Using Statistical Methods To Optimize High-Strength Concrete Performance

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Sound experimental design principles, rather than three-point curves, can be employed to determine high-strength concrete mixture proportions that maximize performance while minimizing cost. Important issues on experimental design and empirical modeling are discussed. Graphical methods illustrate how high compressive strengths can be obtained for a wide range of costs—and how to get the most for your money. An example is given using a classical experimental design. It greatly reduces the number of concrete mixes that otherwise would have been necessary to obtain information on five variables that change simultaneously. This experiment illuminates the effect of the silica fume addition percentage and the high-range water reducer dosage on 28-day compressive strength and raw material cost. Contour plots are provided to demonstrate the usefulness of a statistical approach.

One of the best reasons for using concrete is that it is a forgiving conglomerate. The proper assemblage of cement, sand, stone, water, and admixtures yields a product that has engineering properties useful for various structures. Traditionally, a good mixture proportion is determined by a combination of field experience and simple experimentation. For example, after some simple laboratory batches are made, field mixes are run using three different cement factors and the desired performance is obtained by interpolating from the best-fit curve to the data. Certainly this method will eventually generate a suitable mixture proportion.

However, increased attention to the front end of this process will often pay large dividends when one is ready to make field-sized trial batches to simulate job conditions. When one considers the small price of trial batches compared with production costs, it makes good sense to understand how raw materials work together.

Understanding the effects and synergisms of the materials is particularly important for high-strength concrete mix proportioning. Reducing the water-cementitious ratio as low as possible yet maintaining a certain workability level for the least cost is the goal to be pursued.

The challenge is to ascertain how to assimilate the effects of the cement, aggregates, and admixtures (and their interactions) on each response variable. It will then be possible to execute sound judgment when mix proportions are determined for high-strength concrete.

A statistical strategy will certainly not replace valuable scientific knowledge and experience pertaining to concrete technology. However, experimental design and statistics can be used to complement a researcher's understanding of particular materials. Sound experimental design principles can help determine high-strength concrete mixture proportions that optimize performance and cost.

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THE DILEMMA

Concrete comes from the Latin word *concretus*, which means "grown together." When hydration (the chemical reaction between water and the cementitious material) occurs, the cement paste encompasses the aggregate like glue. To obtain high-strength concrete, the idea is to recreate a large rock by cementing together its broken pieces (*1*).

Scientific knowledge coupled with practical experience usually provides a good starting point. For example, it is known that strength failure occurs in concrete because of limitations associated with (a) Cement paste, (b) Coarse aggregate, and (c) Paste-to-aggregate bond (*2*). Consequently, it is known that the amount and quality of the cement paste have an important bearing on concrete performance. Similarly, the amount, shape, size, and quality of the coarse aggregate significantly affect the engineering properties of concrete. Finally, the importance of the bond between the two cannot be underestimated.

Because concrete is a mixture, there are many ways to assemble its components. The dilemma arises when one considers how to optimally combine the ingredients. ACI 363R-84 comments briefly on optimization (*3*):

> A principal consideration in establishing the desired cement content will be the identification of combinations of materials which will produce optimum strengths. Ideally, evaluations of each potential source of cement, fly ash, liquid admixture, and aggregate in varying concentrations would indicate the optimum cement content and optimum combination of materials. Testing costs and time requirements usually have limited the completeness of the testing programs, but particular attention has been given to evaluation of the brand of cement to be used with the class and source of pozzolan, if a pozzolan is to be used.

Admittedly, when all reasonable combinations of materials are considered, the task of evaluating them can be formidable. Certainly it would be quite impractical to attempt to test more than a small fraction of these combinations. What is needed is an organized strategy that will facilitate navigation through the complex process of optimizing concrete mix proportions.

DESIGNING THE EXPERIMENT

Fortunately, the principles of experimental design provide the framework for an organized strategy. The combination of

careful determination of objectives, experimental design, controlled laboratory testing, thorough analysis, and field evaluation usually yields favorable results.

The first step is to determine what objectives are to be met. This may sound so obvious that it is not worth mentioning. However, many researchers collect data without a clear understanding of their goal. Before work begins, there must be a focused objective in mind. At this point, specific questions should be formulated that can be answered from experimentation. The minimum requirement for success is always knowing what the target is.

As soon as the goals are clear, the researcher can move to the next step—setting up the experimental design or designs. The appropriate experiment will depend on the complexity of the situation, minimum information desired, resources, and time. For example, it may be appropriate to run a simple experiment that compares the mortar compressive strength of several locally available cements. After this experiment has been run, the cement that performs best can be used in a later evaluation. This sequential experimentation serves to reduce the number of variables for more detailed experimentation in the future.

To construct the experimental design, the response variables and the experimental region must be determined. The response variables are simply measured variables that depend on the values of the independent variables. Examples of response variables are initial set time compressive strength and modulus of elasticity. The independent variables are the controlled variables that are intentionally changed to affect the responses. Examples of independent variables are cement factor, water-to-cement ratio, and admixture dosage. The experimental region is the geometric space jointly defined by the independent variables and their respective levels. For example, the effect of the cement factor and water-to-cement ratio on compressive strength is to be evaluated by varying the cement factor between 700 and 900 lb/yd^3 and the water-to-cement ratio between 0.25 and 0.35. The experimental region for this example would simply be a square that contains the following four combinations:

Cement Factor	*Water-to-Cement Ratio*
700	0.25
700	0.35
900	0.25
900	0.35

The interior of this region is where the response values for specific combinations of independent variables are to be predicted. Two levels for a given independent variable allow for the estimation of a linear trend through the experimental region. Three or more levels allow for the estimation of nonlinear trends through the experimental region.

After the number of independent variables and their expected trends have been determined, an efficient way to specify the experimental mixes needs to be ascertained. For example, if five independent variables were under study, with each effect best estimated by a nonlinear trend through the experimental region, then $3^5 = 243$ mixes could be specified, considering all possible combinations. Clearly, however, it would be impractical to conduct this experiment. The correct course of action here would be to conduct an experiment on a relatively small fraction of the total possible combinations. The goal behind experimental design is to obtain the maximum amount of information from the least amount of resources.

Before the experiment can be conducted, some additional information must be obtained. First, it is necessary to understand what the experimental unit (the basic unit to which a treatment is applied) is for each response in the study. Examples of experimental units include compressive strength cylinders, length change beams, and setting time proctors.

Second, it is absolutely critical to ensure that a good estimate of experimental error is available after the study has been run because there is variation in experimental results resulting from known and unknown factors. This variation is sometimes referred to simply as "noise." A simple example may be helpful here. Suppose a large batch of concrete is made and several cylinders are fabricated. After 28 days, when these cylinders are broken, all of them will not have identical compressive strengths. This results from experimental error. In other words, even though all the specimens represent the same concrete there are some nuisance factors that cause the test results to be different. Possibly the loading rate changed or the cylinders were tested slightly off-center. Certainly it is possible that factors not only outside the experimenter's control but also beyond his knowledge influence experimental results. The simple truth is that experimental error is a natural occurrence irrespective of how careful one is in collecting data. As long as the amount of experimental error is understood, a logical determination of the independent variable effects can be made. Without understanding of the experimental error, it is extremely difficult to develop logical inferences from the data. Drawing conclusions from data without knowing the experimental error is like making decisions by flipping a coin in a dark room.

One concept related to estimating experimental error is blocking. It is common to expect certain unavoidable sources of variation. Blocking is a tool to reduce experimental error by grouping similar experimental units with respect to the response variable or variables. In essence, the blocking variable removes one or more sources of variability that would otherwise have been captured in the experimental error. Suppose, for instance, the interest is to compare the compressive strength of two cements. Further suppose that on one day four mixes were made with each cement, and on a second day four additional mixes were made with each cement. Now clearly the day-to-day effect (which includes material, environmental, and personnel changes) would affect the ability to discriminate effectively between the two cements. If this source of variation was removed by mixing all cement on one day, then the comparison between the cements would become more powerful. This is the benefit of adding a blocking variable.

Another important concept relating to experimental design is randomization. Essentially, randomization is a statistical method to distribute the order of experimental runs to prevent systematic bias from contaminating study results. It is helpful to view randomization as insurance against unwanted sources of variation. For example, suppose that one desires to examine the effect of adding 5, 7, 9, and 11 percent of a mineral admixture to the mix design. If this work was performed outdoors in the order of increasing addition percentages, then the influence of temperature may introduce a systematic bias. Here the mixes run early in the morning will, all other things

being equal, have a lower concrete temperature, and hence hydrate more efficiently and yield higher compressive strengths than the mixes run later under higher temperature conditions. Consequently, the effect of the addition percentage would be partially masked by the temperature effect.

Unfortunately, applying randomization is not a natural thing to do. However, suppose in the previous example that the four different addition percentages were mixed throughout the day. If 12 mixes were to be run (three mixes per level), one possible randomized order would be 7, 9, 5, 9, 11, 11, 7, 5, 7, 11, 9, and 5 percent. The effect of temperature would tend to cancel out among the different levels of admixture addition percentages. This is what makes randomization so effective; bias tends to be removed by cancellation, whether external sources of variation can be anticipated or not. Even if no lurking sources of variation exist, the cost of randomization is at most a little inconvenience. On the other hand, it is apparent that using randomization can easily save an experiment that is otherwise doomed to failure. The moral of the story is (*4*)

> Block what you can control,
> Randomize what you cannot control.

CONDUCTING THE EXPERIMENT

After experimental design issues have been addressed, it is time to conduct the experiment. For this step there is no substitute for a team of individuals who are highly skilled in concrete mix proportioning and testing. Those who understand how variations in raw materials (age, grading, composition, specific gravity, etc.), temperature, mixing, handling, compacting, curing, and testing affect performance are necessary. The mixes need to be proportioned to ensure the proper yield. The guidelines given in ACI 211.1, 211.2, and 211.3 (*5*) are helpful for this stage.

After mix proportioning is complete and a set of batch weights is available, the mixes can be made according to the specifications of the experimental design. The air content, slump, and unit weight should always be measured in addition to the other response variables of interest. Each mix should be made in as similar an environment as possible. If not, these uncontrolled environmental variables can confuse the results of a study, rendering an experiment totally worthless. Collecting information without external biases is critical to understanding the data.

ANALYZING THE EXPERIMENT

No matter how much care is taken to execute an experiment, some erroneous values inevitably appear on the data sheets. Before any serious analysis is performed, data should be screened to eliminate numbers that are grossly wrong. However, if there are doubts concerning the validity of any data value, it should be left alone. Simple scatter plots and linear regression analysis as well as common sense will prepare the data for a formal analysis. This process serves to identify simple, detectable errors (such as a transposition of digits or a calculation error).

The next step is to model each response of interest using statistical methods. It is understood that the complex processes involving cement hydration could never be exactly modeled. The idea is to develop models that approximate each response in the region of interest without bias. For example, if the number of days per month was to be modeled, a mathematical model to calculate the value exactly could be employed. However, a simpler yet unbiased model would be using 30.5 as an approximation to the number of days in a month. It is not as good as a more sophisticated model, but it will be correct to within ½ day more than 90 percent of the time. In this situation God's complex model cannot be accessed, so the phenomenon is approximated using mathematical building blocks; generally, the greater the number of building blocks, the better the approximation.

Regression is a powerful statistical technique that is useful in the modeling of data when the exact mathematical relationship between a response variable and a set of independent (predictor) variables is not known. Regression results in a model that optimizes the fit between the data and a criterion. Generally, the model minimizes the squared differences between the actual response values and the values predicted by the regression equation.

After each response has been represented, the statistical model undergoes some "quality" tests. This series of tests checks that the model is an appropriate representation of the data that are being fit. For example, if the model estimated the number of days per month at 40, the model would probably be less than satisfactory because it would always overestimate the correct value. The statistical model sought is one that is not biased in any direction. Consequently, it is crucial to examine predicted values for each data value to ensure that no systematic bias is present. Clearly all models are wrong in some sense, because they are approximations. A statistical model is considered good when it describes a high percentage of the variability without exhibiting systematic deviations from the actual data.

When the models successfully undergo these quality checks, the estimates obtained from these equations assuredly yield reasonably unbiased approximations for each response within the region of interest. An optimization program can now be built. This computer program allows data to be simulated by iterating through each of the independent variables in small steps. The values of the independent variables are plugged into the statistical models yielding predicted values for each response. Raw material costs are also incorporated in this program so that mixes can be generated, each with a specific performance and cost.

Model values are then written to a data set where they can be sorted by various ranges of performance criteria. For example, mixes with a given set time and compressive strength could be sorted by cost. Each point surviving this "final cut" is a suitable candidate to meet the original objectives.

The major assumption is that the statistical models provide adequate approximations to the various responses. Fortunately this assumption can be checked to some extent. However, there are two additional assumptions that must be made. First, it is assumed that no major raw material variations occur. For example, if the study was performed using only Class C fly ash, a substitution involving Class F fly ash could (and does) make a big difference. Second, it must be assumed

that relative performance differences observed in the laboratory also occur in the field. Generally speaking, this is a valid assumption. The only time that this should be a concern is when laboratory work poorly simulates the large scale batches or the actual delivery time involved.

It is also important to consider other variables that are not being measured. For example, a mix may be generated that would yield the necessary performance and cost-effectiveness to be a workable solution. However, the amount of cementitious material may be so high that in practice it would be unworkable. Someone who understands concrete well would be able to eliminate choices that obviously would not be feasible.

The last step is to make and test final mix recommendations. On paper the analytical process yields a scientific choice to improve performance while restricting the cost. The safe course of action at this point is to validate this empirical work by running full-scale batches.

EXAMPLE

Many industries have the luxury of obtaining information on their response variables within a very short period. Unfortunately, with concrete, much of the data on standard hardened properties does not become available until 28 days. Often this precludes sequential experimentation. Consequently, it becomes important to design experiments that carefully blanket the region of interest.

The central composite design is a classical experimental design that enables an experimenter to estimate linear and quadratic trends for each of the independent variables under study. The easiest way to begin to understand these designs is to view them geometrically. Figure 1 presents the experimental region for a three-variable central composite design. The cube portion of this design consists of $2 \times 2 \times 2 = 2^3$ = 8 vertices (corners) that allow the estimation of linear trends for each of the three independent variables. The six axial (star) points protruding from each face of the cube provide five levels for each independent variable. These are included to determine the quadratic (nonlinear) trends present for each variable. The points in the center of each cube are replicates that enable the estimation of pure experimental error. This pure error is used to assess the significance of any model's lack of fit. Any good text on experimental design or response surface methods provides the necessary information to work through the actual design and analysis of this type of experiment. A good statistician would be invaluable here.

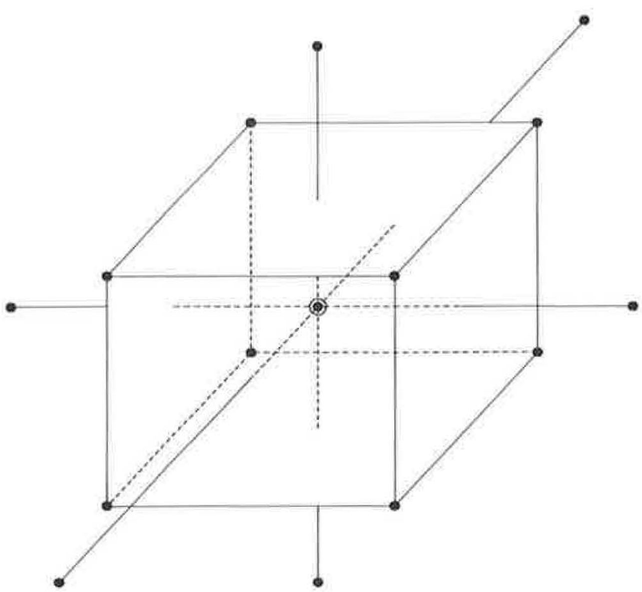

FIGURE 1 Three-factor central composite design.

As an example, Table 1 provides the mix design data for a modified five-variable central composite design, blocked across 3 days. The five factors that were intentionally changed were

- Cement quantity (lb/yd^3),
- Silica fume (addition percent),
- High-range water reducer (HRWR) dosage (oz/100 lb of cement + silica fume),
- Coarse aggregate content (lb/yd^3), and
- Coarse aggregate type or blend.

Before the experiment was run, the cement was tested for false set characteristics using ASTM C 359 and ASTM C 451. Furthermore, the aggregates were analyzed in accordance with ASTM C 29-87, C 40-84, C 127-84, C 128-84, and C 136-84a to determine unit weight, voids, impurities, specific gravity, absorption, and sieve analysis. After this material analysis was performed, the water-to-cementitious ratio was allowed to vary to obtain a slump of 10 $\pm$ 1 in.

The relationship between 28-day compressive strength and the water-to-cementitious ratio for this set of materials is presented in Figure 2. The 28-day compressive strength as a function of cost is depicted in Figure 3. Actually, cost does not predict or determine compressive strength. However, this graph illustrates that one can achieve a particular level of strength for a wide range of raw material costs. The desire here is to produce the intended strength for the minimum cost. A cost-effective combination of raw materials that minimize the water-to-cementitious ratio needs to be determined.

It is admittedly difficult to understand a data table that is in a randomized order. Figures 4–6 represent creative attempts to illuminate the patterns contained in the data. Each of these graphs contains values averaged across coarse aggregate content and limestone percentage of the course aggregate. Therefore, the six axial values are averages of one mix; the cube vertices (corners) are averages of two mixes; and the center is an average of 11 mixes. To understand the HRWR effect, it is necessary to look from the bottom to the top of the diagram. For example, in Figure 4 the 2.5-oz dose yielded a 28-day compressive strength of 6,880 psi. A 15-oz dose yielded average 28-day strengths of 8,660, 9,700, 7,600, and 9,070 psi depending on the amount of cement and silica fume added to the mixture proportions. A 27.5-oz dose provided average compressive strengths of 9,480, 12,210, and 12,950 psi, again depending on the cement and silica fume amounts. A 40-oz dose generated average strengths of 9,600, 11,270, 12,160, and 12,590 psi, whereas a 52.5-oz dose registered 11,780 psi.

If none of the independent variables affected the strength, all the values in the circles would be approximately equal.

TABLE 1 CONCRETE MIXTURE PROPORTIONS—DAY 1

MIX	CEMENT (LB./YD.)	SILICA FUME ADDITION %	HIGH RANGE WATER REDUCER (OZ./CWT.)	COARSE AGGREGATE CONTENT (LB./YD.)	COARSE AGGREGATE BLEND	SAND CONTENT (LB./YD.)
1	650	15	40.0	2050	SILICA GRAVEL	990
2	800	10	27.5	1900	50:50 BLEND	1017
3	950	5	40.0	1750	LIMESTONE	1103
4	650	5	15.0	2050	SILICA GRAVEL	988
5	650	5	15.0	1750	LIMESTONE	1301
6	800	10	27.5	1900	50:50 BLEND	1021
7	950	5	40.0	2050	SILICA GRAVEL	742
8	650	15	40.0	1750	LIMESTONE	1352
9	950	15	15.0	2050	SILICA GRAVEL	277
10	800	10	27.5	1900	50:50 BLEND	1020
11	950	15	15.0	1750	LIMESTONE	665

MIX	SLUMP (IN.)	AIR CONTENT %	WATER/ CEMENT+FUME RATIO	28 DAY COMPRESSIVE STRENGTH	RAW MATERIAL COST A ($/YD.)
1	10.0	1.4	0.31	12428	65.94
2	9.8	2.7	0.30	12322	60.28
3	10.8	2.4	0.26	11515	63.26
4	9.0	1.2	0.40	8652	46.89
5	9.3	3.3	0.44	8657	38.54
6	10.3	2.2	0.31	11773	60.26
7	10.3	1.4	0.25	11019	71.50
8	9.5	2.7	0.33	11892	57.66
9	9.5	1.5	0.36	8341	73.55
10	9.8	1.8	0.29	12166	60.34
11	9.8	1.6	0.38	9803	65.25

A
RAW MATERIAL COSTS ASSUME CEMENT=$60/TON, SILICA FUME=$300/TON, SAND=$5/TON, LIMESTONE=$7/TON, SILICA GRAVEL=$15/TON & HRWR=$6/GALLON.

B
MIX 20 COULD NOT BE MADE TO APPROPRIATE SLUMP SPECIFICATION OF 10" +/- 1"

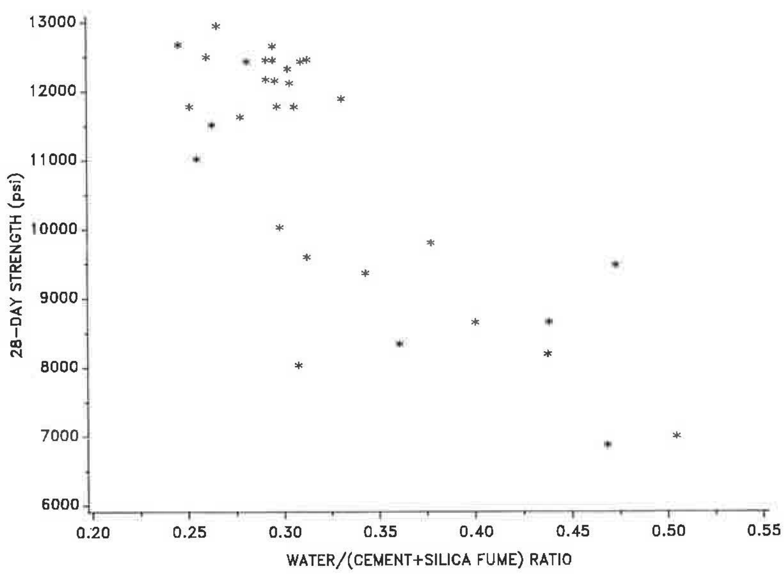

FIGURE 2 Relationship between 28-day compressive strength (psi) and water-to-(cement + silica fume) ratio.

TABLE 2 CONCRETE MIXTURE PROPORTIONS—DAY 2

MIX	CEMENT (LB./YD.)	SILICA FUME ADDITION %	HIGH RANGE WATER REDUCER (OZ./CWT.)	COARSE AGGREGATE CONTENT (LB./YD.)	COARSE AGGREGATE BLEND	SAND CONTENT (LB./YD.)
12	800	10	27.5	1900	50:50 BLEND	1005
13	950	5	15.0	2050	LIMESTONE	638
14	950	5	15.0	1750	SILICA GRAVEL	903
15	800	10	27.5	1900	50:50 BLEND	999
16	650	15	15.0	1750	SILICA GRAVEL	1060
17	650	15	15.0	2050	LIMESTONE	757
18	950	15	40.0	2050	LIMESTONE	658
19	650	5	40.0	1750	SILICA GRAVEL	1381
20 B	650	5	40.0	2050	LIMESTONE	
21	800	10	27.5	1900	50:50 BLEND	1001
22	950	15	40.0	1750	SILICA GRAVEL	859

MIX	SLUMP (IN.)	AIR CONTENT %	WATER/ CEMENT+FUME RATIO	28 DAY COMPRESSIVE STRENGTH	RAW MATERIAL COST A ($/YD.)
12	10.0	2.5	0.29	12453	60.34
13	10.0	2.5	0.34	9366	51.41
14	10.0	2.4	0.30	10029	58.03
15	10.3	2.3	0.30	11777	60.31
16	9.3	1.0	0.44	8192	55.06
17	9.5	1.5	0.50	7004	48.41
18	9.8	2.1	0.26	12498	79.23
19	10.3	2.9	0.31	9597	53.78
20 B	.	.	.	.	.
21	10.0	2.4	0.29	12445	60.32
22	11.0	2.5	0.25	12676	85.67

A
RAW MATERIAL COSTS ASSUME CEMENT=$60/TON, SILICA FUME=$300/TON, SAND=$5/TON, LIMESTONE=$7/TON, SILICA GRAVEL=$15/TON & HRWR=$6/GALLON.

B
MIX 20 COULD NOT BE MADE TO APPROPRIATE SLUMP SPECIFICATION OF 10" +/- 1"

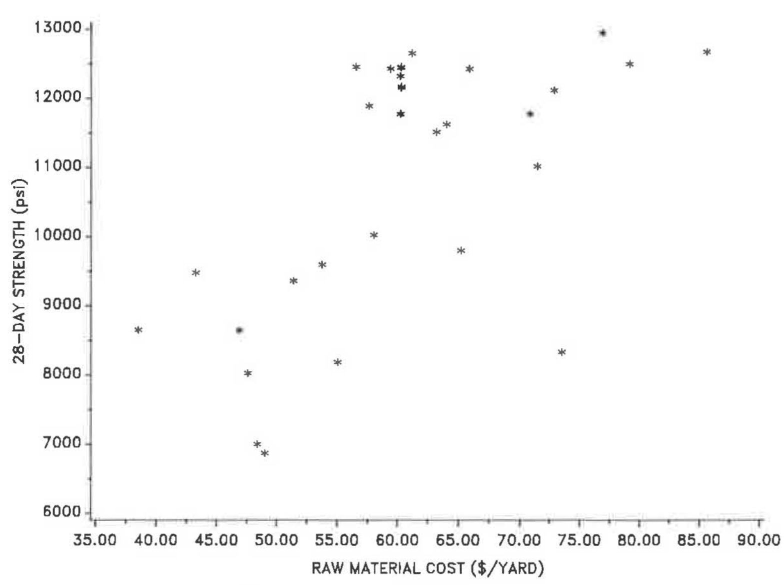

FIGURE 3 Relationship between 28-day compressive strength (psi) and raw material cost ($/yd).

TABLE 3 CONCRETE MIXTURE PROPORTIONS—DAY 3

MIX	CEMENT (LB./YD.)	SILICA FUME ADDITION %	HIGH RANGE WATER REDUCER (OZ./CWT.)	COARSE AGGREGATE CONTENT (LB./YD.)	COARSE AGGREGATE BLEND	SAND CONTENT (LB./YD.)
23	800	0	27.5	1900	50:50 BLEND	1123
24	500	10	27.5	1900	50:50 BLEND	1312
25	800	10	52.5	1900	50:50 BLEND	1094
26	800	10	27.5	1900	LIMESTONE	1033
27	800	10	27.5	1900	50:50 BLEND	1003
28	800	10	2.5	1900	50:50 BLEND	633
29	1100	10	27.5	1900	50:50 BLEND	574
30	800	10	27.5	2200	50:50 BLEND	744
31	800	10	27.5	1900	SILICA GRAVEL	999
32	800	20	27.5	1900	50:50 BLEND	860
33	800	10	27.5	1600	50:50 BLEND	1329

MIX	SLUMP (IN.)	AIR CONTENT %	WATER/ CEMENT+FUME RATIO	28 DAY COMPRESSIVE STRENGTH	RAW MATERIAL COST A ($/YD.)
23	9.8	1.7	0.31	8033	47.60
24	9.0	1.3	0.47	9483	43.28
25	10.5	2.8	0.25	11778	70.88
26	10.0	1.8	0.31	12458	56.59
27	9.8	1.9	0.30	12155	60.32
28	9.0	2.3	0.47	6876	49.02
29	11.0	1.8	0.27	12946	76.96
30	9.8	1.4	0.29	12654	61.25
31	10.0	1.8	0.28	11624	64.05
32	9.5	2.1	0.30	12118	72.92
33	10.0	3.0	0.28	12428	59.47

A
RAW MATERIAL COSTS ASSUME CEMENT=$60/TON, SILICA FUME=$300/TON, SAND=$5/TON, LIMESTONE=$7/TON, SILICA GRAVEL=$15/TON & HRWR=$6/GALLON.

B
MIX 20 COULD NOT BE MADE TO APPROPRIATE SLUMP SPECIFICATION OF 10" +/- 1"

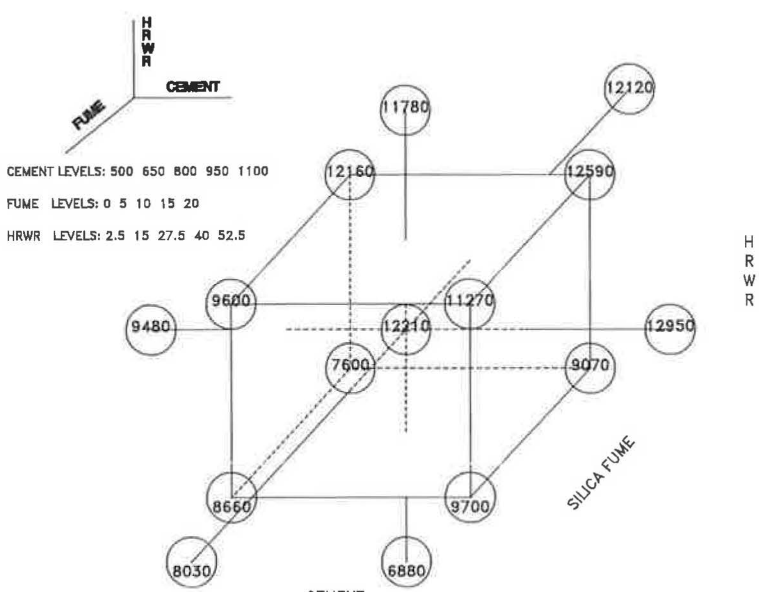

FIGURE 4 Three-dimensional representation of 28-day compressive strength (psi) as a function of cement, silica fume percent, and high-range water reducer dosage.

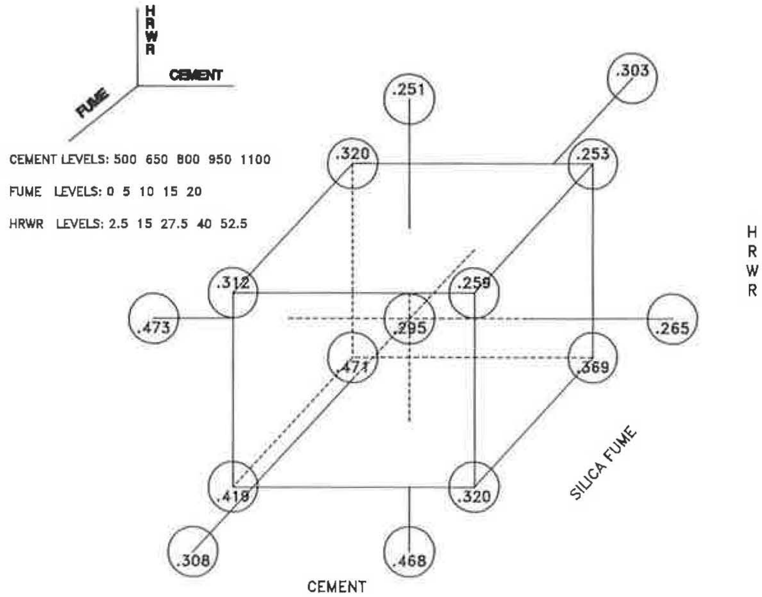

FIGURE 5 Three-dimensional representation of water-to-(cement + silica fume) ratio as a function of cement, silica fume percent, and high-range water reducer dosage.

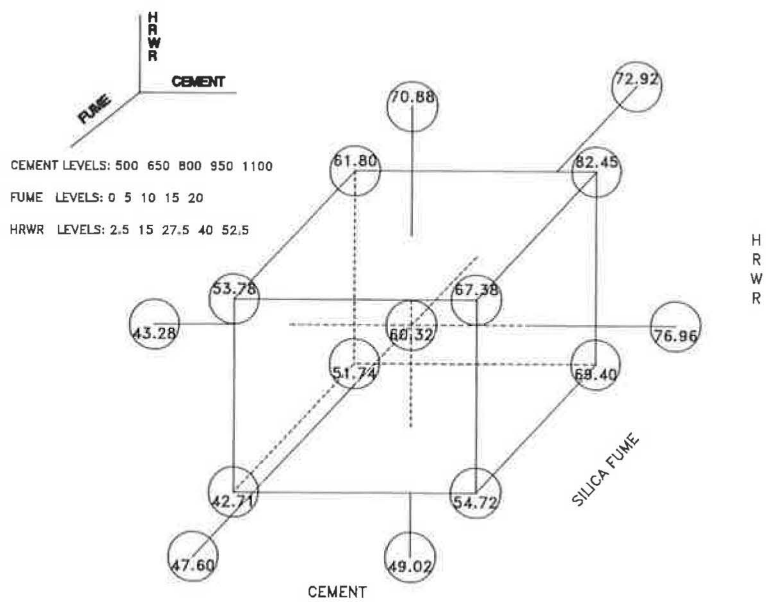

FIGURE 6 Three-dimensional representation of raw material cost ($/yd) as a function of cement, silica fume percent, and high-range water reducer dosage.

However, data in Figure 4 show considerable fluctuation in the compressive strength. In particular, to produce high compressive strength, the selection of the HRWR dose and silica fume addition percentage levels is critical. Too much silica fume without the proper HRWR dosage will hinder performance. Figure 5 illuminates this fact by displaying the water-to-cementitious ratio patterns throughout the region, and Figure 6 displays the raw material costs for these combinations.

For these graphs, actual data values have been used to help visualize what is occurring in this experiment. It is necessary to model the data to approximate the response surface. Probably the best way to understand a response surface is to observe a weather forecaster showing the various temperatures across the United States. These temperatures are analogous to the actual data values obtained from an experiment. Often a contour map that color-codes temperatures by regions is shown. This is, in fact, a response surface. It is these response surfaces that are approximated with statistical models.

Now suppose that the objective is to determine the mixture proportions in the laboratory that will generate 12,000 psi for the least cost. Further suppose that it is known from the lab work, that a mix containing 800 lb of cement and 1,900 lb of

coarse aggregate split 50:50 between silica and limestone will lower the water demanded to optimize performance. Figures 7 and 8 provide the 28-day compressive strength and raw material cost response surfaces, respectively.

The 28-day strength response surface contours illustrate how compressive strength improves as the HRWR dosage and silica fume addition percentage are increased in tandem within the experimental region. Increasing one without the other will only serve to increase cost, not performance. Because both the strength and cost contours increase from the lower left to the upper right, the optimum is easily found to be the area to the lower left of the 12,000–13,000 contour band. This corresponds to a mix containing around 27 oz per hundredweight of an HRWR in combination with 7 ½ percent silica fume added to the 800 lb of cement.

The values contained in the cost contour bands are without variance because cost is a perfect mathematical function of its individual constituents. On the other hand, compressive

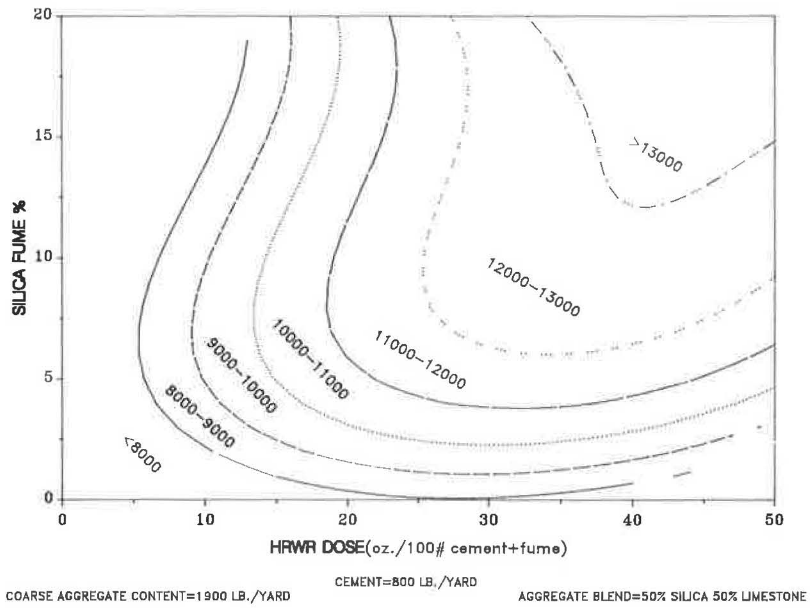

FIGURE 7 Contour plot showing the relationship between 28-day compressive strength (psi) as a function of silica fume percent and high-range water reducer dosage.

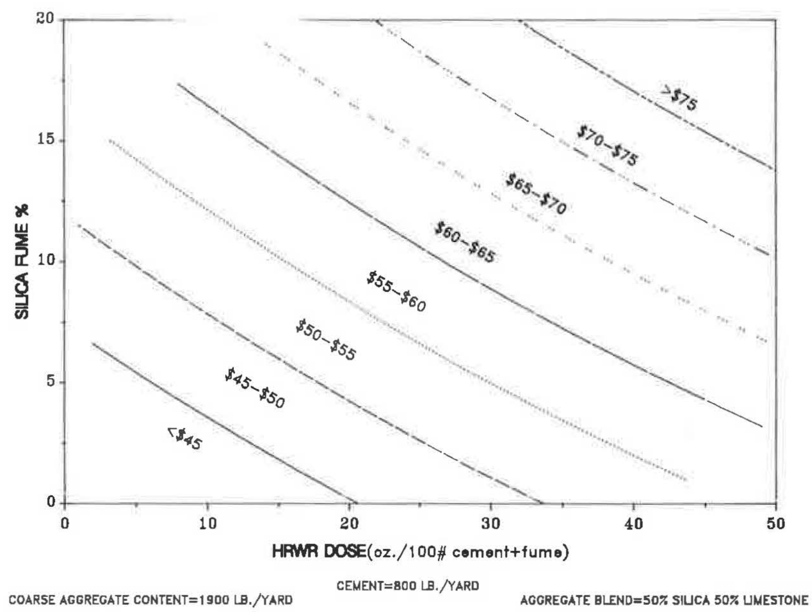

FIGURE 8 Contour plot showing the relationship between raw material cost ($/yd) as a function of silica fume percent and high-range water reducer dosage.

strength is a random variable with a certain level of variance. Consequently, the values contained inside the strength contour bands are predicted values based on the statistical model. Therefore, as stated earlier, it is important to test the recommended mixture proportions to confirm that the desired performance is met.

For this example, it was demonstrated that a graphical approach combined with statistical methods provided a simple solution to a rather complex problem. For more complicated scenarios, a graphical approach would have to be supplemented by an optimization program that would iterate through each of the independent variables.

SUMMARY

The following steps should be taken to optimize high-strength concrete mixes:

1. Discuss specific study objectives,
2. Determine response variables and the experimental region,
3. Develop the experimental design,
4. Conduct the experiment and collect the data,
5. Screen and analyze the data,
6. Build and check statistical models,
7. Create an optimization program,
8. Make mix recommendations, and
9. Test mix recommendations in full-scale batches.

It has been shown how the statistical procedures can facilitate a difficult job. Good analysis coupled with well-chosen graphs can elucidate patterns in the data that would otherwise not be recognized. Sound statistical practices can indeed maximize performance while minimizing cost.

ACKNOWLEDGMENT

The authors wish to acknowledge with thanks the very helpful work of James R. DelGado, manager, for supervising the data collection, and his staff for their diligent work.

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Publication of this paper sponsored by Committee on Mechanical Properties of Concrete.