Estimation of Speeds from Single-Loop Freeway Flow and Occupancy Data Using Cusp Catastrophe Theory Model

ANNA PUSHKAR, FRED L. HALL, AND JORGE A. ACHA-DAZA

Many freeway management systems rely on single-loop detectors, which can measure only flow and occupancy, for information on freeway operating conditions. Although it is possible to estimate average speeds from those data by assuming a constant vehicle length, such estimates are not particularly good. The catastrophe theory model for these variables provides an alternative procedure for estimating average speeds. To apply it, different procedures are needed to calibrate the model. Such procedures are developed and their generality is tested, by applying them first across different days at the same (double-loop) station, then to other double-loop stations, and finally to single-loop stations. The first two tests allow for direct comparisons with measured average speeds; the final comparison can be made only with other estimated speeds, which is done on the basis of speed-flow diagrams. The results suggest that the catastrophe theory estimates are better than those made assuming a constant vehicle length. Estimates based on concurrent nearby measured vehicle lengths are similar to the catastrophe theory estimates on average, but the former overestimate the scatter and the latter underestimate it.

Many current freeway traffic management systems rely on single-loop detectors, which measure only volume and occupancy and can estimate only average speeds. Such systems would benefit from a model that could provide better estimates of average speed from these measurements, thereby avoiding the higher costs of installation and operation of double-loop detectors. Indeed, reliable values of traffic variables, and especially of system speeds or travel times, are becoming increasingly important as an input for intelligent vehicle-highway systems.

A recent paper compared calculations for speeds of 30-sec flow-occupancy observations on freeways given by catastrophe theory with those produced by a number of other traffic flow models (*1*). It was concluded that for the particular data sets used, the catastrophe theory model was better than any of the others. This paper picks up on that conclusion and addresses the question of whether the catastrophe theory model can be used in a freeway management system to provide reliable estimates of speed from single-loop detectors. This paper also attempts to resolve two difficulties noted in that earlier paper. First, the parameters used in the modeling depended heavily on specific extreme observations in each data set being modeled. Second, the model did not work well in all situations; in particular, it did not provide good predictions for queue discharge flow (i.e., where vehicles are accelerating away from a congested or stop-and-go situation).

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BACKGROUND

The background discussion contains three components. First is a brief review of approaches currently used on freeway management systems for obtaining average speeds. Second, the catastrophe theory model is discussed briefly, with particular reference to Acha-Daza's methods for parameter identification (*2*) and potential changes. Third, the research task for this paper is explained.

Today the most widely used method for estimating average speeds at single-loop stations is based on the use of an average vehicle length. It can be shown that space-mean speed can be calculated on the basis of the equation

$$\text{sms} = \frac{\text{vol} \times 100 \times \text{length}}{\text{occ} \times T} \qquad (1)$$

where

sms = space-mean speed (m/sec),
vol = volume measured over time T,
length = average vehicle length plus effective detector length,
occ = occupancy (%), and
T = interval length (20 sec on Highway 401).

Some errors are intrinsic to the use of a constant vehicle length, one of which relates to the effect of the variance of vehicle lengths (*3*). This difficulty is compounded by the fact that the amount of variation in vehicle lengths (and therefore in the error in using Equation 1) varies over lanes and over the day. In this study, this variance is minimized, as data are taken from only the leftmost lane, from which trucks are restricted.

A second method is available for those systems with some double-loop stations among the single-loop ones, namely, using the double-loop data to estimate vehicle lengths that will be arriving at the adjacent single-loop station in the next time interval. Such a system is used on the Highway 401 COMPASS freeway management system.

A third alternative, the catastrophe theory model, was first applied to traffic operations by Navin (*4*) and has subsequently been developed by Hall et al. (*5–8*). The cusp catastrophe model used in these applications can be represented as the three-dimensional surface given by

$$X^3 + aUX + bV = 0 \qquad (2)$$

In converting this mathematical form to the traffic flow situation, it has been conventional to associate speed with the variable X, volume with U, and occupancy with V.

Acha-Daza's advance (*2*) on earlier work was to apply an axes rotation, as was first suggested by Forbes and Hall (*7*), along with an axes translation as proposed by Dillon and Hall (*6*). In doing the translation, Acha-Daza set the origin for the new axes system at the point of maximum observed flow, and the maximum observed occupancy occurring at that flow. The new axes were then rotated about this origin. For simplicity, the flow and occupancy values that determine this new origin will be referred to as the pivot point. The angle (theta) for the rotation was selected as that angle which minimized the number of misclassified observations, that is, the number of data that were classified as congested on the basis of a critical speed but uncongested (left of the *U*-axis) after the rotation, and vice versa. Acha-Daza selected the minimum speed observed at the pivot point as this critical speed. Acha-Daza's paper also uses a data-specific graphical factor to provide numbers of similar magnitude for calculating the trigonometric functions on which the rotation was based.

There are two problems with this method for selecting the pivot point, critical speed, and graphical factor. First, the parameter values are extremely dependent on the particulars of the data set being used. The overall goodness of fit of the model may well depend on whether the particular set of data being analyzed had observations near the "true" pivot point. Second, selecting parameters using the method identified by Acha-Daza makes it difficult, if not impossible, to generalize the results, and especially to identify appropriate parameters for speed estimation at single-loop stations. Hence one of the tasks in this paper is to designate a method for identifying the pivot point and critical speed that is not so data-specific and can therefore produce a more general set of parameters. This problem of data-dependent parameters is not unique to catastrophe theory models: Ceder (*9–11*) encountered similar problems with more conventional models, and data reported by Koshi (*12*) suggest the same difficulties. Four parameters—pivot point occupancy and volume, critical speed, and graphical factor—need to be set via experimentation. Given these four, the remaining three parameters—theta and the values of a and b in Equation 2—are found via analytical procedures.

The choice of best parameters is not simple, even with measured speeds from the paired loops available with which speed estimates from the model can be compared. The difficulty arises because the mean error obtained from the use of the catastrophe theory model is not 0. For this reason several goodness-of-fit measures are used: the mean of the error (the difference between the observed and predicted speed), the standard deviation of the error, and the average of the square of the errors (labeled average difference squared). This last measure is the sum of the squares of the previous two, and it is useful because the other two do not always reach their minima at the same set of parameter values. In addition to these numerical results, plots of predicted speeds versus observed speeds help in discovering trends and biases.

The task of evaluating estimates for single-loop stations is more difficult. Because there are no observed speeds with which to compare estimated speeds, the statistical measures just described cannot be used. Instead, estimated speed-volume plots based on catastrophe theory were compared with similar plots based on other estimation methods. However, it will still not be known which best indicates the realism of the speed estimates. To resolve this difficulty, the estimated speed-volume plots have also been compared with similar plots of observed variables from adjacent double-loop stations.

To summarize, then, there are two specific tasks in this paper. The first is to find a different way of calibrating the model, one that does not rely as heavily on extreme observations in a specific data set. The second task is to investigate how well the resulting parameters can be transferred or generalized—especially to single-loop stations.

AVAILABLE DATA

The data used for this study were made available by the Ministry of Transportation of Ontario (MTO) through its COMPASS traffic management system for Highway 401 in Toronto, which measures volume and occupancy over 20-sec intervals via inductive loops, located at stations 300 to 800 m apart on the freeway. Double-loop stations (roughly one-third of the total) measure speeds over the approximately 4.5 m between the loops. At single-loop stations, average speeds over the 20 sec are estimated using vehicle lengths obtained during the previous time interval at the next upstream double-loop station. In the rest of the discussion, the speed to be estimated is the average speed over the 20-sec interval.

This study involved the analysis of data from only the leftmost lanes on three-lane sections of the express systems. These data were collected for clear spring days in 1993 (May 12, June 3, and June 7). Although there were some differences in grade between the stations chosen, none was particularly steep. Results from five stations are included here (four westbound, one eastbound). Data from six other stations were also analyzed; the results support those presented here. Station identifiers used in this paper are a short form of the 12-character MTO station identifier. The MTO numbering starts from the control center, counting outward both eastbound and westbound. For example, E01W specifies the first station east of the traffic control center, in the westbound roadway.

The westbound stations discussed here lie in the region between Keele Street and Highway 400. W03W and W04W show queue discharge when the transfer lanes from the collector system between W02W and W03W cause upstream congestion. E02W, W02W, and W04W are double-loop stations; W03W is a single-loop station. Data for these stations were collected from 3:00 to 8:00 p.m. The eastbound station, W07E, is a double loop and shows queue discharge flow (QDF) when an entrance ramp creates a bottleneck situation upstream. Its data were collected from 5:00 to 10:00 a.m.

CALIBRATION TO PRODUCE GENERAL PARAMETERS

This section deals with calibration: what is a good general set of parameter values, and how can they be identified? The focus is primarily on selecting an appropriate combination of critical speed and pivot point. Given these values, the methods described by Acha-Daza and Hall (*1*) have been used to find theta and the coefficients a and b. In identifying good parameters, one regular station has been used for the primary analysis. Acha-Daza found problems in applying the model to stations that exhibit QDF (*2*); hence, data for one such station have also been investigated.

Exploratory analyses conducted prior to the ones reported here showed that the pivot point did not need to coincide with the highest observed volume, as was the previous practice. Further, in many cases it *should not* coincide with that point, because often the point with the highest volume does not fall on the line defining the boundary between the congested and uncongested data. A pivot point that would always fall along the dividing line was needed.

If the maximum volume does not necessarily indicate the best pivot point, one might consider using points with lower volumes. One implication of lowering the pivot volume is that some of the transformed points would then have positive *U*-values (i.e., above the cusp of the catastrophe model). In this region outside the cusp, changes in predicted speed will be gradual rather than sudden. In fact, this might prove to be an advantage. With varying occupancy, perhaps smoother changes in speeds are exhibited at higher volumes (such as within QDF) than at lower volumes.

Recognizing that other parameters were unnecessarily data-dependent, a decision was made about the graphical factor, which was applied to the *V*-axis simply to make the *U*- and *V*-axes of similar scale and units. For simplicity and improved transferability, the graphical factor has been made a constant in this analysis, set at 0.25.

In seeking the best set of parameters, it was decided to investigate them independently—that is, first test critical speed using arbitrary pivot points, then, using the best critical speed, test pivot point values. If the initial arbitrary pivot points prove to be too far from the final pivot point, iteration is possible. For these tests, a single site was selected, Station E02W.

Critical Speeds

From the preliminary investigations, two pivot volumes lower than the maximum observed volume were chosen for the tests of critical speed: 13 and 8 veh/20 sec. Selecting constant occupancies at these pivot volumes would most likely create unnecessarily large numbers of misclassified points, thereby affecting the results. As Gilchrist and Hall (*13*) showed, speeds generally vary in bands that move somewhat diagonally across the volume-occupancy graph. The selection of pivot volume and critical speed together determine what the correct pivot occupancy should be. Pivot occupancies were identified to the nearest 0.5 percent to minimize the misclassified data points as critical speeds were changed.

Tables 1 and 2 show the results of the testing of several critical speeds at the two selected pivot volumes. With a pivot volume of 13 veh/20 sec (Table 1), all three criteria (standard deviation, average difference squared, and mean error) were at their minima for a critical speed of 85 km/hr. For a critical speed of 90 km/hr, however, the three measures are similar to their minima. Hence it may be that the results are not particularly sensitive to the critical speed over some range of values, although clearly a value above 70 km/hr is needed. In that respect, this result is consistent with the new speed-

TABLE 1 Effect of Changing Critical Speeds: Station E02W, May 12, Pivot Volume of 13 veh/20 sec

Critical Speed (km/h)	70	75	80	85	90	95
Pivot Occupancy (%)	16.5	16	15	15	14.5	14
Theta (Degrees)	-18.7	-17.3	-17.0	-17.0	-16.0	-16.0
Misclassified Points	6	3	2	2	3	8
a	273.1	245.9	225	184.7	136.4	95.98
b	8597	12133	15611	21910	29782	38460
Mean Error	11.09	7.28	4.50	**0.97**	-1.92	-4.13
Standard Deviation	9.94	9.14	8.5	**7.98**	8.04	8.58
Average Difference²	222.0	136.6	92	**64.6**	68.3	90.78

TABLE 2 Effect of Changing Critical Speeds: Station E02W, May 12, Pivot Volume of 8 veh/20 sec

Critical Speed (km/h)	80	85	90	95	100	105
Pivot Occupancy (%)	9.5	9	8.5	8	7.5	7.5
Theta (Degrees)	-15.0	-16.0	-15.0	-14.0	-13.2	-12.9
Misclassified Points	3	2	3	6	9	15
a	277.0	266.8	227.8	191.6	168.5	144.0
b	39185	43389	47337	52344	59245	67531
Mean Error	7.03	3.68	**1.12**	**-1.12**	-2.99	-6.68
Standard Deviation	8.41	7.98	**7.70**	7.71	8.18	8.88
Average Difference²	120.2	77.04	**60.31**	60.68	75.81	123.46

flow curves in the revised Chapter 3 of the *Highway Capacity Manual* (*14*), which have limiting speeds for uncongested data (i.e., speeds at capacity) ranging from 50 to 60 mph (80 to 100 km/hr).

Given the results of the testing in Table 1, Table 2 uses a higher range of critical speeds for the pivot volume of 8 veh/20 sec. For this volume, critical speeds of 90 and 95 km/hr had the best numerical results, with their measures of error being virtually identical. The results from Table 2 indicate that the best critical speed to use may depend on the particular pivot volume selected but, more importantly, that the error may be relatively insensitive to changes of up to 10 km/hr in the critical speed. For both pivot volumes, a critical speed of 90 km/hr fell in the optimal range. Hence it was chosen as a general value for subsequent analyses.

Varying the Origin

Using the critical speed of 90 km/hr, a range of pivot points was evaluated. Given the pivot volume and critical speed, occupancies were identified that minimized the number of misclassified data. The similarity in theta and in the number of misclassified points (Table 3) confirms that the pivot points lie on approximately the same line. The measures of error all point to a pivot volume of 9 veh/20 sec as being the best value. However, the measures of error for a pivot volume of 10 veh/20 sec are a very close second. Indeed, for the range of 8 to 12 veh/20 sec, the measures of error do not vary more than about 10 percent, and any of those values might be expected to produce reasonable results. Higher values, such as 17 veh/20 sec, are clearly not as good, although they do not have as deleterious an effect as using too low a critical speed (Tables 1 and 2).

Complications When Considering Queue Discharge Flow

Station E02W, used for the analyses in Tables 1 through 3, did not experience QDF but instead had data from within the stop-and-go conditions that constitute a queue on a freeway. In previous work, the catastrophe theory model did not reliably predict speeds at stations exhibiting QDF (*2*). It appeared reasonable to expect that modifying the choice of pivot point might overcome this problem. Several tests were run on one such station (W07E, May 12). It was found that the same critical speed (90 km/hr) produced reasonable results. However, for pivot volume, lower values were optimal: volumes of 5 and 6 minimized the standard deviation, 7 minimized the average difference squared, and 8 minimized the mean error.

In general, any of the volumes from 5 to 10 veh/20 sec produced reasonable results. The error measures are not especially sensitive to small changes of volume within the range of 5 to 10 veh/20 sec, which overlaps with the range of best pivots for E02W (8 to 12 veh/20 sec). Rather than using different parameters for different traffic flow regimes, a single compromise pivot volume was sought to encompass them all. A pivot volume of 10 veh/20 sec was chosen, implying the approximation that has occurred. Although this value is at the top of the range for QDF, comparing predicted and observed speeds for Station W07E (May 12) (Figure 1) gives graphical assurance that this pivot volume works well at this QDF station. For comparison, Figure 2 shows a similar plot of predicted versus observed speeds for the same station based on the use of Equation 1. For that method, length was set to 5.4 m (17.7 ft), the default vehicle length of 4.4 m used by the MTO plus a 1-m effective detector length. Although this figure shows that the method

TABLE 3 Effect of Changing Pivot Point: Station E02W, May 12, Critical Speed of 90 km/hr

Pivot Vol (veh/20-s)	0	6	8	9	10
Pivot Occupancy (%)	0	6	8.5	9.5	11
Theta (Theta)	-15.0	-15.0	-15.0	-15.0	-16.0
Misclassified Points	3	3	3	3	3
a	33.11	188.4	227.8	221.6	210.1
b	48689	54972	47337	42317	37623
Mean Error	2.24	3.71	1.12	**0.61**	-1.05
Standard Deviation	9.55	8.3	7.70	**7.56**	7.57
Ave. Difference^2	97.73	82.66	60.31	**57.63**	58.36

Pivot Vol (veh/20-s) Pivot	11	12	13	15	17
Occupancy (%)	12	13	14.5	16.5	19
Theta (Degrees)	-16.0	-16.0	-16.0	-17.7	-16.0
Misclassified Points	3	3	3	3	3
a	168.1	165.61	136.4	99.46	75.57
b	31003	31121	29782	28461	28181
Mean Error	-1.89	-1.17	1.92	-1.54	-1.62
Standard Deviation	7.86	7.78	8.04	8.46	8.79
Ave. Difference^2	59.43	61.98	68.3	74.01	79.86

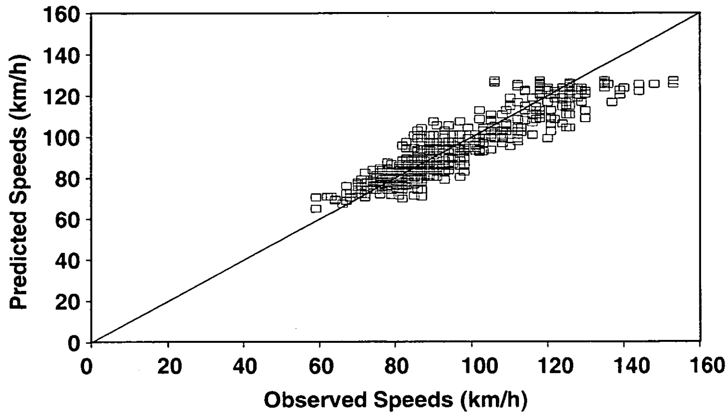

FIGURE 1 Comparison of observed average speeds with those predicted using catastrophe theory model: Station W07E, May 12.

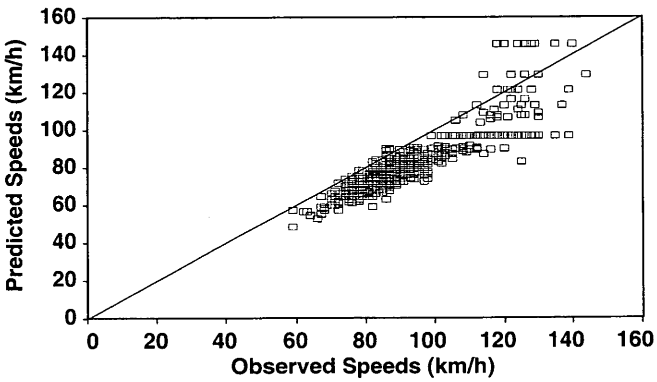

FIGURE 2 Comparison of observed speeds with those predicted on the basis of constant vehicle length: Station W07E, May 12.

produces biased estimates, which could be corrected easily by changing the value of length in Equation 2, other stations showed the opposite bias for the same constant length.

For the rest of the study, a pivot volume of 10 veh/20 sec will be used, with a critical speed of 90 km/hr. The pivot occupancy is the value that minimizes the number of misclassified points and will remain station-specific. Although these new parameters improve on previous work, the catastrophe model still produces a systematic error at some stations, as can be seen in Figure 3.

TESTING PARAMETER TRANSFERABILITY

There are three parts to the analysis of parameter transferability. The first investigates the stability of the parameters over different days at the same station. In previous studies (*9*), one of the troublesome results has been that distinct parameter sets were estimated for different days at the same site. It is essential to be able to overcome this if speed estimates are to be reliable. The second part of the analysis tests whether the ability of the parameter values to be generalized can be extended to other double-loop stations. Applying the values to such stations allows direct matching of speeds estimated using the catastrophe theory model with the measured speeds. The final part of the analysis is the extension to single-loop stations. Although this is the "acid test," it is the hardest to assess, which is why the two preceding sections are included.

Stability of Parameters over Time

The results of testing for parameter stability over different days, as displayed in Tables 4 and 5, were promising. Table 4 deals with a

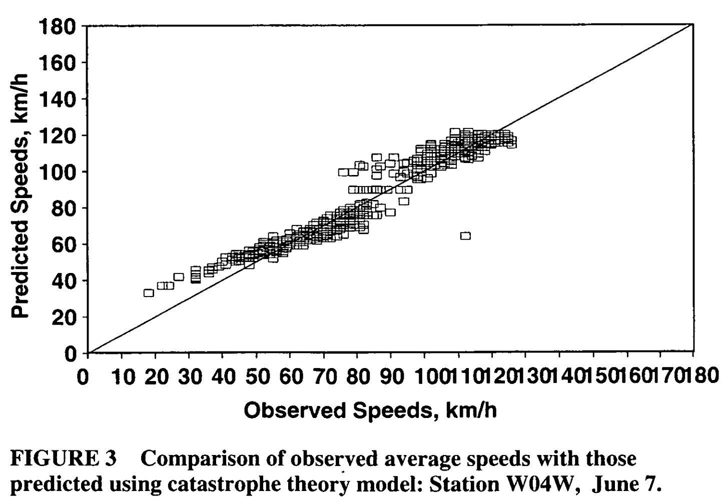

FIGURE 3 Comparison of observed average speeds with those predicted using catastrophe theory model: Station W04W, June 7.

regular station, W02W. The first set of values shows the results based on optimizing for each day separately, given the critical speed and pivot volume. (The rain day had a fairly steady drizzle, resulting in wet pavement, but it was not a heavy rain.) The second set of values shows that when the parameters are averaged across the clear-weather days (and considerably rounded off for *a* and *b*), the resulting values of average difference squared are only marginally higher than for the optimized parameters. Hence it would appear that for regular stations, a single set of parameters can perform well across days, including one with less-than-ideal weather.

For a QDF station, the results are also generally promising, but different in detail (Table 5). In one case, the averaged parameters are slightly worse than the optimized ones, as for the normal station. However, for two of the days, the average difference squared is even lower than it was in the optimized case. (The fact that it might be better than the optimized results signals the hazards of heuristic search and is not an indication of an error in the analysis or printing.) On the second day (June 3), the average difference squared is nearly twice its optimized value, but even so it is only of the same order as the optimized values for the regular station. It seems fair to say that these parameters are stable over time.

Transferability to Other Double-Loop Stations

A further test was done to determine the potential for finding a single set of parameter values that would produce reasonable results when applied to a number of stations. Parameters were determined for four stations, and rough approximations of the average values of these parameters were used in this test. The results (which are not shown in a table) ranged from reasonable to quite poor. For example, considering the increase in average difference squared between the "optimal" parameters and the common ones, reasonable results had error increasing only from 33.1 to 41.6; poor results almost tripled the error, to 144.8 from 54.5.

For another station, a small change in the pivot occupancy (from 11 to 12.5 percent), all other parameters being kept constant, reduced the average difference squared by more than half, from 212.5 to 97.6 (optimized results being 46.1). Hence, it may be best to put some initial effort into calibrating individual stations rather than attempting to find universal parameters, except at stations that are virtually identical.

Speed Prediction at Single-Loop Stations

In applying the catastrophe theory model to speed prediction at single-loop stations, parameters determined at adjacent double-loop stations were used. Because single-loop stations do not have measured speeds, predicted speeds and any observed speed cannot be compared numerically. Instead, to test the relative validity of speeds predicted using catastrophe theory, resulting speed-volume plots were compared with similar plots based on speed values given by two other methods of speed prediction: that given by Equation 1, and the method based on an average vehicle length from the nearest double-loop station, currently used by the MTO and available in the data base.

TABLE 4 Comparison of Parameter Stability Across Days, Station W02W

Critical Speed=90 km/h Pivot Volume=10 veh/20s		Clear Weather			Rain
		May 12	June 3	June 7	May 31
Optimized Results	Occupancy	11	11	11	11
	Theta	-16.0	-16.9	-16.0	-15.0
	a	221.9	258.5	260.0	253.8
	b	52857	52135	54281	55492
	Ave. $Diff^2$	46.87	100.52	54.49	45.52
Averaged Parameters	Ave. $Diff^2$	47.38	102.26	54.72	46.00
Occ = 11 Theta = -16.0 a = 250 b = 53100					

TABLE 5 Comparison of Parameter Stability Across Days, Station W04W

Critical Speed=90km/h Pivot Volume=10veh/20s		Clear Weather			Rain
		May 12	June 3	June 7	May 31
Optimized Results	Occupancy	11	11	11	11
	Theta	-14.0	-14.0	-14.0	-10.0
	a	88.90	133.15	88.34	72.56
	b	25923	26703	22779	18615
	Ave. $Diff^2$	26.65	29.44	33.08	32.30
Averaged Parameters	Ave. $Diff^2$	27.59	50.00	32.25	22.39
Occ = 11 Theta = 14.0 a = 100 b = 25100					

Five single-loop stations were analyzed for the feasibility of speed prediction by catastrophe theory. Here the results for W03W, June 7, are described; they are representative of those obtained for the other stations. Speeds similar to those at W03W would be expected at paired-loop detector W04W, located about 650 m downstream. Figure 4, the speed-flow curve based on speeds measured at W04W, provides a base for comparing the results from W03W.

In Figures 4 through 7, time-connected data, rather than simple scatter plots, are presented. The time-connected plots have three advantages. First, they give an indication of the number of times that a point occurs. Second, they show the distinction in the two regimes of traffic flow, which is not at all obvious from the scatter plot. And third, they show the order in which events occurred, in a general way. All three speed-flow plots for W03W (Figures 5, 6, and 7) indicate three types of operation—congestion, QDF, and uncongested flows—but the quality of the estimates varies considerably. One of the issues is the variation of speeds within uncongested operations. Figure 5 shows the speed-volume plot using the method based on constant vehicle length. Predicted speeds reach almost 200 km/hr and are regularly above 140 km/hr at flows below 10 veh/20 sec (which is a flow rate of 1,800 vehicles per hour). Uncongested speeds generally appear to be above 110 km/hr. A reduction in the vehicle-plus-detector length used would of course lower these numbers, but the combined vehicle and detector length are probably already underestimated. In addition, the scatter in the uncongested data is greater than normally seen with measured speed, as in Figure 4.

Figure 6 shows the plot resulting from predictions based on nearby length estimates. Here the speeds are more believable, surpassing 140 km/hr only a few times. There is a large amount of scatter in the uncongested region. This contrasts markedly with Figure 7, which shows a plot using the speeds predicted using catastrophe theory.

The parameters used in the application of the catastrophe theory model at W03W are a rough average of the parameters determined at Station W02W, just upstream of it, over 2 days. The predictions based on this model show more believable uncongested speeds, but the scatter in the uncongested regime is probably underestimated. Compared with Figure 4, Figure 7 shows too little scatter in the speed estimates, but Figure 5 shows too much. Hence it is not clear

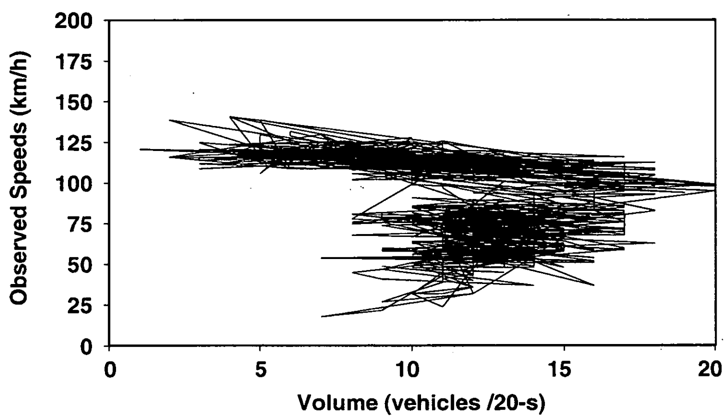

FIGURE 4 Speed-flow diagram using measured speeds: Station W04W, June 7.

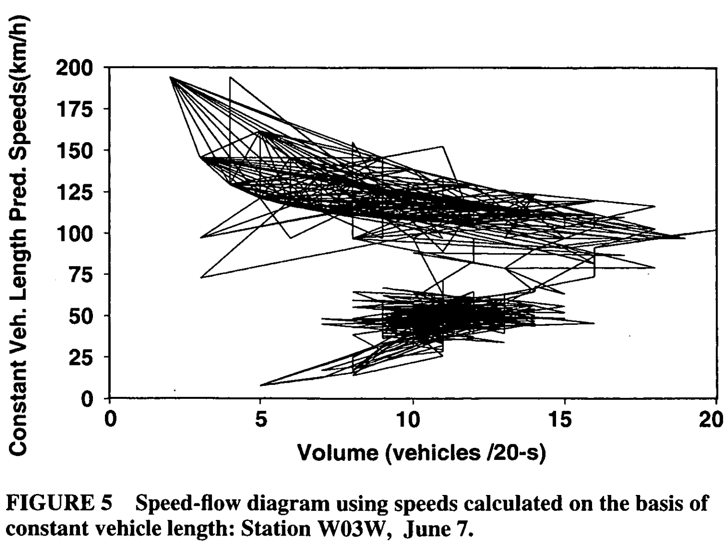

FIGURE 5 Speed-flow diagram using speeds calculated on the basis of constant vehicle length: Station W03W, June 7.

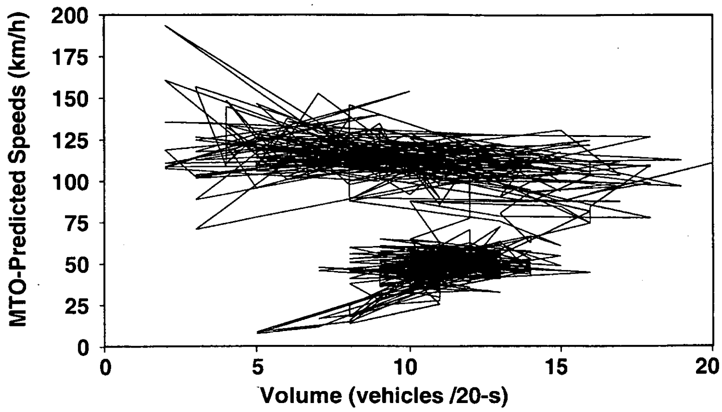

FIGURE 6 Speed-flow diagram using speeds calculated on the basis of vehicle lengths measured at adjacent stations: Station W03W, June 7.

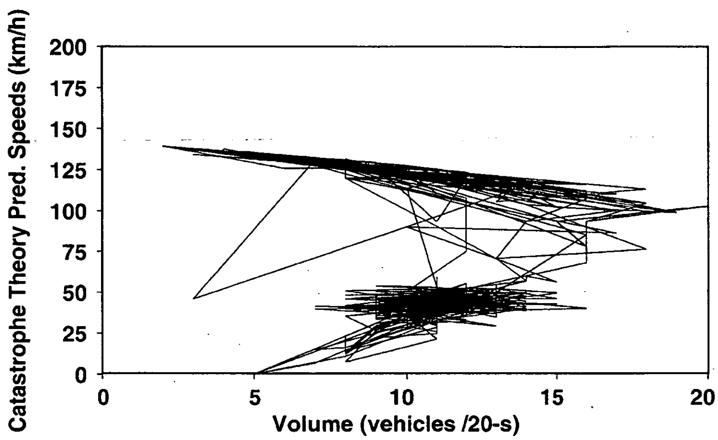

FIGURE 7 Speed-flow diagram using speeds calculated using catastrophe theory model: Station W03W, June 7.

that the catastrophe theory estimates are markedly better than those based on a nearby average. It is, however, clear that they are better than estimates based on a fixed vehicle length, especially below 10 veh/20 sec.

CONCLUSIONS

As part of this effort to use the catastrophe theory model to estimate speeds at single-loop stations, much was learned about the model itself. The axes transformations do not need to be as data-dependent as they were in previous work, and results occasionally improved when constant parameter values were used. A critical speed of 90 km/hr, together with a pivot volume of 10 veh/20 sec, produced reasonable results. The pivot occupancy would be approximately the occupancy that minimizes the number of misclassified data points based on the critical speed. Parameters for a particular station can be transferred quite well across days, but calibration should be fine-tuned across stations.

Applying the catastrophe theory model to speed prediction at single-loop stations gave encouraging results. The predicted speeds were clearly more reasonable, both in terms of levels of speed and scatter in the estimates, than were speeds predicted on the basis of a constant vehicle length. Catastrophe theory predictions of uncongested speeds have less scatter than they should, but speeds based on vehicle lengths measured nearby have more scatter than they should. The question then becomes which kind of error is preferred. Given that an estimate close to the mean is sought, the catastrophe theory model may be preferable.

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REFERENCES

1. Acha-Daza, J. A., and F. L. Hall. Graphical Comparison of the Predictions for Speed Given by Catastrophe Theory and Some Classic

Models. In *Transportation Research Record 1398*, TRB, National Research Council, Washington, D.C., 1993, pp. 119–124.
2. Acha-Daza, J. A. *The Application of Catastrophe Theory to Traffic Flow Variables*. Master's thesis. Department of Civil Engineering, McMaster University, Hamilton, Ontario, Canada, June 1992.
3. Hall, F. L., and B. N. Persaud. Evaluation of Speed Estimates Made with Single-Detector Data from Freeway Traffic Management Systems. In *Transportation Research Record 1232*, TRB, National Research Council, Washington, D.C., 1989, pp. 9–16.
4. Navin, F. P. D. Traffic Congestion Catastrophes. *Transportation Planning and Technology*, Vol. 11, 1986, pp. 19–25.
5. Hall, F. L. An Interpretation of Speed-Flow Concentration Relationships Using Catastrophe Theory. *Transportation Research*, Vol. 21A, No. 3, 1987, pp. 191–201.
6. Dillon, D. S., and F. L. Hall. Freeway Operations and the Cusp Catastrophe: An Empirical Analysis. In *Transportation Research Record 1132*, TRB, National Research Council, Washington, D.C., 1988, pp. 66–76.
7. Forbes, G. J., and F. L. Hall. The Applicability of Catastrophe Theory in Modelling Freeway Traffic Operations. *Transportation Research*, Vol. 24A, No. 5, 1990, pp. 335–344.
8. Acha-Daza, J. A., and F. L. Hall. Application of Catastrophe Theory to Traffic Flow Variables. *Transportation Research B* (in preparation).
9. Ceder, A. *Investigation of Two Regime Traffic Flow Models at the Micro- and Macroscopic Levels*. Ph.D. dissertation. University of California, Berkeley, 1975.
10. Ceder, A. A Deterministic Traffic Flow Model for the Two-Regime Approach. In *Transportation Research Record 567*, TRB, National Research Council, Washington, D.C., 1976, pp. 16–30.
11. Ceder, A., and A. D. May. Further Evaluation of Single and Two-Regime Traffic Flow Models. In *Transportation Research Record 567*, TRB, National Research Council, Washington, D.C., 1976, pp. 1–15.
12. Koshi, M., M. Iwasaki, and I. Ohkura. Some Findings and an Overview on Vehicular Flow Characteristics. *Proc., 8th International Symposium on Transportation and Traffic Theory* (V. F. Hurdle, E. Hauer, and G. N. Steuart, eds.), University of Toronto Press, Ontario, Canada, 1983, pp. 403–426.
13. Gilchrist, R. S., and F. L. Hall. Three-Dimensional Relationships Among Traffic Flow Theory Variables. In *Transportation Research Record 1225*, TRB, National Research Council, Washington, D.C., 1989, pp. 99–108.
14. *Special Report 209: Highway Capacity Manual*, 3rd ed. TRB, National Research Council, Washington, D.C., 1994.

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