Applicability of Resilient Constitutive Models of Granular Material for Unbound Base Layers

M. KARASAHIN, A. R. DAWSON, AND J. T. HOLDEN

Analytical design methods for pavements require the determination of the resilient behavior of each layer. Granular material behavior under traffic loading is nonlinear and stress path dependent. Because the deformation characteristics of the material are significantly affected by stress magnitude and path, stress-strain behavior must be modeled accurately. A wide range of graded granular material types was selected to assess the applicability of stress-strain models. A repeated load triaxial test apparatus, which can cycle deviatoric and cell pressure, was used to test a variety of stress paths for furnace bottom ash, graded washed river sand, sand and gravel, Fontainebleu sand, limestone, and gritstone. Linear and nonlinear regression programs were used to obtain the parameters for five different models. The results indicated that there is no unique model to represent the granular material behavior under all circumstances. Elhannani's model can be used for predicting the response to cyclic deviatoric stress with cyclic cell pressure test data. Using K–Θ and Pappin and Brown models, approximate predictions can be made of axial stiffness under the cycling of both stresses using parameters obtained from more-simple only-cyclic deviatoric stress test data.

Analytical design methods for pavements require the determination of the resilient behavior of each layer. It is well known that granular material behavior under traffic loading is nonlinear and stress path dependent. Although density, degree of saturation, stress history, and grading have some effect on the behavior of granular material, the deformation characteristics of the material are significantly affected by stress magnitude and path. Therefore, it is important to model accurately the stress-strain behavior.

Granular material in the road is generally used in a moist but unsaturated condition. However, this makes it difficult to measure the effective stress in the laboratory. To eliminate this problem, four triaxial tests were conducted on different materials in the dry condition so that the stress-strain characteristics of the material could be obtained in terms of effective stress. Two other triaxial tests were conducted on partially saturated materials and analyzed in terms of total stress.

A range of graded granular material types was selected to assess the applicability of different stress-strain models. A repeated load triaxial test apparatus with a wide range of stress paths was used to test furnace bottom ash, graded washed river sand, sand and gravel, Fontainebleu sand, limestone, and gritstone. From the results, parameters were obtained for several different models developed in the last two decades. The strain behavior of the models under the individual stress paths were predicted, and the predictions were compared with the measured data to assess the performance of those models under different stress paths.

M. Karasahin and A. R. Dawson, Department of Civil Engineering, University of Nottingham, University Park, Nottingham, NG7 2RD England. J. T. Holden, Department of Theoretical Mathematics, University of Nottingham, University Park, Nottingham, NG7 2RD England.

MATERIALS

To see the general behavior of granular material under traffic loading and to test resilient constitutive models on a variety of aggregate types, materials from different origins were tested. The materials selected for the repeated load triaxial test were crushed limestone, sand and gravel, gritstone, graded washed river sand, Fontainebleu sand, and furnace bottom ash. All are more or less commonly used in Europe for base layers, subbase layers, or capping. Materials were tested without changing the grading. The grading curves are shown in Figure 1.

REPEATED LOAD TRIAXIAL TEST APPARATUS

Although it is unable to produce stress conditions representative of the real pavement structure, the triaxial test apparatus has been used for many years to investigate the stress-strain behavior of granular materials. In connection with the repeated loading of granular materials, a triaxial apparatus was developed by Boyce (*1*) at Nottingham University able to cycle both the deviator and confining (chamber) stress. Pappin (*2*) slightly modified some parts of the apparatus to apply tensile stress to the granular material. In 1991 the electronic control system of the apparatus was replaced by a digital control system (*3*). More details about the development of the apparatus can be obtained from work by Boyce (*1*), Pappin (*2*), Boyce et al. (*4*), and Brown et al. (*5*).

The apparatus (Figure 2) is capable of applying an axial load of 3 kN by a 50.8-mm diameter hydraulic actuator at a frequency range of 0–16 Hz. Confining stress is also applied by a hydraulic actuator at a frequency range of 0–2 Hz. This actuator operates a cylinder pump that pressurizes the cell fluid. Silicone oil is used as the cell fluid because of its low density and excellent electrical insulation, which allows on-sample instrumentation to be used without difficulty.

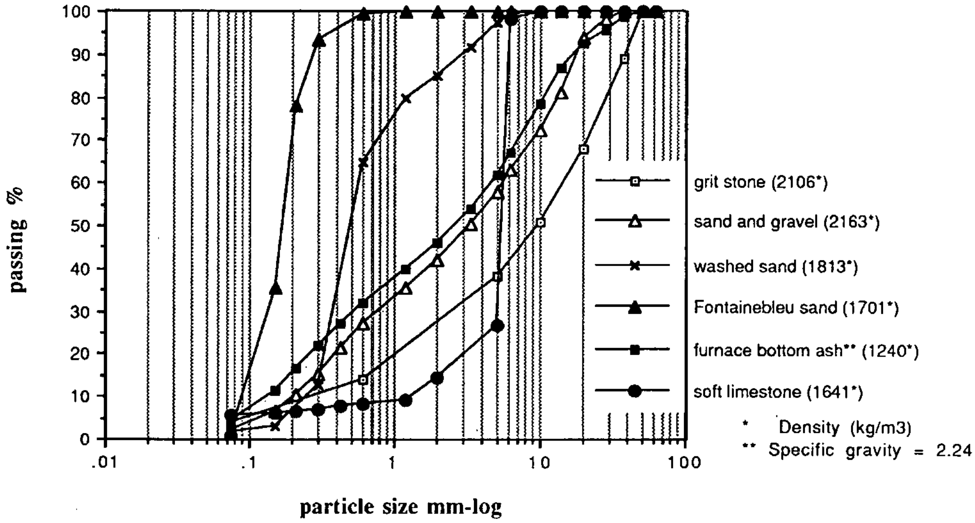

FIGURE 1 Grading curves and densities.

Axial deformations are measured using two linear variable differential transformers (LVDTs) mounted between two pairs of threaded rods. Radial deformations are measured by two hoops incorporating strain gauges fixed to the same rods. The rods are screwed into studs, which are clamped to the membrane and extend a short way into the sample.

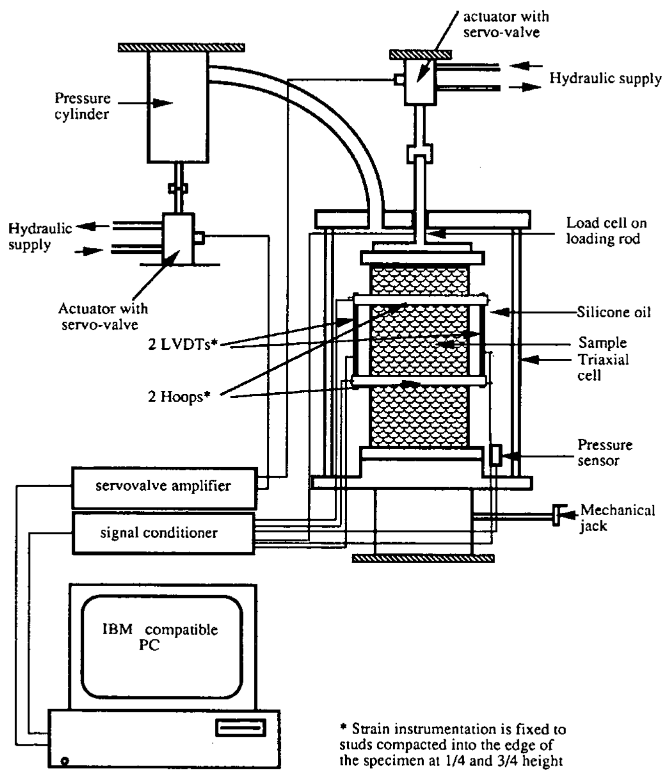

FIGURE 2 Diagram of repeated load triaxial apparatus.

SAMPLE PREPARATION

To form a sample, six layers of material were compacted from bottom to top. A vibrating table was used as a compaction tool, and each layer was subjected to vibration for 15 sec under a surcharge of 30 N (*1,2,6*). A leveling disc was used during the compaction process to apply the compaction evenly across the top of the sample. It was seen from previous experience that fine grains tended to migrate down during the compaction, whereas coarse particles moved up. To prevent this migration, coarse grains were placed in the mold by hand at the bottom before the vibration commenced. Each sample was enclosed in two latex membranes. During the sample preparation, the inner one was held against the porous inner surface of the mold by applying a vacuum. The outer one was added after compaction to cover any possible punctures produced in the inner membrane during the compaction process. An internal partial vacuum was applied to the sample while it was instrumented and before external cell pressure was applied. Before instrumenting the sample, it was visually checked for uniformity. Suspect samples were rejected. The densities of the samples are included in parentheses in the legend of Figure 1.

TEST PROGRAM

Six resilient strain tests were conducted by applying a range of stress paths (Table 1). To determine independently the resilient strain behavior of the plastic strain developed (*6*), 50 cycles of loading were applied for each stress path, and the mean response during the last 5 cycles was recorded. For materials a long way from saturation, the resilient behavior is affected little by loading frequency, so a frequency of 1 Hz was chosen (*7*).

Stress paths of different amplitudes were applied in several stress directions. The applied stress paths can be divided into two groups (Table 1). Type 1 is composed of cyclic deviatoric

TABLE 1 Stress Paths

Cyclic deviatoric stress (Type 1)		Cyclic Both stresses (Type 2)	
Cell pressure (kPa)	Deviatoric stress (kPa)	Cell pressure (kPa)	Deviatoric stress (kPa)
250	0-50	183-167	50-100
250	0-100	150-133	0-50
250	0-150	150-117	0-100
250	0-200	150-100	0-150
200	0-50	100-83	0-100
200	0-100	100-67	0-100
200	0-150	100-33	0-150
200	0-200	100-150	0-50
150	0-50	100-150	0-100
150	0-100	100-150	0-150
150	0-150	100-150	0-200
150	0-200	50-100	0-50
100	0-50	50-100	0-100
100	0-50	50-100	0-150
100	0-50	50-100	0-200
100	0-50		
50	0-50		

(axial) stress paths under a constant cell (confining) pressure. Type 2 is composed of paths in which both cell pressure and deviatoric stress are cycled in phase. This group can better represent real pavement loading.

Many laboratories still use constant confining pressure testing (Type 1) to determine the resilient behavior of the material because this type of equipment is more cost effective when compared with the apparatus, which can cycle both pressures at the same time. The testing procedure here aimed to determine to what extent models could predict the behavior under general stress cycling from data collected under the simplified regime of a constant confining stress (Type 1).

MODELS OF RESILIENT STRAIN BEHAVIOR

Because granular material behavior is markedly nonlinear and stress dependent, nonlinear stress-strain relationships should be used to model the real behavior of pavement structures.

For this particular work, five different granular material models were investigated to fit the data obtained from the repeated load triaxial tests. These models will be introduced briefly. All models are expressed in terms of p (mean normal stress, which is one-third of the bulk stress, Θ) and q (deviatoric or additional axial stress). Material (model) constants are shown by capital letters (A, B, C, etc.).

K–Θ Model

The most commonly used nonlinear elastic model is the so-called K–Θ model (*6*) in which the resilient modulus (M_r in units of stress) is expressed in the form of

$$M_r = A(3p_{\max})^B \qquad (1)$$

which strictly applies to triaxial testing involving the application of relatively small axial repeated loading starting from the $q = 0$ condition.

The model is widely used by pavement engineers to introduce a stress-dependent resilient modulus because it is easy to implement in finite element and backcalculation programs. However, Poisson's ratio in the model is assumed to be constant, and the effect of the deviatoric stress on the resilient properties is not considered. This latter effect is certainly not negligible in pavement engineering (*8*), and therefore, may be used only for low deviatoric stress levels. Clearly, such a limitation is unsatisfactory for pavement applications where, in general, shear stresses are relatively large. The model has been developed from simple laboratory triaxial tests in which the initial deviatoric stress is always zero. This limitation does not apply to tests described in this paper, and the implications of this are discussed later in the paper. Note that constant A must have dimensions controlled by constant B for the equation to be dimensionally correct.

Uzan Model

Uzan (*9*) modified Equation 1 to introduce the effect of deviatoric stress. The modified model is

$$M_r = A(3p_{\max})^B q^C \qquad q > 0.1\sigma_r \qquad (2)$$

where σ_r is radial stress. The problems of constants' dimensions, zero initial deviator stress, and a fixed Poisson's ratio remain.

Pappin and Brown Model

It has been considered useful to separate behavior into shear and volumetric components. For nonlinear behavior, no assumption would then be made regarding a constant Poisson's ratio (*10*). Pappin and Brown (*11*) developed a model framed in this manner—the contour model for granular material behavior. It was designed to model general stress path excursions regardless of the p,q stress state. Mayhew (*12*) concluded that stress path length (which was included in the Pappin and Brown model) had no significant effect on the shear strain behavior. The model could then be rewritten (*7*) in the form

$$\varepsilon_v = \left(\frac{p}{A}\right)^B \left(1 - C\frac{q^2}{p^2}\right) \qquad (3)$$

$$\varepsilon_s = \left(\frac{p}{D}\right)^E \frac{q}{p} \qquad (4)$$

where ε_v, ε_s are volumetric and shear strain, respectively, and material constants A and D have units of stress.

The stress paths in this and the following models are assumed to be from zero to the conditions indicated by p and q. For actual stress paths, the strain is computed by comparing the predicted values for each end of the path. Bulk and shear moduli are usually defined, respectively, on the basis of the

strains as

$$K = \frac{p}{\varepsilon_v} \tag{5}$$

$$G = \frac{q}{3\varepsilon_s} \tag{6}$$

Boyce Model

While Pappin and Brown's (*11*) approach sought to describe observed data, Boyce (*13*) developed a similar G–K model from first principles, using the theorem of reciprocity (*14*), also expressing it in volumetric and shear parts. The model is nonlinear elastic and isotropic. The model is expressed in the form

$$\varepsilon_v = p^B\left[\frac{1}{A} - \frac{(1 - B)}{6C}\frac{q^2}{p^2}\right] \tag{7}$$

$$\varepsilon_s = \frac{p^B}{3C}\frac{q}{p} \tag{8}$$

In this formulation, constants A and C have dimensions controlled by constant B.

It is worth noting that Mayhew (*12*) found that the influence of the mean normal stress (p) on the bulk modulus differs from that on the shear modulus (G), even when the ratio q/p is constant. On this basis, it is evident that B in Boyce's model should be different for the volumetric and shear strain formulations. This approach was taken by Sweere et al. (*15*), Sweere (*16*), and Jouve et al. (*17*) to fit their data into the model. A nonlinear regression analysis revealed that constants B and C should be different for the volumetric and shear strain formulae. The resulting model is the same as the Pappin and Brown model as rewritten by Brown and Selig (*7*) and given in Equations 3 and 4. It has five parameters instead of the three parameters in the original Boyce model.

Elhannani Model

Elhannani (*18*) introduced anisotropy into the original Boyce model, taking the form

$$\varepsilon_v = p_a^{1-B}p^B\left[\frac{1}{A} - \frac{(1 - B)}{6C}\left(\frac{q}{p}\right)^2 - \frac{B}{D}\left(\frac{q}{p}\right)\right] \tag{9}$$

$$\varepsilon_s = p_a^{1-B}p^B\left[\frac{1}{3C}\frac{q}{p} - \frac{1}{D}\right] \tag{10}$$

where p_a is atmospheric pressure (100 kPa) and A, C, and D have units of stress.

All of the models could use the device Elhannani (*18*) introduced that uses atmospheric pressure as a normalizing factor to make stress terms nondimensional. Elhannani used his approach to model behavior on a variety of stress paths that radiated from a single initial stress state at low p,q.

MODEL PARAMETERS

For each model, parameters must be determined from experimental results. Ideally, this determination should be simple, which is possible if the model can be rewritten in a linear form. K–Θ and Uzan (*9*) models may be converted to a linear form by taking the logarithm of both sides of the equations. Hence, any linear regression program for the former and a multivariable regression program for the latter model may be used to find the constants. Considering the Type 1 stress paths, the Uzan (*9*) and K–Θ models result in the same form of Equation 1 because $q = 3p$. Therefore, for the Type 1 paths, the results are presented as K–Θ results.

For the more complex models reviewed here, the BMDP (*19*) statistical package was used. This provides a derivative-free nonlinear regression program with a pseudo-Gauss-Newton iterative algorithm (*19*). For each model, a short program was written in the BMDP code defining the initial values of parameters, boundaries, number of iterations, accuracy limit, and model. The imposed stresses and measured strains for each stress path were then supplied to provide the data set that must be predicted by the program with minimum error. The initial values are important when performing a nonlinear regression analysis. If they are not close to the solution, it is almost impossible to find a feasible solution. Initial values must be adjusted until a reasonable solution is found. Sometimes it is possible to find a local solution that does not satisfy all the data. For this situation, initial values and boundary conditions need to be checked. Sometimes, although the boundary conditions and initial values are changed, no improvement in the results can be seen, raising the possibility that the test data supplied do not fit the model well.

PREDICTIONS WITH THE MODELS

Parameters were obtained for each model using the repeated load triaxial test data. For each model, two different sets of parameters were obtained—one from Type 1 stress paths and one from Type 2 paths—to fit both volumetric and shear strain models where appropriate. Predictions of axial and radial strain have been made using the different stress path data sets, the different materials, and the different models. (For comparison purposes, only axial strain predictions are presented because the resilient modulus, perhaps the most important parameter in pavement design, is a direct function of this strain.) The derived model parameters are given by Karasahin (*20*).

The parameters were then used to predict the data sets from which they had been derived to assess the applicability of the models. Axial strain was also predicted for the Type 2 paths using the parameters obtained from the Type 1 stress path test data. Satisfactory results would indicate that relatively simple cyclic deviatoric stress tests could be used to predict behavior when both stresses are cycled (as in real pavements).

DISCUSSION OF RESULTS

The results are discussed in relation to the following three categories:

1. Predicted axial strain for cyclic deviatoric stress (Type 1 path data to predict the behavior under Type 1 loading);
2. Predicted axial strain for both stresses cycled (Type 2 data to predict Type 2 behavior); and
3. Predicted axial strain under both stresses cycling using the parameters derived from the cyclic deviatoric stress testing (Type 1 data to predict Type 2 behavior).

Predictions of Cyclic Deviatoric Stress Behavior (Type 1)

The results for furnace bottom ash were least successfully predicted. The results shown in Figure 3 show some of the difficulties involved.

The predictions of the K−Θ model (*6*) had a similar pattern for all materials, as shown in Figures 3–5. Values of strain were underpredicted at high levels. Predicted values are almost equal to each other for the same cell pressure, although the deviatoric stress level and hence the measured axial strain were different (sets of points on subhorizontal lines in Figure 3).

The Boyce model predicted a somewhat better match to the measured results, although there was still a small tendency for the effect of increasing deviatoric stress to be underestimated. For the furnace bottom ash, a particularly poor fit was recorded (Figure 3). This lack of fit reflects, in part, the need for both volumetric and shear (and thus axial and radial) strains to be modeled using the same three parameters (see Equations 7 and 8), although only the matching of one strain set is illustrated.

The Pappin and Brown (*11*) model often provides a good fit at low strain levels where granular material has a high resilient modulus. However, for high strain levels, the model greatly underestimated the strain. Nevertheless, the predictions of the model are without significant scatter.

The Elhannani (*18*) model predicts higher axial strains at low strain levels, although at high strain levels, it gave an excellent fit to and prediction of the recorded data.

Prediction of Behavior When Both Stresses Are Cycled (Type 2)

The K−Θ and Uzan models showed a similar pattern for the prediction of axial strain (Figures 6 and 7). Both models gave a reasonable fit to data. The results for the gritstone (Figure 7) were fairly typical of the six materials tested as far as the K−Θ and Uzan models were concerned. The prediction of axial strain by the Uzan model was generally greater than the K−Θ approach, and its average was more nearly on the line of equality. However, both models showed considerable scatter. Nevertheless, the prediction of strain using the K−Θ model was better for the Type 2 loading than for the Type 1 loading. The result for the Fontainebleu sand was exceptional (Figure 6). Neither model gave acceptable predictions.

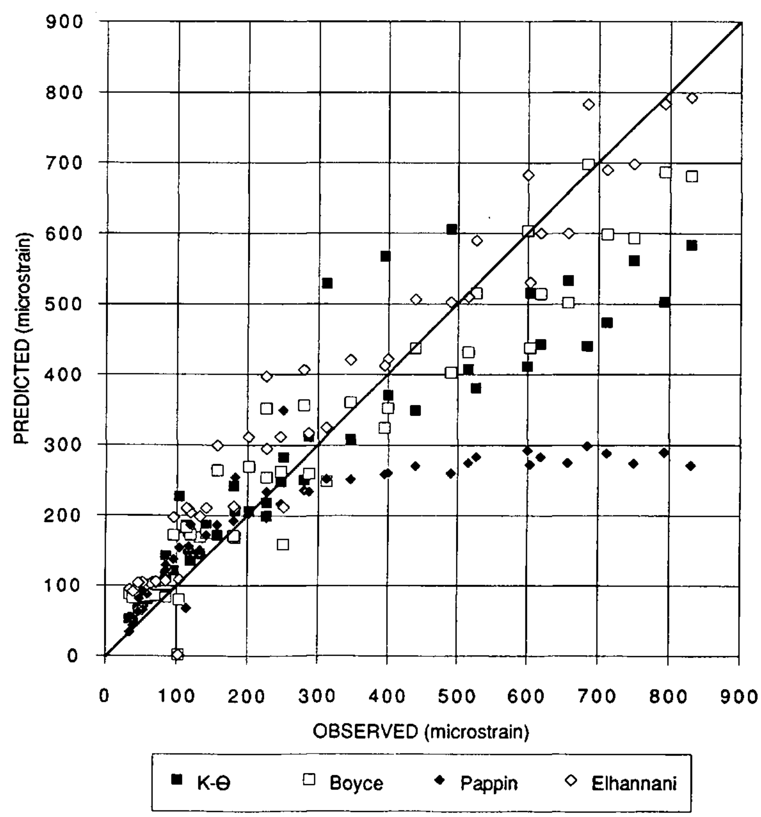

FIGURE 3 Furnace bottom ash—axial strain due to cyclic deviatoric stress.

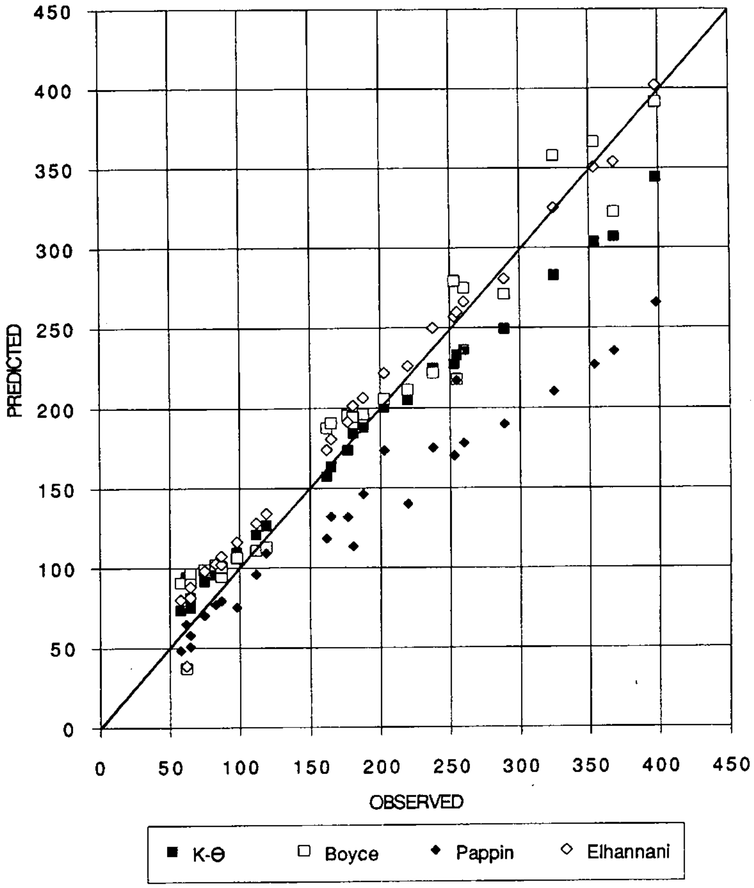

FIGURE 4 Fontainebleu sand—axial strain due to cyclic deviatoric stress.

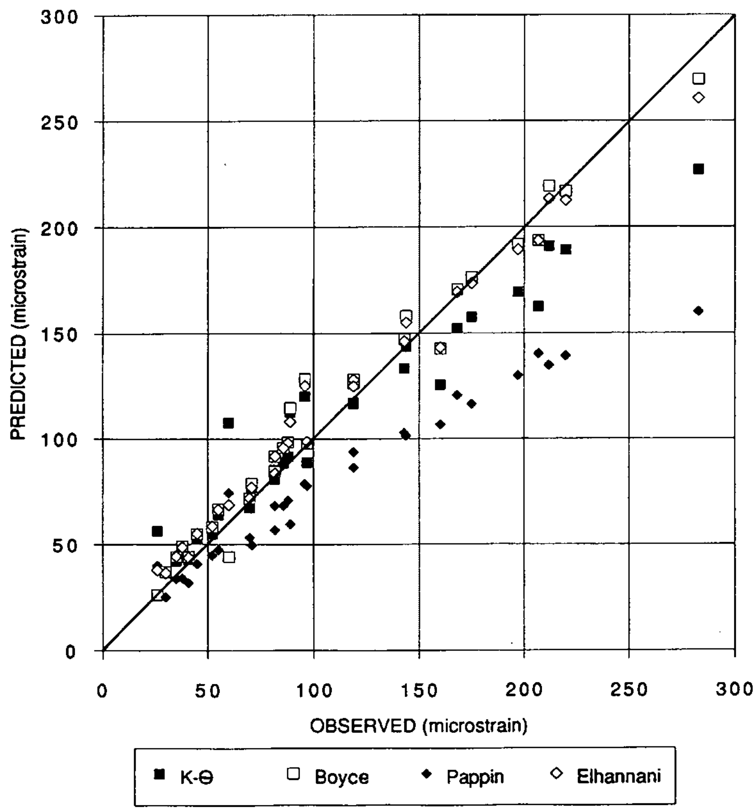

FIGURE 5 Gritstone—axial strain due to cyclic deviatoric stress.

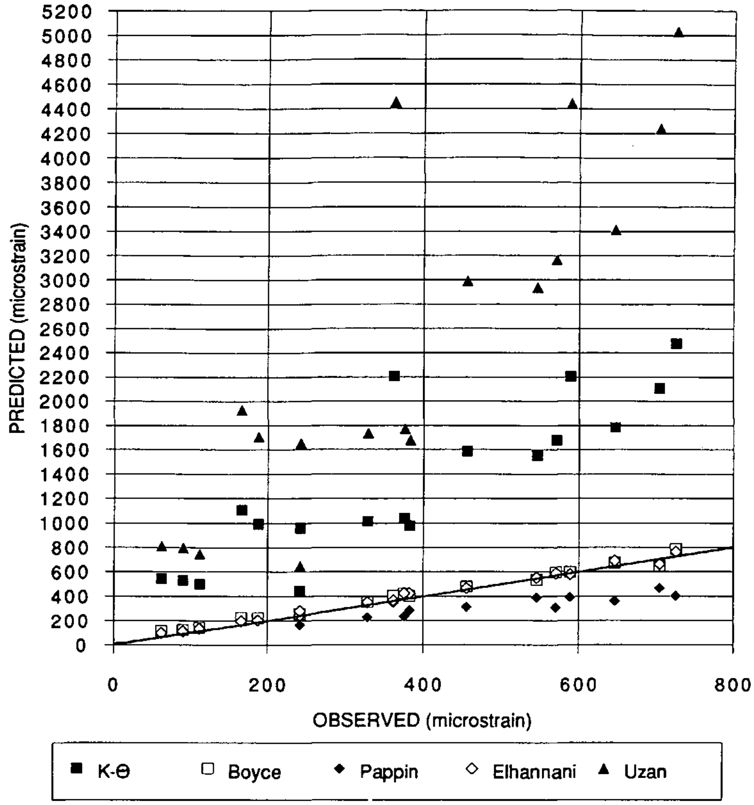

FIGURE 6 Fontainebleu sand—axial strain due to both stresses cycling.

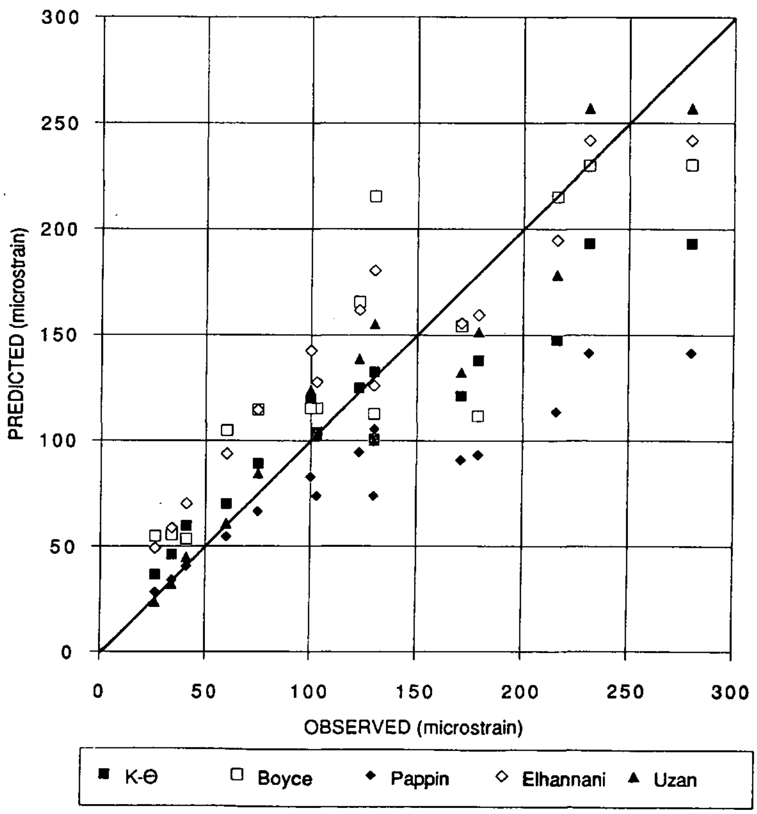

FIGURE 7 Gritstone—axial strain due to both stresses cycling.

The Pappin and Brown model exhibited the same drawback as mentioned in the previous section. In addition, it had greater scatter.

The Boyce and Elhannani models showed almost the same patterns as before, except for the soft limestone for which the Boyce model was unable to predict axial strain accurately. The Elhannani model again provided a good prediction and a relatively nonscattered fit for most aggregates, but it overestimated strain at low levels. In the case of the furnace bottom ash, all the models performed quite well, showing less scatter than that in Figure 3.

Predictions of Behavior under Cycling of Both Stresses Using Parameters Obtained from Cyclic Deviatoric Stress Tests Data (Type 2 Behavior Predicted on Basis of Type 1 Loading)

An aim of the study was to show whether cyclic deviatoric stress test results (which are relatively simple to perform) may be used to predict the stress-strain relationships when both stresses are cycled. This stress-strain is more relevant to the situation found in the pavement.

The Pappin and Brown model generally gave an underestimated prediction of axial strain for sand and gravel, soft limestone, and Fontainebleu sand, which is consistent with its underestimation of strain discussed in the last two sections (compare Figure 8 with Figure 6). For the other three materials, the initial prediction using the Pappin and Brown approach is generally good or a little high at low strains, but it underestimates strain at high levels.

The Boyce and Elhannani models also underestimated strain at low levels for all the materials (see Figure 9). However, the Boyce and the Elhannani models overestimated sand and gravel, furnace bottom ash, soft limestone, and Fontainebleu sand at high strain levels. Hence it appears that the Boyce and Elhannani models are unable to model accurately, on the basis of simple tests, the stiffening of these materials under stress paths closest to those experienced in the pavement.

The Boyce and K–Θ models were the least successful in modeling the behavior of gritstone. For this material the Elhannani model performed best (Figure 9). Predictions that used the K–Θ model were slightly superior to other models when all test results were considered; however, this is because they usually lie about the 1:1 observed:predicted line. In addition, the scatter in predictions of strain under individual stress paths is inconsistent in over-, under-, or on-prediction from material to material. Therefore, the K–Θ model with the parameters derived from the cyclic deviatoric stress tests (Type 1 paths) can be used to predict only approximate behavior of the material under the cycling of both stresses (Type 2 paths).

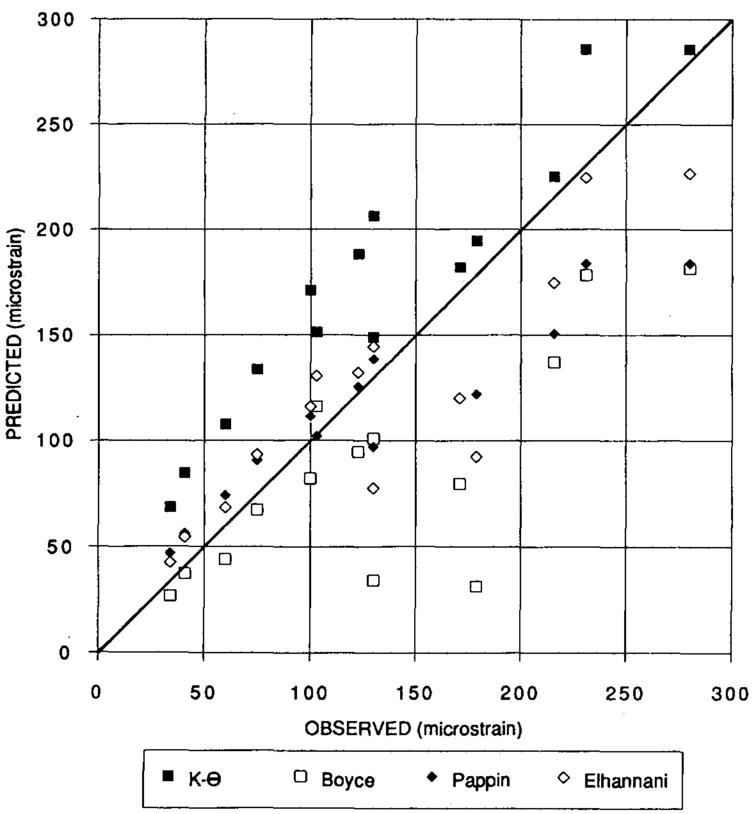

FIGURE 8 Gritstone—axial strain due to both pressures cycling, predicted from simple tests.

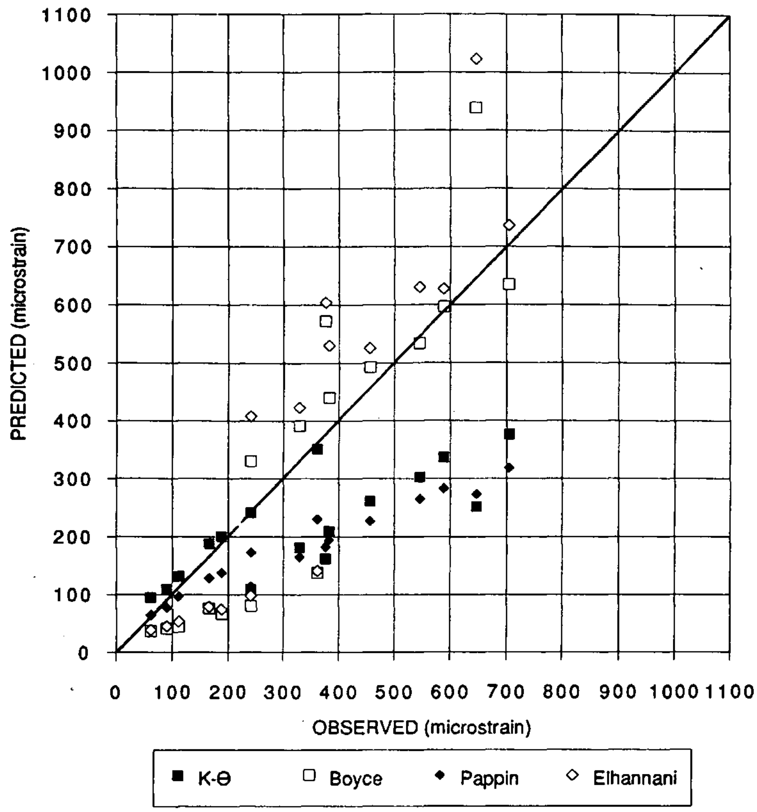

FIGURE 9 Fontainebleu sand—axial strain due to both pressures cycling, predicted from simple tests.

CONCLUSIONS

The testing and modeling program conducted on a range of different aggregate types has demonstrated the following:

1. It is difficult to predict strains (as measured in triaxial testing) by using available resilient constitutive models for aggregates under pavement-type loading.
2. The use of multivariate nonlinear regression programs can be operator-sensitive and may not be an acceptable method for routinely determining model parameters.
3. Of the models available, the Elhannani model (*18*) proved best when applied to cycling deviatoric stress testing.
4. The Elhannani approach is also best for modeling behavior when deviatoric and confining pressures are cycled.
5. There remain considerable uncertainties in using any model, with parameters derived from simple repeated deviator stress testing, for predicting behavior when both stresses are cycled. Of those studied, the K–Θ model (*6*) had the fewest errors.
6. Modeling a range of different types of stress path appears to be the most demanding aspect. In this situation, the Pappin and Brown approach (*11*) has the most value.

These findings can be partially explained by referring to the inherent limitations of the models. The Boyce model (*13*) satisfied the reciprocity theorem (no gain or loss of energy during cycling), and this is certainly invalid for hysteretic materials. By dropping this limitation, Pappin and Brown's model is somewhat better. The limitations of the K–Θ model (in ignoring deviatoric stress effects) have already been described. The improvements of Uzan (*9*) do not appear to be significant. Elhannani's approach, by incorporating an allowance for anisotropy, appears to overcome many of the deficiencies of other models.

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REFERENCES

1. J. R. Boyce. *The Behaviour of a Granular Material under Repeated Loading*. Ph.D. thesis. University of Nottingham, England, 1976.
2. J. W. Pappin. *Characteristics of a Granular Material for Pavement Analysis*. Ph.D. thesis. University of Nottingham, England, 1979.

3. C. K. Chan and J. B. Sousa. State-of-the-Art on Geotechnical Laboratory Testing. *Proc., Geotechnical Congress*, Geotechnical Engineering Division, ASCE, Boulder, Colo., 1991.
4. J. R. Boyce, S. F. Brown, and P. S. Pell. The Resilient Behaviour of a Granular Material under Repeated Loading. *Proc., Australian Road Research Board*, Vol. 8, Session 9, 1976, pp. 8–19.
5. S. F. Brown, M. P. O'Reilly, and J. W. Pappin. A Repeated Load Triaxial Apparatus for Granular Materials. In *Unbound Aggregates in Roads* (Jones and Dawson, eds.). Butterworths, London, England, 1989, pp. 143–159.
6. G. R. Hicks and C. L. Monismith. Factors Influencing the Resilient Response of Granular Materials. In *Highway Research Record 345*, TRB, National Research Council, Washington, D.C., 1971, pp. 15–31.
7. S. F. Brown and E. T. Selig. The Design of Pavement and Rail Track Foundations. In *Cyclic Loading of Soils: From Theory to Design* (O'Reilly and Brown, eds.). Blackie, Glasgow, England, 1991, pp. 249–306.
8. R. W. May and W. W. Witczak. Effective Granular Modulus to Model Pavement Responses. In *Transportation Research Record 810*, TRB, National Research Council, Washington, D.C., 1981, pp. 1–9.
9. J. Uzan. Characterization of Granular Material. In *Transportation Research Record 1022*, TRB, National Research Council, Washington, D.C., 1985, pp. 52–59.
10. L. Domaschuk and N. H. Wade. A Study of Bulk and Shear Moduli of a Sand. *Journal of Soil Mechanics and Foundations Division*, ASCE, 1969, pp. 561–581.
11. J. W. Pappin and S. F. Brown. Resilient Stress-Strain Behaviour of a Crushed Rock. *Proc., International Symposium on Soils Under Cyclic and Transient Loading*, Swansea, England, 1980, pp. 169–177.
12. H. C. Mayhew. *Resilient Properties of Unbound Road Base Under Repeated Triaxial Loading*. TRRL Laboratory Report 1088. TRRL, Crowthorne, Berkshire, England, 1983.
13. J. R. Boyce. A Non-Linear Model for the Elastic Behaviour of Granular Materials Under Repeated Loading. *Proc., International Symposium on Soils Under Cyclic and Transient Loading*, Swansea, England, 1980, pp. 521–542.
14. S. P. Timoshenko and J. N. Goodier. *Theory of Elasticity*, 3rd ed. McGraw-Hill International Editions, Engineering Mechanics Series, 1970.
15. G. T. H. Sweere, A. Renning, and E. Vos. Development of a Structural Design Procedure for Asphalt Pavements with Crushed Rubble Base Courses. *Proc., Sixth International Conference—Structural Design of Asphalt Pavements*, Vol. 1, 1987, pp. 34–49.
16. G. T. H. Sweere. *Unbound Granular Bases for Roads*. Ph.D. thesis. University of Delft, Netherlands, 1990.
17. P. Jouve, J. Martinez, L. L. Paute, and E. Regneau. Rational Model for the Flexible Pavement Deformations. *Proc., Sixth International Conference—Structural Design of Asphalt Pavements*, Vol. 1, 1987, pp. 50–64.
18. M. Elhannani. *Modelisation et Simulation Numerique des Chaussees Souples*. Ph.D. thesis. University of Nantes, France, 1991.
19. W. J. Dixon, M. B. Brown, L. Engelman, M. A. Hill, and R. I. Jennrich. *BMDP Statistical Software Manual*. University of California Press, Berkeley, Calif., 1988, pp. 357–419.
20. M. Karasahin. *Resilient Behaviour of Granular Materials for Analysis of Highway Pavements*. Ph.D. thesis. University of Nottingham, 1993.

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