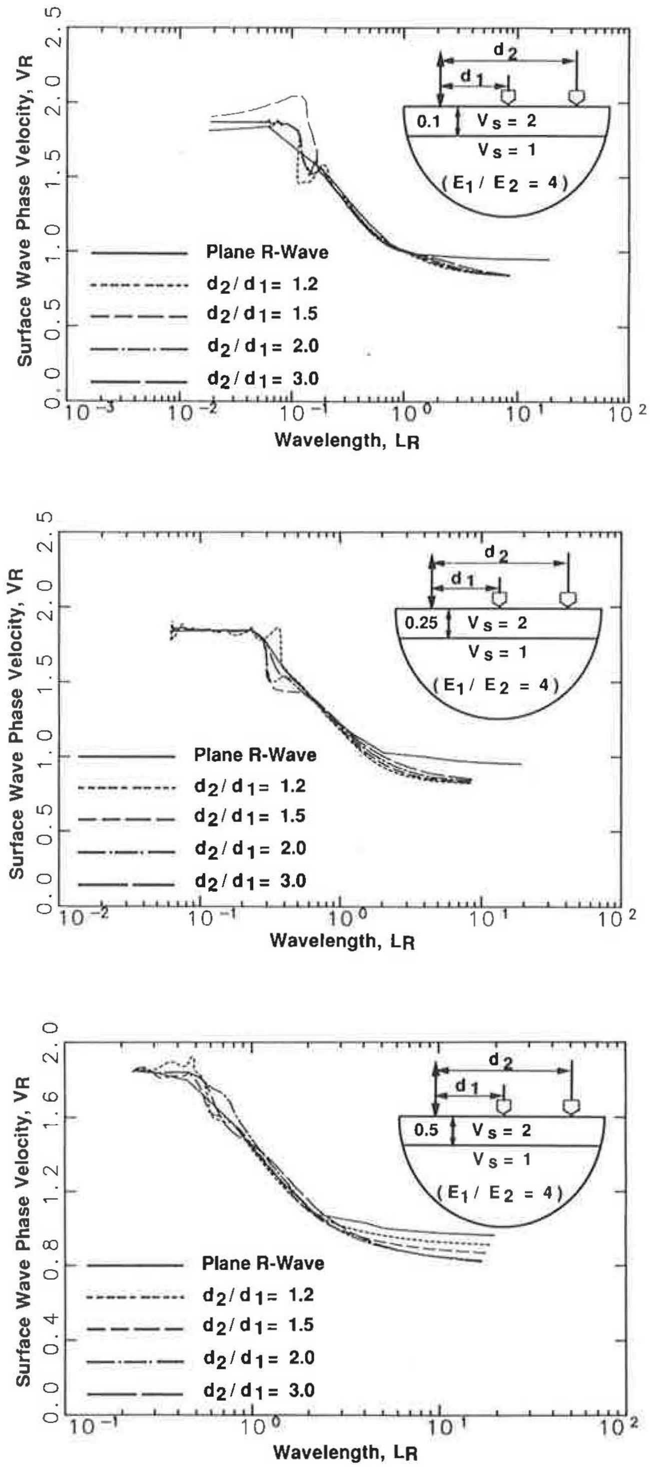

FIGURE 8 Comparison of dispersion curves based on plane Rayleigh waves and combined waves for SASW configurations: E_1/E_2 = 4, d_1 = 1, and H = 0.1, 0.25, 0.5.

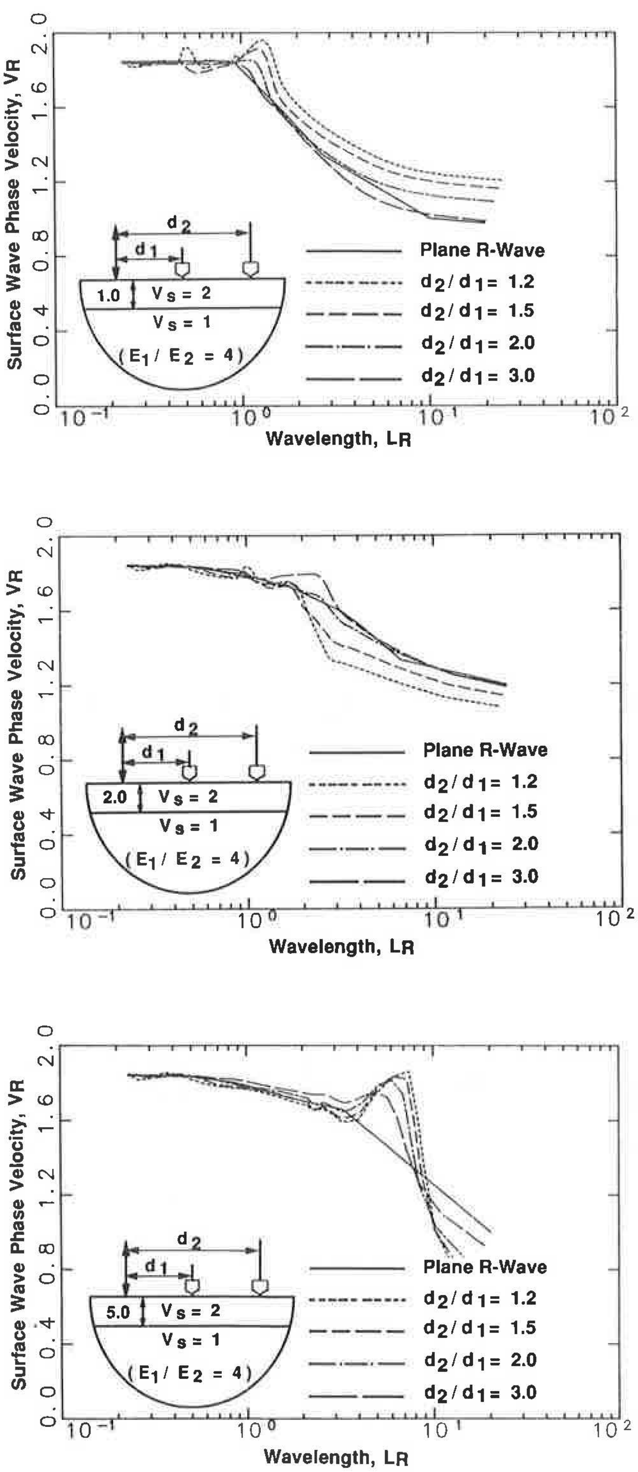

FIGURE 9 Comparison of dispersion curves based on plane Rayleigh waves and combined waves for SASW configurations: $E_1/E_2 = 4$, $d_1 = 1$, and $H = 1.0, 2.0, 5.0$.

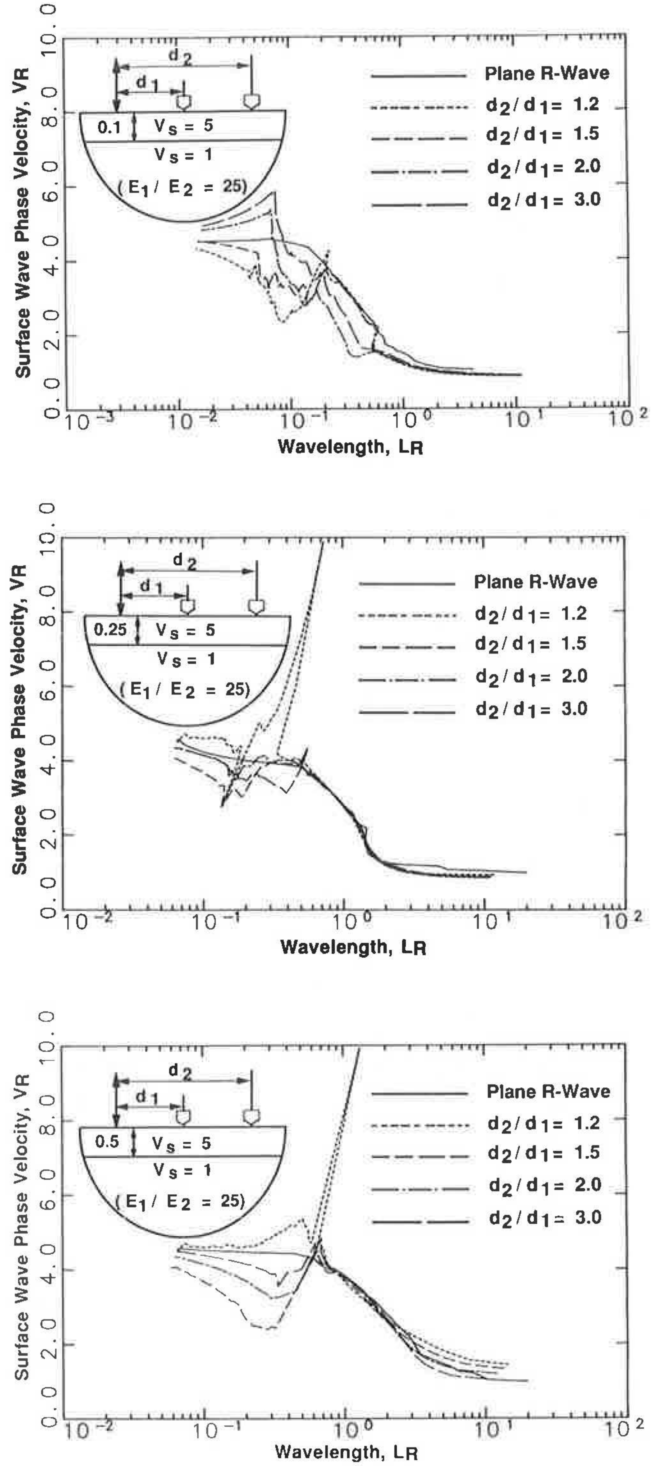

FIGURE 10 Comparison of dispersion curves based on plane Rayleigh waves and combined waves for SASW configurations: $E_1/E_2 = 25$, $d_1 = 1$, and $H = 0.1, 0.25, 0.5$.

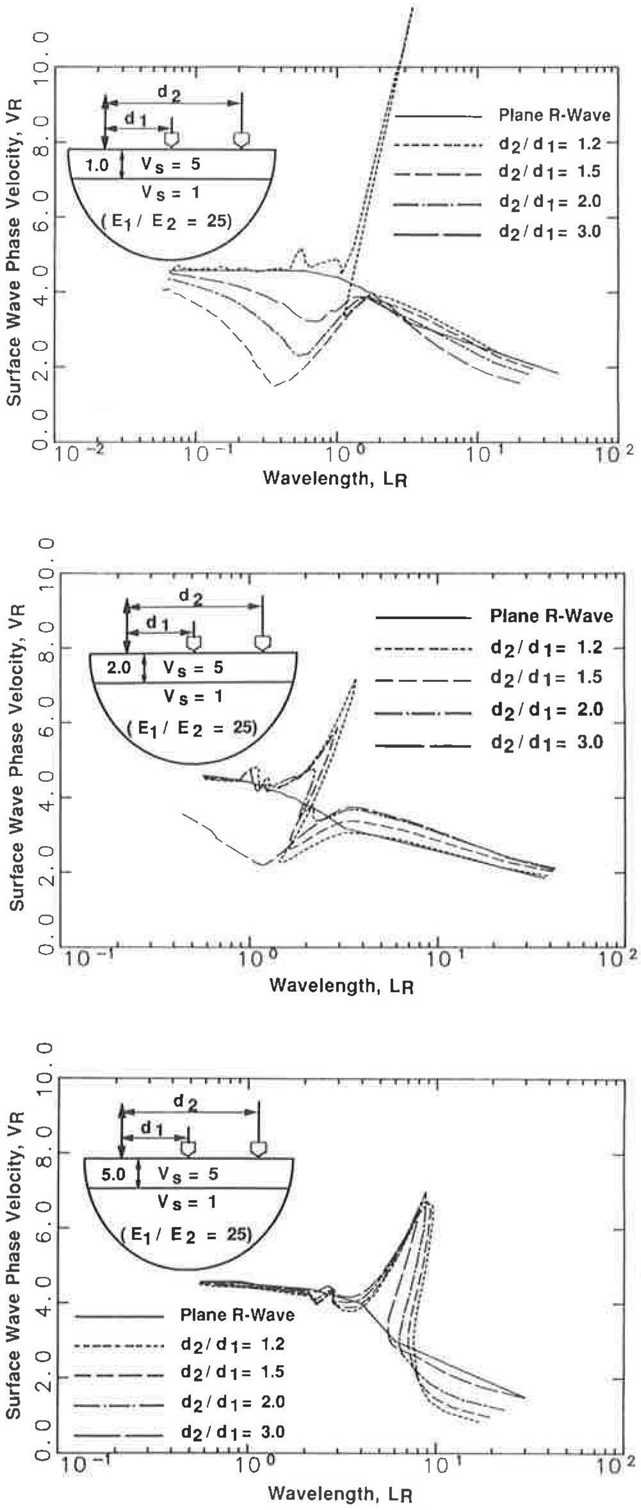

FIGURE 11 Comparison of dispersion curves based on plane Rayleigh waves and combined waves for SASW configurations: $E_1/E_2 = 25$, $d_1 = 1$, and $H = 1.0, 2.0, 5.0$.

The dispersion curve obtained in the field needs to be smoothed if it is going to be assumed to correspond to a plane Rayleigh wave to backfigure the stiffness and thicknesses of the surface layer. However, this smoothing operation is quite straightforward in most cases.

CASE STUDIES

This adaptation of the SASW method for determining the modulus and thickness of the pavement surface layer has been used on many pavement sections in Texas, including 10 sections at the Texas Transportation Institute Annex of Texas A&M University. In all cases, the thicknesses of the pavement surface layer were known. Results from four of these sites are presented in the following sections.

New Highway in Austin, Texas

Tests were performed on a new asphalt concrete pavement about 4 days after placement. The resulting dispersion curve is shown in Figure 12. The surface layer exhibited some frequency dependence as noted by the inclined portion of the initial part of the dispersion curve. The average value of V_R is about 4,500 ft/sec, which results in a Young's modulus of about 2.8×10^8 psf. The thickness was estimated to be 0.51 ft as compared with 0.58 ft measured by cores, a reasonable comparison.

The value of the modulus seems too large in comparison with values determined by conventional laboratory tests. As mentioned earlier, moduli measured at strain levels associated with seismic testing are maximum values. Second, the high frequencies used in the SASW test result in higher values of stiffness for AC material. Tests performed on cores of this material to evaluate the frequency effect are shown in Figure 13. The effect of frequency is significant. If one wanted the modulus at 30 Hz (say to compare with the FWD), then the SASW value would be divided by a factor of about 4; hence the modulus would be 7.0×10^7 psf.

TTI Annex

The experimental dispersion curve calculated from measurements on one test section at the TTI Annex is shown in Figure 14. Using an average value of 5,200 ft/sec for the surface wave phase velocity, a Poisson's ratio of 0.33, and a unit weight equal to 145 pcf, the resulting Young's modulus is determined to be 2.7×10^6 psi for the AC layer. Again, the stiffness of the asphalt concrete is high because of the small strains and high frequencies involved. The thickness of the surface layer is estimated to be 0.42 ft. Cores from the site show the thickness is 0.42 ft, a very good comparison.

Monitoring Changes with Time

To illustrate the usefulness and sensitivity of this approach in testing the surface layer, changes in the stiffness of the layer

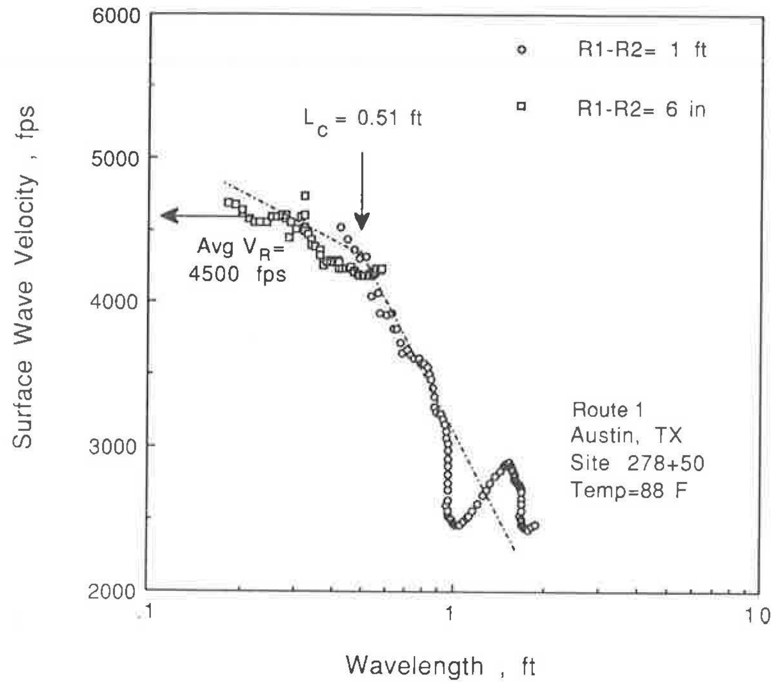

FIGURE 12 Dispersion curve measured on new ACP about 4 days after placement.

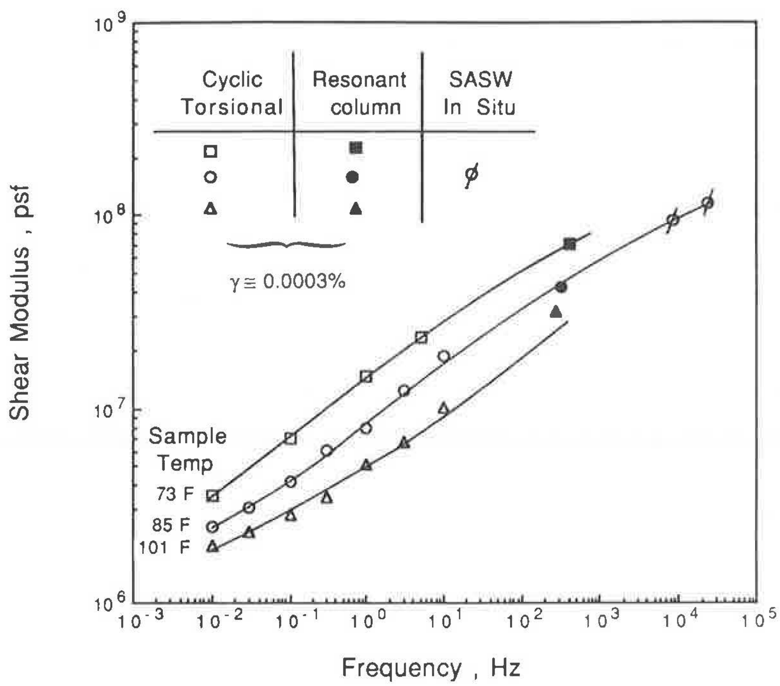

FIGURE 13 Influence of frequency and temperature on the small-strain shear modulus of asphalt concrete: sample cored from a new highway in Austin, Texas.

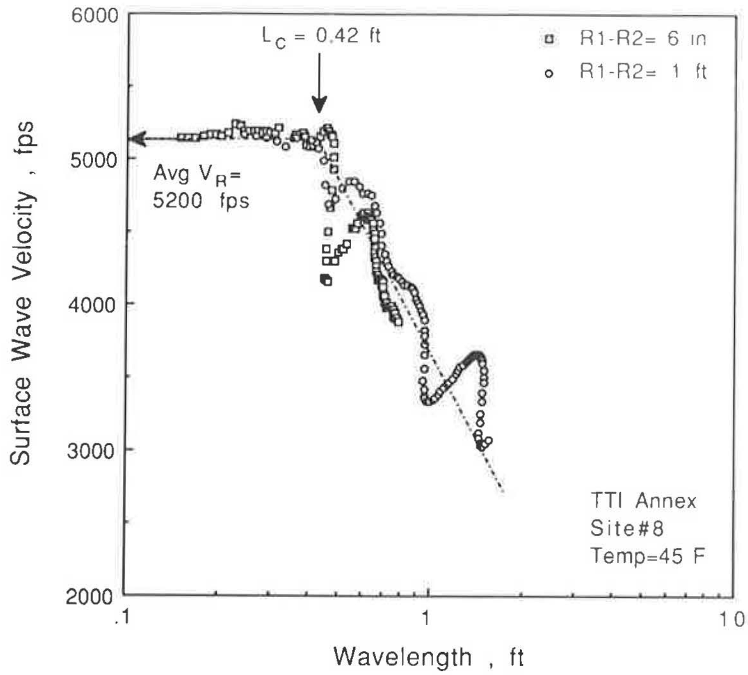

FIGURE 14 Dispersion curve measured on ACP section at the Texas Transportation Institute Annex in College Station.

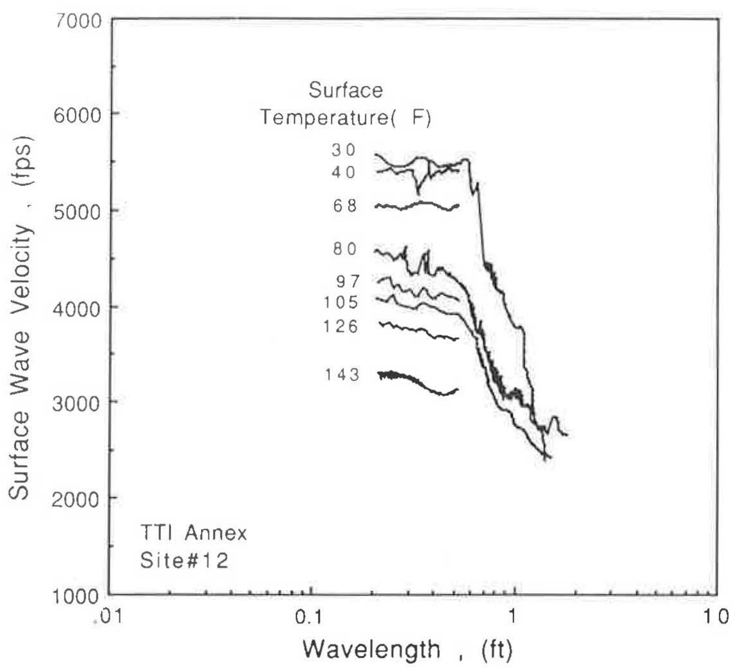

FIGURE 15 In situ measurement of the variation in stiffness of an AC layer with temperature.

with time were measured in situ. The first case, shown in Figure 15, shows the influence of temperature on the stiffness of an AC layer at the TTI Annex. The second case, shown in Figure 16, shows the stiffening of a portland cement concrete layer during curing (*16*). In both cases, the changes in the surface layer were easily measured with a sensitivity unattainable with any other in situ test.

CONCLUSIONS

A new adaptation of the SASW method to determine the moduli of the surface layer of both asphalt concrete and portland cement concrete pavements using surface waves has been developed. This method may also be used to provide a reasonable estimate of the thickness of the pavement surface

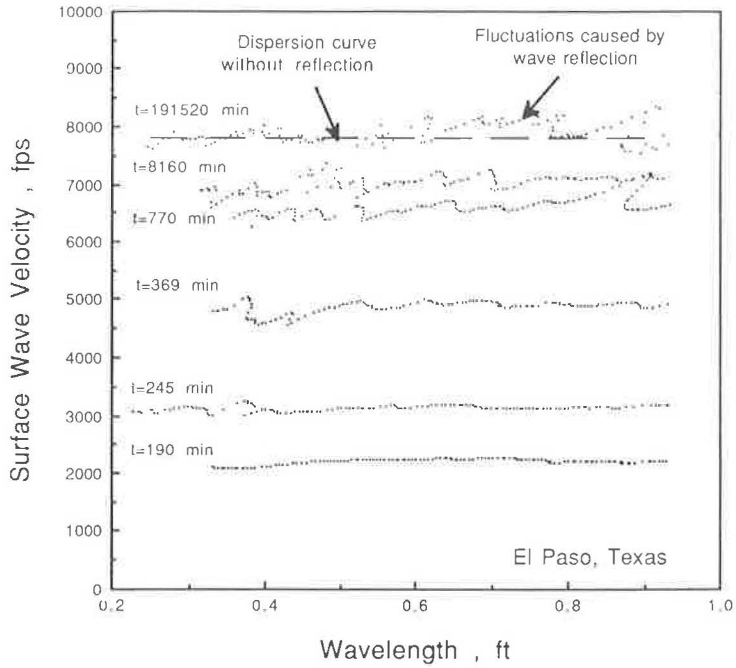

FIGURE 16 Dispersion curves measured on a continuously reinforced PCC pavement section during curing at various times after addition of water to the mix.

layer. The most important features of the technique are the following:

1. Testing can be rapidly performed in the field. At the present time, approximately 5 min is required to perform the test at each location. The time required to conduct the test can be further reduced by automating the placement of the source and receivers.
2. Values of Young's modulus and estimates of the thickness of the pavement surface layer are available immediately in the field. The calculation of these values is based on a simple, straightforward procedure that can be easily implemented.
3. Young's modulus values are calculated using a theoretically sound procedure based on the dispersive property of surface waves. Analytical studies presented herein establish the validity of this approach.
4. Unlike other nondestructive test methods, this technique is very sensitive to the modulus of the pavement surface layer.
5. Because of the small strain levels that exist in seismic testing, measured moduli correspond to maximum values. Also, because of the high frequencies involved, moduli of AC material need to be reduced to compare with moduli evaluated by other nondestructive field tests such as the falling weight deflectometer or Dynaflect.

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