Convergent Algorithm for Dynamic Traffic Assignment

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A link flow formulation and a convergent solution algorithm for the dynamic user equilibrium (DUE) traffic assignment problem for road networks with multiple trip origins and destinations are presented. The link flow formulation does not implicitly assume complete enumeration of all origin-destination paths as does the equivalent path flow formulation. DUE is a temporal generalization of the static user equilibrium (SUE) assignment problem with additional constraints to ensure temporally continuous paths of flow. Whereas SUE can be solved by methods of linear combinations, these methods can create temporally discontinuous flows if applied to DUE. This convergent dynamic algorithm (CDA) uses the Frank-Wolfe method of linear combinations to find successive solutions to DUE while holding node time intervals fixed from each origin. In DUE, the full assignment period of several hours is discretized into shorter time intervals of 10 to 15 min each, for which trip departure matrices are assumed to be known. The performance of CDA is compared with that of a heuristic solution procedure called DTA. CDA can be applied to solving DUE on large networks, and the examples presented show that CDA consistently converges to solutions that closely satisfy the DUE optimality conditions. With computational advances such as parallel computing, CDA can be run in near real-time on large-scale networks and used with in-vehicle route advisory systems for traffic management during evacuations and special events.

Dynamic traffic assignment procedures are needed to evaluate the impacts of alternative travel demand management strategies during peak periods in urban areas and will play a key role in the development of real-time traffic management and in-vehicle route guidance systems. Merchant and Nemhauser (*1*) formulated a system-optimal version of the dynamic assignment problem in which there can be multiple origins but only one destination. A global optimum is difficult to obtain because of the nonconvexity of that formulation (*2,3*). Carey (*4*) shows that the Merchant and Nemhauser formulation satisfies a certain constraint qualification needed for Kuhn-Tucker optimality conditions to exist at the optimum. Carey (*5*) presents an alternative formulation of the Merchant and Nemhauser problem with only one destination that is convex and can be made piecewise linear for solution purposes.

The dynamic user equilibrium (DUE) assignment problem is defined in this paper as follows: Given a set of zone-to-zone trip tables containing the number of vehicle trips departing from and headed toward each zone in successive time intervals of 10 to 15 min each, determine the volume of vehicles on each link in each time interval of a network connecting these zones that satisfy the following two conditions:

1. All paths between a given pair of zones used by trips departing in a given time interval must have equal travel impedances.
2. All paths between a given pair of zones not used by trips departing in a given time interval cannot have lower travel impedances.

These two conditions for DUE given fixed trip departure times, which will be derived later from the link flow formulation of DUE presented herein, are temporal generalizations of Wardrop's conditions (*6*) for static user equilibrium (SUE). Other authors have defined DUE to include variable trip departure times, which are assumed to be fixed in this paper. Janson (*7*) formulates and presents a convergent solution algorithm for the combined problem of dynamic traffic assignment and trip distribution with variable trip departure or arrival times.

Friesz et al. (*8*) present an optimal control theory formulation of dynamic traffic assignment in continuous time for which the equilibrium conditions are a variation of the foregoing conditions. The optimality conditions of their model are that all used paths between any two nodes must have equal impedances at any given instant and unused paths cannot have lower impedances. These conditions allow complete origin-to-destination paths used by trips to have unequal impedances for any given departure time. These conditions are not user optimal, because travelers can switch to paths with lower impedances. Once trips depart on separate paths from a given origin, these paths need not have equal impedances for the remainders of their journeys.

DUE is a temporal generalization of the SUE assignment problem of which SUE is the special case with one long assignment period. In DUE, the full assignment period of several hours is discretized into shorter time intervals of 10 to 15 min each, for which trip departure matrices are assumed to be known. With multiple time intervals, DUE requires nonlinear mixed-integer constraints with "node time intervals" that ensure temporally continuous paths of flow. Although DUE is nonconvex over the domain of feasible node time intervals for trip paths from each origin, DUE is convex with a unique global optimum for any given set of fixed node time intervals. The optimality conditions of DUE are later derived from a qualified statement of DUE in which only temporally continuous paths can be used in the optimum solution according to prespecified node time intervals.

The steady-state flow assumption of SUE allows it to be formulated with all linear constraints. Whereas SUE can be solved efficiently by methods of linear combinations, these methods create temporally discontinuous flows if applied to

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DUE. Janson (9) formulated DUE as a nonlinear mixed-integer program in terms of path flows and described a dynamic traffic assignment (DTA) heuristic that generates approximate solutions to DUE for large networks. DTA is not a convergent solution algorithm for DUE, but instead was designed to produce traffic assignments that tend to satisfy the DUE optimality conditions stated earlier. In test applications, DTA produced both static and dynamic assignments that approximately satisfied the user equilibrium conditions of those problems.

Both SUE and DUE can be formulated equivalently in terms of either path or link flows, but the link flow formulations of these problems do not implicitly assume complete enumeration of all possible paths between zone pairs. More important, the link flow formulation of DUE decomposes directly into two subproblems solved successively by the convergent dynamic algorithm (CDA) presented herein. CDA solves DUE with fixed node time intervals by the Frank-Wolfe (F-W) method of linear combinations and then updates the node time intervals with which to generate the next F-W solution. The procedure terminates when the number of node time interval changes (or other measure of convergence tolerance) is acceptable. Several examples indicate that CDA consistently converges toward optimal DUE solutions. CDA can be applied to large planning networks, and convergence difficulties that might occur on small, specially configured networks are less likely to occur on larger networks in which paths from many origins share common links.

DUE ASSIGNMENT PROBLEM

A temporal generalization of SUE in terms of path or link flows requires that a time interval superscript be added to each link flow variable and impedance function. A departure time superscript must also be added to each origin-specific link flow variable and to each element of the trip matrix Q. Superscripts representing time intervals of link use and trip departure must also be added to each node time interval variable to indicate whether trips departing from origin zone r in Time Interval d reach Node i in Time Interval t. Denoting a link as a node pair ij instead of with an arc subscript k is necessary for the derivation of optimality conditions from this link flow formulation of DUE given by Equations 1 through 10.

$$\text{DUE} \quad \text{Minimize} \sum_{ij \in A} \sum_{t \in D} \int_0^{x_{ij}^t} f_{ij}^t(w)\,dw - \sum_{r \in Z} \sum_{i \in N} \sum_{d \in D} b_{ri}^d \tag{1}$$

subject to

$$x_{ij}^t = \sum_{r \in Z} \sum_{d \leq t} v_{rij}^{dt}\,\alpha_{ri}^{dt} \qquad \text{for all } ij \in A,\ t \in D \tag{2}$$

$$q_{rn}^d = \sum_{t \geq d} \left(\sum_{in \in A} v_{rin}^{dt}\,\alpha_{ri}^{dt} - \sum_{nj \in A} v_{rnj}^{dt}\,\alpha_{rn}^{dt} \right)$$

$$\text{for all } r \in Z,\ n \in N,\ d \in D \tag{3}$$

$$v_{rij}^{dt} \geq 0 \qquad \text{for all } r \in Z,\ ij \in A,\ d \in D,\ t \in D \tag{4}$$

$$\alpha_{ri}^{dt} = (0, 1) \qquad \text{for all } r \in Z,\ i \in N,\ d \in D,\ t \in D \tag{5}$$

$$\sum_{t \in D} \alpha_{ri}^{dt} = 1 \qquad \text{for all } r \in Z,\ i \in N,\ d \in D \tag{6}$$

$$(b_{rj}^d - b_{ri}^d)\,\alpha_{ri}^{dt} \leq f_{ij}^t(x_{ij}^t)\,\alpha_{ri}^{dt}$$

$$\text{for all } r \in Z,\ ij \in A,\ d \in D,\ t \in D \tag{7}$$

$$[b_{ri}^d - (t - d + l)\Delta t]\alpha_{ri}^{dt} \leq 0$$

$$\text{for all } r \in Z,\ i \in N,\ d \in D,\ t \in D \tag{8}$$

$$[b_{ri}^d - (t - d)\Delta t]\alpha_{ri}^{dt} \geq 0$$

$$\text{for all } r \in Z,\ i \in N,\ d \in D,\ t \in D \tag{9}$$

$$b_{rr}^d = 0 \qquad \text{for all } r \in Z,\ d \in D \tag{10}$$

where

- N = set of all nodes,
- Z = set of all zones (i.e., trip-end nodes),
- A = set of all links (directed arcs),
- Δt = duration of each time interval (same for all t),
- D = set of all time intervals in the full analysis period (e.g., eighteen 10-min intervals for a 3-hr peak-period assignment),
- x_{ij}^t = number of vehicle trips between all zone pairs assigned to Link ij in Time Interval t (variable),
- v_{rij}^{dt} = number of vehicle trips departing from origin zone r in Time Interval d assigned to Link ij in Time Interval t (variable),
- $f_{ij}^t(x_{ij}^t)$ = travel impedance on Link k in Time Interval t (variable),
- q_{rn}^d = number of vehicle trips from Zone r to Node n departing in Time Interval d via any path (zero for any node $n \in Z$) (fixed),
- b_{ri}^d = travel time along any path used from origin zone r to Node i by trips departing in Time Interval d (variable), and
- α_{ri}^{dt} = zero-one variable indicating whether trips departing from origin zone r in Time Interval d reach Node i in Time Interval t (henceforth called a "node time interval") (0 = no, 1 = yes) (variable).

This formulation of DUE assumes that a directed network $G(N, A)$ is given, where N is the set of nodes and A is the set of directed arcs or links. Zones (denoted by the set Z) are nodes at which trips originate or terminate. Equation 2 defines the total flow on Link ij in Time Interval t to be the sum of flows departing from any origin r in any time interval d that use Link ij in Time Interval t in order to formulate the objective function as given by Equation 1. It is not necessary to multiply v_{rij}^{dt} by α_{ri}^{dt} in Equation 2, because flows departing from origin r in Time Interval d will only be assigned to Link ij in Time Interval t allowed by the α_{ri}^{dt} term equal to 1 in the nodal conservation-of-flow Equation 3. Equation 3 constrains inflow minus outflow at each node and zone in each time interval to sum to the proper trip departure totals in each time interval between each O-D pair, and Equation 4 requires all link volumes to be nonnegative.

The second part of the DUE objective function is not required in SUE because origin-to-node travel times are not needed to calculate steady-state link volumes. In DUE, origin-to-node travel times are used to determine the node time intervals $\{\alpha_{ri}^{dt}\}$ in Equations 5 through 10. In DUE, each node time interval α_{ri}^{dt} cannot be prespecified with a fixed value because node time intervals are affected by travel times, which are affected by link loadings. The node time intervals are endogenous variables in DUE, creating nonlinear flow conservation constraints and requiring DUE to have additional constraints (Equations 5 through 10) to ensure temporally continuous paths.

Equation 5 defines each node time interval to indicate whether trips departing from origin zone r in Time Interval d reach Node i in Time Interval t. (Strictly speaking, each node time interval value α_{ri}^{dt} can only be 0 or 1, which then indicates the node time interval index t.) Equation 6 requires that there be only one time interval t in which trips departing from a given origin r in a given time interval d can reach a node. The assumption here is that any Link ij incident from Node i can only be used (if used at all) in the time interval t in which Node i is reached by trips departing from origin r in time interval d. Equation 7 determines the origin-to-node travel times, which could be any arbitrary values that optimize the objective function if these times were not maximized by the DUE objective function. Equations 8 and 9 then use the node time interval values to identify time interval indices that are compatible with the path travel times to each node.

Each intrazonal travel time b_{rr}^{d} must be set to zero or to some other fixed value as shown by Equation 10 to prevent the maximization of origin-to-node travel times (or the minimization of negative times) in the DUE objective function from having an infinite solution. Though counterintuitive, the maximization of travel times as given by Equation 1 subject to Constraints 5 through 10 is the correct objective for determining shortest travel time paths in this formulation. The shortest path problem is often formulated to find unit link flows that minimize the use of arc lengths. The second part of Equation 1 plus Equations 5 through 10 constitute the dual of that formulation, which is to maximize zone-to-node path lengths subject to fixed arc lengths. Because many hundreds of vehicles travel each path, the first part of the objective function will dominate the assignment process, whereas the second part causes the correct shortest path travel times to be found. If vehicle units are small, the second part of the objective function can be multiplied by any small constant and it will still achieve its desired result.

According to Equations 7 through 9, links are traversed within the time intervals that trip paths reach their tail nodes. For 10-min intervals, Interval 1 begins at 0 min, Interval 2 begins at 10 min, Interval 3 begins at 20 min, and so forth. If a path reaches a node at the exact beginning of a time interval (to the degree of floating point precision being used), the solution algorithm can be coded to have the path use the link in that time interval rather than in the previous interval. Note that Equations 7 through 9 also work for the static case, albeit unnecessary, so long as the duration Δt of the single time interval exceeds the longest trip length in the network (measured in time). Equations 1 through 10 exactly define SUE if there is only one long time interval and all node time intervals given by array $\{\alpha_{ri}\}$ (disregarding time) are set to 1. Thus, Equations 1 through 10 are a complete formulation of both the static and dynamic assignment problems, with the static problem being a special case.

Figure 1 shows how a node time interval depends on the time at which a path reaches the tail node of a link. The link number is indicated by k, and the time interval of link use is indicated by t. Link 1 is used by this path in Interval 1, although it overlaps into Interval 2. Links 2 and 3 are used in Interval 2, but Link 4 is used in Interval 3, because it begins exactly at the beginning of Interval 3. The portion of Link 1 that overlaps into Interval 2 is discussed next.

If link lengths are generally much shorter than the time interval duration (e.g., less than 20 percent), fractions of paths overlapping time intervals may not be significant. Suppose that a dynamic assignment has a mean link length of M min and a time interval duration of N min. The probability is M/N that a link of M min will overlap time intervals in any path that uses it. When overlap does occur, the average amount of overlap into the next time interval is one-half the average link length, or $M/2$ min. Thus, the average amount that used links overlap time intervals is $M^2/(2N)$, which is one-half the average link length ($M/2$) times the probability that an overlap occurs (M/N). For example, links of 2 min in time intervals of 10 min will, on the average, overlap intervals by 0.2 min, or 10 percent of the link length. The overlapping percentage of total trip length, which accounts for trip volumes on the links, was computed for the examples given later and was always found to fall below this formula's estimate.

Whereas the impedance functions in the SUE objective function are not restricted to travel time alone, inconsistencies could develop between path travel times and node time intervals in DUE if the impedance functions are not strictly measures of time. Equal paths according to a composite cost could reach nodes in different time intervals. The path flow formulation of DUE given by Janson (*9*) avoids this complication by explicitly computing the travel times along every possible O-D path. In the path flow formulation, the impedance functions in Equation 1 can include travel cost factors other than travel time that affect route choice. The path flow formulation only gains that advantage by requiring complete path enumeration.

The derivation of user equilibrium optimality conditions from the preceding formulation is complicated by the nonlinear mixed-integer constraints and integer node time intervals. Although optimality conditions cannot be derived from the general formulation with integer unknowns, they can be derived for a given set of node time intervals to which all temporally continuous paths in the optimal solution to the general problem must conform. Because node time intervals can be uniquely determined from a given set of link volumes, they

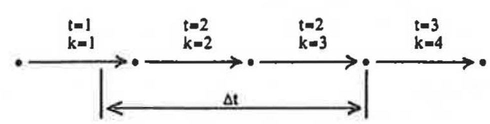

FIGURE 1 Example of time interval overlap at node.

can be assumed to be known in the derivation of optimality conditions for the global optimum.

Equation 11 is the Lagrangian of Equations 1 through 4 with linear constraints and only the first part of the objective function because of fixed node time intervals. Over the domain of variable integer values, Equation 11 is nonconvex, and there are many local optima that are inferior to the global optimum. For a set of fixed node time intervals, the bordered Hessian matrix of Equation 11 is positive definite, which means that there is a unique global optimum with no local optima (*10*). The Hessian matrix is only positive definite so long as each impedance function is a monotonically nondecreasing function of flow on Link ij in Time Interval t alone, which is assumed to be true in the foregoing formulation of DUE.

$$L(X, V, \lambda, \mu, \tau) = \sum_{ij \in A} \sum_{t \in D} \int_0^{x_{ij}^t} f_{ij}^t(w)dw$$
$$- \sum_{ij \in A} \sum_{t \in D} \lambda_{ij}^t \left(x_{ij}^t - \sum_{r \in Z} \sum_{d \leq t} v_{rij}^{dt} \alpha_{ri}^{dt} \right)$$
$$+ \sum_{r \in Z} \sum_{n \in N} \sum_{d \in D} \mu_{rn}^d \left[q_{rn}^d - \sum_{t \geq d} \left(\sum_{in \in A} v_{rin}^{dt} \alpha_{ri}^{dt} - \sum_{nj \in A} v_{rnj}^{dt} \alpha_{rn}^{dt} \right) \right]$$
$$+ \sum_{r \in Z} \sum_{ij \in A} \sum_{d \in D} \sum_{t \in D} \tau_{rij}^{dt}(-v_{rij}^{dt}) \quad (11)$$

The optimality conditions are given by Equations 12 through 14.

$$\partial L/\partial x_{ij}^t \rightarrow f_{ij}^t(x_{ij}^t) = \lambda_{ij}^t \qquad \text{for all } ij \in A,\ t \in D \quad (12)$$

$$\partial L/\partial v_{rij}^{dt} \rightarrow (\mu_{rj}^d - \mu_{ri}^d)\alpha_{ri}^{dt} = \lambda_{ij}^t\, \alpha_{ri}^{dt} - \tau_{rij}^{dt}$$
$$\text{for all } r \in Z,\ ij \in A,\ d \in D,\ t \in D \quad (13)$$

$$\tau_{rij}^{dt}\, v_{rij}^{dt} = 0 \qquad (\tau_{rij}^{dt} \geq 0)$$
$$\text{for all } r \in Z,\ ij \in A,\ d \in D,\ t \in D \quad (14)$$

where $\tau_{rij}^{dt} = 0$ if $v_{rij}^{dt} > 0$, positive otherwise; impedance difference from Node i to Node j via used path versus by Link ij if used or unused in Time Interval t for trips departing from Zone r in Time Interval d.

The last part of Equation 11 ensures nonnegative link flows and results in a third optimality condition given by Equation 14, which requires τ_{rij}^{dt} to be zero if any trips departing from origin zone r in Time Interval d are assigned to Link ij in Time Interval t, and nonnegative otherwise. According to Equation 12, the optimal solution has a unique equilibrium impedance for each link in each time interval. According to Equations 13 and 14, for any given pair of nodes, all used paths from a given origin for a given departure time have the same travel impedance, and any unused path between these nodes cannot have a lower impedance.

The optimality conditions for DUE can be stated similarly to Wardrop's statement of necessary conditions for SUE (*6*). For trips from Zone r departing in Time Interval d, let μ_{ri}^d be the equilibrium travel impedance of used paths to Node i. Also, let λ_{ij}^t be the equilibrium travel impedance of Link ij equal to the right side of Equation 13, and v_{rij}^{dt} be the equilibrium flow on Link ij in Time Interval t of trips from Zone r that depart in Time Interval d. At equilibrium, all paths from Zone r to Node j used by trips departing in Time Interval d have impedance μ_{rj}^d, and no unused path from Zone r to Node n for this departure time can have a lower impedance. These two conditions are given by Equations 15 and 16.

$$\mu_{ri}^d + \lambda_{ij}^t = \mu_{rj}^d \qquad \text{if } v_{rij}^{dt} > 0 \text{ and } \tau_{rij}^{dt} = 0$$
$$\text{for all } r \in Z,\ ij \in A,\ d \in D,\ t \in D \quad (15)$$

$$\mu_{ri}^d + \lambda_{ij}^t \geq \mu_{rj}^d \qquad \text{if } v_{rij}^{dt} = 0 \text{ and } \tau_{rij}^{dt} \geq 0$$
$$\text{for all } r \in Z,\ ij \in A,\ d \in D,\ t \in D \quad (16)$$

Equations 15 and 16 and equivalent to SUE conditions if there is only one time interval for the full analysis period so that the time interval superscripts can be removed from all terms. When SUE is solved by a method of linear combinations, one measure of how close a solution is to equilibrium is the sum of trip impedance differences from shortest path impedances between all zone pairs in each iteration. This measure of convergence is often referred to as the duality gap or impedance gap. In solving DUE, this gap is the sum of trip impedance differences from shortest path impedances between all zone pairs for trips departing in each time interval, which is one way in which the example results presented later are evaluated.

EXAMPLE OF TEMPORALLY DISCONTINUOUS TRIP PATHS

Methods of linear combinations (e.g., F-W and PARTAN), which apply to nonlinear programs with all linear constraints, are used to solve SUE by combining link volumes without regard to when they occur because they are assumed to occur continuously. The difficulty of ensuring temporally continuous flows arises because the proper time interval superscript at each node n is unknown with respect to flows departing from origin r in different time intervals. Although conservation of flow at each node and zone is stated in terms of link flows, Equation 3 would not ensure temporally continuous flows without Equations 5 through 10 defining the values of the integer node time intervals. The summation over intervals $t \geq d$ in Equation 3 only prevents trips from using links earlier than their trip departure times.

In Figure 2, Link k' incident to Node n in Time Interval t may have a short travel time so that its flow also passes onto Link k in Time Interval t. However, the travel time of Link k' or links before it may grow longer because of congestion such that its flow passes onto Link k in Time Interval $t + 1$. As a result, flows that satisfy Constraint Equation 3, but not Equations 7 through 9, can skip over time intervals at any node or even enter nodes later than they exit.

A simple example is given to demonstrate the difference in solving DUE with and without Equations 5 through 10. Figure 2 shows a path from Zone r to Zone s taken by 3,000 vehicles departing in Time Interval 1 between 7:00 and 7:10 a.m., and no trips depart in the only other time interval from 7:10 to 7:20 a.m. Using the BPR impedance function with free-flow travel times and capacities shown for the links, and without Equations 5 through 10 to ensure temporally contin-

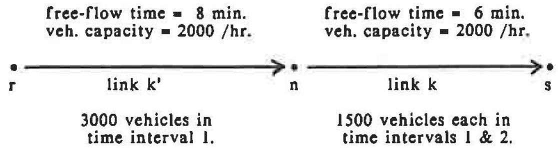

FIGURE 2 Example of temporally discontinuous flows.

uous flows, the optimal solution is to subdivide the flow at Node *n* into two parts of 1,500 vehicles each. One part is assigned to Link *k* in Time Interval 1, and the other part is assigned to Link *k* in Time Interval 2. The F-W method will assign these invalid flows, and once created in the solution, they are never eliminated as the algorithm proceeds. Other examples with two or more routes also show that methods of linear combinations create temporally discontinuous flows when used to solve DUE.

CDA PROCEDURE

In the CDA procedure, DUE is decomposed into two subproblems. The preceding derivation of DUE optimality conditions from Equations 1 through 4 with fixed node time intervals corresponds to the first subproblem, P1, in which the first part of the DUE objective function and Equations 1 through 4 are solved with a fixed set of node time intervals using the standard F-W algorithm or another method of linear combinations. The second part of the DUE objective function and Equations 5 through 10 are then solved in Subproblem P2 to obtain a new set of node time intervals that form temporally continuous shortest paths given the link travel times obtained from solving P1. The iterative process of solving P1 and then P2 terminates when the convergence criterion is satisfied, such as when changes in assigned link flows or node time intervals are acceptably low. Cycling may occur in the node time interval values between successive solutions of the F-W algorithm, but such cycling can be detected and avoided if it occurs. Moreover, such cycling is unlikely to prevent reasonable convergence in larger networks where trips from many origins share common links. An acceptable degree of convergence was obtained in each of the examples given later.

To clarify the explanation of CDA, Subproblems P1 and P2 are formulated below as Equations 17 and 2 through 4, and as Equations 18 and 5 through 10, respectively.

$$\text{P1} \quad \text{Minimize} \sum_{ij \in A} \sum_{t \in D} \int_0^{x_{ij}^t} f_{ij}^t(w)\, dw \tag{17}$$

subject to Equations 2 through 4, where all x_{ij}^t are variable and all α_{ri}^{dt} are fixed in solving P1, and all other values are fixed or variable as they were defined in Equations 1 through 10.

$$\text{P2} \quad \text{Maximize} \sum_{r \in Z} \sum_{i \in N} \sum_{d \in D} b_{ri}^d \tag{18}$$

subject to Equations 5 through 10, where all x_{ij}^t are fixed and all α_{ri}^{dt} are variable in solving P2, and all other values are fixed or variable as they were defined in Equations 1 through 10.

With fixed node time intervals, Subproblem P1 is solved without fixing which links are used but only fixing the time intervals in which links are used by trips depending on their origins and departure times. Subproblem P2 is solved with a label-setting or label-correcting shortest path algorithm adapted for temporally dependent arc lengths. Both types of shortest path algorithms will correctly find temporally continuous shortest paths given dynamic arc lengths with the restriction that vehicles do not pass each other along any link. An equivalent assumption when dealing with aggregate vehicle flows is that vehicles make only one-for-one (or zero-sum) exchanges of places in traffic along any link. This assumption is acceptable and even expected in aggregate traffic models.

CDA converges toward a DUE solution for the following reasons. First, if node time intervals corresponding to a true equilibrium are known, the F-W algorithm will reproduce the equilibrium link volumes from which these node time intervals can be calculated. That convergence proof is equivalent to the convergence proof of the F-W algorithm for SUE. Second, given node time intervals that do not correspond to a true dynamic equilibrium, the F-W algorithm will produce link volumes that shift the node time intervals toward their correct values. For example, if a particular node time interval is too early, then paths to that link will be assigned more traffic by the F-W algorithm such that the node time interval becomes later when recalculated. Oppositely, if a node time interval is too late, paths to that link will be assigned less traffic by the F-W algorithm such that the node time interval becomes earlier when recalculated. Hence, CDA converges toward a set of node time intervals that, when used to assign trips to the network, result in temporal link volumes that give rise to the same node time intervals. A similar case is the convergence proof given by Evans for combined distribution and assignment (*11*) in which a trip distribution is found that when assigned to the network reproduces O-D impedances that give rise to the same distribution.

DTA PROCEDURE

Janson (*9*) developed and evaluated the performance of a heuristic solution procedure for DTA that requires much less memory and computational effort than the just-mentioned approach of successively solving constrained specifications of DUE to which methods of linear combinations can be applied. In example applications, DTA was shown to produce good approximate solutions to DUE for both static and dynamic travel demands. Thus, DTA can serve as an effective means of providing an initial starting solution to the convergent algorithm. The remainder of this section reviews the DTA procedure.

A key assumption of the DTA procedure described here is that route choice decisions are made at the time of trip departure on the basis of projected link impedances that account for changes in travel demand over future time intervals. This assumption offers a great deal of computational efficiency, because the trip departure matrix assigned in each time interval is only a zone-to-zone trip matrix, not a node-to-zone trip matrix as would be required to track trips at nodes through the network by their destinations so that trip paths could be revised en route.

The way in which current link volumes are projected into future time intervals for the purpose of finding trip assignment paths in DTA requires a brief preview. The reason for projecting current link volumes into future time intervals is that some trips departing in later intervals from other origins will use the links concurrently with trips from the present origin for which paths are being found. A premise of DTA is that future link volumes can be adequately estimated from current link volumes on the basis of relative levels of travel demand in future time intervals. These projections are made only for path-finding purposes and are not factored into the actual assignment of link volumes.

The following notation is used to describe the DTA procedure:

x_k^t = assigned volume on Link k in the current Time Interval t after trips departing in Time Intervals 1 through $t - 1$ have been assigned, and while trips departing in Time Interval t are being assigned;

x_k^{t-1} = assigned volume on Link k in Time Interval $t - 1$ (i.e., just before the current time interval) after all trips departing in Time Intervals 1 through $t - 1$ have been assigned;

y_k^{t+n} = projected volume on Link k in interval $t + n$ (where $n \geq 0$ and $t + n \in D$) after trips departing in Time Intervals 1 through $t - 1$ have been assigned and while trips departing in Interval t are being assigned;

$f_k^{t+n}(y_k^{t+n})$ = projected impedance of Link k in the current or future Time Interval $t + n$ computed directly as a function of the projected volume y_k^{t+n}, where $n \geq 0$ and $t + n \in D$; and

Q^t = total number of trips departing from all zones in Time Interval t (i.e., total inflow to the network in Time Interval t).

The superscript of each link volume term always indicates the time interval of link use, whereas the superscript of each trip matrix term always indicates the time interval of trip departure. The DTA procedure assumes that all trip departures are known and fixed for all time intervals and O-D pairs. However, only the trip departure matrix of the current time interval must be stored in random access memory during each iteration of the DTA procedure. The other trip departure matrices can reside in permanent memory until needed.

The steps of the DTA procedure are as follows:

1. Read network data (link-node incidences, free-flow impedances, and practical capacities adjusted to the time interval duration Δt for the link impedance functions). Initialize the link volumes in each time interval to 0, or read in a set of starting volumes if known. Specify as NTREES the number of shortest path trees to be assigned trips from each origin zone in each time interval. Initialize the current time interval t to 0.

2. Increment the current time interval counter t to $t = t + 1$. Read in matrix of trip departures between each O-D pair in Time Interval t.

3. Randomly select an origin zone for which all trips departing in the current time interval t have not yet been assigned. Find a shortest path tree from this origin zone to all other zones on the basis of the projected link impedances in the current and future time intervals. The path search routine finds shortest paths based on link impedances as they are projected to exist during the time intervals in which they are traversed. To minimize array space allocation, each projected link volume and link impedance can be calculated as it is needed in the shortest path routine. The projected link volumes are calculated according to Equation 19, where θ^t is the percentage of Q^t that has not yet been assigned to the network. Define $\omega_{t-1}^{t+n} = Q^{t+n}/Q^{t-1}$ as a measure of systemwide travel demand and projected traffic volumes in interval $t + n$ relative to $t - 1$, and similarly for $t + n$ versus t. The projected volume on Link k in Time Interval $t + n$ is equal to

$$y_k^{t+n} = \theta^t \, \omega_{t-1}^{t+n} \, x_k^{t-1} + (1 - \theta^t) \, \omega_t^{t+n} \, x_k^t$$

$$\text{for all } k \in A, \; n \geq 0, \; t + n \in D \qquad (19)$$

Hence, for each link, the current and projected link volumes are estimated as weighted combinations of the final volume assigned to that link in the previous interval $t - 1$ and the volume assigned thus far in the current interval t, weighted by ratios of total trip departures from all origins in intervals $t - 1$, t, and $t + n$. Each projected link impedance is computed directly from its projected volume using its impedance function.

4. Assign 1/NTREES of the trips departing in Interval t from the current origin to the shortest path tree found in Step 3. Store the assigned link volumes x_k^{t+n} by time of link use for all $n \geq 0$ and $t + n \in D$. If all trips departing from all zones in Time Interval t have been assigned, go to Step 5. Otherwise, return to Step 3 to process the next origin zone.

5. If all departure time intervals in the analysis period have been processed, STOP the program. Otherwise, write out the assigned link volumes for the current time interval t to a disk file and copy the link volumes of each future time interval into the link volume array space of the preceding interval to economize on array space. Return to Step 2.

Trips are assigned from origins chosen in a geographically random order to randomize the order of link loadings. A strategy that reduces random variability in the link volumes from one time interval to the next (as opposed to variations between intervals caused by changes in travel demand) is to find NTREES shortest path trees from each origin in each time interval and to assign 1/NTREES of the trip departures from each origin to each tree. In each time interval, Steps 3 and 4 are repeated NTREES times for all origins chosen randomly without replacement until all trips from all origins in that time interval have been assigned in random order.

The DTA procedure was tested on two networks described later with NTREES values of 2, 3, and 4. As expected, the DTA assignments always improved in terms of satisfying the desired user equilibrium conditions as NTREES was increased. For these networks, loading three trees from each origin in each time interval produced significantly better results than loading only two. However, increasing NTREES from 3 to 4 achieved a much smaller improvement, which indicates a decreasing marginal rate of improvement for the additional burden of finding more trees from each origin for each departure interval. All of the DTA assignments presented later were generated with NTREES equal to 3.

In assigning trips from each origin zone during the tree-by-tree assignment process in DTA, it is unknown how current and future link volumes will be affected by trips assigned from other zones. After each incremental assignment of trips from a given origin in the current time interval, projections must be made of currently assigned link volumes into future time intervals. The technique used in DTA to project current link volumes into future time intervals is to use a weighted combination of current link volumes assigned thus far and final link volumes from the previous interval. This combination is weighted 100 percent toward the just-previous link volumes when assigning the first fraction of trips from the first randomly selected origin and changes to 100 percent of the current link volumes when assigning the last fraction of trips from the last origin. In between, the weight given to current link volumes equals the percentage of total trip departures that have been assigned thus far.

Janson (*9*) evaluated the performance of DTA in several test applications, including comparisons with steady-state equilibrium assignment. When applied to the Pittsburgh network used by Janson et al. (*12*) in a validation of equilibrium assignment, DTA was found to generate link volumes and travel times that compared favorably with both equilibrium assignment and observed link counts and travel times. DTA can also be used to generate a good starting solution for CDA.

COMPARISON OF DTA AND CDA PERFORMANCE RESULTS

DTA and CDA results for steady-state travel demands are first compared with equilibrium assignment for the well-known Sioux Falls network having 76 one-way links, 24 nodes, and 24 origin-destination zones (*13*). Equilibrium assignment results for this network using the standard F-W algorithm have been described by numerous authors, including Fukushima (*14*), LeBlanc et al. (*15*), and Rose et al. (*16*). The standard F-W algorithm was also used to generate the equilibrium assignments with which CDA results are compared in this paper.

Even for cases of steady-state travel demands, the incremental loading of trips causes DTA link volumes to vary from one time interval to the next, and unless perfectly converged, CDA link volumes will also exhibit some variation between time intervals. Thus, for static assignments, it is important (*a*) to examine the degree to which DTA and CDA link volumes vary between time intervals and (*b*) to compare DTA and CDA link volumes over the assignment period with final F-W link volumes. Two measures of variation in DTA or CDA link volumes (called APV1 and APV2) are defined for this purpose. The degree to which DTA or CDA link volumes vary between time intervals is measured by their average percent variation (APV1) from their means as given by Equation 20.

$$\mathrm{APV1} = (100/TX) \sum_{k \in A} \sum_{t \in D} |x_k^t - m_k| \tag{20}$$

where

- APV1 = the average percent variation between time intervals of DTA or CDA link volumes from their means,
- x_k^t = DTA or CDA volume on Link k in Time Interval t,
- m_k = mean DTA or CDA volume on Link k over the time intervals in D, and
- TX = total DTA or CDA link volumes in all time intervals = $\sum_{k \in A} \sum_{t \in D} x_k^t$.

A second measure of DTA or CDA link volume variation (APV2) is used to evaluate the disparity between DTA or CDA and F-W link volumes. APV2 is the average percent variation or absolute difference between DTA or CDA link volumes and final F-W link volumes. APV2 is computed in the same way as APV1, except that (*a*) m_k in APV1 is replaced in APV2 by the final F-W volume on Link k, where each F-W link volume is divided by the number of time intervals in D to represent the same time units as m_k and (*b*) TX in APV1 is replaced in APV2 by the sum of F-W link volumes for the full assignment period.

APV2 is expected to exceed APV1, because APV2 indicates variation from values not derived from the DTA or CDA link volumes. APV2 is affected by both the between-interval variation of the DTA or CDA link volumes and the differences between DTA or CDA mean volumes and F-W volumes. Both APV1 and APV2 are reported for each DTA or CDA static assignment.

How well these assignments satisfy the desired SUE or DUE conditions must also be assessed. One measure of user equilibrium is the size of the SUE or DUE objective function, which is Equation 1 summed over one time interval in the static case. Equation 1 is applied to final F-W link volumes, whereas Equation 1 is computed with the link volumes assigned by DTA or CDA in each time interval. For static assignments, Equation 1 could be applied to average DTA or CDA link volumes over all time intervals, but that would produce a falsely lower objective function and make the DTA or CDA results appear too favorable.

Another standard measure of user equilibrium for an assignment is the duality gap (DG), which is the difference between the sum of assigned trip impedances and the sum of shortest path trip impedances based on the assigned link loadings (*16*). The time dimension of DG, defined by Equation 21 for a dynamic assignment, can be disregarded in computing this measure for a static assignment with only one time period.

$$\mathrm{DG} = (100/TC)\left[\sum_{k \in A} \sum_{t \in D} x_k^t f_k^t(x_k^t)\right] - \left[\sum_{r \in Z} \sum_{s \in Z} \sum_{d \in D} q_{rs}^d c_{rs}^d\right] \tag{21}$$

where

- DG = the duality gap of a dynamic assignment,
- q_{rs}^d = number of trips from Zone r to Zone s departing in Time Interval d via any path,
- c_{rs}^d = shortest path impedance from Zone r to Zone s for trips departing in Time Interval d through a network of assigned link loadings, and
- TC = total trip impedance if all trips were to use their shortest paths through a network of assigned link loadings = $\sum_{r \in Z} \sum_{s \in Z} \sum_{d \in D} q_{rs}^d c_{rs}^d$.

The left-most bracketed term of Equation 21 is the system-optimal objective function of DUE or SUE. The right-most bracketed term, which equals TC, is a strict lower bound on the optimal value of the DUE or SUE objective function for a given feasible solution with no temporally discontinuous paths. The duality gap decreases toward zero, although not strictly monotonically, as the F-W algorithm converges. The duality gap equals zero for a true equilibrium solution in which the impedance of every used path between each pair of zones equals the shortest path impedance. TC is not a strict lower bound on the optimal value of the DUE objective function if it is based on an infeasible solution with temporally discontinuous paths. The percentage of temporally discontinuous node time intervals (called temporal node violations) is reported later for each final solution as one qualification of this lower bound.

Although the convergence rate of the standard F-W algorithm can be improved, these improvements would not affect the comparisons in this paper because the F-W algorithm was run to a high degree of convergence in each case. When applied to the Sioux Falls network to generate assignment results for this paper, the F-W algorithm was halted when the greatest single link volume change was less than 1 percent between iterations. This degree of convergence for the Sioux Falls network required 76 iterations of the standard F-W algorithm starting from free-flow impedances. Comparisons of F-W, DTA, and CDA results can also depend on the initial link volumes or impedances used in the F-W algorithm. The outcomes reported in this paper would not be significantly affected by different F-W starting solutions because of the high degree of F-W convergence required in each case. Rose et al. (*16*) found that final link volumes for the Sioux Falls network had less than a 0.5 percent coefficient of variation between solutions when they applied the 1 percent link volume change stopping criterion to the F-W algorithm with different starting solutions.

An important consideration in developing test problems for DTA and CDA is the time interval duration, or the number of time intervals in the analysis period. An interval duration of 10 min was used in all of the following test cases. This duration was chosen after observing that the mean F-W link impedance for the Sioux Falls network was 6 min. DTA and CDA link volumes show less variation between intervals when the time interval duration is at least four to five multiples of the mean link impedance. The time interval duration would have to be 24 to 30 min to achieve this multiple for the Sioux Falls network. However, most transportation planning networks used in practice generally have shorter links, such as the Pittsburgh network used later in which the mean F-W link impedance is only 0.6 min.

To obtain initial link loadings on the network prior to the first time interval of the analysis period, both DTA and CDA were run to include ½ hr (or three 10-min intervals) of average travel demand in direct proportion to the full trip matrix, but at a lower departure rate than for the peak-hour intervals. Because the majority of trips in both the Sioux Falls and Pittsburgh networks have trip impedances within ½ hr, both DTA and CDA needed roughly ½ hr of initial trip departures for initial link loadings to be obtained. Likewise, each DTA or CDA assignment assigned six time intervals of average travel demand after the six peak-hour time intervals to allow the peak-hour trips to clear the network.

To obtain steady-state assignments from DTA or CDA, a uniform fraction of the 1-hr trip matrix was assumed to depart in each time interval of the analysis period. In DTA, each interval of trip departures was assigned incrementally to three shortest path trees (corresponding to an NTREES value of 3) by executing successive tree-by-tree assignments in each interval and loading one-third of each zone's origins to each tree. Hence, DTA could assign no more than three different paths between each pair of zones in each time interval. For this reason, fewer paths are assigned trips by DTA than by either the F-W or CDA algorithms.

Table 1 presents a comparison of the F-W, DTA, and CDA static assignments for the Sioux Falls network. The total 1-hr link volumes of the F-W, DTA, and CDA assignments were 969, 960, and 972, respectively, which are essentially equal. Trips departing in the six time intervals of the 1-hr analysis period have mean trip impedances of 14.96, 15.84, and 14.91 min for the F-W, DTA, and CDA assignments, respectively. The objective function values of the F-W, DTA, and CDA assignments, which are directly comparable, are 50.05, 50.74, and 50.22, respectively.

The F-W and CDA objective function values are nearly equal, and the F-W algorithm required 23 iterations to achieve a lower objective function value than the DTA procedure. However, the F-W algorithm requires less computational effort than either DTA or CDA. Using the Sioux Falls network and executing all procedures on the same 80486-based microcomputer with all output suppressed, DTA required 12 sec to perform 15 time intervals of dynamic assignment, whereas the F-W algorithm required only 5 sec to execute 23 iterations of equilibrium assignment. CDA required 40 sec to converge to less than a 3 percent maximum link volume change between iterations, which required 57 iterations. Thus, DTA and CDA required 2.4 times and 8 times as much computational effort, respectively, as the F-W algorithm to generate a comparable 1-hr static assignment for the Sioux Falls network. All programs were coded by the author in FORTRAN using many of the same subroutines and compiled with the same compiler.

In order to assess the degree of steady-state equilibrium obtained by DTA and CDA, Table 1 gives the values of APV1, APV2, and DG as defined earlier. The average percent link volume variations between time intervals of the DTA and CDA assignments from their own link volumes means

TABLE 1 SUMMARY OF SIOUX FALLS STATIC ASSIGNMENT RESULTS

Evaluation Measure	F-W	DTA	CDA
Total Link Volume (TX)	969	960	972
Mean Trip Impedance (min)	14.96	15.84	14.91
SUE or DUE Objective Func.	50.05	50.74	50.22
Volume Variation 1 (APV1)	----	3.00%	2.34%
Volume Variation 2 (APV2)	----	5.88%	2.84%
Duality Gap (DG)	----	1.51%	0.426%
Temporal Node Violations	----	7.3%	7.6%
80486 Computation Time	5 sec	12 sec	40 sec

(APV1) were 3.00 and 2.34 percent, respectively. The average percent link volume variations of the DTA and CDA assignments from the final F-W link volumes were 5.88 and 2.85 percent, respectively. The duality gaps of the DTA and CDA assignments are 1.51 and 0.426 percent, respectively. The duality gap of the F-W assignment from its own final shortest path impedances after 76 iterations was 0.2 percent. Temporal node violations are the number of times that shortest paths traverse nodes in incorrect time intervals in the final solution (as a percentage of total node time intervals). DTA had 7.3 percent violations, whereas CDA had 7.6 percent.

Overall, both DTA and CDA produced static assignments for the Sioux Falls network that compared favorably with F-W results. A network of the Pittsburgh eastern travel corridor was used next to examine the performances of DTA and CDA with both static and dynamic travel demands. This network contains 807 one-way links, 372 nodes, and 30 origin-destination zones. Both DTA and CDA were applied to the Pittsburgh network using 10-min intervals, with three initial intervals, six analysis period intervals, and six network-clearing intervals. Both F-W and CDA were run until no single link volume varied by more than 3 percent between iterations. F-W required 33 iterations, but CDA required only 16. However, CDA again used eight times as much CPU time as the F-W algorithm, whereas DTA used only two times as much.

Table 2 compares the F-W, DTA, and CDA static assignment results for the Pittsburgh network. Total 1-hr link volumes in the F-W, DTA, and CDA assignments were 411,178, 408,482, and 410,355, respectively. The objective function values and the mean trip impedances of the three assignments are nearly equal. Although the mean DTA and CDA trip impedances are not much greater than one time interval, many of the routes assigned by DTA and CDA were between 40 and 50 min in length. Table 2 also gives the link volume variations and duality gap measures for the DTA and CDA assignments. The link volume variations are slightly greater in this case than for the Sioux Falls network, whereas the duality gaps are much lower. The duality gap of the F-W assignment from its own final shortest path impedances after 33 iterations was less than 0.1 percent. As far as temporal node violations in the final solutions, DTA had 4.3 percent violations, whereas CDA had only 1.9 percent.

As explained in the previous section, link volumes from an earlier run of DTA can be used as the projected link volumes in future time intervals instead of projecting link volumes on the basis of the current assignment. If the network's structure or travel demands have been altered, link volumes from a previous run may not be usable, depending on the extent of the changes. However, it is instructive to see whether the duality gap of dynamic assignment route impedance differences can be improved by using link volumes from a previous run as prior information. DTA was applied 10 more times to the Sioux Falls static assignment problem, with the link volumes from each run used in the next. Values of DG for these next 10 runs were 1.778, 1.732, 1.478, 1.632, 1.545, 1.419, 1.660, 1.419, 1.718, and 1.502 percent, respectively, with a mean of 1.588 percent. Compared with a value of 1.511 percent for the initial run of DTA described previously for which there were no previous link volumes, using link volumes from previous runs did not significantly affect DG for the Sioux Falls static assignment. This insignificant effect was to be expected for a static assignment, because previous link volumes add no information to the projection formula given by Equation 19 if travel demands are constant.

To test whether previous link volumes improved the duality gap of the DTA assignment with time-varying travel demands, a Sioux Falls assignment was run for six 10-min intervals using trip departure percentages of 12.5, 16.5, 21, 21, 16.5, and 12.5. These percentages were estimated for the Pittsburgh network examples as explained later and used here for example purposes. DTA was applied 10 more times to the Sioux Falls dynamic assignment, with the final link volumes from each run used in the next. Values of DG for these next 10 runs were 3.666, 3.089, 2.722, 2.461, 2.767, 2.214, 2.226, 2.181, 2.564, and 2.652 percent, respectively, with a mean of 2.654 percent. Compared with a value of 3.425 percent for the initial DTA run for which there were no previous link volumes, using link volumes from previous runs improved the duality gap for the Sioux Falls dynamic assignment. However, a low DG value had already been achieved by DTA in the initial run.

The DTA and CDA procedures were also applied to the Sioux Falls and Pittsburgh networks using 6-min time intervals. As expected, both DTA and CDA did not perform as well with 6-min intervals on the Sioux Falls network because the average F-W link length was also 6 min, and both procedures produce better assignments when the time interval duration is at least four to five times greater than the average link length. DTA and CDA produced similar static assignments with both 6-min and 10-min intervals on the Pittsburgh network in which the average F-W link length was only 0.6 min. Thus, these few tests may indicate that DTA and CDA results are not greatly affected by moderate changes in the time interval duration so long as it remains several magnitudes greater than the average link length.

Next, DTA and CDA were applied to the Pittsburgh network with dynamic travel demands over the peak hour. Instead of assigning one-sixth of the peak-hour trip matrix to the network every 10 min, the matrix was assigned in six successive trip departure percentages equal to 12.5, 16.5, 21.0, 21.0, 16.5, and 12.5 percent of the trip matrix. These trip departure percentages are based on travel data collected for a study of highway reconstruction impacts in the Pittsburgh eastern corridor (*17*) and compared with percentages re-

TABLE 2 SUMMARY OF PITTSBURGH STATIC ASSIGNMENT RESULTS

Evaluation Measure	F-W	DTA	CDA
Total Link Volume (TX)	411178	408482	410355
Mean Trip Impedance (min)	11.25	11.23	10.91
SUE or DUE Objective Func.	3884	3873	3885
Volume Variation 1 (APV1)	----	3.84%	0.473%
Volume Variation 2 (APV2)	----	7.28%	3.61%
Duality Gap (DG)	----	0.354%	0.126%
Temporal Node Violations	----	4.3%	1.9%
80486 Computation Time	25 sec	50 sec	200 sec

ported by Hendrickson and Plank (*18*) for work trips commuting to Pittsburgh from the south. These percentages are used in the following example, but they may not be representative of the entire study region or any particular zone within it, because they are based on limited data.

Total 1-hr link volumes assigned in the dynamic case by DTA and CDA to the Pittsburgh network were 404,845 and 401,854, respectively, which are slightly below the total 1-hr link volume assigned by any of the procedures in the static case. However, the mean travel impedances of the DTA and CDA dynamic assignments are 12.3 and 12.1 min, respectively, which are 1 min greater than the mean travel impedances of the static assignments due to nonlinear link impedance functions causing greater than linear increases in trip impedances with increasing travel demands. The objective function values are 3,922 and 3,724 for the DTA and CDA dynamic assignments, respectively.

Additional comparisons between the Pittsburgh dynamic and static assignments are limited to a few of the evaluation measures. Neither of the two link volume variations (APV1 and APV2) is meaningful with time-varying travel demands. The duality gaps of the DTA and CDA dynamic assignments are 0.483 and 0.138 percent, respectively, which are only slightly greater than their respective duality gaps of 0.354 and 0.126 percent for the DTA and CDA static assignments. Concerning temporal node violations in the final solutions, DTA had 6.7 percent violations, whereas CDA had only 4.2 percent. Overall, CDA performed slightly better that DTA in achieving a lower duality gap, but CDA required roughly six times as much CPU time as DTA for the dynamic assignment to converge on this size network.

Table 3 indicates how the total link volumes and mean travel impedances varied over the six time intervals of the Pittsburgh DTA and CDA dynamic assignments. The third and fourth columns give the trip departures in each interval, both in numbers of vehicle trips and as percentages of total 1-hr trip departures. The remaining columns give the mean trip impedances and 10-min link volumes at 10 screenline locations for DTA and CDA, respectively. The screenline locations are on freeways and major arterials along a radius approximately 3 mi from the central business district and essentially capture all significant volumes of traffic approaching downtown Pittsburgh from the eastern communities.

In a previous study using this network, Janson et al. (*12*) found that equilibrium assignment produced reasonable estimates of link volumes but underestimated link impedances, both before and during a major freeway project. That study found the equilibrium assignment link volumes to differ from observed morning peak-hour link volumes at the 10 screenline locations by an APV of 16.2 percent. The level of convergence obtained in the Pittsburgh F-W assignment for this paper is greater than that obtained by Janson et al. (*12*). As a result, the F-W link volumes found here differ from the observed morning peak-hour link volumes at the 10 screenline locations by an APV of 15.1 percent.

By comparison, the DTA static assignment link volumes for the full 1-hr period differed along the screenline from observed traffic counts by an APV of 20.4 percent and from F-W link volumes by an APV2 of 6.0 percent. The CDA static assignment link volumes for the full 1-hr period differed along the screenline from observed traffic counts by an APV of 17.1 percent and from F-W link volumes by an APV2 of 2.5 percent. Thus, CDA produced 1-hr link volumes that are very slightly different from F-W link volumes and only slightly less accurate when compared with actual counts. The average 1-hr screenline crossings from each assignment was exactly 1,000, which is 10 percent different from the observed value of 1,112.

One additional comparison is the extent to which these two procedures achieve similar impedances for alternative routes used between a given origin-destination pair of zones. An examination of used trip paths indicated that four alternative routes connecting a residential zone east of Pittsburgh with the downtown central business district were assigned trips by the F-W algorithm. Routes A and B use all arterial streets, whereas Routes C and D use arterials and portions of a major freeway called the Parkway East. Each route is roughly 8 mi and contains between 19 and 24 links in the coded network. As expected of a highly converged F-W solution, all four routes had the same impedance of 12.3 min (to one decimal place accuracy). The DTA static assignment resulted in average impedances of 12.1, 12.1, 12.5, and 12.2 min for Routes A, B, C, and D, respectively, over the 1-hr assignment period. The CDA static assignment resulted in average impedances of 12.3, 12.1, 12.0, and 12.3 min for Routes A, B, C, and D, respectively, over the 1-hr assignment period. However, the

TABLE 3 SUMMARY OF PITTSBURGH DYNAMIC ASSIGNMENT RESULTS

Time Interval	Time of Day	Trip Depart Prct	Number of Trip Departs	DTA Mean Trip Time	DTA Total Link Volume*	CDA Mean Trip Time	CDA Total Link Volume*
1	6:50	12.5%	3051	10.2 min	1180	10.3 min	1209
2	7:00	16.5%	4027	10.9 min	1478	10.8 min	1511
3	7:10	21.0%	5125	13.1 min	1948	12.9 min	1916
4	7:20	21.0%	5125	14.8 min	2079	14.5 min	2048
5	7:30	16.5%	4027	12.1 min	1699	11.9 min	1680
6	7:40	12.5%	3051	10.5 min	1377	10.4 min	1309

* Total link volume in each interval for the 10 screenline locations.

CDA impedances for these routes were almost exactly constant over the period, whereas the DTA impedances showed significant variation.

Figures 3 and 4 show the travel times of these four routes for trips departing in each interval resulting from DTA and CDA, respectively. Travel times from DTA for these routes are higher and more varied than the travel times from CDA. The travel times all begin at around 11 min (slightly below the SUE travel time of 12 min due to the low initial departure rate of 12.5 percent), rise to between 13 and 14 min, and then decrease to their initial levels as travel demand falls off to a lower, steady-state level. Although observed travel times are not available for each 10-min departure interval, the author's experience with these routes, having lived in the area for 7 years, is that these times underestimate actual times but show relatively valid magnitudes of peak-period variability between intervals. In validating this network for a previous study, Janson et al. (*12*) found that a SUE assignment from the F-W algorithm also tended to underestimate actual impedances along routes where travel time runs had been made.

CONCLUSIONS AND FUTURE RESEARCH

Overall, the F-W, DTA, and CDA procedures were shown to generate similar steady-state assignments in these examples. However, only the CDA procedure steadily converges toward a dynamic equilibrium solution as formulated in this paper. Compared with DTA, CDA was shown to produce assignments that more closely satisfy DUE conditions as measured by the duality gap. Data are unavailable for the networks used in this study with which to validate the DTA and CDA results against observed link counts and travel times in each time interval. The author is arranging to obtain 10-min traffic counts and a coded network from a major metropolitan planning organization for validation purposes.

The link flow formulation of DUE presented in this paper for multiple origins and destinations and the equivalent path flow formulation presented by Janson (*9*) prevent temporally discontinuous flows from entering the solution. An example was given earlier of how methods of linear combinations can easily cause such flows when applied to this problem.

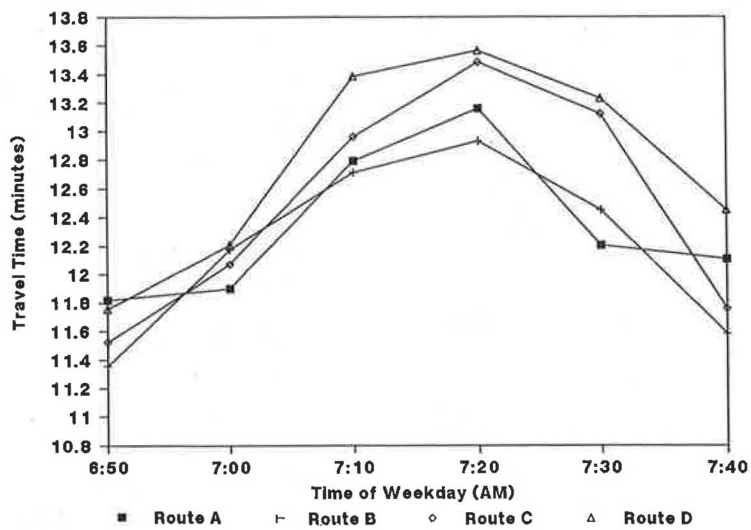

FIGURE 3 DTA impedances of alternative routes.

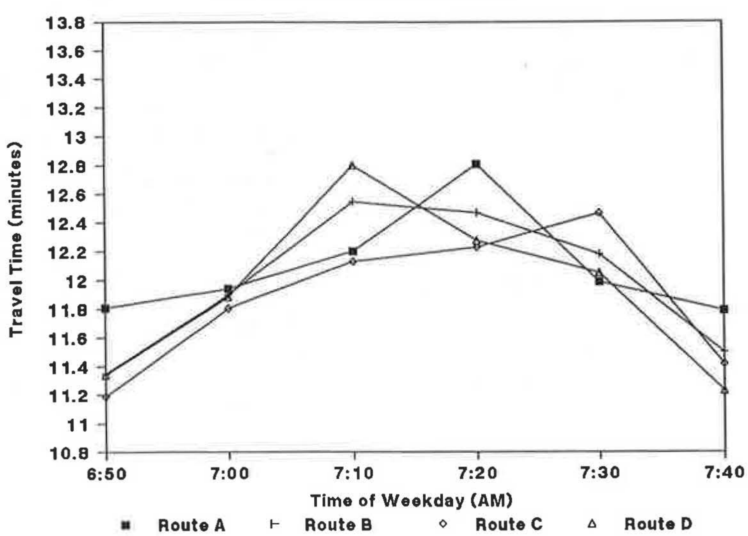

FIGURE 4 CDA impedances of alternative routes.

Hamerslag (*19*) presented results of applying the F-W algorithm to a small network to generate a dynamic assignment but did not evaluate the solution's degree of optimality or temporal continuity. CDA did not exhibit convergence difficulties in the test cases of this paper, and it was relatively quick to converge. With regard to convergence difficulties, Janson and Zozaya-Gorostiza (*20*) show that the F-W algorithm introduces cyclic flows to an assignment that retard its convergence, and this problem will be compounded by the creation of temporally discontinuous flows in dynamic assignments.

The test assignments evaluated in this paper were made with the usual BPR impedance function. Research has been conducted to test whether other functions, such as the Davidson function, may be superior when used in aggregate assignment models (*21*). Hungerink (*22*) suggests a modification of the usual link impedance function as one method of approximating queuing delays in capacity-restrained assignments so that link volumes in excess of capacity affect the impedances of inflow links. Such impedance function alterations might be considered for dynamic assignment procedures, because travel times directly affect the temporal incidences of competing flows on links common to paths from different origins.

A practical advantage of dynamic traffic assignment as formulated and solved in this paper is that it builds directly on the transportation planning data sets and solution algorithms familiar to transportation planners and software developers. DUE and CDA can also be integrated with other travel forecasting procedures. Janson and Southworth (*23*) show how trip departure times can be estimated with dynamic traffic assignment and observed traffic counts on selected links in each time interval. CDA can be run on large networks essentially in real-time using high-speed computers. Thus, CDA is one approach to implementing real-time traffic assignment and route guidance systems on urban transportation networks and to evaluating plans for traffic management during evacuations and special events.

ACKNOWLEDGMENTS

This research was funded by the Director's Research and Development Fund of Oak Ridge National Laboratory and the Federal Emergency Management Agency.

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Publication of this paper sponsored by Committee on Transportation Supply Analysis.