Belief-Function Framework for Handling Uncertainties in Pavement Management System Decision Making

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Belief functions, otherwise known as the Dempster-Shafer theory of evidence, were applied to pavement management system (PMS) decision making. The theory has been advocated by many as a method of representing incomplete evidence of a system's knowledge base. Dempster-Shafer theory has attracted much attention in the artificial intelligence community in recent years because it suggests a coherent approach to aggregate evidence bearing groups of mutually exclusive hypotheses. Two related issues in PMS decision making are examined: (a) the handling of overall uncertainty in project-level and network-level decisions and (b) the handling of incomplete and imprecise data and information. A prototype evidential decision network for pavement management is constructed to illustrate the applicability of the theory. The resulting formulation demonstrates that many of the shortcomings of alternative methods of handling uncertainty may be overcome.

Recently, there has been considerable interest in addressing uncertainties in pavement management systems (PMS) decision making. Attoh-Okine (*1,2*) proposed the use of Bayesian influence diagrams, a type of directed acyclic graphs (DAGs). DAGs express outcomes in terms of combinations of primitive events. In addition, the graphical structure of these models captures the dependency structure among events, enabling the decision maker to exploit conditional independence to reduce specification and computation. Attoh-Okine (*1,2*), using influence diagrams and value-of-information analysis, addressed the question of perfect and imperfect information in PMS decision making under uncertainty. Bayesian influence diagrams provide users with a clear view of the variables in a PMS framework and the relationship between them. Madanat (*3*) used the latent Markov process, which explicitly recognizes the presence of measurement errors in facility condition assessment. Madanat (*3*) uses a methodology "value of more precise information," which allows the decision maker to evaluate various measurement technologies with different precisions and costs and shows how the methodology fits into a PMS framework. Kulkarni (*4*) discussed the application of Markovian decision processes in PMS decision making. Using the fact that the behavior of pavement is not deterministic but probabilistic, Kulkarni developed probability-based decision making in PMS.

Although there are currently several alternatives for addressing uncertainty in pavement management, they have several shortcomings.

1. They have difficulty handling incomplete or conflicting evidence. It is well understood that many data bases for pavement management are quite incomplete. Because data collection for condition assessment can be performed in several ways, conflicting evidence is quite common in pavement management.
2. They have difficulty incorporating updates or corrections in evidence. As new techniques for measurement and data collection emerge, updates in hypotheses relevant to pavement decision making will increase in frequency.
3. They have difficulty addressing the nested or hierarchical nature of hypotheses for pavement management. Hypotheses for pavement management can typically be broken down into subhypotheses. For example, the validity of pavement performance can be split into the validity of data collection and the validity of the prior condition assessments.
4. They have difficulty generating alternative solutions.
5. They have difficulty taking advantage of all available information.

One characteristic feature of the previous model is that the management of uncertainty in the decision-making process is based on conventional probabilities, an assumption of repetitive situations in PMS data collection and measurements that are possible and can be used readily. Unfortunately, at the present stage of PMS data collection and measurements and with the changing nature of pavement condition and the interaction between various pavement condition variables, it will be appropriate to use the belief-function framework in decision making and in addressing uncertainty. This is because the aforementioned factors (pavement condition, data collection and measurements, etc.) are constantly changing. Furthermore, the uncertainties of subjective judgments are also present when a decision must specify an optimal alternative, like in PMS decision making. Therefore, instead of using a fixed sample frame, one must be able to constantly recognize new relationships between frequency and experience. The aim of this paper is to discuss the belief function, otherwise known as Dempster-Shafer theory, in handling uncertainties in PMS decision making.

Fundamental to the belief function is the representation of uncertain knowledge in the form of basic probability assignment in which the probabilities can be assigned directly to subsets of the states of nature and to individual states of nature. The direct consequence of this kind of assignment is that, although the actual probability of any individual subset of the nature may not be specified, its minimum and maximum values will be specified (*5*).

Given pieces of independent evidence, general inferences may be made about what each piece implies. Dempster-Shafer theory of evidence reasoning allows one to combine evidence in a consistent and probabilistic manner. The theory can be applied to obtain a more complete assessment of what the entire body of evidence

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taken as a whole implies. For example, in obtaining condition assessments for pavement sections several pieces of evidence may be compiled, including equivalent single axle loadings, roughness, rutting, and cracking. Each piece of evidence alone may be used to lend credibility or "belief" to a particular hypothesis; however, it is helpful to know what each piece of evidence implies relative to the set of all possible outcomes.

For example, given roughness measurements, a pavement engineer may be 70 percent confident that a section may be in poor condition within a 5-year period. The engineer is therefore 30 percent confident that the roughness measurements tell nothing. In this case it would be wrong to assume that the remaining 30 percent probability contradicts the notion that the section will be poor in 5 years. This remaining probability should be assigned to the complete solution space. In this way evidence in support of a particular hypothesis does not diminish the strength of future evidence rejecting it. If a Bayesian approach had been followed, then the 30 percent probability would have been assigned to the notion of rejecting the hypothesis. A problem then arises if there is future evidence indicating that the section will not be poor within 5 years. Then no matter how strong this evidence might be it cannot carry a weight greater than 30 percent.

BACKGROUND

Formulation of problems by using the theory involves defining the set Θ to contain all possible outcomes or hypotheses about the problem. Θ is commonly referred to as the *frame of discernment*. An example of Θ in the context of pavement condition 5 years from now might be (excellent, good, fair, poor), where each element represents a particular hypothesis. The basic assignment (BPA) has a range of (0,1) and reflects the quantity of belief in a hypothesis. Given the one piece of evidence stated, we can assign values to the sets H_1 and Θ.

H_1 = (poor),
BPA =.70,
Θ (excellent, good, fair, poor), and
BPA = 1 −.70 =.30.

Note that Θ is not the complement of H_1 but encompasses all possible outcomes, including H_1. Therefore, if later tests give strong evidence (e.g., 60 percent) that the section condition will be "good," then there is sufficient probability to reclaim the belief reflected by this evidence.

Next, the theory uses the quantity known as the *plausibility of a given hypothesis* PL(H). The plausibility is the maximum amount of belief possible, given the amount of evidence negating the hypothesis. Specifically, it is obtained by subtracting the BPA associated with all subsets of the complement of the hypothesis ($\bar{H}$).

The next basic element of Dempster-Shafer theory is the belief function, BEL(H). The belief function measures the amount of belief in the hypothesis only on the basis of the observed evidence. Specifically, it is obtained by combining the BPA of H with that of all of its subsets.

The BEL(H) and PL(H) represent lower and upper limits of belief in the hypothesis, respectively, and form the belief interval. These intervals effectively measure the degree in which further evidence might increase the belief in H. Larger intervals reflect a greater uncertainty in the value of BEL(H). In other words, there is a greater opportunity for additional evidence to further substantiate H.

The belief-function approach is linked to conventional probability by considering a multivalued mapping from one space to another (*5*). Figure 1 shows the concept of multivalued mapping. Let Θ represent the parameter space of interest, let θ represent the region of values where the value might lie, and let $\theta \subset \Theta$ represent each individual possible value; T will represent a probability space with the probability density μ_T on it; $\Gamma(t) \subset \Theta$ will represent a multivalued mapping from T to Θ, which means that an observation t in T is equivalent to the observation that the true value of θ is $\Gamma(t) \subset \Theta$. The conventional probability distribution μ_T in T is called *imprecise probability distribution on* Θ.

The belief-function approach (*6*) involves three related representations for belief concerning a topic: the belief function (BEL), the plausibility function (PL), and the basic probability assignment, a generalization of a probability mass distribution. Let the Θ frame of discernment be a set of mutually exclusive and exhaustive hypotheses about some problem domain. A basic probability assignment (bpa) is a function m from 2^Θ, the power set of Θ to (0,1), such that

$$m(\phi) = 0 \tag{1}$$

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{2}$$

The quantity $m(A)$, called A's basic probability number, corresponds to the measure of belief that is committed exactly to hypothesis A in general and not to the total belief committed to A. Hence, a belief function is defined as BEL induced by a bpa m by

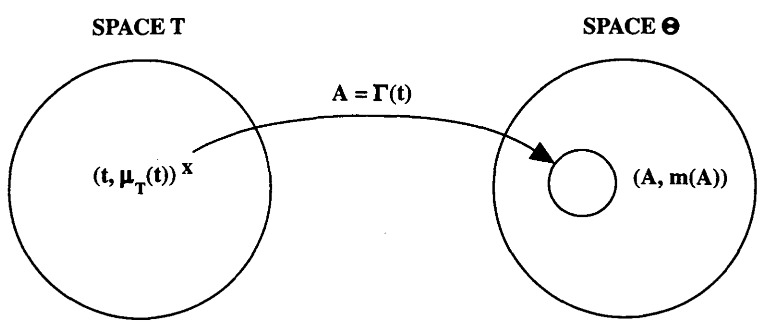

FIGURE 1 Multivalued mapping from T to Θ, which generates a bpa on Θ.

$$\mathrm{BEL}(A) = \sum_{B \subseteq A} m(B)\ (A, B \subseteq \Theta)$$

$$m(A) = \sum_{B \subseteq A} (-1)^{|A-B|}\,\mathrm{BEL}(B) \tag{3}$$

$A - B$ denotes A $\cap$ $\tilde{\mathrm{B}}$, and $|\mathrm{A} - \mathrm{B}|$ denotes the cardinality of this set. m-values either can be assigned directly by the decision maker basis of subjective judgment or they can be derived from compatibility relationships between a frame with known probabilities and the frame of interest. From Equation 3

$$\mathrm{BEL}(A) + \mathrm{BEL}(\bar{A}) \leq 1 \tag{4}$$

a nonadditive formalism. This is different from probability (Pr) theory in which

$$\Pr(A) + \Pr(\bar{A}) = 1 \tag{5}$$

From Equation 4

$$\mathrm{BEL}(A) \leq 1 - \mathrm{BEL}\ (\bar{A}) \tag{6}$$

The quantity $1 - \mathrm{BEL}(\bar{A})$ is called the plausibility of A and is denoted by PL(A). Intuitively, the plausibility of A is the degree to which A is plausible in light of the evidence. A zero plausibility for a hypothesis means that we are sure that it is false, but a zero degree for a preposition means only that we see no reason to believe the preposition.

Notice that each function from {m, BEL, PL] uniquely determines the other two. The equation

$$\mathrm{BEL}(A) + \mathrm{BEL}(\bar{A}) = 1 \tag{7}$$

which is equivalent to

$$\mathrm{BEL}(A) = \mathrm{PL}(A) \tag{8}$$

holds for all subsets of A if and only if BEL's focal elements are all singletons. A subset of A of Θ is called a focal element of BEL if $m(A)$ is greater than 0. By setting $m(\Theta)$ equal to 1 and $m(A)$ equal to 0, for every subset of A of Θ, BEL also satisfies BEL(A) equal to 0 for every subset A; this is called *vacuous belief function.* The BEL indicates no positive beliefs at all to where the truth of Θ lies. This belief-function is appropriate when evidence being considered does not itself tell us anything about which element of Θ is the truth (*6*).

In the belief-function theory, the information about the degree of certainty of an element is represented by the belief interval: [BEL(A) PL(A)]. The belief and plausibility functions denote a lower and an upper bound for unknown probability function. The lower bound represents the degree to which the evidence supports the preposition; the upper bound represents the degree to which the evidence fails to refute the preposition to the degree to which it remains plausible.

If two bpa's on Θ are obtained as a result of two pieces of independent information, they can be combined by using Dempster's rule of combination to yield new bpa's m. The combination can be performed as follows:

$$m(C) = m_1(A) \oplus m_2(B) = \mathrm{K}^{-1} \sum_{A \cap B = C} m_1(A)\, m_2(B) \tag{9}$$

where

$$K = 1 - \sum_{A \cap B = \Phi} m_1(A)\, m_2(B)$$

The second term in K represents the conflict between two items of evidence. If the conflict term is unity, that is, if the two terms contradict each other, K is equal to 0; in such a situation, the two items of evidence are not combinable.

APPLICATION TO PMS DECISION MAKING

The primary advantage of using belief functions in PMS decision making is that each data collection procedure, reliability of various pieces of equipment used for the data collection, pavement performance, and cost analysis can be expressed at a level of detail of its own environment. The ability to represent ignorance concerning the reliability of data collection and the equipment used reduces the likelihood of erroneous interpretation of the overall decision making. Another advantage is that in PMS beliefs are assigned not to a single preposition but to sets of prepositions. Finally, in PMS decision making decision makers have no a priori probabilities of all the variables that form the decision framework.

Figure 2 is a prototype evidential network that represents various objectives in PMS decision making. In this example it is assumed that the overall payoff of the final decision in regard to the maintenance and rehabilitation decision depends on the budget level and the pavement performance. The pavement performance objective depends on data collection procedures, measurements, and how well the previous survey data were interpreted. The objectives are represented with rounded rectangles, and the circular nodes represent relations between the objectives that are of interest to the decision maker. In the present example it was assumed that all the values of the objectives are binary and only "AND" tree relationships may exist among the objectives. Furthermore, it is assumed that there is only one item of evidence for each objective. Figure 3 is an example of an "AND" tree and three nodes. Thus, we will have only one m-value at different objectives. It was assumed that the decision has judgment (although subjective) about the level of support. To determine if the level of payoff is adequate or if there is overall support for payoff on the basis of mutually exclusive evidence, one must aggregate all the evidence to the payoff objective node. This is obtained by propagating the m-values. Because all the objectives are binary we will represent each m-function by triplet [$m(p)$, $m(-p)$, $m(p, -p)$]. For example if $m(p)$ is equal to 0.8, $m(-p)$ is equal 0, and $m(p, -p)$ is equal 0.2, we write mp (0.8, 0, 0.2).

In the present example (Figure 2) there are seven nodes and seven items of evidence. The items of evidence in this present example are considered from the methods and procedures pavement engineers and decision makers relied on to make certain assumptions and decisions on the nodes shown. Table 1 shows this procedure and method. It is assumed that the decision maker has made judgments about the level of support obtained from these procedures and methods for the respective nodes. These values are represented as m-values.

To determine the overall support for each node as a result of aggregating all evidence, we propagate m-values at each node and combine the m-values received by each node from its neighbor with the m-value defined at the node. The combination is done by using Dempster's rules of combination.

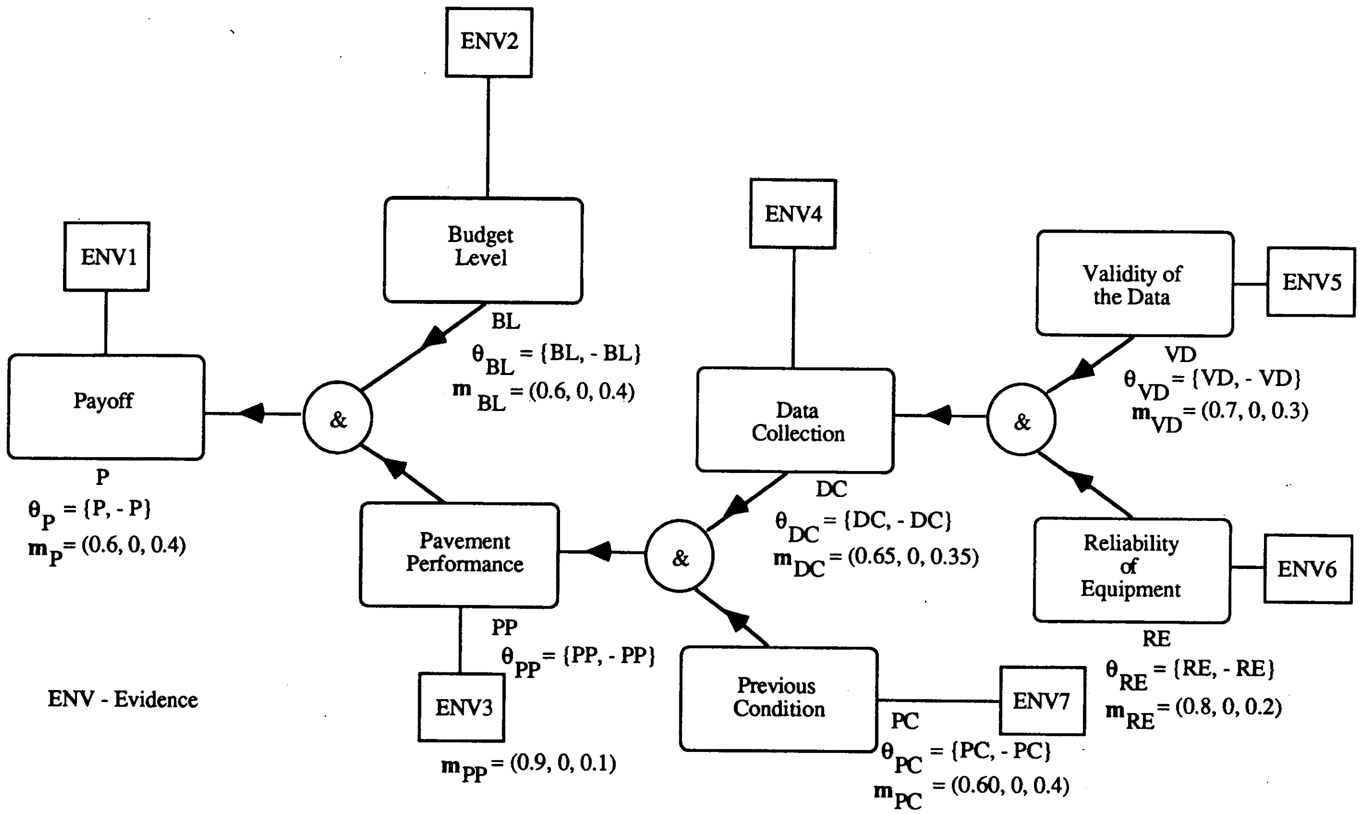

FIGURE 2 Prototype evidential tree for PMS.

In the example we first propagate validity of data (*VD*) and reliability of equipment (*RE*) to data collection (*DC*). This yields $m'_{DC \leftarrow VD+RE}$. The second step is to combine m'_{DC} with m_{DC} to obtain m''_{DC}. The next step is to combine *DC* and *PC* to *PP*, and this yields $m'_{PP \leftarrow DC+PC}$; $m'_{PP \leftarrow DC+PC}$ is then combined with m_{PP} to obtain m''_{PP}. The same steps are used to combine *PP* and *BL* ($m'_{P \leftarrow PP+BL}$), and finally m'_P is combined with m'_P to obtain m''_P. Tables 2 to Table 7 illustrate the propagation and combination of various nodes on the basis of the "intersection tableau" proposed by Gordon and Shortliffe (*7*). In using the intersection tableau, $m_1 \oplus m_2(\phi)$ for any subset is set to be equal to zero. By definition $\Sigma m_1 \oplus m_2\ (X)$ is equal to 1. m''_P is the resulting total *m*-values obtained by all of the mutually

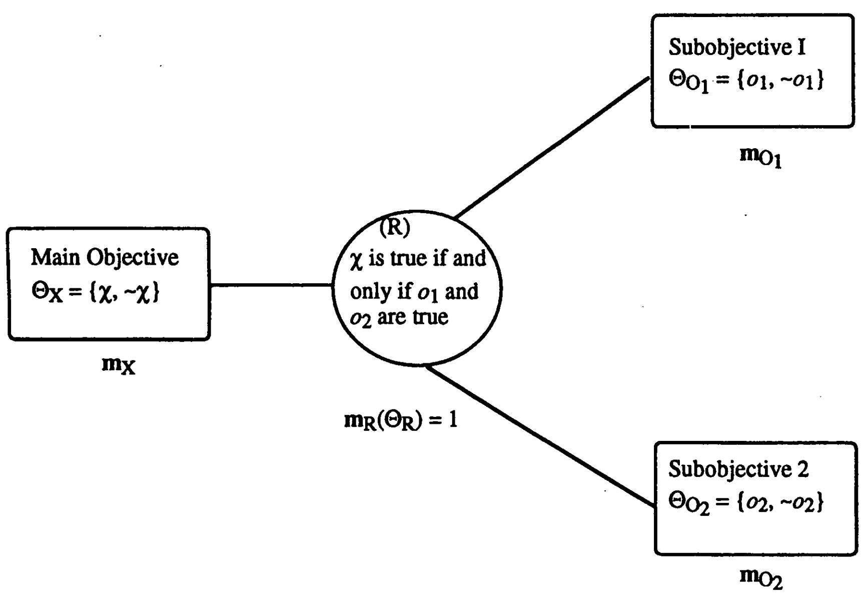

FIGURE 3 "AND" tree with three nodes.

TABLE 1 Potential Sources of Evidence

Evidence Number	Recommended Procedure and Method
1	Prior Years Experience in PMS Decision Making
2	The percentage difference between proposed budget and approved budget
3	Use R-squared obtained from pavement performance equation
4	Frequency of Data Collection
5	Outliers and the distribution pattern should be the major focus
6	They should be based on both operator competence and the reliability of previous collected data.
7	Previous years condition of the pavement based on subjective judgment.

exclusive gathered evidence on the payoff node P. m''_P is (0.072, 0, 0.928).

By definition, the corresponding beliefs are

$$\text{BEL}''_P(P) = 0.072,\ \text{BEL}''_P(-P) = 0,\text{ and }\text{BEL}''_P[(P - P)] = 1$$

and the corresponding plausibilities are

$$\text{PL}''_P(P) = 1 - \text{BEL}''_P(-P) = 1$$

$$\text{PL}''_P(-P) = 1 - \text{BEL}''_P(P) = 0.928$$

The results indicate that there is an overall assurance of 0.072 on the payoff node given the evidence or level of support in the present example that there will be a payoff. $\text{PL}''_P(-P)$ can be expressed as the risk involved in the main objective node (payoff) on the basis of the evidence given.

SUMMARY

This paper illustrates that belief functions can be used in PMS decision making. In designing decision-analytic framework models in PMS, decision makers must formulate relationships between various objectives, incorporate subjective judgment, and pool evidence from various independent sources. The existing analytical tools presently used in PMS decision making do not adequately address such issues in a comprehensive manner.

The belief-function approach provides a more rigorous but straightforward approach to dealing with decision making in PMS with imprecise probabilities and incomplete information from independent sources.

TABLE 2 Combination of m'_{VD} and m_{RE}

m_{VD} / m_{RE}	$(m_{VD} \oplus m_{RE}) \rightarrow m'_{DC}$	
	{VD} (0.7)	θ(0.3)
{RE} (0.8)	φ (0.56)	{RE} (0.24)
θ(0.20)	{VD} (0.14)	θ(0.06)

There is one null entry in the table

∴ $K = 0.56$

$1 - K = 1 - 0.56 = 0.44$, thus

$m_{VD} \oplus m_{RE}\ \{RE\} = 0.24/0.44 = 0.546$

$m_{VD} \oplus m_{RE}\ \{VD\} = 0.14/0.44 = 0.318$

$m_{VD} \oplus m_{RE}\ \{\theta\} = 0.06/0.44 = 0.136$

$m_{VD} \oplus m_{RE}$ is zero for all other subsets θ

TABLE 3 Combination of m'_{DC} and m_{DC}

m'_{DC} / m_{DC}	$(m'_{DC} \oplus m_{DC}) \rightarrow m''_{DC}$		
	{RE} (0.546)	{VD} (0.318)	θ(0.136)
{DC} (0.65)	φ(0.355)	φ (0.207)	{DC} (0.088)
θ(0.35)	{RE} (0.191)	{VD} (0.111)	θ(0.048)

There are two null hypotheses

$K = 0.355 + 0.207 = 0.562$ $1 - K = 1 - 0.562 = 0.438$

$m_{DC} \oplus m'_{DC}\ \{DC\} = \frac{0.088}{0.438} = 0.201$

$m_{DC} \oplus m'_{DC}\ \{RE\} = \frac{0.191}{0.438} = 0.436$

$m_{DC} \oplus m'_{DC}\ \{VD\} = \frac{0.111}{0.438} = 0.253$

$m_{DC} \oplus m'_{DC}\ \{\theta\} = \frac{0.048}{0.438} = 0.110$

$m_{DC} \oplus m'_{DC}$ is zero for all other subsets θ

TABLE 4 Combination of m''_{DC} and m_{PC}

m''_{DC} / m_{PC}	$(m''_{DC} \oplus m_{PC}) \rightarrow m'_{PP}$			
	{DC} (0.201)	{RE} (0.436)	{VD} (0.253)	θ(0.110)
{PC} (0.60)	φ(0.121)	φ(0.262)	φ(0.152)	PC (0.066)
θ(0.40)	{DC} (0.080)	{RE} (0.174)	{VD} (0.101)	θ(0.044)

$K = 0.121 + 0.262 + 0.152 = 0.535$ $1 - K = 0.465$

$m_{PC} \oplus m''_{DC}\ \{PC\} = \frac{0.066}{0.465} = 0.142$

$m_{PC} \oplus m''_{DC}\ \{DC\} = \frac{0.080}{0.465} = 0.172$

$m_{PC} \oplus m''_{DC}\ \{RE\} = \frac{0.174}{0.465} = 0.374$

$m_{PC} \oplus m''_{DC}\ \{VD\} = \frac{0.101}{0.465} = 0.217$

$m_{PC} \oplus m''_{DC}\ \{\theta\} = \frac{0.044}{0.465} = 0.095$

$m_{PC} \oplus m''_{DC}$ is zero for all other subsets θ

TABLE 5 Combination of m'_{PP} and m_{PP}

m'_{PP} / m_{PP}	$(m'_{PP} \oplus m_{PP}) \rightarrow m''_{PP}$				
	{PC}(0.142)	{DC}(0.172)	{RE}(0.374)	{VD}(0.217)	θ(0.095)
{PP}(0.9)	φ(0.128)	φ(0.155)	φ(0.337)	φ(0.195)	{PP}(0.085)
θ(0.10)	{PC}(0.014)	{DC}(0.017)	{RE}(0.037)	{VD}(0.022)	θ(0.010)

$K = 0.128 + 0.155 + 0.337 + 0.195 = 0.815 = 1 - K = 0.185$

$m_{PP} \oplus m'_{PP}\ \{PP\} = \frac{0.085}{0.185} = 0.459$

$m_{PP} \oplus m'_{PP}\ \{PC\} = \frac{0.014}{0.185} = 0.076$

$m_{PP} \oplus m'_{PP}\ \{DC\} = \frac{0.017}{0.185} = 0.092$

$m_{PP} \oplus m'_{PP}\ \{RE\} = \frac{0.037}{0.185} = 0.200$

$m_{PP} \oplus m'_{PP}\ \{VD\} = \frac{0.022}{0.185} = 0.119$

$m_{PP} \oplus m'_{PP}\ \{\theta\} = \frac{0.010}{0.185} = 0.054$

$m_{PP} \oplus m'_{PP}$ is zero for all subsets θ

TABLE 6 Combination of m''_{PP} and m_{BL}

m''_{PP} / m_{BL}	$(m''_{PP} \oplus m_{BL}) \rightarrow m'_P$					
	{PP}(0.459)	{PC}(0.076)	{DC}(0.092)	{RE}(0.200)	{VD}(0.119)	θ(0.054)
{BL}(0.6)	φ(0.275)	φ(0.046)	φ(0.055)	φ(0.120)	φ(0.071)	{BL}(0.032)
θ(0.4)	{PP}(0.184)	{PC}(0.030)	{DC}(0.037)	{RE}(0.080)	{VD}(0.048)	θ(0.022)

$K = 0.275 + 0.046 + 0.055 + 0.120 + 0.071 = 0.567$

$1 - K = 0.433$

$$m''_{PP} \oplus m_{BL}\ \{BL\} = \frac{0.032}{0.433} = 0.074$$

$$m''_{PP} \oplus m_{BL}\ \{PP\} = \frac{0.184}{0.433} = 0.425$$

$$m''_{PP} \oplus m_{BL}\ \{PC\} = \frac{0.030}{0.433} = 0.069$$

$$m''_{PP} \oplus m_{BL}\ \{DC\} = \frac{0.037}{0.433} = 0.085$$

$$m''_{PP} \oplus m_{BL}\ \{RE\} = \frac{0.080}{0.433} = 0.185$$

$$m''_{PP} \oplus m_{BL}\ \{VD\} = \frac{0.048}{0.433} = 0.111$$

$$m''_{PP} \oplus m_{BL}\ \{\theta\} = \frac{0.022}{0.433} = 0.051$$

TABLE 7 Combination of m'_P and m_P

m'_P / m_P	$(m'_P + m_P) \rightarrow m''_{PP}$						
	{BL}(0.074)	{PP}(0.425)	{PC}(0.069)	{DC}(0.085)	{RE}(0.185)	{VD}(0.111)	θ(0.051)
{P}(0.6)	φ(0.044)	φ(0.255)	φ(0.041)	φ(0.051)	φ(0.111)	φ(0.067)	{P}(0.031)
θ(0.4)	{BL}(0.030)	{PP}(0.170)	{PC}(0.028)	{DC}(0.034)	{RE}(0.074)	{VD}(0.044)	θ(0.020)

$K = 0.044 + 0.255 + 0.041 + 0.051 + 0.111 + 0.067 = 0.569$

$1 = K = 0.431$

$$m_P \oplus m'_P\ \{P\} = \frac{0.031}{0.431} = 0.072;$$

$$m_P \oplus m'_P\ \{BL\} = \frac{0.030}{0.431} = 0.070;$$

$$m_P \oplus m'_P\ \{PP\} = \frac{0.170}{0.431} = 0.394;$$

$$m_P \oplus m'_P\ \{PC\} = \frac{0.028}{0.431} = 0.065;$$

$$m_P \oplus m'_P\ \{DC\} = \frac{0.034}{0.431} = 0.079;$$

$$m_P \oplus m'_P\ \{RE\} = \frac{0.074}{0.431} = 0.172;$$

$$m_P \oplus m'_P\ \{VD\} = \frac{0.044}{0.431} = 0.102;$$

$$m_P \oplus m'_P\ \{\theta\} = \frac{0.020}{0.431} = 0.046$$

$m_P \oplus m'_P$ is zero for other subsets of θ

Finally, the framework can be used to quantify the level of uncertainty associated with the payoff node. This is equivalent to the width of the belief interval [$BEL''_P(P)$ $PL''_P(P)$], which is the amount of uncertainty in the main objective payoff node with respect to the items of evidence given. The uncertainty interval associated with the present case study is [0.072, 0.928].

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