Modeling Material Nonlinearity in a Pavement Backcalculation Procedure

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The development of improved procedures for the computation of effective stiffnesses of pavement layers from deflection bowls measured using the falling weight deflectometer (FWD) with particular emphasis given to the behavior of granular materials and subgrades is described. So far, the interpretation of FWD data at the University of Nottingham has been carried out using the computer program PADAL. In this program the pavement is represented by a series of linear elastic layers. The nonlinear properties of the subgrade are incorporated by dividing the subgrade into a series of sublayers, each having a different stiffness based on a stress-dependent elastic model derived from the results of laboratory repeated load triaxial tests. PADAL 2, an updated version of PADAL in which the nonlinearity of the subgrade is approximately modeled in the horizontal direction, has been developed. This modification allows for a more realistic description of the variability of elastic stiffnesses throughout the pavement. A comparison is made between two models incorporated in the back-analysis program to simulate the resilient behavior of fine-grained soils. FENLAP, a new finite element code for the structural analysis of pavements has been developed. The program includes a number of different nonlinear constitutive relationships. The finite element analysis may serve for calibrating the simpler layered elastic analysis approach. A pavement section is analyzed using the two computer programs, PADAL 2 and FENLAP.

The computer program PADAL was developed at Nottingham in 1985 (*1*) for the backcalculation of elastic stiffnesses of the pavement layers from deflection bowls measured using the falling weight deflectometer (FWD). Since then it has been extensively used to evaluate the condition of existing pavements in situ. Meanwhile, research has continued to develop and improve analytical methods for interpreting the results of FWD surveys on road pavements with thick granular layers. To pursue these objectives, two numerical techniques have been considered for the analysis of pavement structures, multilayered elastic systems, and finite elements.

In PADAL the pavement is represented by a series of linear elastic layers. The nonlinear properties of the subgrade are incorporated by dividing the subgrade into a series of sublayers, each having a different stiffness based on a stress-dependent elastic model derived from the results of laboratory repeated load triaxial tests. Extensive use of PADAL over the last 7 years to evaluate in situ pavements has shown this backcalculation technique to be generally satisfactory; however, experience has shown that it has some limitations, especially for pavements with thick granular bases. Recent development has continued to enhance the back-analysis procedure; in particular, consideration has been given to a new constitutive relationship for the subgrade, a nonlinear elastic model for granular materials, and a scheme for taking into account the stress dependency of stiffnesses in the horizontal direction.

With reference to the finite element method, a new computer program for the nonlinear analysis of pavements (FENLAP) has been developed. A pavement section backcalculated through PADAL is recomputed in FENLAP using the elastic stiffnesses obtained in PADAL. The results are used to compare the two programs.

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DEVELOPMENTS ON COMPUTER PROGRAM PADAL

Nonlinear Elastic Subgrade Models

For the structural analysis of pavements, accurate modeling of the behavior of the subgrade is important because the subgrade has a major influence on the pavement performance, particularly when considering deflections under wheel loading. For a realistic description of the subgrade condition, it is desirable to adopt nonlinear stress-strain relationships.

The computer program PADAL, which performs the backcalculation of stiffnesses of the individual pavement layers on the basis of FWD data, uses Brown's nonlinear elastic model for the subgrade, developed from laboratory testing (*2*).

$$E_r = A\left(\frac{p'_o}{q_r}\right)^B \tag{1}$$

where

E_r = resilient modulus;
p'_o = effective mean normal stress caused by overburden;
q_r = deviatoric stress caused by wheel loading;
A = material constant, typically in the range of 20 to 200 MPa; and
B = material constant, typically in the range of 0 to 0.5.

Subsequent laboratory testing of fine-grained soils has resulted in the development of a new model at Nottingham by Loach in a paper by Brown et al. (*3*), expressed by

$$E_r = Cq_r\left(\frac{p'_o}{q_r}\right)^D \tag{2}$$

where C is a material constant, typically in the range of 10 to 100, and D is a material constant, typically in the range of 1 to 2.

This relationship should constitute an improvement to Equation 1, because it was formulated after a comprehensive set of cyclic triaxial tests on samples more representative of soil in the ground. Therefore, Loach's model was incorporated in PADAL. All the essential characteristics of the representation of the subgrade in PADAL have been retained as for the model of Brown et al. (*1*). Hence, the subgrade is divided into five sublayers, and stresses at the middepth of each sublayer are considered for the calculation of stiffnesses and successive adjustments of the material constants, depending on the matching between measured and computed deflections at two outer points of the FWD deflection bowl. Figure 1 shows a typical deflection bowl and corresponding structure with the subgrade subdivision layers.

The only modification in the new algorithm is the implementation of a different procedure for adjusting stiffness between successive iterations. Using Brown's model, the stiffness of the upper layers and the elastic parameters of the subgrade were adopted in each iteration as follows (*1*):

$$E_{ni} = E_{oi}\left(\frac{dc_j}{dm_j}\right)^k \tag{3}$$

$$A_n = A_o\left(\frac{dc_l}{dm_l}\right)^k \tag{4}$$

$$B_n = B_o\left(\frac{dc_m}{dm_m}\right)^k \tag{5}$$

where

E_{ni}, E_{oi} = new and old computed stiffnesses for layer i,
A_n, A_o, B_n, B_o = new and old values of subgrade parameters A and B,
dc_j, dm_j, dc_l, dm_l, dc_m, dm_m = computed and measured deflections at sensors j, l, m, and
k = integer exponent that initially is equal to 1 but may vary as iterations proceed.

Because of the greater difficulty in achieving convergence for Loach's model, it was found necessary to adopt different values for the power index k. The exponent adopted used for the subgrade is half the value previously adopted ($k/2$ in Equations 4 and 5). Hence, the variation of the subgrade stiffness throughout the iterations is much smoother than before, making this approach more stable. However, the reduced power index for the subgrade implies that the convergence rate will be slower. To overcome this, the exponent of the upper layers is increased from k to $2k$ in Equation 3. Even so, the rate of convergence is usually slower for Loach's model than for Brown's model.

Comparison of Nonlinear Subgrade Models

The same pavement was back-analyzed with both Brown's and Loach's models. Details of the structure analyzed are presented in Figure 2. Brown's model took 31 iterations to perform the back-analysis, whereas with Loach's model convergence was reached after only 78 iterations. Additionally, the input parameters C and D used in this example were not very different from the values eventually determined; otherwise the number of iterations required by Loach's model would have been greater.

The output is summarized in Table 1. The results obtained using each of the models are very similar, for both deflections and stiffnesses. These results confirm that PADAL tends to unique solutions even when different constitutive relationships are compared. However, in Loach's model the exponent D of Equation 2 is slightly lower than 1. That is, according to the relationship, the subgrade shows a stress hardening behavior because its stiffness increases with increasing deviatoric stress. This was not expected, since fine-grained soils have always been regarded as stress softening materials.

The experimental data that led to Loach's model were based on tests at high (q/p'_o) ratios compared with those that occur at depth in the subgrade. In fact, for stress pulse durations of 1s and 0.1s (representative of the stress pulses induced in the subgrade by the FWD), Loach used 0.2 for the minimum value of (q/p'_o). However, when using PADAL, this ratio for the lower part of the subgrade was found to be substantially smaller (0.008 in the preceding example). Therefore, the stress conditions that exist in the deep subgrade were not reproduced, so it is possible that the model is not appropriate for these situations.

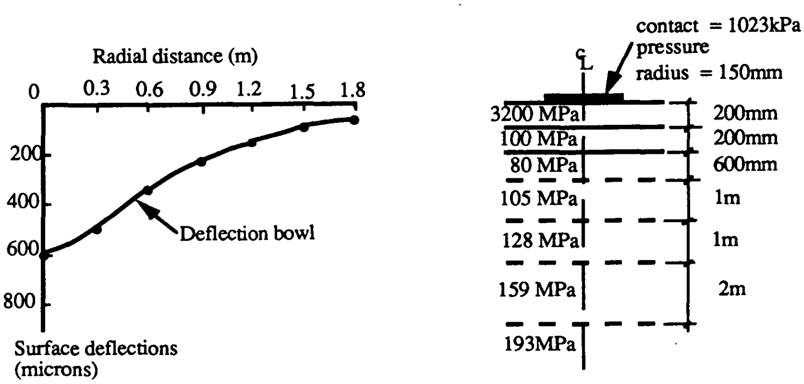

FIGURE 1 Typical deflection bowl (*left*) and corresponding structure (*right*).

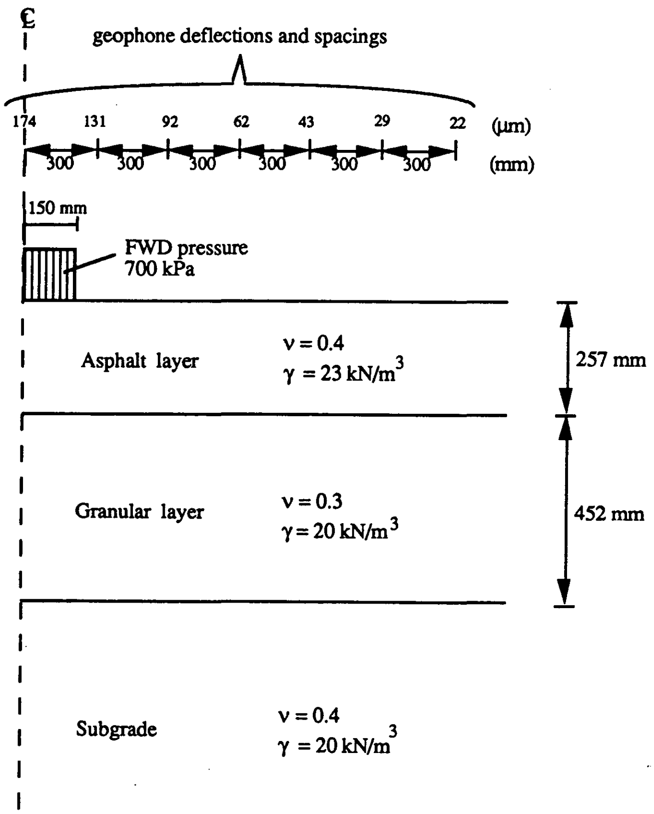

FIGURE 2 FWD data for full-scale pavement.

When an unstressed state is approached, Loach's model tends to predict infinite stiffnesses. However, most soils exhibit a finite maximum stiffness even when tested at low deviatoric stress. This seems to point out the inadequacy of Loach's model for low values of (q/p'_o), which may explain the odd values obtained for the constant D (<1).

Nonlinearity of Subgrade in Radial Direction

The original version of PADAL, despite incorporating a nonlinear elastic relationship for the subgrade, is unable to model nonlinearity in the horizontal direction. This shortcoming is common to all packages that are based on layered elastic analysis, because it is assumed that each layer or sublayer has a unique stiffness. Thus, although the stresses vary with radius, the stress dependency of the materials in the radial direction cannot be reproduced.

For a generic point in the subgrade at depth z and radius r, PADAL assigns a stiffness $E(z,r)$ that is based on the stresses calculated at a point located underneath the load center ($r = 0$) and at the middepth Z of the sublayer considered, that is,

$$E(z,r) = A\left(\frac{p'_o}{q_r}\right)^{B}_{(Z,0)} \tag{6}$$

An approximate procedure has been devised to overcome this restriction, that is, to make the subgrade stiffness variable with radius. Clearly, this procedure can be accurately achieved

TABLE 1 Comparison of Models in PADAL and PADAL 2

	PADAL		PADAL 2	
	Brown's Model	Loach's Model	Brown's Model	Loach's Model
No. Iterations	31	78	36	39
d_2 Error (%)	-2.65	-2.78	-3.16	-2.62
d_3 Error (%)	-2.48	-2.60	-2.69	-2.52
d_6 Error (%)	4.62	4.65	4.49	4.68
E_{as} (MPa)	5041	5009	4814	5068
E_{sb} (MPa)	245	251	337	229
$E_{sg\text{-}l1}$ (MPa)	174	170	148 (220)	180 (162)
$E_{sg\text{-}l2}$ (MPa)	232	232	199 (249)	243 (229)
$E_{sg\text{-}l3}$ (MPa)	301	303	260 (295)	316 (306)
$E_{sg\text{-}l4}$ (MPa)	399	403	345 (372)	418 (411)
$E_{sg\text{-}l5}$ (MPa)	582	587	507 (524)	603 (600)

Notes: $d_1,\ldots,d_7$ = deflections at the geophone locations.

E_{as}, E_{sb}, E_{sg} = stiffnesses of asphalt, sub-base and subgrade layers (subgrade with 5 sub-layers).

Deflections d_1, d_4, d_5 and d_7 were adopted for the matching of computed and measured deflection bowls, hence their errors are negligible.

The "-" sign means that the computed deflection is lower than the measured one.

The subgrade stiffnesses out of "( )" refer to locations beneath the load whereas the values within "( )" refer to a distance of 1.80 m away from the load axis.

only through finite element techniques, so simplifications have had to be adopted for the layered analysis. Essentially, this new approach computes the surface deflection at a certain radial position using a set of stiffnesses for the subgrade corresponding to the stresses in the subgrade at the same radial position. Thus, adopting Brown's relationship, the surface deflection at a distance R from the load center is calculated on the assumption that each sublayer of the subgrade has a stiffness given by

$$E(z,r) = A\left(\frac{p_o'}{q_r}\right)^B_{(Z,R)} \tag{7}$$

This procedure is not entirely correct because a unique stiffness is still assumed for each layer in each deflection computation. Nevertheless, the adoption of stiffnesses taken from the stress state of points directly beneath the location where surface deflection (d) is to be computed can be justified by the following expression:

$$d = \int_{-\infty}^{0} \frac{1}{E} [\sigma_z - \nu(\sigma_r + \sigma_\theta)]\, dz \tag{8}$$

where

E,ν = Young's modulus and Poisson's ratio, respectively,
$\sigma_z, \sigma_r, \sigma_\theta$ = vertical, radial, and tangential stresses, respectively, and
z = depth.

From Equation 8 it can be concluded that the deflection depends mainly on the stiffnesses that exist below the point considered, for the integration variable is z (r is kept constant). However, because the stresses are calculated using constant moduli in the radial direction, there will be an error because the evaluation of stresses does not take into account the variation of stiffness with radius. This limitation is inevitable in layered analysis, in which a unique stiffness must be assigned for each layer to proceed with the computation. In spite of that, the new approach seems to be much more appropriate than the earlier one.

The subgrade stiffnesses used for adjusting each deflection are then different and the analysis is carried out at each stage using a set of subgrade moduli corresponding to the radial position being considered. The stresses required for the calculation of the subgrade stiffnesses are computed every 10th iteration only, because they do not vary significantly between iterations. A summary of the procedure adopted is as follows:

1. Assume initial stiffnesses of individual layers and subgrade constants.
2. Calculate stresses in the subgrade (for first iteration and subsequent 10th iterations only).
3. Calculate deflections at each radial position (using subgrade stiffnesses at corresponding radii for all but first iteration).
4. Compare calculated and measured deflections at each radius.
5. Adjust subgrade constants on the basis of deflection comparison.
6. Having calculated stresses and elastic constants of the subgrade, determine subgrade stiffnesses at selected radial positions.
7. Adjust stiffnesses of upper layers on the basis of deflection match.
8. Repeat Steps 2 through 7 until convergence is achieved.

When convergence has been reached, it is possible to reproduce the spatial variation of the subgrade stiffness, both vertically and horizontally, according to Equation 7.

The iteration process involves only calculation of the stresses at the radial positions where the deflections have been selected for matching purposes. For other locations, the evaluation of subgrade stiffnesses must be preceded by the calculation of the stresses at those locations. By linear interpolation of the stiffnesses already determined at the selected points, estimates of the subgrade stiffness at any radial location can be obtained. These may then be used to calculate the stresses at the same radial position and consequently the actual stiffnesses from Equation 7. The use of linear interpolation is acceptable as variations in stiffness do not significantly affect the values of stress.

The consideration of stresses outside the symmetry axis also implies that the three-dimensional equivalent of the mean normal stress, p, and deviatoric stress, q, should be used. For the axisymmetric system adopted in PADAL, these stresses take the following form:

$$p = \frac{\sigma_z + \sigma_r + \sigma_\theta}{3} \tag{9}$$

$$q = \sqrt{\frac{(\sigma_z - \sigma_r)^2 + (\sigma_z - \sigma_\theta)^2 + (\sigma_r - \sigma_\theta)^2 + 6\tau_{zr}{}^2)}{2}} \tag{10}$$

where σ_z, σ_r, σ_θ are vertical, radial, and tangential stress, respectively, and τ_{zr} is the shear stress in the zr plane.

To obtain convergence when using both Brown's and Loach's models, the subgrade constants (A and B or C and D) were adjusted as shown in Equations 4 and 5, but restricting the value of the power k to 1. For the upper layers, the stiffnesses were adjusted using the same method as in the original PADAL program (Equation 3 with k varying in accordance with the pattern of convergence) (*1*). With this approach convergence was achieved and no special modifications were required for Loach's model.

Comparison of PADAL and PADAL 2

The modifications described earlier were incorporated in a new version of PADAL named PADAL 2. This new program has been tested for the same typical example analyzed earlier, using both Brown's and Loach's models. The results are also presented in Table 1. The computed deflections match quite well against the measured ones (as shown by the low relative errors), and their values were practically equal to the deflections obtained when constant stiffnesses with radius were assumed. The introduction of a radial variation in stiffness has little effect on the overall magnitude of stiffnesses.

Once again, the value obtained for the exponent D of Loach's relationship in this example is <1. This implies that the subgrade stiffnesses decrease with increasing radial distance from the load, contrary to what was expected.

Nonlinear Model for Granular Layers

PADAL is unable to adequately represent the behavior of granular subbases, for it adopts a linear elastic relationship for these materials. To overcome this limitation and provide a more useful package for the back-analysis of pavements for which granular layers are structurally relevant, improved constitutive relationships are necessary. After examining several possibilities, the K-θ model (*4*) was chosen to describe the nonlinear response of unbound granular layers. In this model, the resilient modulus E_r is given by

$$E_r = k_1\theta^{k2} \tag{11}$$

where

- θ = sum of peak values of principal stresses (first stress invariant);
- k_1 = material constant, typically in the range of 100 to 1,000; and
- k_2 = material constant, typically in the range of 0 to 0.8 (these values of k_1 and k_2 apply for stiffnesses and stresses in MPa).

This model has been widely used in structural analysis of pavements because its simplicity makes it very attractive for numerical applications. However, the K-θ model is often inaccurate when compared with more complex resilient models. The simple fact of ignoring the influence of the deviatoric stress on the state of deformation of the material shows its limitations. An assessment of this model led to the conclusion that its performance was generally unsatisfactory (*5*).

Despite these limitations, the K-θ model is perhaps the most suitable nonlinear elastic relationship for granular materials that can be implemented successfully in PADAL, for the following reasons:

1. Only two parameters, k_1 and k_2, are considered in the definition of the material behavior, whereas other models generally involve a greater number of constants. This property is very important in PADAL, since the number of deflections that must be calculated in each iteration should equal the number of material constants to be determined. Thus, an excessive number of material constants will increase the amount of computation and make it more difficult to converge toward a unique solution.
2. The K-θ model assumes a constant Poisson's ratio for the granular layer. Although this assumption is usually regarded as a drawback, this simplification is beneficial to the back-analysis procedure. In fact, the limited number of elastic parameters that can be backcalculated in PADAL implies that only the most important ones, that is, the elastic stiffnesses, can be taken as unknowns. Therefore, a model with stress-dependent Poisson's ratio is inappropriate to be implemented in PADAL.
3. It was shown that, although it is inadequate when predicting stresses and strains in granular layers, the K-θ model gives reasonable results in terms of vertical displacements (*5*). For the back-analysis of a pavement, the requirements of accuracy essentially concern the surface deflection (because the evaluation of the elastic stiffnesses is made on the basis of these deflections).

At the current stage of the research, the method for incorporating the stress-dependent constitutive relationship in PADAL 2 consists of steps similar to those adopted for the subgrade modeling, that is,

1. Division of the granular layer into sublayers, each one having a stiffness consistent with its stress level according to Equation 11.
2. Successive adjustments of the values of parameters k_1 and k_2 of Equation 11, by matching the measured and the calculated FWD deflections at two preselected radial locations.
3. Consideration of the nonlinearity of the granular material in the horizontal direction, by using an approximate approach analogous to the one followed for the subgrade.

Taking into account the range of thicknesses of granular layers that can be found in real pavement structures, it is thought that two sublayers should be sufficient to model the behavior of these materials. Finer subdivision would not result in any significant benefits but would increase the computational procedure.

FENLAP—NEW FINITE ELEMENT CODE FOR PAVEMENT ANALYSIS

General Description

To overcome some of the drawbacks inherent to layered analysis, the mainframe computer program FENLAP (Finite Element Nonlinear Analysis of Pavements) was developed. The program performs a finite element calculation of an axisymmetric solid and is designed for the structural analysis of pavements. A version for use on microcomputers has also been implemented.

The domain is modeled using eight-node rectangular elements, with the loading applied as concentrated forces at the nodes. All nodes along the bottom of the mesh are fixed. All nodes on both sides of the mesh are assumed to be on rollers, allowing vertical displacements but preventing radial displacements (see Figure 3).

FENLAP has been devised to allow for easy incorporation of various stress-strain models. Several material models have been adopted, including linear elastic, Brown's (*2*) and Loach's (*3*) nonlinear models for fine-grained soils, and the K-θ model for granular materials (*4*).

Analysis Procedure

In FENLAP the initial stress conditions are obtained directly from the specific weights of the materials, the level of the water table, the estimated suction values, and the residual lateral stresses. Having determined the initial stresses, the program calculates the starting values of Young's modulus and Poisson's ratio for each nonlinear element that is based on the mean normal effective stress, the deviatoric stress, and the stress-strain relationships adopted. Then, the vehicle loads are applied and the corresponding stresses computed. Using the sum of the initial and load-induced stresses, a new set of elastic parameters is obtained.

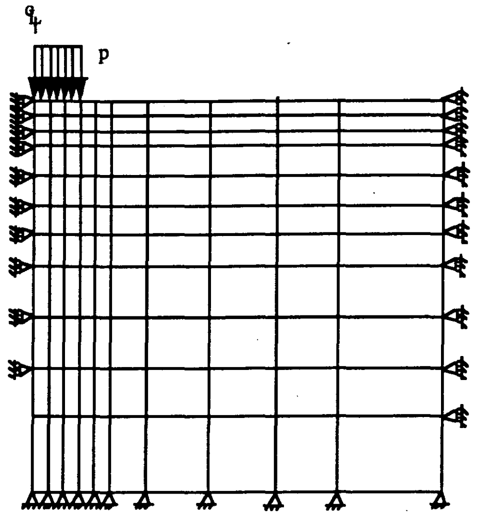

FIGURE 3 Finite element mesh and boundary conditions in FENLAP.

The program continues to iterate until sequential resilient modulus calculations for all nonlinear elements agree within a specified tolerance. Once a stable solution is reached, nodal displacements, strains, and stresses can be printed out.

Knowing the transient stresses and strains, it is possible to obtain equivalent values of elastic stiffness and Poisson's ratio associated with the passing of the wheel load alone. These elastic parameters are referred to as "chord" moduli, for they are obtained from the difference between loaded and unloaded conditions. Chord moduli are particularly useful, for they provide an estimation of the resilient response of the layers (*5,6*) and can be compared directly with the nonlinear elastic stiffnesses determined in the program PADAL.

COMPARISON BETWEEN PROGRAMS PADAL 2 AND FENLAP

Numerical Example

To test the approach introduced in PADAL 2 for modeling the radial variation of stiffness as well as to further validate FENLAP, both programs were used to analyze the same pavement. PADAL 2 is a back-analysis program that returns the values of stiffnesses corresponding to the surface deflections recorded by the FWD, whereas FENLAP is a forward analysis program that evaluates displacements, strains, and stresses in a pavement with given elastic constants. Hence, a comparison between the two programs cannot be made directly; it must be carried out by running program FENLAP on a pavement with a set of elastic parameters obtained through PADAL 2 for the same pavement and then comparing the surface deflections and the subgrade stiffnesses obtained with both methods.

Data obtained from the same full-scale pavement as was used previously was chosen for this comparison. Both Brown's and Loach's models were considered in the finite element analysis. The respective elastic constants of the subgrade determined in PADAL 2 were taken as input parameters in FENLAP. Linear elastic behavior was assumed for the upper layers, their elastic stiffnesses being taken from the PADAL output.

The results obtained using both programs showed a poor match between PADAL 2 and FENLAP solutions. Because the pavement analyzed in FENLAP assumed a rigid boundary at a depth of 10 m, it was decided to rerun PADAL 2 with a rigid layer at that depth. The new elastic parameters were then input to FENLAP and the program was rerun. The calculated deflections at the geophone locations and subgrade stiffnesses are presented in Table 2.

This second match resulted in an improved comparison between the two programs. However, some differences were still found in both stiffnesses and deflections. The following reasons may explain the differences:

1. The finite element method is, by nature, an approximate technique, in which factors such as the mesh size and the convergence criteria influence the results.
2. The approach followed in PADAL 2 to model the stress dependency of stiffnesses in the radial direction is based on a simplification, as described above. Therefore, the procedure is approximate.
3. Although the finite element lower boundary can be simulated in PADAL 2 by assigning a rigid layer at the corresponding depth, PADAL 2 is unable to reproduce the truncation of the domain at a certain radial distance, as it is required by the finite element method.

Nevertheless, the trends of the solutions obtained with both programs are not very different. Hence, despite the limitations point out, the approximate nonlinear modeling procedure adopted in PADAL 2 for the subgrade has been shown to satisfactorily match the output from the more accurate FENLAP analysis.

Finite Elements in Pavement Backcalculation

Because the finite element method has advantages over the layered analysis in modeling the nonlinear behavior of materials, it could potentially provide a framework for a backcalculation procedure. However, finite element techniques use much more computing time than programs that are based on layered analysis. For instance, in the mainframe computer of University of Nottingham, a FENLAP iteration on a typical pavement system can last more than 30 sec (if a mesh is used that is sufficiently fine to guarantee accuracy), whereas a PADAL iteration of the same system takes only about 3 sec. Considering the backcalculation is a trial-and-error process that implies many runs, the difference in computing time between the two approaches would be substantial.

Finite element calculations require a rigid boundary placed at some finite distance below the surface. Although various procedures have been proposed to estimate the depth of a rigid layer (*7,8*), none appears to apply satisfactorily to a wide spectrum of pavements. Hence, unless some geological information enables the approximate location of the bedrock to be known, a subgrade with infinite depth is usually assumed. The assignment of an arbitrary depth for the rigid bottom then can be a potential source of inaccuracy in a finite

TABLE 2 Comparison Between PADAL 2 and FENLAP for Full-Scale Pavement

	FWD	PADAL 2		FENLAP	
	Data	Brown's Model	Loach's model	Brown's Model	Loach's Model
d_1 (μm)	174.0	174.0	174.0	168.5	180.1
d_2 (μm)	131.0	127.3	128.1	122.2	133.6
d_3 (μm)	92.0	89.6	89.9	85.7	94.0
d_4 (μm)	62.0	62.0	62.0	59.5	64.4
d_5 (μm)	43.0	43.0	43.0	41.8	44.1
d_6 (μm)	29.0	30.3	30.4	30.4	30.6
d_7 (μm)	22.0	22.0	22.0	23.2	22.0
$E_{sg\text{-}l1}$ (MPa)	---	179 (219)	228 (163)	183 (220)	228 (162)
$E_{sg\text{-}l2}$ (MPa)	---	209 (233)	264 (220)	221 (241)	267 (216)
$E_{sg\text{-}l3}$ (MPa)	---	239 (254)	297 (269)	258 (276)	295 (254)
$E_{sg\text{-}l4}$ (MPa)	---	275 (285)	338 (322)	302 (307)	332 (304)
$E_{sg\text{-}l5}$ (MPa)	---	331 (337)	402 (393)	329 (326)	438 (431)

Notes: PADAL 2 backcalculates elastic stiffnesses from measured values whereas FENLAP determines the deflections from the stiffnesses obtained using PADAL 2.

The subgrade stiffnesses without "( )" refer to locations beneath the load whereas the values within "( )" refer to a distance of 1.80 m away from the load axis.

element backcalculation scheme, because two different positions of the lower boundary will necessarily lead to different solutions. The shortcoming of finite element programs being limited to solve finite domain problems in both vertical and horizontal directions can be overcome by the use of infinite elements. However, it is doubtful whether the supplementary effort required to consider them would be worthwhile.

In summary, at this stage of the investigation no conclusions can yet be made about the usefulness of a finite element backcalculation method, although the amount of computing time associated would probably rule out its implementation for practical purposes. However, it is certain that finite elements can provide valuable information to assist with a backanalysis procedure, particularly in connection with the following points:

1. To assess the significance of considering nonlinear models for a range of various pavements.
2. To compare the performance of several constitutive relationships tested.
3. To determine average resilient moduli that may describe the global response of the pavement layers in an accurate way.

CONCLUSIONS

The investigations carried out to date essentially have been concerned with developing and improving computational techniques for the nonlinear structural analysis and evaluation of pavements, paying particular attention to the modeling of the stress-dependent constitutive relationships of the foundation layers. The examples given in the paper have been selected as representative of a number of structures investigated. Further validation studies are ongoing, with particular consideration being given to the use of various load levels to determine material nonlinearity.

Loach's relationship for fine-grained soils was incorporated into program PADAL. The results obtained with this model and with Brown's (previously adopted) were similar. However, when applying Loach's relationship to PADAL, the well-known stress softening behavior of clays was not reproduced.

PADAL 2, an updated version of PADAL in which the nonlinearity of the subgrade is approximately modeled in the horizontal direction, was developed. This development allows for a more realistic description of the variability of elastic stiffnesses throughout the pavement. Loach's model was also tested on PADAL 2, but the inconsistency outlined above remained. In view of that, Brown's relationship is considered preferable for modeling the subgrade in PADAL.

Attention was given to the nonlinear modeling of granular layers in PADAL. Among the available models, the K-θ model seems the most adequate for this purpose. FENLAP, a new finite element code for the structural analysis of pavements, was implemented and validated. FENLAP includes a range of various nonlinear constitutive relationships and is a valuable tool for pavement design and evaluation.

The viability and utility of a finite element backcalculation procedure was discussed. Although more appropriate for nonlinear problems, this approach would be much more time consuming than the current one, which is based on layered elastic analysis.

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