New Method of Time-Dependent Analysis for Interaction of Soil and Large-Diameter Flexible Pipe

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Design equations have been developed to predict the pre-yield deflections, stresses, and strains in buried flexible plastic pipes over time. The solutions consider the effects of creep in the pipe material and the surrounding soil and backfill, the water table, arching, and variable bedding conditions. These equations are obtained by regression analysis, and results are generated using a finite element program. The design equations predict pipe deflections that are consistent with those obtained in the field over a period of time. It is shown that the arching of soil surounding a pipe can be quantified to further appreciate its cause and effects. The ratio of the pipe's vertical deflection to its horizontal deflection is shown to be an ambiguous way of defining the structural integrity of nonrigid pipes. Strain level may be a better indicator of the structural integrity of the pipe than pipe deflection, because it considers both the bending moments and the thrust in the pipe wall and can be measured against the allowable strain for that particular pipe material. Vertical pipe deflections predicted by the design equations for different depths of cover as well as for different time periods are shown to match field measurements well.

Modern cities require underground pipelines to provide essential utilities, such as wastewater disposal, potable water, and gas. In recent years there has been a steady increase in the use of flexible pipes as buried conduits despite the fact that much is not understood of soil-pipe interaction, especially their time-dependent behavior. The most common type of flexible pipe is plastic pipe.

Plastic pipes can generally be classified as thermosetting plastic or thermoplastic. The first type uses materials such as glass or sand embedded in a plastic binder. Examples are reinforced thermosetting resin (RTR) pipe, commonly called FRP or GRP (fiber-reinforced plastic or glass-reinforced plastic pipe), and reinforced plastic mortar (RPM) pipe. Thermoplastic pipes are made from materials such as polyvinyl chloride (PVC), high density polyethylene (HDPE), and acrylonitrile butadiene styrene (ABS).

All materials are known to experience a reduction in stiffness with time under an applied load. The reduction in stiffness is usually referred to as relaxation. This property is pronounced for plastic pipe, although it is less obvious in concrete and most metallic pipes. Hence in the design and use of plastic pipes, the ability to predict the effects of relaxation of the pipe and soil on the soil-pipe system is an important consideration.

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The use of flexible pipes toward the middle of the century prompted the development of design procedures. One of the most widely used is Spangler's equation (*1*, pp. 368–369). Time-dependent solutions were attempted rather crudely by using a lag factor to increase deflection with time. A new design procedure that has been developed to predict preyield deflections, stresses, and strains in buried flexible pipes over time is presented. The design equations are obtained by regression analysis, and results are generated by a nonlinear finite element program and are shown to match measured field data well.

ENGINEERING PRACTICE AND CONSIDERATIONS

Background

The soil supports the load above a flexible pipe when it is allowed to deflect and hence generate enough thrust in the soil elements to form an arch. However, a rigid pipe bears more of the load itself as the soil relaxes around it. In the study of soil-pipe interaction, especially soil interaction with flexible plastic pipe, an understanding of the factors influencing the arching of the soil surrounding the pipe is a major objective.

Modeling the Soil-Pipe System

The various ways of modeling the three major components of a soil-pipe system are the following:

1. Trench model: Flexible pipes are usually buried in properly prepared trenches. In a design analysis, there are three distinct soil zones: the in situ soil, which remains undisturbed; the embedment soil of selected and properly compacted fill, which is in contact with the pipe and includes the bedding; and the backfill, which is the disturbed or remolded native soil dumped and nominally compacted above the pipe. Figure 1 shows a typical configuration of a trench. In a proper analysis, each soil zone should be assigned its distinctly different soil properties.
2. Soil model: Design procedures currently in use have linearly elastic soils, nonlinearly elastic soils, and viscoelastic soils. For elastic soils, it is assumed that the stress state will return to the initial state on unloading. In the case of linear elasticity, a linear path is assumed. A nonlinear elastic model

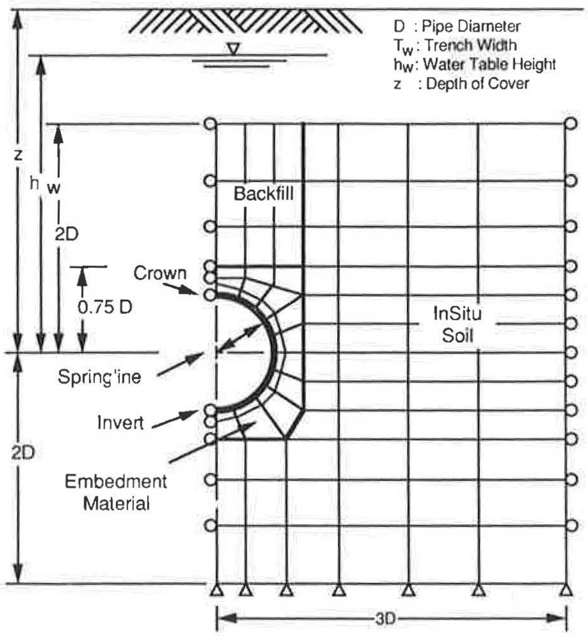

FIGURE 1 Typical configuration of a pipe trench.

that is normally used is the hyperbolic stress-strain model, as reported by Kondner (*2*) and Janbu (*3*) and subsequently used extensively in engineering applications by Duncan et al. (*4*). The use of a nonlinear elastic model allows an unloading-reloading stress path to differ from the loading stress path, hence creating a different soil modulus for a soil that has been remolded and loaded and an undisturbed soil that has undergone unloading and then reloading. In view of this, for a pipe buried in a trench, one should consider assigning the undisturbed in situ soil a larger soil modulus than the disturbed backfill when it is used in an analysis. A viscoelastic model allows properties such as the soil modulus to be time and stress-history dependent.

3. Pipe model: Buried pipes are available with smooth-wall or profile-wall cross sections. The purpose of profile-wall pipes is to achieve a higher stiffness-to-weight ratio than that of smooth-wall pipes. In modeling the material modulus of the pipe, the usual approach is to assume a linearly elastic model. However, because most flexible pipe materials are polymeric, the viscoelastic approach is more appropriate.

FORMULATION OF THE DESIGN EQUATIONS

The design equations were developed for the analysis of a flexible pipe buried in a trench of any width, with or without the presence of groundwater. The hyperbolic stress-strain model was assumed for soils in the three zones (see Figure 1). The design solutions were obtained from a factorial study using CANDE (*5*) (a nonlinear finite element code) to generate a data base of some 720 cases. Before the factorial analysis, the more influential parameters and variables in the soil-pipe system were determined. They were pipe stiffness, which takes into account the size and the material properties of the pipe; the properties of the embedment, the backfill, and the native soil; the depth of cover; soil arching; trench width; and the presence of groundwater. Subsequently, the design equations obtained from the regression analysis were verified by using several sets of field measurements supplied by a pipe manufacturer (*6*) and from the literature (*7*). Predictions that can be obtained using the design equations include (*a*) the pipe vertical deflection with or without groundwater; (*b*) the ratio of the pipe vertical deflection to its horizontal deflection; (*c*) the soil vertical and lateral stress at the springline; (*d*) the soil support modulus, which is assumed to be represented by the soil modulus taken at the springline; (*e*) the bending moment of the pipe wall at the crown; (*f*) the thrust in the pipe wall at the crown; and (*g*) the strain in the pipe wall at the crown, which can be reasonably assumed to be the maximum pipe strain. The elastic design equations were then transformed into a viscoelastic form, which gives these results as a function of time as well.

The following sections present the design equations, which are also used in a microcomputer program called TAMPIPE (Texas *A&M PIPE*).

DESIGN EQUATIONS

Pipe Vertical Deflection

It was initially thought that the equation for the pipe vertical deflection would take the form of Hoeg's equation (*8*). The pipe vertical deflection expressed as a ratio to the average pipe diameter is given by Hoeg as

$$\frac{\Delta D}{D} = \frac{\dfrac{1 - \nu_e}{3(3 - 4\nu_e)} w(1 - k)}{\dfrac{8E_p I_p}{(1 - \nu_p^2)D^3} + \dfrac{(3 - 2\nu_e)(1 - 2\nu_e)E'}{12(3 - 4\nu_e)(1 - \nu_e)}} \tag{1}$$

where

w = uniformly distributed load above the pipe,
k = ratio of the lateral to the vertical loading,
ν_e = Poisson's ratio of the elastic medium,
ν_p = Poisson's ratio of the pipe,
E_p = the elastic modulus,
I_p = the moment of inertia of the pipe wall,
D = the pipe diameter, and
E' = the soil modulus.

It is interesting to note that if the Poisson's ratio of the soil, ν_e, is taken to be 0.315, and the pipe is assumed to have a Poisson's ratio of 0.0, the equation becomes

$$\frac{\Delta D}{D} = \frac{0.131w(1 - k)}{8E_pI_p/D^3 + 0.061E'} \tag{2}$$

which is almost Spangler's equation without the lag factor. It appears that the soil used to obtain Spangler's empirical values had a Poisson's ratio of 0.315 and that the bedding constant in the numerator may well be a function of the Poisson's ratio of the soil. The exclusion of a Poisson's ratio for the pipe material may explain why Spangler's equation is inadequate in modeling buried pipes with very low stiffnesses.

The design equation that has been developed to describe the pipe vertical deflection follows the same form as Equation 1:

$$\frac{\Delta D}{D} = \frac{\dfrac{(1 - \nu_e)}{3(3 - 4\nu_e)}(1 - A_f)\gamma z W_f}{\dfrac{8E_p I_p}{(1 - \nu_p^2)D^3} + \dfrac{(3 - 2\nu_e)(1 - 2\nu_e)E'}{12(3 - 4\nu_e)(1 - \nu_e)}} \tag{3}$$

where

A_f = factor representing the amount of arching,
γ = the unit weight of the soil,
z = the depth of cover to the springline, and
W_f = a factor to correct for the presence of a water table.

This is the form of the model found in TAMPIPE. The soil support modulus E' is the secant modulus of the embedment soil at the springline, given by

$$E' = \left[1 - \frac{R_{fe}(1 - \sin \phi_e)\sigma_x}{(2c_e \cos \phi_e + 2\sigma_x \sin \phi_e)}\right] K_e P_a \left[\frac{(\sigma_y)}{P_a}\right]^{ne} \tag{4}$$

where

c_e = cohesion of the embedment soil,
ϕ_e = angle of shearing resistance of the embedment soil,
K_e = soil modulus number (*4*),
n_e = modulus exponent (*4*),
R_{fe} = failure ratio (*4*),
σ_x = horizontal earth pressure at the springline, and
σ_y = vertical earth pressure at the springline and is approximately equal to the minor principal stress, σ_3.

This stress is represented by the unit weight of the backfill and the embedment soil above the pipe; the depth of cover measured to the springline; the pore water pressure; the pipe stiffness; and the modulus number of the backfill, the embedment, and the native soil. This value can be expressed by

$$\sigma_y = \frac{\gamma z/(144 \times 12)}{C_{f1} \cdot C_{f2} \cdot C_{f3}} \tag{5}$$

where

$$C_{f1} = 1.2 + (5.8 \times 10^{-2} + 4.58 \times 10^{-3} \times 8E_pI_p/D^3 \times \exp[(4.3 \times 10^{-4} - 9.0 \times 10^{-6} \times 8E_pI_p/D^3) \times 14.7K_r]$$

and

$C_{f1} \geq 1.2$
$C_{f2} = 1 + 1.5p_w^{1.42}$
p_w = pore water pressure
$C_{f3} = 1.7588 \times \exp(-1.75 \times 10^{-3} K_r) \times 6.9453 \times \exp(2.10 \times 10^{-3}K_r) \times K_e^{-0.41 \times \exp(6.91 \times 10^{-4}K_r)}$

In this case, K_r refers to a representative soil exponent number that takes into consideration the surrounding soil zones and the trench width and is given by

$$K_r = K_r'[1.0 + (T_w/D - 1.5)(1.1082 + .0016K_e)]$$

$$K_r \geq 0 \tag{6}$$

where

$K_r' = -128.7675 + 1.004K_b + 42K_i/K_b$
$K_r' \geq 0$

σ_x is the lateral earth pressure at the springline and is approximately equal to the major principal stress, σ_1. This value is obtained by multiplying σ_y by a lateral earth pressure coefficient, K_o, which again is a function of the modulus numbers of the soils in the three zones, the pipe stiffness, and the pore water pressure:

$$K_o = (1 - 2.96 \times 10^{-2}K_r^{0.47}p_w) \times (9.3488K_e^{-0.44}K_r^2) \times (1.0122 + 7.11 \times 10^{-4}K_r - 3.4 \times 10^{-7}K_r^2) \times \exp[8E_pI_p/D^3 \times (-0.019048 + 2.28 \times 10^{-5}K_r - 1.14 \times 10^{-8}K_r^2)] \tag{7}$$

Factors Influencing Pipe Deflection

This section indicates how the design equations can be used to analyze installation cases and how the variables affect pipe deflections, using high-density polyethylene pipes as examples.

Pipe Stiffness

This term ($PS = 8E_pI_p/D^3$) is a function of the elastic modulus or relaxation modulus, the diameter of the pipe, and the moment of inertia of the pipe wall. Figure 2 shows the reduction in pipe deflection when the stiffness of a 48-in.-diameter pipe is increased from 1.2 to 4.0 psi. The deflection of an 18-in.-diameter pipe (PS = 11.9 psi) is also shown. The three pipes were installed in a soft native soil [modulus number (K) equal to 68] with the embedment soil compacted to 85 percent Proctor.

Soil Stiffness

Because the soil modulus (E') is stress dependent, the knowledge of the state of stress of the soil around the pipe is of

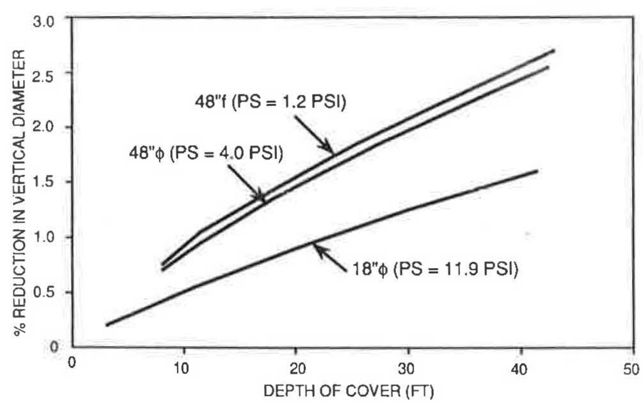

FIGURE 2 Vertical deflections for pipes of different stiffnesses.

great importance. A soil-pipe system with a soft backfill soil (K = 68) results in a higher stress level around the pipe and hence a larger soil modulus. In stiff backfill (K = 1,100), the stress level of the soil elements around the pipe is lower, leading to a lower soil modulus (but the imposed load on the pipe is also smaller). Figure 3 shows how pipe deflections can be reduced by increasing the degree of compaction on the embedment soil. Figure 3 also shows the exceptionally high deflection of a pipe backfilled with soft native soil only, that is, without any bedding material. The soil modulus resulting from the interaction between the different types of native soil and the different degrees of compaction of the embedment soil is shown in Figure 4.

Soil Arching

The degree of soil arching is described by the term A_f, which can take on values ranging from 1.0 to negative values. This term is given by

$$A_f = [1 - (1 - A_{fo})A_{fc}] \tag{8}$$

where

$$A_{fo} = [1 + (K_r - 622.7) \times 4.86 \times 10^{-4}]^{-1}$$

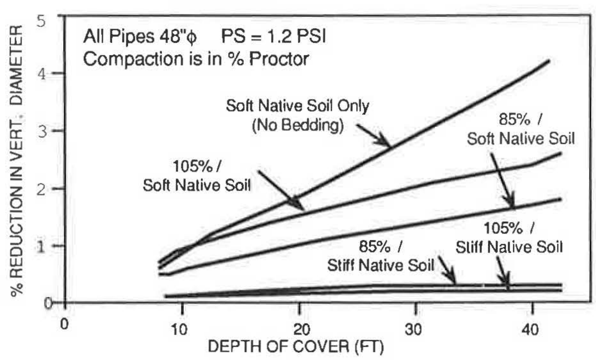

FIGURE 3 Vertical deflections of pipes in different soils.

and

$$A_{fc} = 0.9054 - 1.07 \times 10^{-2}T_w + (8.18 \times 10^{-5} + 6.91 \times 10^{-6}T_w)K_r - [9.61 \times 10^{-5} + 1.30 \times 10^{-5}T_w - (6.75 \times 10^{-8} + 7.32 \times 10^{-9}T_w)K_r]E'$$

Figure 5 shows the arching values that can be obtained for various degrees of compaction in different native soils. It can be seen that in stiff native soil (K = 1,100), the arching value is close to 1.0, indicating that little of the imposed load from above the pipe is transmitted to the pipe. For the pipe surrounded by embedment material of slight compaction and buried in a soft native soil (K = 68), the arching factor is about 0.6. For a highly compacted embedment fill in a soft native soil, the arching factor falls below zero, indicating that the pipe must bear more load than the overburden pressure. It can be seen from the figure that the degree of arching tends to be constant after some depth. It can be shown that a larger trench width reduces soil arching.

Trench Width

When using a flexible pipe, generally, a trench width of about 1½ times the pipe diameter is preferred. It is not economical to overexcavate the trench, but sufficient clearance on both sides of the pipe is required to allow for proper compaction of the fill. As indicated earlier, an increase in the trench width will weaken the arch to be formed. To offset this, the application of a higher degree of compaction to the embedment material is required. In soft native soils, a better way to improve arching and hence reduce deflections is to increase the degree of compaction and not the trench width alone, unless soil exchange is the aim, as when organic soil or marine clay is encountered.

Groundwater

The effects of the presence of a hydrostatic load on the pipe exterior is a highly complicated consideration. In the design equation, W_f is the factor by which the percent pipe deflection for the dry case is multiplied to obtain the resultant deflection.

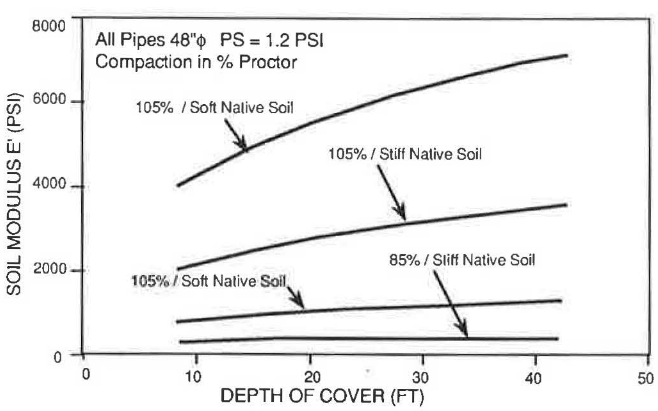

FIGURE 4 Soil moduli for different depths of cover.

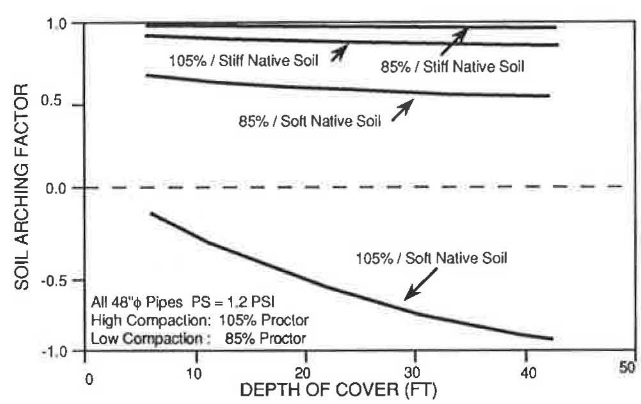

FIGURE 5 Arching factor of a soil-pipe system in different soils.

It is given by

$$W_f = 1 - 0.6718z_w + 2.83 \times 10^{-3}z_w K_r - 4.56 \times 10^6 \times K_r^2 z_w + 0.2520z_w^2 - 3.66 \times 10^{-3}K_r z_w^2 \times 7.84 \times 10^{-6}K_r^2 z_w^2 \quad (9)$$

where z_w is the ratio of the water table height above the springline to the depth of cover. As can be seen from Figure 6, the presence of groundwater will increase deflections only beyond a specific head, which varies with soil modulus. This is probably because beyond this point, the hydrostatic load increases faster than the support offered by the soil for the different depths of cover.

Ratio of Pipe Vertical to Horizontal Deflection

Pipe design engineers have been acutely interested in determining the ratio of the pipe vertical deflection, D_v (an absolute value), to the horizontal deflection, D_h, commonly referred to as the D_v/D_h ratio. This is because it has often been inaccurately presented that all pipes will approach failure if they do not conform to the ellipitical shape, where the D_v/D_h or D_h/D_v ratio is unity. The form of the regression equation that was obtained for the D_h/D_v ratio is

$$D_h/D_v = 1 - D_v/(A_o + D_v) \quad (10)$$

where A_o is a function of the pipe stiffness and the modulus numbers of the soil in the three zones. A regression equation has been obtained for A_o. Figure 7 shows the variation of the D_h/D_v ratio with respect to the pipe vertical deflection for pipes of various stiffnesses. It can be seen that a D_v/D_h ratio is of little value if the pipe deflection is not given at the same time. The horizontal line with the intercept at unity shows the relationship that can be expected for the D_h/D_v ratio and the pipe vertical deflection for a perfectly rigid pipe.

Bending Moment, Thrust, and Strain in the Pipe Wall

The bending moment at the crown (which can be assumed to represent the maximum bending moment) can be expressed

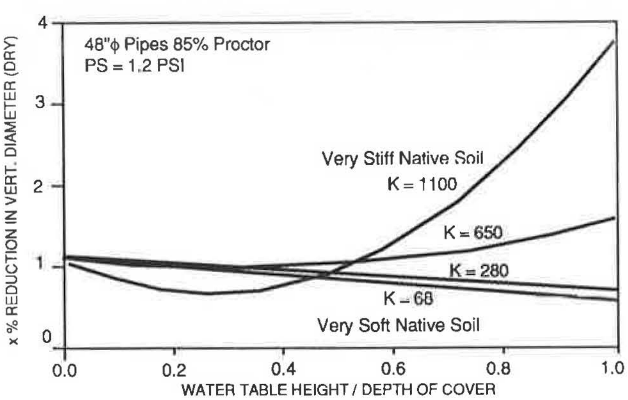

FIGURE 6 Effects of groundwater on pipe deflection.

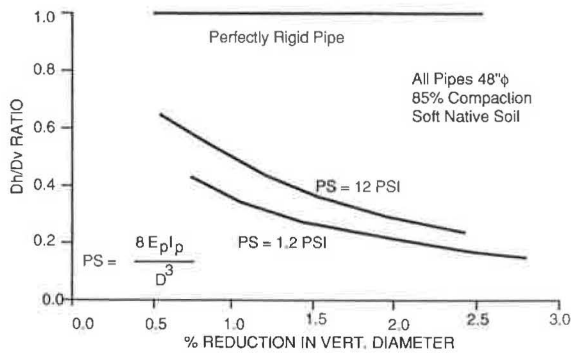

FIGURE 7 Ratio of pipe horizontal deflection to vertical deflection.

as

$$M = D_f 4E_p I_p(\Delta D/D)/D \quad (11)$$

where D_f is the deformation factor, which is given by

$$D_f = M_{c1} \times M_{c2}$$

where

$$\begin{aligned} M_{c1} = {} & (0.2667T_w - 0.6) \times [3.2227 + 5.7 \times 10^{-3}K_r \\ & - 5.3 \times 10^{-6}K_r^2 - (4.4 \times 10^{-3} + 5.8 \times 10^{-6}K_r \\ & - 6.8 \times 10^{-9}K_r^2)K_e] - (0.2667T_w - 1.6) \\ & \times [1.8809 + 1.1 \times 10^{-3}K_r - 7.8 \times 10^{-7}K_r^2 \\ & - (1.4 \times 10^{-3} - 5.5 \times 10^{-7}K_r)K_e] \\ M_{c2} = {} & (0.2667T_w - 0.6) \times [1.3690 - 2.1 \times 10^{-3}K_r \\ & + 1.7 \times 10^{-6}K_r^2 - 2.7 \times 10^{-3}K_e + 1.6 \\ & \times 10^{-5}K_r K_e - 1.3 \times 10^{-8}K_r^2 K_e + 3.5 \times 10^{-3}K_e^2 \\ & - 2.0 \times 10^{-8} \times K_r K_e^2 + 1.6 \times 10^{-11}K_r^2 K_e^2 \\ & + (1.94 \times 10^{-2}K_r - 2.6 \times 10^{-5}K_r^2 - 3.9 \\ & \times 10^{-3}K_e - 1.3 \times 10^{-4}K_r K_e + 1.7 \times 10^{-7}K_r^2 K_e \\ & + 2.9 \times 10^{-5}K_e^2 + 9.9 \times 10^{-8} \times K_r K_e^2 + 1.8 \\ & \times 10^{\ 10}K_r^2 K_e^2)p_w + (-2.4 \times 10^{-2}K_r + 3.3 \\ & \times 10^{-5}\ K_r^2 + 4.9 \times 10^{-4}K_e + 1.7 \times 10^{-4}K_r K_e \\ & - 2.4 \times 10^{-7}K_r^2 K_e - 2.5 \times 10^{-5}K_e^2 - 1.3 \\ & \times 10^{-7}K_r K_e^2 + 2.4 \times 10^{-10}K_r^2 K_e^2)P_w^2] \end{aligned}$$

The thrust at the pipe crown is given by

$$T = (C_1\sigma_x + C_2 p_w \gamma_w z)D/2 \quad (12)$$

where

γ_w = the unit weight of water,
C_1 = 0.7285, and
C_2 = 0.9145.

The maximum pipe wall strain that can be assumed to occur at the crown is defined from the corresponding bending moment and thrust and is given by

$$\varepsilon - \frac{Mc}{2E_p I_p} + \frac{T}{A_p E_p} \quad (13)$$

where c is the distance to the outer fiber from the neutral axis. Figure 8 shows the variation of the strain at the crown at different levels of vertical deflection of a 48-in.-diameter

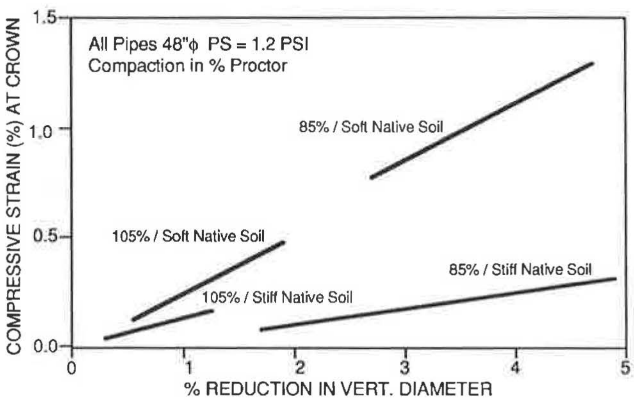

FIGURE 8 Pipe strains at crown for different in situ soils.

(PS = 1.2 psi) HDPE pipe for various installation cases after 1 year. This suggests that at, say, a 5 percent reduction in vertical diameter, the pipe material may or may not have reached yield, implying that a strain criterion rather than a deflection criterion may be more appropriate as a measure of the structural adequacy of a buried pipe. If the yield strain is to be used as the criterion, different types of pipes will be allowed to have different levels of pipe deflections, because the yield strain is different.

TIME-DEPENDENT DESIGN EQUATIONS

Viscoelastic Solutions

To model the elastic modulus as a function of time, the power law is used. It has been used on different types of plastics (*9*, pp. 201–221) as well as for rocks (*10*, pp. 293–301) and for soils (*11,12*). In the power law formulation, the relaxation modulus is given by

$$E(t) = E_1 t^{-m} \tag{14}$$

where E_1 and m are constants peculiar to the material.

The constitutive equation used to describe the stress-strain relationship of nonaging viscoelastic material is given by the following convolution integral:

$$E(t) = \int_{-\infty}^{t} E(t - \tau)\frac{\partial \varepsilon}{\partial \tau}\, d\tau \tag{15}$$

where $E(t - \tau)$ is the relaxation modulus as a function of time t and of the time when the input is applied, τ. The symbol ε refers to the step function strain input.

Biot (*13*) showed that the operational moduli of viscoelastic solutions can be manipulated algebraically as elastic moduli, hence establishing the correspondence rule by which "the classical theory of elasticity may be immediately extended to viscoelasticity by simply replacing their corresponding operators." Lee (*14*) showed that the time variable in the viscoelastic solution can be removed by applying the Laplace transform (LT), thus enabling it to be expressed in terms of an associated elastic problem. This is done by taking the LT of the governing field and boundary expressions with respect to time. The resulting expression is the LT of the viscoelastic solution. Taking the inverse LT of the resulting expression produces the desired result, which is the viscoelastic solution.

Consider a time-dependent response $u(t)$. Using the approximate method of the inversion of the LT as proposed by Schapery (*15*), the viscoelastic solution is given by

$$u(t) = [s\bar{u}(s)]_{s=\beta/t} \tag{16}$$

where s is the variable of integration in the LT. For the case in which the slope of the logarithm of the response versus log (t) is small ($-0.3 \leq m \leq 0.1$), $\beta = 1/2$. This method had been shown to be very accurate (*16*) in approximating the rigorously determined LT. A discussion of this method of time-dependent analysis using elastic solutions for linear and nonlinear materials can be found elsewhere (*17*).

The viscoelastic form of the design equations was developed by using the correspondence principle with the approximate method of the inversion of the LT.

The Laplace-transformed time-dependent pipe vertical deflection is given by

$$\mathcal{L}\left\{\frac{\Delta D}{D}\right\} = \frac{1}{s} \cdot \frac{B_f(1 - A_f)\gamma z W_f}{8E_pI_p/(1 - \nu^2)D^3 + S_cE'} \tag{17}$$

where the Carson transforms of the pipe relaxation modulus as a power law, $E_p(t) = E_{p1}t^{-m_p}$, and for the soil relaxation modulus, $E'(t) = E_1't^{-m_s}$, are as follows

$$\bar{E}' = s_{ms}E_1'\Gamma(1 - m_s) \tag{18}$$

$$\bar{E}_p = s_{mp}E_{1p}\Gamma(1 - m_p) \tag{19}$$

Symbols are as defined earlier with the appropriate subscript to show the material that is described.

In the trench condition where there are three different soil zones, the value of m_s can be estimated using the following regression equation:

$$m_s = \{1 - \exp[-0.47 \times (T_w/D)^{0.31}]\} \times m_e + \exp[-0.47 \times (T_w/D)^{0.31}] \times m_i \tag{20}$$

where m_e and m_i are power law exponents for the embedment and in situ soil, respectively.

The Laplace-transformed time-dependent bending moment in the pipe wall is given by

$$\mathcal{L}\{M\} = 4D_f\bar{E}_pI_p(\Delta D/D)/sD \tag{21}$$

In the preceding development, it was assumed that the parameters related to the loading term, A_f and W_f, are independent of time. This may not be strictly true, but in view of the lack of evidence to the contrary, and because their influence is not critical, they were assumed to be constant. For practical reasons, the Poisson's ratios of the soils and the pipe material were also taken to be constants.

The following sections will discuss results obtained using the viscoelastic solutions and may help explain time-dependent soil-pipe interaction.

TABLE 1 EXPONENTS OF RELAXATION POWER LAW FOR SOILS AND PIPE MATERIALS

Descriptions and m values		Remarks
Allenfarm (ML)	0.106	Texas Soil at Optimum
Moscow (CH)	0.101	Moisture Content
Floydada (CL)	0.079	(18)
Mississippi Delta (CH)	0.082 to 0.104	5 Samples (19)
Louisiana Coast (MH)	0.029 to 0.104	8 Samples (20)
Haney Clay N.C.	0.300 to 0.600	(11)
Seattle Clay O.C.	0.500	(21)
Redwood City Clay	0.250	
Osaka Clay	0.000	
Tonegaw Loam	0.200	
Bangkok Mud	0.200	
Concrete	0.028	
High Density Polyethylene	0.098	
Polyvinyl Chloride	0.031	
Reinforced Plastic Mortar	0.048	

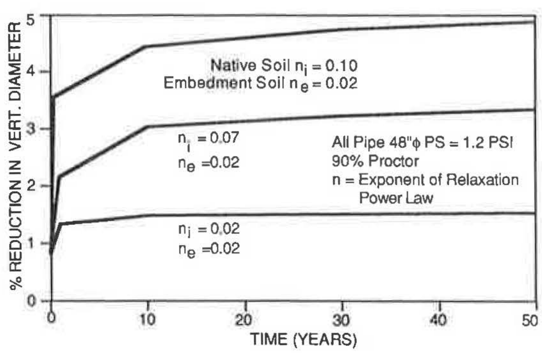

FIGURE 9 Variation of pipe vertical deflection over time.

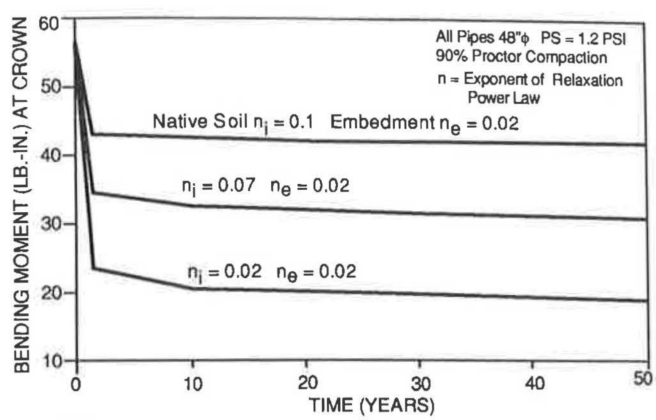

FIGURE 10 Variation of bending moment at crown over time.

Relaxation in a Soil-Pipe System

A flexible pipe deflects noticeably with time because the pipe material as well as the soil surrounding the pipe relaxes. Whether the soil load on a pipe increases or decreases with time depends on the difference between the relaxation rate (exponent of the power law) of the pipe material and the surrounding soil. In the case of a pipe made of stiff material, the soil tends to creep faster than the pipe, and hence the soil load increases. With pipes made of compliant materials, such as HDPE pipe, the pipe material relaxes faster than the soil and, in effect, redistributes the imposed loads back to the soil.

Table 1 gives the values of relaxation rates for various soils and pipe materials.

Pipe Vertical Deflection with Time

Figure 9 shows the predicted percentage reduction in vertical diameter of pipes with coarse-grained embedment materials (compacted to 90 percent Proctor) and buried in in situ (native) soils of the same stiffness ($K = 200$) but of different relaxation rates. The pipes deflect from two to five times the initial deflection during the 50-year period. The exponent of the power law for coarse-grained soils is assumed to be 0.02 (*22*) and for fine-grained soils is usually around 0.1. Silty soils can be assumed to have relaxation rates between the two. The figure also compares the results that can be obtained for *m*-values of 0.02, 0.07, and 0.10 for the different types of in situ soils.

Bending Moment Pipe Strains with Time

Figure 10 shows the variation of bending moment at the crown over time. In these cases, the bending moment at the crown reduces with time. This is because the exponent of the power law of the HDPE pipe material is about 0.098, which is higher than that of the surrounding soil. The converse will happen if a rigid pipe is buried in the same soil.

COMPARISON WITH FIELD MEASUREMENTS

To validate the design equations, field installation conditions as described by soil reports and installation procedures were studied, and the pipe vertical deflections were predicted for various time periods. The predictions were compared with the pipe deflections that were observed during the same period.

The field data were obtained for a variety of sites in the United States from a pipe manufacturer. The site that will be illustrated here is in Kansas. Six 48-in.-diameter HDPE pipe sections (PS = 5 psi), each 20 ft long, were installed as a single line with 18 ft of cover measured to the springline. The soil profile along the line was basically silty clay down to the springline. The modulus number of the backfill soil was $K_b = 100$; the in situ soil was estimated to have $K_i = 200$ and a modulus number $n = 0.4$. The embedment material was compacted to 85 percent Proctor. The soil modulus used in Spangler's equation was 3,000 psi, because crushed rocks were used (*23*) as bedding materials. Five pipe vertical deflections were measured at each section, and readings were taken at 2 days and 1, 2, 6, and 42 weeks after installation.

At the beginning of the installation, the six pipes were buried at different depths of cover ranging from 1 to 18 ft, and the pipe vertical deflections were measured. Figure 11 compares the results of the design equations and Spangler's equation with the average of five pipe vertical deflections per section. The design equations predicted the deflections well. It also appears that the level of compaction may be more than the 85 percent Proctor that was assumed.

The trench was backfilled a depth of cover of 18 ft after 2 days with the exception of the first two sections. The pipe deflections measured during the 10-month period were plotted in Figure 12. A relaxation rate of 0.02 was assumed for the embedment material, and 0.07 was assumed for the silty soil. A lag factor of 1.0 was used for Spangler's equation because lag factors from 1.25 to 1.5 are recommended only if the time periods are on the order of a few years (*23*). It can be seen that the design equations were able to match the deflection behavior. Again, the deflections were slightly more than predicted, probably because of the low compaction assumed for the embedment material. The field data also indicate a nonlinear increase in values over time, just as predicted by the design equations.

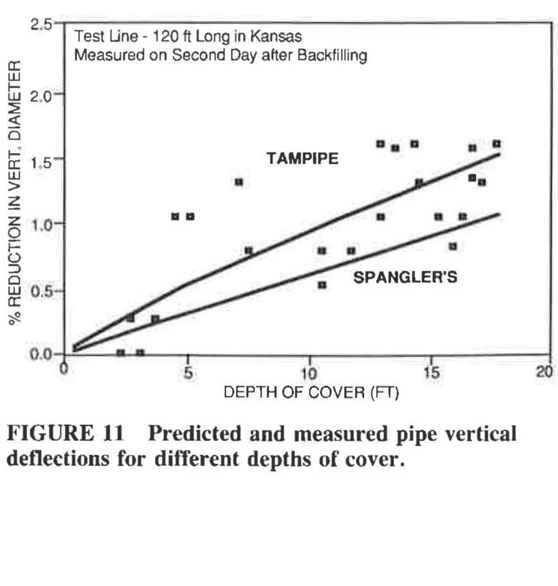

FIGURE 11 Predicted and measured pipe vertical deflections for different depths of cover.

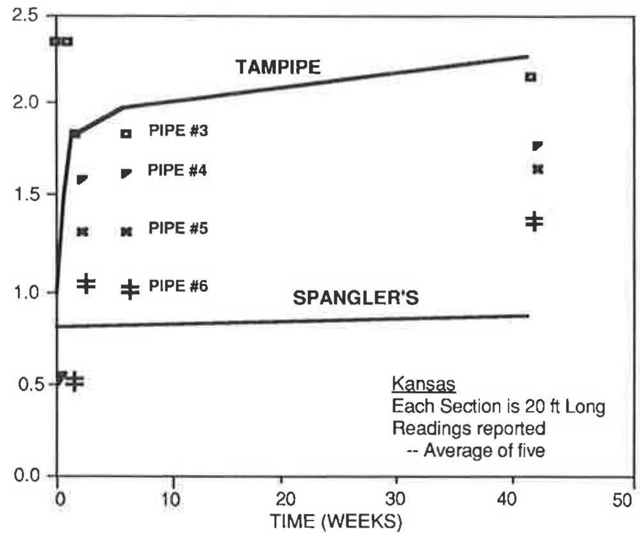

FIGURE 12 Predicted and measured pipe vertical deflection for different time periods.

CONCLUSIONS

Design equations that can be used to determine the deflections, loadings, and strains in flexible pipes and their behavior over time have been presented. The main factors affecting the soil-pipe system were identified. They include pipe characteristics, properties of the different types of soils, arching in the soil, trench width, and presence of groundwater. The time-dependent behavior of the soil-pipe system was also presented, and results were obtained by using the viscoelastic form of the design equations. It was shown that it is possible to quantify the effects of the various factors on pipe deflections over time and that the design equations were able to match field measurements. The ability to describe the details of soil-pipe behavior with a sound engineering approach can be expected to provide a major benefit to the design, construction, and performance of buried flexible pipes.

REFERENCES

1. M. G. Spangler. *Soil Engineering* (3rd ed.). International Textbook Co., Scranton, Pa., 1973.
2. R. L. Kondner. Hyperbolic Stress-Strain Response: Cohesive Soils. *Journal, Soil Mechanics and Foundations Division*, ASCE, Vol. 89, No. SM1, Feb., 1983.
3. N. Janbu. Soil Compressibility as Determined by Oedometer and the Triaxial Tests. *European Conference on Soil Mechanics and Foundation Engineering*, Wiesbaden, Germany, Vol. 1, 1963, pp. 19–25.
4. J. M. Duncan, P. Byrne, K. S. Wong, and P. Mabry. *Strength, Stress-Strain and Bulk Modulus Parameters for Finite Element Analysis of Stresses and Movements in Soil Masses*. Report UCB/GT/80-01, University of California, Berkeley, 1980.
5. M. G. Katona, J. M. Smith, R. S. Odello, and J. R. Allgood. *CANDE—A Modern Approach for the Structural Design and Analysis of Buried Culverts*. Rept. FHWA-RD-77-5, Oct. 1976.
6. *Spiral Engineered Systems*. Chevron Oil, Norcross, Ga., 1985.
7. E. Gaube and W. Mueller. 12 Years of Deformation Measurements on Sewer Pipes from Hostalen GM5010. *Proc., International Conference on Underground Plastic Pipes*, ASCE, New Orleans, La., 1981.

8. K. Hoeg. Stresses Against Underground Structural Cylinders. *Journal, Soil Mechanics and Foundations Division*, ASCE, No. SM1, July 1968, pp. 833–858.
9. *Structural Plastics Design Manual*. ASCE Manuals and Reports on Engineering Practice 63., May 1984.
10. L. Obert and W. I. Duvall. *Rock Mechanics and the Design of Structures in Rocks*. John Wiley and Sons, Inc., New York, 1967.
11. J. K. Mitchell, R. C. Campanella, and A. Singh. Soil Creep as a Rate Process. *Journal, Soil Mechanics and Foundations Division*, ASCE, Vol. 94, No. SM1, 1968, pp. 231–253.
12. R. A. Schapery and M. Riggins. Development of Cyclic Nonlinear Viscoelastic Constitutive Equations for Marine Sediments. *Proc., International Symposium on Numerical Models in Geomechanics*, Zurich, Switzerland, 1982.
13. M. A. Biot. Dynamics of Viscoelastic Anisotropic Media. *Proc., 4th Midwestern Conference*, 1955, pp. 94–108.
14. E. H. Lee. Stress Analysis in Viscoelastic Bodies. *Quarterly Journal of Applied Mathematics*, Vol. 13, 1955, pp. 183–190.
15. R. A. Schapery. Approximate Methods of Transform Inversion for Viscoelastic Stress Analysis. *Proc., 4th U.S. National Congress of Applied Mechanics*, ASME, 1962.
16. T. L. Cost. Approximate Laplace Transform Inversions in Viscoelastic Stress Analysis. *AIAA Journal*, Vol. 2, No. 12, 1964, pp. 2157–2166.
17. K. M. Chua and R. L. Lytton. Short Communication: A Method of Time-Dependent Analysis Using Elastic Solutions for a Non-Linear Material. *International Journal for Numerical And Analytical Methods in Geomechanics*, Vol. 11, No. 4, 1987.
18. A. M. Marimon. *The Effects of Suction and Temperature on the Material Characteristics of Fine-Grained Soils*. Texas A&M University, College Station, 1977.
19. M. Riggins. *The Viscoelastic Characterization of Marine Sediments in Large-Scale Simple Shear*. Ph.D. dissertation, Texas A&M University, College Station, 1981.
20. H. S. Stevenson. *Vane Shear Determination of the Viscoelastic Shear Modulus of Submarine Sediments*. M.S. thesis. Texas A&M University, College Station, 1973.
21. A. Singh and J. K. Mitchell. Creep Potential and Creep Rupture of Soils. *Proc., 7th International Conference of Soil Mechanics and Foundations Division*, ASCE, Vol. SM3, 1970, pp. 1011–1043.
22. K. M. Chua and R. L. Lytton. Time-Dependent Properties of Embedment Soils Back-Calculated from Deflections of Buried Pipes. *Specialty Geomechanics Symposium*, Institute of Engineers, Adelaide, Australia, Aug. 1986.
23. A. K. Howard. The USBR Equation for Predicting Flexible Pipe Deflection. *Proc., International Conference on Underground Plastic Pipes*, ASCE, New Orleans, La., 1981, pp. 37–55.

Publication of this paper sponsored by Commitee on Subsurface Soil-Structure Interaction.