Estimation of Travel Choice Models with Randomly Distributed Values of Time

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The value of time is a key concept in transport planning in terms of the economic valuation of travel time savings and the relative importance of time versus cost in travel forecasting models. A standard method for deriving values of time is to use the trade-off ratio implied by the time and cost coefficients estimated in travel choice models. In actual choice situations, it is impossible to observe all the factors that affect the relative importance of time and cost. Thus, a method for estimating discrete travel choice models was derived and demonstrated with a randomly distributed value of time. In the case studies considered, significant improvements in model fit were obtained when distributed values of time were allowed. In prediction, more realistic responses were found by using the distributed value of time model than by using models with a fixed value of time.

The value of time (VOT) is a key concept in transport planning in terms of the economic valuation of travel time savings and the relative importance of time versus cost in travel forecasting models. A standard method for deriving values of time is to use the trade-off ratio implied by the time and cost coefficients estimated in travel choice models. Such models generally assume that this trade-off ratio is the same for all members or specified groups of the population. That assumption can be relaxed somewhat by allowing the VOT to vary along observed dimensions, such as income, trip purpose, mode of travel, and so forth. Such an approach has been used extensively in major national VOT studies in the United Kingdom (*1*) and the Netherlands (*2*).

In actual choice situations, however, the relative importance of time and cost changes may be influenced by individual-specific tastes and circumstances that cannot be observed. If one cannot model such factors explicitly, it may still be beneficial to try to identify the distribution of their influence across the population. The fraction of the population willing to pay a given amount for a given time savings may be sensitive to the shape and spread of the VOT distribution.

A method for estimating discrete travel choice models with a randomly distributed value of time is derived and demonstrated in this paper. Although the approach can be applied more generally to other distributions, this method assumes a lognormal distribution. The statistical assumptions and implementations of the estimation method are described, and the results of case studies applying the method to three different data sets are presented. The implications of the results, as well as possible extensions and generalizations of the method, are then discussed.

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MODEL

Typical disaggregate methods for estimating travel choice models described by Ben-Akiva and Lerman (*3*) assume that, for a given individual, each choice alternative i has a utility, which can be expressed in the following linear form:

$$U_i = \mu c_i + \eta t_i + \alpha' X_i + \varepsilon_i \tag{1}$$

where

c_i = travel cost of alternative i,
t_i = travel time of alternative i,
X_i = vector of additional observed attributes of individual and of alternative i,
ε_i = influence of unobserved factors affecting utility of alternative i, and
μ,η,α = set of coefficients to be estimated.

In this notation, the implicit value of time is the ratio of the time and cost coefficients, η/μ. If all parameters are normalized by the cost coefficient μ, the form changes to

$$U_i = \mu(c_i + \nu t_i + \alpha^{*\prime} X_i) + \varepsilon_i \tag{2}$$

where ν is the value of time in cost units and α^* is the vector α normalized in cost units.

Written this way, the component between parentheses can essentially be viewed as a generalized cost variable. With the error term going to zero or μ to $-\infty$, this model collapses to a deterministic model where the objective is to minimize a generalized cost of traveling. Further suppose that a subset Z_i of variables contained in vector X_i incorporates attributes assumed to have coefficients that vary proportionally to the time coefficient; that is, they are assumed to follow the same distribution as the value of time. Then the formulation changes to:

$$U_i = \mu[c_i + \beta' Y_i + \nu(t_i + \gamma' Z_i)] + \varepsilon_i \tag{3}$$

where Y_i, Z_i is mutually exclusive subsets of the vector X_i and β, γ is the corresponding subsets of vector α^* (γ is now normalized in time units).

In this last formation, $c_i + \beta' Y_i$ plays the role of a cost composite, and $t_i + \gamma' Z_i$ plays the role of a time composite. It is assumed that the value of time coefficient ν takes a fixed value (the term "fixed" in this paper refers to a single value across the population). A logit choice model among J alternatives then has the following choice probability function for alternative i:

$$P(i|\nu) = \frac{\exp\{\mu[c_i + \beta' Y_i + \nu(t_i + \gamma' Z_i)]\}}{\Sigma_{j=1}^{J} \exp\{\mu[c_j + \beta' Y_j + \nu(t_j + \gamma' Z_j)]\}} \quad (4)$$

where $\exp\{x\} = e^x$.

This equation contains a linear transformation of the systematic utility function of a standard logit model, which can be arrived at by estimating the parameters in the utility function (*1*) and calculating the normalized parameters in Equations 2 and 3 afterward.

Now, relax the previous assumption, and suppose that the value of time takes a random value. The authors postulate that the value is lognormally distributed (i.e., its natural logarithm is normally distributed) across the population, that is:

$$\ln \nu \sim N(\omega, \sigma^2) \quad (5)$$

where ω is $E(\ln \nu)$ = the expected value of the log of the value of time and σ^2 is the variance of the log of the value of time.

The probability density function of ν is then:

$$f(\nu) = \frac{1}{\sigma\nu\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2}\left(\frac{\ln\nu - \omega}{\sigma}\right)^2\right] \qquad \nu > 0 \quad (6)$$

This distribution implies the following properties for the value of time ν:

- Median = $\exp(\omega)$,
- Mode = $\exp(\omega - \sigma^2)$,
- Mean = $\exp(\omega + \sigma^2/2)$, and
- Variance = $\exp(2\omega + \sigma^2)[\exp(\sigma^2) - 1]$.

This is an asymmetric distribution skewed to the left of the mean, with a minimum value of 0 and a tail to the right (Figure 1). Because income levels across the population tend to follow such a distribution, it has sometimes been asserted that values of time will do so as well. In France, for example, lognormal value of time distributions are sometimes used in a binary, deterministic model ($J = 2$ and $\varepsilon_i = 0$ in the notation used above). This model is a special case of the more general model formulation proposed in this paper.

Logit models with random coefficients have been used before (*4*). Our model uses a random trade-off. This is not to be confused with random coefficients. The distribution of a VOT trade-off in a random coefficient logit model would involve a distribution of the ratio of two random coefficients. By using the normalization in Equation 3, the authors addressed this problem in a direct way.

The authors' method is unique in that it assumes a lognormal distribution for a trade-off rather than a coefficient. As far as the authors are aware, the assumption of a lognormal value of time distribution has never been tested empirically. To do so, the authors developed an estimation method that provided estimates of both distribution parameters (ω and σ) and compared the estimation results to those assuming a fixed value of time (standard logit). To calculate logit choice probabilities with a distributed value of time, the authors integrated over the assumed form of the distribution. Combining Equations 4 and 6 and integrating over ν produces:

$$P(i) = \frac{1}{\sigma\sqrt{2\pi}} \times \int_0^\infty \frac{\exp\{\mu[c_i + \beta' Y_i + \nu(t_i + \gamma' Z_i)]\}}{\Sigma_{j=1}^{J} \exp\{\mu[c_j + \beta' Y_j + \nu(t_j + \gamma' Z_j)]\}} \times \frac{1}{\nu}\exp\left[-\frac{1}{2}\left(\frac{\ln \nu - \omega}{\sigma}\right)^2\right] d\nu \quad (7)$$

where μ, ω, σ, β, and γ are the parameters to be estimated.

The parameters can be estimated using a maximum likelihood approach, applying Equation 7 to the observed choice

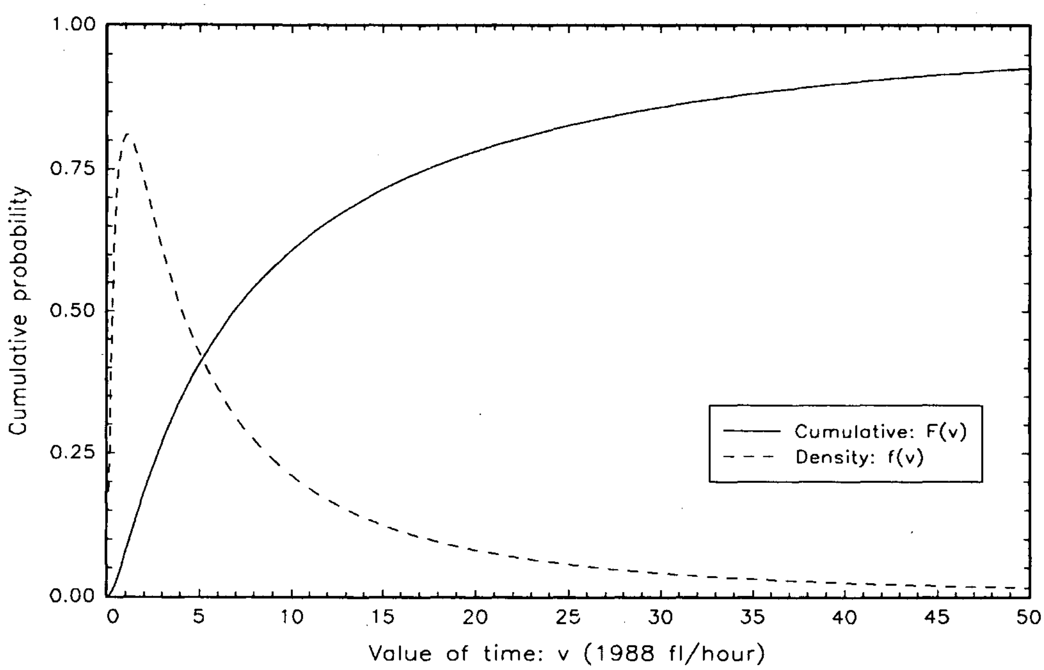

FIGURE 1 Rail SP data—estimated lognormal VOT distribution.

of each individual and maximizing the sum of the logged probabilities across the sample. The log-likelihood function was programmed using the Gauss statistical programming environment. To reduce the computation time required, three extra steps were carried out:

1. A standard logit estimation procedure was first used to provide efficient starting values for the parameters.
2. The analytic first derivatives of the likelihood function were obtained and incorporated into the program.
3. A simple change of variable from $(\ln - \omega)/(\sigma\sqrt{2})$ to y was performed so that a Gauss-Hermite quadrature could be used to compute the integral very efficiently.

(The Gauss-Hermite quadrature is especially designed to evaluate an unbounded integral of the form

$$\int_{-\infty}^{\infty} H(x)e^{-x^2}dx$$

Only a few quadrature points are required for high precision). After this latter transformation, Equation 7 can be rewritten as:

$$P(i) = \int_{-\infty}^{\infty} \times \frac{\exp\{\mu[c_i + \beta' Y_i + \exp(y\sqrt{2}\sigma + \omega)(t_i + \gamma' Z_i)]\}}{\Sigma_{j=1}^{J} \exp\{\mu[c_j + \beta' Y_j + \exp(y\sqrt{2}\sigma + \omega)(t_j + \gamma' Z_j)]\}} \times \frac{1}{\sqrt{\pi}} \exp(-y^2)dy \tag{8}$$

Computation over 8 to 12 quadrature points generally produces accurate results. The Gauss estimation routine was first tested on simulated data generated from a model specification based on a lognormally distributed value of time coefficient. The results were very satisfactory for both binary and polytomous choice settings. By generating very large samples, the authors could verify that the true value of each coefficient was retrieved. This ensures that the maximum likelihood procedure produces consistent estimators. The type of results obtained using data from actual choices is discussed in Case Study Results. To allow a lognormally distributed value of time, a special submodel that coincides with the deterministic binary choice model needs to be introduced.

SUBMODEL

Suppose that $\mu \rightarrow -\infty$. This corresponds to a deterministic choice framework, which is often used to model route choices or to develop traffic assignment procedures, and coincides with a framework that minimizes the generalized cost of traveling. This is an interesting special case that applies to any choice model that may be particularly useful for evaluating the share and the revenue from a toll road. Let us say that only Alternative 1 involves a toll and that the other alternatives are free. In the authors' notation, one can formalize the problem as

$$P(i|\nu) = 1\{i = \underset{j}{\operatorname{argmin}}[c_j + \beta' Y_j + \nu(t_j + \gamma' Z_j)]\} \qquad i = 1, \ldots, J \tag{9}$$

where $1(a)$ is an indicator function equal to 1 if a is true and 0 otherwise. Because Alternative 1 is the only alternative involving a monetary cost and all other alternatives are free, the authors assume that $Y_i = 0$ and $c_i = 0$ for all $i \neq 1$. This sets the cost composite of all free roads to zero. Let j denote the best free alternative:

$$j = \underset{j \neq 1}{\operatorname{argmin}}(t_j + \gamma' Z_j) \tag{10}$$

The random VOT framework can be used to model the choice between the toll road and the best free alternative. The choice probability of the costly alternative can be expressed as

$$\begin{aligned} P(1) &= \text{prob}[c_1 + \beta' Y_1 + \nu(t_1 + \gamma' Z_1) \leq \nu(t_j + \gamma' Z_j)] \\ &= \text{prob}[\nu(\Delta t + \gamma' \Delta Z) \geq c_1 + \beta' Y_1] \end{aligned} \tag{11}$$

where $\Delta t = (t_j - t_1)$ and $\Delta Z = Z_j - Z_1$. Finally,

$$\begin{aligned} P(1) &= \text{prob}\left(\nu \geq \frac{c_1 + \beta' Y_1}{\Delta t + \gamma' \Delta Z}\right) \\ &= \text{prob}\left[\ln \nu \geq \ln\left(\frac{c_1 + \beta' Y_1}{\Delta t + \gamma' \Delta Z}\right)\right] \\ &= \text{prob}\left\{\frac{\ln \nu - \omega}{\sigma} \geq \frac{1}{\sigma}\left[\ln\left(\frac{c_1 + \beta' Y_1}{\Delta t + \gamma' \Delta Z}\right) - \omega\right]\right\} \end{aligned} \tag{12}$$

Because it is assumed that $\ln \nu$ is normally distributed, the following equation can finally be written:

$$P(1) = \Phi\left\{\frac{1}{\sigma}\left[\omega - \ln\left(\frac{c_1 + \beta' Y_1}{\Delta t + \gamma' \Delta Z}\right)\right]\right\} \tag{13}$$

where Φ denotes the standard normal cumulative distribution function. This model formulation corresponds to a binary nonlinear probit model of the choice between the best free alternative and the nonfree alternative.

CASE STUDY RESULTS

Intercity Rail SP Data

In 1987, the Hague Consulting Group conducted a study for the Nederlandse Spoorwegen (NS) on the potential for substitution between car and rail for intercity travel as a function of rail service levels and fares (5). The sample was composed of 235 individuals who had recently traveled by car or rail from the Dutch city of Nijmegen, which is near the German border, to Amsterdam, Rotterdam, or Den Haag, all of which are located about 125 km west. A computer-based home interview was used. Each respondent gave a detailed account of his or her actual journey, including all travel costs, times, and interchanges. The respondent was then asked for his or her perception of making the same journey by the alternative mode. These questions were followed by two stated preference (SP) experiments.

The first SP experiment was designed to measure the relative importance of four rail service attributes: fare, journey time, number of rail-to-rail transfers, and comfort level. The experiment was thus "within-mode," with respondents comparing different rail options. The data from this within-mode experiment were used to test the lognormal value of time estimation procedure. The data had previously been used in tests of joint SP-RP estimation methods (*6,7*), and the authors were confident that it would give reliable results. In addition, standard logit estimation had produced very accurate estimates of the travel time and cost coefficients, and the ratio of the two gave an implicit value of time within a commonly accepted range.

Table 1 compares the standard fixed VOT logit results (estimated using the ALOGIT program) with the results from the Gauss routine assuming a lognormally distributed value of time. In the Lognormal 1 model, the effects of the number of transfers and comfort level are estimated in vector β. In the Lognormal 2 model, these two variables were assumed to have effects proportional to the time coefficient and are thus associated with vector γ. For the fixed VOT results, these coefficients are also shown normalized with respect to the cost and time coefficients for purposes of comparison. (Corresponding *t*-statistics are thus for the ratio of the relevant coefficients).

In the Lognormal 1 model, the log-likelihood has increased by three units with the addition of one parameter. This is a significant increase according to a likelihood ratio test. When the moments of the distribution are calculated, the mean VOT is somewhat higher than for the fixed VOT model (15.5 versus 11.6 fl/hr). The mode and median are much lower than 11.6; however, and the standard deviation is more than twice the mean. The results thus indicate a large "tail" of respondents with high values of time savings. One possible reason for this is that the sample includes both business and leisure travelers, segments that may have quite different values of time. The relative effects of transfers and comfort remain about the same as in the fixed VOT model.

In the Lognormal 2 model, the log-likelihood increases by a further three units, indicating that the effects of comfort and transfers are related to the effect of travel time and should be modeled as following the same distribution as the value of time. The predicted mean and variance of the lognormal distribution are increased somewhat with respect to the Lognormal 1 model. The effects of transfers and the comfort level relative to travel time once again remain close to those of the fixed VOT model. Note that the *t*-ratio for the mean VOT in the lognormal models is lower than that for the fixed VOT model.

To provide a clearer picture of the results obtained, the estimated VOT distribution from the Lognormal 2 model is plotted in Figure 1. The mode, which is the peak of the density function, occurs between 1 and 2 fl/hr. (The scale of the density function is not shown in the figure). The median, the point where the cumulative distribution reaches 0.50, is about 7 fl/hr. According to this distribution, about 70 percent of the sample has a value of time lower than the mean value estimated from the fixed VOT model (11.6 fl/hr), and about 75 percent have a value lower than the mean VOT from the Lognormal 2 model (17.6 fl/hr). These mean values are thus greatly influenced by the 10 percent or so of the population

TABLE 1 Estimation Results for Rail SP Data

	Fixed VOT		Lognormal 1		Lognormal 2	
Variable:	Coef.	(T.St.)	Coef.	(T.St.)	Coef.	(T.St.)
Travel cost (/fl)	μ=-0.149	(19.9)	μ=-0.167	(15.7)	μ=-0.180	(16.6)
Travel time (/hr)	η=-1.722	(10.7)	ω= 1.840	(6.3)	ω= 1.929	(11.5)
			σ= 1.343	(3.5)	σ= 1.369	(5.4)
Transfers (/#)	α_1=-0.326	(5.5)				
(fl/#)	2.197	(5.7)	β_1= 2.278	(6.2)		
(hr/#)	0.190	(5.4)			γ_1= 0.183	(5.8)
Comfort (/level)	α_2=-0.946	(14.6)				
(fl/level)	6.369	(15.9)	β_2= 6.379	(17.4)		
(hr/level)	0.549	(11.0)			γ_2= 0.599	(12.5)
Log-likelihood	-1724.1		-1721.1		-1718.4	
VOT distributions:						
Mean (fl/hr)	11.6	(12.3)	15.5	(3.9)	17.6	(4.3)
Median (fl/hr)	11.6		6.3		6.9	
Mode (fl/hr)	11.6		1.1		1.1	
St.Dev. (fl/hr)	N.A.		34.9		41.3	
Iterations	5		7		14	
Run time (min:sec)	0:19		9:50		25:48	

Sample size: 235 respondents, 2929 observations that is estimated to have very high VOT values in the right-hand tail.

Although Figure 1 gives an idea of how the lognormal distribution differs from the fixed VOT case, it is not immediately evident what the influence will be on predicted choices. An individual with an extremely high value of time still makes only a single choice, even though he or she may have a large influence on the estimate of the mean VOT. Figure 2 shows the results of applying the fixed VOT and the Lognormal 2 models to a binary choice situation where two rail routes have the same number of transfers and comfort level, but where one provides a 30-min time savings relative to the other. Figure 2 shows the change in the predicted fraction choosing the faster alternative as the price difference increases from 0 to 25 guilders, corresponding to an "indifference" VOT of 0 to 50 fl/hr. These time and cost differences are typical of those in the SP data. The lognormal model was applied in each case by numerically integrating the probabilities across the estimated VOT distribution.

Figure 2 shows that the response from the lognormal model is slightly flatter than that of the fixed VOT model. The difference is greatest at the high price end, where the lognormal model predicts that a fraction of the population with high VOTs will still pay for the time savings. Both models have a flat response curve, however, indicating that there is still a good deal of random variation in the choices that is not explained by the VOT distribution. This point is further discussed in the conclusion.

Table 1 also provides an indication of the computer time necessary to estimate the lognormal models with Gauss. Although not nearly as fast as the special-purpose ALOGIT software used to estimate the fixed VOT model, the Gauss lognormal estimation required less than 2 min per iteration on a 33 MHz 80486 microcomputer. Using less precision (fewer points) to evaluate the integrals would result in even faster run times.

Motorway Driver Value of Time SP Data

The second case study uses data collected in 1988 during the Netherlands Value of Time study (*2*). Travelers were intercepted at motorway petrol stations, urban parking areas, train stations, and bus and tram stops at many locations throughout the country. After being asked a number of screening questions, they were sent a self-completion questionnaire that included a number of SP binary choice questions customized to their actual journey. Because the purpose of the study was to estimate accurate, context-specific values of time for evaluation purposes, the SP questions were confined to be "within mode" for their actual mode, and only travel time and travel cost were varied.

The models estimated during the original study contained a number of time-related variables that were assumed to simultaneously influence the value of time. The main effects were related to income, free time available, mode of travel, and the level of congestion for motorway travelers. Travel purpose was used as a segmentation variable, with separate models estimated for "business," "commuting," and "other" purposes. (Note that the SP experiment was conducted so that business VOT would include only the employee's value and not the employer's).

Because the authors were concerned that the large spread in VOT distribution in the Rail SP case study may have been due to the heterogeneity of the sample, the authors decided in this case to focus only on motorway drivers and use the main segmentation and time-related variables to explicitly

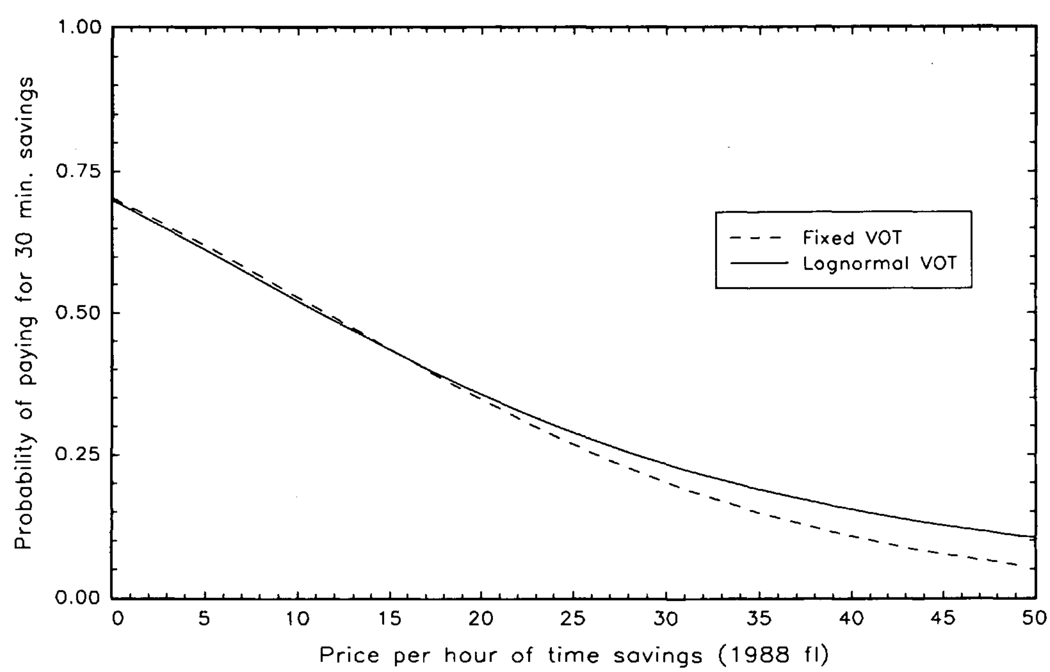

FIGURE 2 Rail SP data—predictions from fixed VOT and lognormal VOT models.

account for many of the person and trip characteristics expected to influence the value of time savings.

Table 2 contains results for fixed VOT and lognormal VOT models for all three segments, using extra travel time variables as functions of income level, number of car passengers, amount of free time available (discretionary time as deduced from self-reported time budgets), and congestion level. The congestion level is the percentage at which the observed traffic speed at the time of intercept was below 120 km/hr. Continuous traffic speed and flow monitors were located near the intercept sites. The extra travel time variables are interaction terms with the base travel time variable and thus by definition are proportionally related to the value of time. They were thus associated with the γ parameters.

For both the fixed VOT and the lognormal VOT models in Table 2, the overall results agree with those of the original study: business and commuting values are higher than those for other purposes, and values markedly increase with income level and decrease with available free time. The effects of congestion and car occupancy are less significant, with the signs varying across the segments.

The most striking results in Table 2 are the huge improvements in the lognormal models relative to the fixed VOT models—70 likelihood units or more with the addition of one parameter for all three segments. This improvement is much greater than in the first case study, suggesting that the lognormal distribution is also appropriate after one has already accounted for as many important observable factors influencing VOT as possible. In contrast to the first case study, the mean VOTs for the lognormal models have t-ratios just as high as for the fixed VOT estimates.

For business and commuting, the mean VOT increases by a factor of more than two in the lognormal models relative to the fixed VOT models. The median VOT is also greater than the estimate from the fixed VOT model. For other purposes, the increase is somewhat less. The lognormal standard deviation is about 1.5 times the mean for all three segments. Note that the proportional effect of income is about one-third less in the lognormal VOT models than for the fixed VOT models for business and commuting. This result suggests that what had been identified as an income effect in the simpler models may be partially due to the correlation of income to other unobserved influences on VOT. The congestion effects also tend to become smaller in the lognormal models.

The lognormal density and cumulative functions for the three segments are shown in Figure 3. The large tails in the distribution for business and commuting at the high-VOT end are evident. Note that the distributions in Figure 3 (and at the bottom of Table 2) are for the "base" VOT only and do not yet include the extra effects of income. When those effects are added, the differences among the segments become more pronounced, as is shown in Figure 4.

TABLE 2 Estimation Results for Motorway VOT SP Data

Segment:	Business		Commuting		Others	
Respondents:	332		218		287	
Observations:	3984		2616		3444	
Fixed VOT	Coef.	(T.St.)	Coef.	(T.St.)	Coef.	(T.St.)
Travel cost (/fl) $\mu =$	-0.319	(17.5)	-0.496	(15.9)	-0.520	(22.2)
Travel time (/hr) $\eta =$	-5.026	(6.9)	-8.009	(6.7)	-5.802	(9.0)
Time-related effects: $\alpha/\eta =$						
Passengers (/#)	+ 9.8%	(1.9)	+1.5%	(0.3)	-5.7%	(3.7)
Income (/Kfl/mth)	+10.2%	(4.2)	+4.1%	(2.6)	+6.9%	(4.9)
Free Time (/hr/day)	- 8.2%	(9.7)	- 6.0%	(7.0)	-2.8%	(5.0)
Congestion (/% delay)	+ 1.3%	(2.3)	+1.1%	(2.2)	-1.7%	(3.0)
Log-likelihood	-1992.1		-1359.2		-1855.0	
Lognormal VOT	Coef.	(T.St.)	Coef.	(T.St.)	Coef.	(T.St.)
Travel cost (/fl) $\mu =$	-0.351	(3.0)	-0.532	(2.5)	-0.483	(3.2)
Travel time (/hr) $\omega =$	3.146	(32.6)	2.995	(27.4)	2.184	(20.3)
$\sigma =$	1.110	(25.2)	1.080	(24.0)	1.055	(27.1)
Time-related effects: $\gamma =$						
Passengers (/#)	+ 9.1%	(2.4)	- 0.2%	(0.5)	-4.2%	(2.6)
Income (/Kfl/mth)	+ 6.2%	(4.8)	+2.7%	(2.5)	+8.7%	(5.1)
Free Time (/hr/day)	- 8.0%	(18.2)	- 6.4%	(13.5)	-2.2%	(3.5)
Congestion (/% delay)	+ 0.0%	(0.2)	+0.7%	(2.0)	-1.3%	(3.5)
Log-likelihood	-1846.5		-1286.8		-1740.2	
Fixed VOT (fl/hr)	15.8	(7.0)	16.1	(6.9)	11.2	(9.5)
Lognormal VOT:						
Mean (fl/hr)	42.8	(8.3)	35.8	(8.3)	15.5	(8.8)
Median (fl/hr)	23.2		20.0		8.9	
Mode (fl/hr)	6.8		6.2		2.9	
St.Dev. (fl/hr)	66.0		53.2		22.2	

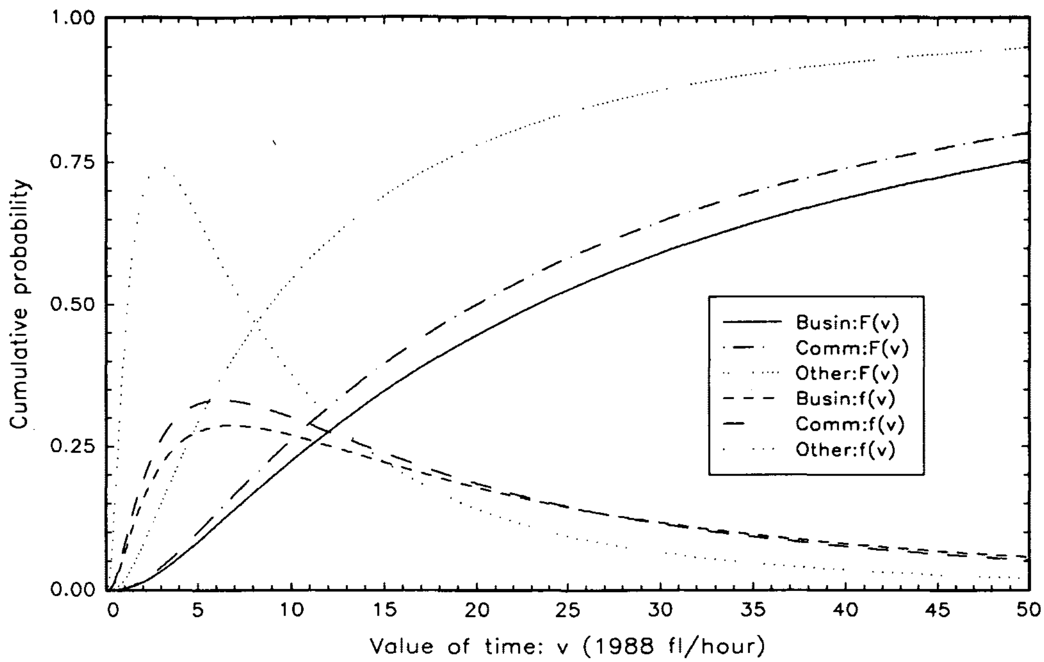

FIGURE 3 Road SP data—estimated lognormal VOT distribution.

Figure 4 was created in the same way as Figure 2, applying both the fixed VOT and the lognormal VOT models to binary choice situations with a 30 min time savings at various price levels. For income, free time, car occupancy, and congestion, the average values found in the SP estimation data for each segment were used. Here, the differences in predictions between the fixed and lognormal models are more pronounced than for the first case study. The reasons for this result are (a) that these models have a better fit (higher scale) and thus the logit prediction curves are steeper and (b) that the spread in the lognormal distribution is greater and thus the lognormal predictions curves are flatter. The fixed and lognormal pre-

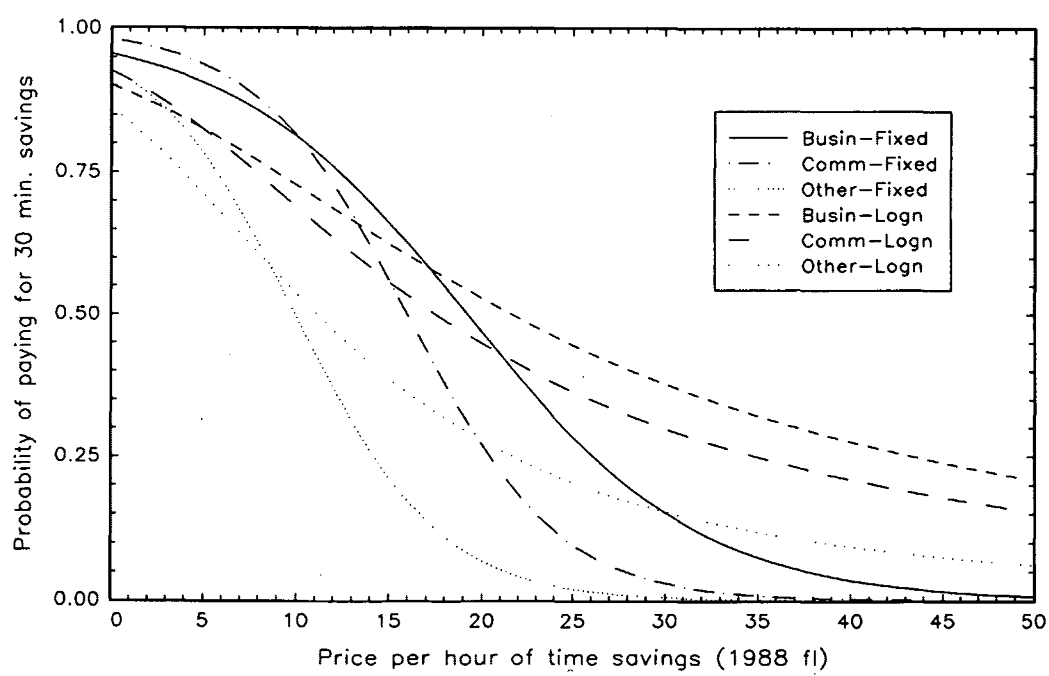

FIGURE 4 Road SP data—predictions from fixed VOT and lognormal VOT models.

diction curves cross each other at price levels of about 18 fl/hr for business, 15 fl/hr for commuting, and 8 fl/hr for other purposes.

Very Fast Train Binary RP Models

The data in this last case study were collected using air, road, bus, and rail intercept surveys in the corridor for the proposed very fast train (VFT) high-speed rail line from Sydney to Melbourne, Australia, in 1988. All time and cost data were network based. For the purpose of homogeneity and for reasons similar to the second experiment, the analysis presented here used only business trips. There are 12,586 such trips, 95 percent of which were by either air or car. The models estimated focus on the binary choice between air and car. There are 10,542 observations with both modes available, 87 per cent of which chose air. The total travel cost averaged over mode users is 110.5 for car and 271.3 for air. The average main mode time expressed in minutes is 698.9 for car and 88.4 for air. Average access and egress time is zero for car users and 152.2 min for air users.

In the lognormal model estimated, access and egress time and total time multiplied by income are specified as time related, and the air constant is specified as cost related. The equivalent ratios are calculated for the multinomial logit (MNL) coefficients. Estimation results are reported in Table 3. Two very interesting points to note regarding these estimates are (a) the log-likelihood increased by 250 units, which represents a huge improvement in the quality of the fit, and (b) the access and egress time coefficient switches to the right sign and becomes significant. The estimated density and cumulative density functions for this example are plotted in Figure 5. Again, when the models were applied in prediction in a manner similar to the previous two examples, the result is that the lognormal models have a flatter response (i.e., lower price elasticity) than the models assuming a single fixed VOT. Figure 6 clearly shows that in the lognormal case, a large proportion of individuals are willing to pay a high price to save 5 hr in travel time.

CONCLUSION

A method for estimating travel choice models that allows a lognormal distribution for the ratio of time and cost effects, instead of assuming a single fixed value across the population, and the testing of the method have been described. A maximum likelihood estimation procedure has been programmed and tested using two different SP data sets and one RP data set.

All case studies showed a significant improvement in model fit when the distribution parameter was added. The spread in the estimated lognormal distributions was found to be large in all cases, with standard deviations exceeding the mean VOT in the SP experiments and large "tails" of the population estimated to have very high VOTs. When the models are applied in prediction, the result is that the lognormal models have a flatter response (that is, lower price elasticity) than the models assuming a single fixed VOT.

Between the two SP studies, the lognormal distribution gave the most substantial improvement in the second one, where a number of observed segmentation variables and VOT effects had already been accounted for in the model specifi-

TABLE 3 Estimation on Results for Car/Air Binary RP Data

	Fixed VOT		Lognormal	
Variable	Coef.	(T.St.)	Coef.	(T.St.)
Total cost	μ=-.0286	(22.3)	μ= -.0796	(7.6)
Air constant	2.118	(12.9)		
Air constant/cost	-74.06		-49.1	(14.1)
Main mode time	η=-.0054	(17.2)	ω= -1.282	(24.4)
			σ= .5258	(30.2)
Access+egress time	.0010	(1.5)		
acc+egr time/time	-.1855		.2693	(3.2)
Total time*income	-.0146	(17.2)		
Time*income/time	2.701		3.108	(9.7)
Log-likelihood	-2396.1		-2148.3	
VOT distributions:				
Mean VOT	0.189		0.319	(18.1)
Median VOT			0.277	(87% of mean)
Mode VOT			0.210	(66% of mean)
St.Dev. VOT			0.180	(56% of mean)
Iterations	8		19	
Run time (min:sec)	1:51		75:16	

Sample size: 10542 observations

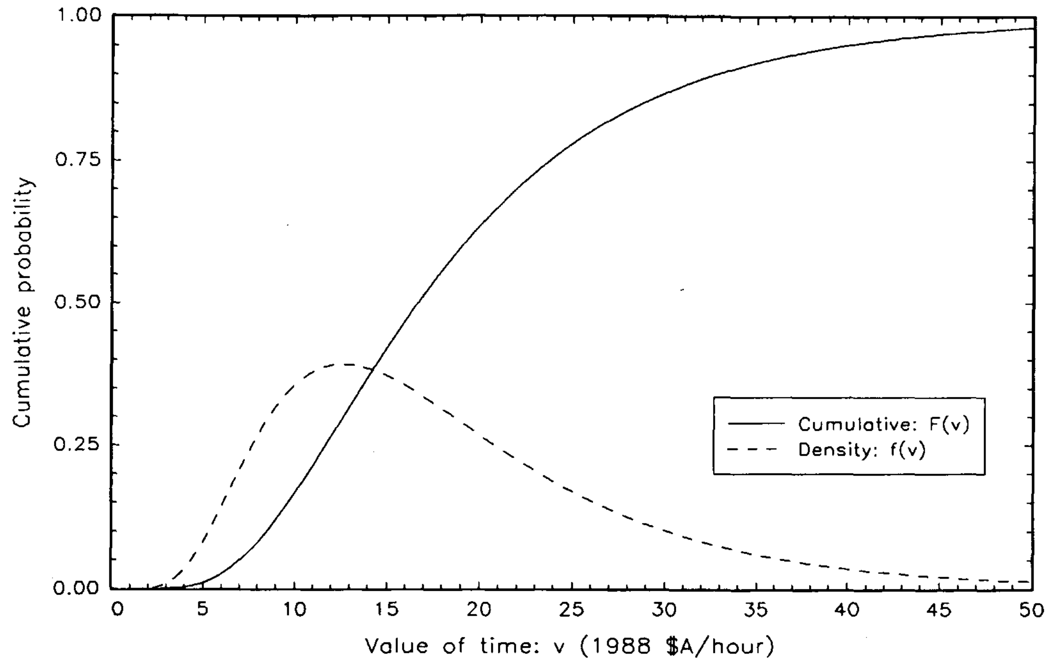

FIGURE 5 Car/air RP data—estimated lognormal VOT distribution.

cation. This result indicated that the distributed VOT estimation should not be used as a substitute for explaining as much variation in the data as possible, but rather as an extra tool for capturing variation that cannot be explained by other means. The goodness of fit in the RP estimation clearly showed the best improvement.

More experience with this approach is necessary before general conclusions can be made. In addition to the case studies reported here, the authors have applied the approach to an urban tolled road versus nontolled road SP experiment (the project for which the approach was actually created), and the results showed a very similar pattern to those of the second case study reported here.

Tests using simulated data may also be useful. The approach described above can measure one specific form of "taste variation" on the time-related parameters. Because estimation techniques have not been available, very little is currently known about the sources of such variation in empirical data.

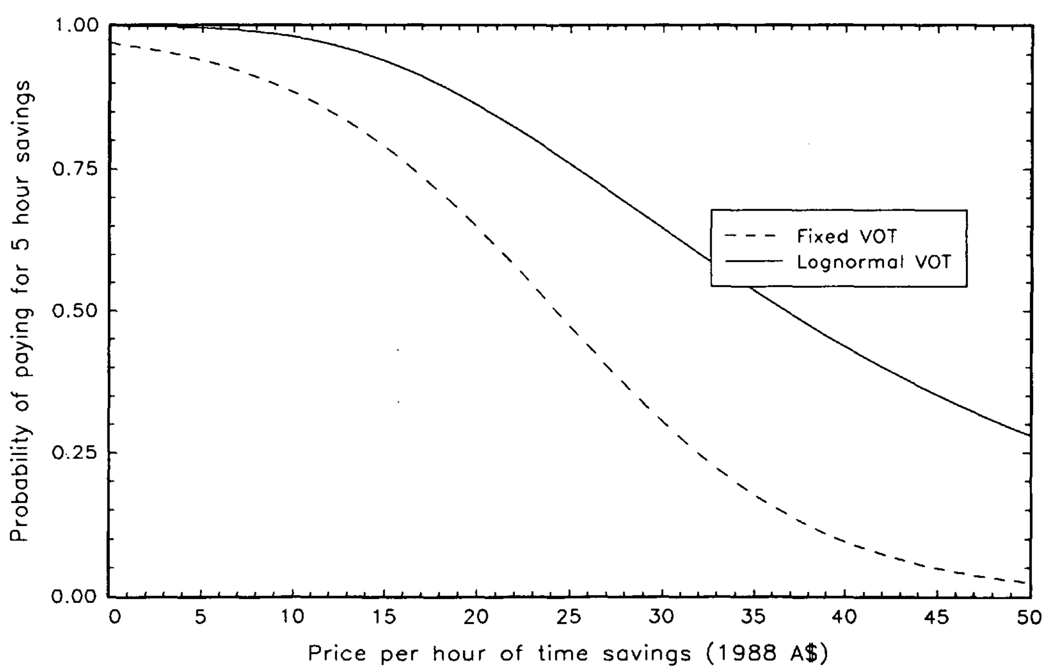

FIGURE 6 Car/air RP data—predictions from fixed VOT and lognormal VOT RP models.

If, for instance, measurement or perception error in the data is somehow related to travel times, it is unclear to what extent this will affect the value of time distribution parameters as opposed to the general random error term. Testing the approach using choice data that are created using various types and amounts of simulated error will be an efficient way of determining its properties.

There are a number of ways in which this approach could be varied or extended. An obvious variation is to assume a VOT distribution other than the lognormal. The authors have adapted the Gauss routine to estimate a normal distribution rather than a lognormal distribution. The routine was then applied to the data used in the case studies here, but the model fit was found to be inferior to that of the lognormal models. The method could also be adapted for other shapes, such as the Gamma or Erlang distributions, or for a semiparametric discrete "mass points" distribution.

A second variation is to apply the method to variables other than the value of time. This can be done easily because the choice of a coefficient as the parameter can be applied to other variables, not just time. If one was to substitute, for example, the coefficient for vehicle emissions level relative to fuel price, then one could estimate a model with a distributed willingness to pay for pollution reduction.

Further extensions would be to include separate distributions on different variables in a single model and to be able to specify the distribution parameters as a function of observable attributes. These additions can be accommodated only in a probit or logit estimation procedure with a very general form. Because the required multidimensional integration is not computationally feasible, an efficient approximation, such as simulated maximum likelihood, is required. Such an approach is currently being developed and tested using a variation on the multinomial probit model (*8,9*).

ACKNOWLEDGMENTS

The authors are indebted to Jean Delons for inspiring many of the ideas behind this research. The authors thank Albert Bouma of Nederlandse Spoorwegen and Henk Kleijn of Rijkswaterstaat for their permission to use the data analysed in the first two case studies described in this paper. The authors also thank two anonymous referees for their valuable comments.

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Publication of this paper sponsored by Committee on Passenger Travel Demand Forecasting.