Thermal Movements in Bridges

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Changes in temperature induce movements in bridges. These movements must be accommodated by bearings or expansion joints, or large internal forces may occur within the structure. These forces may lead to damage and costly repair to the structure if the movements are not properly considered in the design. Present methods for accommodating thermal movements in bridge design are discussed. Experimental results and field observations of thermal movements are summarized, and case studies of bridges which have been damaged by thermal movements are described. A brief overview of recent theoretical research into thermal movements is provided. This leads to conclusions which help bridge engineers better understand thermal movement behavior and may lead to improved performance of bridges under extreme temperatures.

INTRODUCTION

Bridges may be exposed to large variations in temperature. Temperature changes result in expansion or contraction of the bridge, and the resulting movements are accommodated by bridge bearings and expansion joints. Restraint of any component of these movements leads to large internal forces, which must be resisted by the bridge and its substructure. Expansion joints sometimes have serious problems with corrosion and deterioration of the bridge due to water leakage, and as a result bridge engineers frequently minimize the use of these expansion joints and to a lesser extent bridge bearings. In these later applications, they accommodate the movements by deflection or deformation of the bridge piers and abutments. As an alternative, they may design the bridge components very strong to resist the large forces which may result if the movements are restrained.

This paper will review the existing AASHTO (1) design procedures for thermal effects. It will briefly examine the existing evidence regarding temperature effects in bridges, and a number of structures which have developed problems with the existing design methods will be described. These example structures raise questions regarding the accuracy and validity of present design methods, and so the design methods will be briefly compared to more refined analytical results. The comparison will show that the simple design method is very suitable for the vast majority of bridges, but it may be misleading for a few special cases. Guidelines for recognizing these special cases will be discussed, and some factors which must be considered in a more refined analysis will be noted.

AASHTO DESIGN METHOD

Thermal movement design for highway bridges are based on a simple method (1). The United States is divided into two climate zones, and different temperature ranges are used for either steel or concrete bridges within each of these two zones. Metal bridges in the moderate zone vary from 0°F (-18°C) to 120°F (49°C), and in the colder zones, the temperatures vary between -30°F (-34°C) and 120°F (49°C). Concrete bridges use the same two zones, but the temperature range is reduced to approximately 70°F (+30°F and -40°F from the installation temperature) in the moderate zones and 80°F in the colder zones. The smaller temperature range for concrete bridges recognizes the slower rate of heat transfer and greater thermal mass of concrete bridges. Thermal movements are then calculated by a simple uniaxial expansion equation

$$\Delta_T = \alpha L \Delta T , \qquad (1)$$

where α is the coefficient of thermal expansion, L is the expansion length, and ΔT is the temperature change. The bridge bearings and expansion joints are designed for these movements. In other cases the bridge piers and abutments may be integrally constructed with the superstructure (2,3,4) and thermal movements may be accommodated by deflection of the piers or movement of the abutment into the backfill. In a few cases, the bridge will restrict all movement (5,6) and the thermal forces will be resisted within the structure. Extremely large forces, F_T, are possible if all movement is restrained. If the movement is restrained in one direction only, the force is

$$F_T = A\,E\,(\Delta_T/ L) , \qquad (2)$$

where A is the cross sectional area of the restrained elements, and E is the elastic modulus. The force may be much larger if the movement is restrained in two dimensions. A few bridges are designed with combinations of these approaches.

In straight bridges, design engineers consider the movement, Δ_T, or the corresponding force, F_T, as longitudinal effects. That is, the bearings and expansion joints are oriented for longitudinal movement, and the restraining forces are longitudinal forces. Curved bridges cause some additional concern with this simple design procedure. Some engineers place the bearings and expansion joints so that they permit tangential movement at the support as illustrated in Fig. 1a. Other engineers believe that this expansion is uniform and that the curved segment assumes a slightly smaller or larger radius under these thermal conditions. They, therefore, place the bearings and expansion joints so that they are oriented on a chord from the point of fixity of the bridge as illustrated in Fig. 1b. Still other engineers use a combination of these approaches, since they place the bearing on a chord while the expansion joints are set for tangential movement.

This simple design procedure has been used for many years, and has generally produced good results. Experimental observations of thermal movements in bridges suggest, however, that there may be reason to question the validity of these methods for some bridges.

EXPERIMENTAL OBSERVATIONS OF THERMAL MOVEMENTS

Thermal movements were measured and studied in some detail for a number of bridges in England by Emerson (7,8,9). This experimental work showed that thermal movements can be

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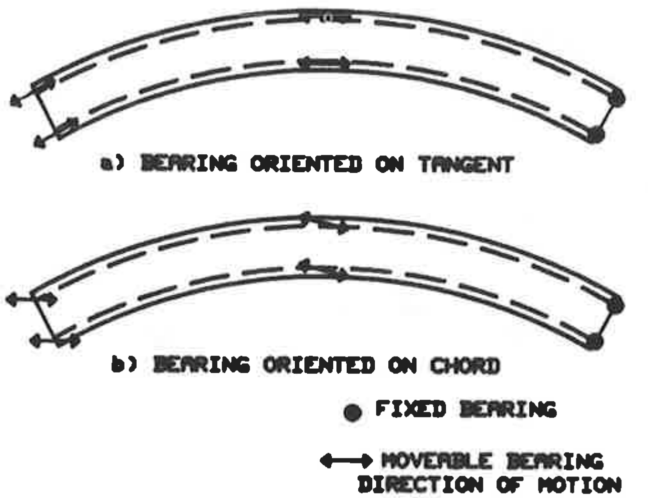

FIGURE 1 Typical orientation and placement of bridge bearings on a curved bridge

divided into two parts. First, there is a daily temperature cycle and the movements associated with this cycle. These daily movements tend to be much larger for steel bridges than concrete bridges, because the steel may experience the full daily range of air temperature while the average temperature of the concrete has much less variation. Concrete bridges have much greater thermal mass and so they do not fully adapt to short term temperature changes as does steel. Direct radiation of the sun may cause daily local temperatures which greatly exceed the air temperature for both steel and concrete. Steel that is exposed to this direct radiation may quickly assume a similar temperature throughout the section because of the conductivity and relatively low thermal mass of the steel. Concrete bridge decks are commonly exposed to direct solar radiation and this affects their daily temperature cycle. However, the average temperatures of the bridge deck seldom approach the ambient temperatures because of the thickness of the deck.

Daily temperature movements are somewhat erratically related to the average bridge temperature. The erratic nature is caused by the slip-stick action caused by friction and other resistance in bearings and expansion joints. The frictional resistance prevents movements until the internal forces are large enough to overcome it, and then a sudden step like movement will occur. The movement will then remain constant until there is an adequate temperature change to overcome the resistance again.

The second part of the thermal movements is caused by the annual temperature cycle. This component is usually considerably larger than the movement due to the daily cycle. Concrete bridges will experience smaller annual movements than steel bridges, because the extreme high and low annual temperatures are of short duration. The mass of the concrete bridge prevents it from responding to very short duration temperature changes, but the relative difference between the annual cycle movements for concrete and steel bridges is much smaller than the relative difference between the daily temperature cycle movements. Movements in concrete bridges are dependent upon a smaller temperature range such as a three day running average of the air temperature.

Experiments have shown that the direction of thermal movement is not always clearly defined. This is particularly true for skewed and curved bridges. Field observations have shown that curved bridges have thermal movements which are neither tangential nor are they on the chord. In some cases the radial component of movement is of similar magnitude to the chord or tangential movement. Field observations have shown that skewed bridges sometimes display transverse movements as well as the longitudinal movements for which they are designed.

A survey of state bridge engineers was made to determine if similar discrepancies between the predicted and actual thermal movements in bridges were noted in the US. The initial response of most bridge engineers was that they observed no problems with temperature movements in bridges, but many of these respondents quickly pulled back from this position after a few moments thought. These engineers noted that they had occasional problems with anchorage pull-out failure, spalling of girders, piers or abutments, and bearing failures. After some thought, they indicated that many of these problems could be related to thermal effects. This initial reaction of bridge engineers represents one of the major difficulties in assessing the thermal movement problem. When bridge engineers note damage in a bridge, their first reaction is to repair it. This leads to an assessment that a component was too weak or defective. Once this assessment has been made, bridge engineers frequently do not consider the cause of the large forces which produced the damage. Only a few engineers directly noted that they had observed temperature movements which were unusual. When these observations were combined, it was clear that the majority of these bridges were sharply skewed or curved bridges. A few typical examples will be discussed in this paper.

CASE STUDIES OF TEMPERATURE MOVEMENTS

Sharply skewed bridges are one area where serious temperature related structural problems sometimes are noted. Figure 2 shows a photograph of a sharply skewed bridge in Richmond, VA. This bridge has 3 spans with simply supported prestressed concrete T-beams. The bridge is approximately 49° from the orthogonal. The beams rest on a bearing pad, and they are set into a recess (approx. 1.5" deep) which restricts the thermal movement to the longitudinal direction. The ends of the simply supported beams were cast into a diaphragm with an expansion gap between adjacent diaphragms.

This particular bridge was built in 1938 and served for a number of years with no observable problems. In the 1960's large transverse movements (in the order of 3" to 6" or 75 to 150 mm) occurred at each interior support and caused severe spalling of the support piers when the girders were forced from the guided path. Figure 3 is a photograph which shows some of the repaired spalled areas of this bridge. Inspection of the bridge showed that dirt had filled the expansion gap between the diaphragms, and the summer expansion of the bridge had caused large internal forces. The granular material developed forces in compression rather than shear and so a large transverse component of force developed as shown in Fig. 4, where f_n is the normal force and f_t is the transverse force. This caused the damage and transverse movement noted in Fig. 3.

FIGURE 2 Photograph of 3 span skewed concrete bridge

FIGURE 3 Repaired spalled concrete bridge piers

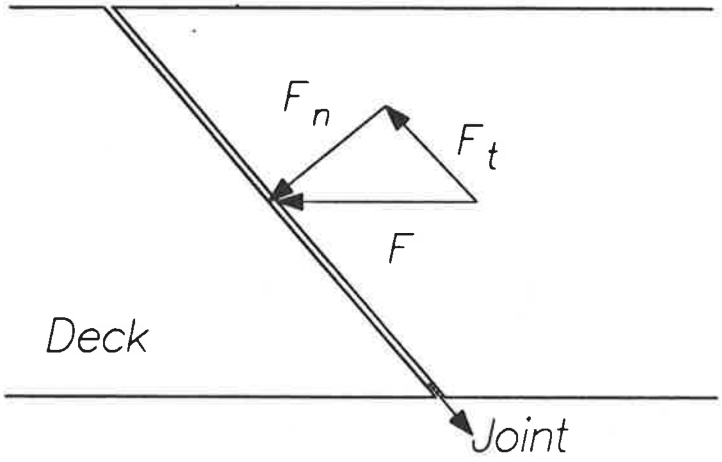

FIGURE 4 Transverse component of force in restrained skew bridges

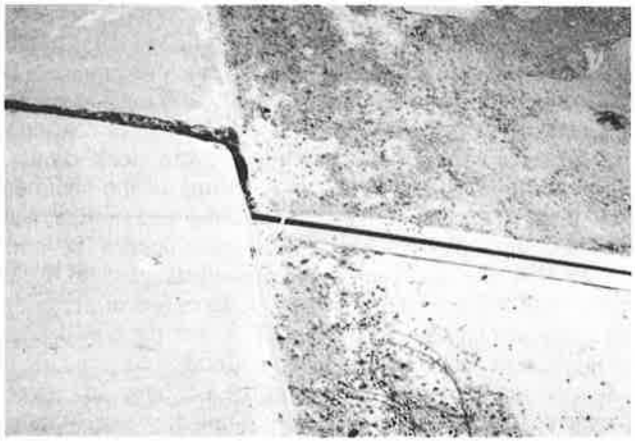

FIGURE 5 Misaligned deck panels due to thermal movement

FIGURE 6 Minor spalling of concrete at piers due to transverse thermal movement

FIGURE 7 Misalignment of curb due to transverse movement of skew bridge

The bridge was repaired by replacing the spalled concrete and by making the deck continuous across the gap. Steel ties also were placed across the gap to transfer the large transverse force between adjacent girders and to prevent the transverse movement and develop the required component of transverse force.

A similar situation can be observed with another sharply skewed bridge in Helena, Montana. This bridge is a 5 span precast concrete girder bridge with simple spans. The bridge deck was replaced with precast deck panels several years ago, and there are small gaps between adjacent panels to accommodate thermal expansion. Each of these panels is misaligned by approximately 1 inch (25 mm) as shown in the photograph of Fig. 5, because of the transverse movements caused by the forces depicted in Fig. 4. The total transverse movement is quite large (6" to 8 " or 150 to 200 mm) over the length of the bridge, but the damage is slight because it is spread over many locations. Nevertheless, there is some minor spalling of the concrete at the piers and supporting elements as shown in Fig. 6. It is possible that this damage may increase if an extremely hot summer occurred.

The Casper Creek Bridge in Wyoming (10) is another sharply skewed bridge with large transverse movements. The transverse movements are more than 8" (200 mm) at the east (fixed) end of the bridge. The bridge has 3 spans of continuous steel girders, and it has sustained severe deck damage, anchorage failure of supports, and spalling of the abutments due to these transverse movements as illustrated in Figs. 7 and 8. This bridge has been analyzed in some detail (10), and it appears that the transverse movement is caused by the expansion of the approach slab into the fixed end of the bridge. This again develops large transverse forces of the type depicted in Fig. 4, and has resulted in considerable damage to the bridge. Numerous other examples of large transverse movements in skewed bridges could be noted.

Curved bridges may also exhibit unusual thermal movements. The Sutton Creek Bridge shown in Fig. 9 in northwest Montana is a sharply curved 3 span continuous steel girder bridge with rocker bearings placed on the tangent of the arc. The bridge was constructed in the early 1970's. The total length of the 3 spans is approximately 577.5 ft (176 m) and so the thermal expansion should be significant. However, an inspection of the bridge bearings in 1988 showed that the bearings were in good condition and unlikely to be frozen, but there was evidence that the rocker bearings over the more slender piers had moved very little if any. This movement was accommodated by the bending of the piers. On the other hand, state inspectors observed that some of the bridge bearings had lifted approximately .25 in. (6 mm) above the piers amid great creaking and groaning on a very hot summer day shortly after the bridge was constructed. This movement is clearly unusual, and it represents a dramatic redistribution of load within the bridge since the bridge girders are 12 ft (3.5 m) high and are quite heavy. This bridge is being analyzed in detail, but the detailed results are not yet finished. However, the preliminary results indicate that the radial movements of the bridge are comparable to the tangential movements and they increase with increase in the internal resistance of the bearings to tangential movements. The results also suggest that the flexibility of the bridge piers is an important factor in the observed movements. This is an important observation, in that bridge engineers seldom consider the deflections of the piers and abutments when designing bridge bearings and expansion joints.

There is other evidence that curved girders frequently experience radial movements. Long curved bridges frequently use finger joints to accommodate expansion and contraction of the deck. The geometry of these joints is well suited for tangential movement, but is poorly suited for differential radial movements. A number of curved bridges in the US have had locked finger joints due to radial movements as illustrated in the photograph of Fig. 10. This locking action has resulted in considerable damage to some bridges (11). Guideways for bridge bearings have sustained similar damage due to unexpected movement in both skewed and curved bridges. Figure 11 shows a guideway that has lost its guiding key and has become misaligned due to these unexpected movements.

There are examples of unexpected temperature movements in straight, orthogonal bridges, but these examples are relatively less common than those for skewed or curved bridges. Figure 12 shows the spalling of a concrete pier produced by thermal shortening of a single span steel bridge girder. The girder rests on an elastomeric bearing with an anchor bolt in a slotted hole. The length of the slotted hole should accommodate the thermal movement. However, it appears that the slot was either of inadequate length or improperly positioned during construction, and spalling has occurred due to pull-out of the anchor bolts. This bridge has a very slight skew (well under 10°), but the skew does not appear to be a contributing factor. Straight, orthogonal bridges have sustained transverse movements due to orientation of the bridge or unusual local conditions. For example, an east-west oriented bridge in a northern latitude exhibits transverse movement due to the low angle of the sun causing direct radiation to the south steel girder. North-south oriented bridges at lower latitudes have exhibited similar problems due to morning or evening sun. There are also a large number of straight and curved bridges with bearing damage where the bearing damage can at least be partially attributed to thermal movements. These transverse movements place severe demands on the bridge bearings and expansion joints, and the demands clearly are not considered in their design. These observations indicate that thermal movements are a significant aspect of bridge behavior, and they cause serious problems if not adequately considered in the design and maintenance of the bridge.

PRELIMINARY ANALYSIS

A series of analyses are being performed to examine thermal movements. Some of the analyses are directed to specific bridges but others are being used for parameter studies. The analyses are not complete and they will be discussed in detail in a later paper. However, some preliminary results will be presented with this paper.

The first step in the analysis of thermal movement is the determination of the temperatures within the bridge as a function of time. Temperature calculations are based on 3 basic heat flow components, radiation, convection, and conduction. Radiation heat transfer consists of long distance transfer of heat from a hot body to a colder one. The sun heats the bridge on sunny days, and the bridge transmits radiant heat to the environment on cold nights. Convection is the transfer of heat from a solid (the bridge) to moving air or fluid. This

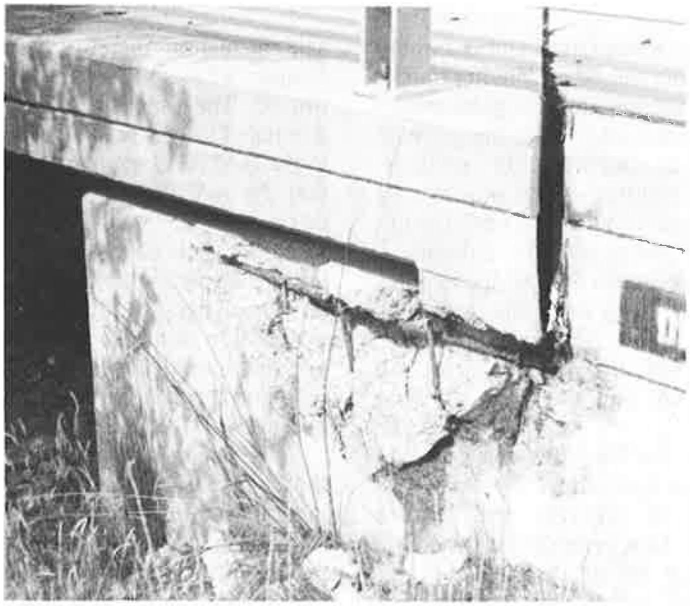

FIGURE 8 Severe spalling of concrete at abutment due to transverse thermal movement

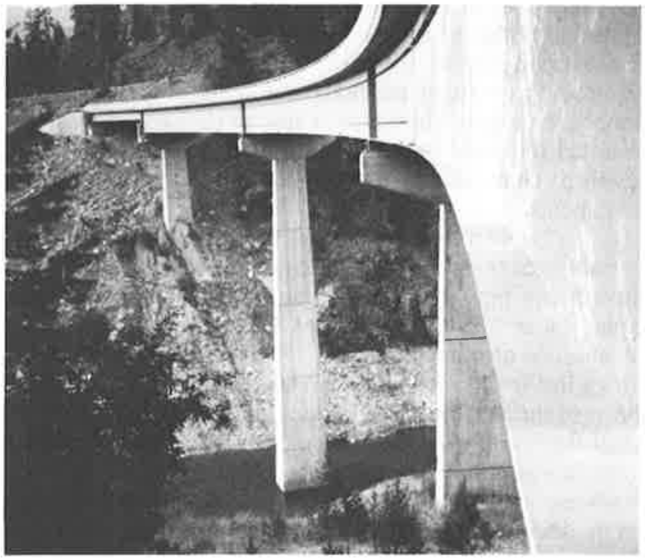

FIGURE 9 Curved steel girder bridge

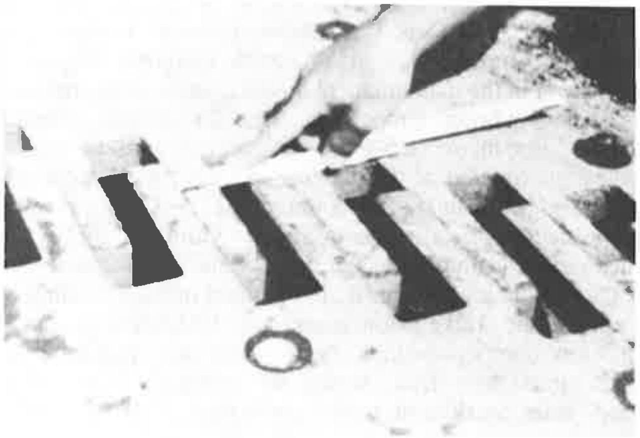

FIGURE 10 Locked finger joints on a curved bridge

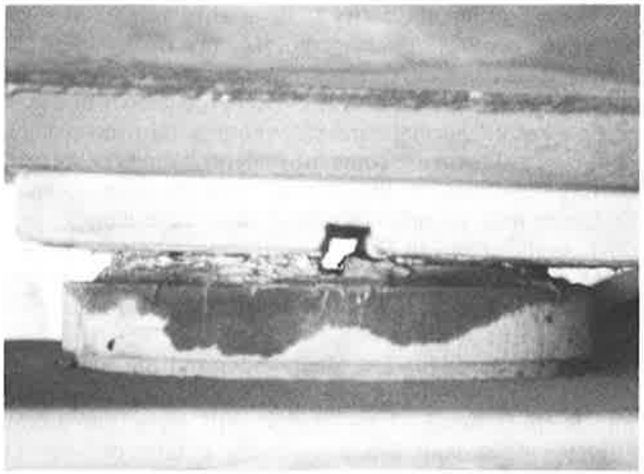

FIGURE 11 Misaligned guideway in a pot bearing due to transverse movement

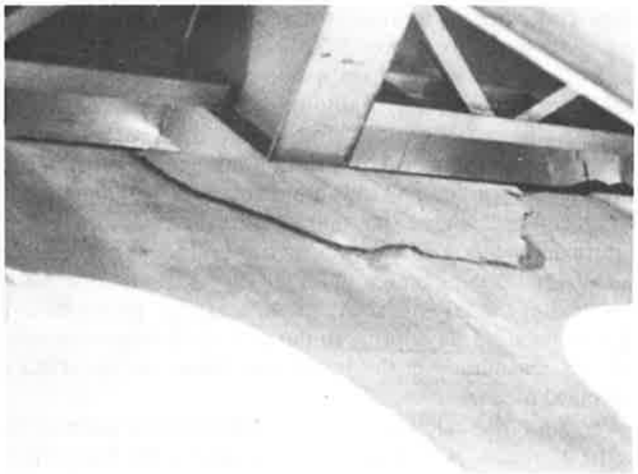

FIGURE 12. Spalling of a concrete pier due to thermal shortening of a steel girder

heat flow is influenced by the air temperature and is largely driven by the wind or by air currents caused by moving traffic. Convection tends to reduce the extreme high temperatures of the bridge during the summer, and it may lower the extreme low temperatures which occur during cold winters. Conduction is the flow of heat within the bridge, since all solid bodies are moving toward a uniform equilibrium temperature in the absence of other outside influences. Accurate determination of the temperature of the bridge requires consideration of all 3 components of heat flow, and it requires other information including the cloud cover, air temperature, wind speed, the angles of the sun, the orientation of the structure with respect to the sun, and the geometry and materials of the bridge.

The horizontal movements of the bridge are primarily related to the average bridge temperature. This average temperature is an integration of the true temperature distribution over the total bridge at a given time. The true distribution may have a minor effect on the thermal movements, but it has a major influence on the temperature dependent deflections and rotations of the bridge girders and the thermal stress in the structure. A series of temperature studies were performed for different bridge types at different locations to evaluate thermal movements. The bridge types included concrete box girders, multiple girder bridges with concrete T-beams, and steel girder bridges with composite deck slabs. The temperature distribution within the bridge may vary significantly with each of these different bridge types. Different bridge types also require different degrees of refinement in the mathematical model to accurately predict the differences in bridge temperature. One dimensional models are adequate for many slab-girder bridges. Figure 13 shows a typical distribution of temperature in a steel girder-composite deck bridge during a hot summer day. The steel girder experiences very little temperature variation through its thickness or depth unless there are unusual circumstances such as the sun shining on a portion of a girder or local shading of a portion of the bridge. Concrete box girder bridges or concrete girder bridges with thick beam webs may require a two dimensional heat flow model to accurately estimate the temperature distribution within the bridge. The more refined model is required since the heat lost by convection from the webs is neglected in a one-dimensional analysis, which results in the prediction of higher bridge temperatures. The air cell in a box girder acts as an insulator and little heat transfer occurs between the inner surface of the webs and the air cell. Hence, the bridge temperatures on a hot summer day are slightly higher for a box girder bridge than a concrete girder bridge. Figure 14 shows a typical temperature distribution within a concrete box girder bridge during a hot summer day. Three dimensional heat flow models are required only in very unusual circumstances where the temperature is expected to vary significantly along the length of the bridge. The heat flow calculations are dependent upon the initial conditions, and the time history analysis sometimes must be performed for approximately 3 days prior to the period of interest to assure that any inaccuracy in the initial conditions do not affect the computed results.

Similar calculations were performed for summer high temperatures and winter low temperatures for the three types of bridge at 11 different locations in the United States. The known air temperature, cloud cover, and local wind speed for the location of interest (12) were used for these conditions. The calculations therefore considered high and low temperature periods where good data was available rather than extreme limits. The ranges of the mean bridge temperatures were computed for the high and low conditions and were compared to the AASHTO temperature range. The comparison suggests that the AASHTO temperature range for steel bridges was fairly similar to that computed for the composite bridge elements. The composite steel girder bridge invariably had a larger temperature range than the concrete bridges, but the difference between the steel and concrete bridge temperature was often smaller than suggested by the AASHTO Specification. The calculations suggest that the AASHTO temperature ranges may be a bit small for some concrete bridges.

After the temperatures are defined, the movements in the bridge can be computed. A wide range of mathematical models were used in these calculations. Simple calculations, such as Equation 1, with the mean bridge temperature provided reasonable accuracy for many simple bridges. However, more refined models were needed in some cases. These more refined models considered factors such as the 3-dimensional deformation of the bridge, the stiffness of piers and abutments, the directional restraint and internal resistance of expansion joints and bridge bearings, the geometry of the bridge and the true temperature distribution. Skewed or curved bridges required the refined methods much more frequently than straight, orthogonal bridges. A few of the typical results will be noted for these bridges, and these results will be used to develop some general conclusions regarding thermal movements.

Skewed bridges frequently have more complex thermal movements and an elementary discussion of these movements may help understand the problem. Thermal expansion as defined in Equation 1, occurs in all directions in the absence of outside restraint. This is depicted for a skew bridge in Fig. 15. This figure shows that the idealized fixed end restraint acts only in the longitudinal direction, and as a result there is horizontal movement in all directions. In this unrestrained condition, there are no thermal forces, and the transverse movements are often of the same magnitude as the longitudinal movement. The greatest movement is across the long diagonal. Unfortunately the fixed end of a real bridge is nearly always restrained in both directions, and bearings at moveable supports are usually guided in the longitudinal direction. Further, the fixity is invariably applied at several points, and so these restraint points may work against one another in restricting thermal movement. These practical restraints prevent some of the movements shown in Fig. 15, and as a result thermal forces develop within the structure. The forces may increase some movements by concentrating the resulting movement to the location with the least resistance. The forces may become very large and cause damage to the bridge as illustrated in Figs. 3-8. It should be noted that the movements increase dramatically once damage initiates, because the bridge no longer has the elastic stiffness to restrain this component of movement.

Skew bridges are often wide, and the unrestrained movements are directly proportional to the bridge dimensions. Thus a short, wide skew bridge will have unrestrained transverse movements which are nearly as large or larger than the longitudinal movement considered in the bridge design, yet they are transverse to the normal bearing placement. Under these conditions the refined theoretical models predict

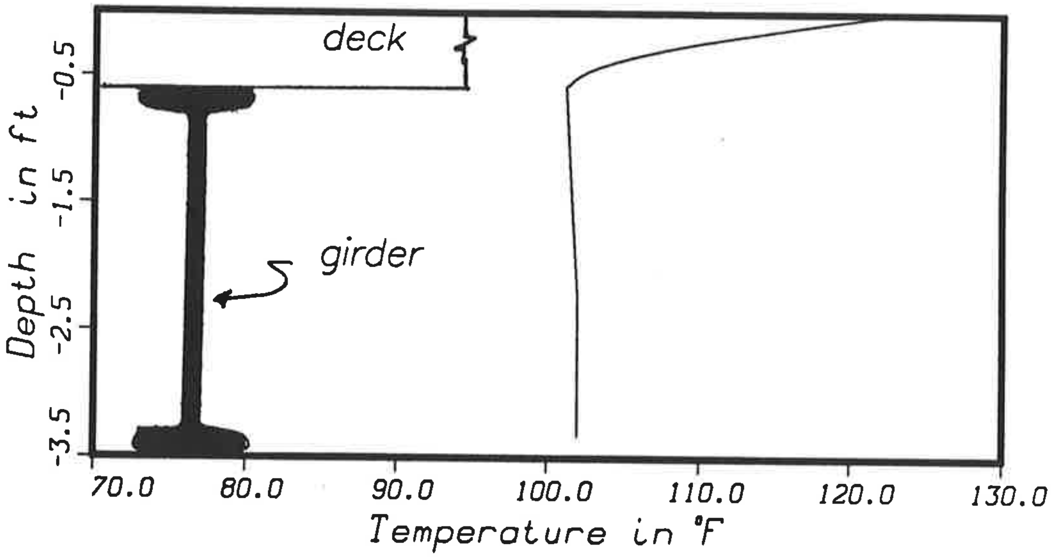

FIGURE 13 Computed temperature distribution on a composite steel girder bridge on a hot summer day

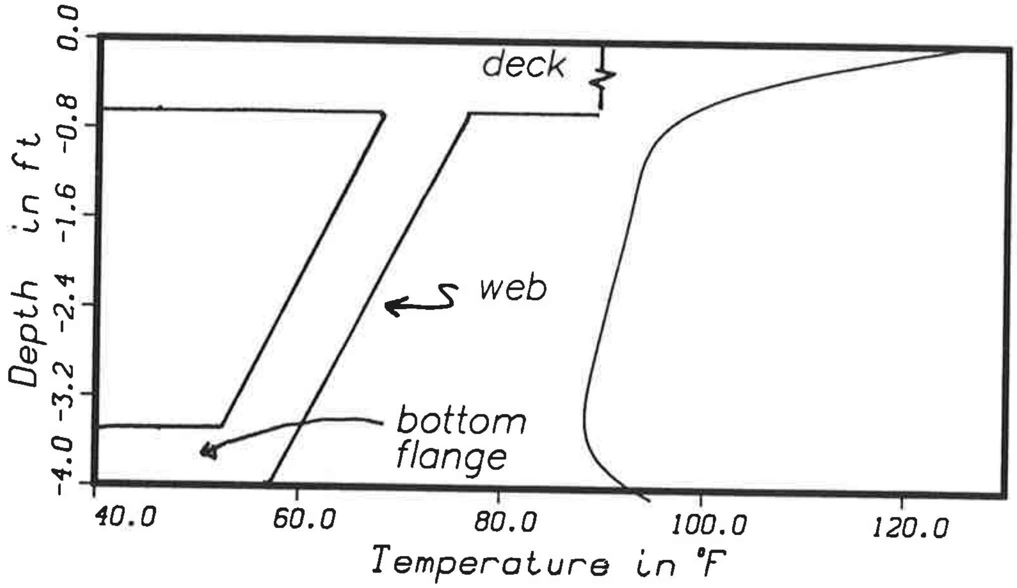

FIGURE 14 Computed temperature distribution on a concrete box girder bridge on a hot summer day

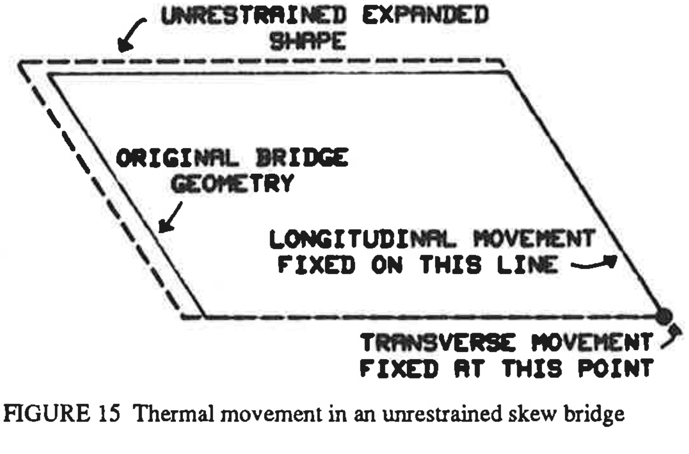

FIGURE 15 Thermal movement in an unrestrained skew bridge

considerable twisting of the bridge girders as illustrated in the lower part of Fig. 16 and an unexpected transverse movement of the bridge as illustrated in the upper part of Fig. 16. If the bridge has stiff diaphragms between girders, the torsional deformation of the girder will be prevented, and the transverse effects will move to other parts of the structure such as the piers or bearings. Increased stiffness results in increased restraint forces and this increases the transverse force component at all interfaces as depicted in Fig. 4. Unfortunately, these forces sometimes find a weakest link in the bridge. This transverse force is the driving mechanism in the damage to skew bridges in earlier figures. When the bearing resistance to longitudinal movement increases, the transverse movement in skewed bridges also increases.

Curved bridges may also illustrate a similar phenomenon. However, they have an additional complication in that engineers do not have a uniform way of placing the bearings. If the curved bridge is a line element with uniform temperature and rigidly fixed at one location, theoretical calculations show that the movement at free supports will be on the chord from the fixed point. However, theoretical calculations show that real bridges often do not obey this simplistic relation. Curved bridges always have at least two girders, and they cease to behave as a line element when this occurs. In addition, the fixed location of the curved girders is often at a pier which is not rigid, and as a result it may have thermal deflection which complicates the direction of movement. The net effect of these observations is that any directional guiding devices at moveable bearings of a curved bridge are almost certain to be oriented in a less than optimal direction. A study conducted on a three span, four girder curved bridge predicted the thermal movements shown in figure 17. The dashed outline in figure 17 is the undeformed shape of the bridge. This figure shows that there is considerable movement of the bridge even at the "fixed" support due to deflection of the piers. The radial movements that would be obtained by orienting the bearings along the chord of this bridge were slightly larger than those obtained by orienting the bearings along the tangent. Thus, guiding devices on curved bridges must be relatively strong or the piers and supporting elements must be relatively flexible if all damage is to be avoided.

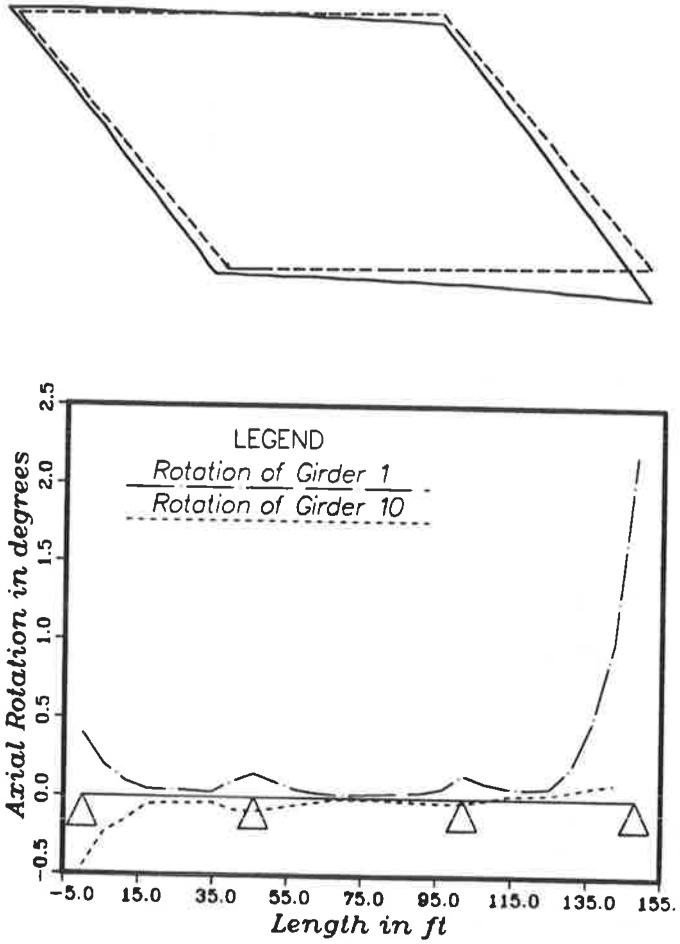

FIGURE 16 Transverse displacements in a skew bridge

PRACTICAL IMPLICATIONS

This paper has provided a brief review of thermal movements observed in real bridges, and it has summarized the results of theoretical studies to examine this behavior. The results are not complete, but a few preliminary observations which have real practical significance can be noted.

1. The present AASHTO temperature ranges for thermal movement calculations appear to be calculated annual temperature range for steel bridges, but they may be a bit small for some concrete bridges. However, concrete bridges are usually designed for additional movements due to creep and shrinkage, and this additional movement allowance probably accounts for any deficiency in the thermal movement calculations.
2. The simple method of predicting thermal movements in Equation 1 is quite reasonable for straight, orthogonal bridges, but more refined calculation methods are needed for some special bridges such as some skewed or curved bridges.
3. Skew bridges with sharp skew angle and with a width that is relatively large compared to their length are most likely to have significant transverse movements and forces and require more refined calculations.
4. Curved bridges with a large width or a tight radius are more likely to require refined thermal movement calculations.
5. Curved bridges which behave as line elements with uniform temperature changes provide a direction of movement which is on a chord from the fixed point of the bridge. However, real curve bridges never precisely satisfy these conditions and as a result always exhibit some other additional thermal movement.
6. The direction of movement for skewed and curved bridges is an element of concern in items 3, 4, and 5, and there is some wisdom in using unguided moveable bearings wherever possible on these bridges.
7. Structural problems caused by thermal movements in bridges are frequently related to lack of maintenance of the bearings and expansion joints or to inadequate control during the construction process. These observations

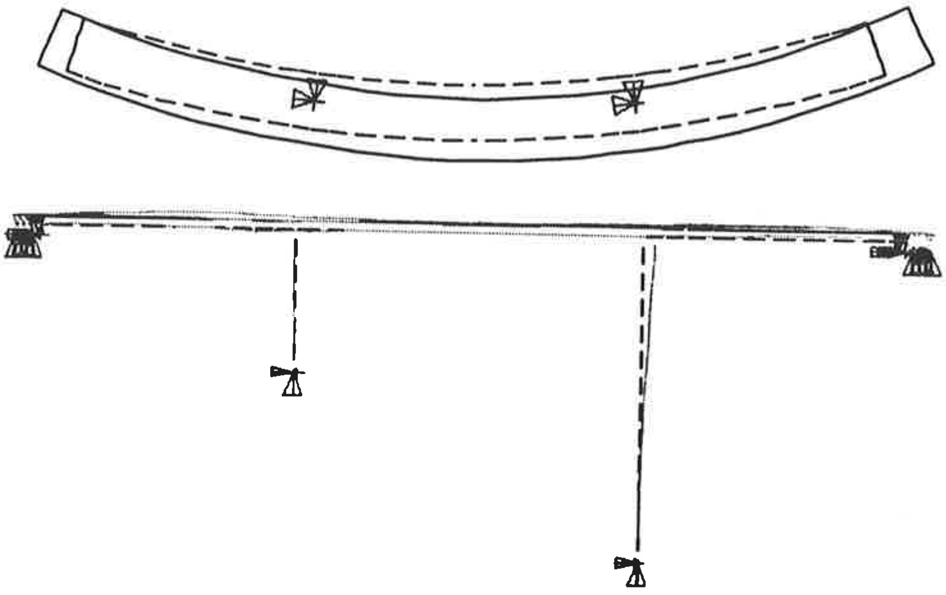

FIGURE 17 Thermal movements of a curved bridge

suggest that maintenance and construction supervision are particularly important in curved and skewed bridges.

8. The uncertainty of the direction of movement in some curved and skew bridges provides increased incentive for the engineer to use integral construction or other forms of jointless construction. However, these alternate methods are not always the panacea that they may seem to some engineers because these alternate structures are often highly restrained and they may attract much larger forces. Thus, they will not always solve the problems associated with thermal movements in complex bridges.

SUMMARY AND CONCLUSIONS

This paper has described some practical observations on thermal movement behavior of bridges. The work is in progress, and further results will be presented in a later publication. Nevertheless, this paper has some practical results which should be useful to the engineering profession. The results are based primarily on analytical studies. These analyses have been correlated to physical observations of bridge behavior, but additional experimental correlation is needed to verify the analytical results.

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