Flexural Cracking in Concrete Structures

Edward G. Nawy

The state-of-the-art in the evaluation of the flexural crack width development and crack control of macrocracks is described. It is based on extensive research over the past 50 years in the United States and overseas in the area of macrocracking in reinforced and prestressed concrete beams and two-way-action slabs and plates. Control of cracking has become essential to maintain the integrity and aesthetics of concrete structures. The trends are stronger than ever—toward better use of concrete strength, use of higher-strength concretes including superstrength concretes of over 20,000-psi compressive strength, use of more prestressed concretes, and increased use of limit failure theories—all requiring closer control of serviceability requirements of cracking and deflection behavior. Common expressions are discussed for the control of cracking in reinforced-concrete beams and thick one-way slabs; prestressed, pretensioned, and posttensioned flanged beams; and reinforced-concrete, two-way-action, structural floor slabs and plates. In addition, recommendations are given for the maximum tolerable flexural crack widths in concrete elements.

Presently, the trend is stronger than ever—toward better use of concrete strength, use of higher-strength concretes including superstrength concretes of 20,000-psi (138-MPa) compressive strength and higher, use of high-strength reinforcement, use of more prestressed concretes, and increased use of limit failure theories—all requiring closer control of serviceability requirements in cracking and deflection behavior. Hence, knowledge of the cracking behavior of concrete elements becomes essential.

Concrete cracks early in its loading history. Most cracks are a result of the following actions to which concrete can be subjected:

1. Volumetric change caused by drying shrinkage, creep under sustained load, thermal stresses including elevated temperatures, and chemical incompatibility of concrete components.
2. Direct stress caused by applied loads or reactions or internal stress caused by continuity, reversible fatigue load, long-term deflection, camber in prestressed systems, and environmental effects including differential movement in structural systems.
3. Flexural stress caused by bending.

Although the net result of these three actions is the formation of cracks, the mechanisms of their development cannot be considered to be identical. Volumetric change generates internal microcracking that may develop into full cracking, whereas direct internal or external stress or applied loads and reactions could either generate internal microcracking, such as in the case of fatigue caused by reversible load, or flexural macrocracking leading to fully developed cracking.

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Although the macrocracking aspects of cracking behavior are emphasized, it is also important to briefly discuss microcracking.

MICROCRACKING

Microcracking can be mainly classified into two categories: (a) bond cracks at the aggregate-mortar interface, and (b) paste cracks within the mortar matrix. Interfacial bond cracks are caused by interfacial shear and tensile stresses caused by early volumetric change without the presence of external load. Volume change caused by hydration and shrinkage could create tensile and bond stresses of sufficient magnitude to cause failure at the aggregate-mortar interface (*1*). As the external load is applied, mortar cracks develop because of increase in compressive stress, propagating continuously through the cement matrix up to failure. A typical schematic stress-strain diagram (Figure 1) shows that the nonlinear relationship developed early in the stress history and started with bond microcracking. Although extensive work exists in the area of volumetric change cracking, the need is apparent for additional work on creep effects on microcracking and also for the development of a universally acceptable fracture theory to interrelate the nonlinear behavioral factors resulting in crack propagation.

It appears that the damage to cement paste seems to play a significant role in controlling the stress-strain relationship in concrete. The coarse aggregate particles act as stress raisers that decrease the strength of the cement paste. As a result, microcracks develop that can only be detected by large magnification. The importance of additional work lies not only in the evaluation of the microcracks, but also in the evaluation of their significance for the development of macrocracks that generate from those microcracked centers of plasticity.

FLEXURAL CRACKING

External load results in direct and bending stresses, causing flexural, bond, and diagonal tension cracks. Immediately after the tensile stress in the concrete exceeds its tensile strength, internal microcracks form. These cracks generate into macrocracks propagating to the external fiber zones of the element.

Immediately after the full development of the first crack in a reinforced-concrete element, the stress in the concrete at the cracking zone is reduced to zero and is assumed by the reinforcement (*2*). The distributions of ultimate bond stress, longitudinal tensile stress in the concrete, and longitudinal tensile stress in the steel are shown in Figure 2.

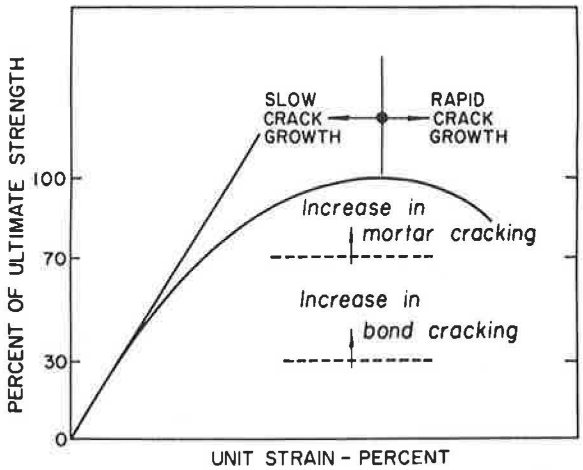

FIGURE 1 Schematic stress-strain diagram of concrete in microcracking.

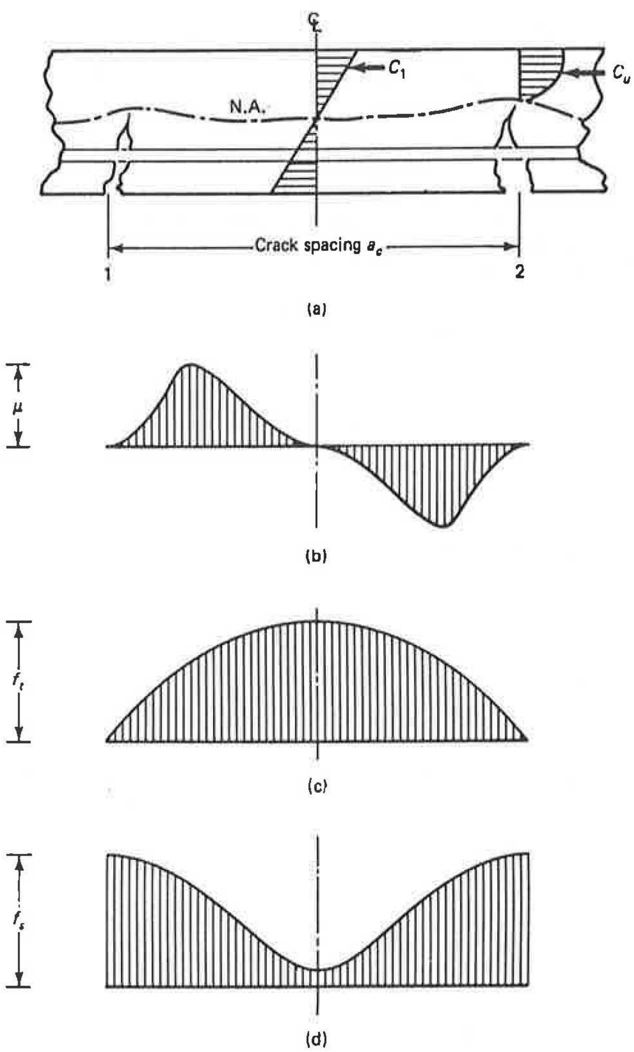

FIGURE 2 Schematic stress distributions (a) between two flexural cracks for (b) ultimate bond stress, (c) longitudinal tensile stress in the concrete, and (d) longitudinal tensile stress in the steel.

In Figure 2, crack width is a primary function of the deformation of reinforcement between adjacent Cracks 1 and 2, if the small concrete strain along the crack interval a_c is neglected. The crack width would hence be a function of the crack spacing, and vice versa, up to the level of stabilization of crack spacing (Figure 3).

The major parameters affecting the development and characteristics of the cracks are percentage of reinforcement, bond characteristics and size of bar, concrete cover, and the concrete stretched area (namely, the concrete area in tension). On this basis, one can propose the following mathematical model:

$$w = \alpha\, a_c^{\beta} \varepsilon_s^{\gamma} \qquad (1)$$

where

w = maximum crack width,
α, β, and γ = nonlinearity constants, and
ε_s = strain in the reinforcement induced by external load.

Crack spacing a_c is a function of the factors enumerated previously, being inversely proportional to bond strength and active steel ratio (steel percentage in terms of the concrete area in tension).

The basic mathematical model in Equation 1 with the appropriate experimental values of the constants α, β, and γ can be derived for the particular type of structural member. Such a member can be a one-dimensional element such as a beam, a two-dimensional structure such as a two-way slab, or a three-dimensional member such as a shell or circular tank wall. Hence, it is expected that different forms or expressions apply for the evaluation of the macrocracking behavior of different structural elements consistent with their fundamental structural behavior (*1–10*).

FLEXURAL CRACK CONTROL IN REINFORCED-CONCRETE BEAMS

Requirements for crack control in beams and thick one-way slabs in the American Concrete Institute (ACI) Building Code

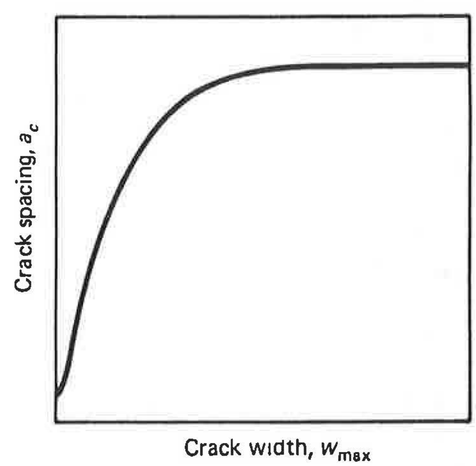

FIGURE 3 Schematic variation of crack width with crack spacing.

(ACI 318) are based on the statistical analysis of maximum crack width data from a number of sources. The following general conclusions were reached:

1. The steel stress is the most important variable;
2. The thickness of the concrete cover is an important variable, but not the only geometric consideration;
3. The area of concrete surrounding each reinforcing bar is also an important geometric variable;
4. The bar diameter is not a major variable; and
5. The size of the bottom crack width is influenced by the amount of strain gradient from the level of the steel to the tension face of the beam.

The simplified expression relating crack width to steel stress is as follows (*4*):

$$w_{\max} = 0.076\ \beta\ f_s(d_cA)^{1/3} \times 10^{-3} \tag{2}$$

where

f_s = reinforcing steel stress, kips/in.^{2} (ksi);
A = area of concrete symmetric with reinforcing steel divided by number of bars, in.^{2};
d_c = thickness of concrete cover measured from extreme tension fiber to center of bar or wire closest thereto, in.; and
h_1 = distance from neutral axis to the reinforcing steel, in.;
h_2 = distance from neutral axis to extreme concrete tensile surface, in.; and
β = h_2/h_1.

A plot relating the reinforcement strength to the ratio of the concrete area in tension to the reinforcement area is shown in Figure 4 for all bar sizes.

In the ACI code, when the design field strength f_y for tension reinforcement exceeds 40,000 psi, cross sections of maximum positive and negative moment have to be so proportioned that the quantity z given by

$$z = f_s\ (d_cA)^{1/3} \tag{3}$$

does not exceed 175 kips/in. for interior exposure and 145 kips/in. for exterior exposure. Calculated stress in the reinforcement at service load f_s (ksi) shall be computed as the moment divided by the product of steel area and internal moment area. In lieu of such computations, it is permitted to take f_s as 60 percent of specified length f_y.

When the strain ε_s in the steel reinforcement is used instead of stress f_s, Equation 3 becomes

$$w = 2.2\ R_i\ \varepsilon_s\ (d_cA)^{1/3} \tag{4}$$

Equation 4 is valid in any system of measurement.

The cracking behavior in thick one-way slabs is similar to that in shallow beams. For one-way slabs having a clear concrete cover in excess of 1 in. (25.4 mm), Equation 4 can be adequately applied if β = 1.25 to 1.35.

Committeé EuroInternationale du Beton (CEB) Recommendations

Crack control recommendations proposed that the European Model Code for Concrete Structures (*9*) apply to both pre-

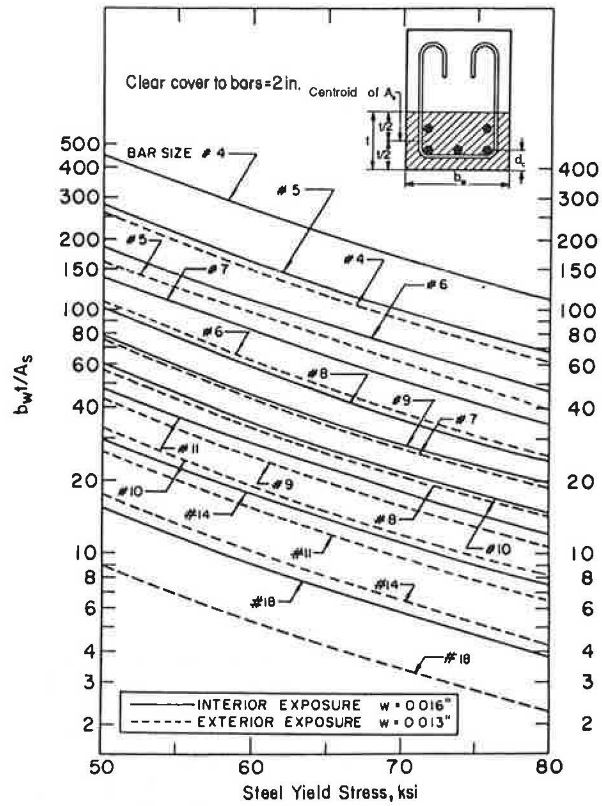

FIGURE 4 Steel reinforcement strength f_y versus ratio of concrete area in tension to reinforcement area for stress level $f_s = 0.6f_y$.

stressed and reinforced concrete can be summarized as follows:

The mean crack width w_m in beams is expressed in terms of the mean crack spacing s_{rm}, such that

$$w_m = \varepsilon_{sm}s_{rm} \tag{5}$$

$$\varepsilon_{sm} = \frac{f_s}{E_s}\left[1 - \chi\left(\frac{f_{sr}}{f_s}\right)^2\right] \leq 0.4\frac{f_s}{E_s} \tag{6}$$

where

ε_{sm} = average strain in the steel,
f_s = steel stress at the crack,
f_{sr} = steel stress at the crack caused by cracking forces at the tensile strength of concrete, and
χ = bond coefficient (1.0 for ribbed bars, reflecting influence of load repetitions and load duration).

The mean crack spacing is

$$s_{rm} = 2\left(c - \frac{s}{10}\right) + \chi_2\chi_3\frac{d_b}{Q_R} \tag{7}$$

where

c = clear concrete cover;
s = bar spacing, limited to $15d_b$;
χ_2 = coefficient that is 0.4 for ribbed bars;
χ_3 = coefficient that depends on the shape of the stress diagram, 0.125 for bending;

$Q_R = A_s/A_t$; and
A_t = effective area in tension.

Depending on arrangement of bars and type of external forces, A_t is limited by a line $c + 7d_b$ from the tension face for beams (in the case of thick slabs, not more than halfway to the neutral axis).

A simplified formula can be derived for the mean crack width in beams with ribbed bars.

$$w_m = 0.7 \frac{f_s}{E_s}\left(3_c + 0.05 \frac{d_b}{Q_R}\right) \tag{8}$$

A characteristic value of the crack width, presumably equivalent to the probable maximum value, is given by $0.7w_m$.

FLEXURAL CRACK CONTROL IN PRESTRESSED, PRETENSIONED, AND POSTTENSIONED BEAMS

The increased use of partial prestressing, allowing limited tensile stresses in the concrete under service and overload conditions while allowing nonprestressed steel to carry the tensile stresses, is becoming prevalent because of practicality and economy. Consequently, an evaluation of the flexural crack widths and spacing and control of their development become essential. Work in this area is relatively limited because of the various factors affecting crack width development in prestressed concrete. However, experimental investigations support the hypothesis that the major controlling parameter is the reinforcement stress change beyond the decompression stage. Nawy et al. have undertaken extensive research since the 1960s on the cracking behavior of prestressed, pretensioned, and posttensioned beams and slabs because of the great vulnerability of the highly stressed prestressing steel to corrosion and other environmental effects and the resulting premature loss of prestress (*11,12*). Serviceability behavior under service and overload conditions can be controlled by the design engineer through the application of the criteria presented in this section.

Mathematical Model Formulation for Serviceability Evaluation

Crack Spacing

Primary cracks form in the region of maximum bending moment when the external load reaches the cracking load. As loading is increased, additional cracks will form and the number of cracks will be stabilized when the stress in the concrete no longer exceeds its tensile strength at further locations regardless of load increase. This condition is important as it essentially produces the absolute minimum crack spacing that can occur at high steel stresses, to be termed the stabilized minimum crack spacing. The maximum possible crack spacing under this stabilized condition is twice the minimum, to be termed the stabilized maximum crack spacing. Hence, the stabilized mean crack spacing a_{cs} is deduced as the mean value of the two extremes.

The total tensile force T transferred from the steel to the concrete over the stabilized mean crack spacing can be defined as

$$T = \gamma\, a_{cs}\, \mu\, \Sigma o \tag{9a}$$

where

γ = a factor reflecting the distribution of bond stress;
μ = maximum bond stress, which is a function of $f_c'^{1/2}$;
a_{cs} = mean stabilized spacing; and
Σo = sum of reinforcing element circumferences.

The resistance R of the concrete area A_t in tension can be defined as

$$R = A_t f_t' \tag{9b}$$

where f_t' = tensile splitting strength of the concrete. By equating Equations 9a and 9b, the following expression for a_{cs} is obtained:

$$a_{cs} = c \frac{A_t f_t'}{\Sigma o\, (f_c')^{1/2}} \tag{10a}$$

where c is a constant to be developed from the tests. The concrete stretched area, namely the concrete area in tension A_t for both the evenly distributed and nonevenly distributed reinforcing elements, is shown in Figure 5. With a mean value of $f_t'/(f_c')^{1/2} = 7.95$ in this investigation, a regression analysis

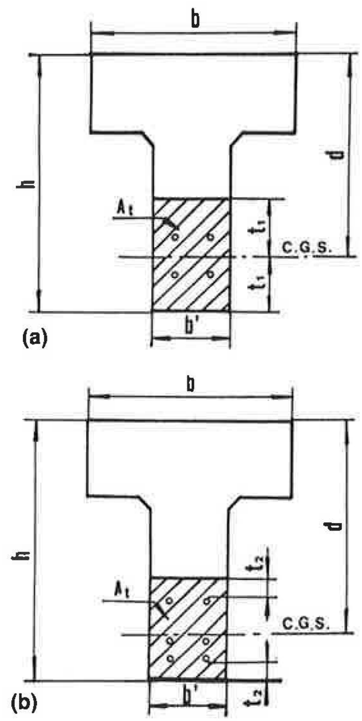

FIGURE 5 Effective concrete area in tension (a) for even distribution of reinforcement in concrete, and (b) for noneven distribution of reinforcement in concrete.

of the test data resulted in the following expression for the mean stabilized crack spacing:

$$a_{cs} = 1.20A_t/\Sigma o \tag{10b}$$

Crack Width

If Δf_s is the net stress in the prestressed tendon or the magnitude of the tensile stress in the normal steel at any crack width load level in which the decompression load (decompression here means $f_c = 0$ at the level of the reinforcing steel) is taken as the reference point, then for the prestressed tendon

$$\Delta f_s = f_{nt} - f_d \tag{11}$$

where

f_{nt} = stress in the prestressing steel at any load beyond the decompression load, and
f_d = stress in the prestressing steel corresponding to the decompression load.

The unit strain $\varepsilon_s = \Delta f_s/E_s$. It is logical to disregard as insignificant the unit strains in the concrete caused by the effects of temperature, shrinkage, and elastic shortening. The maximum crack width as defined in Equation 1 can be taken as

$$w_{\max} = ka_{cs}\varepsilon_s^{\alpha} \tag{12a}$$

where k and α are constants to be established by tests, or

$$w_{\max} = k'a_{cs}(\Delta f_s)^{\alpha} \tag{12b}$$

where k' is a constant in terms of constant k.

Expressions for Pretensioned Beams

Equation 12a is rewritten in terms of Δf_s so that analysis of the test data of all the simply supported test beams in this work leads to the following expression at the reinforcement level:

$$w_{\max} = (1.4 \times 10^{-5})a_{cs}(\Delta f_s)^{1.31} \quad \text{(in.)} \tag{13}$$

Linearizing Equation 13 for easier use by the design engineer leads to the following simplified expression of the maximum crack width at the reinforcing steel level:

$$w_{\max} = (5.85 \times 10^{-5}) \frac{A_t}{\Sigma o} (\Delta f_s) \tag{14a}$$

and a maximum crack width (in.) at the tensile face of the concrete:

$$w'_{\max} = (5.85 \times 10^{-5}) R_i \frac{A_t}{\Sigma o} (\Delta f_s) \tag{14b}$$

A plot of the data and the best-fit expression for Equation 14a are shown in Figure 6 with a 40 percent spread (which is reasonable in view of the randomness of crack development and the linearization of the original expression in Equation 13).

Expressions for Posttensioned Beams

The expression developed for the crack width in posttensioned bonded beams that contain mild steel reinforcement is

$$w_{\max} = (6.51 \times 10^{-5}) \frac{A_t}{\Sigma o} (\Delta f_s) \tag{15a}$$

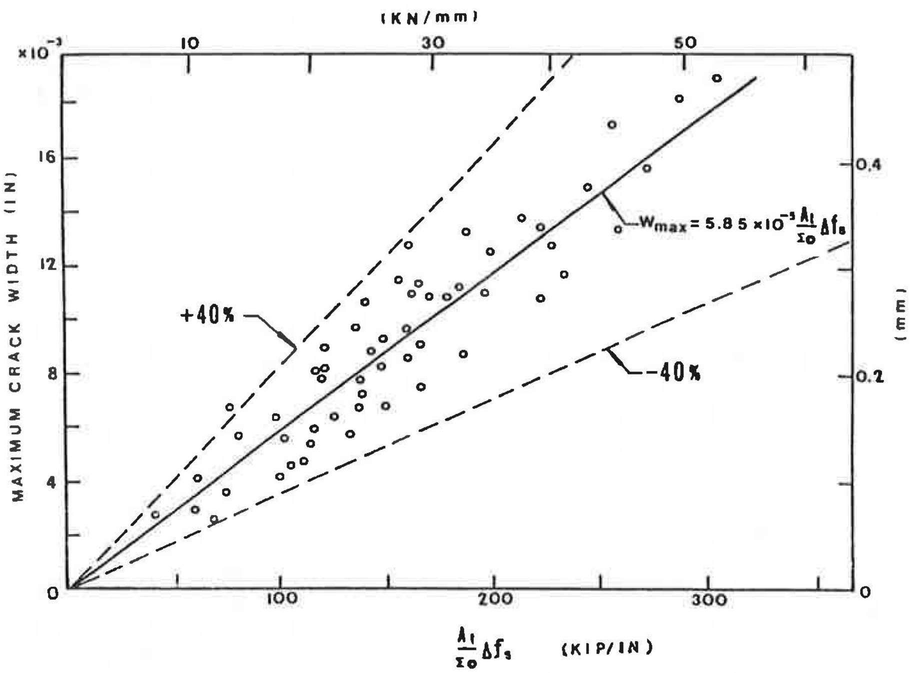

FIGURE 6 Linearized maximum crack width versus $(A_t/\Sigma o)\ \Delta f_s$ for pretensioned beams.

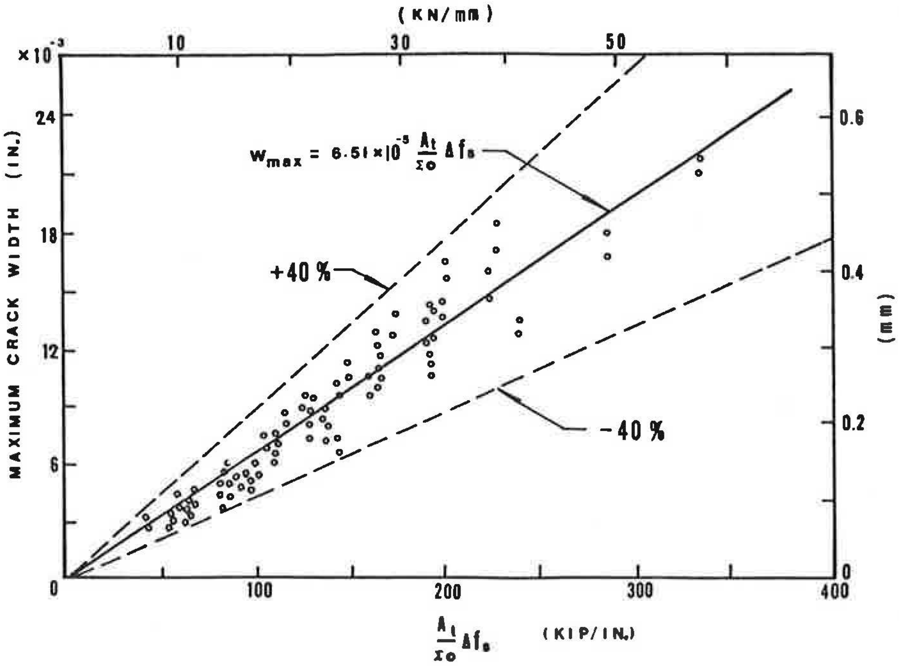

FIGURE 7 Linearized maximum crack width versus $(A_t/\Sigma o)\ \Delta f_s$ for posttensioned beams.

for the width at the reinforcement level closest to the tensile face, and

$$w_{max} = (6.51 \times 10^{-5}) R_i \frac{A_t}{\Sigma o} (\Delta f_s) \tag{15b}$$

at the tensile face of the concrete lower fibers.

For nonbonded beams, the factor 6.51 in Equations 15a and 15b becomes 6.83.

A plot of the data and the best-fit expression for Equation 15a are shown in Figure 7.

A typical plot of the effect of the various steel percentages on the crack spacing at the various steel levels Δf_s is shown in Figure 8. In this plot, crack spacing stabilizes at a net stress level of 36 ksi.

Other Work on Cracking in Prestressed Concrete

On the basis of the analysis of results of various investigators, Naaman (*8*) produced the following modified expression for partially prestressed pretensioned members

$$(w_{max}) = \left[42 + 5.58 \frac{A_t}{\Sigma o} (\Delta f_s) \right] \times 10^{-5} \quad \text{(in.)} \tag{16}$$

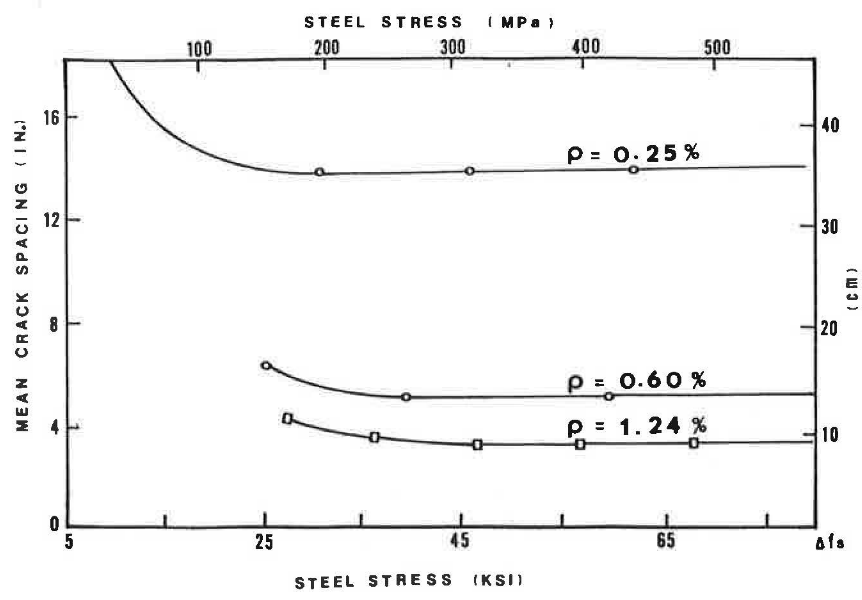

FIGURE 8 Effect of steel percentage on mean crack spacing in prestressed beams.

This regression expression is close to Equation 14. When plotted against the experimental results of the various researchers, it gives a best fit as shown in Figure 9.

The author's equations and the CEB–Fédération Internationale de la Précontrainte (FIP) equations can be compared using similar notations.

Nawy: $$w_{\max} = \zeta\left(\frac{A_t}{\Sigma o}\right)\Delta\sigma_p$$

CEB–FIP: $$w_{\max} = \left(k\frac{\phi}{\rho_t} + \alpha_0\right)\frac{\Delta\sigma_p}{E_s}\left[1 - b\left(\frac{\Delta\sigma_{pr}}{\Delta\sigma_p}\right)^2\right]$$

These equations are similar assuming $1/\Sigma o = \lambda(\phi/\rho_t)$, where ϕ is the diameter of the bar, λ is a multiplier, k and b are experimental parameters, and α_0 and $\Delta\sigma_{pr}/\Delta\sigma_p^2$ are terms in the CEB–FIP expression not of major significance that are accounted for by using $\zeta = 6.51$ for the posttensioned beams in the author's expressions.

The study by Meier and Gergely (*10*), concentrating on the area of concrete in tension and the nominal strain in the concrete at the tensile face, does not yield a reliable prediction of the crack width. In particular, it does not account for the actual stress in the steel reinforcement and depends on measurements of strain at the concrete surface that are difficult to reliably evaluate.

FLEXURAL CRACK CONTROL IN TWO-WAY-ACTION SLABS AND PLATES

Flexural crack control is essential in structural floors where cracks at service load and overload conditions can be serious, such as in office buildings, schools, parking garages, industrial buildings, and other floors where the design service load levels exceed those in normal-sized apartment building panels and also in all cases of adverse exposure conditions.

Flexural Cracking Mechanism and Fracture Hypothesis

Flexural cracking behavior in concrete structural floors under two-way action is significantly different from that in one-way members. Crack control equations for beams underestimate the crack widths developed in two-way slabs and plates, and do not tell the designer how to space the reinforcement. Cracking in two-way slabs and plates is controlled primarily by the steel stress level and the spacing of the reinforcement in the two perpendicular directions. In addition, the clear concrete cover in two-way slabs and plates is nearly constant [¾ in. (19 mm) for interior exposure], whereas it is a major variable in the crack control equations for beams. The results from extensive tests on slabs and plates by Nawy et al. demonstrate this difference in behavior in a fracture hypothesis on crack development and propagation in two-way plate action. As shown in Figure 10, stress concentration develops initially at the points of intersection of the reinforcement in the reinforcing bars and at the welded joints of the wire mesh, that is, at grid nodal points, thereby dynamically generating fracture lines along the paths of least resistance, namely, along A_1B_1, A_1A_2, A_2B_2, and B_2B_1. The resulting fracture pattern is a total repetitive cracking grid, provided that the spacing of the nodal points A_1, B_1, A_2, and B_2 is close enough to generate this preferred initial fracture grid of orthogonal cracks narrow in width as a preferred fracture mechanism.

If the spacing of the reinforcing grid intersections is too large, the magnitude of stress concentration and the energy absorbed per unit grid is too low to generate cracks along the reinforcing wires or bars. As a result, the principal cracks follow diagonal yield-line cracking in the plain concrete field away from the reinforcing bars early in the loading history. These cracks are wide and few.

This hypothesis also leads to the conclusion that surface deformations of the individual reinforcing elements have little effect in arresting the generation of the cracks or controlling their type or width in a slab or plate of two-way action. In a

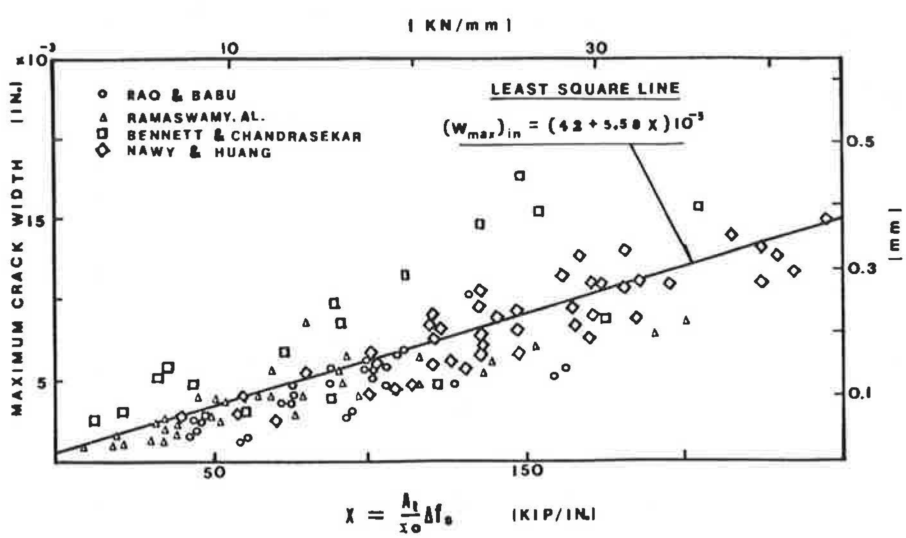

FIGURE 9 Reinforcement stress versus crack width (best-fit data of several investigators).

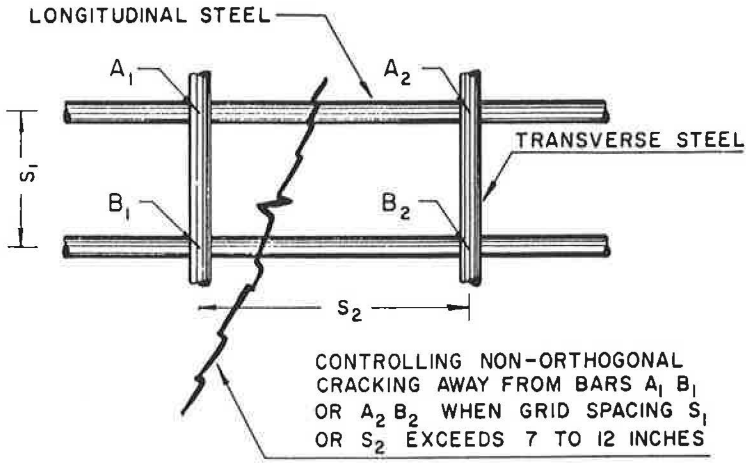

FIGURE 10 Grid unit in two-way-action reinforcement.

similar manner, one may conclude that the scale effect on cracking behavior during two-way action is insignificant, because the cracking grid would be a reflection of the reinforcement grid if the preferred orthogonal narrow cracking widths develop. Therefore, to control cracking in floors with a two-way action, the major parameter to be considered is the reinforcement spacing in two perpendicular directions. Concrete cover has only a minor effect, because it is usually a small constant of value 0.75 in. (19 mm).

For a constant area of steel determined for bending in one direction, that is, for energy absorption per unit slab area, the smaller the spacing of the transverse bars or wires, the smaller should be the diameter of the longitudinal bars. The reason is that less energy has to be absorbed by the individual longitudinal bars. When the magnitude of fracture is determined by the energy imposed per specific volume of reinforcement acting on a finite element of the slab, a proper choice of the reinforcement grid size and bar size can control cracking into preferred orthogonal grids.

This hypothesis is important for serviceability and reasonable overload conditions. In relating orthogonal cracks to yield-line cracks, the failure of a slab ultimately follows the generally accepted rigid-plastic yield-line criteria.

Crack Control Equation

The basic Equation 1 for relating crack width to strain in the reinforcement is

$$\bar{w} = \alpha a_c^{\beta}\, \varepsilon_s^{\gamma} \tag{17}$$

where

a_c = crack spacing,
ε_s = unit strain in the reinforcement, and
α, β, γ = constants.

The effect of the tensile strain in the concrete between the cracks is neglected as insignificant.

As a result of the fracture hypothesis, the mathematical model of Equation 17, and the statistical analyses of the data for 90 slabs tested to failure, the following equation for crack control emerged:

$$w = K_{\beta} f_s \left(\frac{d_{b1} s_2}{Q_{i1}}\right)^{1/2} \tag{18}$$

where

w = crack width at concrete face caused by flexural load, in.;
k = fracture coefficient, in.^{2}/lb;
β = ratio of the distance from the neutral axis to the tensile face of the slab to the distance from the neutral axis to the centroid of the reinforcement grid;
f_s = actual average service load stress level, or 40 percent of the design yield strength, ksi;
d_{b1} = diameter of the reinforcement in Direction 1 closest to the concrete outer fibers, in.;
s_2 = spacing of the reinforcement in the perpendicular Direction 2, in.;
A_s = area of steel per foot of width of concrete, in.^{2};
c_1 = clear concrete cover measured from the tensile face of the concrete to the nearest edge of the reinforcing bar in Direction 1, in.; and
Q_{i1} = active steel ratio, given by $A_s/12(d_{b1} + 2c_1)$.

Direction 1 is the direction of the reinforcement closest to the outer concrete fibers; this is the direction for which the crack control check is to be made.

The quantity whose square root is taken is termed the "grid index," and can be transformed as follows:

$$G_I = \frac{d_{b1} s_2}{Q_{i1}} = \frac{s_1 s_2 d_c}{d_{b1}} \frac{8}{\pi} \tag{19}$$

in which s_1 is the spacing of the reinforcement in Direction 1.

For uniformly loaded, restrained, square slabs and plates of two-way action, $k = 2.8 \times 10^{-5}$ in.^{2}/lb. For concentrated loads or reactions, or when the ratio of short span to long span is less than 0.75 but greater than 0.5, $k = 2.1 \times 10^{-5}$ in.^{2}/lb. For a span aspect ratio of less than 0.5, $k = 1.6 \times 10^{-5}$ in.^{2}/lb.

Although β varies in value between 1.20 and 1.35, the intermediate value $\beta = 1.25$ was used to simplify the calculations.

Subscripts 1 and 2 generally pertain to the two directions of reinforcement. Detailed values of the fracture coefficients for various boundary conditions are presented in Table 1.

A graphical solution of Equation 18 is shown in Figure 11 for f_y = 60,000 psi (414 MPa) and $f_s = 0.4 f_y$ = 24,000 psi (165.5 MPa) for rapid determination of the reinforcement size and spacing needed for crack control.

Permissible Crack Widths in Concrete Structures

The maximum crack width that a structural element should be permitted to develop depends on the particular function of the element and the environmental conditions to which the structure is liable to be subjected. Table 2 from the ACI Committee 224 report on cracking serves as a reasonable guide on the permissible crack widths in concrete structures under the various environmental conditions that are normally encountered.

The crack control equation and guidelines presented are important not only for the control of corrosion in the reinforcement but also for deflection control. The reduction of the stiffness EI of the two-way slab or plate caused by orthogonal cracking when the limits of permissible crack widths in Table 2 are exceeded, can lead to excessive deflection, both short-term and long-term. Deflection values several times those anticipated in the design, including deflection caused by construction loading, can be reasonably controlled through camber and control of the flexural crack width in the slab or plate. Proper selection of the reinforcement spacings s_1 and s_2 in the perpendicular directions as discussed in this section, and not exceeding 12 in. center to center, can maintain good serviceability performance of a slab system under normal and reasonable overload conditions.

In most cases, the magnitude of crack widths increases in long-term exposure and long-term loading. The increase in crack width can vary considerably in cases of cyclic loading, such as in bridges, but the width increases at a decreasing rate with time. In most cases, a doubling of crack width after several years under sustained loading can be expected.

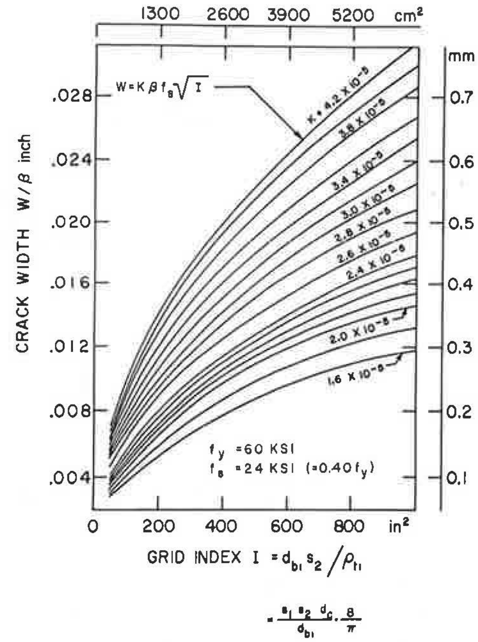

FIGURE 11 Crack control reinforcement distribution in two-way-action slabs and plates for all exposure conditions: f_y = 60,000 psi, f_s = 24,000 psi (= $0.40f_y$).

CONCLUSIONS

With the aid of the expressions summarized in the following, the design engineer and the constructor can limit the flexural

TABLE 1 FRACTURE COEFFICIENTS FOR SLABS AND PLATES

Loading type[a]	Slab shape	Boundary condition[b]	Span ratio,[c] S/L	Fracture coefficient 10^{-5} K
A	Square	4 edges r	1.0	2.1
A	Square	4 edges s	1.0	2.1
B	Rectangular	4 edges r	0.5	1.6
B	Rectangular	4 edges r	0.7	2.2
B	Rectangular	3 edges r, 1 edge h	0.7	2.3
B	Rectangular	2 edges r, 2 edges r	0.7	2.7
B	Square	4 edges r	1.0	2.8
B	Square	3 edges r, 1 edge h	1.0	2.9
B	Square	2 edges r, 2 edges h	1.0	4.2

[a]Loading type: A, concentrated; B, uniformly distributed.

[b]Boundary condition: r, restrained; s, simply supported; h, hinges.

[c]Span ratio: S, clear short span; L, clear long span.

TABLE 2 MAXIMUM PERMISSIBLE FLEXURAL CRACK WIDTHS

Exposure condition	Crack width in.	Crack width mm.
Dry air or protective membrane	0.016	0.41
Humidity, moist air, soil	0.012	0.30
De-icing chemicals	0.007	0.18
Seawater and seawater spray; wetting and drying	0.006	0.15
Water-retaining structures (excluding nonpressure pipes)	0.004	0.10

macrocrack width that develops in concrete systems. By limiting the width to within the permissible levels presented in Table 2 in accordance with the prevailing environmental conditions, it would be possible to prevent or considerably minimize long-term corrosion deterioration and also maintain the aesthetic behavior of the various elements of the system.

1. Reinforced-Concrete Beams and Thick One-Way Slabs

$$w_{max} = 0.076\beta f_s(d_cA)^{1/3} \times 10^{-3}$$

or

$$z = f_s(d_cA)^{1/3}$$

where f_s is in ksi and z is not to exceed a value of 145 kip/in. for exterior exposure or 175 kip/in. for interior exposure.

2. Prestressed, Pretensioned Beams
   a. Steel reinforcement level

$$w_{max} = (5.85 \times 10^{-5})\frac{A_t}{\Sigma o}(\Delta f_s)$$

   b. Tensile face of concrete

$$w'_{max} = (5.85 \times 10^{-5})R_i\frac{A_t}{\Sigma o}(\Delta f_s)$$

3. Prestressed Post-Tensioned Beams
   a. Steel reinforcement level

$$w_{max} = (6.51 \times 10^{-5})\frac{A_t}{\Sigma o}(\Delta f_s)$$

   b. Tensile face of concrete

$$w_{max} = (6.51 \times 10^{-5})R_i\frac{A_t}{\Sigma_o}(\Delta f_s)$$

For nonbonded beams, the factor 6.51 becomes 6.83.

4. Two-way Action Structural Slabs and Plates

$$w_{max} = K\beta f_s\,(G_1)^{1/2}$$

where

$$G_1 = \frac{s_1 s_2 d_c}{d_{b1}}\frac{8}{\pi}$$

Values of coefficient K are presented in Table 3.

Some useful metric unit equivalents are presented below:

Customary Unit	*Metric Unit*
1 in.	25.4 mm
1 ft	0.305 m
1 in.^{2}	645.16 mm^2
1 in.^{3}	16 387.06 mm^3
1 in.^{4}	416.231 mm^4
1 psi	6.895 Pa
1 ksi	6.895 MPa
1 lb	4.448 N
1 kip	4448 N
1 lb/ft	14.594 N/m
1 kip/ft	14.594 kN/m
1 kip-in	113 N-m
1 $\sqrt{f'_c}$ psi	0.083036 $\sqrt{f'_c}$ MPa

TABLE 3 *K* VALUES FOR FULLY RESTRAINED SLABS AND PLATES

Slab/Plate Conditions[a]	K
Uniformly loaded, square	2.8×10^{-5}
At concentrated loads and columns	2.1×10^{-5}
$0.5 < l_s/l_l < 0.75$	2.1×10^{-5}
$l_s/l_l < 0.5$	0.6×10^{-5}

[a]For simply supported slabs multiply these values by 1.6. Interpolate K values for intermediate span ratios l_s/l_l or for partial restraints at the boundaries such as cases of end and corner panels of multipanel floor systems. l_s and l_l are the short and long spans of the two-way slab or plate, respectively.

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