Centrifugal Modeling of Consolidation Phenomena

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Geotechnical centrifugal modeling is a technique whereby centrifugal accelerations are used to create prototype stresses within a small-scale model. Diminishing prototype geometry drainage path lengths in the model also permit rapid simulation of large prototype time frames. Centrifugal modeling of consolidation behavior is reviewed. Centrifugal modeling examples presented are large strain (self-weight) consolidation of reclamation schemes involving waste clays, determination of consolidation behavior parameters, observation of consolidation behavior, and computer model verification. The reviews indicate that centrifugal modeling is a feasible technique for assessing geotechnical problems involving consolidation.

Traditional design and analysis of geotechnical engineering problems are based on small-scale laboratory tests or in situ test correlations to develop parameters for material behavior, which are then used in various analytical solutions. The analytical techniques currently used include conventional 1-D Terzaghi theory and 2-D and 3-D finite element methods and finite difference approaches. The validity of these techniques must be verified to ensure that limitations of small-scale laboratory tests used to define soil properties are adequate. In addition, mathematical modeling to produce the observed laboratory or field response is difficult because the behavior of soils is greatly influenced by factors such as (*a*) water content, density, and soil structure; (*b*) previous stress history; (*c*) stresses imposed by boundary conditions; and (*d*) a nonlinear, hysteritic, and time-dependent behavior. Since it is virtually impossible to evaluate all these effects in laboratory tests or by means of nonlinear mathematical models or analytical techniques, the unknown degree of uncertainty must be covered by safety factors or overly conservative estimates of soil properties.

The various limitations of precise design and analysis techniques make full-scale testing attractive; however, because of the large soil masses and weights of material required to include gravity forces, and time frame involved, full-scale tests are seldom performed to evaluate consolidation behavior. Consequently, soil modeling, in a manner that includes gravity force, offers an attractive and needed verification method.

BASIC CENTRIFUGAL MODELING THEORY

The basic concept of centrifuge model testing is to create a scale model similar in geometry, material properties, and boundary conditions to the full-scale prototype, and to subject the scale model to an acceleration such that the increase in self-weight stresses matches those at corresponding points in the prototype. Thus applying Newton's second law ($F = ma$), the stress σ simply becomes the force F divided by the area A or $\sigma = F/A$ and $\sigma = F/A = ma/A$.

Since $m = W/g$, where W = weight and g = gravity, then $\sigma = Wa/Ag$. Considering that unit weight γ = weight volume $= W/Ah$, where h = height of material, then $\sigma = \gamma Aha/Ag = \gamma ha/g$. At the earth's surface $a = g$, and thus $\sigma = \gamma h$, which is the familiar equation for vertical geostatic stresses in a horizontal soil deposit. If we wish to model this soil deposit by a scale of n (i.e., h/n), then $\sigma = \gamma ha/ng$, and we see that the stress is only $1/n$th of the prototype. Therefore to achieve similitude, the acceleration must be increased by a factor of n, such that $\sigma = \gamma hna/ng = \gamma ha/g$. Thus, the first law of centrifugal modeling may be stated as follows:

> If soils with identical friction, cohesion, and density are formed into two geometrically similar bodies, one of a prototype of full scale and one a model of $1/n$th, and if the $1/n$th scale model is accelerated so that the self-weight increases n times, the stresses at corresponding points are then similar.

If one considers dissipation of pore water pressure,

$$t_p = Th^2/cv$$

where

t_p = time in prototype,
T = time factor,
h = thickness of layer, and
cv = coefficient of consolidation.

In the case of a model of $1/n$th scale $t_m = T(h/n)^2/cv$, where t_m = time in model, then $t_m = t_p/n^2$. Thus the second law of centrifugal modeling can be stated as follows:

> Once the excess pore pressure distribution has been made to correspond in model and prototype, all subsequent primary flow processes of pore water are correctly modelled after time t_m in the model that is less than time t_p in the prototype in the ratio of the square of the scale factor n. (*1*)

From these considerations, centrifugal modeling is most attractive for examining consolidation phenomena. For example, a 1/100th scale model at 100 g models time as 1 min = 6.9 prototype days or 52.6 min = 1 year prototype.

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OBJECTIVE

The objective of this paper is to review and examine centrifugal modeling of consolidation behavior.

Centrifugal modeling is becoming more in vogue with over 50 geotechnical modeling centrifuges worldwide. At this time centrifugal modeling appears akin to numerical modeling of two decades ago. It is a feasible technique for examining geotechnical problems but has not entered the mainstream of geotechnical engineering.

LARGE STRAIN (SELF-WEIGHT) CONSOLIDATION

Reclamation schemes involving dredged materials, mine waste clays, and so forth require predictions of consolidation rates and final consolidation levels for estimating storage capacities of contaminant areas or for considering alternative disposal techniques (e.g., sand/clay mixes, surcharge capping, or stage filling). Current large strain computer programs can predict some boundary conditions (homogeneous deposits, uniform surcharges) but are limited when considering nonuniform (layered) deposits and stage filling (*2*). In addition, input parameter determination for such soft soils requires specialized laboratory testing (e.g., slurry columns, CRS slurry consolidometers, and pump flow).

Bloomquist and Townsend (*3*) provide one of the few centrifuge–numerical model prototype verifications for a waste phosphatic clay. In the study, a "modeling of models" centrifuge test series at 40, 60, and 80 g replicating the same prototype was performed to ascertain the time scaling exponent. Although pore pressure dissipation scales as n^2, sedimentation scales as n. Thus for cases where sedimentation/consolidation occurs, the exponent varies between 1.0 and 2.0. These centrifuge results in Figure 1 show the time scaling exponent for solids contents (s) between 14 percent ($e = 16$) and 20 percent ($e = 11$) [$e = G(1 - s)/s$]. Using these centrifugal test–devised exponents, a comparison was made with a 2.79- × 4.27-m metal prototype tank test that self-weight consolidated from an initial solids content of 12.6 to 21.2 percent over a 403-day period. Using Somogyi's finite strain consolidation program (*4*), a numerical prediction was also made. These comparisons are shown in Figure 2 and reveal an excellent agreement.

Centrifuge to Obtain Consolidation Properties

Large deformation (finite strain) self-weight consolidation equations (*5,6*) differ from Terzaghi consolidation in that k, m_v, and layer thickness are not constant during consolidation. Therefore, nonlinear σ-e and k-e relationships are required as input for computer codes.

Takada and Mikasa (*7*) used centrifugal modeling to obtain these nonlinear relationships. In their tests 30-cm-diameter by 100-cm-thick specimens of clay at different water contents but approximately twice their liquid limit were "cured" several days at 1 g to minimize "hindered settling" before accelerating at 150 g for 1,000 to 5,000 min. By monitoring the specimen surface settling rate ($\dot{s}$) for different e_o values, from Mikasa's theory (*6*) $k = \frac{\dot{s}}{n}\gamma_w/\gamma$ a k-e relationship as shown in Figure 3 was obtained. Upon stopping the centrifuge, an

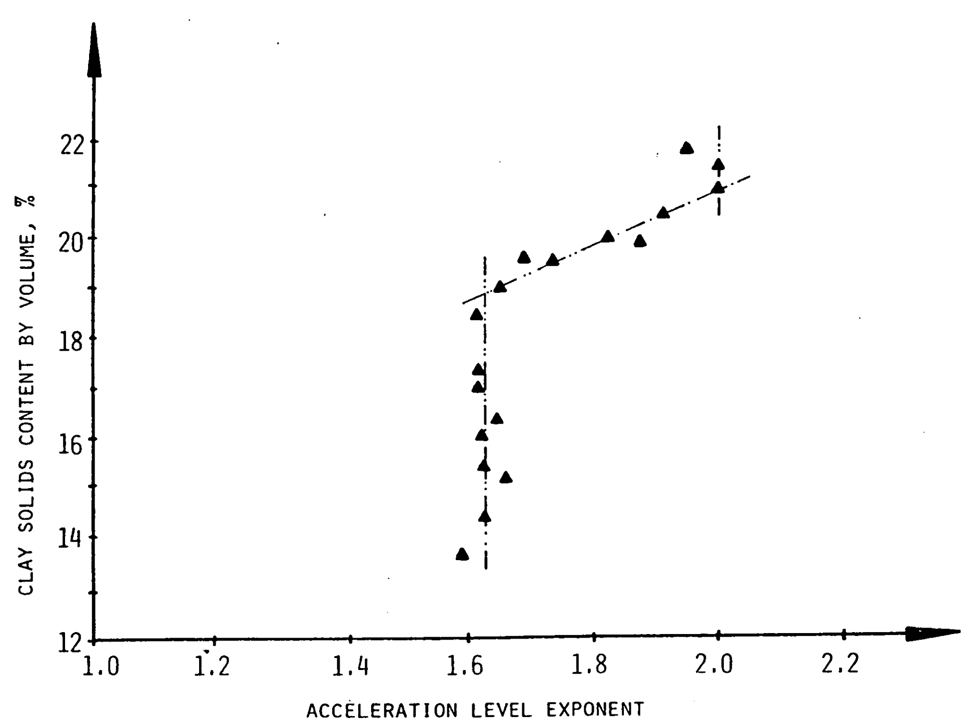

FIGURE 1 Time scale exponent versus solids content (*3*).

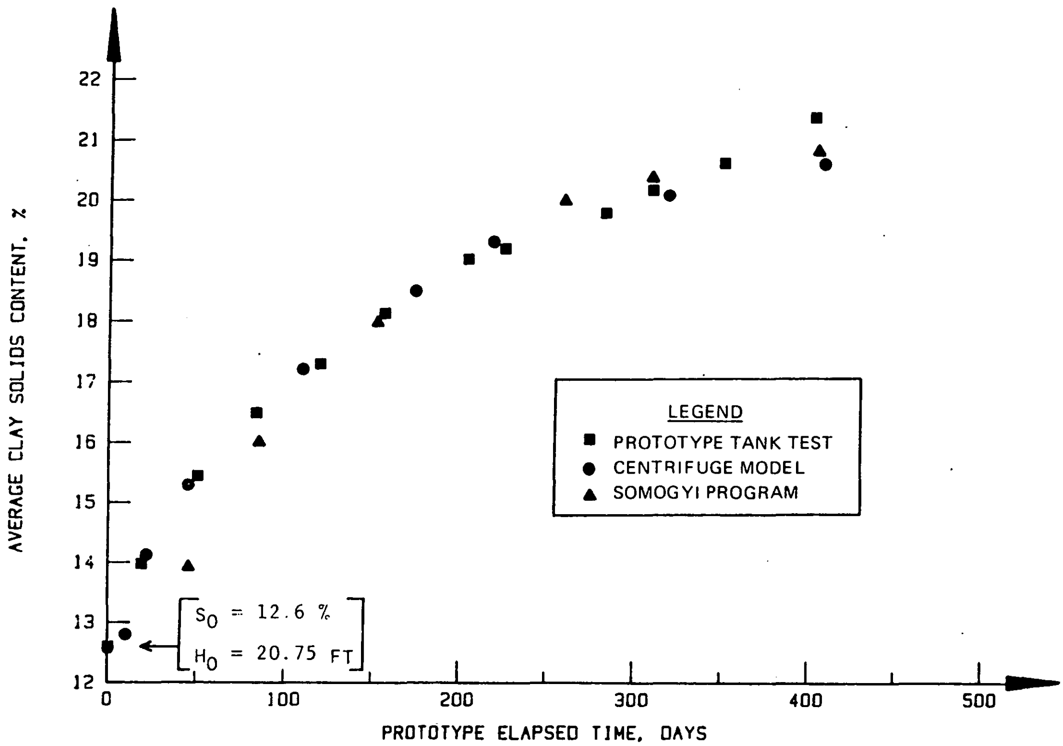

FIGURE 2 Comparisons of prototype tank test, centrifugal model, and numerical prediction (3).

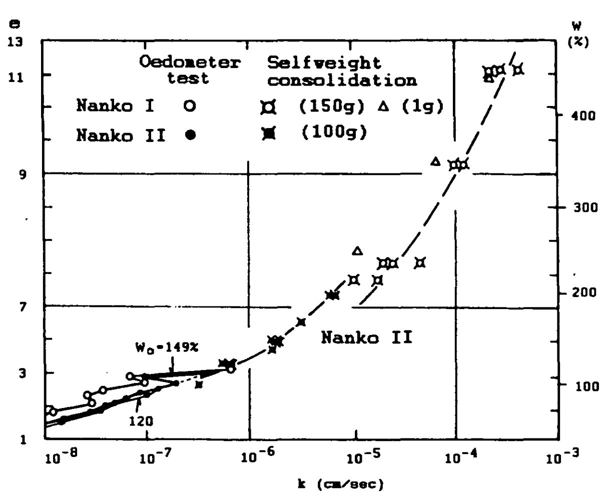

FIGURE 3 Permeability versus void ratio (7).

undisturbed 5-cm soil column was sampled throughout the specimen length using a thin-walled tube, and water content determinations were made for every 2- to 3-mm slice. The effective overburden pressure P' was obtained by integrating $n\gamma'$ from the specimen surface to a specific depth. Thus values of e–log P' relationships as presented in Figure 4 were obtained. This relationship is not unique for very low P' values.

To circumvent the multiplicity of centrifuge tests using Takada and Mikasa's approach, the University of Florida (UF) used an approach based on measurements of pore pressure and void ratio with depth and time during a centrifuge test (8). The void ratio values were obtained by using small individual 5-cm-diameter subsample tubes and periodically stopping the centrifuge, removing a tube to determine the water content distribution. The permeability values were calculated from the pore pressure measurements. Figures 5 and 6 present the e–log P' and e–log k relationships for centrifuge tests on a waste phosphatic clay as compared with 1 g constant rate of deformation consolidometer (CRD) tests. The compressibility curves show good agreement for the virgin zone of the curve, particularly when the slower deformation rate is used for the CRD test. Unfortunately, the centrifuge permeability values are a half order of magnitude greater than the CRD tests, which is probably due to difficulties in measuring pore pressures along the side of the centrifuge container.

Phenomenological Observations

Townsend et al. (9,10) used centrifugal modeling to evaluate disposal schemes to enhance densification/consolidation phosphatic waste clays and thus reduce storage area. Specifically, (a) use of flocculents, (b) sand/clay mixes, and (c) surcharging were investigated by centrifuge models. It was deemed that centrifugal modeling would be easier than numerical modeling in determining how waste clays responded to these treatments due to the difficulty and uncertainties of input parameters for the latter. Figure 7 shows the flocculent effectiveness in increasing the sedimentation rate. Although 1-g bench tests (not shown) suggested that flocculents were not beneficial and the final solids content achieved is reduced because of the formation of large flocs, the centrifuge tests belie this conclusion and show final solids contents only slightly lower than un-

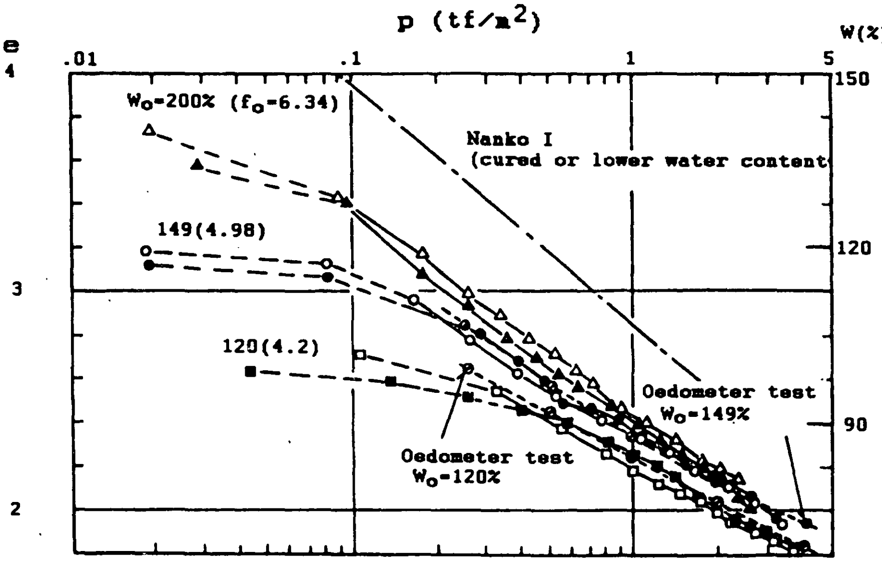

FIGURE 4 Void ratio–log *P* relationships (*7*).

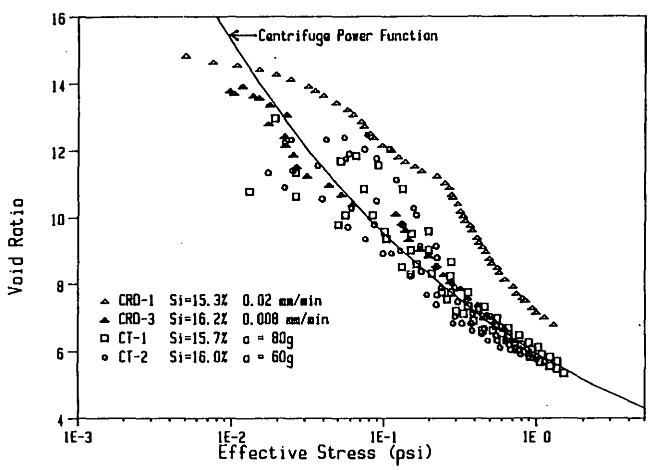

FIGURE 5 Compressibility relationships (1 psi = 6.9 kPa) (*8*).

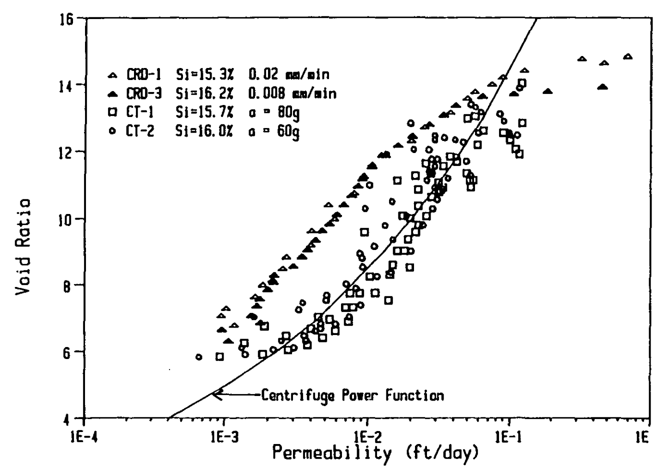

FIGURE 6 Permeability relationships (1 ft/day = 3.53 × 10^{-6} m/sec) (*8*).

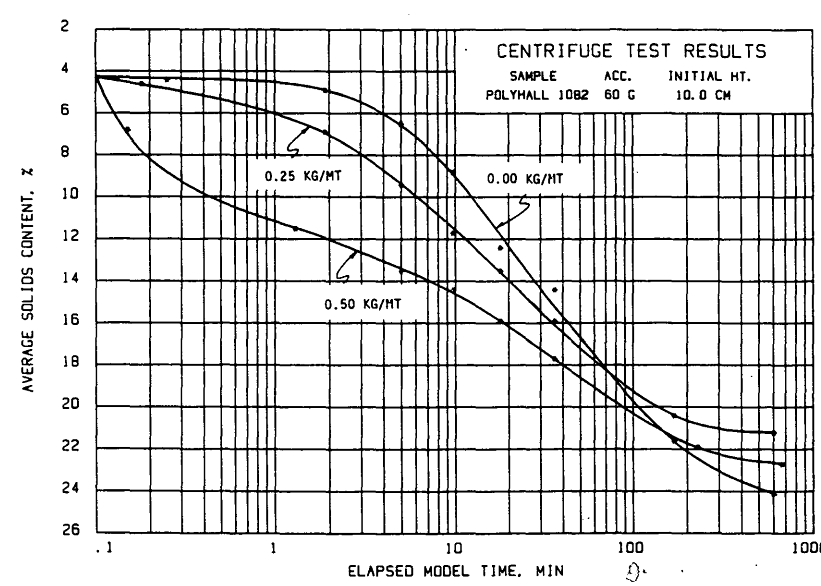

FIGURE 7 Effect of Polyhall flocculent on solids content (*10*).

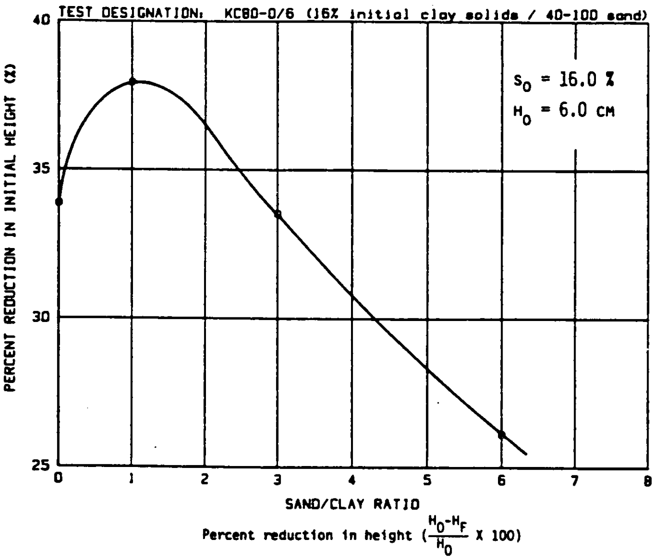

FIGURE 8 Percent reduction in height versus sand/clay ratio (*9*).

treated nonflocculated (dosage = 0.00 kg/MT) solids contents. When the agglomerated flocs are subjected to field stress conditions, they collapse, but they never achieve higher solids contents than untreated clay. Thus treatment with flocculents gains sedimentation rates at the expense of lower solids contents or increased storage areas.

One disposal scheme is to mix trailings sands with the waste clays, thereby increasing the unit weight and subsequent consolidation. Figure 8 shows that an optimum sand/clay mix ratio (SCR) exists at 1:1 ≈ 2:1, and that SCRs greater than 3:1 produce less consolidation. This observation is attributed to interference by the sand grains at higher SCRs preventing densification of the waste clay within the sand matrix. (One concern in these model tests was segregation of the sand particles due to centripetal forces invalidating the models. This was avoided by maintaining a sufficient clay viscosity.)

Surcharging waste clay ponds by flowing a sand/clay cap onto the pond's surface to increase effective stresses was a "mixed bag." Several considerations for applying this scenario were apparent: (*a*) bearing capacity of the underlying clay sufficient to support the surcharge, (*b*) field considerations of actually placing a surcharge on very soft clays, (*c*) large time frames required for consolidation because of impeded drainage from crusting at the sand cap interface, and (*d*) the occupation of disposal space by an incompressible cap. Figure 9 compares the effects of an SCR surcharge cap in a 6-m waste clay pond at ≈ 24 percent solids over which a 3.2-m surcharge is placed. Although the 6:1, 3:1, and 1:1 SCR mixes produced increasing solids contents of 35.2, 32.2, and 31.5 percent, respectively, in the underlying clay, this increase in solids content did not translate into considerable reductions in interface height because of the incompressibility of the sand.

Croce et al. (*11*) used seven centrifugal model tests to evaluate the influence of vertical sand drains on consolidation behavior. The prototype modeled was a single drained 7.5-m clay deposit into which 1.9-m-diameter drains were placed on a 10.1-m spacing. Three successive 1.8-m sand fills (γ = 18.3 kN/m^3) would be placed on the clay surface. The model tests were reduced to a single equal strain consolidometer cylinder with an equivalent diameter of 11.4 m as shown in Figure 10. Vertical, radial, or combined flow conditions could be imposed on the model. Figure 11 and Table 1 present the settlement-time results and show that the final observed settlements are in agreement with those calculated from odometer tests. (The scatter was explained by initial height variations.) Traditionally, the combined degree of consolidation is estimated as $U_{rx} = 1 - (1 - U_R)(1 - U_x)$ using superposition of Terzaghi's theory. Thus c_v was evaluated by fitting the log

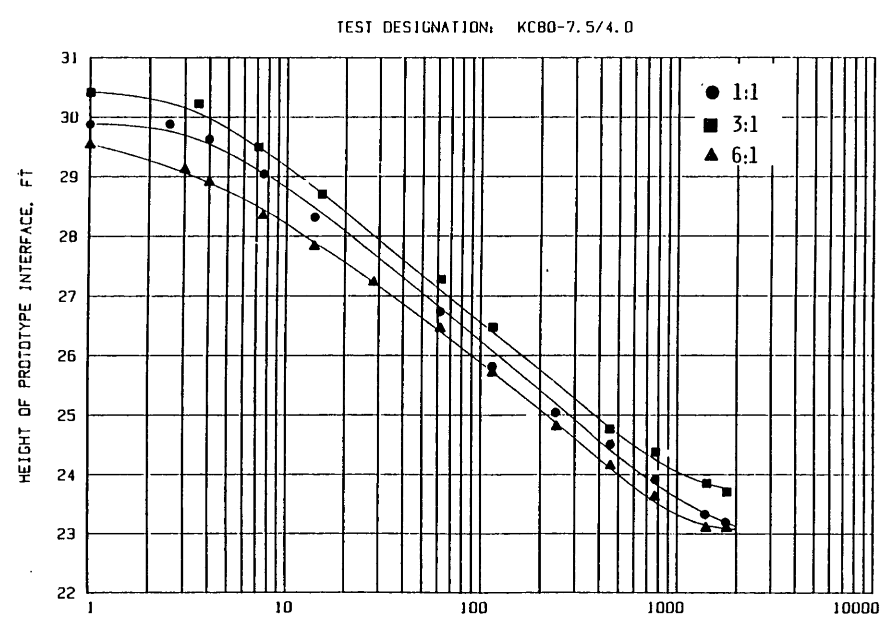

FIGURE 9 Prototype interface versus model time for 4-cm cap (*9*).

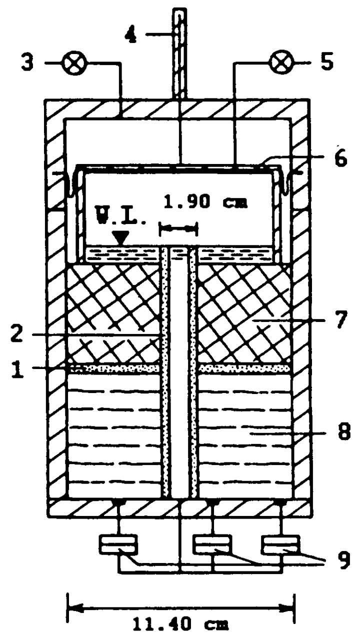

FIGURE 10 Sand drain model apparatus: 1, porous (or impervious disc); 2, porous (or impervious disc); 3, load pressure; 4, LVDT; 5, back pressure; 6, loading ram with convoluted rubber membrane; 7, piston; 8, clay sample; 9, differential pressure transducers (*11*).

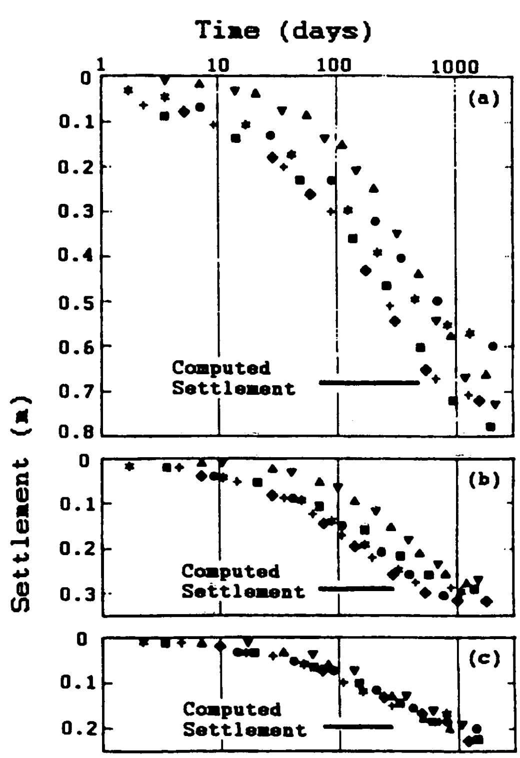

FIGURE 11 Settlements versus prototype time (*11*).

TABLE 1 Model Tests (*11*)

Symbol	Test #	Pore Water Flow	Height (cm) Initial	Step 1	Step 2	Step 3
●	1	Vert.	7.47	6.87	6.52	6.32
■	2	Vert.	7.57	6.79	6.49	6.27
▲	3	Rad.	7.20	6.54	6.26	6.06
▼	4	Rad.	7.58	6.86	6.59	6.39
◆	5	Comb.	7.43	6.71	6.39	6.20
+	6	Comb.	7.43	6.71	6.39	6.20
✳	7	Comb.	7.12	6.54	6.25	6.08

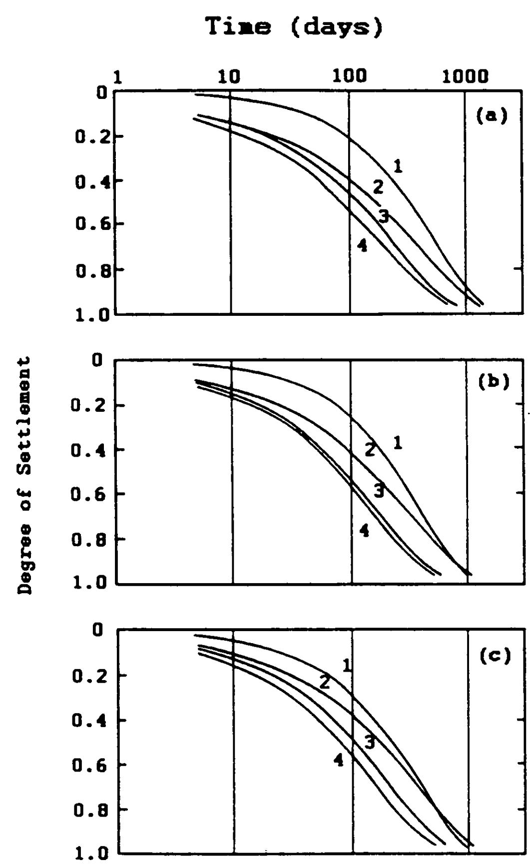

FIGURE 12 Analysis of combined flow settlements (see Table 1) (*11*).

settlement data, while c_v was fitted using Barron's theory (*12*). Figure 12 shows an analysis of combined flow settlements and reveals that the calculated rate of settlement (Curve 4) is always faster than the measured (Curve 3); thus linear superposition overpredicts settlement rates. (Curves 1 and 2 of Figure 12 are for radial or vertical flow only, respectively.)

Computer Model Verification

Mitchell and Liang (*13*) conducted a "modeling of models" test series using accelerated levels of 75 to 97 g of an

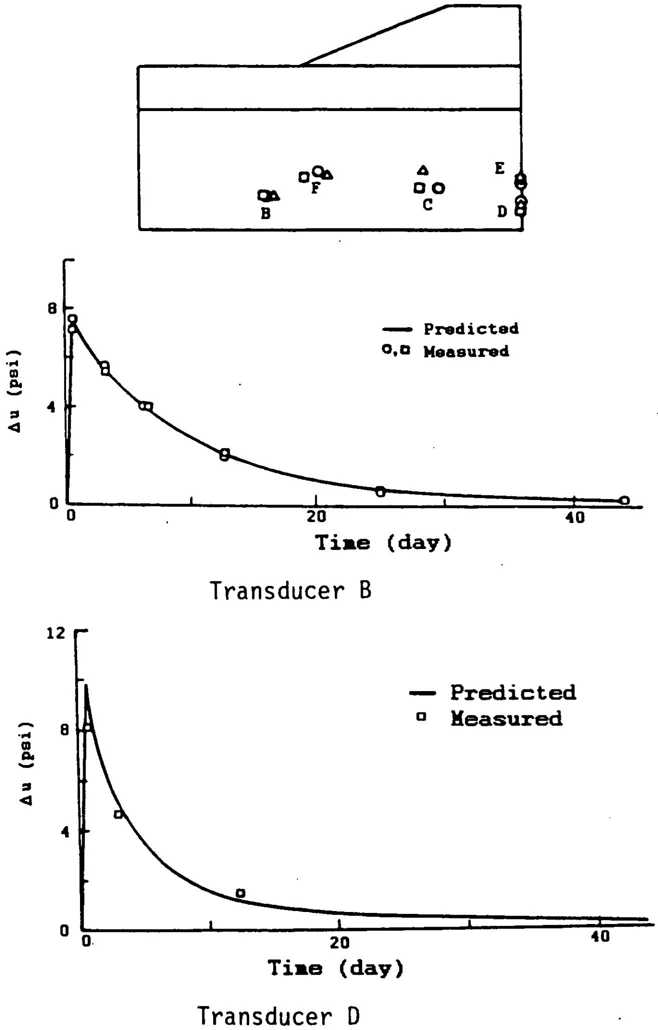

FIGURE 13 Comparisons of pore pressure response (*13*).

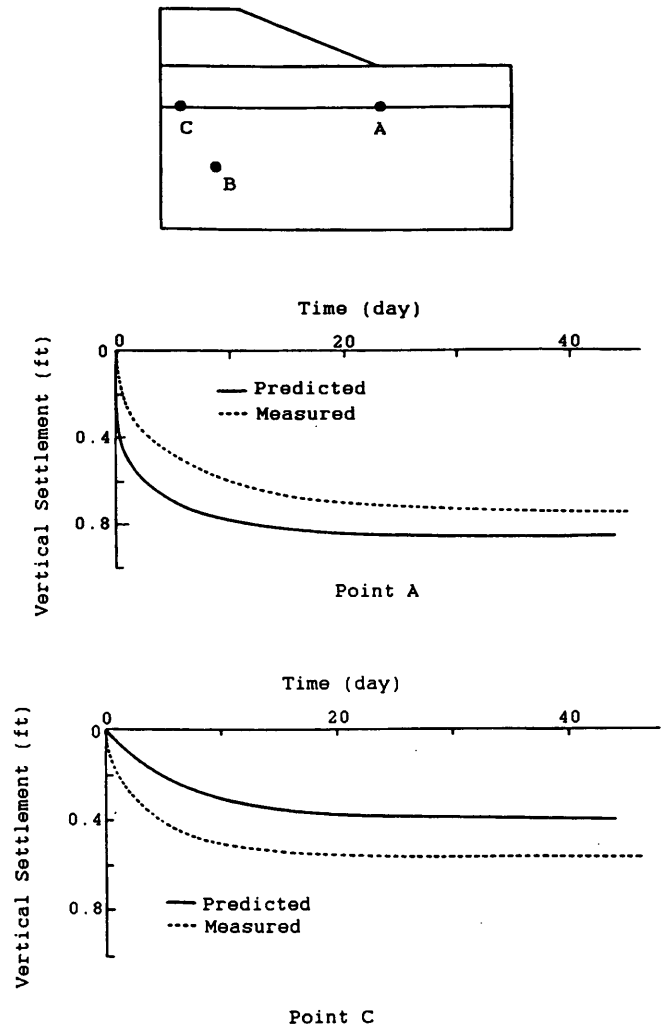

FIGURE 14 Comparison of measured and predicted vertical settlements (*13*).

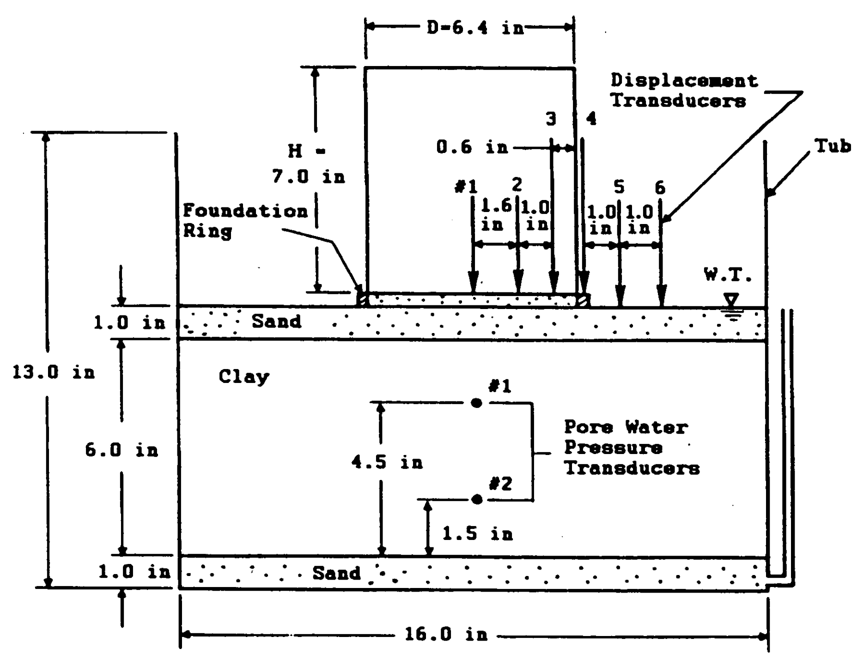

FIGURE 15 Dimensions of model oil storage tank (*14*).

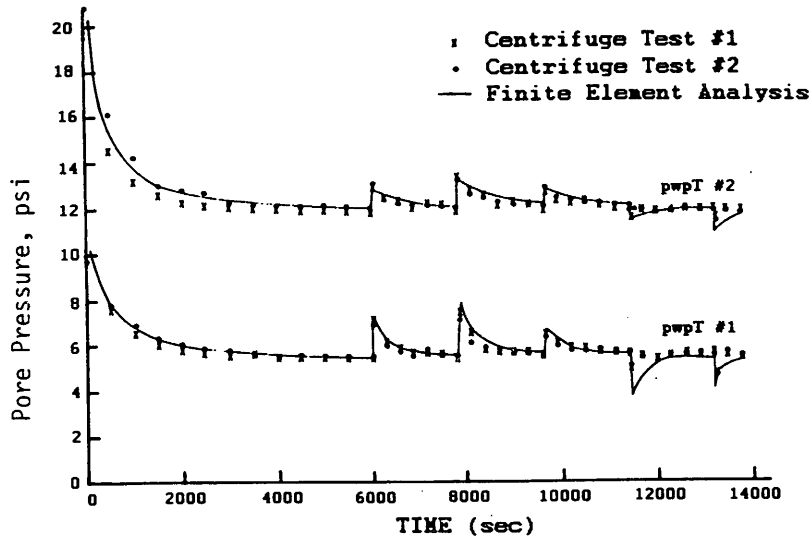

FIGURE 16 Pore pressure response to loading and unloading (*14*).

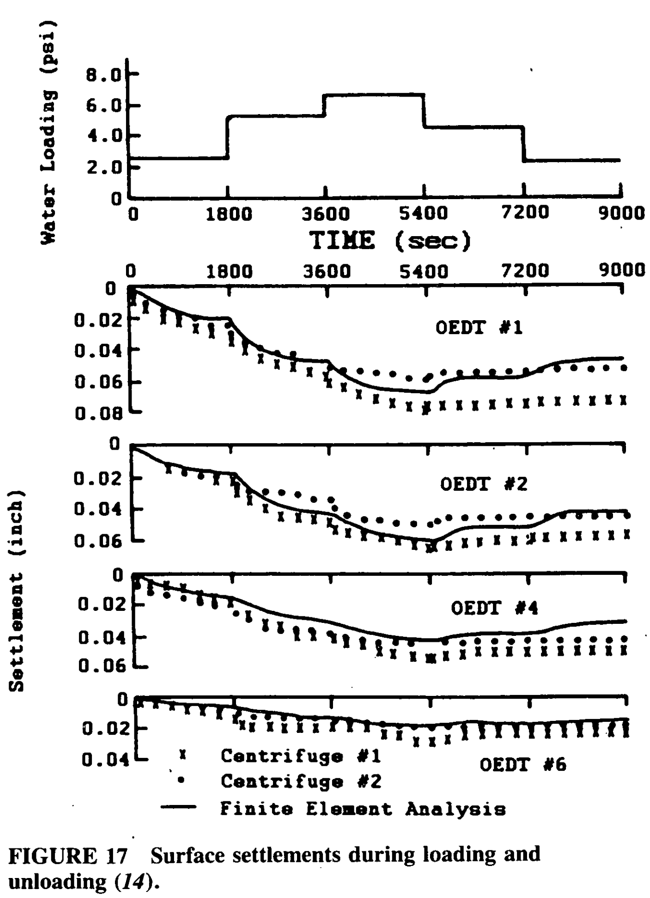

FIGURE 17 Surface settlements during loading and unloading (*14*).

embankment-type soil structure to evaluate a two-dimensional FEM consolidation program, CON2D. The prototype condition modeled was essentially a single drained kaolinite clay deposit 10 m (33 ft) thick overlain by a 3.4-m (11-ft) sand blanket upon which was placed a sand embankment 4.6 m (15 ft) high with 2.5:1 slopes. Figure 13 presents the measured and predicted pore pressures versus time using a time-scaling of n^2. The comparison shows that both the initial loading increment and subsequent rate of dissipation are closely predicted. A comparison of several vertical settlement locations in Figure 14 shows that the immediate settlements were underpredicted in areas near the toe and overpredicted beneath the central portions of the embankment.

Shen et al. (*14*) performed centrifugal model tests at 60 *g* of a prototype oil storage tank 9.8 m (32 ft) in diameter founded on a soft double-drained kaolinite deposit 9.1 m (30 ft) thick as shown in Figure 15. Testing consisted of loading (filling) and unloading (draining) the storage tank. Numerical modeling used a bounding surface plasticity model involving 18 input parameters, which had been incorporated into 2-D and 3-D FEM codes. Figure 16 presents the model time pore pressure predictions, whereas Figure 17 presents the surface settlements. As can be seen the agreements are good.

CONCLUSIONS

Centrifugal modeling is a powerful technique for assessing consolidation phenomena in that prototype geostatic stresses are applied to the soil, and the reduced model geometry accelerates pore pressure dissipation and thus consolidation time. The technique is applicable for assessing behavior such as sand drains, self-weight consolidation, and consolidation of flocculated soils. The technique also provides a feasible method for verifying numerical computer codes and assumptions.

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