Building Transportation Analysis Zones Using Geographic Information Systems

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A model is developed to aggregate transportation analysis zones (TAZs) using fuzzy set theory and spatial analysis tools found in geographic information systems (GIS). The purpose of the model is to provide analysts with standardized mathematical and computerized approaches for network design. Approaches for modeling zonal homogeneity are compared, and a model for evaluating zone shape is presented. Implementation of these models is discussed for Arc/Info and Atlas GIS. The focus of the work described is on aggregating TAZs, but the model applies equally well to creating TAZs from smaller units like census blocks.

In order to model travel demand by the Urban Transportation Planning Process (UTPP), transportation analysis zones (TAZs) must be developed. This zone structure is used in transportation planning and forecasting models at regional and subregional scales. However, when conducting site impact analysis, if the region being modeled is large, planners often use subarea focusing to perform detailed analyses of a smaller area. By aggregating zones outside the specific area of interest, organizations can save considerable time and expense. Aggregating zones is also helpful when working with sketch networks, which have a lower level of detail than typical representations of actual road layouts. Although accurate disaggregate data are preferred in transportation modeling and forecasting, privacy rights along with excessive cost of data collection and processing often restrict agencies from using disaggregate data. Research indicates that gravity model accuracy need not be significantly affected by aggregation (*1–3*). However, a planner must exercise caution in aggregating zones to form new zones because zonal characteristics, such as homogeneity, affect model output (*4–7*).

To help minimize the introduction of error into transportation planning models, various criteria for delineating and aggregating zones have been suggested (*8,9*). These criteria are summarized here.

1. Make zones as homogeneous as possible;
2. Maximize interaction between zones;
3. Avoid irregular or elongated shapes;
4. Avoid creating zones within zones;
5. Use census boundaries as much as possible;
6. Employ other political, historical, and physical boundaries as needed;
7. Aggregate only adjacent zones;
8. Construct zones so that roughly equal numbers of trips are generated and attracted between each pair of zones; and
9. Establish a maximum number of trip ends per zone.

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Different analytical approaches have been developed to create and aggregate zones according to these criteria. Techniques reported in the literature emphasize maximizing interaction between zones (*10*), minimizing information loss (*11*), and measuring proximity (*12*). Some planners do not use these criteria but base the zonal structure on the road network (*13*). All methods vary in their degree of automation.

For the most part, these techniques have been applied at the experimental level. In practice, planners often create new zones by exercising professional judgment. Experience and understanding of an area are invaluable to planners. However, nonquantifiable and sometimes subjective decisions result. Therefore, there is a need to develop a standard procedure that minimizes the adverse effects of aggregation on model output. In addition, standardization would permit better comparison of results among agencies and over time.

Developing a process for aggregating TAZs within a GIS framework promotes standardization. Since transportation data bases are increasingly being built in GIS, the GIS seems a logical place in which to design and aggregate TAZs. Furthermore, GIS graphical capabilities greatly facilitate visual analysis of different aggregations. Since many regional transportation planning agencies are adopting GIS technology, a method that uses this technology may be more readily accepted into practice. Several researchers are linking transportation modeling with GIS (*14–17*). The purpose of this paper is to (a) demonstrate the use of spatial analysis tools in a GIS by modeling some of the criteria stated earlier and (b) present a fuzzy C-varieties (FCV) algorithm as an alternative to a thematic mapping approach to model the homogeneity criterion. The conceptual model for aggregating TAZs using GIS is shown in the flowchart in Figure 1.

Several researchers have applied fuzzy set theory in transportation studies (*18–21*) but not in travel demand modeling. The FCV algorithm presented here, developed in the early 1980s by Gunderson and Jacobsen, is applicable to developing and aggregating TAZs because it provides the planner with greater ability to obtain homogeneous zones based on simultaneous analysis of several planning variables without presuming information about the data base.

The use of thematic mapping procedures to create homogeneous TAZs has not been widely explored either. GIS software packages typically offer users a variety of automated and manual algorithms for defining class ranges for each variable. A separate thematic layer may be developed for each individual variable. The combination of these layers identifies boundaries of homogeneous zones.

Three GIS packages have been used in the development of this paper, namely, Arc/Info, TransCAD, and Atlas GIS. Work reported using fuzzy set analysis used Arc/Info and TransCAD, pri-

marily. Discussions on thematic map classification routines relate directly to Atlas GIS. All of the algorithms or procedures identified here are relevant to all packages.

HOMOGENEITY EVALUATION

Thematic Mapping Procedure

A thematic mapping approach to identifying homogeneous areas differs from the fuzzy clustering approach, or any multidimensional modeling approach, in that each variable is considered independently. With multidimensional algorithms, the question "How similar are Area A and Area B?" is answered by considering all variables at one time. With the thematic mapping approach this question is asked as many times as there are variables describing Areas A and B. The thematic mapping approach is similar to the fuzzy clustering approach in that users must define the number of classes (clusters, ranges) used to aggregate (or group) data.

A danger with using the thematic mapping approach is that no areas with similar characteristics may be identified. For example, suppose a data base consists of three (m = 3) variables, such as population density, employment density, and average income. Further, suppose that each of these three variables is classified into three groups (n_i = 3, for i = 1, 2, 3), such as high, medium, and low. (This example is simplified by assuming that all variables are classified into the same number of groups. Actually, each variable may be classified into any number of groups, n_i, where $n_i < > n_j$.) There are n^m, or 3^3 = 27, possible combinations of values (combined groups) resulting from the overlay of individual layers. The number of possible combinations when $n_i\ n_j$, $\forall(i,j)$ is $G = n_1 * n_2 * \ldots * n_m$. Area 1 may have high population density, high employment density, and high income, or high population density, high employment density, and medium income, and so forth. An example of this is shown in Figure 2. Each of the three variables was classified into four groups such that 25 percent of the observations fell in each group (quantiles). Adjacent cells in the same class are joined into single homogeneous areas. However, when these areas are overlaid on each other, the original 16-cell grid appears.

Another challenge with using the thematic mapping approach to identify homogeneous zones is the selection of the appropriate model for classifying variables. For instance, four automatic and five manual methods are available in Atlas GIS. The four automatic data classification algorithms are as follows:

- Quantiles: ranges for data are determined such that each range contains the same number of observations.
- Equal size: ranges are determined that are equal in size. Range size = (max − min)/number of classes
- Standard deviation: ranges are one standard deviation in size around the mean.
- Optimal: ranges maximize the goodness-of-variance fit that minimizes variance within ranges and maximizes variance between ranges.

The effect of these different algorithms is shown in Figure 3, for the employment density layer. Table 1 presents descriptive statistics generated for each of these ranging approaches. (Data for these demonstrations come from the Salt Lake City regional planning data base. The spatial structure has been simplified by putting it into a 16-grid cell structure.)

Fuzzy Clustering Approach

Fuzzy sets are groupings or classes of objects whose boundaries are not fixed. The concept of fuzziness differs from probability. Fuzzy set theory deals with definition (albeit, fuzzy) of data, not with accuracy. Thus, the ability to describe non-binary aspects of the world is enhanced. Instead of an object being a member of one class and not of others, it receives partial membership in many or all classes. This allows greater flexibility in assigning meaning to fuzzy classes, such as low, medium, and high income.

Fuzzy clustering was selected over other clustering methods because it handles data outliers better. The most common clustering procedure is hierarchical clustering. This method separates n objects into n clusters, then $n - 1$ clusters, then $n - 2$ clusters, and so forth. Once a sample point is assigned to a cluster, it cannot be reassigned to another cluster. This is similar to many taxonomies in which each group is a subset of another group, except for the highest-order cluster. Hierarchial clustering works well with compact and well-separated classes, but it does not do well with sample points that are outliers or fall between two compact centers. If this is the case, outliers need to be eliminated from the data set, which is not permissible with TAZs.

A cluster represents a group of zones with similar demographic characteristics. Each cluster has a center, which represents the average values of each of the socioeconomic variables used to describe the cluster. The FCV algorithm locates a zone in

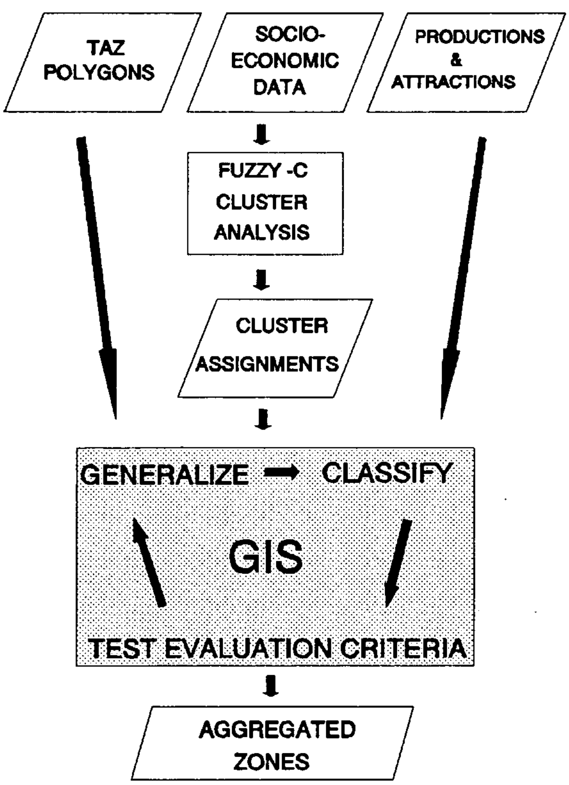

FIGURE 1 Flowchart of TAZ aggregation model.

n-dimensional space where n is the number of socioeconomic attributes used to characterize a zone. Through an initial guess and a series of iterations, the program identifies the n-dimensional coordinate centers of the clusters. The function of the initial guess is to identify the zones that are most dissimilar. Users determine the number of iterations to be performed by specifying a maximum number of iterations, such as 50, and a tolerance on minimum change in membership to be achieved between iterations, such as 0.005.

Instead of the rigid, classical clustering approach of assigning a given zone to whichever cluster it is most similar to, the FCV gives each zone a degree of membership to each cluster on the basis of socioeconomic distance from the cluster center. A membership of 0 means that the particular zone is very dissimilar to the cluster center (average) in terms of its demographic characteristics. A value of 1, on the other hand, indicates that a zone is very much like the center values. However, as with all other clustering techniques, the user must predefine the number of clusters to search for in the data. In the approach described here, the planner specifies different numbers of clusters until each zone has a membership of 80 or 90 percent, for example, in one of the clusters.

The algorithm does not force a class structure on the data. By calculating eigenvalues and the class centers, the algorithm also permits the user to determine the within- and between-class variation. This provides a way to describe how homogeneous the zones really are. Another benefit of knowing the class centers and eigenvalues is the ability to differentiate the importance of certain variables in describing homogeneity. Extraneous variables that do not contribute to the formation of unique clusters may then be excluded from this part of the analysis.

Figure 4 shows three- and four-cluster assignments of zones from the FCV algorithm using population density, employment density, and park and recreational land use density from the Salt Lake City data base. Table 2 contains descriptive statistics for these clusters.

GIS Tools

Regardless of the approach used to define homogeneous areas, the use of certain tools or procedures in GIS packages is required. These spatial analysis tools include classification, evaluation of spatially based criteria, and generalization. A flowchart of a model developed using Arc/Info is shown in Figure 5. TAZ polygon coordinates and FCV or thematic cluster assignments create the data base needed to perform the analysis described here.

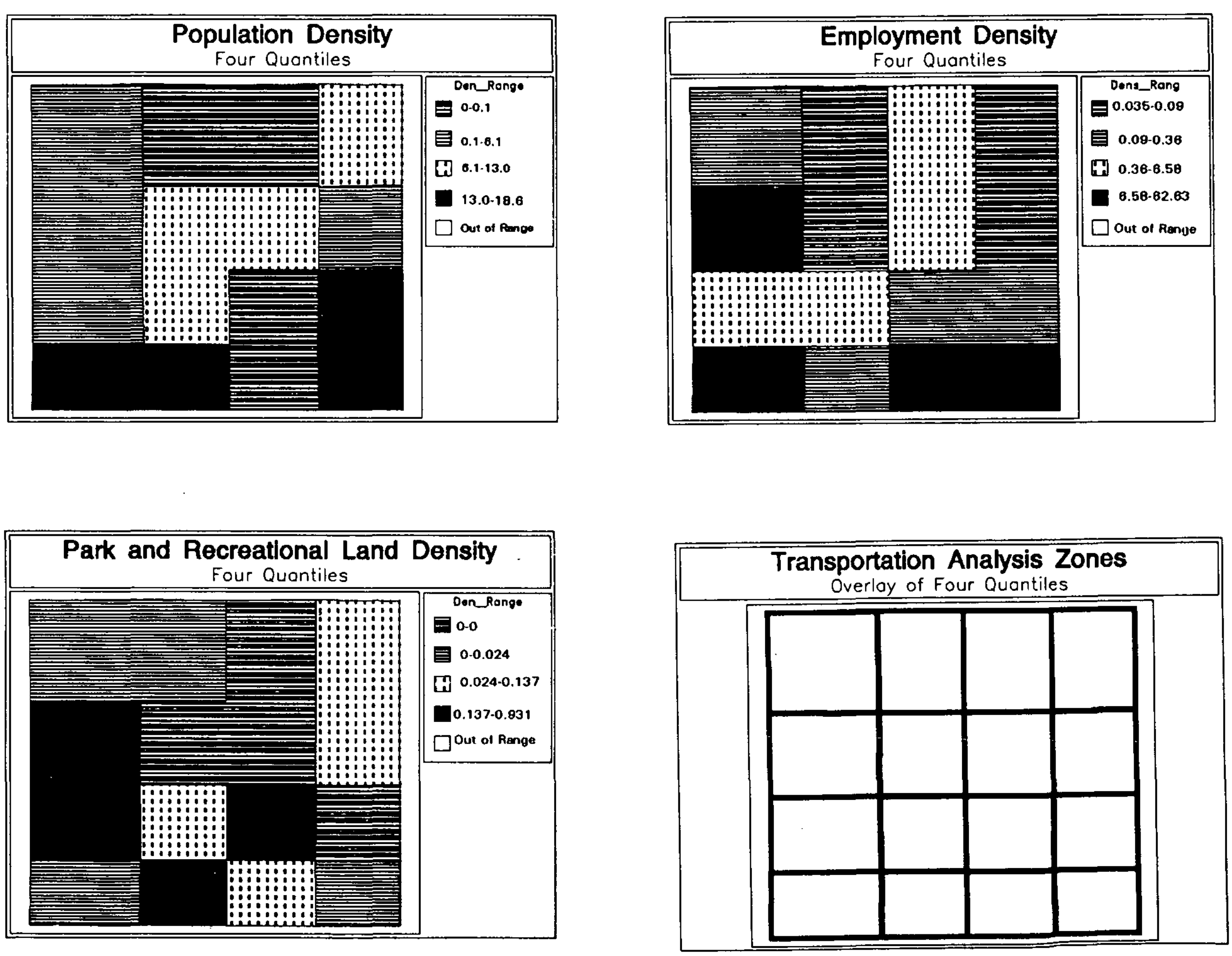

FIGURE 2 Thematic overlay of three variables.

Currently, a routine programmed using Arc Macro Language (AML) which calls executable C programs, has been developed for the fuzzy clustering approach. After invoking this routine, the user specifies the coverage (i.e., TAZ polygons), the item (i.e., cluster assignments) to evaluate, and the value (i.e., zones in Cluster 1, 2, . . .) of the item. Users may aggregate on the basis of a single cluster number or several cluster numbers.

A method for applying the thematic approach is described for Atlas GIS. A separate layer of the base geographic file is created for each variable used in the analysis. This can be accomplished by either opening a geographic file several times under different names (File, Geographic, UseAs) or selecting all features in a geographic layer and writing the selected features to a new file (Select, Layer, File, Geographic, Tools, Write). For each layer, the attribute file structure is changed to contain a blank integer field in which to store the class range value for a zone. (File, Attribute, Tools, Structure). A thematic ranged fill map is generated using the /Replace option. Users must specify the number of ranges and the ranging method (as described earlier). Adjacent areas in the same class can be merged into a single area using the Operate, Union command. Users must specify how each attribute value is to be aggregated to the new zone (i.e., copy first value, leave blank, average, or sum). These new geographic layers are saved and, once all variables have been treated, merged into one geographic file. Separate layers are overlaid, two at a time, to form the final map using the Operate, Union command.

ZONE SHAPE EVALUATION

As mentioned previously, several criteria exist for aggregating zones. The FCV model and thematic mapping address the homogeneity criterion. Analysis of fractal dimensions is used to address shape and compactness criteria. A fractal is defined as

> Objects (or sets of points, or curves, or patterns) which exhibit increasing detail ("bumpiness") with increasing magnification. Many interesting fractals are self-similar. B. Mandlebrot informally defines fractals as "shapes that are equally complex in their details as in their overall form. That is, if a piece of a fractal is suitably magnified to become of the same size as the whole, it should look like the whole, either exactly, or perhaps only after slight limited deformation." (*22*,p.380)

Fractal dimensions are used here to quantify the relationship between the area and perimeter of a polygon. The fractal dimension of a polygon is calculated as

$$f_d = \frac{2 * \ln(T_1)}{\ln(A)} \quad (1)$$

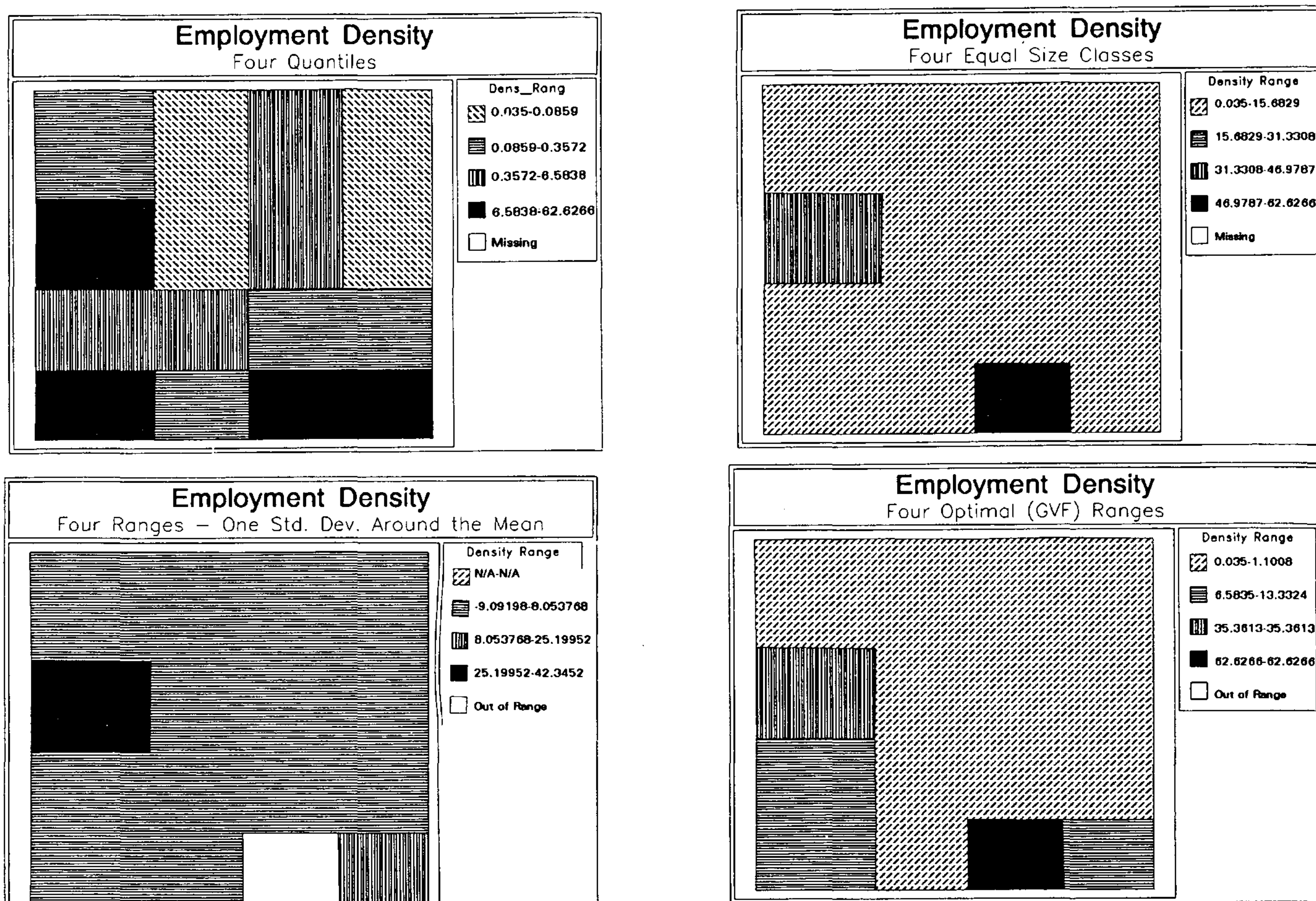

FIGURE 3 Results from automatic thematic ranging techniques.

TABLE 1 Range Statistics for Employment Density Layer

Employment Density Thematic Ranges					
Quantiles					
Class	Minimum	Maximum	Percent Observations in Class	Class Mean	Class Standard Deviation
1	0.035	0.0859	25.0	0.0697	0.0234
2	0.0859	0.3572	25.0	0.203	0.1111
3	0.3572	6.5838	25.0	2.232	2.843
4	6.5838	62.6266	25.0	29.617	25.117
Equal Size					
1	0.035	15.6829	87.5	2.2052	3.989
2	15.6829	31.3308	0	N/A	N/A
3	31.3308	46.9787	6.25	35.361	0
4	46.9787	62.6266	6.25	62.627	0
Standard Deviation					
1	N/A	N/A	0	N/A	N/A
2	-9.092	8.0538	81.25	1.349	2.475
3	8.0538	25.1995	6.25	13.332	0
4	25.1995	42.3452	6.25	35.3614	0
Optimal					
1	0.035	1.1008	68.75	0.3461	0.377
2	6.5838	13.3324	18.75	9.0218	3.744
3	35.3613	35.3613	6.25	35.36	0
4	62.6266	62.6266	6.25	62.627	0

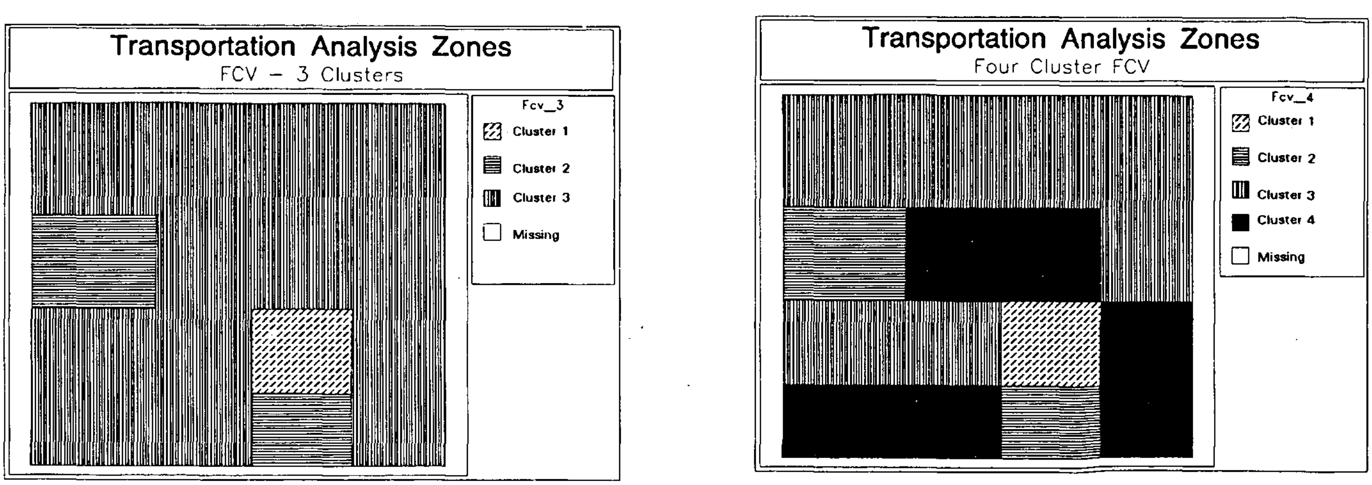

FIGURE 4 TAZs from fuzzy clusters.

where A equals the area of the polygon, and T_1 equals the perimeter divided by $2\sqrt{\pi}$. Note that this equation is scaled, so the fractal dimension of a circle, the most compact geometric shape, is 1 instead of 0.

The approach programmed in an Arc/Info model first identifies all patches in the coverage, where a patch is defined as a group of adjacent zones having the same value of the item being analyzed. The user provides a threshold value, between 0 and 2, that is used to test the shape criterion. A threshold value of 0 forces the shape of the generalized polygons to be more compact than the shape of the original, ungeneralized polygon. A threshold value of 2 allows all polygons to merge regardless of their combined shape.

Within a patch, the model calculates the fractal dimension of each polygon. The polygon with the highest fractal dimension is selected, and an improvement function is calculated for each of its neighbors. The improvement function evaluates the change in fractal dimension if the polygon pair is generalized (i.e., the shared boundary line dissolved). This function is specified as

$$\delta f = 2 * f_{A_1A_2} - f_{A_1} - f_{A_2} \tag{2}$$

where

$f_{A_1A_2}$ = fractal dimension of $A_1 \cap A_2$,
f_{A_1} = fractal dimension of A_1, and
f_{A_2} = fractal dimension of A_2.

If δf − threshold value ≤ 0, then an improvement occurs and the boundary between the polygon pair may be dissolved. The program selects the pair with the greatest improvement in fractal dimension to aggregate.

A simple example of the generalization process is demonstrated using the four polygons shown in Figure 6. Table 3 gives calculation results for the first iteration of this example. Polygon A_4 has the highest fractal dimension in this patch so it is considered first. The fractal dimension of the intersection of A_4 with each of its neighbors (A_2 and A_3) is determined. The change in fractal dimension is determined using Equation 2 and reveals that dissolving the shared boundary between A_4 and A_2 is appropriate.

If the starting rule is changed, so that one begins by looking at the most compact polygon, then one would consider merging A_3

TABLE 2 Fuzzy Clusters Descriptive Statistics

FCV With 4 Clusters					
Cluster	Minimum	Maximum	Percent	Mean	Std. Dev.
1 Emp. Den.	0.18	0.18	6.25	0.18	0
Pop. Den.	0.04	0.04		0.04	0
%Pk & Rec	0.93	0.93		0.93	0
2	35.36	62	12.5	48.99	13.63
	0.00	6.05		3.03	3.03
	0.14	0.21		0.17	0.04
3	0.03	6.58	43.75	1.25	2.21
	0.00	8.48		2.66	3.31
	0.00	0.14		0.05	0.04
4	0.08	13.3	37.5	3.65	4.99
	10.56	18.0		15.72	2.97
	0.00	0.18		0.03	0.07
FCV With 3 Clusters					
Cluster	Minimum	Maximum	Percent	Mean	Std. Dev.
1	0.18	0.18	6.25	0.18	0
	0.04	0.04		0.04	0
	0.93	0.93		0.93	0
2	35.36	62	12.5	48.99	13.63
	0.00	6.05		3.03	3.03
	0.14	0.21		0.17	0.04
3	0.03	13.3	81.25	2.36	3.95
	0.00	18.6		8.69	7.24
	0.00	0.18		0.04	0.06

with A_1 or with A_4. The improvement function shows that A_3A_1 is the best overall pair to aggregate. To further verify the use of the improvement function in this procedure, the authors have shown that aggregating A_3A_2 is the worst option at the initial stage.

A second iteration of this example is given in Figure 7 and Table 4. Polygons A_2 and A_3 will be generalized at the end of this iteration if the threshold value is less than 0.050764.

The aggregation process continues until all polygons in a patch have been tested. After each decision to generalize a zone pair, essentially a new map is created and the procedure repeated for the remaining polygons. Although this method increases processing time, it ensures that the order of processing polygons does not influence the results. For example, suppose a polygon (*A*) with the highest fractal dimension fails the test to dissolve its border with any neighbor. However, after generalizing other polygons (*C,D,E,,,*) in the patch adjacent to the original polygon (*A*'s) neighbor (*B*) but not (*A*) itself, it is discovered that (*A*'s) border with (*B*) should be dissolved. The program must be able to consider aggregating (*A*) at each iteration for the model to be valid.

Experimentation on the study area is required to establish a threshold value. An evaluation of the distribution of fractal dimensions of the original polygons will indicate the degree of compactness of the original map. However, it is necessary to examine how the distribution of the length of common borders affects merging polygons.

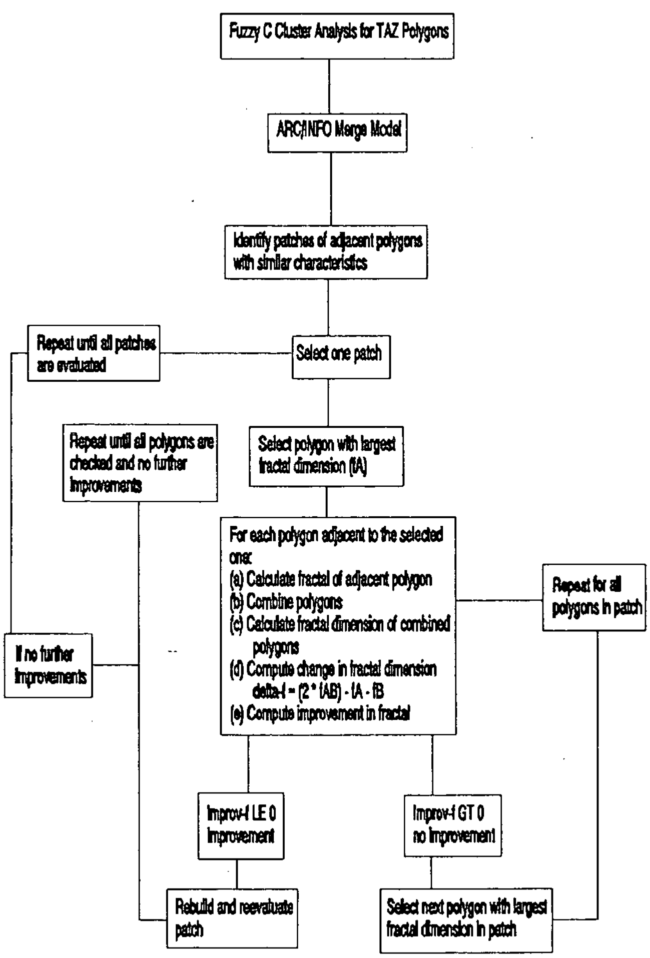

FIGURE 5 Flow chart of Arc/Info based aggregation model.

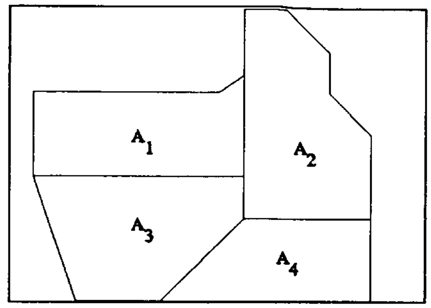

FIGURE 6 Example polygon patch for aggregation.

CONCLUSIONS

GIS is a very useful tool for defining TAZs according to specific criteria established in traffic theory. However, the use of GIS does not preclude the need for experienced, trained transportation planners in the design process. In fact, even the most sophisticated models require sound judgment to produce meaningful results.

This paper explores the use of GIS in addressing homogeneity and shape criteria for developing TAZs. Other TAZ modeling criteria may be incorporated into this model, such as evaluation of intrazonal trips. An AML, script, or C-procedure may be programmed to determine whether the number of intrazonal trips in a merged zone pairs falls below a user-specified tolerance. The tolerance level established here considers the appropriate ratio of intrazonal trips to interzonal trips for the merged pair. If a pair of polygons is chosen to be aggregated from the shape test, a trip distribution routine calculates the intrazonal and interzonal trips associated with the aggregated pair. A pair must pass this test, as well as the shape test, to be generalized.

Another possible routine to be added to the model considers the size (i.e., area) of the candidate pair for generalization and the centroid-to-centroid distance from this pair to the study area site. The farther a pair is from the study area, the larger it may be.

As mentioned earlier, one benefit of integrating FCV and GIS spatial analysis tools is that it standardizes a procedure for developing and aggregating zones. A process that is fairly well automated and quantitatively based decreases the excessive time requirements and subjectivity usually present. This model incor-

TABLE 3 Results of Example, Iteration 1

Polygon	Area	Perimeter	Fractal Dimension	Improvement Function
A_1	253.0	70.61	1.081312	
A_2	300.0	74.14	1.066115	
A_3	287.5	69.95	1.053578	
A_4	200.0	64.14	1.093010	
A_4A_2	500.0	108.28	1.100378	0.041631
A_4A_3	487.5	105.81	1.097409	0.048232
A_1A_3	540.5	90.56	1.029957	-0.074970
A_2A_3	587.5	134.09	1.139608	0.159523

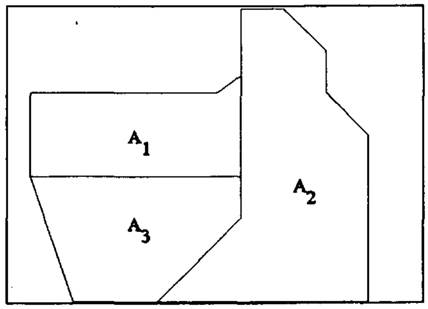

FIGURE 7 Second iteration map.

TABLE 4 Results of Example, Iteration 2

Polygon	Area	Perimeter	Fractal Dimension	Improvement Function
A_1	253.0	70.61	1.081312	
A_2	500.0	108.28	1.100378	
A_3	287.5	69.95	1.053578	
A_2A_1	753.0	154.89	1.140449	0.099208
A_2A_3	787.5	139.95	1.102360	0.050764

porates the experience and judgment of the user using thresholds and tolerance levels. The planner also must decide how many FCV clusters or thematic ranges and the ranging method to use, and therefore the degree of homogeneity.

The evaluation criteria need to be put into algorithmic form and ranked in terms of sequence and perhaps weight. In addition, the street system will be overlaid on zones aggregated by the given criteria and road patterns used to aid in defining zones. Further research plans also include running a gravity model with aggregate and disaggregate data and comparing the results.

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REFERENCES

1. DeCorla-Souza, P., and N. J. Grubb. Network Focusing: A Tool for Quick-Response Subarea Analysis. *ITE Journal*, Vol. 61, No. 9, 1991, pp. 33–38.
2. Horowitz, A. J. Subarea Focusing with Combined Models of Spatial Interaction and Equilibrium Assignment. In *Transportation Research Record 1285*, TRB, National Research Council, Washington, D.C., 1990, pp. 1–8.
3. Wildermuth, B. R., D. J. Delaney, and K. E. Thompson. Effect of Zone Size on Traffic Assignment and Trip Distribution. In *Highway Research Record 392*, HRB, National Research Council, Washington, D.C., 1972, pp. 58–75.
4. Fotheringham, A. S., and D. W. S. Wong. The Modifiable Areal Unit Problem in Multivariate Statistical Analysis. *Environment and Planning A*, Vol. 23, 1991, pp. 1025–1044.
5. Baass, K. G. Design of Zonal Systems for Aggregate Transportation Planning Models. In *Transportation Research Record 807*, TRB, National Research Council, Washington, D.C., 1981, pp. 1–6.
6. Griffith, D. A., and K. G. Jones. Explorations into the Relationship Between Spatial Structure and Spatial Interaction. *Environment and Planning A*, Vol. 12, 1980, pp. 187–201.
7. Openshaw, S. Optimal Zoning Systems for Spatial Interaction Models. *Environment and Planning A*, Vol. 9, 1977, pp. 169–184.
8. Garber, N. J., and L. A. Hoel. *Traffic and Highway Engineering*. West Publishing Co., New York, 1988.
9. Sammons, R. Zone Definition in Spatial Modeling. In *Resources and Planning* (B. Goodall and A. Kirby, eds.), Pergamon Press, Oxford, 1979, Ch. 4.
10. Masser, I., and P. J. B. Brown. An Empirical Investigation of the Use of Broadbent's Rule in Spatial System Design. In *Spatial Representation and Spatial Interaction* (I. Masser and P. J. B. Brown, eds.), Martinus Nijhoff, Boston, Mass., 1978, Ch. 3.
11. Batty, M. Speculations on an Information Theoretic Approach to Spatial Representation. In *Spatial Representation and Spatial Interaction* (I. Masser and P. J. B. Brown, eds.), Martinus Nijhoff, Boston, Mass., 1978, Ch. 6.
12. Miller, D. G., and N. L. Nihan. The Subarea Focusing Concept for Trip Distribution in the Puget Sound Area. In *Transportation Research Record 610*, TRB, National Research Council, Washington, D.C., 1976, pp. 37–43.
13. Barrett, W. Building a Regional Transportation Model with GIS Software. *Proc., 1991 GIS for Transportation Symposium,* 1991, pp. 117–126.
14. Anderson, L. D. Applying Geographic Information Systems to Transportation Planning. In *Transportation Research Record 1305*, TRB, National Research Council, Washington, D.C., 1991, pp. 113–117.
15. Bates, C. A., and T. Schwetz. Linking Transportation Planning and Geographic Information Systems. *Proc., 1990 Annual Conference of the Urban and Regional Information Systems Association*, Vol. 1, 1990, pp. 174–185.
16. Patterson, P. E. An Evaluation of the Capabilities and Integration of Aggregate Transportation Demand Models with GIS Technologies. *Proc., 1990 Annual Conference of the Urban and Regional Information Systems Association*, Vol. 4, 1990, pp. 330–342.
17. Prastacos, P. Integrating GIS Technology in Urban Transportation Planning and Modeling. In *Transportation Research Record 1305*, TRB, National Research Council, Washington, D.C., 1991, pp. 123–130.
18. Shaw, S. Design Considerations for a GIS-Based Transportation Network Analysis System. *Proc., GIS/LIS '89*, Vol. 1, 1989, pp. 20–29.
19. Chun, L. L., et. al. Freeway Ramp Control Using Fuzzy Set Theory for Inexact Reasoning. *Transportation Research*, Vol. 24A, 1990, pp. 15–25.
20. Kim, K. W., and S. Amirkhanian. Evaluation of Effectiveness of Antistrip Additives Using Fuzzy Set Procedures. In *Transportation Research Record 1323*, TRB, National Research Council, Washington, D.C., 1991, pp. 112–122.
21. Zhang, Z., N. Singh, and W. R. Hudson. A Comprehensive Ranking Index for Flexible Pavement Using Fuzzy Sets Model. In *Transportation Research Record 1397*, TRB, National Research Council, Washington, D.C., 1993.
22. Pickover, C. A. *Computers, Pattern, Chaos and Beauty: Graphics from an Unseen World*. St. Martin's Press, New York, 1990.

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