Passenger Flow Distributions at Airports

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Models available for passenger traffic flow patterns at air terminals are reviewed, and a theoretical framework is proposed. Analogous models in other areas of engineering applications are investigated, and it has been attempted to take advantage of the prevailing experience in those fields. The relevance of the research into passenger flow distributions is explained and applications of such models described. The proposed model provides a sound starting point for queueing theory applications leading to analysis of congestion, delay, and travel times. The passenger behavioral aspects, such as the desire to reach the terminal early and apprehension related to missing flights, are taken into account. Data collected at Sydney International Airport have been used in comparison of the models.

This paper covers the model development phase of an ongoing research project carried out at the University of New South Wales, Sydney, Australia, into traffic flow distributions of airports. An in-depth discussion of modeling techniques available for handling the distribution of arrival and departure traffic on the road interface at airports is provided. Areas of application of the passenger arrival distribution, from the point of view of planners and traffic engineers, are described to highlight the relevance of this particular research project. The methodology developed in this paper has potential application to other types of modal interchanges where at least one mode is related to long-distance travel.

In conventional transport planning, the land use activity pattern of transport demand centers is taken into account to determine the traffic generation and attraction characteristics related to the transport center. A similar approach can be readily adopted in modeling the traffic flow of modal interchanges. For example, in a work by Wirasinghe and Vandebona (*1*) a regression method provided a relationship between the size of mass-transit stations and the intensity of the access passenger demand. A better estimate of the passenger traffic demand, however, may be made by considering the interrelationship and transformation of traffic flows among various modes linked to the particular interchange. This paper covers the modeling aspects that characterize that phenomenon.

The research work has two main areas of application: (a) terminal size estimation and (b) design of access road networks. Both require knowledge of passenger arrival and departure time distributions.

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BACKGROUND

Terminal Size Models

Airport size estimation typically relies on the intensity of passenger throughput data and magnitude of air traffic serving the terminal. The passenger throughput, with a level of service criteria, allows the estimation of the design size of the terminal area. The space requirements per person under varying level of service conditions have been reported elsewhere (*2–4*). Different levels of service are classified according to an alphabetic letter grade similar to that of highway engineering. A qualitative service classification scheme has been documented by various air transport authorities (*3,5*).

A simplistic method, which can be adopted in the sizing process, is to multiply a representative space requirement by the number of potential users. A refined calculation, however, will have to take into account the physical layout of the activities in the terminal building and the stochastic flow of passengers. Figure 1 shows a flow diagram of activities that are followed by passengers and aircraft at a typical airport. The figure shows that passengers progress through a series of holding areas for different activities. It is possible to estimate the average and maximum queue length at each of the activity areas by adopting the appropriate queueing model. The queue length measures reflect the passenger accumulation in the particular zone of activity. The space requirement for each zone of activity is computed, on the basis of space standard, for the particular activity under a specified level of service.

Consider that the maximum accumulations in each of the activity areas, because of a particular aircraft, are given by $S_1, S_2, S_3, \ldots$, and S_n. If the space standards are given by $a_1, a_2, a_3, \ldots$, and a_n for respective activity areas, the total space requirement is given by A^*, which can be computed as follows:

$$A^* = S_1 \cdot a_1 + S_2 \cdot a_2 + S_3 \cdot a_3 + \ldots + S_n \cdot a_n \qquad (1)$$

The maximum accumulations ($S_1, S_2, S_3 \ldots$) reflect the operational aspects of the zone of activity. For example, if the movement from one activity space to another can be performed more quickly by increasing the processing rate, the queue processing slopes will be relatively steeper and thereby yield smaller accumulation values.

Additional space for passenger handling has to be allocated for passenger movement in links between various sections of the airport. Aspects of walking distance and its impact on the terminal design have been discussed elsewhere (*1,6–8*). In an ideal design it may be possible to neglect the distances passengers have to negotiate in various concourses, but geometric constraints often necessitate including such links, which yield significant walking distances.

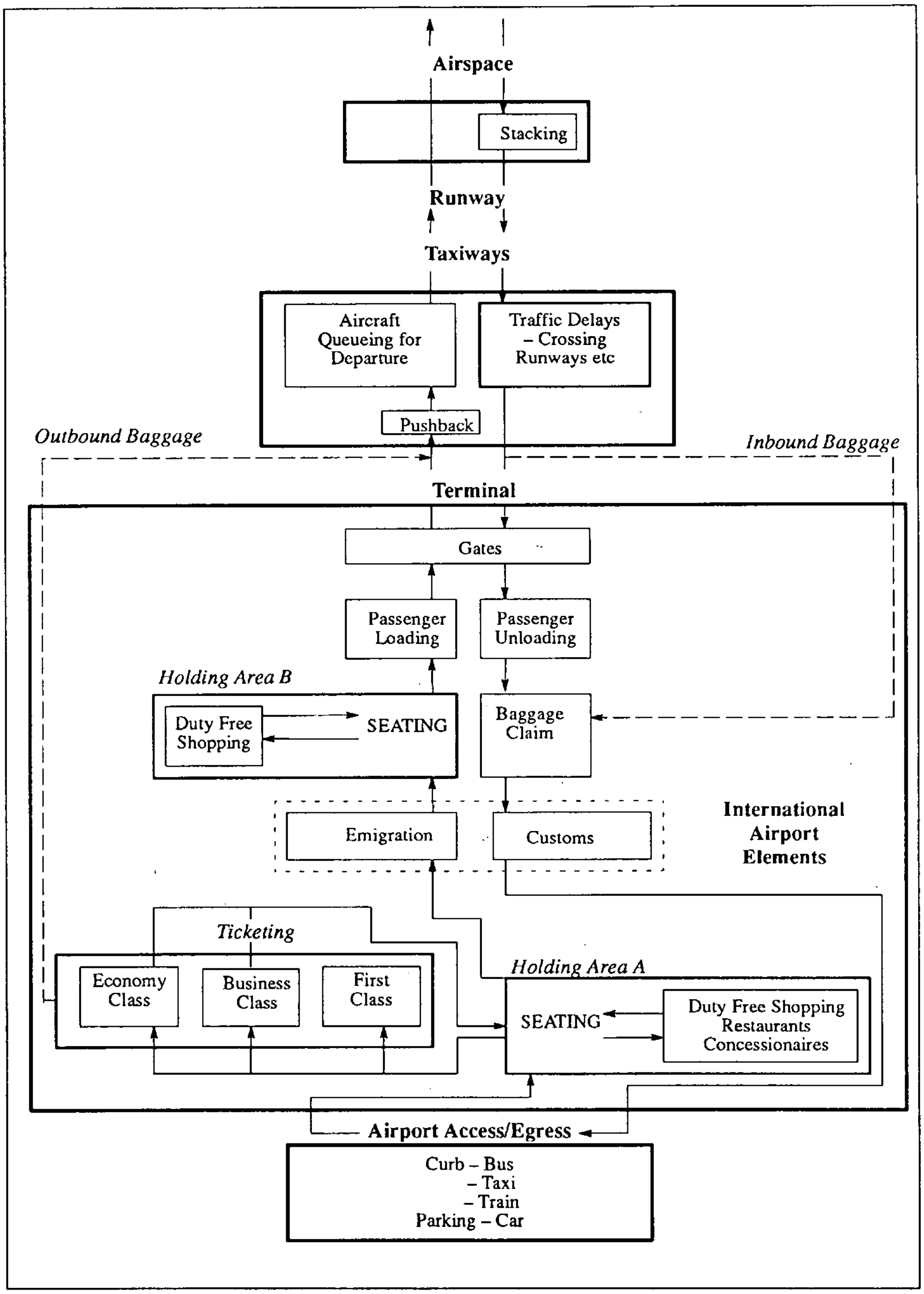


FIGURE 1 Airport activities.

It is also important to note that, as shown in Figure 1, the journey toward the aircraft consists of a different series of queues compared with the passenger journey from the aircraft.

Another technique available to handle the terminal sizing problem is simulation modeling (*9*). In simulation modeling, it is customary to adopt negative exponential service times and interarrival times in the queueing process. However, Bhatti (*10*) warns against the adoption of this particular distribution in queueing processes at airports on the basis of research work carried out in Nigeria. Thus there is a need for more field data to verify the range and applicability of the above models.

Airport Access Traffic Distribution

The other area of application of passenger arrival and departure time distributions is the development of models to estimate the temporal traffic distributions at airport access street networks. The aircraft arrivals and departures attract and generate traffic on landside modes serving passengers, well wishers, and employees of the airlines and airport. Knowledge about passenger arrival distribution and aircraft departure helps to better estimate vehicular traffic flows on access roads. Estimation of access road traffic flows is important for short-term purposes (area-wide traffic signal coordination) and long-term applications in planning of the road network.

An attempt to develop a regression model to forecast traffic flow on the access road on the basis of airline schedules at the Dallas–Fort Worth regional airport was reported by Dunlay and Wierzig (*11*). The inbound and outbound level of traffic prediction models were linear and took into account the number of aircraft enplanements and deplanements in 3 hr of airport operation (*11*).

At the Department of Transport Engineering of the University of New South Wales a number of graduate research projects have investigated aspects of access to airports. For example, Hupp (unpublished data) collected information related to the access and parking profiles at the Sydney Mascot Airport. Applications related to rail links were investigated by Doust (unpublished data), using a case study of a proposed remote airport. Recent work carried out by Pribadi (unpublished data) and Waluja (unpublished data) was aimed at developing computer models based on network algorithms and travel time formulations to estimate the travel times and level of congestion on the access road network.

Passenger Arrival and Departure Models

Models available for arrival and departure time distributions are reviewed. Work in this field provides descriptive explanations (*9,12*). Attempts to explain the above distributions in the form of mathematical relationships have been documented in works by Bhatti (*10*), Dunlay and Wierzig (*11*), and Tosic (*13*). It is evident that the models in this particular area can be classified into three main categories: descriptive, analogy, and regression models.

Descriptive Models

In some literature, the arrival passenger frequency distribution, relative to the aircraft departure time, is simply stated in a descriptive manner. In the descriptive method two main features have been associated with the distribution. The first part of the flow density distribution consists of a generally increasing function. The second part consists of a density function decreasing at a much faster rate than in the increasing period. Figure 2 shows the descriptive model in a graphical

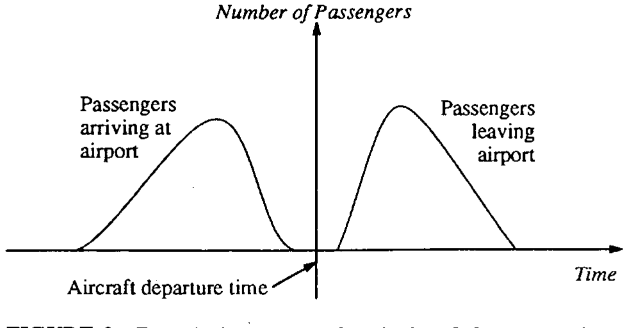


FIGURE 2 Descriptive model of arrival and departure time distribution.

manner. The frequency distribution of passenger arrival at curbside is indicated by the curve on the left-hand side of the figure, assuming that the aircraft is scheduled to depart at time zero. Note that the skew is to the left in this particular curve. The curve on the right-hand side represents the expected form of the distribution of passengers who are leaving the aircraft and the airport.

A detailed qualitative explanation about the spread of the passenger arrival time distribution at airports has been provided in a work by Odoni and De Neufville (*9*). It is argued that the arrival time distribution depends on the variability of travel time on the access road network and an individual's response to such variability. Because of renewed security procedures, some airports in Europe and North America cause the passengers to arrive at a predefined time before departure of the aircraft. The result is smaller space allocation per person because of the extra time in the airport imposed on the passengers.

Analogy Model

A useful analogy to the passenger distributions is found in hydrology. In particular, the runoff model includes concepts similar to the dispersion of passengers disembarking from an aircraft. The dispersion characteristics of passengers leaving aircraft is conceptually similar to the dispersion of water from a rain storm on a terrain with low permeability. Despite this similarity, the authors have been unable to find recent literature that adopted this analogy in airport landside traffic modeling. An early attempt to show the applicability of the unit hydrograph concept to data from a Brisbane, Australia, airport is documented in a work by Davidson et al. (*14*). Details of an applicaton of hydrograph analogy using data from Tullamarine Airport in Melbourne, Australia, are provided by Apelbaum and Richardson (*15*) and Apelbaum (*16*). However, as mentioned before, this particular analogy has not received further attention by researchers, perhaps because of lack of data to evaluate the method.

However, previous research work in which an attempt has been made to model the variability of travel time on road and public transport can also be considered relevant in applying this analogy. Dandy and McBean (*17*) provide a useful summary of previous research in this area and identified 12 research projects in which the travel time variations were monitored. The reported travel time surveys covered travel by car, bus, train, and by foot. About 25 percent of the research projects have concluded a log-normal distribution as a suitable model framework for travel time variations.

An advantage of pursuing this analogy is that modeling concepts related to different sizes of aircraft and various schedule patterns can be handled by exploiting the normalizing method referred to as unit hydrograph method in hydrological work. The runoff models are well established worldwide and an authoritative guide is provided by Pilgrim (*18*).

Table 1 presents the main similarities between the runoff model and passenger distribution model by identifying the corresponding features. Physical characteristics included in a runoff model, such as terrain slope and length of stream,

TABLE 1 Runoff Analogy Applied to Traffic Flow Discharged From a Demand Center

Traffic Flow	Runoff
Passenger disembarkation	Storm
Speed of movement in airport	Slope of the terrain
Free speed on the road	Slope of the main stream
Distance from airport	Length of the stream
Congestion attributes of the access roads	Channel roughness

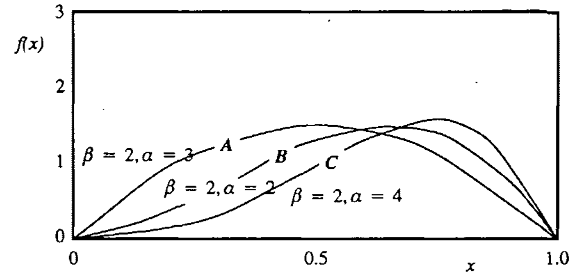


FIGURE 3 Beta distribution.

correspond to walking speed at the airport, vehicle speed on access roads, and distance from the airport.

To verify the applicability of the analogy it is necessary to monitor traffic flows on access roads to airports, relative to the aircraft arrivals. If it can be proved that the hydrograph analogy holds, then synthesizing traffic flow formulations for similar airport environments may be possible. In hydrological work, the synthesizing method has been adopted for ungauged catchments since the 1930s (*18*).

Regression (Curve Fitting) Models

Dunlay and Wierzig (*11*) document a regression-based model from the airline schedules at the Dallas–Fort Worth regional airport to forecast the traffic flow on access roads. Note that the researchers' attempt to fit theoretical probability distributions to various measurements of time showed a reasonable fit with gamma distribution for the vehicular flows bringing passengers to the airport relative to the departure time of the aircraft. (However, regression parameters have not been reported.) Corresponding distribution for vehicles carrying passengers from the airport is a log-normal shape. The applicability of the theoretical probability distributions at other airports has not yet been reported because of a lack of data.

The reasons previous researchers overlook beta distribution in attempts to find a standard probability distribution fit to the field data are not clear. The beta distribution has characteristics relevant for modeling in the area of passenger flow distributions. The gamma distribution and the log-normal distribution contain a long tail—difficult to explain in a traffic distribution model. On the other hand, the beta distribution can achieve the shape explained in the descriptive model because it contains a definite lower bound and upper bound in the independent variable axis. For example, consider the beta distribution using the conventional notation:

$$f(x) = \begin{cases} \dfrac{\Gamma(a + \beta)}{\Gamma(a)\Gamma(\beta)} x^{a-1}(1 - x)^{\beta-1} & 0 < x < 1 \\ 0 \quad \text{elsewhere} & \end{cases} \tag{2}$$

The frequency distributions are shown in Figure 3 for three different parameter value settings. A distribution similar to curve *C* in Figure 3 matches the descriptive model explained previously for access traffic. An advantage of the beta distribution is that it eliminates the possibility of a passenger arriving at an infinite time before departure of the aircraft. Such possibilities exist with the adoption of log-normal or gamma distributions and are therefore not representative of reality. However, the tediousness of the integration process to determine the cumulative beta distribution is a disadvantage of the beta model.

A number of students from the University of New South Wales have conducted surveys to investigate the suitability of the described models to Sydney Mascot Airport and other modal interchanges (Jayasinghe, unpublished data). A regular polynomial distribution of order 4 is seen to provide a better fit to data for Interstate travelers at railway and coach terminals in Sydney compared with distributions such as exponential, gamma, normal, and log-normal.

In general, the goodness of fit is improved with the increasing order of polynomials. The fourth-order polynomial was selected in this project as a compromise between the complexity of the resulting formulation and the degree of fit with field data.

Attempts to fit a similar fourth-order polynomial distribution to data obtained from the behavior of enplaning air travelers at the Sydney International Airport yield an equation of the following form:

$$Y_E = 2.399 - 0.318t + 8.888 \times 10^{-3}t^2 - 62.58 \times 10^{-6}t^3 + 0.129 \times 10^{-6}t^4 \tag{3}$$

Y_E is the percentage of passengers predicted in a 3-min period (compared with total boardings) around time t min before the departure time for the flight. For deplaning passengers the corresponding equation follows:

$$Y_D = 0.727 - 0.562t + 56.08 \times 10^{-3}t^2 - 1.213 \times 10^{-3}t^3 + 7.464 \times 10^{-6}t^4 \tag{4}$$

A weakness of the polynomial models is that unacceptable negative values result in troughs within the function.

MODIFIED POLYNOMIAL METHOD

A polynomial form developed by the authors attempts to satisfy the characteristics of the descriptive models and maintain mathematical simplicity essential for developing the cumulative distribution for further queueing theory analysis.

The model characteristics identified are:

- Frequency distribution of passenger arrival time at the airport relative to the aircraft departure time is skewed to the left for embarking passengers, whereas the skew is to the right for disembarking passengers.
- There is a distinct earliest and latest time related to arrival of embarking passengers and departure of disembarking passengers.

Consider the function $y_1(t)$ in Figure 4. This particular function has the quadratic equation:

$$y_1(t) = a_1t^2 + b_1t + c_1 \qquad 0 \le t \le T \tag{5}$$

In the equation, t represents the time variable. a_1, b_1, and c_1 are coefficients. It is evident that $c_1 = 0$ as the function passes through the origin. Therefore, the function can be rewritten as:

$$y_1(t) = a_1t^2 + b_1t \qquad 0 \le t \le T \tag{6}$$

Then consider the function $y_2(t)$ a quadratic formulation described in the same range but with a function value of zero at time T. $y_2(t)$ is given in the following:

$$y_2(t) = a_2(T - t)^2 + b_1(T - t) \qquad 0 \le t \le T \tag{7}$$

The product of the above two functions yields a distribution that agrees well with the descriptive model. This particular function follows:

$$R(t) = y_1(t) \cdot y_2(t) \qquad 0 \le t \le T \tag{8}$$

By expanding, it can be shown that

$$R(t) = a_1t^2(a_2\alpha - b_2\beta) + b_1t(a_2\alpha - b_2\beta) \tag{9}$$

where

$\alpha = t^2 - 2Tt + T^2$

and

$\beta = t - T$

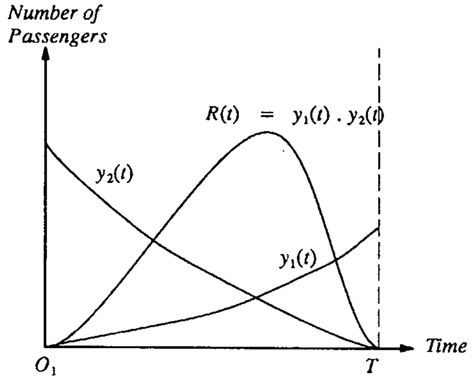


FIGURE 4 Representation of polynomial model.

There are a number of important characteristics of the above function for arrival and departure time distributions. They are explained in the following section.

When skew to the right is required, as in the distribution of inbound passenger arrival times, the coefficients of these equations take the following pattern:

$a_1 > 0,\ b_1 = 0,\ a_2 > 0$ and $b_2 > 0$

or

$a_1 > 0,\ b_1 \ge 0,\ a_2 = 0$ and $b_2 > 0$

or

$$a_1 > a_2 \text{ and } b_1 = b_2 \tag{10}$$

On the other hand, when skew to the left is required, as in the distribution of outbound passenger arrival times, the coefficients of these equations take the following pattern:

$a_1 > 0,\ b_1 > 0,\ a_2 > 0$ and $b_2 = 0$

or

$a_1 = 0,\ b_1 > 0,\ a_2 > 0$ and $b_2 \ge 0$

or

$$a_1 < a_2 \text{ and } b_1 = b_2 \tag{11}$$

Table 2 gives the range of skewness of the $R(t)$ function for different combinations of parameter values. The skewness is computed in the conventional manner by calculating the first, second, and third moments of the particular function. The table provides a useful guide for selection of the parameter values when the skewness expectations are known from field information.

Furthermore, the cumulative distribution, $A(t)$, can be developed by integrating the function $R(t)$.

$$A(t) = \int_0^T y_1(t) \cdot y_2(t)\, dt \qquad 0 \le t \le T \tag{12}$$

By expansion,

$$A(t) = a_1t^3(a_2\alpha_{n=5} - b_2\beta_{n=4}) + b_1t^2(a_2\alpha_{n=4} - b_2\beta_{n=3}) \tag{13}$$

where

$$\alpha_n = \frac{t^2}{n} - \frac{2Tt}{n-1} + \frac{T^2}{n-2}$$

and

$$\beta_n = \frac{t}{n} - \frac{T}{n-1}$$

Thus a relatively elegant cumulative curve is available by adopting the above model. For an inbound stream of passengers, $A(t)$ is typically the total number of passengers uplifted by the aircraft.

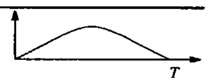
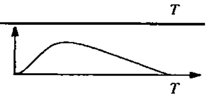
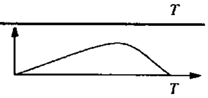
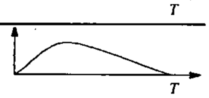
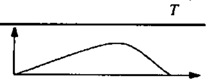
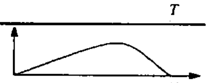
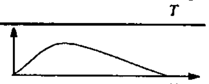
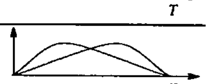
Through manipulation of the above equations, the average time $\bar{t}$ can be determined. This is shown below.

$$\bar{t} = \frac{T^2a_1(Ta_2 + 3b_2) + b_1T(2Ta_2 + 5b_2)}{Ta_1(2Ta_2 + 5b_2) + b_1(5Ta_2 + 10b_2)} \tag{14}$$

Parameters for the above equations were derived using a least-square regression method. A partial differential technique was used to implement the least-square regression. For enplaning passengers:

$$y_{1E}(t) = 9.42t^2 - 8.38t \tag{15}$$

TABLE 2 Upper and Lower Bounds of Skewness

Parameters	Range of Skewness (s_k) and Shape of $R(t)$	
$a_1 = a_2 = 0$	0 *symmetric*	
$b_1 = b_2 = 0$	0 *symmetric*	
$a_1 = 0$	$-\frac{2}{7} \le s_k \le 0$	
$a_2 = 0$	$0 \le s_k \le \frac{2}{7}$	
$b_1 = 0$	$-\frac{\sqrt{7}}{4\sqrt{2}} \le s_k \le 0$	
b_2	$0 \le s_k \le \frac{\sqrt{7}}{4\sqrt{2}}$	
$a_1 = b_2 = 0$	$0 \le s_k \le \frac{2}{7}$	
$a_2 = b_1 = 0$	$-\frac{2}{7} \le s_k \le 0$	
$a_1, a_2, b_1, b_2 \ne 0$	$-\frac{\sqrt{7}}{4\sqrt{2}} \le s_k \le \frac{\sqrt{7}}{4\sqrt{2}}$	

and

$$y_{2E}(t) = 0.0354 \times 10^{-6}(85 - t)^2 - 0.3900 \times 10^{-6}(85 - t) \tag{16}$$

Similarly, for deplaning passengers, the corresponding equations take the following form:

$$y_{1D}(t) = 0.12t^2 - 2.50t \tag{17}$$

$$y_{2D}(t) = 0.0641 \times 10^{-6}(223 - t)^2 - 0.0203 \times 10^{-6}(223 - t) \tag{18}$$

The graphical forms of the above models are shown in Figures 5 and 6. The proposed polynomial method appears to underestimate the peak values. Even the regular polynomial model performs poorly closer to peak demand but provides marginally better answers in that region. As expected, the proposed model provides fewer estimates of unrealistic negative values from peaks compared with the regular polynomial model.

BEHAVIORAL INTERPRETATION

It has been attempted to establish by using field data the model properties useful in applications. Model calibration using field data currently being collected at the Sydney International Airport were also provided.

The $R(t)$ function is a product of a decreasing function and a monotonically increasing function. In practical terms, these two quadratic functions may be likened to the increasing sense of urgency that encourages passengers to arrive early and a decreasing liberty to arrive after a specified time. The sense of urgency, $y_2(t)$, and degree of liberty, $y_1(t)$, are combined to provide the arrival distribution, $R(t)$. This method of incorporating passenger concerns in traffic flow distribution is a novel concept developed during this research project.

Similarly, for deplaning passengers, the function $y_1(t)$ corresponds to the increasing sense of urgency to leave the terminal before a given time. Function $y_2(t)$ can be interpreted

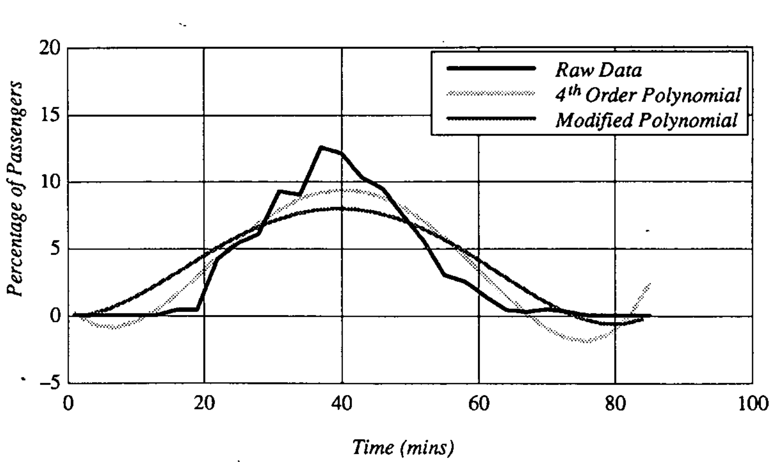


FIGURE 5 Comparison of enplanement models.

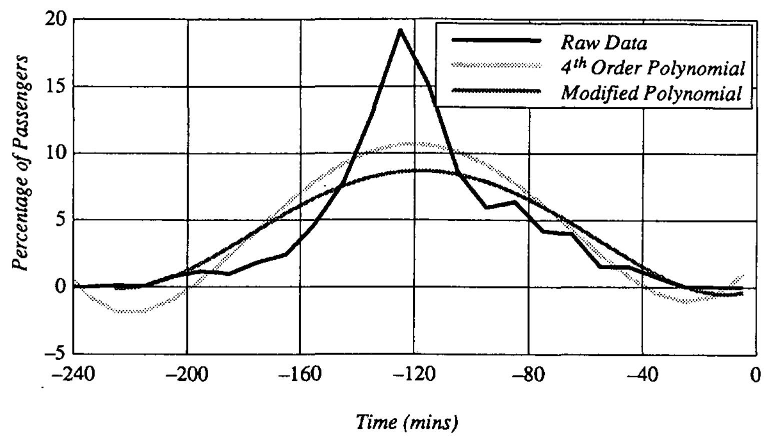


FIGURE 6 Comparison of deplanement models.

as the decreasing amount of liberty to leave the airport after the specified time.

The form of the functions $y_1(t)$ and $y_2(t)$ is assumed to be quadratic at this stage. It is possible to achieve the desired skewness with linear formulations for $y_1(t)$ and $y_2(t)$ but with little degree of freedom to manipulate curve fitting applications. On the other hand, adopting formulations of higher than second order may only introduce unnecessary computational complications with little improvements in accuracy.

CONCLUSIONS

It has been shown that the passenger flow distribution at modal interchanges, such as airports, is important for terminal sizing computations and estimation of access road traffic conditions. The terminal sizing problem is solved by applying queueing theory, level of service concept, and passenger flow distribution. On the other hand, it is seen that improved estimates of access road traffic conditions are available by analyzing the passenger flow distribution resulting from the traffic transformation effect of the modal interchange.

The passenger flow distribution, in terms of the arrival and departure times of passengers relative to the relevant aircraft movement, is presented in some literature only in a descriptive manner. This paper provides a review of passenger flow distribution models and identifies some modeling options overlooked by previous researchers. It is shown that the proposed analytical model can include the behavioral response of passengers toward the system characteristics.

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