Low-Frequency Vibropile Driving and Prediction of Dynamic Tip Resistance of Piles

M. A. SATTER

Experimental tests have been conducted on mainly open-ended pile models driven into fine-grained sand (soil) by low-frequency vibratory excitations. The minimum input force amplitude that causes a pile to penetrate the soil and the optimum amplitude that ensures the quickest penetration of the pile to the ultimate depth have been determined. Postoptimum force levels have been found to induce a high impactive reaction at the pile tip (i.e., dynamic tip resistance), and this phenomenon has been investigated. A mathematical theory to predict the dynamic tip resistance is proposed. Piles with various tip configurations have been tested.

It is economically important to optimize pile-driving parameters, especially the input power level and the rate of penetration, to achieve the ultimate depth of the pile. It was shown in an earlier study (*1*) that it is possible to use pile records to predict optimum driving parameters. The study was conducted using a model pile with a closed tip driven into loose, fine-grained sand (soil). The vibratory excitation frequency was kept in the range 5 to 60 Hz, substantially below the fundamental longitudinal frequency of the pile. At this range of excitation the elastic deformation of the pile was negligible, thus the pile was considered as a rigid body in the relevant mathematical formulations. A minimum input vector force was found that caused the pile to begin to penetrate the soil very slowly. Likewise, there was an optimum level of input force that ensured the quickest penetration of the pile to the ultimate depth. Furthermore, it was observed that if the input force exceeded the optimum level corresponding to the applied surcharge (Figure 1), no further penetration of the pile was effected. Instead, the pile underwent steady-state vibration, including impactive reaction (dynamic reaction) from the soil.

It was clearly established that the soil reaction was that of a nonlinear cubic spring. A nonlinear equation of motion of the pile was developed, which enabled the accurate prediction of the pile's tip resistance from the pile's dynamical records. The existence of dynamic reaction between the pile and soil has been reported by other researchers (*2,3*).

In vibropile driving, vibrocompaction causes the mechanical properties of soil around the pile to change (*4,5*). The driving parameters and the shape of the pile tip determine the degree of compaction and hence the stiffness of the soil, which in turn influences the dynamic tip resistance of the pile. For the same soil under identical driving inputs, vibrocompaction is expected to be different for differently shaped pile tips. In other words, the degree of compaction of soil will not be the same for closed and open piles, so their dynamic tip resistances can be expected to be different. This paper investigates the depth of penetration and the corresponding level of the optimum force and then studies the dynamic resistance of mainly open piles under the postoptimum condition. Open pile tips with circular and elliptical geometries have been used. The results are compared with those of a closed-tip pile.

Yuasa Battery Ltd., 60/1 Purana Paltan, 2nd Floor, Dhaka, Bangladesh. Current affiliation: Department of Mechanical Engineering, Shiraz University, Shiraz, Iran.

EXPERIMENTS AND RESULTS

Laboratory tests have been conducted on model piles with open tips, but a closed-tip pile was included in the tests for comparison purposes. The soil consists of brown subangular sand from Shiraz, Iran. The granular size of sand corresponds to the 16/40 U.S. standard sieve size. The model pile data are as follows:

- Material: mild steel,
- Length: 0.8 m,
- Outside diameter: 0.019 m, and
- Inside diameter: 0.017 m.

The fundamental longitudinal frequency of the pile is 950 Hz, and the equivalent dynamic mass including that of the pile is 0.7 kg. Among the piles tested, the length and the dynamic mass were the same but the embedded tips were either circular or elliptical. For the piles with elliptical tip openings, the embedded end was tapered to various lengths. The dimensions of the tapers appear in Table 1 and Figure 2.

In the experimental setup (Figure 1), the pile under consideration was connected to a shaker that was fastened with a steel rope and counterbalanced by a suitable weight. The rope was supported by two pulleys affixed to a frame. Frictional force between the rope and the pulleys was kept to a minimum, as was the eccentricity between the shaker and the pile. The excitation frequency was kept at 40 Hz, which was found to be convenient for the pile-soil combination. As is the practice in vibratory pile driving, the combined weight of the shaker, pile, and attachments is balanced by a counterweight, and then a small bias weight, called a static surcharge,

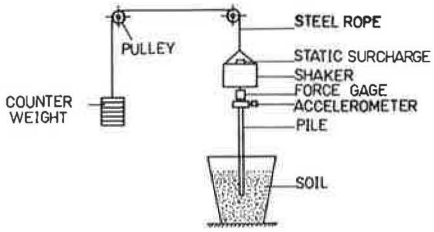

FIGURE 1 Schematic diagram of experimental setup.

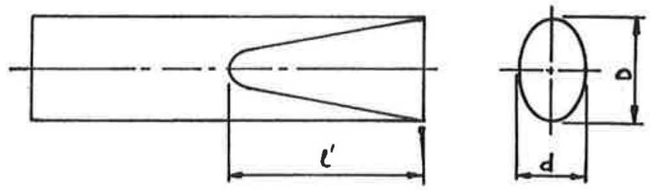

FIGURE 2 Geometry of tapers for Table 1.

is added (Figure 1). The static surcharge is necessary for a pile to penetrate the soil. Static surcharges from 0 to 26.7 N were used for the pile models tested.

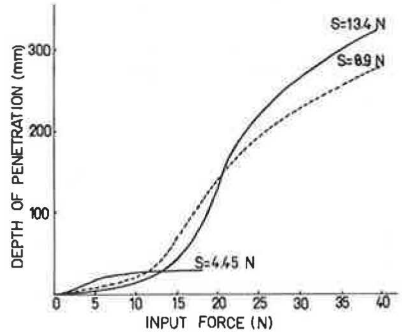

FIGURE 3 Depth of penetration against input force level (open pile).

Depth of Penetration

The records of soil penetration of various piles against input forces are shown in Figures 3–7. The depth of penetration depends on both the input force level and the applied static surcharge. Generally, the pile achieves deeper penetration for a heavier static surcharge, but this necessitates a higher input force level. The embedded tip configuration of a pile has a marked effect on the depth of penetration. For an applied surcharge of 8.9 N, the maximum depths of penetration achieved by the open and closed piles are about 320 and 35 mm, respectively (Figures 3 and 4). Piles with tips lying between the fully open and fully closed configurations will achieve intermediate depths of penetration (Figures 5–7). The three piles (A, B, and C) in Table 1 have almost equal tip openings but different taper lengths. The maximum depths of penetration of the piles with taper lengths of 100, 50, and 35 mm are 90, 165, and 192 mm, respectively, for the applied surcharge of 8.9 N. Thus, the pile with a shorter taper penetrated deeper into the soil than the piles with longer tapers.

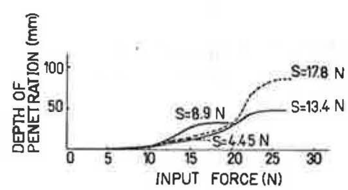

FIGURE 4 Depth of penetration against input force level (closed pile).

Minimum and Optimum Force Levels

Experiments were also conducted to determine the minimum and optimum levels of input force necessary for vibropile driving. The minimum force level is defined here as the force that causes the pile, under a certain static surcharge, to begin penetrating very slowly into the soil. The optimum force level, however, is defined as the input force under which the pile achieves the ultimate penetration within the shortest possible time. If the input force level is increased beyond the optimum

TABLE 1 DIMENSIONS OF TAPER (mm)

Type of Pile		length (ℓ') of taper									
		0	10	20	30	40	50	60	80	100	120
Pile A	D	25.11	--	23.99	-	22.81	-	21.89	20.93	19.52	19.52
	d	7.63	--	9.92	-	11.91	-	13.22	14.31	16.52	19.52
Pile B	D	25.31	24.31	23.19	21.91	20.63	19.61	19.50	-	-	-
	d	7.21	9.10	10.81	12.49	14.29	16.31	19.50	-	-	-
Pile C	D	25.21	24.29	22.62	20.78	19.50	-	-	-	-	-
	d	7.31	8.68	10.69	14.58	19.50	-	-	-	-	-

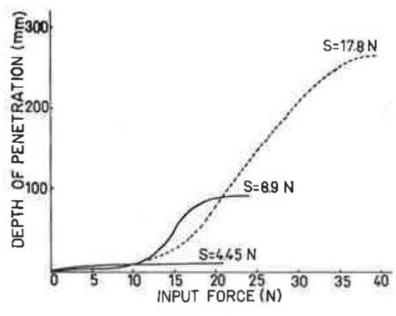

FIGURE 5 Depth of penetration against input force level (Pile A, Table 1).

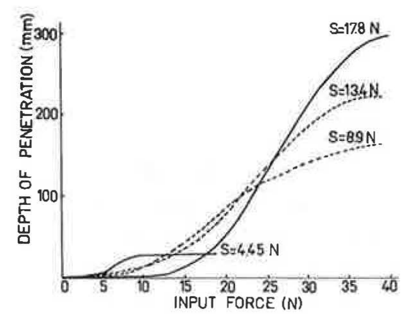

FIGURE 6 Depth of penetration against input force level (Pile B, Table 1).

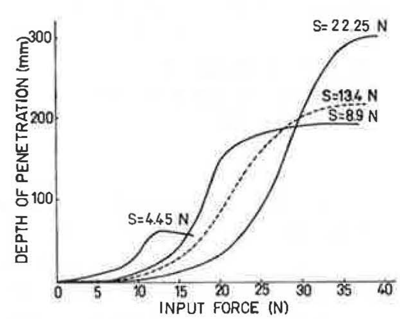

FIGURE 7 Depth of penetration against input force level (Pile C, Table 1).

level, no appreciable increase in the depth of penetration takes place. The minimum and optimum force levels for various tip configurations of the piles are shown in Figure 8.

For all the piles tested, the minimum input force level is the same, and its value should be equal to the applied static surcharge—or slightly greater than it, if some friction is present. Thus, the relationship between the minimum force level and the applied static surcharge is linear, irrespective of the tip configuration. But the optimum force level depends greatly on the shape of the pile tip. A linear relationship between the optimum force level and the applied static surcharge can be observed for a closed pile; the constant of proportionality is given by $\alpha = F_o/S = 2.45$. With a slightly open pile, the optimum force level increases rapidly; the greatest increase occurs for a fully open pile, which also achieves the deepest

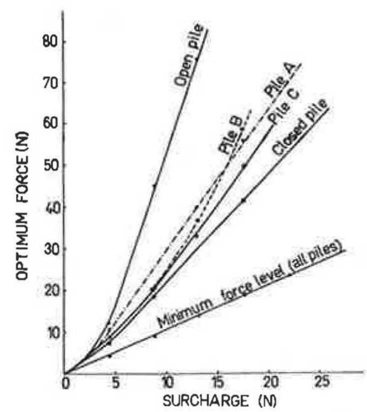

FIGURE 8 Minimum and optimum force levels against static surcharge.

penetration. For a low surcharge domain, the relationship between optimum input force and applied surcharge appears to be nonlinear. This is probably due to the predominance of a small friction force that is present. The effect of the tip opening on the optimum force level and the respective depths of penetration are clearly seen in Figure 8. For example, for an applied surcharge of 8.9 N, the optimum force levels for closed, semiopen (Pile A), and open piles are 18, 28, and 46 N, and the depths of penetration are 4, 6.5, and 17 mm, respectively.

Dynamic Tip Resistance

As stated earlier, a pile under a certain static surcharge penetrates the soil to an ultimate depth under the influence of the optimum force. If the applied force is then increased beyond the optimum level and the surcharge is not increased, the pile does not penetrate the soil further. Instead, the pile undergoes steady-state vibration, and the dynamic tip resistance at this condition is investigated here.

The theoretical basis for the measurement of tip resistance is explained as follows: because the pile undergoes a rigid body motion, the tip resistance may be expressed simply by

$$TR = F(t) - m\ddot{x}$$

where

TR = tip resistance,
$F(t)$ = input force,
m = effective mass of the pile and its attachments, and
$\ddot{x}$ = instantaneous acceleration of the pile.

This equation neglects skin friction of the pile. The loss of accuracy for this assumption is found to be insignificant. A small clearance is probably created around the pile during the steady-state vibratory condition, so the effect of skin friction becomes negligible. There is no tip resistance ($TR = 0$) while the pile vibrates outside the soil; therefore $F(t) = m\ddot{x}$. In the experimental setup, a force gage and an accelerometer may be used to measure the input force [$F(t)$] and the pile inertia ($m\ddot{x}$). The signals of the transducers may be adjusted and

combined to produce a null resultant for the unembedded pile. Phase distortion was kept to a minimum. The signals of tip resistance for various piles are similar, and only a representative set of signals is plotted on an enlarged scale in Figure 9. The results are summarized in Table 2. The results for a closed pile have been included for comparison.

In Figure 9, the phase difference between the input force (Curve A′) and the inertia (Curve B′) of the pile while it is vibrating outside the soil is 180 degrees, and their algebraic sum (A′ plus B′) is zero. However, while the embedded pile undergoes steady-state vibration, the inertia signal (Curve B) changes its phase by almost 180 degrees because of soil reaction, and it reinforces the input force signal (Curve A). The resulting signal (Curve A + B) shows a series of peaks spaced by the period of excitation.

For any type of pile (open or closed), a higher static surcharge warrants a greater input force level, which in turn induces greater tip resistance. The optimum input force level of an open pile is normally higher than that of a closed pile.

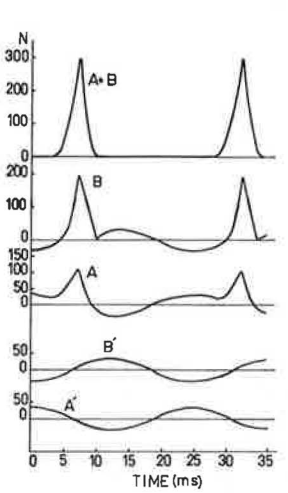

FIGURE 9 Dynamic tip resistance of an open pile, S = 6.7 N, F = 40 Hz.

This is because an open pile penetrates deeper than a closed pile under identical conditions. The tip resistance of an open pile, however, does not increase in the same proportion with respect to the applied force as the tip resistance of a closed pile does. For example (Table 2), the tip resistance of an open pile is only 197 N for the applied force of 49 N, whereas the tip resistance of a closed pile is 107 N for the applied force of 13.7 N; the static surcharge in both cases is 6.7 N. In this instance, the ratio (open/closed) of the applied force levels is 3.58, whereas the ratio of the tip resistances is 1.84. This indicates that for a fixed set of driving parameters, the closed pile will experience greater tip resistance than an open pile will. Also from Table 2, it is clear that, depending on the tip opening area, a semiopen pile will experience tip resistance greater than that of an open pile but less than that of a closed pile.

THEORY

When the experimental results of the dynamic tip resistance of an open pile are compared with those of a closed pile, it becomes apparent that there are similarities in the nature and duration of the reaction between the pile and soil. A pile-soil model is shown in Figure 10 in which the soil is represented

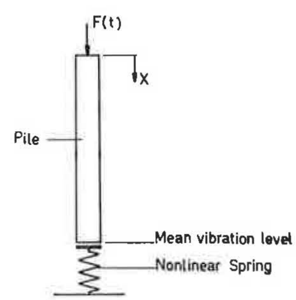

FIGURE 10 Theoretical model of pile vibration.

TABLE 2 COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

Type of Pile	Static Surcharge S(N)	Frequency (Hz)	Applied Force F_o(N)	ACC (Expt.) (g)	Expt. TR max(N)	TR max(N) (Eq. 5')	Discrepancy
Closed Pile	6.7	40	13.7	10	107	111.3	+4%
	17.8		39.3	25	276	285	+3%
Open Pile	6.7	40	49	14.0	197	196	-0.5%
Pile A	8.9	40	24	9.4	116	120	+3%
	17.8		49	25.0	328	300	-8%
Pile B	8.9	40	22	11.9	140	140	0%
	13.4		24	12.5	153	148.5	-3%
Pile C	8.9	40	26	7.5	105	105	0%

by a nonlinear cubic spring. The pile-soil reaction takes place only during the downward motion from the mean vibrational level of the pile. The tip resistance of a pile can then be written as

$$TR = R[H(t - t_1) - H(t - t_2)]x^3 \quad (1)$$

where

R = dynamic soil parameter to be determined,
x = pile displacement,
$H(t - t_1)$ = unit step function, and the expression in brackets represents a filter function.

Neglecting the skin friction, the steady-state vibrational motion of the pile may be expressed by

$$m\ddot{x} + R[H(t - t_1) - H(t - t_2)]x^3 = S + F_o \sin \omega t \quad (2)$$

where

S = static surcharge,
F_o = input force amplitude, and
ω = excitation frequency.

Equation 2 is nonlinear (*6,7*), and its approximate solution is assumed to be $x(t) = a \sin \omega t$, where a is the amplitude of vibration to be determined. Adding $\omega^2 x$ to both sides of Equation 2 and substituting $x(t) = a \sin \omega t$ into the right-hand side obtains

$$\begin{aligned}\ddot{x} + \omega^2 x = {} & \frac{S}{m} \\ & + \left\{\frac{F_o}{m} + \omega^2 a - \frac{3Ra^3}{4m}[H(t - t_1) - H(t - t_2)]\right\} \\ & \times \sin \omega t + \frac{Ra^3}{4m}[H(t - t_1) \\ & - H(t - t_2)] \sin 3\omega t \end{aligned} \quad (3)$$

To avoid the secular term (*7*) in Equation 3, impose the condition

$$\frac{F_o}{m} + \omega^2 a - \frac{3Ra^3}{4m}[H(t - t_1) - H(t - t_2)] = 0 \quad (4)$$

Equation 4 is significant because it relates the two important unknown quantities a and R. Normally, a can be measured and hence R can be expressed in terms of other measurable quantities. Thus, for $t_1 \leq t \leq t_2$,

$$R = (4F_o + 4m\omega^2 a)/3a^3$$

and

$$Ra^3 = (4F_o + 4m\omega^2 a)/3 = TR \quad (5)$$

After deleting the secular term, Equation 2 may be reduced to

$$\ddot{x} + \omega^2 x = \frac{S}{m} + \frac{Ra^3}{4m}[H(t - t_1) - H(t - t_2)] \sin 3\omega t \quad (6)$$

From Satter and Ghahramani (*1*), the solution of Equation 6 is given by

$$\begin{aligned} x(t) = {} & \frac{S}{m\omega^2} + \left(a - \frac{S}{m\omega^2}\right) \sin \omega t \\ & - \frac{Ra^3}{32m\omega^2}(\sin \omega t + \sin 3\omega t) \end{aligned} \quad (7)$$

The acceleration

$$\begin{aligned} \ddot{x}(t) = {} & \frac{S}{m} - \omega^2\left[\left(a - \frac{S}{m\omega^2}\right) \sin \omega t \right. \\ & \left. - \frac{Ra^3}{32m\omega^2}(\sin \omega t + 9 \sin 3\omega t)\right] \end{aligned} \quad (8)$$

In Equation 8, the constant acceleration term is added to account for the static surcharge.

As mentioned earlier, the inertia signal changes its phase by 180 degrees during pile-soil reaction; hence, in computing the tip resistance from Equation 5, the following restrictions must be observed:

$$TR = F(t) - m\ddot{x} \qquad \text{for } t_1 \leq t \leq t_2 \quad (9)$$

and

$$TR = 0 = F(t) + m\ddot{x} \qquad \text{for } t < t_1 \text{ or } t > t_2 \quad (10)$$

In the case $TR = 0$, amplitude (a) must be calculated from $a = F_o/m\omega^2$. The dynamic tip resistance may be calculated easily from Equation 5.

DISCUSSION OF THEORETICAL AND EXPERIMENTAL TIP RESISTANCE

Both theoretical and experimental values of tip resistance for various piles are summarized in Table 2. The theoretical results have been calculated from Equation 5, but the values of pile acceleration required for the calculations are those obtained experimentally. It is clear that the theoretical and experimental values of the tip resistances for various piles have good agreement. With the exception of one reading (Pile A), the theoretical results are within 4 percent of the experimental values. For the open pile, the theoretical tip resistance value is within 0.5 percent of the experimental value. It is noted from Equation 5 that the tip resistances do not depend directly on the applied static surcharge but on the applied force level. Surcharge, however, influences the tip resistance indirectly because it controls the level of applied optimum force to the pile.

CONCLUSIONS

This study has provided several useful observations about low-frequency vibropile driving. As expected, for a fixed set of static surcharges, an open pile will achieve the greatest depth of penetration and a closed pile will achieve the least; a semiopen pile will achieve intermediate depth. The minimum force level

that the shaker must supply for the pile to begin penetration does not depend on the tip configuration (open or closed). The optimum force level at which the pile achieves the ultimate depth of penetration corresponding to a certain applied static surcharge is generally greater for the open pile than that for the semiopen or closed pile. The geometry of the pile tip has a marked effect on dynamic tip resistance. Under the same driving conditions, an open pile experiences the least tip resistance, a closed pile experiences the greatest, and a semiopen pile experiences levels in between. The pile-soil reaction occurs during the downward motion of the pile from the mean vibration level, and its duration is about one-quarter of the period of excitation. The dynamic tip resistance is proportional to the cube of the pile's steady-state displacement, and it does not depend directly on the applied static surcharge. The knowledge of the input force amplitude and pile inertia level are the measured quantities necessary to commute the pile tip resistance from the theory provided in the paper. The theoretical and experimental results have good agreement.

ACKNOWLEDGMENTS

The author expresses his thanks to A. Ghahramani and N. Mostaghel of Shiraz University for useful discussions, to the reviewers for useful suggestions for improving the paper, and to Shiraz University for provision of facilities. Thanks are also due to Mrs. Shaterpouri for typing the manuscript.

REFERENCES

1. M. A. Satter and A. Ghahramani. Prediction of Tip Resistance from Pile Dynamics. *Iranian Journal of Science and Technology* (in preparation).
2. F. Rausche, F. Moses, and G. G. Goble. Soil Resistance Predictions from Pile Dynamics. *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 98, No. SM9, 1972, pp. 917–937.
3. R. H. Scanlan and J. J. Tomko. Dynamic Prediction of Pile Static Bearing Capacity. *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 95, No. SM2, 1969, pp. 583–604.
4. Y. Koizumi et al. Field Tests on Piles in Sand. *Soils and Foundations*, Vol. 11, No. 2, 1971, pp. 29–49.
5. D. D. Barkan. *Dynamics of Bases and Foundations* (translated from Russian by L. Drashevska). McGraw-Hill, New York, N.Y., 1962.
6. I. I. Bykhovsky. *Fundamentals of Vibration Engineering*, (translated from Russian by V. Zhitomirsky). Mir Publishers, Moscow, Russia, 1972.
7. Y. Panovko. *Elements of the Applied Theory of Elastic Vibration* (translated from Russian by M. Konyaeva). Mir Publishers, Moscow, Russia, 1971.