Maximum Bearing Stress of Concrete in Doweled Portland Cement Concrete Pavements

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Hundreds of numerical calculations were conducted using the component model developed recently to test the accuracy of existing design procedures. It has been found that the maximum bearing stress of concrete, under the critical dowel, cannot be accurately predicted by the "effective length" assumption that is currently used in engineering analysis. A detailed discussion to analyze the potential problems and the causes of the problems is presented. Errors in computed values of maximum bearing stress can also affect the prediction of joint faults in pavement performance models.

Smooth round dowel bars have been employed as load transfer devices in jointed concrete pavements for a long time. Many experimental and analytical studies have been conducted to develop and improve the design procedure for dowels. Before the 1960s, the most influential analytical models were developed by Timoshenko et al. (*1*) and Friberg (*2*), and some significant experimental studies of dowelled slabs were conducted by Teller et al. (*3,4*), Finney et al. (*5*), and Keeton et al. (*6*). The complete review of these studies and the application in engineering design can be found in the thesis by Snyder (*7*) and the report by Heinrichs et al. (*8*).

Researchers have concluded that the maximum concrete bearing stress is the most important parameter to be determined in portland cement concrete (PCC) pavement joint design. Currently the maximum bearing stresses of concrete under dowels are required to be equal to or smaller than the concrete bearing strength. Furthermore, the level of the bearing stress has direct effects on the accumulation of joint faulting, which is a very important parameter in the performance of PCC pavements (*8*).

The procedure of Friberg (*2*) can be generally divided into two steps to determine the maximum bearing stress. The first step is to predict the maximum shear force acting on the critical dowel bar. The second step is to calculate the maximum bearing stress on the concrete under the critical bar by using the maximum shear force obtained in the first step.

The first step is based on three assumptions:

1. A certain percentage of the total load is transferred across the dowelled joint. The range varies from 0 to about 50 percent, depending on the quality of the joint, pavement structural parameters, and the load type. Because 50 percent would be a maximum bearing stress, Heinrichs et al. (*8*) suggest using 45 percent.
2. The dowel shear forces are linearly distributed along the joint (see Figure 1).
3. An "effective load transfer length" (L_1) was assumed to be 1.8*l* by Friberg (*2*), and all dowels located farther than L_1 from the load center do not contribute to transferring load. The radius of relative stiffness of the slab was determined to be *l* by the formula

$$l = \left[\frac{Eh^3}{12(1 - \mu^2)k}\right]^{1/4} \text{ (in.)} \tag{1}$$

where

E = elasticity modulus of concrete (psi),
μ = Poisson's ratio of the concrete,
h = thickness of the concrete slab (in.), and
k = modulus of the subgrade.

Using these assumptions, the maximum shear force acting on the critical dowel bar can be calculated. As mentioned above, L_1 = 1.8*l* was proposed by Friberg (*2*). In the second step, the shear force on the dowel is assumed known. The model of Timoshenko et al. (*1*) gives a procedure to predict the behavior of a steel bar embedded in "pure elastic" concrete. On the basis of the Timoshenko theory, Friberg (*2*) derived the maximum bearing stress (σ_{max}) formula:

$$\sigma_{max} = \psi\delta_0 \tag{2}$$

where

$$\delta_0 = \frac{P_1(2 + \beta J_0)}{4\beta^3 E_s I} \tag{3}$$

in which

P_1 = "maximum" shear force acting on the dowel, predicted in the first step (lb);
J_0 = width of the joint opening (in.);
E_s = modulus of elasticity of the dowel bar (psi);
I = moment of inertia of dowel bar cross section = 0.25 × π $(D/2)^4$ (in.^{2});
D = diameter of the dowel bar (in.);
β = $(\psi\ D/4E_sI)^{0.25}$ (1/in.); and
ψ = dowel-concrete interaction coefficient (lb/in.^{3}).

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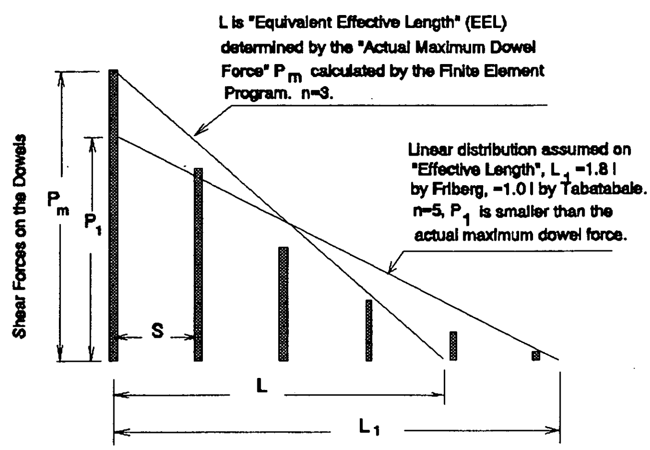

FIGURE 1 Effective length (L_1) and equivalent effective length (L).

After conducting a three-dimensional finite element analysis for a dowel embedded in an elastic concrete space, Tabatabaie et al. (*9*) proposed to use the following formula to determine the maximum bearing stress directly:

$$\sigma_{max} = \frac{(800 + 0.068E)}{D^{4/3}}(1 + 0.355J_0)P_1 \quad \text{(psi)} \tag{4}$$

where E is concrete elastic modulus in kpsi and P_1 is the maximum shear force acting on the critical dowel as determined previously (*9,10*) by using the ILLISLAB program as follows:

$$P_1 = \alpha S P_t \tag{5}$$

in which

- α = 0.0091, for edge load;
- α = 0.0116, for protected corner load (with tie bar on shoulder);
- α = 0.0163, for unprotected corner load (without tie bar on shoulder);
- S = dowel spacing (in.); and
- P_t = total load (kp).

On the basis of the results produced by using the finite element program ILLISLAB, Tabatabaie et al. (*9*) concluded, "Only the dowels within a distance 1.0l from the center of the load are effective in transferring the major part of the load." It is obvious that this assumption is more conservative than that of Friberg. Henrichs et al. (*8*) proposed to use L_1 = 1.0l instead of 1.8l in the first step to predict the maximum shear force acting on the critical dowel; then he used Friberg's model (Equation 3) to determine the maximum bearing stress.

SOME COMMENTS

The Effective Length

As summarized earlier, the determination of the maximum bearing stress can be divided into two steps. First, the effective length (EL) was introduced to determine the maximum shear force acting on the critical dowel. However, the only purpose of the first step is to determine the maximum shear force on the dowel. If the effective length assumption is good, the maximum shear force P_1 predicted by using the assumed effective length should be identical or close to the actual maximum shear force P_m. Figure 1 indicates that when the distribution of the dowel shear forces is very nonlinear, the procedure could cause significant error ($P_1 < P_m$ in most cases).

Comparison of a Few Numerical Examples

Table 1 presents the maximum bearing stresses calculated by using Friberg's model (Equations 2 and 3, effective length L_1 = 1.8l and L_1 = 1.0l), the model of Tabatabaie et al. (Equation 4), and the component dowel bar model [Guo et al. (*11,12*)], which have been installed in computer program JSLAB-92 on the basis of the original JSLAB (*13*). The pavement analyzed is presented in Figure 2. The parameters used are h = 25.4 cm (10 in.), Ψ = 0.4065 × 10^6 MPa/m (1.5 million psi/in.), E = 31.005 GPa (4.5 million psi), and J_0 = 0.635 cm (0.25 in.).

Table 1 demonstrates that the results acquired by using different models are different. It is important to understand what would cause these discrepancies.

Effects of the Subgrade Modulus

Figure 3, taken from Figure 44 of Heinrichs et al. (*8*), indicates that the maximum bearing stress increases when the subgrade modulus k increases. The same conclusion can also be obtained by analyzing Equations 2 and 3 (see also Table 1). However, this conclusion is difficult to interpret. It does not seem logical that as subgrade support at the joint increases, all other factors being equal, the maximum bearing stress would also increase.

Effects of Concrete Modulus E

Equation 4 indicates that the maximum bearing stress increases as the concrete modulus increases when the other parameters remain the same. This conclusion does not agree with the results obtained from Friberg's model. As discussed earlier, the higher concrete modulus means that the loaded side has greater load resistance capability, so that the total load and the maximum load transferred by the critical dowel should be reduced, not increased as suggested by Equation 4.

Since development in the 1940s, the effective length concept has been widely used in PCC pavement design. It may be worthwhile to reinvestigate the concept to possibly improve the design procedure.

EQUIVALENT EFFECTIVE LENGTH

Based on the EL assumption, if the total load is known and the percent of the total load transferred is assumed, the maximum shear force can be calculated by the following formulas.

TABLE 1 Calculated Maximum Bearing Stresses by Various Models

k \ D	1.905 cm (0.75 in)	3.175 cm (1.25 in)	4.445 cm (1.75 in)
Friberg's model, L = 1.8 l			
13.55 MPa/m(50 pci)	16.4/2387	6.51/945	3.56/516
54.2 MPa/m(200 pci)	22.1/3208	8.75/1270	4.78/694
135.5 MPa/m(500 pci)	26.65/3868	10.56/1532	5.77/837
Friberg's model, L = 1.0 l			
13.55 MPa/m(50 pci)	26.91/3907	10.66/1547	5.82/845
54.2 MPa/m(200 pci)	35.44/5143	14.03/2037	7.67/1113
135.5 MPa/m(500 pci)	41.08/5962	16.27/2361	8.89/1290
Component dowel bar model			
13.55 MPa/m(50 pci)	35.54/5158	19.4/2815	13.53/1964
54.2 MPa/m(200 pci)	30.72/4459	17.07/2478	11.66/1692
135.5 MPa/m(500 pci)	26.5/3846	15.36/2229	10.5/1524
Tabatabaie's model			
k=any values	21.43/3111	10.84/1574	6.92/1005

NOTE: Values are in MPa (psi).

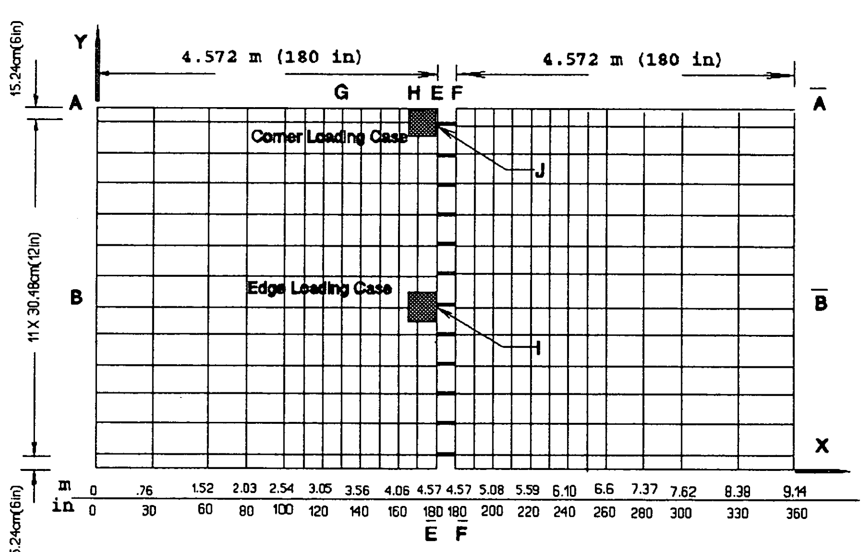

FIGURE 2 Finite element mesh and two load cases.

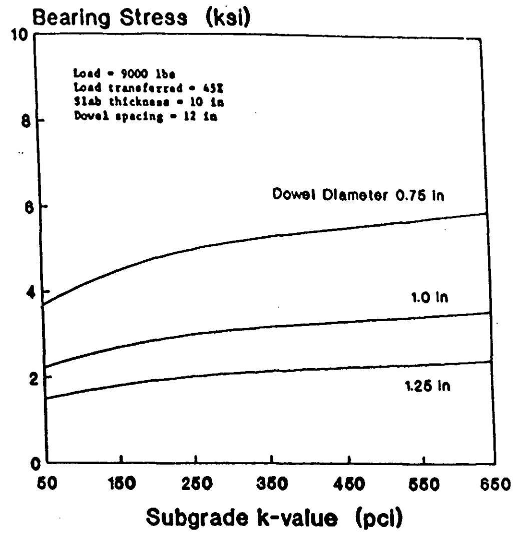

FIGURE 3 Effects of subgrade modulus on maximum bearing stress (8).

When the load is located in the middle of the joint (defined as edge loading),

$$P = \frac{P_t C}{2n + 1 - \dfrac{Sn(n + 1)}{L}} \tag{6}$$

When the load is located at an end of the joint (defined as corner loading),

$$P = \frac{P_t C}{n + 1 - \dfrac{n(n + 1)S}{2L}} \tag{7}$$

where

- P = maximum shear force and is equal to P_1 in Friberg's model (Figure 1);
- n = number of effective dowels on one side of bar under load (number of total active bars is $2n + 1$ for the edge loading case and $n + 1$ for the corner loading case);
- S = dowel spacing;
- C = percent of total load transferred across joint; and
- P_t = total load.

Some of the above parameters are shown in Figure 1.

Because the most important parameter in the first step of the current design procedure is maximum shear force, the EEL may be defined as a length L that can be determined by Equation 6 or 7, depending on the edge or corner loading case. The maximum shear force and the percent of the total load transferred can be calculated by an appropriate finite element program as discussed by Guo et al. (*11,12*). According to definition, n is not the total number of the effective dowels on one side of the bar under the load; it is only the number of the dowels that significantly transfer load. As shown in Figure 1, n is 5 under the EL definition but is only 3 under the EEL definition.

The EEL formulas can be obtained by solving L in Equations 6 and 7 as follows (P is equal to P_m when the EEL definition is applied). When the load is located at the edge of the joint,

$$\frac{L}{l} = \frac{P_m n(n + 1)\, S}{l[(2n + 1)P_m - P_t C]} \tag{8}$$

When the load is located at the corner of the joint,

$$\frac{L}{l} = \frac{P_m n(n + 1)\, S}{2l[(n + 1)P_m - P_t C]} \tag{9}$$

In any case the following formula must be satisfied:

$$n \le \frac{L}{S} \le (n + 1) \tag{10}$$

There exists a significant difference between the concept of the EL and the EEL. The EL is an assumed value for predicting the maximum shear force that may be either more or less than the actual value. The EEL is the value calculated by using the "actual maximum shear force" predicted by the finite element method so that when it is substituted back in Equation 6 or 7 the predicted maximum shear force must be equal to the actual force calculated by the finite element program. Friberg (*2*) proposed $L_1/l = 1.8$ and Tabatabaie et al. (*9*) suggested $L_1/l = 1.0$, whereas both assumed L/l constant. However, Equations 8 and 9 indicate that L/l, where L is corresponding to the actual maximum shear force of the dowels, is a function of n, S, P_m, P_t, and C, as well as the radius of relative stiffness l.

Some Characteristics of EEL

JSLAB-92 [a computer program modified from the JSLAB version 1986 developed by Tayabji et al. (*13*)] was employed to calculate the maximum shear force on the dowel bar. The finite element mesh is given in Figure 2. The numerical analyses were conducted for two loading cases. The first is a 40-kN (9,000-lb) load with tire pressure of 344.5 kPa (50 psi) acting at Node I in Figure 2. The loading area is 30.5 × 38.1 cm^2 (12 × 15 $in.^2$), and this case is defined as edge loading. The second is the same type of load acting at Node J in Figure 2, and this case is defined as corner loading. Using the calculated maximum shear forces and the percentage of total load transferred across the joint, L/l may be calculated by using Equations 8 through 10.

By using the EL assumption, the higher subgrade modulus always reduces the l-value (Equation 1) and then reduces L_1 ($L_1 = 1.81$ or $L_1 = 1.01$) and the number of effective dowels n (Equation 10). Because the percent of the total load transferred is assumed constant, the maximum shear force and bearing stress will always increase as shown in Table 1. However, Figure 4 indicates that L/l increases when the k-value increases. Figure 5 shows that the total load transferred decreases as the k-value increases. Figures 4 and 5 explain that the increase of subgrade modulus does not have to increase the maximum bearing stress.

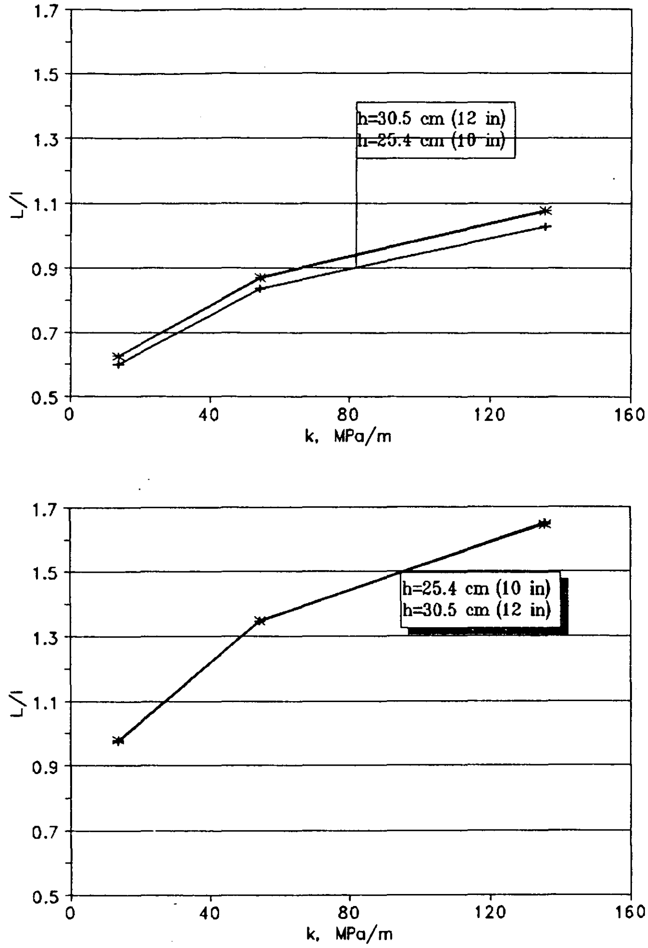

FIGURE 4 Effects of subgrade modulus on EEL: *top*, load at corner; *bottom*, load at edge.

Relation Between EEL and *l*

Figure 6 plots the 12 examples with the same dowel-concrete interaction coefficient (0.407 × 10^6 MPa/m = 1.5 × 10^6 psi/in.), dowel diameter (3.175 cm = 1.25 in.), and joint opening (0.635 cm = 0.25 in.), but with different slab thickness and subgrade modulus for both edge and corner loading cases. For the same radius of the relative stiffness value *l*, the *L* value could be very different. Therefore, the rather wide bandwidth suggests that the assumptions of L/l = 1.8 or 1.0 might not be an appropriate assumption to accurately predict the maximum shear forces on the critical dowel and the maximum bearing stress of the concrete.

EFFECTS ON MAXIMUM BEARING STRESS

When Friberg (*2*) developed the dowel bar analytical model in 1940s, it was impossible to analytically predict the maximum shear force acting on the critical dowel precisely; hence, he proposed the approximate but simple procedure for the dowel bar design. Since development of the finite element method and the application of high-speed computers, more options now exist for analyzing the load transfer mechanism. For example, it is not necessary to divide the entire analysis procedure into two steps. As discussed by Guo et al. (*11*,*12*), the component model of a dowel bar can be installed into a finite element program to calculate the responses of each dowel, including the distribution of bending moments, shear forces, the relative displacements of the beam, and the bearing stresses of the concrete. The results are calculated with comprehensive consideration of all inputs simultaneously and without additional assumptions such as effective length and percent of total load transferred. In this section, more numerical examples will be given to analyze the effects of different variables on the maximum bearing stress of the critical dowel. All results presented in this section were calculated for a single tire with 40 kN (9,000 lb) acting at the corner of the slab (Figure 2).

Effects of Slab Thickness and Subgrade Modulus

Figure 7 shows that the bearing stress decreases when the slab thickness increases. Four curves showing the effects of the subgrade modulus on the maximum stress are presented in Figure 8 and indicate that the maximum stress decreases when the subgrade modulus (*k*-value) increases. This conclusion is different from the results presented previously (*8*,*9*) (see also

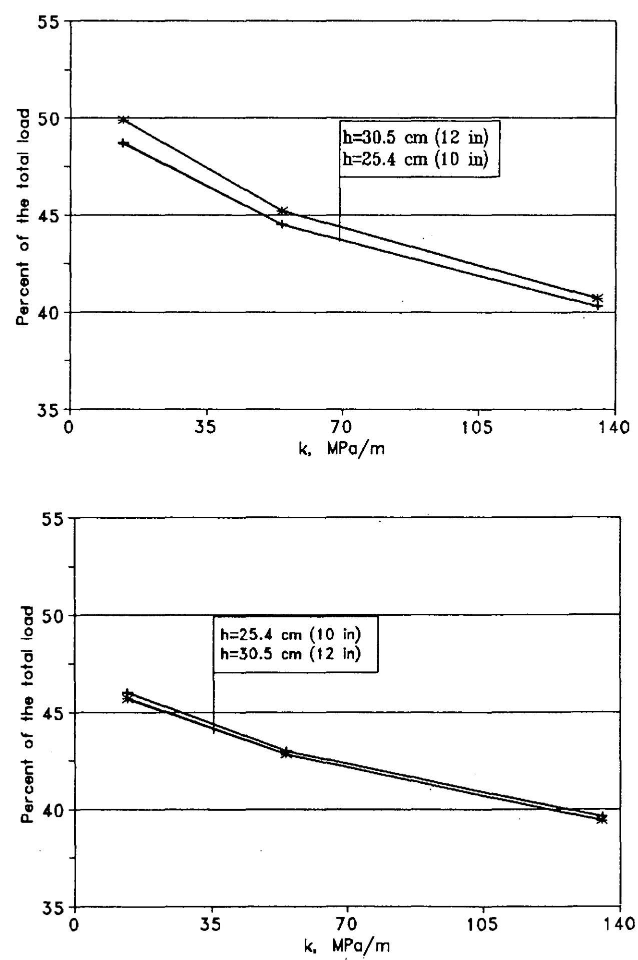

FIGURE 5 Effects of subgrade modulus on load transfer efficiency: *top*, load at corner; *bottom*, load at edge.

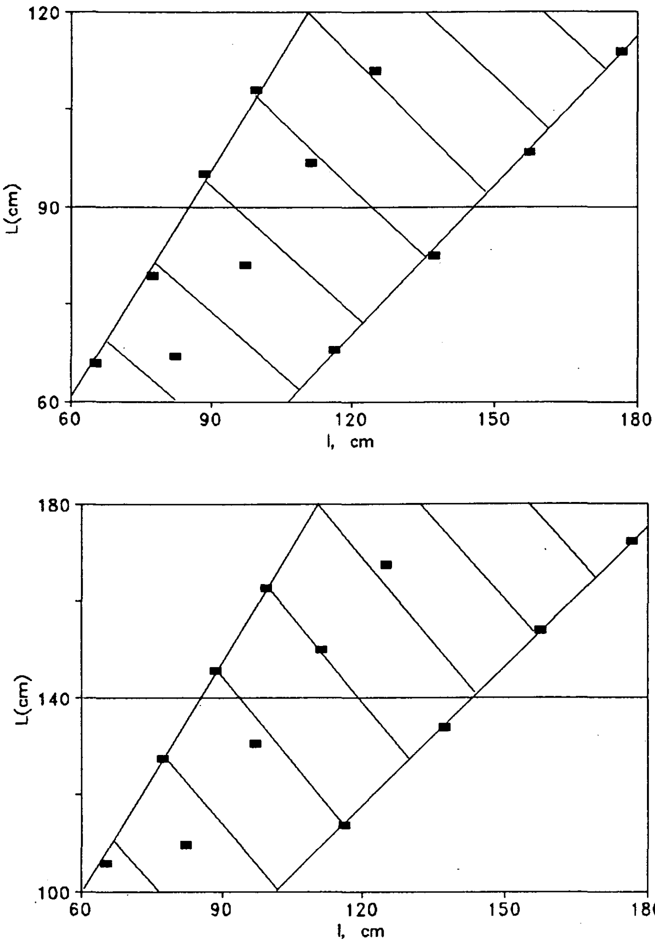

FIGURE 6 Relationship between EEL and *l* (radius of relative stiffness): *top*, load at corner; *bottom*, load at edge.

Figure 3 and Table 1). It is believed that the historical discrepancy was caused by using the effective length assumption, which sometimes cannot accurately describe the maximum bearing stress characteristics.

Effects of Dowel Diameter and Width of Joint Opening

Figure 9 indicates that the maximum bearing stress of the concrete is very sensitive to the dowel's diameter (D), which might be the most sensitive parameter of all. The smaller diameter can cause a dramatic increase in the maximum stress. This finding qualitatively has good agreement with those found in previous formulas (*2,9*) (Equations 2 through 4). Both Figures 9 and 10 indicate the insensitivity of the maximum bearing stress due to the variation of width of the joint opening. Using Equation 4 to predict the maximum bearing stress, the increase of joint opening from 0.635 cm (0.25 in.) to 1.905 cm (0.75 in.) yields about a 16 percent increase in the maximum bearing stress, $[0.355 \times (0.75 - 0.25)/(1 + 0.355 \times 0.25)] \times 100 = 16.3$. However, by using the dowel bar component model in JSLAB-92, the increase is only 1.2 percent for the D = 1.905-cm (0.75-in.) dowel, 3.3 percent for the

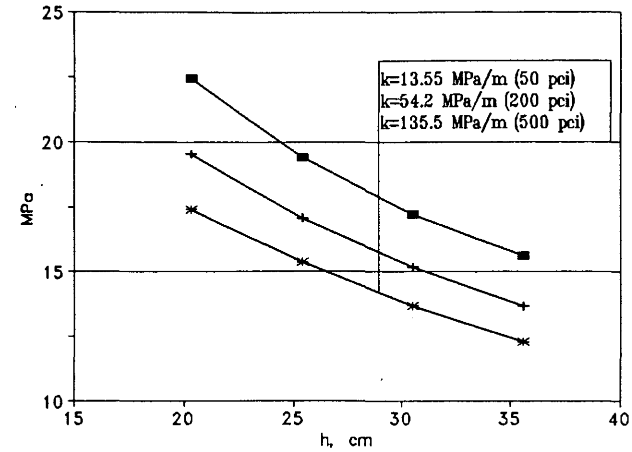

FIGURE 7 Effects of slab thickness on maximum bearing stress.

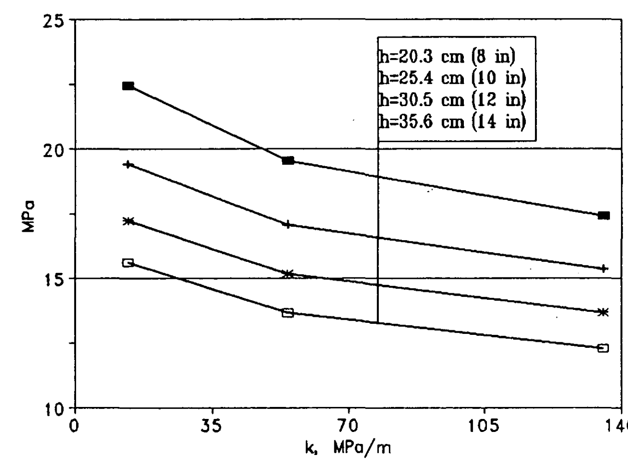

FIGURE 8 Effects of subgrade modulus on maximum bearing stress.

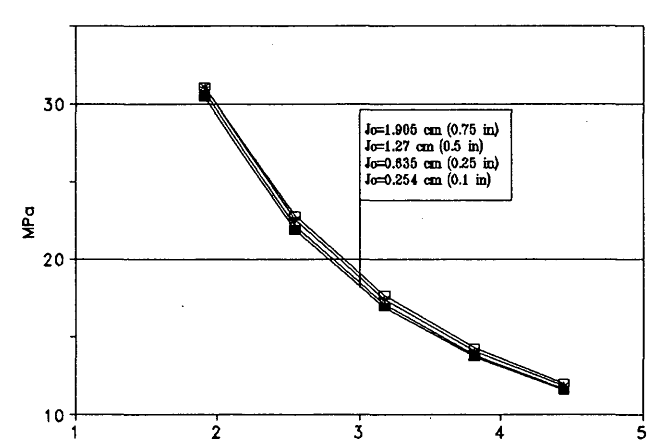

FIGURE 9 Effects of dowel diameter on maximum bearing stress.

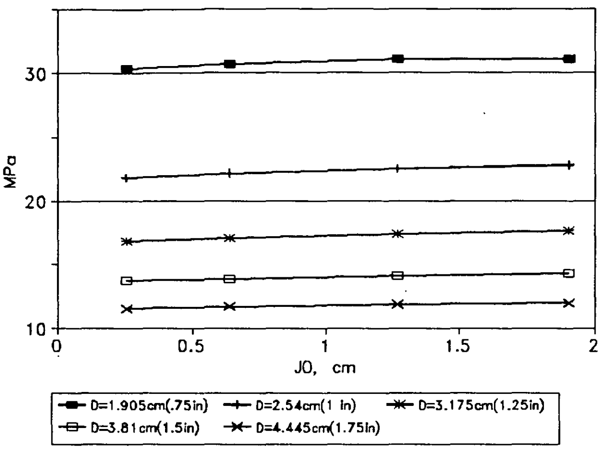

FIGURE 10 Effects of width of joint opening on maximum bearing stress.

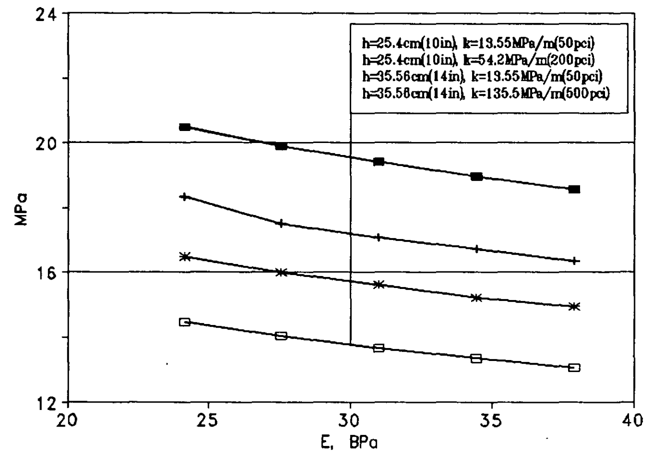

FIGURE 12 Effects of concrete elasticity modulus on maximum bearing stress.

D = 3.175-cm (1.25-in.) dowel, and 2.7 percent for the D = 4.445-cm (1.75-in.) dowel.

Effects of Concrete Elasticity and Dowel-Concrete Interaction Coefficient

Figure 11 presents the maximum stress curves versus the dowel-concrete interaction coefficient Ψ. As summarized previously (*5*), the values of Ψ measured by various investigators varied from 81.3 to 2331 GPa/m (0.3 × 10^6 to 8.6 × 10^6 lb/in.^{3}). In practice, 406.5 GPa/m (1.5 × 10^6 lb/in.^{3}) is often used. However, Ψ is a function of such variables as concrete properties, dowel bar diameter, slab thickness, dowel length, and dowel looseness. Figure 11 shows that the higher Ψ corresponds to the higher maximum bearing stress and implies that the dowel in a deteriorated joint or with significant looseness would have a smaller maximum bearing stress.

Figure 12 shows that the maximum bearing stress decreases when the concrete elasticity modulus increases. As discussed earlier, this is understandable because the higher E-value indicates that the stronger loaded slab can withstand more of the load and will distribute less load across the dowels to the unloaded slab. Because the role of the higher E-value is similar to that of the higher k-value of the subgrade, both should reduce the amount of load transferred across the joint. However, Equation 4 indicates that the higher E-value would cause higher maximum bearing stress. The discrepancy might be partially caused by the application of the dowel stiffness matrix in ILLISLAB and the component stiffness matrix in JSLAB-92 (*11,12*). The former produces a nonequilibrium element, which occasionally leads to unreasonable results.

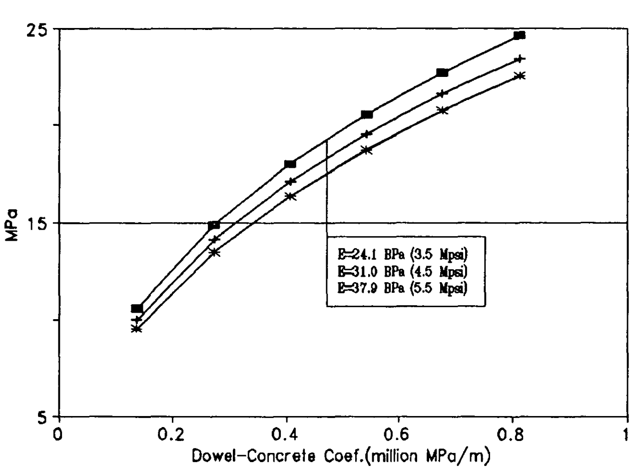

FIGURE 11 Effects of dowel-concrete interaction coefficient on maximum bearing stress.

SUMMARY

A general principle has been drawn from analyzing the results of hundreds of numerical examples: The higher values of dowel diameter, slab thickness, concrete modulus, and subgrade modulus can reduce the maximum bearing stress of the concrete under the critical dowel. The maximum bearing stress is not sensitive to the width of joint opening but is very sensitive to the dowel-concrete interaction behavior (which is difficult to control).

The discovery of an error in the stiffness matrix of dowel bars used in some finite element programs and of the inappropriate utilization of the joint effective length made it necessary to reevaluate some design procedures for the dowel system. The following are the major findings in this paper.

- Equation 4 could yield some questionable results. The maximum bearing stress does not increase proportionately with the increase in concrete elasticity E, and it is also not sensitive to the width of joint opening, as indicated in Equation 4.
- The maximum bearing stress of the critical dowel increases as the subgrade modulus decreases. This finding, different from the conclusion presented in the historical literature, suggests that the most critical season in the year for the maximum bearing stress is spring because the thawing reduces the subgrade modulus in the wet-frozen region. The thawing

effect (subgrade softening) could cause a 10 to 20 percent difference in the maximum bearing stress.

- The use of the effective length, which began in the 1940s and was modified at the end of the 1980s, underestimates the maximum bearing stress in some cases. The equivalent effective length concept presented herein has been developed to prove that the EL assumption needs more study.
- The most critical dowel is the one under a tire load nearest the corner. The maximum bearing stress could be two times that of the critical dowel stress under the tire load at the edge of the joint.
- Because of the significant difference between the results obtained from the existing and the developed models in predicting the maximum bearing stresses, it is suggested that all empirical models that use Equation 4 to calculate the maximum bearing stress, and then to predict the joint faulting, should be reevaluated before being employed in engineering projects.

ACKNOWLEDGMENT

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DISCUSSION

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In the pursuit of improved mechanistic design procedures for doweled PCC pavements, the "component" dowel formulation introduced by the authors (*3,4*) is certainly a rigorous and well-founded contribution. Furthermore, results presented in the paper are invaluable in clarifying several crucial issues related to the design of doweled joints. Intense research activities at the University of Illinois (UI) in the last 5 years have also focused on unraveling the complex interactions arising at such joints (*14,15*). The purpose of this discussion is to complement the authors' work by providing some additional information from the UI studies.

An important conclusion reached by the authors is that "the maximum bearing stress (σ_{max}) . . . increases as the subgrade modulus (k) decreases," thereby contradicting "the historical literature." The authors correctly point out that the finite element (FE) program J-SLAB does not confirm the trend presented in a sensitivity plot (Figure 2) by Heinrichs et al. (*8*). However Figure 2 was compiled using computer program PFAULT, and not the UI FE program ILLI-SLAB. Whereas the latter is primarily an analytical tool, PFAULT was intended to facilitate the application of a mechanistic-empirical design algorithm. As such, PFAULT incorporates a number of reasonable and necessary—at the time—assumptions, including two alluded to by the authors, that is, that the transferred load efficiency (C) is assumed to be 45 percent and that the "effective length" (L) is assumed to be equal to the radius of relative stiffness (l) of the slab-subgrade system. From a design point of view (*14*), another significant assumption pertains to the value of the "dowel-concrete interaction coefficient" (Ψ) which is set to 1.5 million lb/in.^{2}. When ILLI-SLAB is used to verify the authors' results in Table 1, it is found that, indeed, as k increases, σ_{max} decreases. The source for the discrepancy in trends noted by the authors is, therefore, not the result of a difference between the J-SLAB and ILLI-SLAB FE programs, but rather the consequence of the semiarbitrary assumptions made in PFAULT to incorporate knowledge obtained using the FE method into a practical design algorithm, which would not necessitate the execution of any demanding FE code. Table 2 clearly indicates that neither the C- nor the L-assumptions in PFAULT are satisfied in the cases considered by the authors. The contradiction identified by the authors points to the need to exercise caution when empirical (or even mechanistic-empirical) algorithms are used in sensitivity analyses, particularly concerning variables for which they afford little resolution. As Heinrichs et al. (*8*) pointed out, "the (PFAULT) solution is more sensitive to those variables related to the dowel bars used (i.e., dowel diameter, spacing, and modulus of support) than to pavement system characteristics (e.g., slab modulus and thickness, subgrade modulus, and joint width)."

It is also observed in Table 2 that ILLI-SLAB results can be considerably different from those obtained using J-SLAB. It is not readily apparent, however, that such differences are exclusively due to the "component" dowel formulation introduced in J-SLAB. Similar discrepancies had been observed in comparisons with results from an earlier version of J-SLAB and were ascribed to "differences in the FE idealizations adopted in each of these programs" (*15*). J-SLAB was found to accentuate "the relative importance of the bending load transfer mechanism compared to the shear mechanism by 5 to 20 times." Note that the ILLI-SLAB results in Table 2 were obtained assuming that the dowels are pure shear load transfer devices (no moment transfer), which is consistent with the prevailing understanding of in situ phenomena (*16*). It would be very enlightening as to the net effect of the introduction of the "component" dowel formulation if the authors presented a comparison between results obtained using the original and their modified versions of J-SLAB.

The authors do not address the development of a design algorithm that would dispense with the need to execute an FE code on a case-by-case basis. Setting $L = nS$ in Equations 6 and 7, these become identical to the formulas presented by Ioannides et al. (*17*); that is, an assumption about the size of L is still required. Equations 8 and 9 cannot be used for this purpose because they involve the maximum dowel shear force (P_m), which would be available only if the FE code were executed. Such an execution would, of course, determine L precisely as well, thus dispensing with the need for an assumption about its size. The authors note that (L/l) "is a function of n, S, P_m, P_t, and C, as well as the radius of relative stiffness, l." The large number of variables warrants the use of dimensional analysis as done by Ioannides and Korovesis (*14*). A much simpler solution to this problem was recently proposed by Khazanovich and Ioannides (*18*), who suggested estimating P_m by the following expression:

$$P_m = B \frac{1 - \mathrm{LTE}_\delta}{1 + \mathrm{LTE}_\delta} \Delta_f \tag{11}$$

In this expression LTE_δ is the load transfer efficiency in terms of deflections, B is set equal to the composite (springs-in-series) shear stiffness of the composite dowel [given by Equation 2 previously (*14*)], and Δ_f is the free-edge (or free-corner) deflection at the critical location. LTE_δ may be measured in the field or may be estimated using the *S*-shaped curve in Figure 1 (*14*) for edge loading; a similar curve may also be compiled for corner loading. Δ_f may be calculated using the appropriate Westergaard equation (*19*) or simply set equal to the sum of the deflections measured in the field on the loaded and unloaded sides of the joint at the critical location. The last column in Table 2 confirms that this method applies, assuming that dowels are pure shear load transfer devices.

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TABLE 2 Verification of Authors' Results for Corner Loading

RUN	k pci	D in.	P_T lb	P_m lb	ILSL σ_{max} psi	JSLAB σ_{max} psi	C %	f_{dc}	L/l	δ_L mils	δ_U mils	LTE_δ	B lb/in.	P_m* lb
1	50	0.75	3957	2273	5987	5158	43.96	0.574	0.565	37.17	33.04	0.889	5.51E+05	2273
2	200	0.75	3616	2086	5494	4459	40.17	0.576	0.795	16.52	12.73	0.770	5.51E+05	2086
3	500	0.75	3278	1900	5004	3846	36.42	0.579	0.993	10.18	6.737	0.661	5.51E+05	1900
4	50	1.25	4010	2743	2861	2815	44.55	0.684	0.438	36.09	34.13	0.945	1.40E+06	2743
5	200	1.25	3751	2584	2695	2478	41.67	0.688	0.613	15.55	13.70	0.881	1.40E+06	2584
6	500	1.25	3509	2430	2535	2229	38.98	0.692	0.765	9.332	7.592	0.813	1.40E+06	2430
7	50	1.75	4027	3033	1728	1964	44.74	0.753	0.377	35.70	34.52	0.966	2.56E+06	3033
8	200	1.75	3794	2882	1642	1692	42.15	0.759	0.526	15.18	14.06	0.925	2.56E+06	2880
9	500	1.75	3586	2742	1562	1524	39.84	0.764	0.655	8.997	7.927	0.881	2.56E+06	2741

Note: P_T = total transferred load by all dowels;
C = transferred load efficiency = $(P_T/P_t) \times 100\%$;
f_{dc} = critical distribution factor = (P_m/P_T);
Δ_L = deflection on the loaded side of the joint at the critical location;
Δ_U = deflection on the unloaded side of the joint at the critical location;
LTE_δ = (Δ_U/Δ_L);
P_m* = Value of P_m predicted by Eq. (11).

AUTHORS' CLOSURE

It is a pleasure to read the discussion by Ioannides, who corroborates the findings through his own research; in particular he states that ILLISLAB has been used to verify that as subgrade modulus k increases, the maximum bearing stress σ_{max} decreases.

The authors agree with the statement in the discussion: J-SLAB (*13*) "was found to accentuate 'the relative importance of the bending load transfer mechanism compared to the shear mechanism by 5 to 20 times.' " It has also been found that the maximum dowel bending moments calculated by the original J-SLAB could differ from the modified JSLAB-92 by more than 10 times. This discrepancy was caused by the non-equilibrium stiffness matrix employed in the J-SLAB program. It is believed that the component model (*11*,*12*) installed in JSLAB-92 has the ability to more appropriately consider the contribution of shear and bending in the dowel design (*11*,*12*).

Equation 11 could be a very good form for estimating the maximum shear force in the dowels. When test data are available (δ_U, δ_L, and then LTE_δ), the equation can be used to estimate the maximum shear force in a straightforward manner. However, it seems that Equation 11 should be used with special care where the dowels exhibit significant looseness, which is often the condition in the field. For example, one limit case is the joint that has entirely lost its load transfer capability, namely, $\delta_U = 0$, then $LTE_\delta = 0$ and $P_m = B\Delta_f$; this result is greater than the shear force of any dowel in perfect condition. In fact, P_m should be zero in this case, not the maximum.

The considerable difference between the results by ILLI-SLAB and JSLAB-92 in Table 2 might be caused by the various dowel bar models used in the two programs. As discussed previously (*11*,*12*), the dowel stiffness matrix used in ILLI-SLAB cannot satisfy the equilibrium conditions [see Equations 3 through 43 in Tabatabaie et al. (*9*)]. Further study is needed to quantitatively evaluate the effects of using the nonequilibrium stiffness matrix in a computer program.

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