Analysis of Energy Dissipation Caused by Snow Compaction During Displacement Plowing

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Energy dissipation during displacement plowing of snow caused by snow compaction in front of the plow is estimated. Depending on the speed of the plow, a plastic wave will form in front of the plow, thereby compressing the snow before removal. This compression wave dissipates energy provided by the prime mover. An approximate solution of the compression zone was developed to determine the significance of the energy dissipation caused by a plastic wave. The results indicate that energy losses may be substantial during high-speed plow operations. A parametric analysis of parameters involving plow geometry, operating speeds, and snow properties was conducted. Conditions under which energy losses caused by compression may be either reduced or eliminated are clearly identified.

The power requirements for displacement plowing of snow may be roughly divided into two categories: (a) power required to cast the snow off to the side, and (b) power dissipated as a result of snow compaction in front of the plow. The latter requirement has received considerably less attention in the literature as the primary objective of plowing snow is the actual removal of material from the roadway. Furthermore, these problems are fundamentally different in that the first is essentially a hydrodynamic problem in which the snow is in a fluidized state, whereas the second requires a detailed knowledge of high-rate mechanical properties of snow.

Power is dissipated when plowing snow as the result of compaction of the snow caused by a plastic (nonlinear) wave running out in front of the plow. The presence of a plastic wave in snow resulting from impact loading has been documented analytically and experimentally. Wakahama and Sato (*1*) conducted drop weight impact tests of snow using a 1-kg plate with impact velocities of 2.5 to 5 m/sec. High-speed photography, in conjunction with streak lines on the sample, was used to determine the wave speeds. The photography clearly indicates a nonlinear wave extending out from the impact plate. For instance, for an impact velocity of 4.3 m/sec, the plastic wave velocity was found to range from 6.5 m/sec for an initial snow density of 200 kg/m^3 to 12 $\pm$ 2 m/sec for an initial density of 400 kg/m^3.

On the other end of the loading spectrum, Napadenski (*2*) conducted stress wave experiments in snow by explosively loading flier plates into a snow sample. Results of these data show plastic wave speeds reaching as high as 170 m/sec. Brown (*3–5*) has done a substantial amount of theoretical work aimed at studying nonlinear waves in snow using a volumetric constitutive law. The theoretical results for wave speeds were found to be in agreement with data presented by Napadenski.

The dynamics of displacement plowing of snow can be considered similar to those of drop weight impact testing as well as explosively induced stress wave experiments. The main difference is that there is a continuous energy supply available to the driver plate, i.e., the plow itself. Furthermore, the operating speeds of a displacement plow will lie somewhere in between those of the low-velocity impact tests of Wakahama and Sato (*1*) and the high-rate explosive loadings of Napadenski (*2*). Therefore, it is reasonable to assume the existence of a plastic wave running in front of a displacement snow plow.

Under the assumption of a plastic wave does exist, estimating the amount of energy dissipated by compression of the snow as the wave passes through the material is desirable. For analysis of the compression zone, an analytical formulation has been developed on the basis of an assumed one-dimensional deformation.

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ANALYSIS OF THE COMPRESSION ZONE

Consider a displacement plow moving forward with velocity V_0 as shown in Figure 1. The region to be analyzed is taken from the compression zone boundary forward into the undisturbed snow. This region is defined by the Lagrangian (reference) coordinate X. Figure 1a is defined at time $t = 0$. Hence, the material in the positive X direction is in its undisturbed state.

At a later time, t^*, the plow has reached the snow particles originally located at $X = 0$, Figure 1b. The total zone of compression referenced to Lagrangian coordinates is given by X_c.

Modified Sheet Model

First, a modified sheet model analysis is performed to predict the forward pressure applied to the snow by the plow. The sheet model in general neglects the effects of snow compaction and surface friction or bonding. These assumptions produce a forward pressure on the plow given by

$$p = \rho_0 * V_0^2 [\cos^2 \beta(1 - \cos \phi)] \quad (1)$$

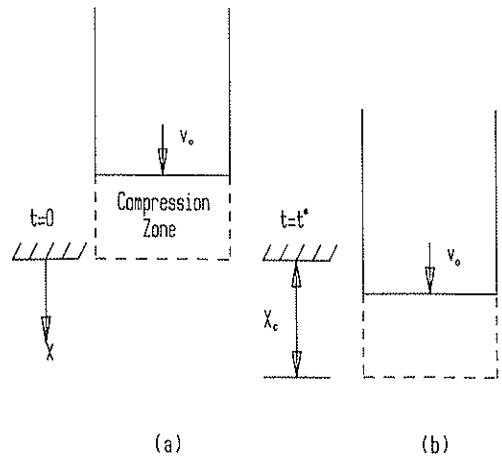

FIGURE 1 Schematic of the compression zone leading a displacement plow.

where β is the swivel angel, ϕ is the cutting angle, and ρ_0 is the initial density, as shown in Figure 2.

Now it is assumed that a compression wave is running in front of the plow, thereby compacting the snow before removal. The compression wave imparts a forward velocity to the snow before the arrival of the plow. The sheet model may still be used to predict the forward pressure acting on the plow by modifying the density and the velocity. Let V denote the velocity of the plow with respect to the snow after compression, and ρ the corresponding density of the snow. From conservation of mass,

$$\rho_0 V_0 = \rho V \tag{2}$$

Hence, allowing for compression, the velocity of the snow with respect to the plow is given by

$$V = (\rho_0/\rho) V_0 \tag{3}$$

On substitution of the relative velocity of Equation 3 and the corresponding density into Equation 1, the forward pressure predicted by the sheet model becomes

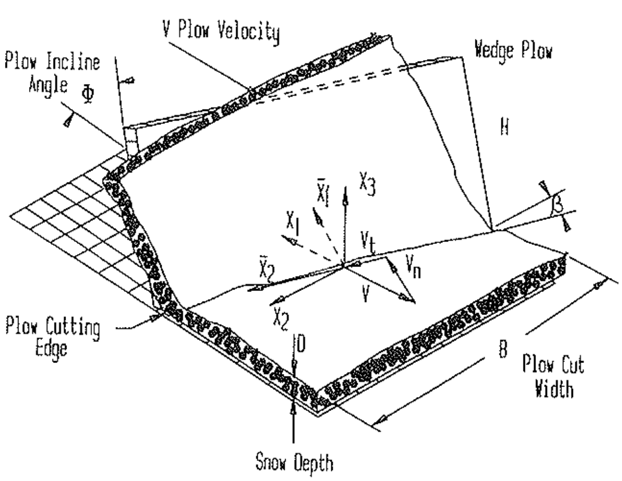

FIGURE 2 Simplified plow model.

$$p = \rho(\rho_0/\rho)^2 V_0^2 [\cos^2 \beta(1 - \cos \phi)] \tag{4}$$

Therefore, from Equation 4 for a given plow velocity, V_0, the forward pressure acting on the snow may be computed as a function of the density, as shown in Figure 3. In this figure, the forward pressure applied to the snow by the plow actually decreases with increasing density caused by a compression. This decrease may be attributed to the decrease in the relative velocity of the plow with respect to the snow.

Power Requirements for Compressing Snow

Now consider the power dissipated by compacting snow in the compression zone. The volumetric property of snow is expressed in terms of a dimensionless density ratio given by

$$\alpha = \rho_m/\rho \tag{5}$$

where ρ_m is the density of the matrix material (ice) and ρ is the density of the snow.

As shown in Figure 1a, the deformation is assumed to be strictly one dimensional. Hence, the Cartesian components of the deformation gradient are given by

$$F_{iJ} = \begin{vmatrix} \dfrac{\partial x}{\partial X} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{6}$$

where x represents the deformed coordinate and X represents the reference coordinate.

The Lagrangian form of conservation of mass is given by

$$\frac{\rho_0}{\rho} = \det \mathbf{F} = \frac{\partial x}{\partial X} \tag{7}$$

Equation 7 may be expressed in terms of the density ratio as

$$\frac{\alpha}{\alpha_0} = \frac{\partial x}{\partial X} \tag{8}$$

The stress power (power dissipation) per unit *undeformed* volume is given by $\mathbf{T} \cdot \cdot \dot{\mathbf{F}}$ where $\mathbf{T}$ is the First Piola-Kirchhoff stress tensor. This tensor is related to the Cauchy (engineer-

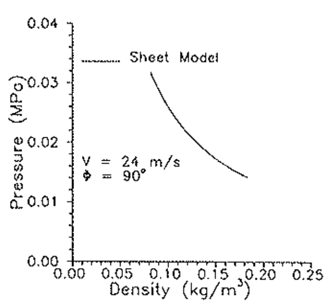

FIGURE 3 Pressure versus density at the cutting edge of a plow on the basis of a modified sheet model analysis.

ing) stress tensor by

$$\mathbf{T} = \frac{\alpha}{\alpha_0} \mathbf{F}^{-1} \mathbf{t} \tag{9}$$

where $\mathbf{t}$ is the Cauchy stress. Noting Equations 7 and 8, the components of $\mathbf{T}$ are

$$T_{ij} = \begin{vmatrix} t_{11} & t_{12} & t_{13} \\ \frac{\alpha}{\alpha_0} t_{12} & \frac{\alpha}{\alpha_0} t_{22} & \frac{\alpha}{\alpha_0} t_{23} \\ \frac{\alpha}{\alpha_0} t_{12} & \frac{\alpha}{\alpha_0} t_{22} & \frac{\alpha}{\alpha_0} t_{33} \end{vmatrix} \tag{10}$$

For the assumed deformation, the axial stress is to a good approximation equal to the hydrostatic pressure, p. Therefore, differentiating Equation 6 with respect to time and noting Equations 8 and 10, the stress power may be expressed as

$$P = p(\dot{\alpha}/\alpha_0) \tag{11}$$

The total energy dissipated per unit volume in compressing the snow is obtained by integrating the stress power over time, i.e.,

$$E = \int_0^t p \frac{\dot{\alpha}}{\alpha_0} ds \tag{12}$$

Finally, the power dissipated by the compression wave is obtained by multiplying Equation 12 by the volumetric flow rate of snow removal.

In order to close the equations, a volumetric constitutive law must be specified providing a relation between the pressure p and the density ratio α. The form taken is

$$p = F \ln (\dot{\alpha}/A_1) \tag{13}$$

where

$$F = \frac{C_1 \rho_0 (\alpha/\alpha_0)^{C_2 + C_3 \rho_0}}{B_1 \alpha}$$

and A_1, B_1, and C_i ($i = 1, 2, 3$) are constants. The constitutive law is a simplified version of a microphysical model developed by Brown (*6*) and works well for snow of low density.

Approximate Solution

An exact solution of the energy dissipation given by Equation 12 requires knowledge of the volumetric rate of deformation $\dot{\alpha}$ as the pressure wave advances through the material. This information may only be determined by carrying out a complete solution of the wave propagation problem in the compression zone. Such an analysis is currently in progress and is based on a two-dimensional finite difference wave propagation code for finite deformation of snow.

In order to obtain some immediate quantitative results, an approximate solution has been formulated by assuming a constant value for the volumetric rate of deformation. In reality, $\dot{\alpha}$ varies in a continuous fashion as the compression wave advances. Fortunately, the constitutive behavior of snow is only weakly rate dependent, as demonstrated by Equation 13 in which the pressure is shown to vary with the natural log of $\dot{\alpha}$. As a result, an order of magnitude error in the value of $\dot{\alpha}$ through the compression zone results in only a 10 percent difference in the total energy dissipated in the approximate solution. Hence, the results predicted by assuming $\dot{\alpha}$ is constant throughout the deformation are felt to be a good measure of the exact analytical solution.

The approximate solution of the compression problem is begun by assuming values for the plow velocity, plow cutting angle, and initial snow density. The modified sheet model is then used to determine a curve for pressure versus density similar to that shown in Figure 3. Next, a curve for pressure versus density predicted by the constitutive law is generated using Equation 13 and superimposed on the sheet model analysis, as shown in Figure 4. The energy dissipated per unit volume is then determined from Equation 12, in which the integration is carried out until the pressure and density predicted by the constitutive law agree with the corresponding values predicted by the modified sheet model. These values are determined by the crossover point shown in Figure 4.

The rate of deformation $\dot{\alpha}$ was determined by assuming

$$\dot{\alpha} = \alpha_0 \dot{g} \tag{14}$$

where $\dot{g}$ is the ratio of the plow velocity divided by the length X_c of the compression zone. Physically, Equation 14 corresponds to the deformation rate for a spatially homogeneous uniaxial compression test. The length X_c of the compression zone was taken to be 0.5 m for all calculations; this assumption in turn fixes $\dot{\alpha}$, because of Equation 14. As stated previously, the assumed value for $\dot{\alpha}$ has little effect on the energy dissipation.

There are conditions under which the two curves shown in Figure 4 do not intersect. This situation occurs when the initial pressure predicted by the constitutive law is greater than that of the sheet model. Therefore, the critical pressure needed to cause compression is not achieved and the snow behaves in a sheet-like manner during removal.

RESULTS

In what follows, estimates of power requirements for plowing snow caused by the compression wave leading the plow are

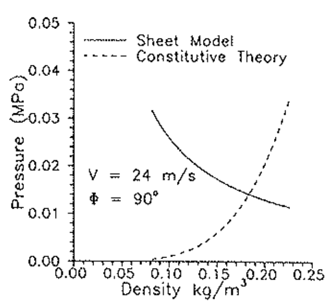

FIGURE 4 Pressure versus density as predicted by the sheet model and the constitutive law.

described. Power is expressed in units of horsepower, the most familiar unit applied to vehicles. Parameters that are varied are limited to the initial density, velocity, and plow cutting angle.

In the following data, the plow width was assumed to be 3.33 m with a swivel angle of β = 33 degrees, Figure 2. The plow velocities considered were allowed to vary from 9 to 30 m/sec (20 to 66 mph). This range covered the entire spectrum of low-speed city plowing up to high-speed highway operations. The snow depth is assumed to be 15 cm. A range of initial snow densities was considered from 60 to 160 kg/m^3. This range represents low densities that may be encountered in a fresh mountain snowfall up to higher values associated with wet snow or snow that has been allowed to sinter for some time. For instance, on February 13, 1990, approximately 20 cm of snow fell at Laramie, Wyoming. Laramie is located at an elevation of 2190 m and has an extremely dry climate. Density measurements taken for the snowfall were found to range from 76 to 108 kg/m^3 for three different samples.

Figure 5 shows a plot of horsepower versus velocity for an initial density of 60 kg/m^3 and cutting angles of 90 and 70 degrees, respectively. The figure shows that for either case, the power lost to compression is negligible for velocities under 9 m/sec (20 mph). However, as the velocity increases beyond 9 m/sec the power lost to compression increases significantly. For instance, at 24 m/sec (54 mph) the total power dissipated reaches 17 hp for the vertical cutting angle. The lost power represents a significant portion of the total power available to the prime mover. The figure also shows a reduction in the energy dissipation is achieved by laying the cutting angle back to 70 degrees. For instance, the horsepower drops from 17 to 13 hp at 24 m/sec, representing a reduction of 24 percent in the power requirements for these particular high-speed plowing conditions.

Figures 6–8 show plots similar to Figure 5 for initial densities of 80, 120, and 160 kg/m^3. The figures indicate trends similar to those of Figure 5. In general, there is little or no energy dissipated for plowing speeds up to 9 m/sec (20 mph). Also, the benefits from the reduced cutting angle become more significant as the initial density increases. For instance, at 24 m/sec the power decreases from 17 to 13 hp for low-density snow of 60 kg/m^3. In contrast, for an initial density of 160 kg/m^3, the power drops from 57 to 41 hp, representing a reduction in energy requirements of nearly 28 percent.

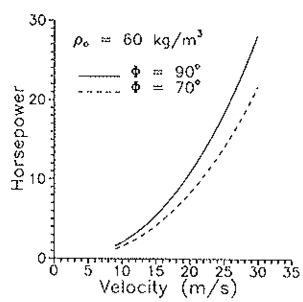

FIGURE 5 Horsepower versus velocity for cutting angles of 70 and 90 degrees and an initial density of 60 kg/m^3.

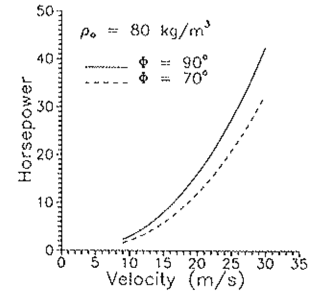

FIGURE 6 Horsepower versus velocity for cutting angles of 70 and 90 degrees and an initial density of 80 kg/m^3.

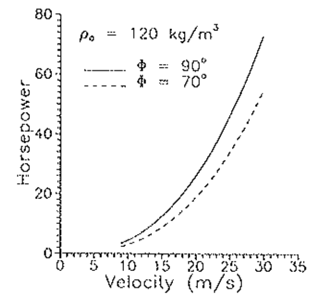

FIGURE 7 Horsepower versus velocity for cutting angles of 70 and 90 degrees and an initial density of 120 kg/m^3.

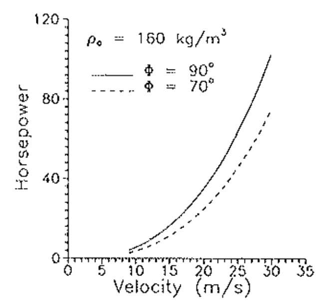

FIGURE 8 Horsepower versus velocity for cutting angles of 70 and 90 degrees and an initial density of 160 kg/m^3.

Figure 9 shows a parametric plot of power requirements versus velocity for initial densities of 60, 80, 120, and 160 kg/m^3, respectively. The figure dramatically shows the effect of initial density on the energy requirements.

Figure 10 shows a parametric plot of power versus velocity for cutting angles ranging from 50 to 90 degrees at an initial density of 80 kg/m^3. The results indicate that significant energy reductions are possible by laying the plow angle back to 50 degrees. For example, at 24 m/sec (54 mph) the power consumed by compression decreases from 25 hp for ϕ = 90

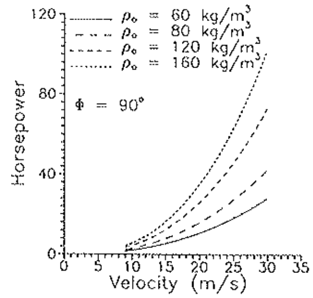

FIGURE 9 Parametric plot of horsepower consumed by compression versus velocity for various initial densities.

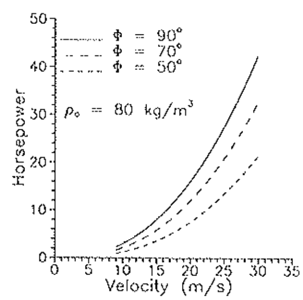

FIGURE 10 Parametric plot of horsepower consumed by compression versus velocity for various cutting angles.

degrees to 12 hp for ϕ = 50 degrees, representing a 52 percent reduction in energy dissipation.

Figure 11 shows a plot of the final density versus velocity for initial densities ranging from 60 to 160 kg/m^3. The density changes shown in the figure represent measurable changes caused by the advancing compression wave. Hence, it may be possible to verify these changes experimentally.

CONCLUSION

The results of the approximate solution discussed in the previous section indicate that energy dissipation caused by a compression wave is negligible at speeds below 9 m/sec (20 mph). However, energy dissipation may be significant at higher speeds typically found in highway plow operations.

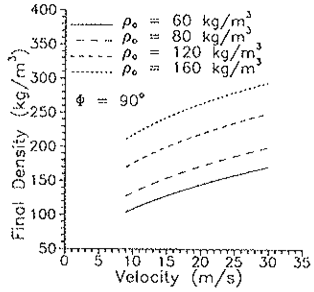

FIGURE 11 Parametric representation of the final density as a function of velocity for various initial densities.

The significant parameters affecting energy dissipation caused by compression are the initial snow density, plow cutting angle, and plow velocity. Of these parameters, only the cutting angle is readily varied without affecting plow operations. The data suggest that by laying the cutting angle back, significant reductions in energy dissipation are possible. Furthermore, in addition to energy dissipation, the compression wave leading the plow adversely effects the cast distance as the exit velocity of the snow is reduced.

The primary adverse effect of laying the cutting angle back lies in safety. In particular, proper tripping of the blade when the plow encounters hidden obstacles becomes increasingly difficult as the angle is laid further back.

A similar analysis is currently underway to estimate compression losses for plowing snow that has been precompacted by vehicle traffic. The primary variable that must be changed in the analysis is the initial density. For this case, densities typically will lie in the range of 350 to 450 kg/m^3. For instance, initial densities and compressed densities were taken for a snowfall in Laramie, Wyoming, on February 13, 1990. The data for the vehicle-compacted snow ranged from 354 to 400 kg/m^3. The densities resulting from vehicle compaction are consistent with results from plate sinkage tests for a deep snow cover obtained by Hansen (*7*).

Work is continuing on developing a fundamentally sound experimental procedure to measure the density changes caused by a compression wave leading the plow. An approach currently being investigated is to use high-speed photography, in conjunction with streak lines placed on the snow, to observe the advancing compression wave. The density and pressure changes caused by the wave may then be determined from conservation of mass and momentum principles in the form of the Rankine-Hugoniot relations (*1*).

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