Rapid Determination of Layer Properties of Pavements from Surface Wave Method

DEREN YUAN AND SOHEIL NAZARIAN

The spectral analysis of surface waves (SASW) method has been used as a nondestructive test method for determining elastic modulus profiles of pavement systems. To perform the test, a disturbance is applied to the pavement surface to generate stress waves that propagate mostly as surface waves of various wavelengths. The waves are monitored and captured with a data acquisition system. Signal and spectral analyses are used to determine a dispersion curve (variation in phase velocity with wavelength). The last step is to determine the elastic modulus of various layers from the dispersion curve. Several alternatives are available. First, a trial-and-error (forward modeling) process can be carried out. Second, optimization techniques can be used. Finally, generalized inverse theory can be implemented. In most applications, the backcalculation of layer moduli is accomplished using a manual trial-and-error matching process between the theoretical and experimental dispersion curves. Unfortunately, this process is rather time consuming and requires engineering judgment. To improve this aspect of SASW testing, a backcalculation technique that is based on the generalized inverse theory is developed. The technique provides a fast and automated procedure for simultaneously determining layer elastic moduli and thickness of pavements. In addition, some description of uncertainty in the backcalculated results is provided. The development of this algorithm is discussed and the speed and accuracy as well as the limitations of the algorithm are demonstrated. An actual field case history is included to exhibit the usefulness of the method in actual field testing.

As a nondestructive test method, the spectral analysis of surface waves (SASW) method has been used to determine elastic modulus profiles of pavements (*1,2*). The SASW method is based on the dispersive characteristic of seismic surface waves in layered media. To perform the test, three major steps are followed: (a) field testing, (b) determination of experimental dispersion curve, and (c) determination of stiffness profile.

The in situ testing consists of generating and detecting surface waves by affecting the surface of the pavement and monitoring and capturing the motion of the pavement surface at several points. The captured signals are manipulated using the Fourier and spectral analyses to determine the cross-power spectra and the coherence functions.

Determining the experimental dispersion curve consists of processing the cross-power spectra and the coherence functions to construct an experimental dispersion curve. This curve represents the dependence of phase velocity on frequency or wavelength.

Determining the elastic modulus profile from the experimental dispersion curve is the last but the most time-consuming and complicated step. This process is known as inversion or backcalculation. To accomplish this task, a manual trial-and-error (forward modeling) procedure has been successfully employed for years (*3*). A merit of the forward modeling procedure is that common sense notions can be incorporated into the trial-and-error stage. This, however, implies that a fairly experienced person is needed to conduct this task efficiently. Therefore, one of the major shortcomings of the procedure is probably the time required to reduce the data. To remedy this shortcoming, an automated procedure is desirable.

To turn the SASW method to a practical testing equipment, all aspects of testing and data reduction should be automated. A device for conducting the field tests rapidly (in about 1 min) is under development for SHRP (*4*, p. 128). An algorithm for automatically determining the dispersion curve has been developed by Desai (*5*). Finally, an automated backcalculation process has been developed and is described in this paper. The process, which is based on the general linear inverse theory (*6,7*), provides an automated algorithm for rapidly determining modulus profiles from dispersion curves. At the same time, the algorithm yields information needed for uncertainty analysis of backcalculated results. Synthetic (theoretically derived) data and actual case studies are included to illustrate some aspects of the strengths and limitations of the process.

BACKGROUND

Any algorithm for determining the modulus profile of a pavement system from an experimental dispersion curve contains two fundamental components. First, a proper algorithm that is based on wave propagation theory is needed to construct a theoretical dispersion curve from a trial profile. Second, an algorithm is needed to minimize the error between the theoretical dispersion curve and the experimental one. Each step will be discussed.

Shown in Figure 1 is an idealized cross section of a pavement. To apply the theory of wave propagation to this pavement section, two major simplifying but realistic assumptions are usually made. First, it is assumed that only the in-plane waves (P- and SV-waves) are involved so that the problem can be approximated as a plane strain problem. Second, it is assumed that the system is composed of a stack of flat layers with homogeneous and isotropic properties and that each layer extends horizontally to infinity (as compared with the dimension of source-receiver configuration). The effects of these two assumptions either are minor (*8*) or can be minimized with proper setup in field testing (*9*). Because of these as-

Department of Civil Engineering, University of Texas at El Paso, El Paso, Tex. 79968.

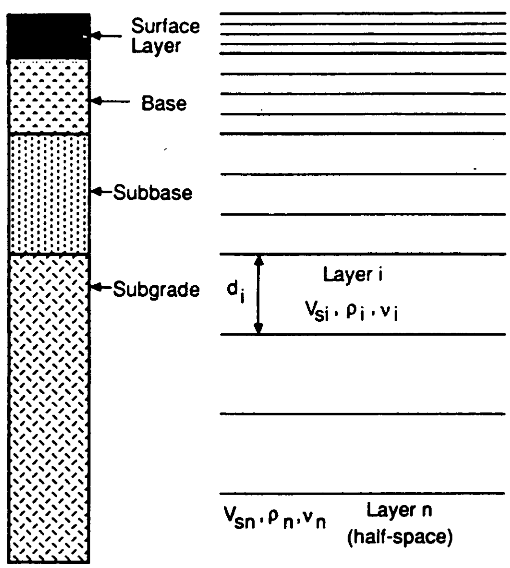


FIGURE 1 Idealized profile of pavement system: *left*, traditionally assumed layering; *right*, sublayering assumed in SASW analysis.

sumptions, the solution to the wave equation reduces to a simple two-dimensional problem in the Cartesian coordinate system.

With these assumptions, the parameters needed to define the properties of each layer are thickness (h), Young's modulus (E), Poisson's ratio (ν), and mass density (ρ). Alternatively, Young's modulus can be replaced by shear or S-wave velocity (V_s). One way of demonstrating this relationship is

$$E - 2\rho V_s^2(1 + \nu) \qquad (1)$$

Among these parameters, shear wave velocity (or Young's modulus) has the dominant effect on the dispersion characteristics of surface waves, and the effects of density and Poisson's ratio are rather small. Nazarian has numerically shown that the effects of these two parameters, in most practical cases, are less than 5 percent (*3*). Therefore, to simplify the inversion process, one can assume that density and Poisson's ratio for each layer are known parameters. Reasonable values can be assigned to these parameters on the basis of past experience. Shear wave velocity of each layer is the only active pavement strength parameter that should be backcalculated.

To estimate layer thicknesses, two procedures can be followed. They can be either assumed to be known or backcalculated. If layer thicknesses are assumed as known parameters, it is necessary to include many layers (on the order of 10) in the trial profile to minimize the biased effect of interfaces on resulting profile. This is necessary because the interpretation of layer thickness is done by determining layer interfaces with significant modulus contrast. If both thicknesses and moduli need to be simultaneously determined, only a few (on the order of five) layers will be required.

THEORETICAL DISPERSION CURVE

In the development of the theory of surface waves and its application to determining shear wave velocity or stiffness profiles, much attention has been paid to the situation in which velocity or stiffness generally increases with depth. This situation typically covers most geophysical and geotechnical investigations. However, for the reverse case, such as pavements, where modulus decreases with depth, very limited theoretical analyses have been made. These analyses are mainly reflected in the works of Jones (*10*), Vidale (*11*), and Nazarian (*3*). Therefore, it is thought that a brief discussion regarding the characteristic and computation of dispersion for pavements is necessary.

Dispersion curves for soil deposits demonstrate a continuous relationship between phase velocity and frequency; that is, phase velocity is a continuous function of frequency or wavelength. In the contrary, a characteristic of the theoretical dispersion curve for a layered system resembling a pavement structure is that the curve is composed of multiple branches or limbs (see Figure 2, which is based on a Poisson's ratio of 0.25).

The existence of discontinuities in dispersion curves was first recognized by Sezawa and Kanai (*12*). On the basis of both theoretical analyses and actual observations, Jones (*10*) and Vidale (*11*) reported this characteristic of dispersion curves for pavement constructions.

To apply properly any analytical backcalculation technique, the theoretical dispersion curve should be very accurately determined. Several methods have been suggested for the computation of the theoretical dispersion curve. These methods include transfer matrix method (*13,14*), stiffness matrix method (*15*), finite element method (*16*), and finite difference method (*17*). However, the results from most of these methods in original form (as applied to pavement systems) face problems with accuracy or with numerical difficulties, or both.

In the work presented in this paper, the transfer matrix method as modified by Dunkin (*18*) has been chosen to calculate the theoretical dispersion curves. This method was selected because of its flexibility and potential accuracy. The computer program used is based on the major routines of program INVERT developed by Nazarian (*3*) with major im-

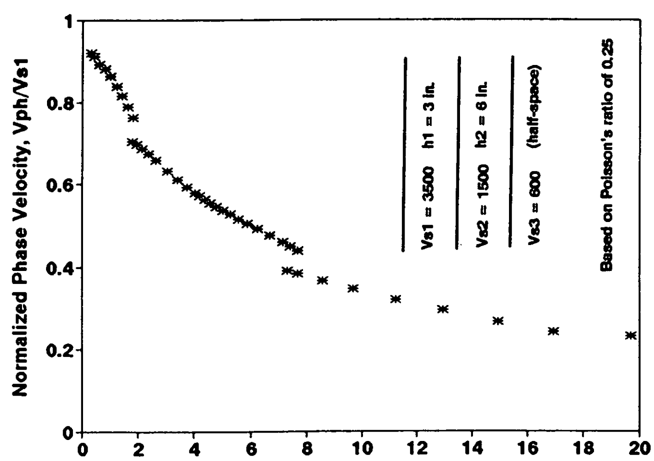


FIGURE 2 Theoretical dispersion curve for typical pavement section.

provements in mode isolation and root refinement. These refinements are discussed elsewhere (*3*) and are omitted from here for brevity.

INVERSION PROCESS

Description of Problem

Let us assume that a series of SASW tests has been carried out on a pavement section with M assumed layers. As a result, an experimental dispersion curve containing N data points has been constructed; that is, at each frequency, f_i, phase velocity, V_i^{obs}, is determined. To initiate the backcalculation process, an initial shear wave velocity profile, $\mathbf{p}^0$, is assumed. Mathematically, $\mathbf{p}^0$ is a vector whose elements, p_j^0, contain the shear wave velocity and thickness of each layer. The initial profile $\mathbf{p}^0$, is then used to determine the theoretical phase velocity, V_i^{thr}, at each frequency, f_i. The goal is to determine a vector $\mathbf{p}^f$ that minimizes the difference between the theoretical and observed data.

The inversion process can be categorized as a nonlinear problem with respect to the unknowns to be determined. To simplify the procedure, the problem is linearized. As a result, several iterations are necessary before the final profile is obtained. This procedure is defined below.

The initial profile, $\mathbf{p}^0$, is used to calculate the theoretical phase velocity, V_i^{thr}, and the partial derivatives, $\partial V_i^{thr}/\partial p_j^0$, at each frequency. The governing equations of the inversion problem can then be expressed as

$$V_i^{obs} - V_i^{thr} = \sum_{j=1}^{K} \frac{\partial V_i^{thr}}{\partial p_j} \Delta p_j \qquad i = 1, \ldots, N \tag{2}$$

or in matrix form

$$\mathbf{A}\Delta\mathbf{p} - \Delta\mathbf{c} \tag{3}$$

where

- $\Delta\mathbf{c} = (\Delta c_1, \Delta c_2, \ldots, \Delta c_N)^T$ where $\Delta c_i = V_i^{obs} - V_i^{thr}$;
- $\Delta\mathbf{p} = (\Delta p_1, \Delta p_2, \ldots, \Delta p_K)^T$ is defined as a correction or modification vector that is added to vector $\mathbf{p}^0$ to determine the vector of unknowns $\mathbf{p}^i$ for the next iteration. In the vector, K equals M, if layer thicknesses are assumed to be fixed, and $2M - 1$, if both shear wave velocity and thickness of each layer need to be determined simultaneously; and
- $\mathbf{A} = N\text{-}\times\text{-}K$ matrix of partial derivatives whose elements $A_{ij} = \partial V_i^{thr}/\partial p_j^0$.

Vector $\mathbf{p}^i$ considered as the solution to the unknowns when it yields a vector $\Delta\mathbf{c}$ in Equation 3 that is sufficiently small. This is elaborated in the next section.

The choice of the number of experimental dispersion data points (parameter N) depends on the quality of the field data and the number of layers (parameter M) in the trial profile. In practice, it is assumed that N (number of dispersion data points) is greater than K (number of unknowns). Thus, the problem defined by Equation 3 is overconstrained. Typically, parameter N is limited to 20 to 40 data points. As indicated before, parameter M is limited to about five layers when both thickness and stiffness should be determined and is about 10 when only stiffness has to be determined.

Construction of Shear Wave Velocity Profile

In general, Equation 3 cannot be solved through calculating the conventional inverse of matrix $\mathbf{A}$, $\mathbf{A}^{-1}$. Matrix $\mathbf{A}^{-1}$ exists only if $\mathbf{A}$ is square and nonsingular. An approach to solving Equation 3 is to construct its normal or Gaussian-Newton equation. This results in the classical least-squares solution

$$\Delta\mathbf{p} = (\mathbf{A}^T A)^{-1}\,\mathbf{A}^T\Delta\mathbf{c} \tag{4}$$

subject to minimization of $(\Delta\mathbf{c} - \mathbf{A}\Delta\mathbf{p})^T(\Delta\mathbf{c} - \mathbf{A}\Delta\mathbf{p})$ with respect to $\Delta\mathbf{p}$. This solution is also known as the optimization solution.

The computation of matrix $\mathbf{A}^T\mathbf{A}$ may involve numerical inaccuracy that can be troublesome when the number of dispersion data or the number of layers is large. To avoid this drawback, the singular value decomposition of a matrix (*19*) approach has been used to develop the generalized inverse solution of Equation 3.

The decomposition of matrix $\mathbf{A}$ leads to a product of three matrices

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^T \tag{5}$$

where

- $\mathbf{U} = N\text{-}\times\text{-}K$ matrix whose columns are eigenvectors, $\mathbf{u}_j$ ($j = 1, \ldots, K$), of length N associated with the columns (observations) of $\mathbf{A}$,
- $\mathbf{V} = K\text{-}\times\text{-}K$ matrix whose columns are eigenvectors, $\mathbf{v}_j$ ($j = 1, \ldots, K$), of length K associated with the rows (parameters) of $\mathbf{A}$, and
- $\mathbf{S} = K\text{-}\times\text{-}K$ diagonal matrix with diagonal entries, s_{jj} ($j = 1, \ldots, K$), which are the nonnegative square roots of the eigenvalues of symmetric matrix $\mathbf{A}^T\mathbf{A}$ and known as the singular values of $\mathbf{A}$.

By substituting Equation 5 into Equation 4 and utilizing the orthonormal property of $\mathbf{U}$ and $\mathbf{V}$ [i.e., $\mathbf{U}^T\mathbf{U} = \mathbf{V}^T\mathbf{V} = \mathbf{V}\mathbf{V}^T = \mathbf{I}$ (unit matrix)], it is easy to show that

$$\Delta\mathbf{p} = \mathbf{V}\mathbf{S}^{-1}\mathbf{U}^T\Delta\mathbf{c} \tag{6}$$

This expression gives the generalized inverse solution of Equation 3.

Adding $\Delta\mathbf{p}$ to $\mathbf{p}_0$ yields an updated profile from which a new set of phase velocities and a new set of partial derivatives can be calculated. This procedure is repeated until Δc_i's (elements of vector $\Delta\mathbf{c}$ described in Equation 3) are "sufficiently small." At this time, a profile that satisfies the given data is found. The term sufficiently small is defined in the next paragraph.

The convergence of successive iterations is monitored by the following root-mean-squares (*RMS*) error criterion:

$$\varepsilon = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\Delta c_i} \tag{7}$$

The iteration procedure is terminated when ε reaches an acceptably small value or when all elements of vector $\Delta\mathbf{c}$ are within the standard error bounds of each experimental datum or other prespecified limits.

In many applications, a physically acceptable shear wave velocity profile cannot be determined from Equation 6 because of the lack of numerical stability; that is, a small change in the data can lead to unacceptably large changes in the derived profile. The remedy to this problem is as follows.

Let us rewrite Equation 6 in the from of a linear combination or weighted sum of eigenvectors, $\mathbf{v}_j$,

$$\Delta\mathbf{p} = \sum_{j=1}^{K} \frac{\mathbf{u}_j^T \Delta\mathbf{c}}{s_j} \mathbf{v}_j \tag{8}$$

Vectors $\mathbf{v}_j$ are finite in magnitude and increasingly oscillatory as j increases. Hence, the stability of the solution is controlled by the magnitude of $\mathbf{u}_j^T\Delta\mathbf{c}/s_{jj}$. As j increases, s_{jj} becomes significantly smaller than $\mathbf{u}_j^T\Delta\mathbf{c}$. This can result in very large numbers, which can cause instability in the solution. To remedy this instability, the singular value decomposition algorithm has been combined with Marquardt's technique as described by Jupp and Vozoff (*20*). In this approach, s_{jj} in Equation 8 is replaced by $(s_{jj} + \gamma/s_{jj})$. The solution can then be written as

$$\Delta\mathbf{p} = \sum_{j=1}^{K} \frac{s_{jj}}{s_{jj}^2 + \gamma} \mathbf{u}_j^T \Delta\mathbf{c}\mathbf{v}_j \tag{9}$$

where γ is a "small" (in comparison with the largest singular value) positive number. Parameter γ, known as Marquardt's factor, dampens or eliminates the undesirable effects that small singular values can have on the solution.

Evaluation of Uncertainty

In addition to the representative shear wave velocity profile, the level of uncertainty associated with each unknown parameter is determined. To evaluate uncertainty in the profile, the variance or standard error associated with each parameter is determined according to the statistics of experiment errors (*7*). The stability constraints shown in Equation 9 have to be taken into account. The variance of each parameter can then be estimated through the following formulation:

$$\text{var}[\Delta\mathbf{p}_j] = \sigma^2 \sum_{k=1}^{K} \left[V_{jk} \frac{s_k}{(s_k^2 + \gamma)} \right]^2 \tag{10}$$

where V_{jk} are the elements of matrix $\mathbf{V}$, and σ^2 is associated with the random errors in the experimental data. The approximate value of σ^2 is given by Draper and Smith (*21*):

$$\sigma^2 = \frac{1}{N - K} (\Delta\mathbf{c} - \mathbf{A}\Delta\mathbf{p})^T (\Delta\mathbf{c} - \mathbf{A}\Delta\mathbf{p}) \tag{11}$$

The square root of each variance gives the standard error for the corresponding profile parameter.

The uncertainties obtained are caused by only the random error in the field data. Other factors, such as errors in the assumed profile (Poisson's ratio and mass density), simplifying assumptions in the algorithm, and systematic errors in the observed data, cannot be included. Therefore, these estimates should be considered as the lower limits of the level of uncertainty.

PARAMETRIC STUDY

In this section, results from the analysis of a set of "synthetic" (theoretically determined) dispersion data are presented to illustrate the applicability and limitation of the inversion process presented. The dispersion data correspond to a three-layer pavement section. The profile consists of 3 in. of asphalt concrete (AC) (shear wave velocity of 3,000 ft/sec), over 6 in. of high-quality base (shear wave velocity of 2,000 ft/sec), over subgrade (shear wave velocity of 1,000 ft/sec). A Poisson's ratio of 0.33 and a uniform mass density were assigned to each layer.

Twenty-one data points were theoretically calculated using the pavement properties described above. These dispersion data were then input into the inversion program to backcalculate the shear wave velocity profile.

As the dispersion data are theoretically calculated, they are not contaminated with systematic errors or random scatter usually contained in the field data. The same theoretical algorithm used for calculating the dispersion curve was used as a subroutine in the inversion program. In addition, the exact properties of each layer are known. As such, any differences between the real and backcalculated profiles are a result of the weaknesses of the inversion algorithm.

Various seed profiles were used to examine the convergence in each simulation. In most instances, the same results were achieved. However, a reasonable setup of initial profile is important to reduce the computation time. In the remainder of this section, several different testing scenarios are considered and studied. In almost all cases, the final results were obtained in less than 1 min using an 80386-based 20-MHz personal computer.

Simulation 1

In Simulation 1, it was assumed that the number and thicknesses of layers were known. Only shear wave velocity for each layer had to be determined. The initially assumed (seed) and the backcalculated profiles are shown in Table 1. The backcalculated profile is identical to the true profile. The "observed" (the synthetic dispersion is called that for simplicity) and the "final" (refers to the dispersion curve obtained

TABLE 1 True, Seed, and Backcalculated Values of Shear Wave Velocities, Simulation 1

Layer Number	Thickness(in.)	Shear Wave Velocity (fps)		
		True	Seed	Backcalculated
1	3.0	3000.	2500.	3000.
2	6.0	2000.	1500.	2000.
3	∞	1000.	500.	1000.

Data Misfit (%): Min. = 0.0, Max. = 0.0, Average = 0.0

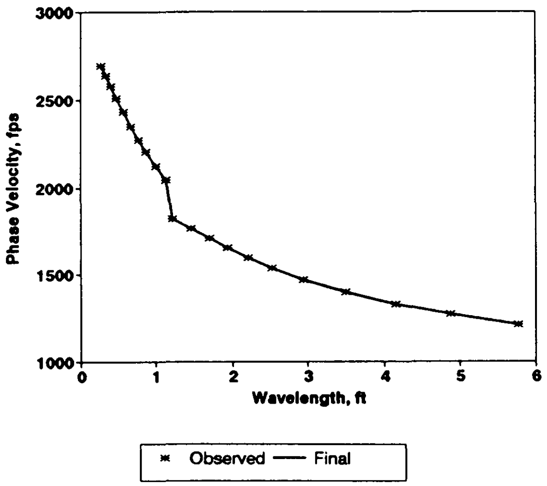
from the profile after the completion of inversion) dispersion curves are compared in Figure 3. For each data point at a given wavelength, the percent data misfit is calculated by simply determining the relative errors between the observed and final dispersion data, $\Delta c_i / V_i^{obs}$. The maximum, minimum, and average misfit among the 21 data points are also given in Table 1. The complete convergence in data matching after backcalculation demonstrates the effectiveness and accuracy of the process.

The same exercise with the same trial profile but using less (eleven and nine) observed data points were repeated. The results obtained were the same as those given in Table 1.

Simulation 2

In Simulation 2, it was assumed that only the number of layers was known. Shear wave velocity for each layer was backcalculated with the arbitrarily fixed layer thicknesses. The seed and backcalculated profiles are shown in Table 2. At a first sight, the differences between the true and backcalculated shear wave velocities may seem small. However, as indicated before, the dispersion data used in the inversion process are "accurate" (i.e., error- and scatter-free). Therefore, the differences are rather significant. This emphasizes that, similar to any other nondestructive deflection testing (NDT) method, to obtain an accurate stiffness profile, the thickness of the layers should be accurately known.

The observed and final dispersion data are compared in Figure 3 and Table 2. The two dispersion curves deviate significantly from one another. The maximum misfit is more than 8 percent.



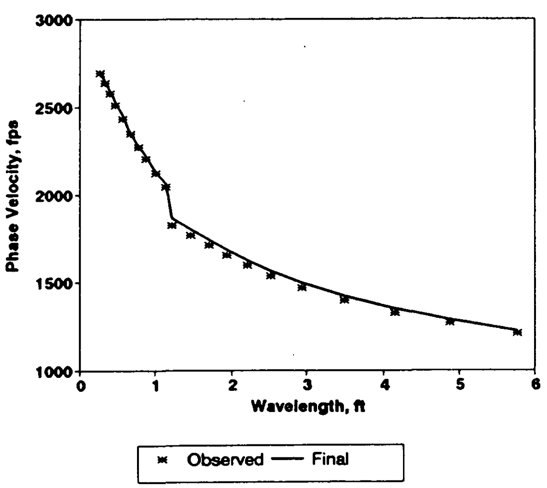


FIGURE 3 Comparison of observed and final dispersion curves: *top*, Simulation 1; *bottom*, Simulation 2.

Simulation 3

In Simulation 3, the same seed velocities and layer thicknesses used in Simulation 2 were used. However, both thickness and shear wave velocity for each layer were determined through the inversion process. The results from this simulation are reflected in Table 3. Once again, the thicknesses and velocities are perfectly determined. The mismatch between the observed and final dispersion curves is practically zero. The results obtained in this simulation demonstrate the effectiveness and accuracy of the backcalculation technique.

Simulation 4

Simulation 4 corresponds to a practical situation usually encountered. It was assumed that thicknesses and shear wave velocities of layers, as well as the number of layers, are all unknown. Thicknesses and shear wave velocities were estimated by assuming that the profile contains five layers. The thicknesses of the layers in the seed profile were assumed to be equal.

TABLE 2 True, Seed, and Backcalculated Values of Shear Wave Velocities, Simulation 2

Layer Number	Thickness (in.)		Shear Wave Velocity (fps)		
	True	Seed	True	Seed	Backcalculated
1	3.0	3.6	3000.	2500.	2915.
2	6.0	4.8	2000.	1500.	1870.
3	∞	∞	1000.	800.	1093.

Data Misfit (%): Min. = 0.06, Max. = 8.20, Average = 1.16

TABLE 3 True, Seed, and Backcalculated Values of Shear Wave Velocities, Simulation 3

Layer Number	True		Seed		Backcalculated	
	h (in.)	V_s (fps)	h (in.)	V_s (fps)	h (in.)	V_s (in.)
1	3.0	3000.	3.6	2500.	3.0	3000.
2	6.0	2000.	4.8	1500.	6.0	2000.
3	∞	1000.	∞	800.	∞	1000.

Data Misfit (%): Min. = 0.02, Max. = 0.08, Average = 0.05

The final profile after inversion is included in Table 4 and is compared with the true profile in Figure 4. The thickness and velocity of the surface layer are determined relatively accurately. The thickness of the second layer of the true profile is determined accurately, provided that the second and third layers of the backcalculated profile are combined. This is justified because the velocities of the second and third layers of the final profile are rather close. The velocity of the half space is predicted rather well, if the fourth layer is ignored.

The velocity of the fourth layer from the final profile is significantly higher than the actual value. This corresponds to one of the weaknesses of the present inversion process. This discrepancy can be attributed to the change of branch patterns (five layers against three layers) in dispersion curves. Means of resolving this problem are being studied.

Simulation 5

In every simulation represented up to this point, the result was obtained on the assumption that the Poisson's ratio and density for each layer are known. The effects of misestimation of Poisson's ratio on backcalculated profile is demonstrated here.

TABLE 4 True, Seed, and Backcalculated Values of Shear Wave Velocities, Simulation 4

Layer Number	True		Seed		Backcalculated	
	h (in.)	V, (fps)	h (in.)	V, (fps)	h (in.)	V, (fps)
1	3.0	3000.	3.0	2500.	3.07	2966.
2	6.0	2000.	3.0	2000.	1.06	2153.
3	∞	1000.	3.0	1500.	4.83	2025.
4			3.0	1000.	1.44	1206.
5			∞	800.	∞	1036.

Data Misfit (%): Min. = 0.35, Max. = 2.18, Average = 1.26

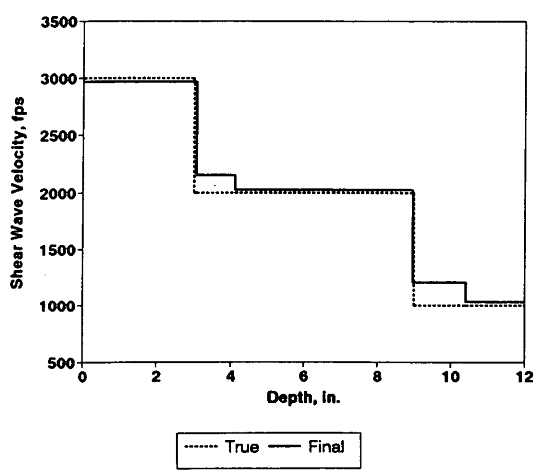


FIGURE 4 Comparison of true and final shear wave velocity profiles from Simulation 4.

TABLE 5 True, Seed, and Backcalculated Values of Shear Wave Velocities, Simulation 5

Layer Number	Thickness (in.)	Shear Wave Velocity (fps)		
		True	Seed	Backcalculated
1	3.0	3000.	2500.	3006.
2	6.0	2000.	1500.	2092.
3	∞	1000.	800.	1002.

Data Misfit (%): Min. = 0.51, Max. = 3.63, Average = 1.45

A Poisson's ratio of 0.25, rather than 0.33, was assumed for all layers. For simplicity, the number and thicknesses of layers have been assumed to be known parameters. The backcalculated velocities are shown in Table 5.

Decreasing the Poisson's ratio from 0.33 to 0.25 should naturally result in an increase of about 1.5 percent in the velocity of each layer. It can be seen that velocities of all layers are predicted closely. This confirms that Poisson's ratio has a minor influence on the backcalculated results.

CASE STUDY

A series of tests was carried out on Highway US-69 in Angelina County near Lufkin, Tex. The cross section of the road consisted of two 12-ft-wide driving lanes and two 8-ft-wide paved shoulders. The pavement section reportedly consisted of a 6-in.-thick AC layer. On the basis of construction drawings, the AC layer consisted of three different lifts placed at various times. The base course consisted of 10 in. of flexible ash base, below which the subgrade existed. The results from one site are discussed here.

The dispersion curve from the site is shown in Figure 5. To backcalculate elastic moduli, 31 data points were sampled from the experimental dispersion curve. The automated method suggested by Desai (5) was used to select these representative data points. A relatively large number of data is incorporated in the inversion process so that the discontinuity in the experimental dispersion curve can be sufficiently defined. The

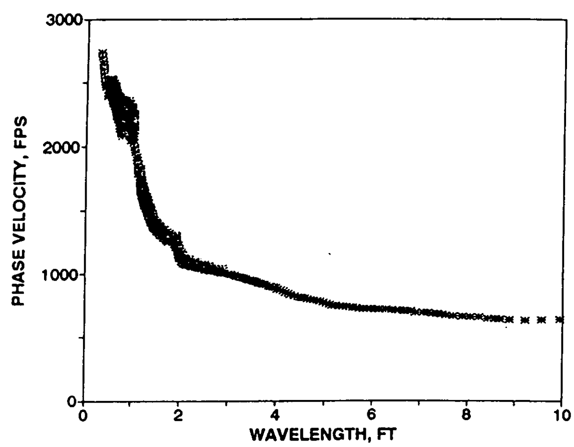


FIGURE 5 Dispersion curve from flexible pavement section tested.

statistical standard errors in the data range from 15 to 50 ft/sec (equivalent to a relative error of about 2 percent).

The seed profile is shown in Figure 6. The selection of the proper seed profile is rather simple for the top layer and the half space. A convenient method for determining the seed values for other layers is given by Nazarian (*3*). The final profile after simultaneous (thickness and velocity) inversion is also presented in Figure 6.

The idealized dispersion curve used in the inversion process is compared with the theoretical one obtained from the back-calculated profile in Figure 7. The two curves compare quite favorably showing the appropriateness of the final profile.

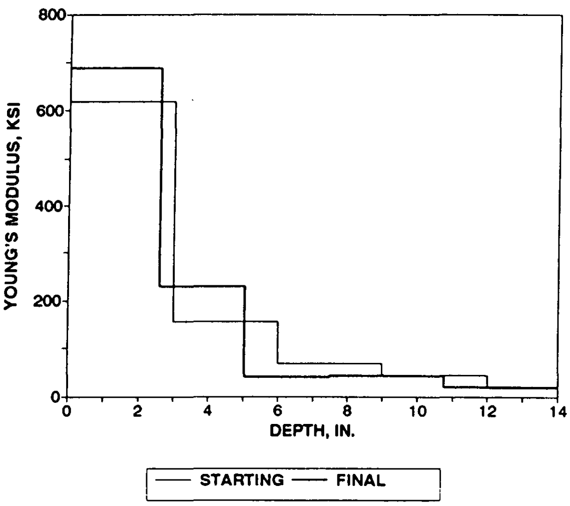


FIGURE 6 Seed (starting) and final modulus profiles.

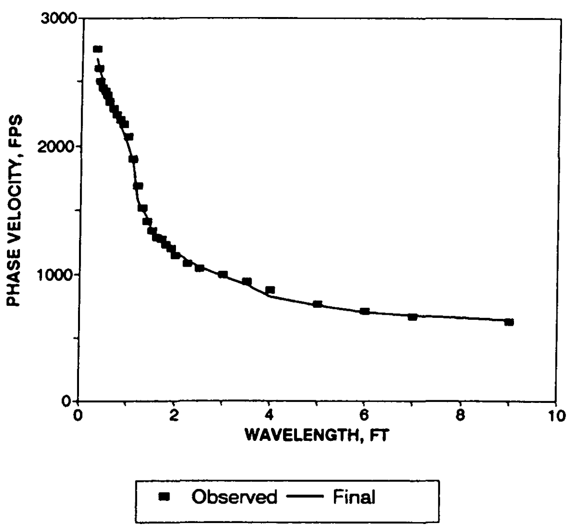


FIGURE 7 Comparison of observed and final dispersion curves.

The final profile along with the estimated 95 percent confidence interval bounds associated with the modulus and thickness are shown in Figure 8. The confidence interval bounds are rather narrow, corresponding to small scatter in the field data and good fit between the dispersion curves.

The thickness of the AC layer is estimated at about 2.6 in., underlying a softer layer down to a depth of about 5.5 in. The thickness of the base layer was found to be about 6 in. also. As mentioned before, the as-built thickness of the AC layer was reported as 7 in. On the basis of actual coring after the completion of the analysis, it was found that the AC layer was severely stripped. Actually only about 2.5 to 3 in. of the core could be recovered and the rest of the AC layer had lost its binding agent.

The FWD tests were also carried out at the site. Because of time constraints, the drop load was limited to 10,000 lb (nominally). Program MODULUS 4.0 (*22*,p.72) was used to backcalculate the moduli. Three different pavement profiles were used. In the first profile, the layering obtained from construction drawings was used. In this case, the thickness of the AC and base were assumed to be 7 and 10 in., respectively. The backcalculated moduli from this layering are reported in Table 6. Also shown in the table are the moduli obtained

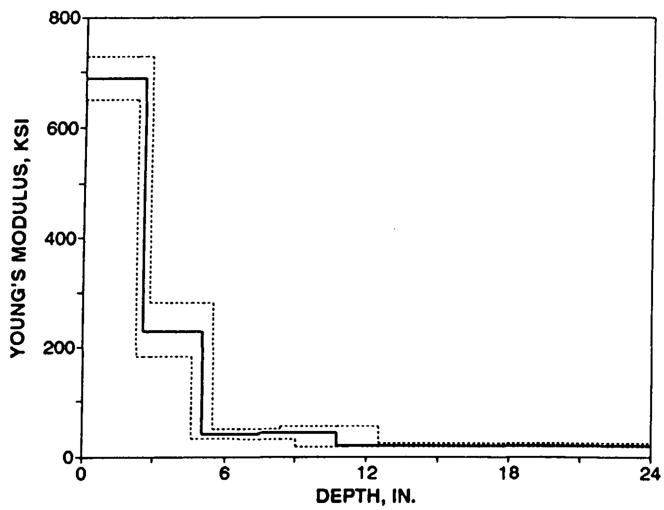


FIGURE 8 Final modulus profile with 95 percent confidence bounds.

TABLE 6 Comparison of Moduli Obtained from SASW and FWD

	Modulus, ksi				
	AC*		Base	Subgrade	Absolute Average
Method	Layer 1	Layer 2			Errors (%)
SASW	688	229	41	21	1.8
FWD (Construction Drawings)*	193		6	21	0.9
FWD (SASW)*	500	286	4	21	1.3
FWD (Coring)*	323	273	4	21	1.0

* Source for Determining Layer Thickness

from SASW tests. The modulus of the subgrade from both methods are comparable; however, the moduli of the base and AC layers are quite different. It would be difficult to derive any conclusions from the FWD moduli of the top two layers, because the actual layer thicknesses were significantly different from those assumed in the backcalculation process.

The second pavement profile used was the one obtained from the SASW tests. In this case, the modulus values are much closer. The modulus of base seems unreasonably low (4 ksi). The reason for this matter is not known at this time.

The final pavement profile used was the one obtained from the actual coring of the site. The thickness of the intact AC layer was reported as 2.5 to 3 in. and the thickness of the base was reported between 6 and 7 in.

This profile yields moduli that are similar to those of the SASW profile. This result is expected because the layering from the SASW tests and coring are relatively close.

SUMMARY AND CONCLUSIONS

On the basis of the general inverse theory, a backcalculation technique for estimating layer properties of pavements from surface wave dispersion data has been developed. The theoretical principle and numerical considerations of this technique are briefly described and discussed. Results from both synthetic (theoretically derived) and actual field dispersion data are presented. These preliminary and limited results demonstrate that the technique may be an effective tool for estimating elastic modulus profiles of pavement systems. More work is under way to improve and accelerate the method.

Because a pavement structure contains only a few material layers, and certain prior information about them is always available, simultaneous backcalculation for both layer thicknesses and elastic moduli may provide more reliable results.

ACKNOWLEDGMENTS

This work has been mainly funded by the Texas Department of Transportation, and the authors would like to thank that organization. Robert C. Briggs and Jeff Jackson have been the project coordinators. The Strategic Highway Research Program is also acknowledged. The case history was carried out for that organization under Project H-104B. The efforts of Bob Briggs to reduce the FWD data are highly appreciated.

REFERENCES

1. S. Nazarian and K. H. Stokoe, II. *In Situ Determination of Elastic Moduli of Pavement Systems by Spectral-Analysis-of-Surface-Waves Method (Practical Aspects)*. Research Report 368-1F. Center for Transportation Research, University of Texas, Austin, 1986.
2. S. Nazarian and K. H. Stokoe, II. *In Situ Determination of Elastic Moduli of Pavement Systems by Spectral-Analysis-of-Surface-Waves Method (Theoretical Aspects)*. Research Report 437-2. Center for Transportation Research, University of Texas, Austin, 1987.
3. S. Nazarian. *In Situ Determination of Elastic Moduli of Soil Deposits and Pavement Systems by Spectral-Analysis-of-Surface-Waves Method*. Ph.D. dissertation. University of Texas, Austin, 1984.
4. S. Nazarian, M. Baker, and R. E. Smith. *Measurement Concepts and Technical Specifications of Seismic Pavement Analyzer*. Center for Geotechnical and Highway Materials Research, University of Texas, El Paso, 1991.
5. M. Desai. *An Automated Method for Construction of Surface Wave Dispersion Curve*. M. S. thesis. University of Texas, El Paso, 1991.
6. R. A. Wiggins. The General Linear Inverse Problem: Implications of Surface Waves and Free Oscillations on Earth Structure. *Reviews of Geophysics and Space Physics*, Vol. 10, 1972, pp. 251–285.
7. D. D. Jackson. Interpretation of Inaccurate, Insufficient and Inconsistent Data. *Geophysical Journal of the Royal Astronomical Society*, Vol. 28, 1972, pp. 97–109.
8. I. Sanchez-Salinero, J. M. Roësset, K.-Y. Shao, K. H. Stokoe, II, and G. J. Rix. Analytical Evaluation of Variables Affecting Surface Wave Testing of Pavements. In *Transportation Research Record 1136*, TRB, National Research Council, Washington, D.C., 1987, pp. 86–95.
9. J. C. Shue, K. H. Stokoe, II, and J. M. Roësset. Effect of Reflected Waves on SASW Testing of Pavements. In *Transportation Research Record 1196*, TRB, National Research Council, Washington, D.C., 1989, pp. 51–61.
10. R. Jones. Surface Wave Technique for Measuring the Elastic Properties and Thickness of Roads: Theoretical Development. *British Journal of Applied Physics*, Vol. 13, 1962, pp. 21–29.
11. R. F. Vidale. *The Dispersion of Stress Waves in Layered Media Overlying a Half Space of Lesser Acoustic Rigidity*. Ph.D. thesis. University of Wisconsin, 1964.
12. K. Sezawa and K. Kanai. Discontinuity in the Dispersion Curves of Rayleigh Waves. *Bulletin of Earthquake Research Institute*, Vol. 13, 1935, pp. 237–244.
13. W. T. Thomson. Transmission of Elastic Waves Through a Stratified Solid Medium. *Journal of Applied Physics*, Vol. 21, 1950, pp. 89–93.
14. N. A. Haskell. The Dispersion of Surface Waves in Multilayered Media. *Bulletin of the Seismological Society of America*, Vol. 43, 1953, pp. 17–34.
15. E. Kausel and J. M. Roësset. Stiffness Matrices for Layered Soils. *Bulletin of the Seismological Society of America*, Vol. 71, 1981, pp. 1743–1761.
16. J. Lysmer and L. A. Drake. A Finite Element Method for Seismology. In *Methods in Computational Physics*, Vol. 11, Academic Press Inc., New York, N.Y., 1972, pp. 181–216.
17. M. M. Hossain and V. P. Drnevich. Numerical and Optimization Techniques Applied to Surface Waves for Backcalculation of Layer Moduli. *Nondestructive Testing of Pavements and Backcalculation of Moduli*, ASTM Special Technical Publication 1026, Baltimore, Md., 1989.
18. J. E. Dunkin. Computation of Modal Solutions in Layered, Elastic Media at High Frequencies. *Bulletin of Seismological Society of America*, Vol. 55, 1965, pp. 335–358.
19. G. H. Golub and C. Reinsch. Singular Value Decomposition and Least Squares Solution. *Numerische Mathemetik*, Vol. 14, 1970, pp. 403–420.
20. D. L. B. Jupp and K. Vozoff. Stable Iterative Methods for Inversion of Geophysical Data. *Geophysical Journal of the Royal Astronomical Society*, Vol. 42, 1975, pp. 957–976.
21. N. R. Draper and H. Smith. *Applied Regression Analysis*. John Wiley and Sons, New York, N.Y., 1980.
22. J. Uzan, T. Scullion, C. H. Michalek, M. Parades, and R. L. Lytton. A Microcomputer-Based Procedure for Backcalculating Layer Moduli from FWD Data. Research Project 1123-1. Texas Transportation Institute, Texas A&M University, College Station, 1988.