Evaluation of Culvert Deformations Using the Finite Element Method

SUNIL SHARMA AND JAMES H. HARDCASTLE

A finite element analysis was performed to study the stability of a rib-reinforced, low-profile, long-span steel arch culvert located at Hayden Creek, Hayden Lake, Northern Idaho. The culvert, which is 3.42 m high and has a span of 10.52 m, was designed on the basis of empirical methods. The culvert suffered some unexpected deformations (sag) during the first few months after installation. Because of the limitations of conventional, empirical analysis methods, the finite element method was used to model the complex soil-structure interaction conditions at the culvert and to assess the ability of the culvert to accommodate future design loads safely. The CANDE program, which was developed for the Federal Highway Administration, was selected for this study because it was adjudged as providing the most flexible and realistic treatment of the culvert soil interaction. The soil-structure model used to analyze the Hayden Creek culvert is described. Factors of safety for compression and plastic hinge formation (inelastic buckling) are computed from the finite element analysis results; the limitations of the approach are presented. A nonlinear, finite element analysis requires a large amount of processing time. Previously this was an expensive proposition, but with the availability of fast desktop computers, the finite element method may be used regularly with appropriately selected linear or nonlinear parameters to gain a general insight into the deformation behavior of culverts.

A rib-reinforced, low-profile, long-span steel arch culvert was installed at Hayden Creek, Hayden Lake, Northern Idaho, in January 1986. The culvert, which is 3.42 m high and has a span of 10.52 m, was designed on the basis of empirical methods. Initially the dimensions of the structure were well within the 2-percent limit recommended by the steel supplier's specifications (*1*). Between January 13, 1986, and March 19, 1986, a sag or flattening developed in the plates located in the eastern half of the structure's roof (L. E. Wolf, unpublished data).

The maximum amount of deformation occurred at a location approximately 9 m from the inlet of the structure (S. D. Powell, unpublished data). These deformations indicated a maximum deflection of about 0.175 m at the crown and about 0.100 m to 0.125 m of foundation settlement. The calculations made by the steel supplier using the AASHTO criteria and the estimated longer top radius of 8.60 m in place of the 7.14-m design radius indicated that the flattened arch retained a factor of safety (FOS) against plastic hinge formation of 3.0 (S. D. Powell, unpublished data). A substantial part of the dimensional and elevation changes were experienced in the first 68 days after construction, and at no time since the measurements were initiated has the apparent rate of settlement exceeded the initially observed rate. As the rate of deformation had reduced considerably, a study to determine the design capacity of the deformed culvert was initiated to determine whether the existing culvert could perform satisfactorily. Because of the limitations of conventional empirical analysis methods, the finite element method was used to model the complex soil-structure interaction conditions at the culvert and to assess the future ability of the culvert to accommodate the design loads safely.

Several finite element computer programs are available to perform this type of analysis, including FINLIN (*2*), SSTIP, NLSSIP (*3,4*). However, the CANDE program (*5–8*), which was developed for the Federal Highway Administration, was selected for this study because it was adjudged as providing the most flexible and realistic treatment of the culvert soil interaction (*9,10*). The soil-structure model used to analyze the Hayden Creek culvert is described. In addition, the factors of safety for compression and plastic hinge formation are computed from the finite element analysis results, and a discussion of the limitations of the approach follows.

Department of Civil Engineering, University of Idaho, Moscow, Idaho 83843.

NUMERICAL MODEL

Figure 1 shows the mesh that was used to analyze the full section for cases where differential settlement of the foundation could be appropriately simulated. A half-section mesh, shown in Figure 2, was also used for some preliminary analyses because solutions could be obtained much more quickly than with the full section. The plane strain meshes used in this study were not investigated for solution convergence and accuracy because they have a form that is similar to meshes used for previous analyses by Katona et al. (*7,11*). Both meshes are fixed at the lower boundary, that is, the location of bedrock, and are restrained in the horizontal directions at the two side boundaries. The meshes for the full and half sections consisted of 352 and 167 elements and 360 and 176 nodes, respectively.

CONSTRUCTION SEQUENCE

It has been shown by Katona (*7*), Leonards et al. (*10*), and McVay and Selig (*12*) that the construction sequence must be modeled during the analysis phase to obtain a realistic assessment of deformations and stresses. This is simulated by "adding" the soil above the springline in layers, thus effectively accounting for the modulus—the overburden stress dependency. The sequential construction effects generally dom-

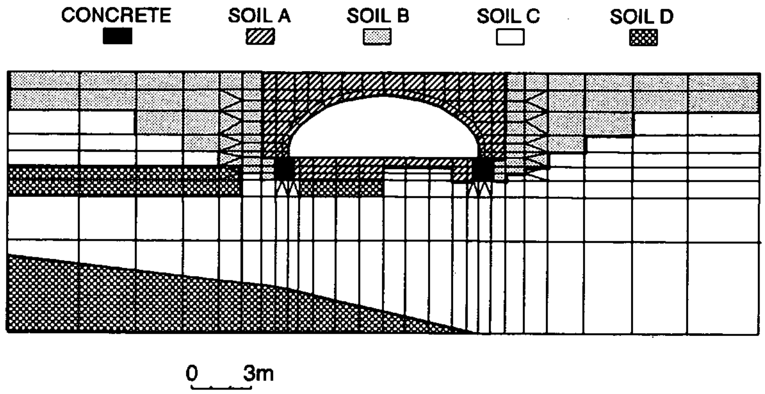


FIGURE 1 Full-section mesh and soil units used for finite element analysis.

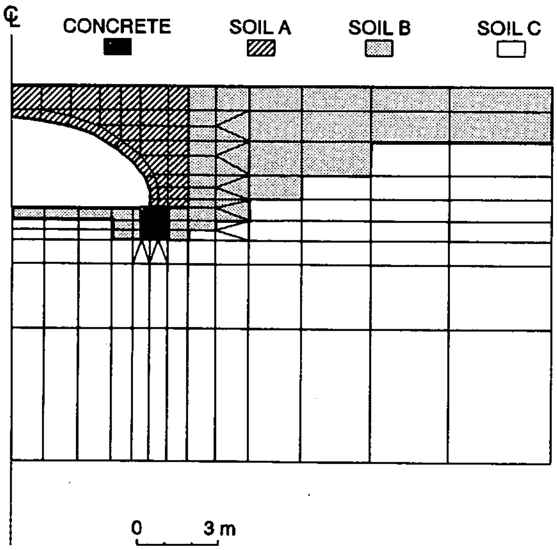


FIGURE 2 Half-section mesh and soil units used for finite element analysis.

inate the behavior of large span culverts with a shallow cover similar to the Hayden Creek culvert (*13*).

In this study the construction sequence was simulated by the following six steps, which are illustrated in Figure 3:

1. "Build" soil to foundation level (Level A in Figure 3);
2. Erect the culvert;
3. Place the compacted soil to the level of the springline (Level B);
4. Place the compacted soil to Level C;
5. Place the compacted soil to just above the crown (Level D); and
6. Place the compacted soil to the final grade (Level E).

Additional steps were included in the analysis to apply the live loads simulated by a line load applied above the crown at finished grade elevation.

ANALYSIS

For this study, the analysis considered three sets of model parameters corresponding to good, average, and poor soil conditions. This range was selected to investigate the possible influence of a variety of potential soil conditions that may be expected to affect the behavior of the installed culvert.

Model of Subsurface Profile

The subsoil profile was developed from the data collected from three soil borings performed at the site. Figures 1 and 2 show the distribution of soils used to analyze the full and half sections, respectively. Estimated model parameters for the soil zones are presented in Table 1. The model parameters were selected for three different soil conditions from the values suggested by Duncan et al. (*14*) for the appropriate soil types as follows:

- Good conditions were selected on the basis of optimum conditions during the compaction and backfilling operation. This category is considered to be an upper bound for the strength and compressibility of the soils. With these values, lower stresses and deformations are expected in the culvert.
- Average conditions were based on an estimate of the actual conditions that may exist at the site. These values were based on studies of similar soils from the area and results of the laboratory tests performed on disturbed or remolded samples obtained from the soil borings. The deformation results from the analysis are expected to be similar to values that may have been predicted during the design of the culvert.
- Poor conditions were based on the assumption that the subsoils and the compacted backfill have low strength and high compressibility. Such parameters are expected to generate large stresses and deformations in the culvert due to a lack of lateral support that is essential for such large-span structures.

These ranges of parameters (Table 1) were selected on the basis of a soil classification and simple laboratory tests that had been reported during the initial site exploration program.

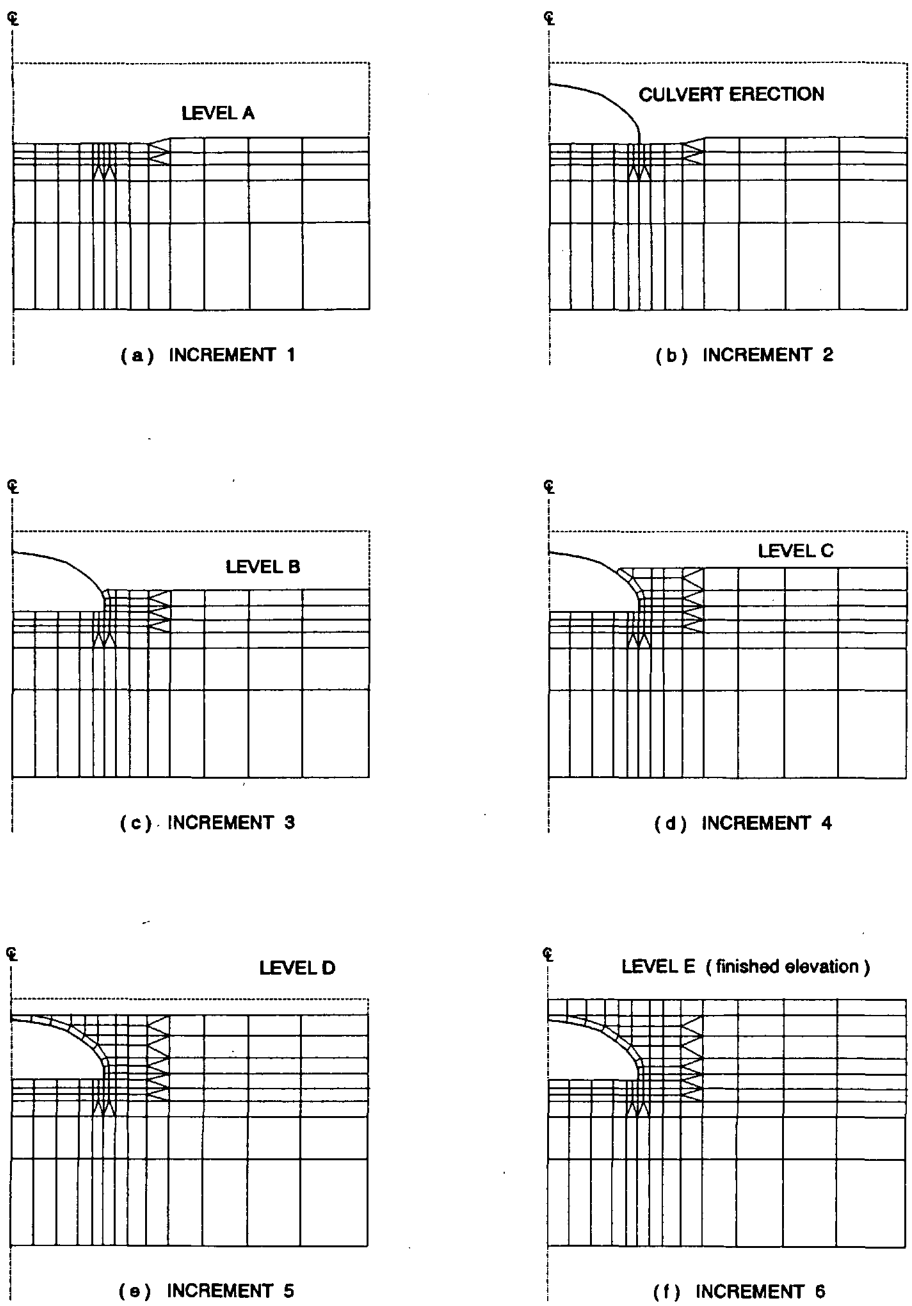


FIGURE 3 Incremental sequence used to simulate construction for finite element analysis.

The general parameters were initially selected for a preliminary assessment. A more elaborate and complete soil sampling and testing program would have been undertaken if the initial analyses had suggested marginal factors of safety.

LIVE LOADS

The AASHTO live load may typically be represented by a line load of 88 or 68 kN/m for a culvert with a 0.300-m or 0.600-m cover as suggested by Duncan (*13*). These line loads are estimated using the Boussinesq elastic theory and will produce the same peak stress at the top of the culvert as do two HS-20 truck trailers with single rear axles side by side on a two-lane road (*13*). For this study, several different line loads were selected to investigate the influence of possible overloading during or soon after construction.

RESULTS

The finite element analysis provides the following results at the pipe element nodes: displacements, thrust forces, bending moments, and shear forces. The thrusts and bending moments

TABLE 1 Soil Parameters for Duncan-Chang Model (*15*)

PARAMETER	SOIL A			SOIL B			SOIL C			SOIL D
	Good	Average	Poor	Good	Average	Poor	Good	Average	Poor	Average
K Modulus No.	800.0	700.0	450.0	600.0	500.0	300.0	600.0	500.0	300.0	200.0
n Modulus Exponent	0.5	0.4	0.45	0.5	0.35	0.4	0.5	0.35	0.4	0.35
c (kPa) Cohesion Intercept	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\varphi°$ Friction Angle	50.0	50.0	50.0	45.0	40.0	40.0	45.0	38.0	38.0	30.0
R_f Failure Ratio	0.55	0.7	0.6	0.6	0.7	0.65	0.6	0.7	0.65	0.85
G Poisson's Ratio	0.25	0.3	0.3	0.3	0.33	0.35	0.3	0.33	0.35	0.35
F Poisson's Ratio Parameter	0.05	0.05	0.1	0.07	0.05	0.1	0.07	0.07	0.1	0.1
d Poisson's Ratio Parameter	10.0	5.0	5.0	5.0	5.0	4.0	5.0	5.0	4.0	4.0
γ (kN/m^3) Unit Weight	19.65	19.65	19.65	18.39	15.51	16.51	9.43	9.43	9.43	7.86

controlled the ability of the culvert to support the gravitational and design live loads. The shear forces were not expected to affect the stability of the culvert and thus were excluded in the computation of the factors of safety. The factor-of-safety values were calculated as follows:

Factor of safety against "pure" compression failure:

$$\text{FOS} = \frac{P}{A\sigma_y} \tag{1}$$

Factor of safety against plastic hinge formation (*13*):

$$\text{FOS} = \frac{1}{2} F_1[\sqrt{(F_1^2F_2^2 - 4)} - F_1F_2] \tag{2}$$

where

A = cross-sectional area of culvert,
S = section modulus,
σ_y = yield stress of culvert,
F_1 = P_p/P,
F_2 = M/M_p,
P = thrust force in culvert,
P_p = $A\sigma_y$, the axial force at yield,
M = bending moment, and
M_p = plastic moment resistance, $1.5\sigma_y S$ (estimated).

More than one plastic hinge is required for the formation of an unstable mechanism. In this case, the factor of safety against the formation of the *first* hinge is expected to give a conservative estimate of a plastic failure that will be caused by the formation of several such hinges. The possibility of *elastic buckling* was not considered for this analysis because it was not expected to be a controlling factor for a culvert of this size and a cover of about 0.600 m (*13*).

Results of the full sections analyses are presented according to the pipe node numbering system shown in Figure 4. The numbering of the nodes increases in a clockwise direction or from the left to the right.

Nonlinear Analyses

To obtain a general understanding of the effects of reasonable variation in soil parameters and culvert geometry (deformed versus ideal) within a reasonable expenditure of computational effort, a series of preliminary "runs" were performed using constant elastic moduli. However, nonlinear analyses are expected to provide a better simulation of the anticipated behavior of the culvert and supporting soils. Thus, following the preliminary elastic analysis, nonlinear analyses were performed using the subsoil models presented in Figures 1 and 2 and the parameters given in Table 1. A summary of the computed FOS values is presented in Table 2, and the results are discussed later.

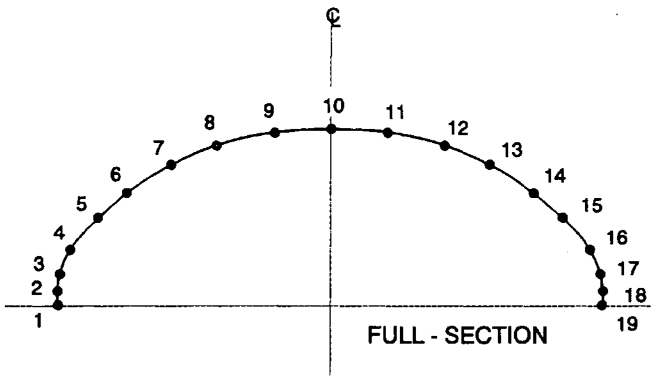


FIGURE 4 Location of pipe nodes for the full-section analyses.

DISCUSSION OF RESULTS

Half-Section Analyses

The nonlinear cases represent a realistic simulation of the culvert-soil interaction and are expected to provide the most reliable results. These results, summarized in Table 2, indicate FOS values ranging from 6.2 to 6.7 for thrust failure and 3.8 to 5.4 for plastic hinge formation with zero live loads. In the case of the half section, the poor soil conditions generated the lowest FOS values.

Full-Section Analyses

On the basis of results from the half-section analysis, two full-section cases were also analyzed to investigate the effects of a soft layer under the west footing. The FOS values are summarized in Table 2, and a discussion of the predicted deflections, thrust forces, and bending moments follows.

Good Soil Conditions

The horizontal and vertical deflections of the culvert for the good soil conditions are presented in Figure 5. For the full-section case, the effect of the soft layer leads to nonsymmetric deflections and further complicates their interpretation. However, the "peaking" is again evident from the y-deflection values. Also, the inclusion of the soft layer results in larger vertical settlements at the west footing, as expected, in comparison with that of the east footing. This differential settlement amounts to about 32 mm. The horizontal deflections are complex and difficult to interpret due to the inherent interaction between the culvert and surrounding soils.

The thrust forces are presented in Figure 6 (*top*) for the last two construction sequences and for a live load of 88 kN/m (Inc −7). For the dead load case (Inc −6), a minimum thrust force of 1000 kN/m is predicted at the crown, and a maximum force of 277 kN/m occurs at node 17, which is close to the west springline. Generally, the dead load thrust increases from the crown (node 10) to nodes 5 and 15, and then reduces slightly down to the footing level. This variation may be caused by the differential settlement which results in the development of positive arching and an apparent reduction in the dead loads. The application of the 88-kN/m live load increased the thrust at all nodes. A maximum increase of 93 kN/m was predicted at node 14, and the overall maximum thrust of 339 kN/m was computed at node 17, which is close to the east springline.

The bending moments for this case are shown in Figure 6 (*bottom*) and indicate that the west portion of the culvert near the springline has the higher bending stresses. (*Positive bending moments are defined as those generating tension on the outside face of the culvert.*) It can be seen that due to the residual effects of the peaking, the bending moments at the crown are positive under dead-load conditions, but the applied live load tends to generate negative bending moments. These moments increased from a maximum dead load value

TABLE 2 Results of the Nonlinear Analysis

	LOAD CASE DESCRIPTION	LIVE LOAD ($N/m \times 10^3$)	FACTOR OF SAFETY Thrust	FACTOR OF SAFETY Plastic Hinge
1.	Good soils; Half-section.	0	6.7	5.4
		88	5.5	4.5
2.	Average soils; Half-section.	0	6.6	5.1
		88	5.7	4.3
3.	Poor soils; Half-section.	0	6.2	4.6
		44	5.8	4.3
		88	5.3	3.8
4.	Good soils; Full-section	0	6.5	3.8
		88	5.3	3.2
5.	Poor soils; Full-section.	0	6.3	4.7
		88	5.4	3.9

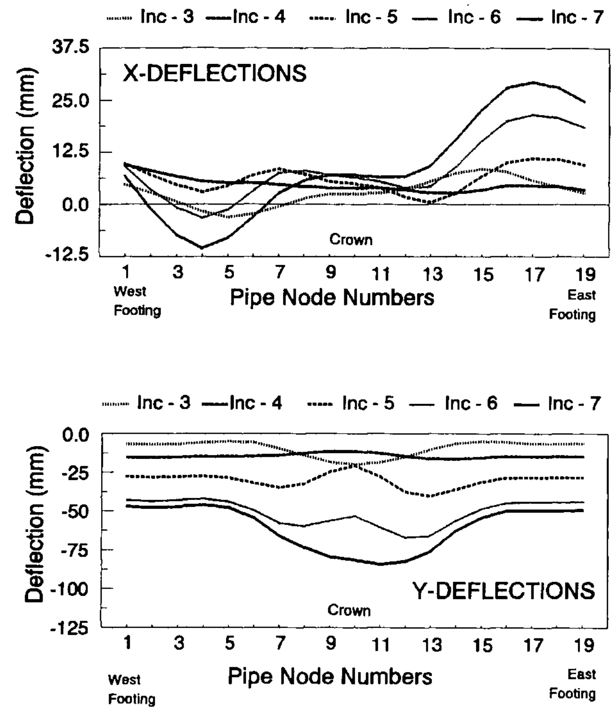


FIGURE 5 Horizontal and vertical deflections for good soil (full section).

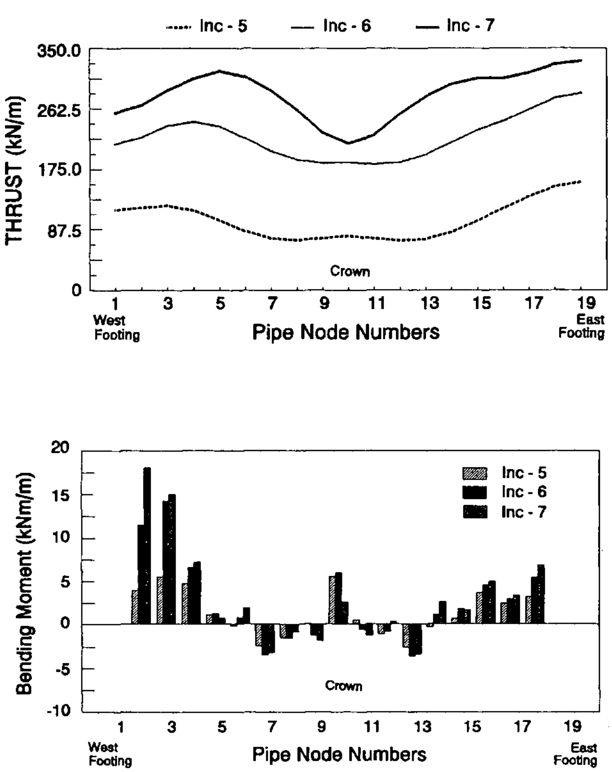


FIGURE 6 Thrust forces and bending moments for good soil computed by finite element analysis (full section).

of 14 195 N·m/m (node 3) to 17 980 N·m/m (node 2) with the application of the live load. The bending moments induced by the live load are significant for this case, with a predicted 57 percent increase at the west springline (node 2). However, this bending moment is not expected to cause any instabilities because there is an FOS of 3.2 against formation of a plastic hinge. The live load effects on the east side are less pronounced and increase bending moments by only a small amount.

Poor Soil Conditions

Results of the final analysis using the full-section mesh and poor soil condition parameters are presented in Figures 7 and 8. For this case, the contrast between the poor soils and the soft layer was marginal, and only a small amount of differential settlement was predicted by the finite element analysis. The peaking effects were successfully simulated during the construction sequence. The x-deflections are again difficult to interpret, but there is a reasonable symmetry between the predicted deformations at the east and west nodes of the culvert.

The computed thrust forces are similar to the previous case and ranged from 183 kN/m, close to the crown, to a maximum value of 286 kN/m at the east footing level for the dead-load case (Inc -6). The predicted bending moments are smaller than the good soils case with maximum values of 8 188 N·m/m and 12 149 N·m/m for the dead load and live load cases. Again the application of live loads was significant, leading to a 48-percent increase in the bending moment at the west springline.

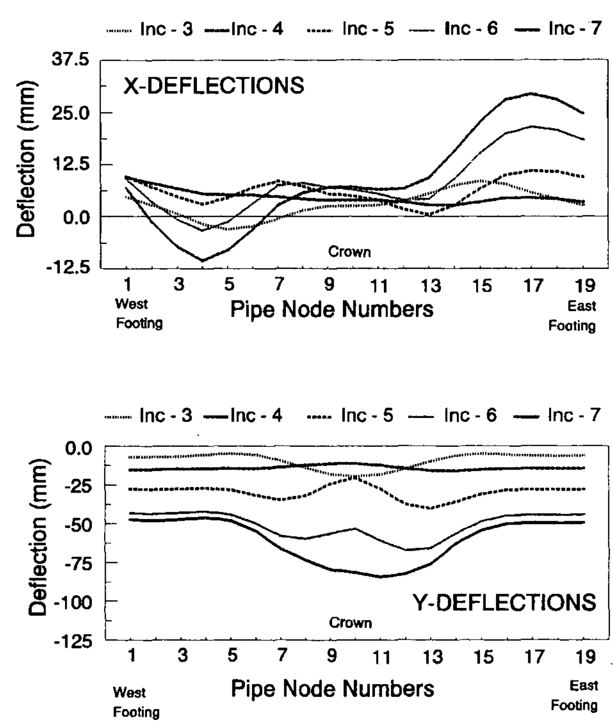
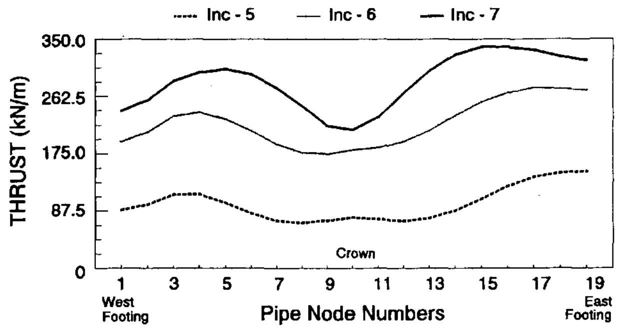


FIGURE 7 Horizontal and vertical deflections for poor soil (full section).



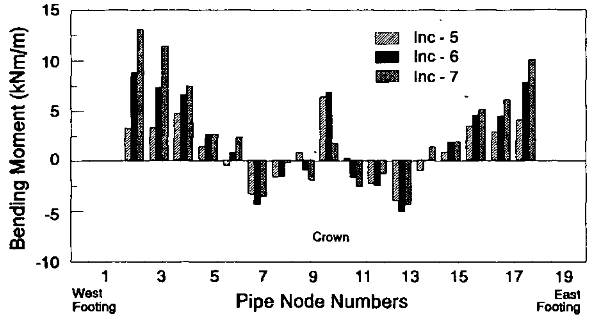


FIGURE 8 Thrust forces and bending moments for poor soil computed by finite element analysis (full section).

SUMMARY OF RESULTS

On the basis of the finite element analysis of the Hayden Creek culvert, the following minimum factors of safety were determined from several cases:

	Factor of Safety	
	Thrust	*Plastic Hinge Formation*
Dead load only	6.2	3.8
Live load = 88 kN/m	5.3	3.2

The values given for a live load of 88 kN/m represent a conservative estimate of the HS-20-44 wheel loading. Minimum values for the design live load represented by a HS-20-44 wheel are likely to be less than 88 kN/m and can be expected to result in larger factors of safety than reported above. These values are greater than the design factors of safety recommended by AASHTO.

The deformations computed by the finite element analysis were smaller than the maximum observed values. The computed values of 75 to 100 mm vertical deformation at the crown (from Figures 7 and 8) were considerably less than the reported value of 175 mm. Similarly, the calculated footing settlements of 50 to 60 mm were less than the reported value of 100–125 mm. The relative displacement of the crown with respect to the footings was reported within the range of 50 to 75 mm. This relative deformation appears to be in close agreement with the values predicted by this analysis.

CONCLUSIONS

The following points summarize the finite element analyses:

- Slight deformations in the shape of the culvert are not expected to significantly affect the stability of the culvert as the factors of safety are likely to be greater than 2.0.
- Although direct comparisons are not really possible, the factors of safety determined using the linear soil properties were greater than those of the nonlinear soil analyses.
- For the half section, nonlinear analysis, the poor soil condition parameters generated the lowest FOS.
- For the full section, nonlinear analysis, the contrast between the good soil conditions and the soft layer under the west footing gave the lower factors of safety.
- The finite element analysis was unable to predict deformations that closely agreed with observations. However, the relative deformations with respect to the footing elevation were predicted within reasonable accuracy.

Finally, it should be recognized that the large amount of processing time required by a linear or nonlinear finite element analysis can be readily provided by desktop computers. In view of the tremendous decrease in computational expenses, the finite element approach can be used to analyze the deformation behavior of culverts. However, considerable effort is still required to determine the subsoil conditions and assign the appropriate parameters for the constitutive model. It is hoped that the knowledge gained with this increased familiarity will lead to increased understanding of the behavior of culverts.

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