System Optimization of Failure, Constitutive Modeling, and Strengths of Concrete and Other Geological Materials Using Genetic Algorithm

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An application of genetic algorithms (GAs) to the system optimization of failure, constitutive modeling, and strengths of concrete and other geological materials is presented. GA is a relatively new, general purpose, optimization algorithm that applies the rules of natural genetics to explore a given search space. Knowledge of the basic constitutive properties of concrete and other geological materials is needed to analyze service load characteristics, design, and evaluate strengths. GA can be used to evaluate parameters of a concrete constitutive modeling, which is based on the theory of plasticity. All the parameters are constants for an ultimate (failure) yielding condition. GA also can be used to evaluate parameters of tensile and compressive strengths for frictional materials such as igneous, sedimentary, and metamorphic rocks; ceramics; mortar; polymer concrete; porous limestone; river gravel; dense limestone; and cemented soils. Such parameters are constants for failure (strengths) conditions. Numerical results indicate that GA is capable of optimizing the system parameters quickly and accurately. Resulting parameter values agree well with previous studies.

A constitutive law or model represents a mathematical model that describes the behavior of a material. In other words, a constitutive law simulates physical behavior that has been perceived mentally. The main advantage of establishing a mathematical model is to apply it to solve (complex) events quantitatively. Therefore, the power of a constitutive model depends on the extent to which the physical phenomenon has been understood and simulated.

In this paper, a constitutive model based on the theory of plasticity, described elsewhere (*1–8*), is used that can be used to characterize the stress-deformation behavior of concrete and geological materials. The model allows for factors such as stress hardening, volume changes, stress paths, cohesive and tensile strengths, and variation of yield behavior with mean pressure. To establish the constitutive model, determination of material constants (parameters) is very important. The only rational way to determine parameters to define the constitutive model is to conduct appropriate laboratory and field tests.

GA is a powerful search procedure based on the mechanics of natural selection. It uses operations found in natural genetics to guide it through the paths in the search space. It provides a means to search poorly understood irregular spaces. Because of its robustness, GA has been applied successfully to a variety of function optimizations, self-adaptive control systems, and learning systems.

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This study uses GAs to solve parameters of a constitutive model and strength models. Test data (*1–6*) was used to check the effectiveness of GA. Finally, parameters obtained by GA are compared with various methods in literature (*1–6*).

CONSTITUTIVE MODEL

Theoretical development of the hierarchical model approach and application to soil, rock, and concrete behavior is given elsewhere (*1–6;9–12*). Application of the model for geological materials, including comprehensive modeling and verifications for various geological materials, is discussed in words by Salami (*1–5*). The hierarchical concept provides a framework for systematic development of models with progressively complex responses: isotropic associative hardening, isotropic nonassociative hardening, anisotropic hardening and strain-softening. As a result, the concept can be sufficiently simplified in terms of material constants that are determined from laboratory tests (*1–6*).

A compact and specialized form, F, of the general polynomial representation (*1–5;9*), adopted herein to describe both the continuous yielding and ultimate (failure) yield behavior, is given by

$$F \equiv J_{2D} - F_b F_s \tag{1}$$

where

J_{2D} = second invariant of the deviatoric stress tensor;
S_{ij} = total stress tensor, σ_{ij};
F_b = basic function; and
F_s = shape function.

The function F is a continuous function in the stress space, and the final curve represents the ultimate behavior. In expanded form, Equation 1 is written as

$$F(J_1, J_{2D}, J_{3D}) \equiv J_{2D} - \left(\frac{-\alpha}{\alpha_0^{n-2}} J_1^n + \gamma J_1^2\right)(1 - \beta S_r)^{-1/2} \tag{2}$$

where

$J_1 = \sigma_1 + \sigma_2 + \sigma_3$, the first invariant of σ_{ij};
S_r = stress ratio = $(J_{3D})^{1/3}/(J_{2D})^{1/2}$, which can also be the Lode angle;
J_{3D} = third invariant of S_{ij}
α, n, γ, and β = response functions;

$\alpha_0 = 1$ stress unit; and
$m = -1/2$ response function.

As a simplification, γ and m are assumed to be constants, whereas β is expressed as a function of mean pressure, J_1, to account for the observed yield behavior of geological materials (*1–5;10–12*).

$$\beta = \beta_0 e^{-\beta_1 J_1} \tag{3}$$

Here, β_0 and β_1 are constants. The constitutive model is developed to represent a wide range of materials. Based on Equation 2, the developed model is proposed to describe both failure and yielding of the concrete and geological materials. The model agrees with the experimental evidence regarding the shapes of yield surfaces on various planes. Moreover, ultimate failure and yielding are defined by a single yield surface.

TENSILE STRENGTH EXPRESSION, f_t

The proposed analytical expression of uniaxial strength f_t, as a function of cylindrical compressive strength for frictional materials, which is based on the experimental results found elsewhere (*13;1–6;14–16*), is given as

$$f_t = -mP_a\left\{\frac{f_c'}{p_a}\right\}^n \quad \text{(compression positive)} \tag{4}$$

where m and n are dimensionless numbers, and p_a is atmospheric pressure in the same units as those of f_t and f_c'. Values of m and n have been determined by using GA for several frictional materials and are listed in Table 1.

TENSILE STRENGTH, f_{sp}

On the basis of experimental results (*1–6;14*) and also Equation 4, the split tensile strength of concrete is given as

$$f_{sp} = -\lambda P_a\left\{\frac{f_c'}{p_a}\right\}^\eta \quad \text{(compression positive)} \tag{5}$$

TABLE 1 Values of Parameters *m* and *n* Obtained by GA for Various Types of Concrete and Frictional Materials for Equations (*4–6*)

MATERIALS	m	n
Mortar, $*f_t$	0.77	0.65
Cemented Soils, f_t	0.39	0.84
Ceramics, f_t	0.70	0.77
Igneous Rock, f_t	0.52	0.72
Metamorphic Rock, f_t	0.21	0.83
Sedimentary Rock, f_t	0.19	0.79
Plain Concrete, f_t	0.61	0.73
Plain Concrete, $**f_{sp}$	$\lambda = 0.69$	$\eta = 0.65$
Plain Concrete, $***f_r$	$\alpha = 0.91$	$\beta = 0.62$
Porous Limestone, f_{sp}	$\lambda = 2.13$	$\eta = 0.43$
Porous Limestone, f_r	$\alpha = 1.96$	$\beta = 0.54$
River Gravel, f_{sp}	$\lambda = 2.28$	$\eta = 0.42$
River Gravel, f_r	$\alpha = 2.32$	$\beta = 0.53$
Dense Limestone, f_{sp}	$\lambda = 0.21$	$\eta = 0.83$
Dense Limestone, f_r	$\alpha = 0.54$	$\beta = 0.76$

$*f_t$ = Direct Tensile Strengh $**f_{sp}$ = Split Tensile Strengh
$***f_r$ = Beam Flexural Tensile Strength

where λ and η are dimensionless numbers, and p_a is atmospheric pressure in the same units as those of f_{sp} and f_c'. Values of λ and η have been determined by using GA for several frictional materials and are listed in Table 1.

FLEXURAL TENSILE STRENGTH, f_r

On the basis of experimental results (*1–6;14*) and also Equation 4, the flexural tensile strength is given as

$$f_r = \alpha P_a\left\{\frac{f_c'}{p_a}\right\}^\beta \quad \text{(compression positive)} \tag{6}$$

where α and β are dimensionless numbers, and p_a is atmospheric pressure in the same units as those of f_r and f_c'. Values of α and β have been determined by using GA for several frictional materials and are listed in Table 1.

Polymer concrete materials identified in Figure 1 are used to find parameters of Equation 4 for different temperatures. The values of parameters obtained by GA are presented in Table 2.

GA

GA is a general purpose, optimization algorithm with a probabilistic component. It provides a means to search poorly understood,

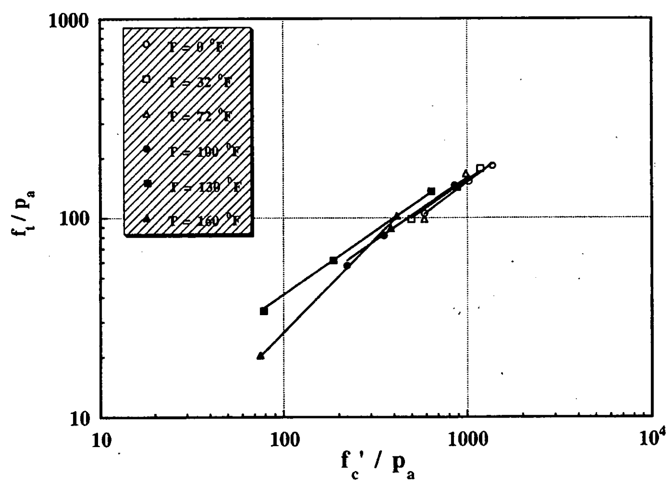

FIGURE 1 Comparison of experimental and predicted results for polymer concrete at different temperatures.

TABLE 2 Values of Parameters *m* and *n* Obtained by GA for Polymer Concrete Materials for Equation 4.

TEMPERATURE (^{0}F)	m	n
T = 0	1.29	0.69
T = 32	1.35	0.69
T = 72	2.05	0.63
T = 100	1.93	0.64
T = 130	1.69	0.68
T = 160	0.35	0.64

irregular spaces. GA has been applied successfully to a variety of function optimization, parameter searches, and machine learning applications. Holland (*17*) originally developed GA and provided its theoretical foundations. GA was developed to simulate some of the processes observed in natural evolution, a process that operates on chromosomes (organic devices for encoding the structure of living beings) instead of on living beings. Natural selection links chromosomes with the performance of their decoded structures. The processes of natural selection cause those chromosomes that encode successful structures to reproduce more often than those that do not. Recombination processes create different chromosomes in children by combining material from the chromosomes of the two parents. Mutation may cause the chromosomes of children to be different from those of their parents.

GA appropriately incorporates these features of natural evolution into computer algorithms to solve difficult problems in the way that nature has done, through evolution. GA requires the problem to be maximized (or minimized) in the form of an objective (cost) function. In GA, a set of variables for a given problem is encoded into a binary string, or any other coding structure, analogous to a chromosome in nature. These strings are converted to a numerical value and then linearly mapped over the range allowed for the variable. The value is then used to evaluate the objective function, yielding a "fitness." GA selects parents from a pool of strings (population) according to the basic criterion of survival of the fittest. It reproduces new strings by recombining parts from the selected parents in a random manner.

Repopulation of the next generation is done using three methods: reproduction, crossover, and mutation. Reproduction means simply that strings that are highly fit should receive multiple copies in the next generation, whereas strings with low fitnesses receive fewer if any copies. Crossover refers to splitting a string into two parts at a randomly generated crossover point and recombining it with another string that has been split at the same crossover point. The procedure serves to promote changes in the best strings, those that will produce higher fitnesses. Mutation is the random alteration of a bit in the string, which will assist to preserve diversity in the population of strings.

To explain the mechanisms of GA, a few terms need to be defined. Because binary strings are considered, a notation must be developed to denote similar subsets (schemata). A schema is a similarity subset that has strings containing similarities at some bit positions. Furthermore, the format can be expanded with the introduction of a wild card character, *, in addition to the binary set {0,1}. For example, the set {0001,0101,0011} can be described by the similarity template 0**1. By using this notation, a schema's order and defining length can be specified. For a given schema, *h*, its order $o(h)$ is defined as the number of fixed bit positions within that schema. The defining length of a schema, $\delta(h)$, is the distance between the outermost fixed positions of a schema. For example, the schema 01****0 has order 3, defining length 5, and can represent 16 different individuals.

With these definitions, one can present the fundamental theorem of GA, the schema theorem (*18*). The schema theorem enables the calculation of lower bound on the expected number of a particular schema, *h*, following reproduction, crossover, and mutation. The theorem is stated as

$$\lambda(h, t+1) \geq \lambda(h, t)\frac{f(h)}{x}\left[1 - p_c\frac{\delta(h)}{l-1} - p_m o(h)\right] \tag{7}$$

where

λ = expected number of schemata,
t = generation index,
l = overall string length,
$f(h)$ = average fitness of those strings representing the subset h,
x = average fitness of the entire population, and
p_c and p_m = the crossover and mutation probabilities, respectively.

The schema theorem states that a schema will grow when it is short, has low order, and has above average fitness.

GA has many advantages over other methods. Currently most literature defines three main types of search methods: calculus-based, enumerative, and random (*19*). Calculus-based methods can be divided into two classes: indirect and direct. For indirect methods, local extrema are determined by finding where the gradient of the objective function is equal to zero. Direct methods follow along the objective function in a direction related to the local gradient. Both classes share two main disadvantages that greatly limit their usefulness. First, both are local in scope; that is, if a function has multiple local maxima, the method may drive toward one of these values without ever approaching the global maxima. The second deficiency of calculus-based methods is their dependence on the existence of derivatives. However, many real-world functions are discontinuous and noisy and do not work well with a method that prefers smooth, continuous functions.

Enumerative methods offer an attractive advantage, simplicity, but that advantage carries a high cost. Enumerative schemes take a discretized search space and examine the objective function at every point. Although simple in technique, the brute force method is quite inefficient, and its execution time becomes too long as the search space becomes larger.

The last method is the random search method. In many respects, this method might perform as poorly as an enumerative method because of its inefficiency. Randomly searching through a space and saving the best results can be time-consuming as the space becomes large.

The primary advantage of GA is its robustness. GA works through function evaluation, not through differentiation or other such means. Whereas GA begins with a randomly generated set of points, it exploits the information contained in those points to drive it through the search space. Because GA is based on function evaluation, it can be applied to all type of optimization problems: linear, nonlinear, discontinuous, and discrete, as long as the problem is properly coded.

Because of its robustness, GA has been used in optimization problems as diverse as image analysis by Grefenstette and Fitzpatrick (*20*), gaming strategy by Axelrod (*21*), and the traveling salesman problem by Homaifar (*22*). Another practical engineering example of GA's application is Goldberg's study (*23*) of a system of 10 pipes and 10 pumping stations. Also, expert systems can be improved on by using GA, as shown by Davis and Coombs (*24*).

PROBLEM DESCRIPTION AND METHODOLOGY

The constitutive model and strength models that were used have some material constants. Determination of such constants for any material requires a comprehensive series of laboratory tests with a number of loading, unloading, and reloading cycles. In this paper, GA is used to determine material parameters: n, γ, β_0, and β_1 and at the ultimate yielding conditions of constitutive model, and *m, n* for strength models. The materials that were tested elsewhere (*1–8*) are used in this research.

The basis for software used in this work is the Simple Genetic Algorithm program developed by Goldberg (*19*). The program is rewritten in C language to allow the evaluation of multiple parameters. Multiple parameters are incorporated by dividing each string up into substrings that represent different variables. For example, a string of length 18 could be used to represent three substrings of length 6, each of which could represent a different parameter. Obviously, using multiple parameters requires increasingly longer string lengths, if each variable has an acceptable precision.

The main purpose of this computational experiment is to examine the ability of GA to perform a multiparameter objective function optimization on a real-world problem.

The strings for the GA implementation were formed by concatenating the encoded value of each parameter. The string lengths were chosen as follows: (*a*) a step size of 0.001 was chosen for all parameters of model, and (*b*) the string lengths were chosen to provide appropriate ranges for all parameters. The constraint for choosing the string lengths is given by

$$2^l - 1 = \frac{\text{range}}{\text{stepsize}} \tag{8}$$

where l is string length. The resulting string lengths of parameters for failure, constitutive modeling, and tensile strengths are 8, except n which is 10.

A measure of performance is derived to effectively and accurately compare the performance of the GA. The error, measured over the entire simulation period, is described as

$$E = \sqrt{\sum_i (y_i - x_i)^2} \tag{9}$$

where y is actual data, and x is GA data.

Derivation of the fitness function for the GA is one of the difficult and crucial portions of this study. The fitness function should be formulated to discriminate among different strings. Initial experiments were conducted using a simple mean square error measure

$$\text{Fitness} = \frac{1}{\text{Error}} \tag{10}$$

However, the results provided only a mean square fit, so the fitness function was changed to

$$\text{Fitness} = \frac{\text{number of matched points}}{E} \tag{11}$$

where a matched point is a GA point within a specified tolerance.

Roulette wheel selection and single point crossover are used throughout the experiments. The variables that had to be defined for GA are population size, crossover probability, mutation probability, and maximum number of generations. Their respective values are 1,000, 0.60, 0.01, and 50. Figure 2 shows the flowchart for the implementation. Note that the stopping criterion is usually given in terms of a threshold in improvement or total number of generations. Population sizing is an important requirement in GA. Populations must be large enough to provide adequate diversity. However, the larger the population, the greater the number of calculations.

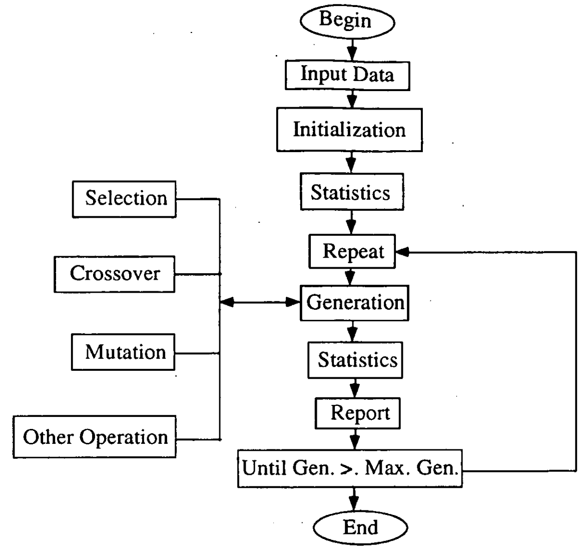

FIGURE 2 The main structure of GA implementation.

ANALYSIS

In this study, parameters for frictional materials, including plain concrete and soapstone, are determined by using GA. Three different cases are considered:

- Case 1: Cemented Soils, Ceramics and Mortar; Igneous, Metamorphic and Sedimentary Rocks are used to analyze the relationships between uniaxial cylindrical compressive strength f_c' and uniaxial tensile strength f_t, as given in Equations 4–6. The parameter values obtained by GA for these materials are shown in Table 1. Graph of typical results is shown in Figure 1 for polymer concrete materials under different temperatures, and demonstrate that parameter values obtained by the GA are compared very well with the experimental data.
- Case 2: The growth (hardening or softening) function α equals zero in Equation 2 of the ultimate condition for concrete materials. Then Equation 2 becomes

$$F(J_1, J_{2D}, J_{3D}) = J_{2D} - \gamma J_1 (1 - \beta S_r)^{-1/2} = 0 \tag{12}$$

where $\beta = \beta_0 e^{-B_1 J_1}$

The ultimate constants, n, γ, β_0, and β_1, are obtained by GA when the solutions are reached after 100 generations. Original data obtained by Salami are presented in Table 3 and Table 4, for plain concrete and soapstone, respectively. The GA solutions, along with Salami's solutions, are reported in Table 5. Parameter values obtained by the GA are compared with those obtained by Salami's method (*6*); the precision is 0.068.

- Case 3: The observed ultimate (failure) and preyielding surface for four different values of α is shown in Figure 3. Figures 3(a) and 3(b) express the triaxial compression and simple shear stress path, respectively, where all are assumed to be in an octahedral plane (*1–6*). The GA solutions for triaxial compression and simple shear parameters based on different values of α are given in Tables 6 and 7, respectively. Graphs of typical results are provided

TABLE 3 Ultimate Data for Plain Concrete (*6*)

Load Path	Spec.	σ1 (ksi)	σ2 (ksi)	σ3 (ksi)	J1 (ksi)	√J2D (ksi)
Conventional Triaxial Compression (CTC)	B3	4.8977	0.3944	0.3944	5.6865	2.6000
	B1	10.864	1.3944	1.3944	13.653	5.4675
	B2	14.345	2.3944	2.3944	19.134	6.8999
	B4	17.765	3.3944	3.3944	24.554	8.2971
Triaxial Compression	E1	6.4144	0.3844	0.3844	7.1832	3.4814
	E2	8.7944	0.6944	0.6944	10.183	4.6765
SS*	K1	9.8144	4.8944	-0.0256	14.683	4.9200
Triaxial Extension (TE)	N2	6.6923	6.6923	-0.2013	13.183	3.9800
	N1	8.2694	8.2694	-0.356	16.183	4.9799
	N3	9.7142	9.7142	-0.2451	19.183	5.7500
RTE**	R1	7.8944	7.8944	4.9444	20.733	1.7032

1.0 psi = 6.89 KPa
* SS = Simple Shear and ** RTE = Reduced Triaxial Extension
Compression is Positive.

TABLE 4 Ultimate Data for Soapstone (*6*)

Load Path	Spec.	σ1 (ksi)	σ2 (ksi)	σ3 (ksi)	J1 (ksi)	√J2D (ksi)
Conventional Triaxial Compression (CTC)	T4	1.3533	0.1533	0.1533	1.6599	0.6928
	T2	5.7143	1.1533	1.1533	8.0209	2.6333
	T1	7.9873	2.1533	2.1533	12.294	3.3683
	T3	10.578	3.1533	3.1533	16.885	4.2868
Triaxial Compression (TC)	S3	4.4873	0.9863	0.9863	6.4599	2.0213
	S1	6.3353	1.5623	1.5623	9.4599	2.7557
	S2	7.5473	2.4563	2.4563	12.460	2.9393
Simple Shear (SS)	P3	3.5623	2.1533	0.7453	6.4609	1.4085
	P2	4.9293	3.1533	1.3773	9.4599	1.7760
	P1	6.7013	4.1533	1.6063	12.461	2.5475
Triaxial Extension (TE)	M3	2.8963	2.8963	0.6683	6.4609	1.2863
	M2	4.1083	4.1083	1.2443	9.4609	1.6535
	M1	5.4613	5.4613	1.5373	12.460	2.2655

1.0 psi = 6.89 KPa
Compression is Positive.

TABLE 5 Comparison of Material Parameters for Plain Concrete and Soapstone by Using Two Different Methods for Various Stress Paths (*6*)

		n	γ	β0	β1
Plain Concrete	Salami (6)	7.0	0.113	0.844	0.027
	GA	7.0	0.110	1.830	0.030
Soap-Stone	Salami (6)	7.0	0.047	0.749	0.047
	GA	7.0	0.050	1.460	0.060

in Figures 3 through 6, demonstrating that GA parameters provide a well-shaped curve that closely matches the given curves.

CONCLUSIONS

GA has been applied to find material parameters for a complex failure and constitutive model for concrete material. It also has been applied to find strength material parameters for frictional materials such as concrete, polymer concrete, rocks, and ceramics. Although under most circumstances a multiparameter, multiobjective function optimization problem would be considered a difficult task, GA could handle it successfully. Although the model used here to represent a constitutive model is a relatively simple one, the procedure described would be the same for complex models. Results of assorted runs agreed with experimental and single parameter optimization results. Of course, realistic failure and constitutive models would include many more parameters, but for GA that would only involve increasing the string length to incorporate the additional parameters. The ability of GA to handle a problem of that nature could make it a very important tool in the future of constitutive modeling for geological and engineering materials.

The correlation between the experimental results and analytical predictions was very good and provides a simple approach for developing tensile strength models, failure, and constitutive models for frictional materials.

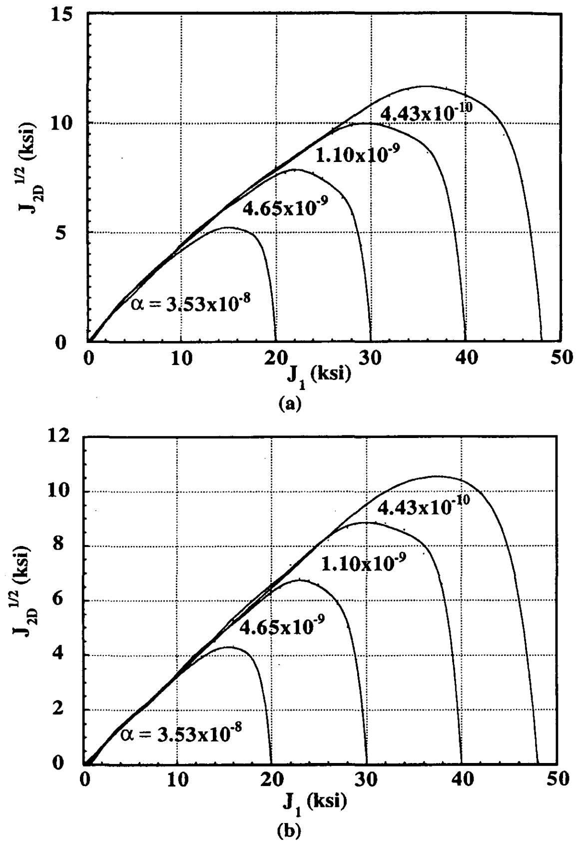

FIGURE 3 Observed ultimate (failure) and preyielding surface in $\sqrt{J_{2D}} - J_1$ plane for (*a*) triaxial compression test and (*b*) simple shear test. (1.0 psi = 6.89 kPa).

TABLE 6 Material Parameters for Plain Concrete for Triaxial Compression Tests by GA Method

TC	α	n	γ	β0	β1
Plain Concrete (Figure 3)	4.43e-10	7.04	0.13	0.56	0.03
	1.10e-09	7.09	0.15	0.56	0.07
	4.65e-09	7.09	0.15	0.56	0.05
	3.53e-08	7.12	0.16	0.56	0.10

TABLE 7 Material Parameters for Plain Concrete for Simple Shear Tests by Using GA Method

SS	α	n	γ	β0	β1
Plain Concrete (Figure 3)	4.43e-10	7.00	0.11	0.39	0.17
	1.10e-09	7.02	0.12	0.42	0.04
	4.65e-09	7.00	0.11	0.41	0.03
	3.53e-08	7.00	0.11	0.12	0.66

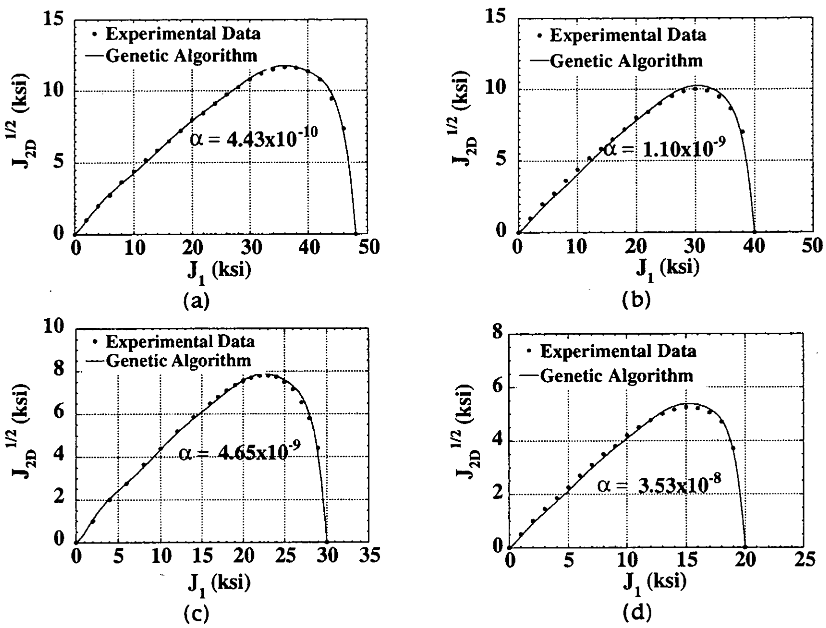

FIGURE 4 Comparison of predicted and experimental results in ultimate and preultimate envelopes in $\sqrt{J_{2D}} - J_1$ planes for triaxial compression test for various α for plain concrete. (1.0 psi = 6.89 kPa).

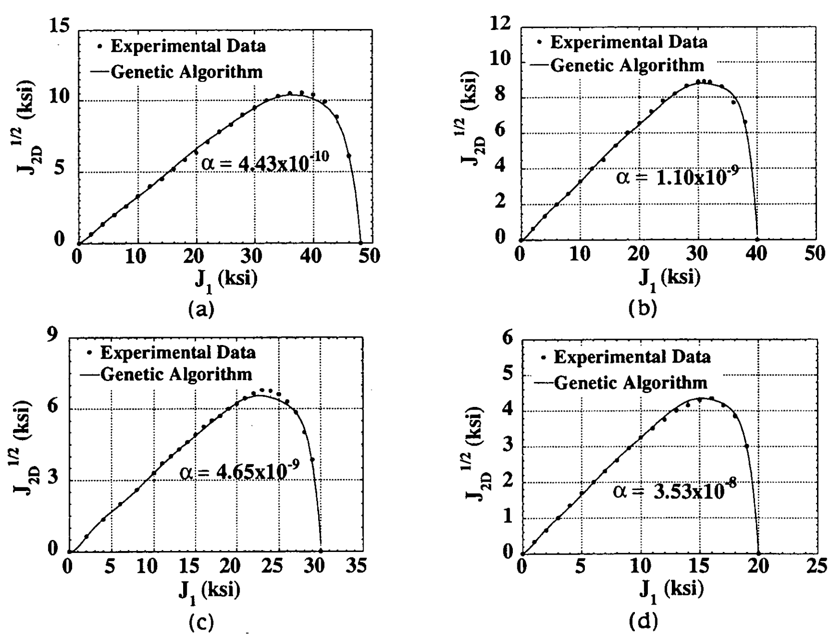

FIGURE 5 Comparison of predicted and experimental results in ultimate and preultimate envelopes in $\sqrt{J_{2D}} - J_1$ planes for simple shear test for various α for plain concrete. (1.0 psi = 6.89 kPa).

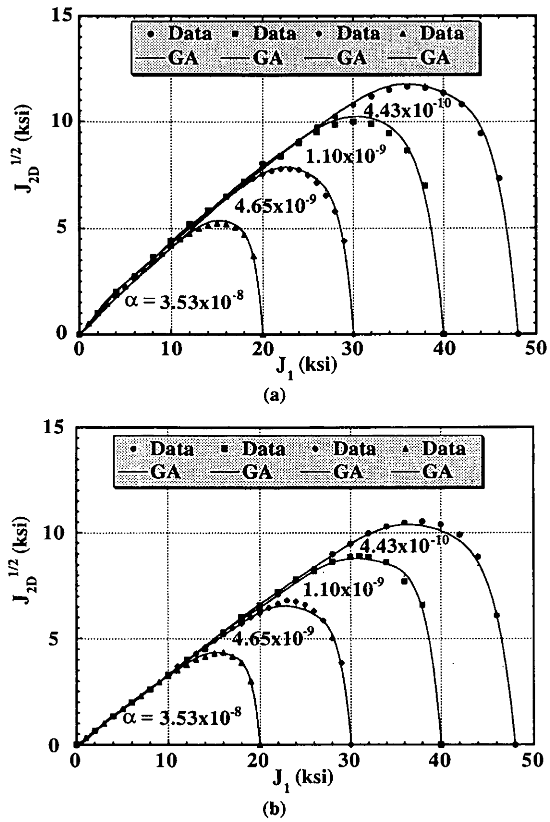

FIGURE 6 Comparison of predicted and experimental results in ultimate and preultimate envelopes in $\sqrt{J_{2D}} - J_1$ planes for (*a*) triaxial compression and (*b*) simple shear tests for various α for plain concrete. (1.0 psi = 6.89 kPa).

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