TRANSPORTATION RESEARCH
RECORD

No. 1428

Aviation

Public-Sector Aviation Issues
Graduate Research Award Papers 1992–1993

A peer-reviewed publication of the Transportation Research Board

TRANSPORTATION RESEARCH BOARD
NATIONAL RESEARCH COUNCIL

NATIONAL ACADEMY PRESS
WASHINGTON, D.C. 1994

Transportation Research Record 1428
ISSN 0361-1981
ISBN 0-309-05506-7
Price: $19.00

Subscriber Category
V aviation

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Foreword

The papers in this volume are reports on research topics chosen by graduate students selected for awards from a nationwide competition under the Seventh Graduate Research Award Program on Public-Sector Aviation Issues (1992–1993). The program is sponsored by the Federal Aviation Administration and administered by the Transportation Research Board. Its purpose is to stimulate thought, discussion, and research by those who may become managers and policy makers in aviation. The papers were presented at the 73rd Annual Meeting of TRB in January 1994. The authors, their university affiliations, faculty research advisors, and TRB monitors are as follows.

Leola B. Ross, a Ph.D. candidate in economics at Southern Methodist University, made a dynamic analysis of oligopolistic behavior in the domestic airline industry. Her faculty research advisor was Kathy J. Hayes of the Department of Economics, Southern Methodist University. TRB monitors were Francis P. Mulvey of the U.S. General Accounting Office and John W. Fischer of the Congressional Research Service, Library of Congress.

Ila Semenick, a Ph.D. candidate in economics at Rice University, analyzed domestic airline technical efficiency scores and their implications for future industry structure. Her faculty advisor was Robin C. Sickles of the Department of Economics, Rice University. TRB monitors were Gerald S. McDougall of Southeast Missouri State University and J. Bruce McClelland of Dornier Aviation (North America, Inc.).

Guy M. Smith, a Ph.D. candidate in education at Montana State University, evaluated self-analysis as a strategy for learning crew resource management in undergraduate flight training. His faculty advisor was John W. Kohl, Department of Education, Montana State University. TRB monitors were Lemoine V. Dickinson, Jr., of Failure Analysis Associates, Inc., and Richard F. Pain of TRB.

James R. Valentine, a Ph.D. candidate in mechanical engineering at the University of Utah, studied airfoil performance in heavy rain. His faculty advisor was Rand Decker, Civil Engineering Department, the University of Utah. TRB monitors were Hubert C. Smith of the Pennsylvania State University and C. W. Kauffman of the University of Michigan.

Dynamic Analysis of Oligopolistic Behavior in the U.S. Airline Industry

LEOLA B. ROSS

The recent history of the airline industry has exhibited relentless price wars of national proportion begun by failing airlines desperate to fill their planes. However, price reductions and sporadic discounting are often observed intermittently on scattered routes from time to time. If substantial discounts are offered, these episodes may also be classified as less publicized (or covert) price wars. An arbitrary rule is described that classifies the most traveled routes between the second quarter of 1990 and the third quarter of 1992 as experiencing or not experiencing a price war on the basis of distribution of prices. The classification scheme is helpful in characterizing market behavior during price wars and normal periods. The causes and effects of price wars are assessed, and special attention is given to the relationship between price wars and concentration. The analyses are conducted in the context of an economic theory that depicts price wars as a normal reaction to changing market conditions when a specific type of equilibrium characterizes an industry. The most profound result is that price wars do not increase market concentration as successfully as more cautious price reductions taken during normal periods.

The recent history of domestic airlines has been marked by mergers, takeovers, failed airlines, volatile ticket prices, and price wars. Whereas the airline industry is among the most studied in the past decade, domestic airline price wars have not been the central focus of economic research. Examination of these price wars during the early 1990s is timely and significant with regard to both the economic literature and the political arena.

Past economic studies focused largely on static models aimed at describing airline industry behavior at a point in time. For example, Borenstein (*1*) links airport dominance and route concentration to high fares and argues that increased concentration of this nature should lead to even higher fares. The General Accounting Office (*2*) published a similar, more detailed static model seeking to capture the effects of certain barriers to entry, market share, and congestion on airfares. It found that a single variable does not a have a large effect on prices but that in combination the factors studied can significantly increase airfares. That study enjoyed the contribution of a tremendous amount of data, which enriched the explanatory power of the results substantially. A recent study, which was more parsimonious in its use of data, was done by Evans and Kessides (*3*). They found evidence that airport concentration was a strong determinant of fares on a given route. They further concluded that for the quarter they studied route dominance was relatively unimportant in explaining higher prices. The contrasting results of these studies affirm the need for a dynamically based model to explain pricing behavior. In a later section of the paper, reference is made to a Chow test of structural change across time periods. This test confirms that pricing behavior has not been the same across time, which suggests a possible explanation for differing results in previous papers.

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Economic studies focusing on the evolving nature of the airline industry are less numerous than single-period studies. Morrison and Winston (*4*) study entry and exit patterns as affected by hubbing and route fares. They find that airlines tend to shy away from airports where other airlines have hubs because of limited gates. They, like Evans and Kessides, find a strong correlation between airport concentration and high prices. However, they predict that hubbing should eventually decrease fares, since hubs allow increased airline connectedness and contact with competitors so that airlines should be able to compete with each other more effectively. Kim and Singal (*5*) examined the dynamic nature of prices during the merger wave of the mid-1980s. They identify the price changes on routes affected by specific mergers, compare them with price changes on routes unaffected by those mergers, and find that the elimination of the noncooperative failing airline allows the remaining airlines to collude more successfully. Furthermore, they suggest that multimarket contact helps airlines maintain a less-than-competitive arrangement and that the competition observed shortly after deregulation is less likely under the evolving market structure. However, since 1988, the airlines seem to have entered a new era of short-term price wars and collusive periods, in contrast to the predictions of Kim and Singal. Why has the stability they predicted broken down? Or does this recent trend actually reflect a different kind of equilibrium that has until now not been considered?

The model described here will show that pricing behavior varies not only over time but over routes. The causes and effects of price wars are modeled and evaluated to demonstrate that the airlines reflect both competitive and collusive behavior at various times and on various routes. It is shown that, regardless of hub and route dominance, lagging demand can trigger destructive competition and that certain types of routes will be more prone to price wars than others. Furthermore, there are clear winners and losers from price wars, and the toughest battles are fought on the routes with the most at stake.

PRICE WAR EQUILIBRIUM

A growing theoretical literature has been devoted to explaining the dynamic nature of imperfect markets. It has been recognized since Stigler (*6*) that the static models of collusive cartel, Cournot-Nash equilibrium, or Bertrand competition do not sufficiently explain the behavior of firms existing in such markets. Whereas we know that a cartel is an unstable arrangement at best, empirically we observe that highly concentrated industries are likely to behave like any one of these classic models (including cartel) at some time. In the past decade, game theorists have developed dynamic models to portray more realistically the actions of oligopolists who learn from the past and plan for an uncertain future. The Green and Porter model (*7*) is particularly applicable to recent airline behavior, since it describes an oligopoly that goes through periods of sustained collusion and intense competition. They describe a "Nash equilibrium" (the most profitable choice for a firm, given the most likely reactions of its competitors) of strategies that determine a firm's behavior over an infinite time horizon. In their model each firm will price at a normal (or collusive) level unless sales drop too low. If this happens, the firms will assume that some other firm is cheating (or discounting too much) and will respond by dropping prices to punish the cheating firm for some time. Thus, the dynamics of the industry will be characterized by firms bouncing back and forth between normal behavior and price wars. Green and Porter point out that a drop in sales need not be the result of a cheating party; it could be caused by a drop in consumer demand or some other factor. Thus, the price wars recently exhibited by the airlines could be based on the pricing practices of various airlines (perhaps, for example, the value pricing scheme of American Airlines) or simply a shrinking consumer demand for travel.

The primary difference between the Green and Porter model and the structure of the airline industry is the multimarket nature of the airlines. Recall that Kim and Singal suggested that such multimarket contact should allow the airlines to maintain collusive behavior without the threat of excessive competition, whereas Morrison and Winston indicated that this multimarket countact should, in fact, increase the competitiveness of the airlines. Adding multimarket contact to the Green and Porter model complicates matters somewhat. If an airline lowers prices on one route, what is to prevent the other airlines from abandoning that route altogether and lowering prices on some other route where it has a comparative advantage? Such behavior would lead to market segmentation, and then both firms would emerge as monopolists (or at least dominant carriers) in their respective markets. Casual empiricism suggests that this does not frequently occur, or does it? Southwest Airlines has successfully carved a niche by forcing other carriers to lower prices substantially or drastically reduce service on Southwest's routes. Whereas it is not clear that every airline could be a Southwest, it is curious that more have not tried to copy the success of their most profitable adversary.

A rigorous examination of the Green and Porter model, extended to multiple markets, reveals that though each airline could be more profitable as a monopolist, the lure of invading other routes may be too strong for a segregated market to be sustained. This is true if the markets still show some evidence of contestability in this industry and the only defense against an invasion by a competitor is limit pricing. Therefore, once a price war erupts, a spillover into another market only serves to extend the price war rather than to segregate the market or drive out competitors. The resulting equilibrium (not unlike the one described above) is a sequence of normal prices occasionally interrupted by an industrywide price war.

Since this theory predicts that price wars are not likely to disappear, where they are likely to occur and how they affect market structure are important issues in developing public policy or assessing market performance.

FREQUENCY ANALYSIS

The previously described theoretical model does not indicate whether price wars should lead to increased or decreased market concentration. It might seem counterintuitive that a price war would leave a market in the same condition in which it began. If no airline gains customers at the expense of a competitor, then one might question the rationality of starting a price war. To motivate the empirical model and to make explicit the effects of a price war on market concentration, consider a simple comparison of changes in concentration in price wars and in "normal" periods.

To analyze the frequency of anything concerning price wars, one must first define a price war. A price war may be characterized by public announcements by the airlines and newspaper headlines, or they may be more obscure. In fact, a price war may occur on only one or two routes for several time periods or half of the domestic routes for a single time period. With this in mind, the nature of the distribution of prices for a particular route should be analyzed to confirm or deny that a price war is occurring. Unfortunately, to ask this of the data, a "rule" must be imposed as to the inclusion or exclusion of a particular route at a particular time, and this rule will be unavoidably arbitrary. The rule chosen is as follows:

1. Calculate, by route and date, the maximum price charged and divide by 5 to determine the percentage of tickets sold below 20 percent of the maximum price.
2. Compare this percentage in each period with the percentage in the previous period to determine the percent change in the percentage.
3. Conclude that a price war is in effect if the percent change is more than 25 percent.
4. Conclude that the price war is still in effect if there was a price war last period and the percent change this period does not "substantially" change (does not decrease any more than 10 percent).
5. Call the period "normal" if a price war is not in effect by the preceding two steps.

To measure route concentration, the Theil coefficient (an entropy-based measure), TC = $\Sigma_a s_a \ln s_a$, is calculated for each route at each time period in the sample (where the market share of Airline a on a route is s_a). TCs were calculated with all the data for the 100 most traveled airports. The measure more commonly used for industry case studies is the Herfindahl index, HI = $\Sigma_a s_a^2$, since it is more widely known and understood. Both of these indices possess some properties relating them to economic theory. For example, HI may be linked to a firm's ability to price above marginal cost in a particular setting, and TC may be used to draw some conclusions about the detachment of upper management from the actual production process [a more detailed description of these properties is given by Hannah and Kay (*8*, p. 27)]. However, both of these relationships are shaky at best, and neither lends itself to a reduced form regression. From a practical perspective, the difference between the two measures is that HI places most of its weight on the largest firm share on the route or airport or industry in question, whereas TC places its emphasis on the dispersion of the firm's respective shares. [For a description of these properties see Theil (*9*).]

TC is distinguished from an arbitrary index such as the Herfindahl by virtue of its origins in statistics and information theory [for a discussion of these origins see Slottje (*10*, pp. 63–66)]. The entropy class of indices measures the deviation of a particular distribution (in this case the distribution of firm shares) from a hypothesized null distribution (in the case of concentration the implied null is a symmetric market). If market shares are insignificantly different from the null distribution, TC will be distributed χ^2 with number of firms less two degrees of freedom (*11*). Hence, divergence of a TC from its null is governed by a well-known distribution, so that statistical inference is possible and its usefulness is maximized. Further, Hayes and Ross (*12*) show that these properties may also be used to construct a directed divergence statistic for conducting inference test of the similarities of concentration among routes and time periods.

The calculated frequency of increased concentration during price wars and normal periods is given in Table 1. In the first column, restrictions are placed on the percent increase in concentration. We begin by considering the event of any increase at all during and after price wars and find that an increase occurs in or out of a price war with almost equal probability. As we tighten our definition (requiring from a 5 to a 50 percent increase in concentration), it is clear that the occurrence of a price war does not increase the probability of large changes in concentration either but exhibits a consistent difference. Clearly, this analysis is based on our arbitrary price war rule, and we have no confidence intervals to substantiate these conclusions. However, the similarities in percentages between price wars and normal periods appear to be robust to the percent increase in concentration. These similarities can be easily observed in Figures 1 and 2.

DATA

The data used for the construction of this frequency analysis and the model to follow have been extracted from two data banks maintained by the Department of Transportation—the *Origin and Destination Survey* (Data Bank 1A or DB1A) and the *T100 Do-*

TABLE 1 Frequency Analysis of Price Wars and Increases in Concentration by Route

	Frequency of Increased Concentration (this periods behavior)		Frequency of Increased Concentration (last periods behavior)	
% Increase in Concentration	During Price War (%)[a]	Normal Period (%)[b]	After Price War (%)[a]	Normal Period (%)[b]
0	3039	3724	2336	4427
	(53.86)	(47.48)	(49.97)	(50.25)
5	2159	2555	1611	3103
	(38.27)	(32.57)	(34.46)	(35.22)
10	1610	1812	1193	2229
	(28.54)	(23.10)	(25.52)	(25.30)
15	1217	1361	894	1684
	(21.57)	(17.35)	(19.12)	(19.11)
22	992	1080	714	1358
	(17.58)	(13.77)	(15.27)	(15.41)
25	817	887	583	1121
	(14.48)	(11.31)	(12.47)	(12.72)
50	382	404	234	552
	(6.77)	(5.15)	(5.01)	(6.27)

[a] Percent based on routes exhibiting a price war.
[b] Percent based on routes not exhibiting a price war.

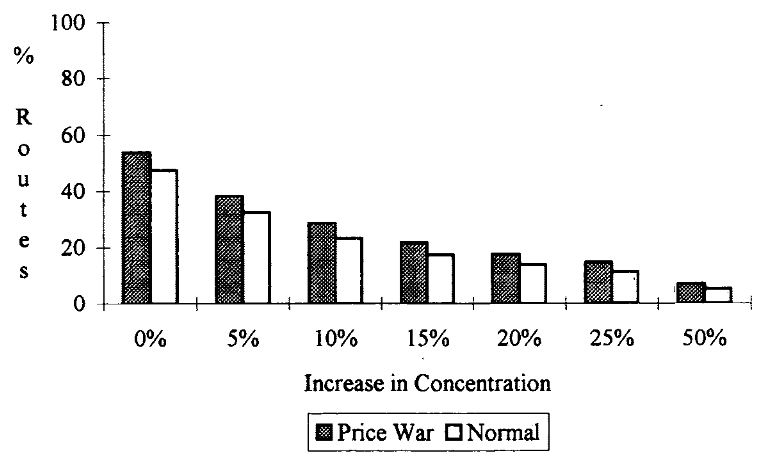

FIGURE 1 Frequency of increased concentration during a price war.

mestic Segment Data (Data Bank 28DS or T100). These data banks are available from the Volpe National Transportation Systems Center in Cambridge, Massachusetts, or from the National Archives in Washington, D.C., for older data. The DB1A is a random 10 percent survey of all tickets issued for flights within the United States and is published on a quarterly basis. The T100 contains data reported by U.S. carriers operating nonstop service within the United States and is published monthly. The following types of tickets are removed from the sample:

1. Any ticket with one or more segments of first-class travel,
2. Any tickets that are not either one-way or round-trip,
3. Any tickets with more than one change of plane per direction of travel,
4. Tickets with any origin or destination outside the United States,
5. Interline tickets (tickets where services are provided by more than one carrier), and
6. Any tickets that were less than \$10 or more than \$750 each way (or \$20 and \$1,500 round-trip, respectively) (these are assumed to be frequent flier tickets, chartered flights, or input errors).

There are 1,226 routes selected from these two data sets to use for these analyses. These are the only routes that are present in both data sets for all the time periods among the top 100 airports in the United States and represent roughly 30 percent of all tickets in the DB1A. The use of the T100 somewhat restricts the choice set of routes since it is a segment-based data source. For an observation to occur on T100 there must be a nonstop flight between the endpoints. The use of the hub-and-spoke system by most major carriers has reduced the number of airports having nonstop flights between them. Thus, to ensure a balanced panel of routes, the data set is reduced. Conversely, the DB1A has observations on almost any combination of segments imaginable between various endpoints. If the statistical tests were restricted to variables extracted from DB1A, the number of routes in the sample would be considerably larger. However, information such as number of flights scheduled and load factor is only available from the T100. These variables enlarge the set of independent variables and should not be ignored when analyzing pricing behavior. However, inconsistencies in the interpretation of the variables extracted from these data sets may arise, given their differences.

The most recent 11 quarters of data were used for the analysis (1990:1 through 1992:3). The price equation below was estimated by route pair and time. (A route pair is listed in alphabetical order such that, for example, flight DFW-LGA is the same as LGA-DFW and is called DFWLGA. This is common in the literature and is necessary to prevent duplication of observations in light of the high percentage of round-trip tickets purchased.)

EMPIRICAL MODEL

The frequency analysis gave insight into the effects of price wars on some minimum change in concentration. To more thoroughly examine the influence of route characteristics on market behavior, a system of equations describing changes in the bottom quintile of prices, changes in concentration, and the absolute price level is defined. All three of these may be determined by the characteristics of the route and market. Price levels are defined by a reduced form of demand and supply conditions. Price changes are often responses to slackening demand or the behavior of competitors (both are suggested by our theory). Finally, route concentration changes may be the result of price changes, shifts in consumer demand, or the concentration of the endpoints of travel. Therefore, the following system of equations is suggested:

$$\begin{aligned}\text{PERCHANG}_{it} = \beta_{10} &+ \beta_{11}\text{LNSCHED}_{it} + \beta_{12}\text{LNPRICE}_{it}\\ &+ \beta_{13}\text{ROUTHEIL}_{it} + \beta_{14}\text{APTHEIL}_{it}\\ &+ \beta_{15}\text{LFLAG}_{it}\end{aligned} \quad (1)$$

$$\begin{aligned}\text{ROUCHANG}_{it} = \beta_{16} &+ \beta_{17}\text{PERCHANG}_{it} + \beta_{18}\text{APTHEIL}_{it}\\ &+ \beta_{19}\text{LFLAG}_{it} + \beta_{21}\text{LNPASS}_{it}\\ &+ \beta_{22}\text{LNSCHED}_{it}\end{aligned} \quad (2)$$

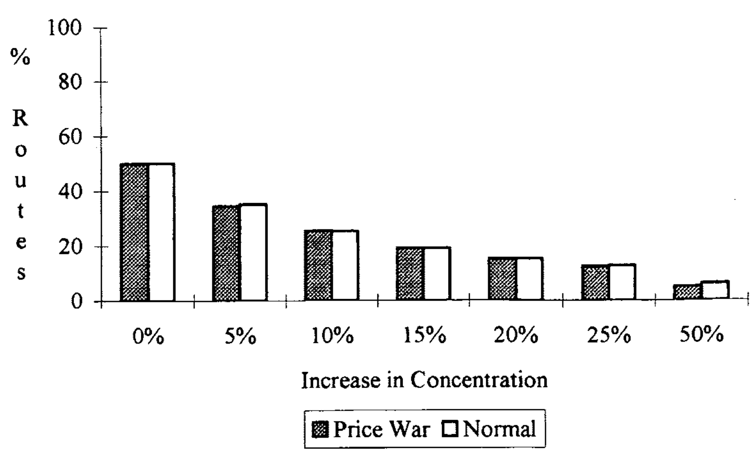

FIGURE 2 Frequency of increased concentration after a price war.

$$\text{LNPRICE}_{it} = \beta_{23} + \beta_{24}\text{LNPASS}_{it} + \beta_{25}\text{PERSTOP}_{it} + \beta_{26}\text{PERROUND}_{it} + \beta_{27}\ \text{ROUTHEIL}_{it} + \beta_{28}\text{APTHEIL}_{it} + \beta_{29}\text{LOADF}_{it} + \beta_{30}\text{LNDIST}_{i} \quad (3)$$

The endogenous variables are as follows:

- PERCHANG$_{it}$ = (PERCENT$_{it}$ − PERCENT$_{it-1}$)/PERCENT$_{it-1}$, where PERCENT$_{it}$ is the percentage of tickets sold at 20 percent or less than the maximum price, is a measure of price volatility and is expected to increase ROUCHANG in price wars and normal periods (source DB1A).
- ROUCHANG$_{it}$ = (ROUTHEIL$_{it}$ − ROUTHEIL$_{it-1}$)/ROUTHEIL$_{it-1}$ is a measure of market structure volatility (ROUTHEIL is defined later) (source DB1A).
- LNPRICE$_{it}$ is the natural logarithm of the average price of a ticket on route *i* at time *t*. The predicted effect of this variable on PERCHANG is positive in normal periods and negative during price wars (source DB1A). (Since all variables henceforth, except LNDIST, are indexed over route and time, the subscripts will be dropped from the following descriptions.)

The exogenous variables are as follows:

- LNSCHED is the natural logarithm of the total number of nonstop flights scheduled for a route. This variable is related to the frequency of fights and thus reflects the possibility that a route may be rather competitive and, thus, exhibit more activity over time. It is expected to increase both PERCHANG and ROUCHANG during normal periods and price wars (source T100).
- ROUTHEIL is the Theil concentration index of the route traveled, $\Sigma_a\ s_a \ln s_a$, where s_a is the proportion of passengers Airline *a* serves on the route. This is a measure of route concentration and is expected to positively affect LNPRICE and decrease PERCHANG at all times (source DB1A).
- APTHEIL = (AP1THEIL + AP2THEIL)/2, where AP1THEIL is the Theil concentration index of the airport first listed in the route pair and AP2THEIL is the Theil concentration index of the airport listed second in the route pair, measures concentration and is expected to increase LNPRICE and ROUCHANG and decrease PERCHANG at all times (source DB1A).
- LFLAG = LOADF$_{t-1}$, where LOADF (load factor) is the percentage of available seats occupied on nonstop flights. This variable reflects both past and current demand. The lagged load factor is expected to be instrumental in stirring activity when planes are empty, thus decreasing both PERCHANG and ROUCHANG in normal periods, but it might have an opposite effect on ROUCHANG during price wars. The effect of the current load factor on LNPRICE should be negative during normal periods and price wars (source T100).
- LNPASS is the natural logarithm of the total number of passengers in the sample flying the route. This is an indicator of highly established routes thus decreasing ROUCHANG; newer, less-traveled routes are likely to be more contestable. However, since it might also imply economies of scale, it might decrease LNPRICE (source DB1A).
- PERSTOP is the percentage of passengers experiencing a change of planes. This indicates a route that is starting or ending at a nonhub airport and, thus, is expected to increase costs and LNPRICE as well (source DB1A).
- PERROUND is the percentage of passengers flying round-trip on a route. A large percentage of round-trip tickets might imply more pleasure travel as opposed to business travel and a more elastic price resulting in lower LNPRICE at all times (source DB1A).
- LNDIST is the natural logarithm of the great circle distance in official statute miles between the origin and destination of airports. Greater distance is expected to increase both costs and LNPRICE for both models (source T100).

A monotonic logarithmic transformation of the variables (such as distance, total passengers, average price, and number of scheduled flights) with magnitudes out of line with the other variables is taken.

There exists explicit simultaneity in the system of equations, and therefore three-stage least squares estimation is appropriate. The time-series nature of the data is ignored in the error structure for three reasons. Since the cross section (1226) is far greater than the time series (10 after lagging some variables), it is likely that the pooled sample closely resembles a cross-sectional data set. Whereas it is possible that some autocorrelation exists in the error structure of the LNPRICE equation, as prices are expected to have some inertia, it is doubtful that this is a problem in the first two equations, since a change in concentration or a price war in this period does not imply similar behavior next period. And finally, as will be explained shortly, the data set is split into two subsets that are independent of time and route. Therefore, to draw comparisons between the entire data set and the two subsets, one regression technique must be used, and it is not possible to treat the two subsets as panel data when they are completely unbalanced. The balanced nature of the original data set, however, is essential for determining the values of lagged variables (PERCHANG, ROUCHANG, and LFLAG) and is instrumental in assessing the importance of dynamic change in the market structure.

RESULTS

The system of equations from the previous section was estimated three times. Initially, the model was estimated allowing for no variation in parameters across routes or periods. This is referred to as the combined model. Next, the data set was segregated by time period so that a general test for time-invariant behavior could be conducted. It is clear that this is not the case. Therefore, the conflicting results of previous studies, which where discussed earlier, may be partially explained by differences in the time periods used by the authors. Since this model was only estimated to demonstrate this point and is not the focus of this research, the parameter estimates are not reported. However, the by-equation Chow test results are reported in Table 2. (This test for structural change is done by-equation since there is no Chow test defined for a system of equations. The residuals used for these tests are from the two-stage least squares step of the three-stage least squares procedure.)

In the final estimation the pooled data set was separated into two categories: price war and normal. This separation disregards the panel nature of the data set since each route/period observation is categorized by the price war rule described in an earlier section of this paper. The regression results for these two models are reported with the combined model in Table 3. Again, a Chow test was conducted confirming that separating the data in this way

TABLE 2 Results of Chow Tests of Structural Change

	H_0: Time Invariant Structure F-Statistic (DF-num, DR-den, Critial Value)	H_0: Price War Invariant Structure F-Statistic (DF-num, DR-den, Critial Value)
Equation (1)	4.48	119
	(54, 12200, 1.32)	(6, 12248, 2.10)
Equation (2)	6.65	4.64
	(54, 12200, 1.32)	(6, 12248, 2.10)
Equation (3)	43.38	52.88
	(72, 12180 ,1.22)	(8, 12244, 1.94)

significantly improves the fit of the model. The *F*-statistics are also reported in Table 2. Since it is shown that the combined model is incorrect, a discussion of the results is unnecessary. They are reported so that one can observe how a model ignoring the effects of price wars can give results contrary to the segregated models.

The results of the normal period and price war models are often conflicting and for some variables are counterintuitive. First, consider the PERCHANG equation. LNSCHED leads to increases in price volatility during normal periods, thus indicating a push toward price wars. However, during a price war LNSCHED takes the opposite sign, indicating that price wars on frequently departing flights may be less severe. ROUTHEIL and APTHEIL are associated with decreasing PERCHANG in normal periods, indicating an ability to sustain prices more effectively when concentration is higher. Conversely, the positive coefficients during price wars indicate that if a price war breaks out it will be more severe. Perhaps this is an indication that these routes are contestable; this

TABLE 3 Three-Stage Least Squares Estimation Results

	Expected (PW,N)	During a Price War	Normal Period	Combined Model
(1) PERCHANG				
INTERCEPT		40.99^b	0.85^b	16.94
LNSCHED		-0.30^a	0.02^b	-0.21^b
LNPRICE		-4.26^a	-0.33^b	-1.47^b
ROUTHEIL		0.77	-0.11^b	0.48^a
APTHEIL		6.71^b	-0.14^b	3.07^b
LFLAG		-4.89^b	0.08^b	-2.27^b
(2) ROUCHANG				
INTERCEPT		2.36^b	1.43^b	0.29
PERCHANG		-0.06^b	0.51^a	0.09^b
APTHEIL		0.39^a	0.27^b	-0.22
LFLAG		-0.35	0.17	0.39^a
LNPASS		-0.23^b	-0.13^b	-0.17^b
LNSCHED		0.04	0.002	0.04^b
(3) LNPRICE				
INTERCEPT		4.68^b	4.63^b	4.62^b
LNPASS		-0.05^b	-0.11^b	-0.09^b
PERSTOP		0.11^b	0.03^b	0.19^b
PERROUND		0.07^a	0.44^b	0.32^b
ROUTHEIL		-0.12^b	-0.19^b	-0.16^b
APTHEIL		0.11^b	0.19^b	0.17^b
LOADF		-0.16^b	-0.61^b	-0.43^b
LNDIST		0.13^b	0.20^b	0.18^b
Cross Model Correlation				
(1) & (2)		0.245	-0.098	-0.343
(1) & (3)		-0.014	0.233	-0.233
(2) & (3)		-0.050	-0.019	-0.019

[a]Significant at the 5% level.
[b]Significant at the 1% level.

is consistent with the theory described above. The positive coefficient on LFLAG is counterintuitive because it suggests that fuller planes exhibit more discounted fares during normal periods. However, we intuitively observe that empty planes increase the severity of a price war should it erupt. Again, this is consistent with the theory in the second section. LNSCHED and LFLAG exhibit similar effects on ROUCHANG.

The negative coefficient on PERCHANG in the ROUCHANG equation is curious. This seems to indicate that price wars slightly increased market shares for smaller airlines at the expense of the larger airline, creating a more symmetric market. The positive coefficient during normal periods suggests that covertly discounting some fares without starting a price war gives more market share to the larger airlines. Put simply, price wars do not increase market concentration. This is consistent with the simple frequency analysis described earlier, but the relationship is not revealed so explicitly. These results suggest that if a relatively small airline tries to increase its market share by starting a price war, it may have some minimal success, and these price wars may be a useful market mechanism for keeping the dominant carriers in check.

The most surprising results, from an economic perspective, are in the LNPRICE equation. For example, PERROUND was expected to decrease prices because of pleasure travel. However, we show that an increase is actually the case. Perhaps one-way tickets are dominated by lower-priced commuter flights (consider the New York, Boston, Washington shuttles as an example). Further, the negative impact of ROUTHEIL on prices seems unusual. Is it possible that this is an indication of limit pricing on highly concentrated routes? This explanation is consistent with the theory given earlier.

As a whole, the results of this regression are informative. Factors that reduce discounting in normal periods imply increased intensity when price wars occur. Similarly, factors that increase discounting in normal periods imply less intense price wars. It is also apparent that small, covert reductions in price during normal periods will increase route shares for the larger airlines better than rapid changes that set off price wars, whereas smaller carriers can gain some market share during price wars.

CONCLUSIONS

The use of a system of simultaneous equations is particularly instructive in evaluating the causes and effects of volatile prices in the airline industry. We have confirmed that what pushes a route into a price war, such as frequently departing flights that fill up quickly, may also act to reduce the severity of a price war. Characteristics that reduce normal-period price discounting, such as dominance of routes and at airports, may intensify a price war. Most significant, the advantages of covert price reductions by larger airlines are affirmed by the changing sign of PERCHANG when regressed on ROUCHANG. This demonstrates that the incentive to cheat in normal periods is very strong for small airlines seeking to improve their market share.

Many aspects of economic theory have been affirmed, indicating that perhaps the airlines are, in fact, in an equilibrium that is characterized by both collusive and competitive behavior. The suggestion that the multimarket contact can successfully reduce competition is as correct as the alternative. In this sense, past research that focused on dynamic change in the airlines is certainly superior to static reduced-form models that ignore the importance of change in this industry. However, one must appeal to economic theory to successfully interpret empirical results. The results of this research indicate that price wars are likely to occur for some time and that market concentration may go up or down on the basis of the frequency of these price wars and the ability of the airlines to stay in the game. Since this industry is so sensitive to demand conditions, price wars may become less common if consumer demand becomes stronger. If demand improves, the traveling population will look forward to higher fares and a less competitive market.

ACKNOWLEDGMENTS

The author expresses gratitude to FAA for its sponsorship of the Graduate Research Awards Program and to Southern Methodist University for its support. The author is indebted to Herman Beriens, Joseph Hirshberg, Esfandier Maasoumi, Daniel Slottje, and especially Kathy Hayes of Southern Methodist University and to John Fischer, Larry Jenney, and Francis Mulvey of the Transportation Research Board for many helpful comments and suggestions. Any errors are the responsibility of the author.

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Movements of Domestic Airline Technical Efficiency Scores over Time: Implications for Future Industry Structure

ILA SEMENICK

The volatile nature of the domestic airline industry has received much attention since deregulation in 1978. The large number of failures, mergers, bankruptcy filings, and operating loss reports has raised concerns that the future is bleak in terms of the number of carriers that will survive and prosper. Economic theory suggests that it is vital for firms to operate efficiently to compete. To avoid falling behind competitors, firms need to imitate any advances in efficiency-enhancing technology made by others in the industry. A panel data set of 11 domestic airlines, followed quarterly from 1970 to 1990, and three methods that are currently being pursued in the efficiency-measurement literature are used to explore the movements of technical efficiency over time in the industry. The analysis indicates that the efficiency scores of the carriers in the sample exhibit long-term relationships and move closer together over time.

The past two decades have proven highly disruptive to the American airline industry. The impetus behind the trend toward a more concentrated market structure was economic deregulation of the airline industry in 1978 by the Civil Aeronautics Board (CAB). Under regulation, firms had an incentive to select an inefficient combination of inputs, since the only means of competition was through service, which often meant "too many planes on too many routes" (*1*). It was widely believed that once the barriers to entry instituted by regulation were removed, such distortions would be eliminated and industry performance and efficiency would improve. Thus, whereas overcapitalization may have been the correct decision during regulation, on deregulation carriers found themselves with fleet configurations and labor commitments that were no longer optimal and had to be modified because of the intensified level of competition. The structure of fares, quality of service, and pace of modernization of airline capital have, therefore, changed dramatically.

Furthermore, substantial changes are likely to continue during the coming decade. One powerful force that will propel further change is the enormous growth of demand for airline services. Airlines have become a vital component of the world travel industry with passenger travel doubling since the U.S. airline industry was deregulated. A second stimulus to change is the increasingly competitive international market. For example, the prospect of integration of the European Community will remove current economic barriers in Europe; this deregulation will affect the airline industry and lead to the negotiation of new international agreements and the possibility of trans-Atlantic mergers.

In the light of these domestic and international challenges, the ability of U.S. carriers to operate efficiently is critical to their prospects for prosperity or survival. The goals of this research are to evaluate the performance of each domestic airline in the sample over time using alternative measures of relative technical efficiency and to use these measures to address the question of future industry composition. The identification of efficiency differentials among American carriers provides a means of ranking the airlines relative to one another through time. The ranking provides a way of ascertaining which carriers may be headed for trouble. In addition, at the managerial level, these measures indicate the success, or lack thereof, in performance enhancement. At the industry level, the hypotheses of cointegration and convergence of these efficiency scores over time can be tested to predict future industry movements.

Cointegration occurs when two variables do not move too far from one another although individually they move unpredictably through time. Convergence occurs when two variables move closer together over time. Theory suggests that the time series of efficiency scores for the airlines should move together (cointegration) or closer together (convergence) as technological advances become diffused throughout the industry. This argument is based on the assumption that the efficiency advances made by one carrier can be adopted by another; namely, improved technology is a public good available to any firm wishing to use it. Failure to exhibit cointegration or convergence would be indicative of a firm's inability to capitalize on this public good. Rigorous identification of the underlying reasons for differences in efficiency and the presence or absence of cointegration-convergence between carriers is the subject of future research.

DATA

The original Good-Sickles data set has been updated and consists of quarterly observations of 11 domestic carriers from 1970 to 1990 with a Department of Transportation (DOT) Group III classification. This category consists of certified carriers with the largest total annual operating revenues. Smaller carriers are categorized as Group I or Group II. The sample includes American (AA), Continental (CO), Delta (DL), Eastern (EA), Frontier (FL), Ozark (OZ), Piedmont (PI), Trans World Airlines (TW), United (UA), USAir (US), and Western (WA). The primary source of the data is the CAB Form 41 reports. The DOT's reporting requirements are extensive, and as of 1970 the data are rigorously audited to maintain a high degree of accuracy. Form 41 is therefore a rich and definitive source of data for industry analysis.

The input and output accounts of the Form 41 schedules were aggregated into four broad input indices and one output index (*2*).

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The input indices are capital (K, the number of aircraft), labor (L, an aggregate of pilots, flight attendants, mechanics, passenger and aircraft handlers, and other labor), energy (F, gallons of aircraft fuel), and a residual designating materials (M, which includes items such as advertising, supplies, outside services, passenger food, and maintenance materials). The aggregate output variable available for use is the quantity of revenue output (RTM, revenue ton-miles, which includes both passenger and cargo operations).

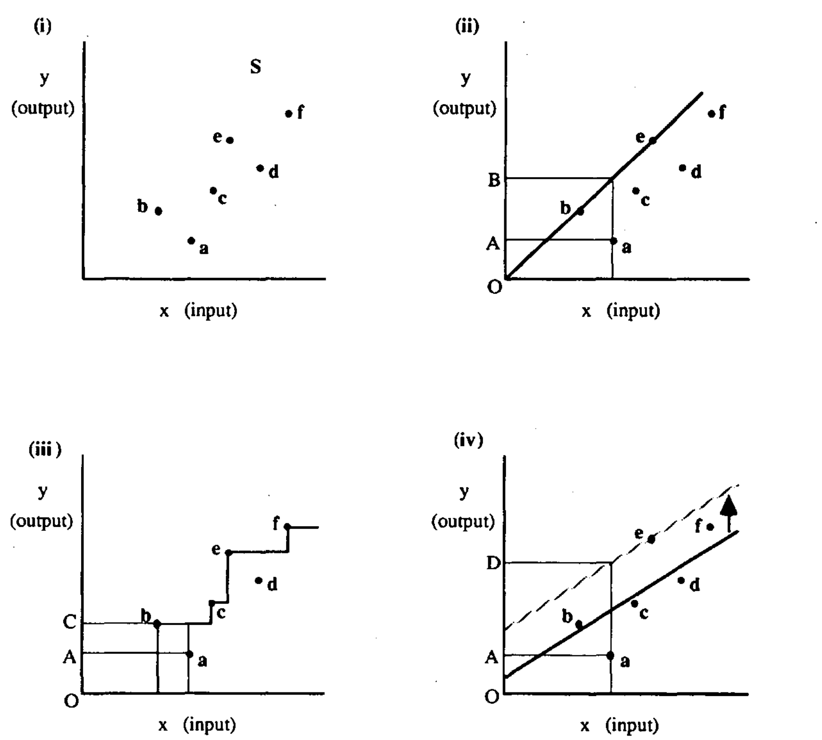
Two airline output and two capital stock characteristics are also calculated. The former characteristics are aircraft stage length (STAGE), which describes the average length of route segments (obtained by dividing aircraft miles by flights), and load factor (LOADF), which provides a measure of service quality and is often used as a proxy for service competition. A small average stage length means the carrier's aircraft spend only a short part of each flight at an efficient altitude. A low load factor, indicative of a large number of planes on a particular route, indicates high service quality. Deregulation has switched the focus from service quality (i.e., large number of flights) to price competition, causing load factor to increase as service has declined. The latter characteristics are the average size of the carrier's aircraft (KSIZE) and the percentage of a carrier's fleet that is jet (PJET). These two variables provide measures of the potential productivity of capital. For example, as the average size of a carrier's aircraft increases, more services can be provided without a proportionate increase in resources such as flight crews, passenger and aircraft handlers, and landing slots. On the other hand, the percentage of jets provides a measure of aircraft speed. Jets require proportionately less flight crew resources than turboprops because jets fly approximately three times as fast.

METHODS

Three methods currently pursued in the efficiency-measurement literature will be used to model technical inefficiency. The first two approaches, data envelopment analysis (DEA) and a variant of DEA called free disposable hull (FDH), differ from the third methodology, stochastic frontier analysis (SFA), in that the latter is based on statistical regression techniques.

Efficiency Measurement

Assume a panel data set where, for each time $t = 1, \ldots, T$, there are $n = 1, \ldots, N$ firms in the sample each consuming $j = 1, \ldots, J$ different inputs to produce $k = 1, \ldots, K$ different outputs. Assume that there exists a production set that can be constructed using all input and output observations from all time periods. The production technology, **S**, is thus defined as all possible combinations of inputs and outputs that are feasible, where feasibility means that the inputs can produce the outputs. For example, assuming only one time period and a one-input, one-output activity, a set **S** may be shown as in Figure 1 (i); Points a through f are input-output combinations observed of hypothetical firms.



Points a-f are hypothetical firms in a one-input, one-output industry.

FIGURE 1 Hypothetical example of different frontiers: (i) production technology, S; (ii) DEA frontier (constant returns to scale); (iii) FDH frontier; and (iv) SFA regression (constant returns to scale).

Efficiency measures are calculated as the distance from a production frontier. In general this distance is calculated in one of two ways in input-output space: either "horizontally," called input-based measurement since outputs are held constant, or "vertically," called output-based measurement since inputs are held constant. An output-based distance function holds inputs constant and expands outputs as much as possible without exceeding the boundaries or frontier of S. Similarly, an input-based function holds outputs constant and contracts inputs as much as possible without exceeding the boundaries of S. Under constant returns to scale, it does not matter which approach is chosen, since the values obtained from these two approaches are simply reciprocals. This study assumes constant returns to scale, which occur when outputs can be doubled by doubling inputs, since research on returns to scale in the airline industry has found that they satisfy this condition (*3*).

The next step is to define the boundary of S by using DEA, FDH, or SFA.

DEA

The first method, DEA, was introduced to economics by Charnes et al. (*4*) and has since found a multitude of applications including banks (*5–7*), the military (*8*), public schools (*9*), and hospitals (*10*). One reason for the proliferation of DEA applications is that it is a linear programming method that does not require price information. This is an empirical advantage since often the only data available are physical units of inputs and outputs. Other reasons for its widespread appeal are that it requires neither the assumption of cost minimization or profit maximization nor the specification of a production function. Furthermore, the computation of the relative efficiency for each firm under study, which may have multiple inputs and outputs, is easily executed on any computer with linear programming capabilities.

DEA, as its name suggests, creates an "envelope" of observed production points. It provides for flexible piecewise linear approximations to model the "best-practice" reference technology. Its flexibility lies in the ability to place constraints on the linear program to account for constant, decreasing, increasing, or variable returns to scale. Measures of technical efficiency levels are then developed for firms that operate inside this data envelope.

The output-based efficiency score for an observation of inputs and outputs for a firm at a particular time is obtained from a linear programming operation carried out for every carrier in every time period. In the simple hypothetical one-period, one-input, one-output, six-firm example, this process creates a production technology frontier as shown in Figure 1 (ii). Firms b and e are efficient and have scores of 1; the other firms are inefficient and have scores less than 1. For example, Firm a's output-based score will be its vertical distance from the frontier given by the ratio OA/OB < 1.

FDH

FDH was recently developed by Deprins et al. (*11*). FDH has an additional advantage over DEA because it imposes one less restriction on the data: it does not require that convex combinations of every observed production plan be included in the production set. Therefore, whereas DEA creates a piecewise linear best-practice frontier, FDH creates a best-practice frontier resembling a staircase.

Figure 1 (iii) shows an FDH production frontier. Because linear combinations of observed productions are not allowed under FDH, Firm a's technology is now compared with only Firm b's technology rather than with a combination of Firm b and Firm e technology. As a result, Firm a's output-based score is given by the ratio OA/OC < 1. Note that under FDH (as compared with DEA) more firms are efficient (Firms b, c, e, and f are all now on the frontier and have scores of 1) and inefficient firms' scores are nearer to 1 because they are closer to the FDH frontier. In other words, firms do better using the FDH rather than the DEA framework.

Deprins et al. (*11*) claim that FDH is more valuable for managerial decision making than either DEA or SFA. This assertion is based on the fact that an FDH efficiency measure is relative to an observed point on the frontier. DEA and SFA allow the measure to be relative to a hypothetical point on the frontier, since both the DEA and SFA techniques allow for convex combinations of observed points to be included in the production set. Hence managers can look at an actual rather than a theoretically possible alternative to modify current practices and improve performance.

SFA

SFA, the classical statistical approach, specifies efficiency relative to a stochastic production function. Unlike the linear programming techniques that have no particular functional form to describe their boundary, SFA requires an a priori specification of the technology (*12–14*). Furthermore, this measure of efficiency is fundamentally different from the preceding linear programming techniques because, rather than comparing a firm with a best-practice or efficient frontier, it compares a firm with an average technology. Schmidt (*15*) labels this result "paradoxical" given the usual definition of a production function as maximizing output given a set of inputs. He points out, however, that this approach may be preferred because it allows standard types of statistical inference. In this respect, SFA provides a useful counterpart to the linear programming approaches.

The technology is specified as a Cobb-Douglas stochastic frontier production function (*14*). Using data from all time periods and for all firms, the natural logarithm of output is regressed on the natural logarithms of inputs, firm characteristics, and firm-specific dummies as well as a random error term. Firm-specific dummies (variables that have the value 1 for a particular firm and 0 for all other firms) are also interacted with time to capture variation over time. The coefficients of the dummies capture the firm-specific effects and are used to calculate the relative technical efficiency scores.

Figure 1 (iv) shows the production function under this approach. Since a regression, by definition, runs through the mean of the data and does not lie atop the observed points, it is not a frontier in the same sense as DEA and FDH; rather it can be thought of as a "statistical frontier" (*15*). Efficiency scores are calculated by determining the most efficient firm [Firm e in Figure 1 (iv)] and then measuring the other firms relative to it. This is achieved by shifting the estimated frontier up to the most efficient firm. In Figure 1 (iv) this shifted line is dotted and passes through Point e. Thus, Firm a's output-based score will be OA/OD < 1.

Cointegration

Once the various efficiency scores are obtained, the next step is to test whether they exhibit cointegration or convergence. Cointegration analysis examines the existence of long-term relationships between two variables each of which moves unpredictably through time. Such variables are called nonstationary. Cointegrated variables cannot move too far from one another. In contrast, a lack of cointegration suggests that the variables have no long-term link. To remain competitive, carriers would attempt to follow each other's efficiency advances, and as a result the efficiency scores should follow each other in the long run. Lack of cointegration of a firm's efficiency scores with those of its counterparts may indicate the firm's inability to capitalize on technology that the other carriers are using.

Before testing for cointegration it is necessary to test whether each carrier's time series of efficiency scores is nonstationary, since this analysis is not relevant if the series is stationary. Given nonstationarity, the cointegrating regression is estimated. Specifically, one carrier's efficiency score time series is regressed on a constant and another carrier's efficiency score time series. If the two time series are cointegrated, any linear combination of them will be stationary, and the residuals from the regression will also be stationary.

Engle and Granger (*16*) considered several tests to evaluate the null hypothesis of no cointegration and recommended two. One approach, popular because of its simplicity, is the Cointegrating Regression Durbin-Watson (CRDW), which tests whether the Durbin-Watson statistic of the cointegrating regression is significantly different from 0. It is a characteristic of a regression in which the residuals are nonstationary to have a DW statistic near 0. Thus, if a calculated DW exceeds the critical value, the null hypothesis of no cointegration is rejected in favor of cointegration.

The second test involves applying the augmented Dickey-Fuller method to the residuals obtained from a cointegrating regression. The simplest form of this test is based on the regression $\epsilon_t - \epsilon_{t-1} = a + b * \epsilon_{t-1}$, where ϵ_t is the error term in period t. If the coefficient of the lagged error term, b, is statistically significant, the error series is stationary, and a long-term relationship exists between the two variables in the cointegrating regression. Thus, the null hypothesis of no cointegration (H_0: $b = 0$) can be rejected in favor of cointegration.

Convergence

Whereas cointegration tests determine whether two nonstationary variables are tied together in a long-run equilibrium relationship, convergence tests determine whether there is a closing of the gap over time between inefficient and efficient carriers. Convergence theories are currently being pursued in the economic growth literature to determine whether productivity growth rates among countries have been converging over time. This theory can be extended to test how efficiency in the domestic airline industry has proceeded over the past two decades.

This hypothesis is tested two ways. The first measures the dispersion of the efficiency scores over time using the coefficient of variation. If convergence is present, the carriers' scores should cluster together more closely as time progresses. The second regresses the carriers' average growth rates in technical efficiency on a constant and the carriers' efficiency scores at the beginning of the sample period. An inverse correlation between the growth rate and the original efficiency score indicates convergence. In other words, the higher a firm's original 1970 level of efficiency, the slower that level should grow. The reason for this phenomenon again lies in the public good nature of technology, which means that there are spillover effects from leader carriers to follower carriers as the laggards learn from the innovators (*17*).

Efficiency Score Computation

The DEA and FDH linear programming computations were carried out using quarterly data from 1970 to 1990. The four inputs (K, L, M, and F) and one output (RTM) were used. Raw distance scores from each technique were then regressed on the characteristic variables (STAGE, LOADF, KSIZE, and PJET) as well as the dummies used in the SFA regression. Predicted values were obtained from each of these two regressions and normalized. Normalization is necessary to obtain values between 0 and 1 and is achieved by determining the largest predicted score from each time period and dividing it into the predicted scores for all airlines in that time period. This two-step procedure is necessary to control for differences in input and output characteristics that the stochastic frontier model includes as additional regressors (*18*). This modification allows for the comparison between the linear programming and stochastic frontiers results. The SFA regression was also carried out and the normalized scores determined.

Figure 2 shows the values of all three approaches for each airline over time. The notation on the time axis is year and quarter; for example, 70I refers to the first quarter of 1970. Not all graphs span the entire time period: Frontier ends 86II because it merged into People Express in 1985, which merged into Continental in 1987; Ozark ends 86III because it merged into TWA in September 1986; Piedmont and Western end 86IV because the former was absorbed by USAir in 1989 whereas the latter was acquired by Delta in December 1986. In some cases the data end before the actual mergers (several years for Piedmont) because after merger announcements are made, data reporting accuracy sometimes declines, and it was decided that a more conservative approach to data collection should be adopted.

RESULTS

General Observations

The SFA lines are much less volatile than the DEA and FDH plots because they are based solely on a linear regression. Furthermore, SFA consistently has only two break points for all carriers: one at 82II and one at 86IV. The first break point occurs when the industry leader, in terms of SFA efficiency measurement, switches from Frontier to Ozark. The second occurs when Ozark is absorbed by TWA leaving USAir the industry efficiency leader.

Whenever one carrier leapfrogs another to become industry leader under SFA, a break point will result because of the linear nature of the method and because the efficiency scores are now measured relative to a different airline. Frontier has the highest raw SFA efficiency score in each period until 82II and therefore has a normalized score of 1 in each of these periods. However, Frontier's raw score declines during this period, allowing other carriers' normalized efficiency scores to rise relative to it. Even-

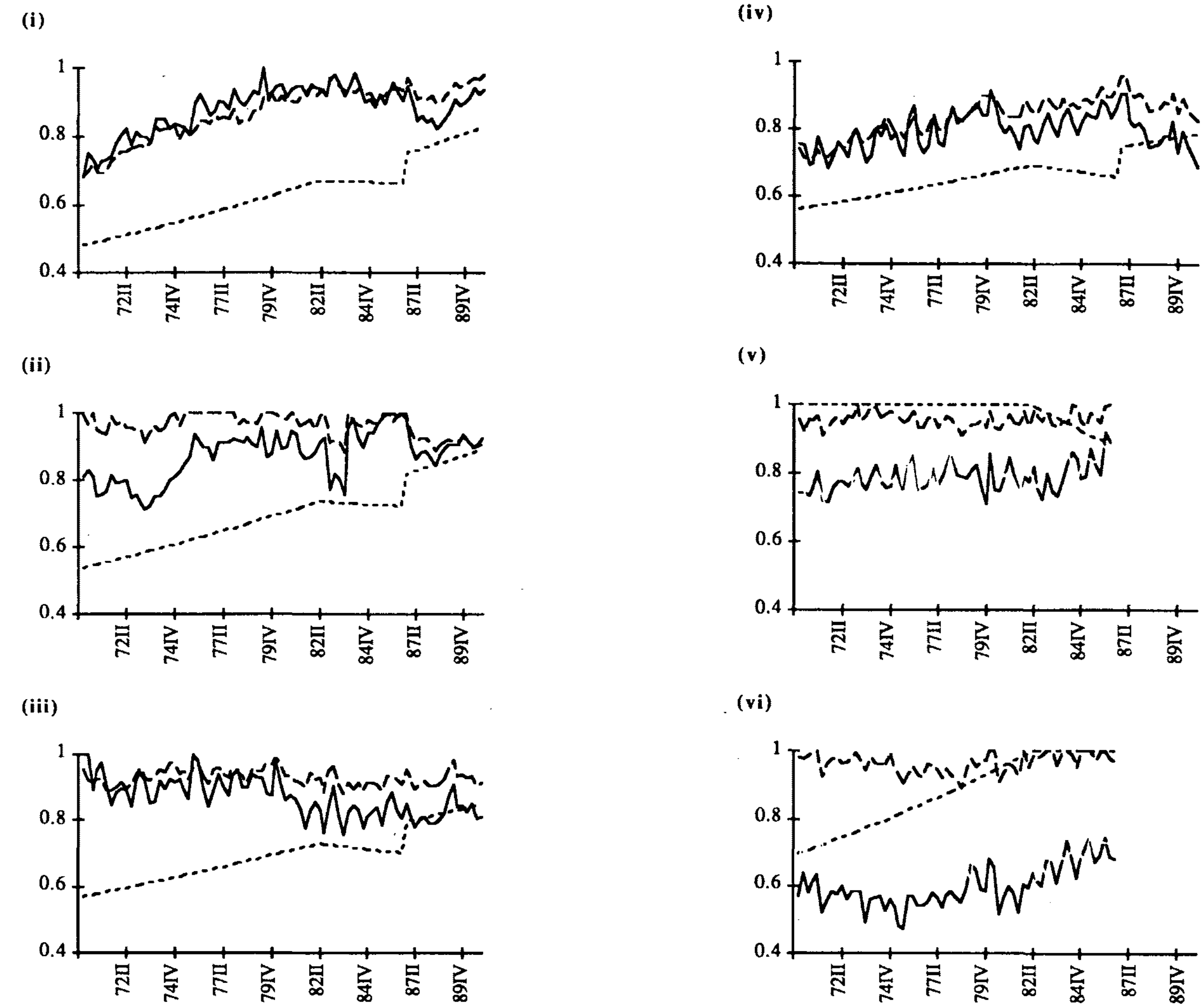


FIGURE 2 Time series of efficiency scores for each method and each airline: (i) American, (ii) Continental, (iii) Delta, (iv) Eastern, (v) Frontier, (vi) Ozark, (vii) Piedmont, (viii) Trans World, (ix) United, (x) USAir, and (xi) Western. (*continued on next page*)

tually Ozark catches up to and becomes more efficient than Frontier in Period 82II. After this Ozark's raw efficiency score grows much faster than its competitors, so their relative scores drop off. Frontier's and USAir's SFA lines drop off more quickly than the other carriers because their raw efficiency scores are declining over time.

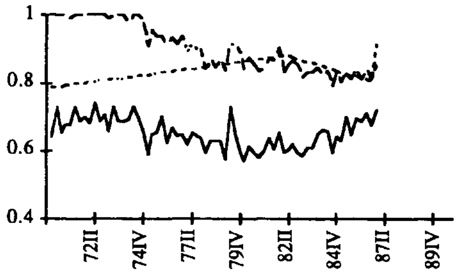
In addition, the downturn in the SFA scores beginning in 1982 would probably not have occurred if capacity ton-miles instead of revenue ton-miles were used as the measure of output. The carriers were still moving approximately the same number of seats the same number of miles, so capacity was constant, but the percentage of seats filled declined, causing revenue ton-miles to fall. The carriers may have been operating as efficiently as before with respect to capacity ton-miles, but with respect to revenue ton-miles they were producing much less output with the same amount of input.

Finally, when Ozark merged with TWA, USAir became the leader in 86III because its raw scores were the highest among the remaining carriers. With USAir as the new leader, an upward-sloping line results as was the case under Frontier's leadership. Again, this occurs because USAir's raw scores are declining while the other carriers' raw scores are increasing over time, causing the other carriers' normalized scores to grow very quickly between 86III and 90IV.

A possible explanation for the declining raw efficiency scores of Frontier and USAir can be found in the convergence hypothesis, which states that laggards grow faster than leaders because it is easier to imitate than to innovate. As a result, firms like Frontier and USAir, which start out with the highest levels of SFA raw efficiency, grow more slowly or decline because the others in the sample are merely catching up to those with the more efficient technology.

Now consider the DEA and FDH plots, which follow each other much more closely than SFA because they are both based on linear programming. Correlation analysis reveals a relatively strong positive relationship between DEA and FDH (correlation coefficient = 0.369) compared with the relationship between DEA and SFA (correlation coefficient = −0.521) and between FDH and SFA (correlation coefficient = 0.134). Table 1 presents the correlation between each pair of methods for each airline and indicates the strong positive relationship between DEA and FDH. Associations between DEA and SFA and between FDH and SFA are much more

(vii)



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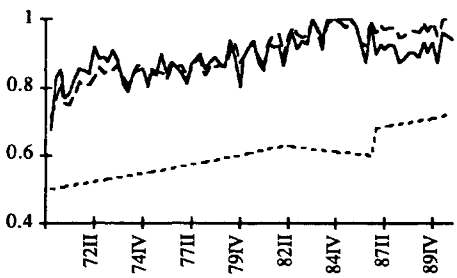

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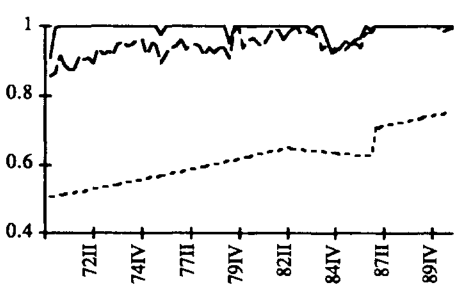

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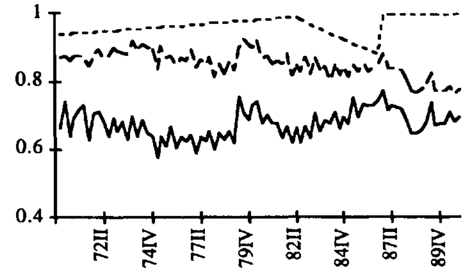

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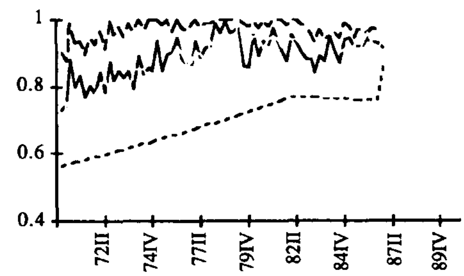


Legend:
Solid Line: DEA
Dashed Line: FDH
Dotted Line: SFA

The efficiency score is on the vertical axis and time is on the horizontal axis.

FIGURE 2 *(continued)*

unpredictable from carrier to carrier. The linear programming, best-practice frontier techniques are apparently measuring technical efficiency in a different way from the statistical, average technology approach.

Some overall trends among the various lines are illuminating. The Big Three carriers—American, Delta, and United—are performing well. All three methods indicate that American's efficiency has been improving and that United's performance has remained strong throughout the past two decades. According to FDH and SFA, Delta's performance has remained steady or has improved, whereas DEA indicates only a slight downward trend. Another interesting result is apparent in the graph for Eastern, the only airline in the sample to fail. Both DEA and FDH indicate a deterioration in efficiency starting in the late 1970s. At that time Eastern began to experience labor unrest, which continued until Eastern's demise in early 1991.

Consider also the trends of the four firms that were merged or were facing merger into larger carriers in 1986. In general the scores for Frontier, Ozark, Piedmont, and Western were high or increasing, or both, just before this period. Ozark was recognized as the most profitable and best-managed carrier in the industry while it was operating, and this would have made it attractive to its competitors. Another influential factor in these mergers was the recession, during which carriers were not able to fill their planes because demand was no longer increasing as fast as it had in the past. Ozark and the other three carriers, however, had fleets of smaller aircraft and were not as adversely affected as the larger carriers. Thus, they would have been attractive to the larger carriers, who wanted to acquire the smaller carriers' capital equipment.

Cointegration

Since cointegration analysis can only be performed on nonstationary time series, tests for this characteristic are first performed on each carrier's time series of efficiency scores. The time series were found to be nonstationary for all carriers under all three methods with four exceptions: United and USAir DEA scores and Continental and Western FDH scores.

For the cointegration analysis of the SFA series, 110 cointegrating regressions are performed (each of the 11 carriers is regressed on one of the other 10 carriers). The simplest test for cointegration, the CRDW, indicates that cointegration does not exist between any two carriers. The second cointegration test also indicates no long-term relationships between most of the pairs of carriers with one exception: a pattern of cointegration exists between Ozark and the other carriers. This suggests that there was a leader-follower relationship between Ozark and the other carriers (consistent across all three efficiency-measuring technologies) that wished to emulate Ozark's position as the most profitable in the industry.

TABLE 1 Spearman Correlation Coefficients

Carrier		FDH	SFA
American:	DEA	0.773 (0.0001)	0.570 (0.0001)
	FDH		0.898 (0.0001)
Continental:	DEA	0.400 (.0002)	0.497 (.0001)
	FDH		-0.382 (.0003)
Delta:	DEA	0.674 (.0001)	-0.672 (.0001)
	FDH		-0.138 (.2112)
Eastern:	DEA	0.744 (0.0001)	0.247 (0.0235)
	FDH		0.728 (0.0001)
Frontier:	DEA	0.580 (.0001)	-0.305 (.0128)
	FDH		-0.200 (.1068)
Ozark:	DEA	0.739 (.0001)	0.601 (.0001)
	FDH		0.231 (.0595)
Piedmont:	DEA	0.534 (0.0001)	-0.663 (0.0001)
	FDH		-0.600 (0.0001)
Trans World Airlines:	DEA	0.859 (.0001)	0.639 (.0001)
	FDH		0.862 (.0001)
United Air:	DEA	0.289 (.0076)	0.024 (.8254)
	FDH		0.818 (.0001)
USAir:	DEA	0.165 (0.1341)	-0.064 (0.5626)
	FDH		-0.335 (0.0018)
Western:	DEA	0.664 (.0001)	0.566 (.0001)
	FDH		0.175 (.1527)

(Probability > |R| under H_0: Rho = 0.)

These results are particular to the SFA method because of its linear nature. The SFA time series do not change direction often enough (only twice in our analysis) to determine whether the carriers are indeed following each other. Furthermore, the linear nature means that even the slightest difference in slope will reject the existence of comovement.

DEA and FDH, however, present the opposite conclusion. There are 72 pairs of carriers if United and USAir are excluded from the cointegration analysis of the DEA series. The two cointegration tests find 70 and 65 long-term relationships, respectively. If a 10 percent significance level is adopted instead of a 5 percent level, the second test yields 69 cointegrated pairs out of 72. Similarly, for FDH, omitting Continental and Western, the two tests yield 56 and 57 cointegrated pairs out of the possible 72. This is an overall acceptance rate of 86 percent.

Convergence

The convergence results also support the theory that technological advances become dispersed throughout the industry. Table 2 gives the coefficient of variation for each year and each method. For each of the three methods the amount of dispersion in 1990 is less than in 1970, which indicates convergence in technical efficiency. However, the coefficients for both DEA and FDH reach their lowest value in 1987 before rising through the remaining periods. This result may be attributable to the loss in 1986 of four carriers in the sample. The absorption of these competitors may have reduced the pressure among the survivors to continue their efforts to keep up with each other.

The second test of convergence involves the regression of growth rates on a constant and the initial efficiency levels. This also supports convergence. Figure 3 shows the carriers' average growth rates versus their initial levels. A negative relationship can be detected for all three methodologies. When a regression line is estimated for each method, the slope is negative and significantly different from 0 in all cases.

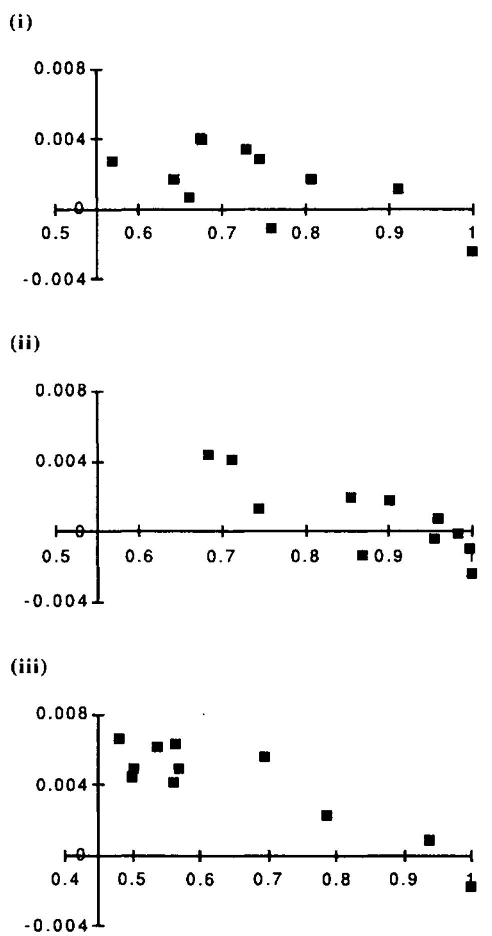
CONCLUSIONS

Economic theory suggests that, as an industry becomes more competitive, it becomes more important for a firm within that industry

TABLE 2 Coefficients of Variation

Year:	DEA:	FDH:	SFA:
1970	15.32	12.06	26.73
1971	14.64	10.83	25.78
1972	14.28	9.02	24.86
1973	15.53	8.34	23.96
1974	15.19	7.26	23.09
1975	18.09	7.57	22.25
1976	17.23	6.71	21.44
1977	17.23	6.47	20.67
1978	17.62	6.31	19.94
1979	15.57	5.03	19.26
1980	16.78	5.27	18.62
1981	16.87	5.59	18.04
1982	16.95	6.10	17.50
1983	15.87	5.97	17.04
1984	14.50	6.45	16.64
1985	13.20	6.19	16.31
1986	12.23	6.01	15.55
1987	9.89	5.39	12.30
1988	12.20	7.30	11.75
1989	11.21	7.36	11.23
1990	12.65	8.37	10.77

to perform efficiently relative to other firms if it is going to survive. This theory suggests two time patterns. First, the efficiency scores of the firms within the industry should not move too far from one another. If efficiency-enhancing technological advances made by one firm are not adopted by another firm, the two firms' efficiency scores will move apart. As a result, the firm that fails to follow innovations will eventually be driven out of the industry because its inputs are not being efficiently converted into outputs. Thus there is an incentive to keep up with movements of efficiency exhibited by other firms. This phenomenon is called cointegration. Second, the efficiency scores of the firms within the industry should also exhibit convergence over time. In other words, the scores should move closer together as firms realize that success in an increasingly competitive environment requires that they close efficiency gaps and become more alike in technical efficiency. To determine whether domestic airline carriers exhibit these two characteristics, three methods of measuring technical efficiency were performed. In general the hypotheses of cointegration and convergence were supported, indicating that the carriers are adopting efficiency advances made within the industry.



The average growth rate of efficiency is on the vertical axis and the initial 1970 efficiency score is on the horizontal axis.

FIGURE 3 Relationship between average growth rate of technical efficiency and initial levels: (i) DEA, (ii) FDH, and (iii) SFA.

These results are suggestive with respect to the direction of future industry structure. First, conventional wisdom holds that the firms remaining in the industry were able to do so because they adjusted to the increasing competitive pressure, whereas those that failed were not able to adapt. This observation is supported by the empirical evidence presented here. For example, Eastern's efficiency scores declined sharply before its demise in 1991. In addition, smaller carriers that exhibited strong or improving efficiency in 1986 were absorbed by the larger carriers, which found their performance and fleet configurations attractive. Finally, each of the remaining carriers has a general time pattern that is steady or increasing over time, and each of these carriers is still in the industry.

Furthermore, it is generally accepted that deregulation has led to more efficient use of resources in the industry. The evidence of cointegration and convergence provides empirical evidence to support this belief. As the firms have followed one another and become more alike, the industry's efficiency level has improved. The average efficiency under DEA was 0.789 in 1970, compared with 0.862 in 1990. For FDH the values are 0.882 and 0.917, respectively, and for SFA the values are 0.653 and 0.829, respectively. It can be argued that this is a positive effect of deregulation that most likely will continue into the future.

A final point concerns the applicability of this analysis to other industries. In particular, other transportation sectors such as trucking could be similarly studied.

ACKNOWLEDGMENTS

The author thanks the following people and organizations: FAA for sponsoring the Graduate Research Award Program; TRB program coordinator Larry Jenney and TRB monitors Bruce McClelland of Dornier Aviation and Gerald McDougall of Southeast Missouri State University for their time and insightful comments; and Dr. Good of Indiana University, Marty Langelan of the Office of the Secretary, Department of Transportation, and Gloria Laury of the Office of Airline Statistics, Department of Transportation, for their time and for going out of their way to help. Special thanks go to the author's advisor, Dr. Sickles, who has supported and encouraged her in all her academic and research efforts.

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Evaluating Self-Analysis as a Strategy for Learning Crew Resource Management in Undergraduate Flight Training

GUY M. SMITH

College aviation programs are in a unique position to provide crew resource management (CRM) training to meet industry demands for pilots with a high degree of competence in interpersonal skills. CRM training is usually a student's first exposure to crew operations, requiring the college to modify airline training to create meaningful learning for inexperienced pilots. Research with airline pilots has found that line-oriented flight training (LOFT) was most effective for teaching CRM. LOFT is best when airline crews debrief themselves using self-analysis to evaluate their CRM performance. The study investigated whether undergraduate flight students could effectively learn CRM skills by using self-analysis of LOFT as a debriefing strategy, despite their inexperience with crew operations. Eight men and two women completed CRM and LOFT training. Self-analysis was randomly inserted into their training using an alternating treatments research design. Crew effectiveness was assessed by measurements of crew attitudes, observations by trained observers, crew reflections on their performance, and communications analysis. It was found that at least one self-analysis session was effective for each crew, and overall gains were noted for two of the five crews. Self-analysis was effective when crews had the prerequisite technical skills and was ineffective if technical skills were lacking or if the scenario was too complex. Results suggest that self-analysis should not be applied universally in undergraduate flight training, but it is a valuable supplementary strategy to focus attention on personalities, roles, team dynamics, or specific CRM skills.

Sophisticated machines demand master operators with finely tuned motor skills, the ability to execute complex procedures, and an extensive information base. Modern aircraft require that professional pilots stretch far beyond these technical skills into the milieu of cognitive, behavioral, social, and organizational psychology, where interpersonal skills and teamwork are equally important. Statistics indicating that 70 percent of worldwide accidents in the public air transport sector are caused by flight crew actions (*1*) affirm that team skills are vital. The ideal airline candidate is a technical expert and a master of teamwork. For most of this century, however, pilot selection and training were based on technical proficiency alone. Airlines recognized this deficiency, poured substantial investments into human factors research, and developed advanced training programs such as crew resource management (CRM).

It is argued that CRM is advanced training and is not appropriate for beginning students, who should concentrate on "stick and rudder" skills. Others contend that teamwork is an indispensable pilot skill and that it is a disservice to students to postpone crew training until they reach the airlines (*2*). European ab initio programs, in which nonpilots are taught from the beginning to be airline pilots, have successfully included CRM in initial flight training for years (*3*). College aviation programs are in a unique position to develop effective CRM training for initial flight students.

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LINE-ORIENTED FLIGHT TRAINING

The Federal Aviation Administration (*4*) developed three guidelines for an effective CRM program for airlines operating under Federal Aviation Regulations (FAR) Parts 121 and 135:

- The course content should emphasize CRM skills.
- Students should experience and practice these skills.
- Students should get feedback on their CRM performance.

To make these guidelines applicable to undergraduates, a content model, concerned with transmitting information and skills, was insufficient. An experiential or process model, concerned with providing resources to help learners acquire CRM skills, was required. Moreover, to evaluate the outcomes of this model, the primary effectiveness measure had to be performance.

There are many CRM instructional methods to choose from. Of the 16 listed by Sams (*5*), the most effective for airline pilots was line-oriented flight training (LOFT), an experiential learning method in which flight crews fly a complete scenario in a high-fidelity simulator in real time. Airlines achieved striking results with LOFT, but systematic research was necessary to ensure that LOFT is also effective in teaching CRM to undergraduate students.

Self-analysis is a discovery learning strategy based on the theory of objective self-awareness (*6*). It proposes that self-focusing stimuli often force objective appraisals of oneself that may lead to attitude and behavior changes. Self-analysis of LOFT, in which the debriefing is led by the crew themselves, has been a highly effective technique for improving CRM performance in airline pilots (*7*). For college crews, self-analysis could give powerful insights into CRM performance, offsetting some of their inexperience (*8*).

DEVELOPMENT OF AN EFFECTIVE COLLEGE CRM PROGRAM

For most undergraduates, LOFT is their first exposure to nonroutine, high-stress, high–work load, and emergency situations requiring teamwork. To expand technical skills into higher-order CRM skills, students must be actively involved in each stage of the learning process (*9*). Active learning strategies gave direction to a college CRM program that progressed through three distinct phases: learning sessions where CRM skills were introduced, practice sessions where CRM skills were exercised, and feedback sessions where behaviors were reinforced or corrected.

Learning sessions were content sessions that used an active cooperative learning method called jigsaw (*10*). Students read assigned material and then share information with their crew member by discussing case studies, analyzing accident reports, and writing team response papers.

Practice sessions were LOFT simulator exercises that required students to actively use CRM skills in an operational environment. They were flown in real time without assistance and were videotaped from start to finish.

Feedback sessions were debriefing periods during which two distinct methods were used: conventional debriefing and self-analysis. Conventional debriefing was not an active learning strategy; feedback was immediate, quantifiable, and objective. Instructors provided most of the input (*11*). Self-analysis debriefing, as an active learning strategy, gave students responsibility for their own debriefing. Self-analysis debriefings were postponed for 2 days while videotapes, verbatim transcripts, and communications analysis were being prepared as objective material for their exploration (*9*).

RESEARCH DESCRIPTION AND METHODOLOGY

Objectives

The purpose of this study was to determine whether undergraduate flight students could effectively learn CRM by using self-analysis of LOFT as a debriefing strategy, despite their inexperience with crew operations. Performance was selected as the measure of effectiveness. The jigsaw learning sessions and LOFT practice sessions were common to all crews, but debriefing sessions (conventional or self-analysis) were distinctive so that differences in CRM performance could be measured.

Design

The research design was an alternating treatments design (*12*), a type of single-subject design. Subjects were alternately exposed to a nontreatment (conventional debriefing) and a treatment (self-analysis of LOFT training). Repeated measurements of attitudes, effectiveness, performance, and self-reporting were taken to determine whether differences in performance could be noted. The alternating treatments design was selected because the population was small (five crews). The performance of each crew was analyzed independently, and any comparisons between crews were speculative and noninferential. There was no attempt to generalize from this research to any other population.

Subjects

The subjects were 12 students enrolled in the CRM course in spring 1993. Before any CRM instruction, each student completed a questionnaire to document pilot experience, education, and exposure to CRM. They were instructed in the LOFT simulator individually and evaluated on their technical flying skills. Students of equal skills were assigned to permanent crews to balance the crew technical skill level. Of the 12 students, 9 were fully qualified to be research subjects, 2 were unqualified and excluded, and 1 (Ed) was marginally qualified and included because each crew requires two people. Eight men and two women were teamed as follows:

- One crew with above-average skills (Alex/Art),
- Two crews with average skills (Betty/Bob and Carl/Cathy),
- One crew with mixed skills (Dan/Dave), and
- One crew with below-average skills (Ed/Eric).

Each crew completed five sessions of CRM and LOFT training; two sessions of self-analysis debriefing were randomly inserted into their training.

LOFT Scenarios

The LOFTs were flown in a Frasca model 142 twin-engine flight simulator with scenarios based on FAR Parts 91 and 135 operations requiring commercial pilot skills. Instrument flight rules were required throughout. No scenario forced students to choose a solution that would violate regulations. Flights took place in the United States intermountain Northwest, an area that requires extreme vigilance because of mountainous terrain and intermittent radar coverage. Unfamiliar airports and routes were chosen. Flights were designed to last 45 min, including 15 min of normal work load followed by an occurrence triggering a high–work load phase.

LOFT 1 was designed as a crew training session because it was their first crew experience. The scenario required normal crew interactions for instrument flight; there were no critical occurrences. There were two similar legs allowing each student the opportunity to fly as captain. Two crews (Alex/Art and Carl/Cathy) received self-analysis debriefing.

LOFT 2 was a communications exercise concentrating on the CRM skill of advocacy. The scenario was a medical support flight that was requested to divert because of an urgent need for blood replacements. It required crew interaction and radio communication to choose a divert airport that was above weather minimums and could deliver the required blood. Self-analysis debriefing was used for Betty/Bob, Dan/Dave, and Ed/Eric.

LOFT 3 was a decision-making exercise focusing on the CRM skills of prioritizing and analyzing alternatives. The crew was on a long-distance flight that encountered arrival deadlines, departure delays, and unsuitable weather at the destination. It required consideration of operational commitments, weather complications, and fuel constraints. Self-analysis debriefing was used for Alex/Art and Carl/Cathy.

LOFT 4 was designed as a situational awareness exercise to emphasize the CRM skills of situation monitoring and cross-checking. While transporting high-priority medical supplies, minor mechanical difficulties progressively developed into a total loss of electrical power. The scenario required attentive monitoring of the aircraft's capabilities and awareness of external factors: weather, operational requirements, navigation capabilities, and alternatives. Communication with air traffic control and radar services was lost about 30 min after takeoff. Self-analysis debriefing was used for Betty/Bob, Dan/Dave, and Ed/Eric.

LOFT 5 was a team management exercise highlighting the CRM skills of work load assessment and management. The crew was exposed to operations in a high-density (Class B airspace) environment where the weather was unsuitable for the destination but above minimums for several nearby alternatives. The crew lost communication with air traffic control, requiring crew interaction and leadership skills to select a course of action from a large number of alternatives. Because of the complex airspace, marginal weather, and faulty radios, LOFT 5 became known as the "LOFT from Hell."

Analytical Instruments

A repeated measures strategy was used to evaluate crew effectiveness via converging sources of data (*13*). Each measurement used five evaluation methods to assess different aspects of effectiveness. Reliability was maximized by collecting data from these five sources and establishing that they converged on a global measure of effectiveness (*13*).

The cockpit management attitudes questionnaire (CMAQ) is a 25-item Likert scaled instrument measuring attitudes that are an indirect indication of crew performance (*14*). It was completed by each crew (scored by consensus) after each LOFT as a measure of the effectiveness of the strategy (conventional or self-analysis). The CMAQ was factored into three subscales: communication and coordination, command responsibility, and recognition of stressor effects (*15*).

The LINE/LOS checklist (LLC) is an evaluation of a crew's performance of CRM skills by trained observers (*16*). It was scored immediately after each LOFT by two instructors who used extensive field notes and deliberations to reach consensus scores. The checklist consists of two global ratings and eight crew effectiveness markers that are indicators of crew performance (*17*).

Communications analysis is a measure of crew interaction and coordination that reflects trends in flight crew performance (*18,19*). Communications analysis started at the beginning of the high–work load phase and lasted for exactly 30 min. Using a procedure adapted from Foushee and Manos (*18*), cockpit communications were transcribed verbatim, and each statement or phrase was coded into 1 of 20 categories of communication. Two coders worked independently on all of the transcripts, and a point-by-point comparison established an interrater reliability of 81 percent. Four categories that have been related to performance were used as measures of crew effectiveness: total communications, commands by the captain, acknowledgments by the first officer, and observations by both crew members (*20*).

The CRM survey is a survey of crew reactions to their training experience that was completed by consensus after each LOFT. Responses were factored into six categories to obtain students' views on the value of LOFT as a training technique, the quality of the LOFT scenario, the work load imposed by the LOFT scenario, the ratings of the LOFT instructor, a self-evaluation of overall performance, and a self-report on use of CRM skills (*21*).

The lessons learned is another crew report of 10 lessons that they learned from each training phase. Students reflected on the entire experience, listed their CRM lessons learned, and specified the source of learning for each lesson.

RESEARCH FINDINGS

Data analysis in this single-subject design involved inspection and analysis of graphic presentations (*12*). To summarize the graphs, a variant of the nonparametric sign test was used to show magnitude and direction of a change (*22*). Tables of findings list only those factors that showed gains for self-analysis that were more than one standard deviation higher than the preceding conventional session.

Alex/Art

Table 1 gives factors exhibiting measurable gains in performance for Alex/Art after self-analysis sessions. The Alex/Art crew was above average in technical skills and well matched. There was, however, a significant difference in experience; Art was a low-time private pilot, whereas Alex was an active flight instructor. Alex struggled with role definition, thinking of himself as a flight instructor and recognizing that he was expected to perform as a

TABLE 1 Gains After Self-Analysis Sessions (Alex/Art)

	1ST SELF-ANALYSIS (LOFT #2)	2ND SELF-ANALYSIS (LOFT #4)
LINE/LOS CHECKLIST		
OVERALL TECHNICAL EFFECTIVENESS	+	++
OVERALL CREW EFFECTIVENESS	nc	++
CRM SURVEY		
SCENARIO QUALITY	+++	+
WORKLOAD IMPOSED	++	++
INSTRUCTOR RATING	++	--
COMMUNICATIONS ANALYSIS		
TOTAL COMMUNICATIONS	+++	+
COMMANDS (BY CAPT)	+	+++
ACKNOWLEDGEMENTS (BY FO)	+	++
OBSERVATIONS (CAPT & FO)	++	+

Standard deviations since the previous observation:
- - - = <-2, - - = -1<-2, - = 0<-1, **nc** = no change,
+ = 0>1, + + = 1>2, + + + = 2>3

crew member. The crew grappled with role definition in both self-analysis sessions, resulting in keener awareness of CRM issues. Their LLC showed that self-analysis increased both technical and CRM skills, consistent with their concern for "looking good." The CRM survey showed that self-analysis imposed greater work loads but resulted in higher-quality scenarios. Their rating of instructors decreased after the second self-analysis session, indicating that they preferred conventional debriefing. Communications analysis showed gains in all four categories in both sessions, the strongest evidence that self-analysis motivated this crew. Lessons learned focused on team building, though that CRM skill was not formally taught until LOFT 5. They recorded their principal learning sources as LOFT and self-analysis. It appears that self-analysis made an important contribution to their learning experience.

Betty/Bob

Table 2 gives factors exhibiting measurable gains in performance for Betty/Bob after self-analysis sessions. The Betty/Bob crew had difficulty disregarding the research and concentrating on learning. They were also reluctant to "suspend reality" and accept the realism of the simulator. More important, crew dynamics was a possible hindrance to their learning. Bob was confident, capable, and occasionally patronizing. Betty was equally capable but more acquiescent; her voice inflections exhibited some sensitivity to his manner. Evidently, these dynamics were more apparent to the instructors and were not a concern the crew discussed in self-analysis sessions. Their first self-analysis session was very successful, with every item on the LLC and three CRM survey items showing strong gains. Their progress was strongly supported by two communications analysis items. These gains contrasted sharply with a significant decline in the second self-analysis session. There was no link between these declines and self-analysis, who was captain, or responses on the CRM survey. However, lessons learned gave evidence that the crew was struggling with crew dynamics:

- "Do not assume that your partner knows what you mean."
- "Share decision making. Don't let captain override the crew."
- "The need for CRM skills was not practiced. We used a lot of nonverbal communication and that was a mistake."
- "First officer learned to wait for captain decisions or make verbal suggestions before taking action."

They showed gains in communications analysis, contrary to the other measures, suggesting that perhaps they were making improvements in crew dynamics. Betty/Bob concentrated their lessons learned on situational awareness and communication. They primarily learned from LOFT; only 10 percent of their learning was attributed to self-analysis. The data suggest that self-analysis had limited value for this crew in learning CRM skills.

Carl/Cathy

Table 3 gives factors exhibiting measurable gains in performance for Carl/Cathy after self-analysis sessions. The Carl/Cathy crew was matched in skills and compatible in personality, performing well as a male/female crew. Preoccupation with technical details such as crew coordination, radio communication, and checklists limited their ability to absorb CRM skills. After the first self-analysis session the LLC indicated negative results for their comprehension of crew concepts. Contrary to this outcome, the crew recorded an increase in usage of CRM skills in the CRM survey. Their perceived gains were mostly in technical areas, confirming that they were unable to recognize CRM skills at that point. Communications analysis showed a notable increase in total communications, usually an indication of increased performance. Foushee and Manos (*18*) warn that more communication among flight crew members does not necessarily translate into better performance. The crew worked hard but did not know what to do. Initially, self-analysis provided few answers; the crew needed an explicit role model, someone with considerable experience in crew operations to demonstrate effective crew performance.

Deliberately modifying procedures, the instructor closely monitored their second self-analysis session to circumvent digressions into technical discussions. It became a hybrid between self-

TABLE 2 Gains After Self-Analysis Sessions (Betty/Bob)

	1ST SELF-ANALYSIS (LOFT #3)	2ND SELF-ANALYSIS (LOFT #5)
LINE/LOS CHECKLIST		
COMMUNICATIONS/DECISION BEHAVIOR	+++	--
TEAM BUILDING AND MAINTENANCE	+++	--
WORKLOAD MGMT/SITUATION AWARENESS	+++	-
OVERALL TECHNICAL EFFECTIVENESS	++	--
OVERALL CREW EFFECTIVENESS	+++	-
CRM SURVEY		
SCENARIO QUALITY	++	--
SELF-EVAL OF PERFORMANCE	++	--
SELF-REPORT ON CRM SKILLS	+++	-
COMMUNICATIONS ANALYSIS		
TOTAL COMMUNICATIONS	-	+++
COMMANDS (BY CAPT)	+++	--
OBSERVATIONS (CAPT & FO)	+++	++

Standard deviations since the previous observation:
- - - = <-2, - - = -1<-2, - = 0<-1, **nc** = no change,
+ = 0>1, + + = 1>2, + + + = 2>3

analysis and conventional debriefing, herein referred to as guided self-analysis. Guided self-analysis manifested strong gains in the LLC and in communications analysis, indicating that it was an effective learning method. The crew reported lessons learned in situational awareness and in technical areas. They learned mostly from LOFT debriefings and self-analysis. Although self-analysis, as designed for this research, indicated marginal gains for this crew, guided self-analysis was more effective.

Dan/Dave

Table 4 gives factors exhibiting measurable gains in performance for Dan/Dave after self-analysis sessions. The Dan/Dave crew was mismatched in skills; Dan was above average and Dave was below average. They had steady gains in effectiveness for both self-analysis sessions, regardless of who was captain. However, they disliked self-analysis and sometimes requested conventional debriefing with the instructor. The CMAQ showed gains in recognizing stressors, and their LLC gave the most persuasive confirmation that self-analysis was effective. In the CRM survey they rated self-analysis high, despite their stated dislike of the method. Communications analysis supported the gains of the first self-analysis session. In the lessons learned, the crew documented the best variety of lessons: decision making, situational awareness, teamwork, and communications. Their learning sources were predominantly LOFT and self-analysis. Self-analysis was noticeably effective as a learning agent for this crew.

Ed/Eric

Table 5 gives factors exhibiting measurable gains in performance for Ed/Eric after self-analysis sessions. The Ed/Eric crew had a positive attitude and were exceptionally conscientious. Both had excellent academic records but below-average technical skills. In the first self-analysis session, the crew realized that their communication was poor; subsequently they focused exclusively on communication and registered partial gains in the CMAQ, the LLC, and the CRM survey. Communications analysis strongly corroborated their concentration on communication and indicated considerable progress in that area. Beginning in LOFT 4, the crew experienced scenario complexity that was beyond their technical ability. As difficulty increased, effectiveness measurements, particularly communications analysis, document a laborious and mostly futile journey from textbook knowledge (theory) to practical skills. LOFTs 4 and 5 were "lost communications" incidents, in which they did not use CRM skills because they were "in over their heads" with scenarios that were too difficult for their skill level. The lessons learned for Ed/Eric focused on communication and team building. They reported that most of their lessons were learned from LOFT; self-analysis accounted for only 14 percent of lessons learned. Self-analysis was effective in the first session but proved ineffectual when their technical skills were deficient.

SUMMARY OF LESSONS LEARNED

Table 6 is a compilation of lessons learned for all crews. Each crew focused lessons learned on a specific CRM skill, and four crews had a CRM skill they neglected:

Crew	*Focus*	*Area of Neglect*
Alex/Art	Team building	None
Betty/Bob	Situational awareness	Team building
Carl/Cathy	Situational awareness	Team building
Dan/Dave	Decision making	Communication
Ed/Eric	Communication	Decision making

Crews were asked to name the source of learning for each lesson learned. Without exception, LOFT proved to be a valuable learning source, an indication that these students learned CRM by doing it. Self-analysis was a valuable learning source for three crews, indicating that it also had value. The strongest support for self-analysis came from Dan/Dave, who frankly acknowledged that they did not like doing self-analysis but attributed 40 percent of their learning to it.

TABLE 3 Gains After Self-Analysis Sessions (Carl/Cathy)

	1ST SELF-ANALYSIS (LOFT #2)	2ND[a] SELF-ANALYSIS (LOFT #4)
LINE/LOS CHECKLIST		
COMMUNICATIONS/DECISION BEHAVIOR	--	+++
TEAM BUILDING AND MAINTENANCE	--	+++
WORKLOAD MGMT/SITUATION AWARENESS	--	+++
OVERALL TECHNICAL EFFECTIVENESS	--	+++
OVERALL CREW EFFECTIVENESS	--	++
CRM SURVEY		
SCENARIO QUALITY	++	++
WORKLOAD IMPOSED	++	+
SELF-REPORT ON CRM SKILLS	++	nc
COMMUNICATIONS ANALYSIS		
TOTAL COMMUNICATIONS	+++	-
COMMANDS (BY CAPT)	-	+++

Standard deviations since the previous observation:
- - - = <-2, - - - = -1<-2, - = 0<-1, **nc** = no change,
+ = 0>1, + + = 1>2, + + + = 2>3

[a]Guided self-analysis session.

TABLE 4 Gains After Self-Analysis Sessions (Dan/Dave)

	1ST SELF-ANALYSIS (LOFT #3)	2ND SELF-ANALYSIS (LOFT #5)
CMAQ		
RECOGNITION OF STRESSOR EFFECTS	nc	++
LINE/LOS CHECKLIST		
COMMUNICATIONS/DECISION BEHAVIOR	+	++
TEAM BUILDING AND MAINTENANCE	++	++
WORKLOAD MGMT/SITUATION AWARENESS	++	++
OVERALL TECHNICAL EFFECTIVENESS	--	++
OVERALL CREW EFFECTIVENESS	+	+++
CRM SURVEY		
WORKLOAD IMPOSED	++	--
SELF-EVAL OF PERFORMANCE	++	++
SELF-REPORT ON CRM SKILLS	++	---
COMMUNICATIONS ANALYSIS		
ACKNOWLEDGEMENTS (BY FO)	++	-
OBSERVATIONS (CAPT & FO)	+++	+

Standard deviations since the previous observation:
- - - = <-2, - - = -1<-2, - = 0<-1, **nc** = no change,
+ = 0>1, + + = 1>2, + + + = 2>3

DISCUSSION OF RESULTS

For every crew, the CMAQ had only slight variability and provided essentially no evidence for effectiveness of self-analysis. Relationships between attitudes and performance have been validated for airline crews (*23*), but the instrument may be unsuitable for undergraduates because they lack crew experience on which to base attitudes. Also, the CMAQ was scored by a crew as a consensus measure of crew attitude, though it was designed as an individual instrument. A "crew attitude" may not even exist. It is also conceivable that the CMAQ showed small variations because it was completed so often (every 2 weeks) and crews remembered previous responses. For these reasons, the CMAQ did not render an acceptable measure of self-analysis effectiveness.

The LLC was probably the most objective measure of effectiveness because it required systematic data collection of CRM skills distinct from technical performance. Consensus grading compelled justification for every grade and reduced the possibility of grading by instinct, crew reputation, or preferred results. Of all the measurements taken, the LLC is the best summary of the study. It shows significant gains in 5 of the 10 self-analysis sessions.

The CRM survey was designed as a self-analysis instrument. One factor in the survey, self-report on CRM skills, is probably the most direct measure of self-analysis. Three of the five crews showed a step increase in this factor after the first application of self-analysis, but none reported gains in the second session. It appears that crews became more discerning and critical as they gained awareness of CRM skills. For self-analysis sessions, two other trends were evident in the survey: instructor ratings declined and work load imposed increased. Crews apparently preferred conventional debriefing with the instructor; the extra work was perceived as a negative feature of self-analysis.

Multiple measures of effectiveness were used because each data source has its strengths and weaknesses. A data source has merit if it consistently validates or disproves the results from other measures. In communications analysis, frequencies are an equivocal measure of effectiveness because communication must be interpreted within a task, environment, or interpersonal context (*24*). In this study, three of the four communications categories

TABLE 5 Gains After Self-Analysis Sessions (Ed/Eric)

	1ST SELF-ANALYSIS (LOFT #3)	2ND SELF-ANALYSIS (LOFT #5)
CMAQ		
COMMUNICATIONS AND COORDINATION	++	+
LINE/LOS CHECKLIST		
TEAM BUILDING AND MAINTENANCE	++	--
CRM SURVEY		
SCENARIO QUALITY	++	---
WORKLOAD IMPOSED	++	nc
COMMUNICATIONS ANALYSIS		
COMMANDS (BY CAPT)	++	---
ACKNOWLEDGEMENTS (BY FO)	+++	-
OBSERVATIONS (CAPT & FO)	+++	-

Standard deviations since the previous observation:
- - - = <-2, - - = -1<-2, - = 0<-1, **nc** = no change,
+ = 0>1, + + = 1>2, + + + = 2>3

confirmed results of other measures. However, "total communications" was not consistent as a measure of effectiveness. It appears that well-intentioned crews, in an effort to practice communications skills, "talked" more but "communicated" less.

The two women who participated in the study were as professional and competent as the men, indicating that women belong in aviation and should be encouraged to participate equally with men in all domains of the industry. Crews in this study found that the cockpit can be a confining and sometimes emotional environment and that male/female relationships can add CRM issues that must be considered. Further research is needed to understand perceptions of male dominance, male/female dynamics, and the seniority of captains regardless of age, sex, and often skill. These issues are compelling reasons why CRM should be included in initial flight training: to educate men and women to the paradigm that men and women are equal and that performance, not gender, is the decisive factor.

Because they involved "lost communication," LOFTs 4 and 5 were particularly difficult, especially for the less skilled. All crews experienced some difficulty with lost communication, and two deliberately chose to violate regulations in a lost communication situation. All students were cognizant of textbook answers, but LOFT required them to convert their knowledge into appropriate action without assistance or feedback. LOFT elicits higher-order thinking, just as do life's situations, providing another argument for introducing LOFT in initial flight training.

Reflection on the crews that struggled with role definition and crew dynamics reveals an important difference between airline CRM training and undergraduate training. Airline crews are expected to have resolved such issues beforehand, but these contentions are natural learning encounters for college students. The outcome for Alex/Art was positive because self-analysis made them aware of the role definition problem and they struggled with it, though it was not totally resolved. On the other hand, self-analysis did not expose the crew dynamics issue to Betty/Bob, so it was not addressed forthrightly and the outcome is uncertain. It proved insufficient for the researcher to document the problem; education should have overridden research, and the issue should have been addressed so the students could resolve it.

CONCLUSIONS

None of the crews rated self-analysis highly, suggesting that they preferred conventional debriefing to self-analysis. Evidence weighed against self-analysis as a stand-alone strategy for teaching CRM to undergraduate flight students. The results are characteristic of initial flight students, who are accustomed to more guidance and rely heavily on feedback from instructors to evaluate their performance. However, there are sufficient data supportive of self-analysis, especially for experienced crews, that self-analysis should not be rejected. Self-analysis seems to be more effective as a supplemental strategy to be used when certain conditions exist. Further research is needed to determine the circumstances (personalities, team dynamics, experience, etc.) that would make it successful. Self-analysis appears to gain effectiveness as students accumulate experiences with crew operations.

The LLC reported the observer's overall evaluation of both technical and CRM performance; the CRM survey reported each crew's self-evaluation of technical and CRM performance. Concerning technical performance, crews' assessment of gain through self-analysis matched the observer's appraisal in 70 percent of cases. Concerning performance of CRM skills, crews' evaluation of gain through self-analysis matched the observer's assessment in only 50 percent of cases. Despite their focus on CRM skills, these students were more adept in evaluating their changes in technical performance than in assessing variations in crew effectiveness.

TABLE 6 Summary of Lessons Learned for All Crews

Lessons-Learned						
CREW	Communication	Decision Making	Sit. Aware	Team Building	Tech	
A/A	-	-	-	++		
B/B	+	-	++	--		
C/C	-	-	++	--	+	
D/D	--	++	-	-		
E/E	++	--	-	+		
Where Learned						
CREW	Debrief	Instructor	LOFT	Preflight	Rdgs	Self-analysis
A/A	-	-	++	-	+	+
B/B	-	-	+++	-	-	-
C/C	+	- -	++	-	--	+
D/D	-	-	+++	-	-	+++
E/E	-	-	+++	-	-	-

Standard deviations from the mean:
- = 0<-1 -- = -1<-2 --- = -2<-3
+ = 0> 1 ++ = 1> 2 +++ = 2> 3

The objective of LOFT is to provide crew members with the opportunity to practice both technical and CRM skills in a realistic scenario. The scenarios for this research were created, field tested, and evaluated by experienced aviators on the basis of perceived skills of commercial pilots. "Realistic and reasonable" for designers may not be viable for the students. In retrospect, two unanticipated factors may have influenced the results: students needed more low–work load time in all scenarios, and LOFTs 4 and 5 were too difficult for most of the crews. With the exception of Dan/Dave, overall technical performance in LOFT 5 was deficient, making it difficult to determine whether outcomes were attributable to self-analysis or to the scenario itself. Future research should recognize that college students need acclimation to crew operations; scenarios should be uncomplicated and should include significant low–work load periods. Guidelines and scenarios developed for airline pilots may not be appropriate for undergraduate flight students.

Throughout this research the focus has been on CRM skills, leaving the impression that CRM skills are superior to or more desirable than technical skills. A high degree of technical proficiency is essential for safe and efficient flight operations (*4*). In this study, crews with lower technical ability had considerable difficulty learning CRM skills. In 8 of 10 self-analysis sessions, differences in technical skills reflected analogous variations in CRM skills. CRM skills were not taught in isolation, confirming the conventional wisdom that mature technical skills are essential for developing CRM skills. This finding confirms the value of LOFT and self-analysis of LOFT as training technologies that integrate technical and CRM training.

RECOMMENDATIONS

Because one crew centered on technical discussions, guided self-analysis, a combination of self-analysis and conventional debriefing, was used. It produced strong gains for them in the LLC, suggesting that a research design using guided self-analysis may be more effective than self-analysis alone for undergraduate flight students. Research would be complicated; differences between guided self-analysis and conventional debriefing are less distinct.

Participants in this study were sometimes frustrated because they did not always know "the right way" to do things. They had difficulty applying theory to practice in the LOFT, and self-analysis did not furnish a standard for comparison. This inadequacy suggests that a research design in which self-analysis is preceded by role modeling to illustrate effective crew performance would be more appropriate. Students could observe role models on videotapes or role plays, but the best training would be achieved by flying a LOFT scenario with a pilot experienced in crew operations.

For thorough training, students swapped roles between captain and first officer in each scenario. This is an inferior design for research because crew performance could vary significantly with the captain. Assigning the more experienced crew member to be captain for the entire study would be better for research and would strengthen training because the concept of seniority would be established. That option was not possible in this study because students required exposure to both roles in a single-semester course. In further research, CRM could be taught in two semesters with beginner students flying first officer and experienced ones flying captain. A potential benefit is that experienced students could provide a role model for novice students. Research should also determine whether a student with one semester of LOFT experience is an adequate role model.

ACKNOWLEDGMENTS

The LOFT training described was conducted at the Rocky Mountain College Flight Simulator Laboratory in a Frasca Model 142 ground trainer. The work was funded by the 1992–1993 Graduate Research Award Program on Public-Sector Aviation Issues sponsored by FAA and administered by TRB. The project was conducted under the leadership of John W. Kohl, faculty advisor, and monitored by Larry L. Jenney and Richard F. Pain of TRB and by Lemoine V. Dickinson of Failure Analysis Associates. Research assistance was provided by MaryJo O. Smith, who transcribed 25 LOFT scenarios, facilitated data reduction, and reviewed endless drafts.

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Airfoil Performance in Heavy Rain

JAMES R. VALENTINE

In recent years microbursts have been implicated in several major aviation accidents. Since microbursts are often accompanied by heavy rainfall, an interest in airfoil performance in rain has arisen. As raindrops strike the leading edge of an airfoil, small droplets are splashed back into the airflow field, and an uneven water film forms on the airfoil surface. Both phenomena have been hypothesized to contribute to a degradation of airfoil performance in rain that may be manifested as a decrease in lift, an increase in drag, and premature stall. The splashed-back droplets are accelerated by the airflow field. Thus droplet drag acts as a momentum sink to deenergize the boundary layer, while the uneven water film effectively roughens the airfoil surface. A numerical, two-way momentum coupled, two-phase flow scheme for the evaluation of the effect of splashed-back droplets on a NACA 64-210 airfoil section in cruise configuration is described. A thin-layer Navier-Stokes computational fluid dynamics code is coupled with a Lagrangian particle tracking scheme to determine the two-phase flow field in an iterative manner. Noninteracting, nondeforming, and nonevaporating spherical particles representing statistical distributions of raindrops are tracked through the curvilinear body-fitted grid used by the airflow code. A simple model is used to simulate raindrop impacts and the resulting splashback on the airfoil surface. Results are compared with wind tunnel test results.

On July 9, 1982, Pan American World Airways Flight 757, a Boeing 727, encountered a microburst upon taking off from New Orleans International Airport and crashed, killing 153 persons. Estimates of rainfall rates encountered by the aircraft range up to 144 mm/hr (*1*). Serious investigations of heavy rain effects on aircraft performance had begun only a few years earlier, and researchers reported that significant airfoil performance penalties (decreased lift, increased drag, and earlier stall) may occur at rainfall rates of 150 mm/hr or greater (*1*). The primary cause of the accident was the microburst wind patterns, but it was unknown whether rain had also played some role. It is possible that a rain-induced premature aerodynamic stall could occur before the aircraft stall warning system was activated. Concern over this accident led to a Federal Aviation Administration (FAA) and National Academy of Sciences (NAS) study of the hazards of wind shear for aircraft that are landing or taking off (*2*). The study analyzed 27 wind shear–related aircraft accidents and incidents that had occurred between 1964 and 1982 and concluded that the most dangerous types of wind shears are the downdraft and outflow microbursts associated with convective storms. Since these storms are often accompanied by heavy rainfall, one of the report's recommendations was the continued investigation of the aerodynamic performance of aircraft in heavy rain. The most recent analysis of aircraft performance in heavy rain (*3*) had been developed from experimental studies of rough airfoils and low-speed water drop splashes and had not been validated by wind tunnel simulations of airfoils or aircraft in rain.

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AIRFOIL AERODYNAMICS

A typical streamline pattern around an airfoil at a relatively low angle of attack (α) is shown in Figure 1a. As α is increased, lift also increases until a maximum is reached at the stall angle of attack (α_{stall}). At stall, there is a rapid decrease in lift and an increase in drag due to massive separation of the flow on the upper surface of the airfoil, as shown in Figure 1b. The airfoil shown in Figure 1 is in cruise configuration; for landing and taking off, leading edge slats and trailing edge flaps are extended to increase lift and to delay stall to a higher angle of attack. In this paper, lift and drag are measured in terms of the normalized (or nondimensionalized) quantities, lift and drag coefficiencts (c_l and c_d).

Experimental results have indicated that an airfoil in heavy rain may be subject to a decrease in maximum lift, an increase in drag, and earlier onset of stall (at a lower α). These effects are most pronounced in high lift configurations with flaps and slats deployed. Since high lift configurations are used in takeoffs and landings when there is little margin for error, the adverse effects of rain may have the most serious consequences in these cases. It is unlikely that heavy rain by itself will cause an accident. However, it may be a contributing cause when other factors such as wind shears are present.

MICROBURSTS

A microburst is a short-lived, thunderstorm-induced local downdraft. As the vertical downdraft winds encounter the ground, a strong horizontal outflow is produced around the downdraft core. Initially, an aircraft encountering a microburst experiences a strong headwind and increases in airspeed and lift as it enters the outflow region, possibly prompting the pilot to decrease thrust or pitch or both. After the aircraft passes the downdraft area, however, the outflow becomes a tail wind, and the resultant decrease in airspeed causes a decrease in lift, often with dire consequences if the aircraft is landing or taking off and especially if thrust or pitch was reduced on the initial headwind encounter. In a study of 75 microbursts the average change in wind velocity encountered by the aircraft was 47 knots, whereas a maximum of almost 100 knots was measured (*4*). It was estimated that the aircraft experienced this velocity change over 20 to 40 sec. Microbursts are often accompanied by heavy rainfall. Thus flight through a microburst may be complicated by the adverse effects of rain on aircraft aerodynamics.

HEAVY RAIN ACCIDENTS

Other accidents and incidents in addition to the Pan American World Airways Flight 757 accident have occurred during very heavy rainfall. Several before 1982 are mentioned by Luers and

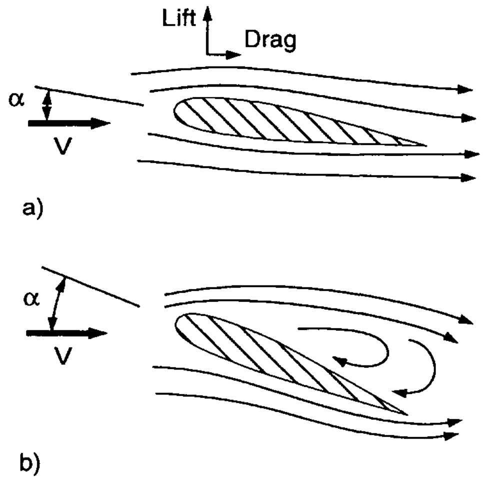


FIGURE 1 Streamline patterns for the flow around an airfoil in cruise configuration: (*a*) airfoil at low angle of attack (α), (*b*) stalled airfoil at high angle of attack (α).

Haines (*5*) and two of these, an Eastern Airlines Flight 066 accident at JFK International Airport on June 24, 1975, and an Eastern Airlines Flight 693 incident at William B. Hartsfield Atlanta International Airport on August 22, 1979, are analyzed. In both cases, Luers and Haines estimate that the aircraft involved may have encountered rainfall rates of 300 mm/hr. These rates could induce a significant aerodynamic performance penalty. However, in neither case did the National Transportation Safety Board (NTSB) report account for rain effects. Another accident occurred on August 2, 1985, when Delta Airlines Flight 191 crashed after encountering a microburst during an intense thunderstorm as it approached Dallas–Fort Worth International Airport for landing. Weather radar indicated a rainfall rate of up to 114 mm/hr, and witnesses described the aircraft as emerging from a wall or curtain of water immediately before ground impact (*4*). Brandes and Wilson (*6*) report that it is not uncommon for radar measurements of rainfall rates to be in error by more than a factor of two and found that in heavy rainfall radar may underestimate the rainfall rate. Thus, a rainfall rate of 114 mm/hr may be less than that actually encountered by the aircraft.

Luers and Haines (*5*) suggest that pilots be made aware of the possibility that aerodynamic stall can occur in heavy rain above the usual stall speed and before the aircraft stall warning system activates. They advise that high angle of attack microburst recoveries, which sacrifice airspeed for altitude, be avoided in favor of an attempt to increase airspeed at a slower climb rate, thus avoiding a rain-induced premature stall. Although they are somewhat controversial, such high angle of attack recoveries have been recommended to pilots of jet-powered aircraft in microburst encounters (*7*).

PHYSICS OF AN AIRFOIL IN RAIN

Several mechanisms have been hypothesized as contributing to the degradation of airfoil (or aircraft) performance in heavy rain. The main ones are the loss of aircraft momentum due to collisions with raindrops, the effective roughening of the airfoil surface due to the presence of an uneven water layer, and the loss of boundary layer air momentum due to the splashback of droplets into the airflow field as raindrops strike the airfoil surface. Bilanin (*8*) has also considered the evaporation of droplets near the airfoil surface and concluded that this process does not significantly affect airfoil performance.

This paper describes a numerical scheme to model the loss of boundary layer air momentum due to splashed-back droplets. As raindrops strike an airfoil, and "ejecta fog" of splashed-back droplets forms at the leading edge, as shown in Figure 2. It has been hypothesized that the acceleration of these droplets in the boundary layer by the airflow field may act as a momentum sink for the boundary layer, resulting in a decreased airflow velocity. Deceleration of the boundary layer can lead to a loss of lift, premature separation and stall, and an increase in drag. By evaluating the boundary layer momentum sink (or source) term, modifications of the boundary layer flow and the resulting change in airfoil performance can be evaluated.

Beneath the ejecta fog layer, a thin water film forms on the airfoil surface because of the fraction of the raindrop that is not splashed back. The thickness of the water film has been measured in small-scale wind tunnel investigations to be of the order of 0.1 mm or less (*10*) and has been estimated at full scale to be about 1 mm or less (*11*). Raindrop impact craters and suface waves in the water film effectively roughen the airfoil surface. The adverse effect of this rougher surface on aerodynamic performance has been analyzed in detail by Haines and Luers (*11*). As the water film is carried downstream, rivulets form on the back portion of the airfoil. With increasing angle of attack, the extent of the water film decreases on the upper surface and increases on the lower surface. When stall is reached, the rivulets disappear and pooling of water occurs on the separated portion of the airfoil.

CHARACTERISTICS OF RAIN

Ground-level rainfall rates are generally measured in terms of millimeters or inches of water accumulation per hour. The heaviest recorded ground rainfall occurred during an intense thunderstorm in Unionville, Maryland, on July 4, 1956, when a rainfall rate of 1874 mm/hr was recorded for a period of approximately 1 min

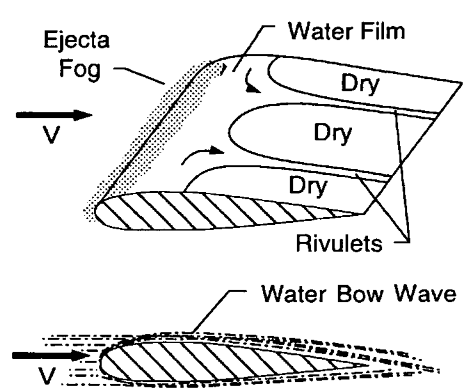


FIGURE 2 A cruise-configured airfoil in rain showing the ejecta fog and water surface film (*9*).

(*12*). Typically, ground-level rainfall rates are much lower than this, with the heaviest rainfalls occurring for short periods of 30 sec or less. Dunham (*13*) has estimated that at any location in the subtropical maritime southeastern United States, a total of approximately 2 min of 200 mm/hr or heavier rainfall can be expected during 1 year.

During a thunderstorm, significantly higher rain intensities than those at ground level can be expected at a higher altitude. The rain measurement parameter used above ground level is liquid water content (LWC) or the mass of water per unit volume of air. LWC is also important in wind tunnel testing, since the same value must be used in small-scale tests as is measured in an actual rainstorm (*8*). Roys and Kessler (*14*) have taken airborne measurements of LWC within several Great Plains thunderstorms and reported an average value of 8.7 g/m^3 and a peak value of 44 g/m^3. At the time and location of the peak airborne measurement, however, ground-based radar indicated a rainfall rate of only 37.6 mm/hr (corresponding to an LWC of about 1.14 g/m^3), possibly because of the small size of the region of extremely intense rain.

The drop size distribution of ground-level rain can be approximated by the expression

$$N(D_p) = N_0 \exp(-\Lambda D_p) \qquad (1)$$

where $N(D_p)$ is the number of raindrops of diameter D_p (in mm) per cubic meter of air per diameter interval and N_0 and Λ are empirically determined parameters dependent on rainfall rate and the type of rainstorm. (*15*). Marshall and Palmer's (*15*) values of $N_0 = 8 \times 10^3$ m^{-3}mm^{-1} and $\Lambda = 4.1 \times R^{-0.21}$, where R is the rainfall rate (mm/hr), have been used commonly for continuous rain, but values of $N_0 = 1.4 \times 10^3$ m^{-3}mm^{-1} and $\Lambda = 3.0 \times R^{-0.21}$ have been found more appropriate for heavy thunderstorm rain (*16*). Raindrop diameters generally range up to about 6 or 7 mm with the larger drop sizes most prevalent in heavier rainfalls.

A relationship between LWC and ground-level rainfall rate can be derived by multiplying the raindrop diameter distribution given by Equation 1 by the mass of the raindrop, then integrating over the range of drop diameters. Assuming a maximum raindrop diameter of 7 mm, the average LWC of 8.7 g/m^3 measured by Roys and Kessler (*14*) corresponds to a rainfall rate of about 546 mm/hr at ground level. In experimental and analytical analyses of aircraft performance in heavy rain, LWCs corresponding to rainfall rates of 500 mm/hr to 2000 mm/hr are commonly used.

HISTORY OF AIRCRAFT HEAVY RAIN STUDIES

The first study of heavy rain effects on aircraft flight was performed by Rhode (*17*) in 1941. He concluded that the most severe performance penalty experienced by a DC-3 flying through a rainstorm with LWC of 50 g/m^3 was due to the loss of aircraft momentum caused by collisions with raindrops. It was estimated that this effect could result in a decrease in airspeed of up to 18 percent, but the duration of the rain would not be sufficient to pose a significant hazard to an aircraft at a cruising altitude of 5,000 ft. Aircraft landing and taking off in heavy rain were not considered; these operations were not routine at that time during low-visibility conditions. Rhode recognized that the surface of the aircraft may be effectively roughened by rain but noted that insufficient test data existed to evaluate this effect.

The current interest in heavy rain effects began with reports by Luers and Haines (*3,5,11*) in 1982. Four mechanisms that could potentially degrade aircraft or airfoil performance in heavy rain were identified: (*a*) the momentum lost by the aircraft due to collisions with raindrops, (*b*) the added weight of a thin water film on the surface of the aircraft, (*c*) the added roughness due to the uneven surface of the water film, and (*d*) a change in pitching moment caused by raindrops striking the aircraft unevenly. The first three were analyzed, and the third appeared to have the most effect on aircraft performance. The added weight of a water film was inconsequential, and the loss of aircraft momentum due to raindrop impacts may be measurable for an aircraft landing or taking off in a torrential rainfall but would not present a significant hazard by itself. However, an effectively rougher aircraft surface due to an uneven water film could have a profound effect on the performance of the aircraft. Estimates of this effect on the aerodynamic performance of a Boeing 747 for various rainfall rates were made. By assessing the roughness of the water layer and comparing the results with correlations for flat plates and airfoils with fixed roughness elements (which were not available at the time of Rhode's study), the increase in drag, decrease in maximum lift, and decrease in stall angle of attack were evaluated. Appraisals of aircraft performance penalties were made for rainfall rates varying from 100 to 2000 mm/hr. Estimates of drag increases ranged from 5 to 30 percent, decreases in maximum lift from 7 to more than 30 percent, and decreases in stall angle of attack from 1 to 6 degrees. The highest penalties were predicted for the highest rainfall rates.

During the last 10 years, wind tunnel investigations of heavy rain effects on airfoil performance in rain have been conducted. There have been two main categories of investigations: those in which the boundary layer on the dry airfoil is predominantly laminar and those in which the boundary on the dry airfoil is tripped to turbulence near the leading edge. The rain effect on a laminar flow airfoil has been mimicked by tripping the boundary layer to turbulence on the dry airfoil, whereas the rain effect on an airfoil with a turbulent boundary layer appears to result from premature flow separation.

An early laminar boundary layer test was conducted with a Rutan VariEze, a small canard-configured sport aircraft (*18*). Pilots of similar aircraft had reported control difficulties in rain (*19*). The canard surface is used for pitch control and is designed to promote laminar flow. It can be very sensitive to any surface roughness that may cause turbulence. In a full-scale wind tunnel investigation, it was discovered that the rain effect is appoximately equivalent to tripping the canard surface boundary layer to turbulence without rain.

Hansman and Barsotti (*20*) examined the performance of a small-scale laminar flow Wortmann FX67-K170 airfoil (similar to those used on sailplanes) with various surface coatings of different wettability in simulated rain. A wettable surface is one on which water spreads out and forms a thin film, whereas an unwettable surface is one on which water tends to form beads. An unwettable surface should develop a larger effective roughness in rain because of the beading effect, and an airfoil with this surface could be expected to sufffer a larger performance penalty. In these experiments, the performance of a waxed (low wettability) Wortmann FX67-K170 airfoil suffered a larger decrease in lift and increase in drag in simulated rain than one with a more wettable unwaxed surface. The rain effect could be partially simulated by tripping the boundary layer to turbulence on the dry airfoil. However, there was also a rain-induced effective change in the camber of the airfoil (evidenced by a decrease in the zero lift angle of attack) that could not be duplicated with a turbulent boundary layer. This apparent change in airfoil shape may have been a result of the very small scale of the airfoil (6-in. chord length) and the inability to scale the surface film and splashback appropriately.

Hansman and Craig (*21*) conducted small-scale tests of three airfoils at wind tunnel speeds low enough (low Reynolds numbers) that the boundary layer can be assumed predominantly laminar. The three airfoils tested were a NACA 0012 airfoil similar to those used as horizontal stabilizers on general aviation aircraft, a NACA 64-210 airfoil characteristic of the type used for many modern transport aircraft, and a Wortmann FX67-K170 airfoil. At low angles of attack, the performance of each airfoil was degraded in simulated rain, with the Wortmann FX67-K170 airfoil, which is designed for laminar flow, suffering the largest penalty, a decrease in lift of up to 25 percent. This performance loss could be mimicked by tripping the boundary on the dry airfoil to turbulence. The NACA 0012 and NACA 64-210 airfoils both exhibited a delayed stall in rain, a result that could be expected if the laminary boundary layer was tripped to turbulence.

There have also been wind tunnel tests of airfoils with turbulent boundary layers, a condition more closely resembling the actual flow around a general aviation or transport type airfoil. NACA 0012, NACA 64-210, and NACA 23015 airfoil sections and wings have been used in experiments (*9,13,22,23*). The overall results of these tests indicate that an airfoil with a turbulent boundary layer in heavy rain may be subject to a decrease in maximum lift, an increase in drag, and premature stall. The effects are most pronounced in high-lift configurations with flaps and slats deployed.

Typical results are those of Bezos et al. (*9*) for a NACA 64-210 airfoil section in simulated wind tunnel rain. A 2.5-ft-chordlength airfoil section mounted between two endplates was tested in both cruise and high-lift configurations, with the boundary layer tripped to turbulence near the leading edge. In cruise configuration, simulated rain resulted in a decrease in maximum lift of up to 17 percent and an increase in drag at constant lift of up to 71 percent. In high-lift configurations with a leading edge slat and a double-slotted trailing edge flap deployed, a decrease in maximum lift of up to 18 percent, an increase in drag at constant lift of up to 40 percent, and a decrease in stall angle of attack of up to 8 degrees were measured. The airfoil showed a greater sensitivity to rain in the high-lift configuration than in cruise configuration. In general, the largest performance penalties were measured at the highest wind tunnel velocities (largest Reynolds numbers) and for the largest values of LWC. The effect of surface wettability was investigated in the high-lift configuration, but no significant change in the performance penalty was measured, in contrast to the results of Hansman and Barsotti (*20*) for a laminar flow airfoil.

Thus in turbulent boundary layer investigations the airfoil performance penalty in rain is most severe at high angles of attack and appears to be due to a rain-induced premature boundary layer separation that can result from either (or both) an effectively rougher airfoil surface or boundary layer momentum loss to splashed-back droplets. The penalty is more pronounced in high-lift configurations, in heavier rainfalls, and at higher air velocities.

The primary value of small-scale wind tunnel experiments lies in the extrapolation of the results to full scale, and Bilanin (*8*) has examined scaling laws for this purpose. Geometric scaling problems were among several difficulties noted. In small-scale wind tunnel investigations the thickness of the water surface film and the splashback process probably will not be scaled by the same factor as the airfoil itself. Thus the airfoil shape may be effectively changed, as was observed by Hansman and Barsotti (*20*). The water surface film will probably be too thick at small scale. Thus slots between flaps or flaps and the main body of the airfoil will be blocked more than at full scale, possibly resulting in a wind tunnel overprediction of the actual performance penalty in rain. Bilanin (*8*) has shown that the value of LWC must be conserved between small and full scale, but the drop diameters must be scaled. Scaling of drop diameters in a wind tunnel investigation reduces the downward velocity of the raindrops, and this in turn affects the incidence angles and locations where drops strike the airfoil. Because of these scaling difficulties, NASA has developed a facility for large-scale testing of a NACA 64-210 airfoil (with a chord length of 10 ft) at the Langley Research Center (*24*).

In addition to the analytical and experimental studies of airfoil performance in heavy rain, there have been numerical investigations. Calarese and Hankey (*25*) added a body force term to the Navier-Stokes equations because of droplet drag and calculated the resulting pressure distribution on a NACA 0012 airfoil for the two limiting cases of very fine rain (small drop diameters) and very coarse rain (large drop diameters). The flow was treated as a continuous, homogeneous rain-air mixture with a set of conservation laws for each phase. For coarse rain, no appreciable change in performance was determined, but for very fine rain, an increase in lift was predicted because of the increase in density of the mixture over that of air alone. This analysis neglected the effect of splashes and surface roughness, however. Kisielewski (*26*) added a force due to droplet drag to the Euler equations and used a flux vector splitting scheme to solve for the resultant flow field around a NACA 0012 airfoil section. He was unable to duplicate the performance penalty measured experimentally for similar rain conditions, however, and recommended that investigations of the effects of surface roughness and splashback be carried out. Donaldson and Sullivan (*27*) estimated the momentum sink experienced by the boundary layer due to splashed-back droplet drag and added it to a boundary layer code. They concluded that a rainfall rate of 500 mm/hr may be sufficient to induce premature stall of a commercial transport aircraft. Bilanin et al. (*28*) also evaluated the effect of splashed-back droplet drag on the boundary layer and reached a similar conclusion—that this deenergization of the boundary layer could cause an early separation. However, they noted that the effectively rougher surface of the wet airfoil can also play a role in this process and that relative importance of these two mechanisms is unknown.

NUMERICAL METHOD

The numerical scheme used in this project models the two-phase flow of rain (particulate phase) and air (fluid phase) over an airfoil. Two approaches are commonly used to model fluid-particle flows. These models have been reviewed by Decker and Schafer (*29*) and Durst et al. (*30*), among others. The "two-fluid" or Eulerian model treats both the fluid and dispersed particle phases as continuous and solves the appropriate conservation equations for each flow. Interphase exchanges of mass, momentum, and energy are included as source terms in the appropriate conservation equations. This model is most easily implemented when particles are of a uniform size.

The "tracking" or Lagrangian approach involves solving a set of Eulerian conservation equations for the continuous fluid phase, then solving Lagrangian equations of motion to determine particle trajectories. A one-way momentum coupled model assumes that the particle motion is influenced by the fluid phase through drag but that the fluid flow field is unaffected by the presence of particles. A fully two-way coupled model, as used here, accounts for the two-way exchange of momentum (and mass and energy if applicable) between the particle and fluid phases through inclusion of source terms in the fluid conservation equations.

The present model consists of a thin-layer Navier-Stokes code for the calculation of the airflow field and a particle tracking scheme for determination of raindrop trajectories. The two-phase flow field is evaluated with a particle-source-in cell technique (*31*), as shown in Figure 3. The fluid and particle fields are initially calculated, then the fluid phase is updated, this time accounting for particle effects, and the particle trajectories are recalculated in the new fluid flow field. The process is repeated until a stationary solution is reached. Interphase momentum coupling is through drag forces. Drag forces acting on particles influence the particle trajectories, and when sufficient numbers of particles move with velocities other than the fluid velocity, the fluid flow field is influenced by particle drag. Raindrop impacts on the airfoil surface and the resulting breakup and splashback of droplets into the airflow field are modeled in the particle-tracking algorithm.

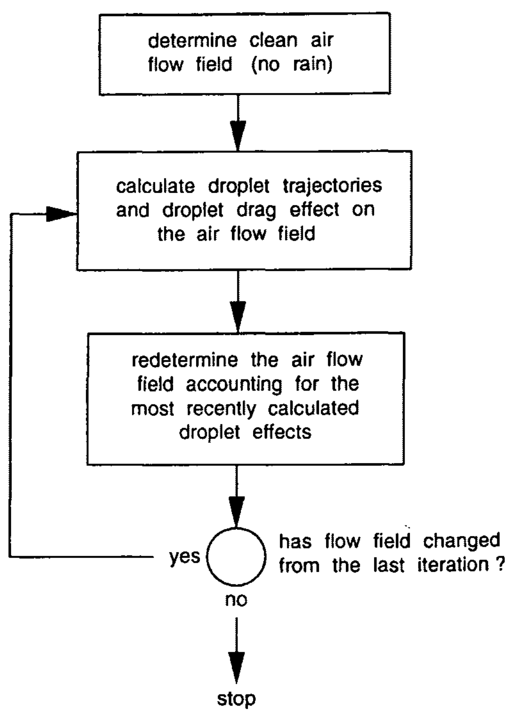


FIGURE 3 Particle-source-in cell algorithm (*31*) for the determination of two-phase particle/fluid flows. Beginning with a clean airflow field (no particles), the particle trajectories and momentum source/sink terms for the airflow field are determined, then the airflow field is updated, accounting for the particle effects. The process is repeated until the fields are unchanged between successive iterations.

Airflow Field Determination

The motion of the fluid (air) phase is governed by the incompressible Navier-Stokes equations. The airflow field is determined with FMC1, a three-dimensional flux-splitting code for the thin-layer approximation of these equations, the details of which have been reported previously (*32*). The code has been modified to account for interphase momentum coupling by adding a momentum source term due to particle drag to the right-hand side of the Navier-Stokes equations.

For numerical determination of the flow field around an arbitrary shape such as an airfoil, a grid that conforms to the body surface is generally used. In this case, an O-H grid is used around a NACA 64-210 airfoil section, a spanwise cross section of which is shown in Figure 4. Grid dimensions are 45 normal to the surface (ξ) by 3 spanwise (η) by 143 circumferential (ζ). This grid defines a curvilinear $\xi\eta\zeta$ coordinate system that is used to track particles. There is no variation of the flow field in the spanwise direction.

Particle-Tracking Algorithm

Raindrops are represented by nonevaporating (no mass coupling between the phases), noninteracting (no collisions between drops), and nondeforming spherical particles (drag on a sphere is easily determined) subject only to drag and gravity forces. In reality, raindrops will deform because of shear stresses as they enter the airfoil boundary layer, but, on the basis of a Weber number criterion (*33*), breakup of the drops should not occur.

Particle trajectories are determined by Newton's second law of motion. The particle equation of motion can be written in nondimensional form as

$$\frac{d\overline{\mathbf{V}}_p}{dt} = \frac{3\rho c C_D|\overline{\mathbf{V}} - \overline{\mathbf{V}}_p|}{8 r_p \rho_p}(\overline{\mathbf{V}} - \overline{\mathbf{V}}_p) + \frac{c}{\mathbf{V}_\infty^2}\,\overline{\mathbf{g}} \tag{2}$$

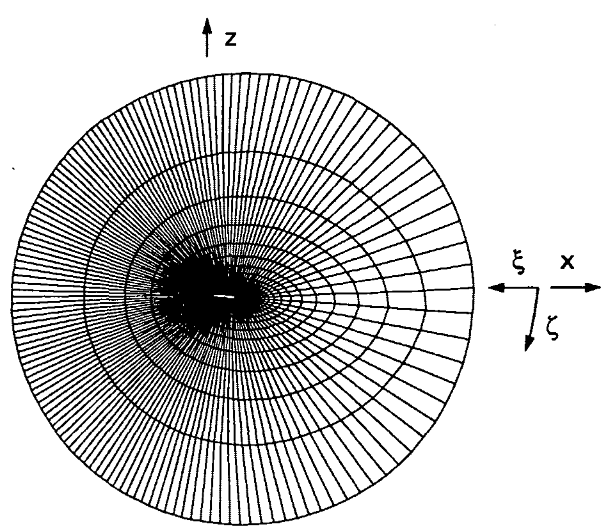


FIGURE 4 Spanwise cross section of the computational grid around a NACA 64-210 airfoil. Grid dimensions are 45 normal to the surface (ξ) by 3 spanwise (η) by 143 circumferential (ζ). *y* is spanwise.

where

$\bar{\mathbf{V}}$ and $\bar{\mathbf{V}}_p$ = air and particle velocity vectors,
ρ and ρ_p = air and particle material densities,
r_p = particle radius, and
$\bar{\mathbf{g}}$ = acceleration of gravity.

The first term on the right-hand side of Equation 2 represents the drag force acting on a particle, and the second term represents the gravitational force. For consistency with the airflow field, the variables in Equation 2 are nondimensionalized in the same manner as the Navier-Stokes equations; velocities are scaled by the free stream air velocity $\mathbf{V}_\infty$, lengths by the airfoil chord length c, and time by c/V_∞. The drag coefficient in Equation 2 can be represented over a wide range of particle Reynolds numbers by (34)

$$C_D = \max\{0.44, \frac{24}{Re_p}(1 + 0.15Re_p^{0.687})\} \tag{3}$$

where the particle Reynolds number is defined in terms of the nondimensional velocities $\bar{\mathbf{V}}$ and $\bar{\mathbf{V}}_p$ as

$$Re_p = \frac{\rho|\mathbf{V}_\infty(\bar{\mathbf{V}} - \bar{\mathbf{V}}_p)|2r_p}{\mu} \tag{4}$$

A second particle trajectory equation is a chain rule expression for the contravariant particle velocity

$$\frac{d\bar{\xi}_p}{dt} = \bar{\xi}_x u_p + \bar{\xi}_y v_p + \bar{\xi}_z w_p \tag{5}$$

where $\bar{\xi}_p = (\xi_p, \eta_p, \zeta_p)$ is the particle position in the curvilinear coordinate system and u_p, v_p, and w_p are the Cartesian components of the particle velocity. The metric vectors in Equation 5 are defined as

$$\bar{\xi}_x = (\xi_x, \eta_x, \zeta_x) \qquad \bar{\xi}_y = (\xi_y, \eta_y, \zeta_y) \qquad \bar{\xi}_z = (\xi_z, \eta_z, \zeta_z) \tag{6}$$

which are evaluated at the particle position through linear interpolation between the values at adjacent grid points. The subscripts x, y, and z in Equation 6 indicate partial differentiation with respect to the subscripted variable. At grid points, the metrics ξ_x, η_x, ζ_x, ξ_y, and so forth are evaluated with second-order accurate finite differences as described by Anderson et al. (35).

Equations 2 and 5 are two-vector, first-order ordinary differential equations that can be integrated to determine a particle trajectory. Following the example of Crowe et al. (31), Equation 2 is integrated analytically. Over a small time setup of particle travel, the fluid velocity and the particle Reynolds number are assumed approximately constant. Integration of Equation 2 then yields

$$\bar{\mathbf{V}}_p^{n+1} = \bar{\mathbf{V}}^n - (\bar{\mathbf{V}}^n - \bar{\mathbf{V}}_p^n)\exp(-D^n\Delta t) + \bar{\mathbf{g}}\frac{c\,[1 - \exp(-D^n\Delta t)]}{\mathbf{V}_\infty^2 D^n} \tag{7}$$

where superscripts refer to time level and

$$D^n = \frac{3\rho c C_D |\bar{\mathbf{V}}^n - \bar{\mathbf{V}}_p^n|}{8 r_p \rho_p} \tag{8}$$

Equation 5 is integrated numerically with a modified Euler scheme (36). The contravariant particle velocity, $d\bar{\xi}_p/dt$, is first calculated at the current particle position and the current time level n and then is used to predict the next particle position and contravariant velocity at time level $n + 1$. The particle position is advanced using the average of the two velocities

$$\bar{\xi}_p^{n+1} = \bar{\xi}_p^n + \left(\frac{d\bar{\xi}_p}{dt}\right)_{\text{ave}} \Delta t \tag{9}$$

with a time step based on a particle residence time of four steps in the current cell.

Modeling of Rain

The drop size distribution of natural rain can be approximated by Equation 1. For modeling purposes, this continuous spectrum of drop diameters is divided into four discrete intervals each of length $\Delta D_{p,i}$. The number density of raindrops in each interval, $N(\Delta D_{p,i})$, can be calculated by integrating Equation 1 over the interval. Then the average diameter of raindrops in the interval, $D_{p,i}$, can be determined.

Particles are entered into the computational domain from discrete locations around the boundary with an initial horizontal velocity equal to the free stream velocity $\mathbf{V}_\infty$ and an initial vertical velocity determined by equating the gravity and vertical drag forces. Each entry location j has an associated area A_j, so the raindrop number flow rate from entry location j for diameter interval $\Delta D_{p,i}$ can be expressed as

$$\dot{N}_{ij} = N(\Delta D_{p,i})\,(\bar{\mathbf{V}}_{p\infty,i} \cdot \bar{A}_j) \tag{10}$$

where $N(\Delta D_{p,i})$ is the raindrop number density for diameter interval i, and $(\bar{\mathbf{V}}_{p\infty,i} \cdot \bar{\mathbf{A}}_j)$ is the dot product of the free stream velocity of particles of average diameter $D_{p,i}$ and the normal vector to area A_j. Thus for each drop size interval $\Delta D_{p,i}$ and each entry location j, one particle of average interval diameter $D_{p,i}$ is tracked through the domain and has associated with it a raindrop number flow rate $\dot{N}_{ij}$.

Interphase Coupling

Particle drag acts as the momentum coupling between the fluid and particulate phases. It is explicitly accounted for in the particle equation of motion, but a momentum source/sink term must be added to the Navier-Stokes equations to account for the particle drag effect on fluid motion. The momentum source/sink term is determined by tabulating the particle drag throughout the flow field.

Nondimensional particle drag distributions are collected on a per volume basis for each grid cell as

$$\bar{\mathbf{F}}_{\text{drag}} = \frac{1}{V_{\text{cell}}}\sum_{i,j}\left[\frac{1}{2}\pi\frac{r_p^2}{c^2}C_D|\bar{\mathbf{V}} - \bar{\mathbf{V}}_p|(\bar{\mathbf{V}} - \bar{\mathbf{V}}_p)\right]\dot{N}_{ij}\,\Delta t_{p,ij} \tag{11}$$

where velocities are averaged over the time step, V_{cell} is the nondimensional volume of the cell (scaled by the cube of the airfoil chord length), the particle drag coefficienct C_D is determined by

Equation 3, $\dot{N}_{ij}$ is the number flow rate associated with the particle from Equation 10, and $\Delta t_{p,ij}$ is the residence time of the particle in the cell. The bracketed term in Equation 11 represents the nondimensional drag force acting on the particle, and the sum is over all particles that traverse the cell for all diameter intervals i and all particle entry locations j. The vector quantity $\overline{\mathbf{F}}_{drag}$ determined in Equation 11 represents the coupling between the air and particle fluids and is subtracted from the right-hand side of the Navier-Stokes equations to account for the particle effect on fluid motion.

Splashback Model

Modeling of raindrop impacts on the airfoil surface presents a very complex problem, and little literature exists on the characteristics of these types of impacts. Raindrops strike the airfoil at high velocities and at angles varying from perpendicular (high-incidence impact) to nearly tangential (low-incidence impact). Some fraction of the mass of the incident drop is splashed back as droplets, and the remainder is incorporated into the liquid surface film. The fraction of mass splashed back and the diameters, initial velocities, and directions of the splashed-back droplets all affect the momentum sink experienced by the boundary layer. These characteristics of the splash are functions of the incidence angle and velocity of the incoming raindrop, and all change during the duration of splash. Obviously, it will be very difficult, if not impossible, to accurately model this phenomenon, so a relatively simple model is used. This model captures enough of the major characteristics of the splashback process that it can be used to predict, at least qualitatively, a part of the performance degradation experienced by an airfoil in rain.

Feo (*37–39*) has experimentally observed some features of the splashback process. The raindrop impact model used in particle-tracking code and shown in Figure 5 is somewhat loosely based on his observations. For a perpendicular impact (β = 90 degrees), 5 percent of the mass of the incident drop is splashed back over an angular range of θ = 120 degrees centered about the surface normal. Splashed-back droplets have a radius of 10 μm and an initial velocity equal to the velocity of the incident raindrop. For a tangential impact (β = 0 degrees), the angular range of splashback (θ), the initial velocity of the splashed-back droplets, and the fraction of the incident drop mass splashed back all go to zero, whereas the radius of a splashed-back droplet goes to 50 μm. A linear variation is assumed between these two extremes, with the splashback always centered about the surface normal.

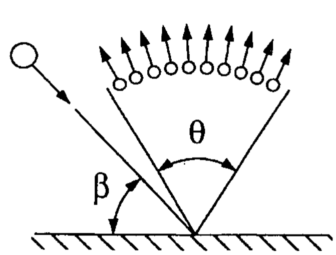


FIGURE 5 Splashback model. Droplets are splashed back over an angular range θ, which decreases linearly from 120 to 0 degrees as the incidence angle β decreases from 90 degrees for a perpendicular impact to 0 degrees for a tangential impact.

RESULTS

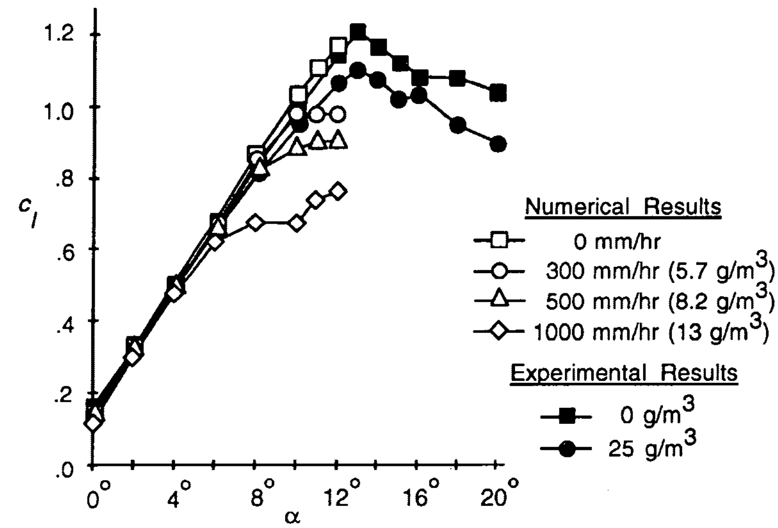
Numerical results are presented for a cruise-configured, 1-m chord length NACA 64-210 airfoil at a Reynolds number of Re = 2.6×10^6 (corresponding to a free stream air velocity of $\mathbf{V}_\infty \approx$ 38 m/sec) and for rainfall rates of 0, 300, 500, and 1000 mm/hr. An eddy viscosity turbulence model (*32*) is activated near the leading edge of the airfoil to simulate a turbulent boundary layer. Turbulent particle dispersion is not considered. The airflow code does not appear to predict stall accurately for flow over the dry airfoil, so numerical results are limted to angles of attack below the stall angle of attack of 13 degrees determined experimentally (*9*).

Figure 6 shows plots of lift coefficient (c_l) versus angle of attack (α) and lift coefficient (c_l) versus drag coefficient (c_d). Wind tunnel results for the same airfoil at the same Reynolds number are also plotted. The numerical results show a decrease in lift and an increase in drag at higher angles of attack, with the penalty becoming more severe as the rainfall rate increases. Very little loss of airfoil performance is evident at low angles of attack, indicating that the loss is apparently due to premature flow separation. Although the rainfall rates used in the numerical simulations correspond to much lower LWCs than those used in the wind tunnel experiments, the performance penalty is larger, probably due to inaccuracies in the raindrop splashback model. Thus, the numerical scheme predicts a rain-induced airfoil performance penalty qualitatively similar to that measured experimentally, but the magnitude of the penalty is overpredicted.

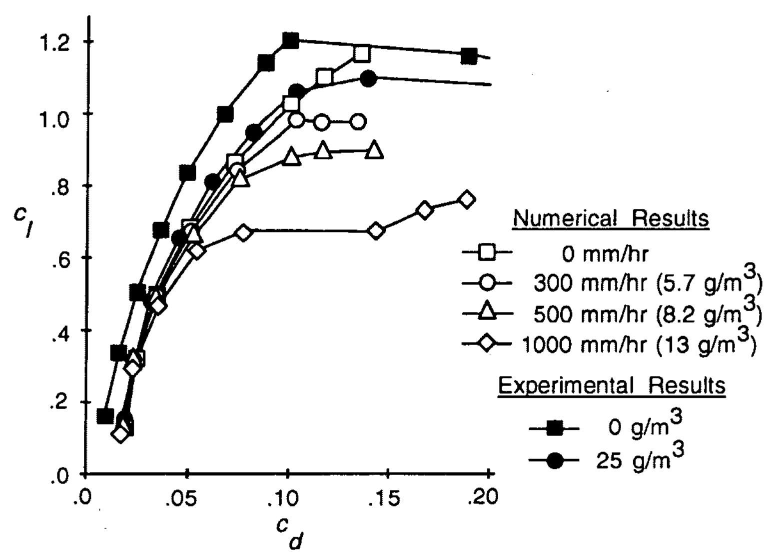
Some features of the c_l versus c_d plot shown in Figure 6b are worth noting. First, the numerical scheme overpredicts the drag determined experimentally somewhat; lift was determined more accurately. At higher angles of attack and for a fixed value of c_l, rain causes an increase in drag. Thus the airfoil is less efficient in rain at these angles of attack. Finally, the experimental results indicate an increase in drag even at lower angles of attack that is not exhibited in the numerical results. This may be due to the effectively rougher airfoil surface, which is not modeled in the numerical scheme.

Figure 7 shows streamline patterns around the airfoil at the highest rainfall rate of 1000 mm/hr. For a rainfall rate of 1000 mm/hr, there is no separation at an angle of attack of 4 degrees, but at 8 degrees a separated region has formed near the trailing edge of the airfoil. When the angle of attack is increased to 12 degrees, massive separation has occurred on the upper surface of the airfoil and the airfoil appears to have stalled. In the absence of rain, there is no obvious separation of the flow at any angle of attack up to 12 degrees. A similar pattern could be seen for increasing rainfall rates at a constant angle of attack; as the rain increases in intensity, a separated region will grow, and the airfoil may eventually stall.

These results show a rain-induced airfoil performance penalty exhibited by a decrease in lift and an increase in drag. The performance penalty results from premature flow separation and is more severe at higher rainfall rates and higher angles of attack. Although the performance loss determined numerically shows the



a) Lift coefficient (c_l) vs. angle of attack (α).



b) Lift coefficient (c_l) vs. drag coefficient (c_d).

FIGURE 6 Numerically determined plots of lift (c_l) and drag (c_d) coefficients for various rainfall rates in mm/hr with corresponding LWC shown in g/m^3. Experimental results (*9*) are shown for comparison purposes.

same overall patterns that have been observed experimentally, it is greater in magnitude.

CONCLUSIONS

A particle-tracking code for an arbitrary curvilinear coordinate system has been developed and incorporated in a two-way momentum coupled scheme to numerically evaluate the performance degradation of an airfoil in heavy rain. Results show a rain-induced performance loss due to premature flow separation, although the magnitude of the loss is overestimated relative to experimental measurements. However, the method shows promise for development of a more accurate predictive tool for the evaluation of airfoil performance in rain.

Some recommendations for further research are as follows: (*a*) numerical experiments to study the effect of variations in the splashback model and possibly improve it, (*b*) revision of the airflow code or incorporation of a different code into the scheme so that stall and poststall behavior are more accurately predicted for the dry airfoil (and presumably for the airfoil in rain also), and (*c*) inclusion of the effective increase in airfoil roughness due to a water film. Variation of parameters in the model may help in the understanding of the splashback process and its role in airfoil

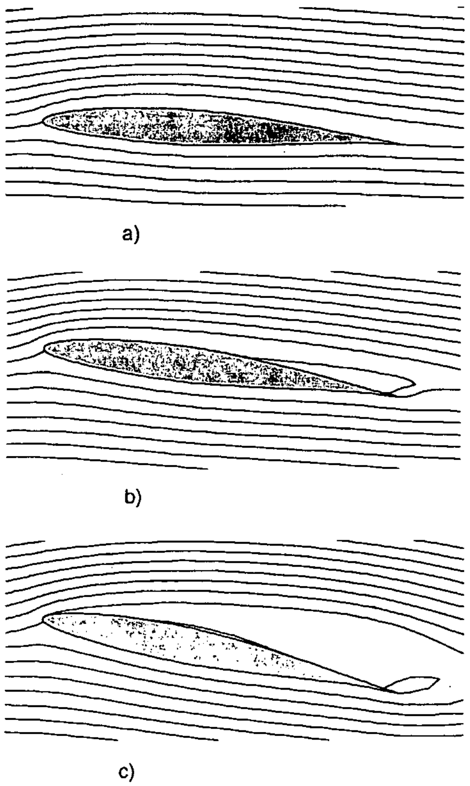


FIGURE 7 Numerically determined streamlines around an airfoil for a rainfall rate of 1000 mm/hr showing the increasing separation of the flow as the angle of attack (α) is increased: *(a)* α = 4 degrees, *(b)* α = 8 degrees, and *(c)* α = 12 degrees. Without rain, no obvious separation occurs at these angles of attack.

performance loss. To more accurately model the splashback process, experimental or analytical studies of the splashback process may be required. The apparent inability of the airflow code to accurately predict stall and poststall behavior is a problem, since premature stall is an important rain effect on airfoil performance. Finally, since the airfoil performance degradation in rain is apparently largely due to two effects, the boundary layer momentum loss to splashed-back droplets studied here and the effectively rougher airfoil surface, it may be useful to include both phenomena in the model.

It may also be advisable to educate pilots on the detrimental effect that very intense rain can have on aircraft performance. Although there seems to be an effort to train pilots in microburst avoidance and recovery techniques, the effect of the heavy rain that often accompanies a microburst appears to be largely overlooked.

ACKNOWLEDGMENTS

This paper was written for the Transportation Research Board's Graduate Research Award Program VII. The author thanks TRB for providing the opportunity to write the paper and the following people for their help and comments: Larry Jenney of TRB, Rand Decker of the University of Utah, Hubert C. Smith of Pennsylvania State University, and C. W. Kauffman of the University of Michigan. The particle-tracking code and two-phase flow model were developed under a NASA contract.

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