Track Modulus: Its Meaning and Factors Influencing It

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Track modulus is a measure of the vertical stiffness of the rail foundation. Another parameter, track stiffness, is a measure of the vertical stiffness of the entire track structure. Both are related to the track performance. In order to provide a basis for assessing and modifying track performance, the definitions of track modulus and track stiffness are reviewed, and four methods of determining track modulus or track stiffness are discussed. On the basis of analysis with a track structure model, the effects of superstructure and substructure factors influencing track modulus are illustrated and the means of altering track modulus are suggested. Finally, the relationship between the track modulus and the track performance is discussed. The subgrade soil conditions are shown to have the greatest influence on track modulus and stiffness. Next in importance are the combined ballast-subballast thickness and the vertical tie-fastener stiffness.

Track modulus provides a measure of the vertical stiffness of the rail foundation. It is a measure of the structural condition of the track, and, as such, is related to the track performance. Track modulus is generally considered to be an important parameter, although it is seldom measured and its magnitudes are unknown (or at best roughly known) for most sections of railway track.

Track modulus and the related track stiffness are important parameters, for which the optimum values are neither too low nor too high. Thus, effort should be made to quantify these parameters and understand the relationship between track modulus or stiffness and track performance. The effects of factors influencing track modulus or stiffness also need to be defined in order to provide a basis for assessing and modifying track performance.

The objectives of this paper are (*a*) to define track modulus and track stiffness, (*b*) to discuss the means of determining track modulus and track stiffness, (*c*) to illustrate the effects of factors influencing their values, and (*d*) to relate the track modulus to the track performance.

DEFINITIONS OF TRACK MODULUS AND STIFFNESS

Consider a rail as a continuous beam resting on an elastic foundation (Figure 1). A concentrated vertical force, P, from the train wheel will produce vertical rail deflection, δ, which is maximum (δ_m) beneath the wheel. The track stiffness, k, is defined as follows:

$$k = \frac{P}{\delta_m} \tag{1}$$

The track foundation modulus, u, commonly termed the track modulus, is defined as the supporting force per unit length of rail per unit deflection, that is,

$$u = -\frac{q}{\delta} \tag{2}$$

where q is the vertical foundation supporting force per unit length.

The differential equation for the model in Figure 1 is

$$EI\frac{d^4\delta}{dx^4} = q = -u\delta \tag{3}$$

where

E = the rail Young's modulus of elasticity,
I = the moment of inertia of the rail, and
x = the horizontal distance along the rail, measured from the applied load point.

Based on the solution to this differential equation, the maximum deflection, δ_m, is given by

$$\delta_m = \frac{P\beta}{2u} \tag{4}$$

where

$$\beta = \left(\frac{u}{4EI}\right)^{\frac{1}{4}} \tag{5}$$

The maximum rail bending moment, M_m, is given by

$$M_m = \frac{P}{4\beta} \tag{6}$$

The maximum supporting line force, q_m, is given by

$$q_m = u\delta_m \tag{7}$$

The maximum rail seat force, Q_m, can be estimated on the basis of Equation 7 by

$$Q_m = q_m S \tag{8}$$

where S is tie center-to-center spacing.

Substitution of Equations 1 and 5 into Equation 4 and rearrangement gives the relationship between track stiffness, k, and track (foundation) modulus, u, as follows:

$$u = \frac{(k)^{\frac{4}{3}}}{(64EI)^{\frac{1}{3}}} \tag{9}$$

Note that by substitution of Equation 9, Equations 6, 7 and 8 may be defined in terms of k rather than u. The difference between k and

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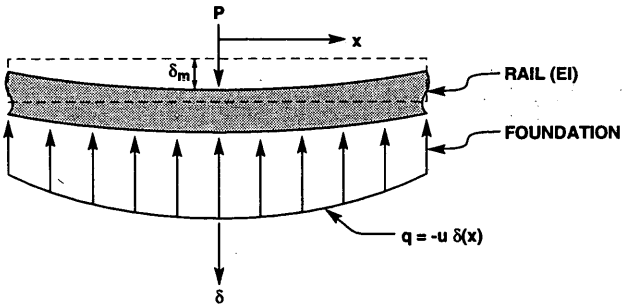


FIGURE 1 Beam on elastic foundation model.

u is that k includes the effect of the rail EI, whereas u represents only the remainder of the superstructure (fasteners and ties) and the substructure (ballast, subballast, and subgrade).

METHODS OF MODULUS/STIFFNESS DETERMINATION

Four methods will be discussed for determining track modulus and track stiffness:

1. Deflection basin test,
2. Single load point test,
3. Multiple axle vehicle load test, and
4. Calculation from track model.

The first three involve field measurements on track. The fourth involves a computer model which requires properties of the different track components. Variations of these methods have been discussed by previous researchers (*1–6*).

The determination of track stiffness/modulus in general is based on the deflection measurements on one rail under its associated loads, but with both rails simultaneously loaded the same amount at opposite ends of the same tie.

Deflection Basin Test

The deflection basin method is based on the vertical equilibrium of forces acting on the rail (Figure 2). This equilibrium is expressed as

$$\sum P = \int_{-\infty}^{\infty} q(x)dx$$

or

$$\sum P = \int_{-\infty}^{\infty} u\delta(x)dx \qquad (10)$$

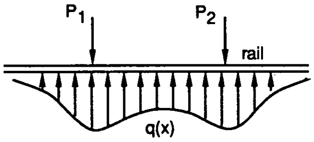


FIGURE 2 Vertical equilibrium of rail.

If u is considered constant, Equation 10 becomes

$$\sum P = u \int_{-\infty}^{\infty} \delta(x)dx$$

or

$$\sum P = uA_\delta \qquad (11)$$

where A_δ = the deflection basin area, that is, the area between the original rail position and the deflected rail position.

If the vertical deflection of each tie is measured, then Equation 11 may be written as follows:

$$\sum_{j=1}^{n} P_j = uS \sum_{i=1}^{m} \delta_i \qquad (12)$$

where

S = center-to-center tie spacing,
n = number of wheels, and
m = number of ties which deflect under the vertical loads.

The track modulus, u, is obtained from either Equation 11 or 12 using the deflection basin method. In this case P is the total vertical force.

If the load-deflection relationship were linear as illustrated in curve a in Figure 3, then the track modulus, u, would be independent of the value of P. Typically, the load-deflection relationship is not linear. The value of u will then depend on the load level. A simplified case is illustrated by curve b in Figure 3, in which the initial portion represents taking up slack in the track. The deflection basin method may be used to determine u using any two force levels with the lower force level removing the slack. The u is determined by obtaining the area change due to the force change as follows:

$$\sum_{j=1}^{n} (P_h - P_0)_j = u(A_{\delta_h} - A_{\delta_0}) \qquad (13)$$

or

$$\sum_{j=1}^{n} (P_h - P_0)_j = uS \sum_{i=1}^{m} (\delta_h - \delta_0) \qquad (14)$$

where P_h, P_0, δ_h, and δ_0 are as defined in Figure 3.

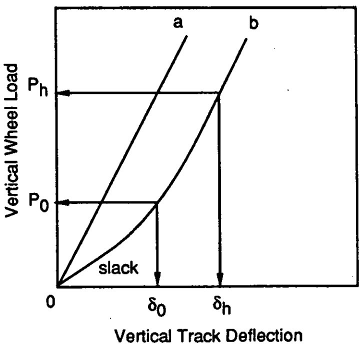


FIGURE 3 Schematic of track load–deflection curves.

Single Load Point Test

The basin test requires a large number of measurements and so is time-consuming and difficult. An easier method is the single load point test as represented in Figure 1. With this method the track stiffness is determined from Equation 1 and the track modulus is determined from Equation 9.

To account for non-linearity, such as slack, Equations 1 and 9 can be revised using this formula:

$$k = \frac{P_h - P_0}{\delta_{mh} - \delta_{m0}} \tag{15}$$

Multiple Axle Vehicle Load Test

The most convenient way to load the rail is to use railway vehicles which normally have 2 or 3 axle bogies. The deflection basin method may be used to calculate track modulus. Alternatively, wheel load superposition analysis may be performed using the single load point method. Kerr (*4*) has proposed such an approach. The disadvantage of his approach is that it references the loaded deflection to the unloaded deflection and hence includes the slack. This method, however, has been extended to use two-wheel load levels as shown by El-Sharkawi (*7*). His method uses a computer track model GEOTRACK (*8*) to do the superposition of axles and handle two load levels.

Calculation from Track Model

A computer model such as GEOTRACK can be used to predict track stiffness and track modulus using values for properties of each of the components. GEOTRACK (*8*) includes the rails and ties as beams connected with a fastener and resting on a multilayer elastic system representing the ballast, subballast and subgrade. Because accurate values of all these properties are difficult to obtain, this method is most useful for estimating the effects of each of these variables on track stiffness and track modulus. This application is very valuable, however, because a knowledge of values of k and u does not permit one to distinguish the factors causing them nor to assess how to alter them. Such information is needed for maintenance planning.

EFFECT OF TRACK COMPONENTS ON TRACK MODULUS

Parametric Study of All Components

The application of the GEOTRACK model to the analysis of track modulus and the verification of the analytical results by the field measurements of track modulus in a few revenue track sites have been reported by Stewart and Selig (*3*) and Stewart (*6*). The main conclusions about the influence of track parameters on the track modulus from those studies can be summarized as follows:

1. Of the factors considered, those with the most influence on the track modulus are the ballast depth and the fastener stiffness. An increase in both a ballast depth and a fastener stiffness generally leads to an increase of track modulus.
2. Track maintenance (rail surfacing and ballast tamping) does not have much effect on the track modulus.

However, these past parametric studies were not complete, in that the effects of varying the subgrade characteristics were not included. Thus, a more comprehensive parametric study on track modulus using the GEOTRACK model is given in this paper. Table 1 gives the nominal values of material properties for the input to the GEOTRACK analysis. Table 2 gives the ranges of these variables considered in the parametric study. While each variable is changed in the GEOTRACK analysis, the other variables are assigned the nominal values given in Table 1. The values given in these two tables are based on values given in other references (*3, 6, 8–10*). Although the resilient moduli of ballast, subballast, and subgrade are stress-dependent, they are assumed constant in each analysis for the purpose of simplicity.

Since, in most situations, subgrade is not a uniform half-space of the same soil type, the influence of subgrade stratification is also considered in the parametric study. In this paper the subgrade is simplified as a layer of variable modulus with different thicknesses overlying a hard bedrock.

In the model the "fastener stiffness" is the term given to the vertical compressibility between the top of the rail and the bottom of the tie. For the wood tie track the major contributor is the tie compressibility, while for the concrete tie track the major contributor is the rail seat pad.

Figure 4 shows the comparison of the influences of the track components on the track modulus from the GEOTRACK analysis. The horizontal line represents the track modulus for the nominal concrete tie track, as defined by the properties given in Table 1. The

TABLE 1 Nominal Track and Subgrade Parameters for GEOTRACK Analysis

Variable	Value			
<u>Rail Properties</u>				
E - GPa (ksi)	207	(30)		
I - m^4 (in.^4)	3.95e-5	(94.9)		
Cross area - m^2 (in.^2)	8.61e-3	(13.4)		
Gauge - m (in.)	1.50	(59.3)		
Mass - kg/m (lb/yd)	67.7	(136.2)		
<u>Tie and Fastener</u>	<u>Concrete</u>		<u>Wood</u>	
Base width - m (in.)	0.273	(10.8)	0.229	(9.0)
Base length - m (in.)	2.59	(102)	2.59	(102)
Cross area - m^2 (in.^2)	5.59e-2	(86.6)	4.06e-2	(63.0)
E - GPa (ksi)	31	(4,500)	10	(1,500)
I - m^4 (in.^4)	2.42e-4	(582)	1.07e-4	(257)
Mass - kg (lb)	363	(800.0)	114	(250.0)
Spacing - m (in.)	0.61	(24.0)	0.50	(19.5)
Fastener stiffness - kN/mm (kips/in.)	175	(1,000)	70	(400)
<u>Ballast</u>				
Density - Mg/m^3 (pcf)	1.76	(110)		
Poisson's ratio	0.3			
Modulus - MPa (ksi)	276	(40)		
Ko	1.0			
Thickness - m (in.)	0.30	(12.0)		
<u>Subballast</u>				
Density - Mg/m^3 (pcf)	1.92	(120)		
Poisson's ratio	0.35			
Modulus - MPa (ksi)	138	(20)		
Ko	1.0			
Thickness - m (in.)	0.15	(6.0)		
<u>Subgrade</u>				
Density - Mg/m^3 (pcf)	1.92	(120)		
Poisson's ratio	0.35			
Modulus - MPa (ksi)	41	(6)		
Ko	1.0			
Thickness - m (in.)	infinite			
<u>Bedrock (if necessary)</u>				
Density - Mg/m^3 (pcf)	2.24	(140.0)		
Poisson's ratio	0.1			
Modulus - MPa (ksi)	6890	(1,000)		
Ko	1.0			

TABLE 2 Range of Variables for Parametric Study

Variable	Lower bound		Nominal		Upper bound	
Track type	Wood		Concrete		--	
Tie spacing - m (in.)	0.46	(18)	0.61	(24)	0.76	(30)
Fastener stiffness - kN/mm (kips/in.)	26	(150)	175	(1,000)	350	(2,000)
Ballast E_r - MPa (ksi)	138	(20)	276	(40)	551	(80)
Subballast E_r - MPa (ksi)	69	(10)	138	(20)	276	(40)
Subgrade E_r - MPa (ksi)	14	(2)	41	(6)	138	(20)
Ballast thickness - m (in.)	0.15	(6.0)	0.30	(12.0)	0.61	(24.0)
Subballast thickness - m (in.)	0.15	(6.0)	0.15	(6.0)	0.46	(18.0)
Subgrade thickness - m (ft)	1.22	(4.0)	infinity		--	

Note: Dash (--) indicates "not applicable".

numbers or letters in the figure represent the upper and lower bounds of the variables considered. The changes in track modulus caused by the change in each individual variable are indicated by vertical lines.

In Figure 4, the track moduli for the nominal concrete tie track and the nominal wood tie track are 25 MPa (3,640 psi) and 21 MPa (3,090 psi), respectively. The difference is 16 percent relative to the concrete tie track. The majority of this difference can be explained by the difference in fastener stiffness between the concrete tie track and the wood tie track. If a fastener stiffness of 70 kN/mm (400 kips/in.), assumed for the nominal wood tie track, is used to replace the value of 175 kN/mm (1,000 kips/in.) assumed for the nominal concrete tie track, the track modulus obtained for the concrete tie track is then 22 MPa (3,190 psi). The relative difference between these two types of track with the same fastener stiffness then becomes only 4.7 percent. This remaining difference is the result of the effect of the difference in tie bending stiffness and tie spacing. Thus, theoretically, the major source of a lower track modulus for wood tie tracks than for concrete tie tracks is from the vertical resilient compression of the wood ties.

The remaining parametric comparisons in Figure 4 are conducted one variable at a time only with the concrete tie track.

In Figure 4, a decrease in tie spacing causes a slight increase in track modulus. However, the increase is insignificant for the change of spacing from 0.76 to 0.46 m (30 to 18 in.). On the other hand, track modulus increases significantly with increasing fastener stiffness. As can be seen in this figure, the track modulus increased by 70 percent for the increase in fastener stiffness shown.

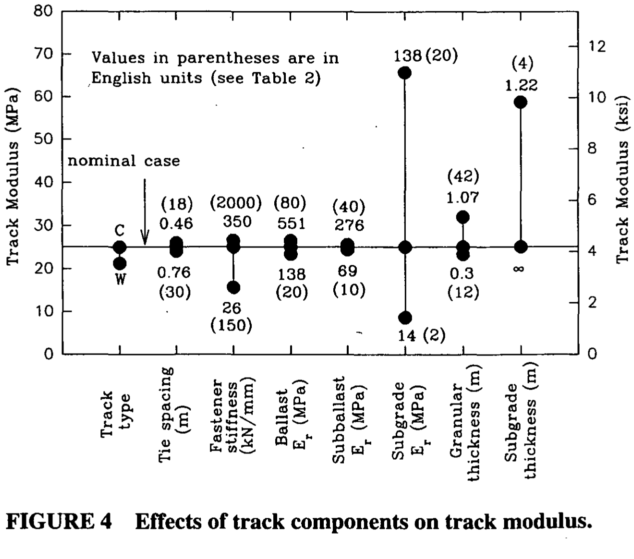


FIGURE 4 Effects of track components on track modulus.

Increasing ballast modulus or subballast modulus causes a slight increase of track modulus. However, the dominant factor influencing the track modulus is the subgrade resilient modulus. As demonstrated in Figure 4, a 10-fold increase in subgrade resilient modulus from 14 to 140 MPa (2,000 to 20,000 psi) leads to an increase in track modulus of approximately a factor of 8. Compared with the subgrade, the ballast and subballast can be considered to have very little effect on track modulus. The reasons for this are that the ballast and subballast layers are thin compared to the influence depth of the subgrade, and the ballast and subballast moduli do not vary as much as the subgrade modulus.

As demonstrated in Figure 4, the thicknesses of both the granular layer (ballast + subballast) and the subgrade layer overlying the bedrock affect the track modulus, though the latter has a much greater effect. An increase in the granular layer thickness leads to an increase in track modulus. On the other hand, an increase in the subgrade layer thickness leads to a decrease in track modulus, because the subgrade modulus is lower than that of the ballast, the subballast and the bedrock.

Interaction of Layer Thickness and Modulus

Although Figure 4 shows the effect of each individual factor on track modulus independently, it does not show how the major factors interact with each other in affecting track modulus. Figures 5 and 6 are presented to complement Figure 4 in studying the change of track modulus caused by the change of both material modulus and the corresponding material layer thickness. The values of the unvaried parameters are given in Table 1.

Figure 5 shows the influence of the granular layer thickness and the resilient modulus of the granular material on track modulus. In this figure, both the ballast and the subballast are assumed to have the same modulus, and the granular layer thickness is the sum of ballast layer thickness and subballast layer thickness. As the figure shows, for the change of granular layer thickness from 0.30 to 1.07 m (12 to 42 in.), the track modulus increases from 24 to 34 MPa (3,460 to 5,000 psi). The effect of the resilient modulus of the granular material on track modulus with different granular layer thickness is shown by using three moduli for the granular material. The lower bound is 138 MPa (20,000 psi), the nominal value is 276 MPa (40,000 psi), and the upper bound is 551 MPa (80,000 psi). The

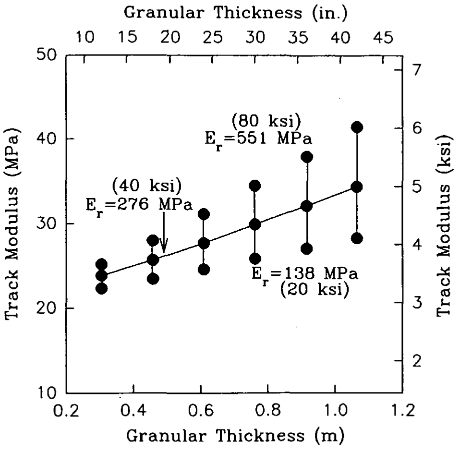


FIGURE 5 Effects of granular layer thickness and modulus on track modulus.

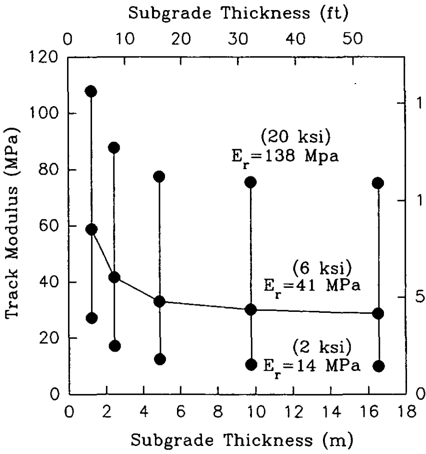


FIGURE 6 Effects of subgrade layer thickness and modulus on track modulus.

change of track modulus caused by the change of granular material modulus is indicated by the vertical lines. As this figure demonstrates, the sensitivity of track modulus to the modulus of the granular material increases with increasing thickness of the granular layer.

Figure 6 shows the effects of the subgrade thickness and the subgrade resilient modulus on track modulus. As already indicated in Figure 4, an increase in the subgrade thickness leads to a decrease in track modulus. However, as Figure 6 shows, the reduction rate of track modulus decreases as the thickness of the subgrade layer increases. Also the major contribution of a subgrade to track resiliency is from the layer down to approximately 5 m (16 ft) from the bottom of the subballast layer. However, the subgrade modulus effect on track modulus is always significant, independent of the thickness of subgrade layer.

Field Measurements

Tables 3 and 4 summarize field measurements of track modulus (*6, 7*). The measurements of track modulus were conducted using the single load test method as described previously, except for in Broken Bow, for which the multiple axle vehicle load test was used. The track conditions and subgrade soil conditions were also investigated using conventional soil investigation methods and some geotechnical in-situ techniques such as the electric cone penetration test and the dilatometer test.

TABLE 3 Field Test Results of Track Modulus for Concrete Tie Tracks

Test	Track Modulus,	Subgrade conditions		
site	u - MPa (ksi)	Layer no.	E_r -MPa (ksi)	Thickness - m (in.)
Leeds	(Tamping)	1	18 (2.6)	1.52 (60)
	before: 27 (3.9)	2	61 (8.8)	2.01 (79)
	after: 23 (3.3)	3	48 (7)	infinite
Aberdeen	38 (5.5)	1	131 (19)	0.61 (24)
		2	21 (3.1)	3.20 (126)
		3	117 (17)	infinite
Lorraine	(Tamping)	1	23 (3.4)	0.86 (34)
	before: 48 (7.0)	2	76 (11)	1.98 (78)
	after: 52 (7.6)	3	145 (21)	0.81 (32)
		4	179 (26)	infinite

TABLE 4 Field Test Results of Track Modulus for Wood Tie Tracks

Test	Track Modulus,	Subgrade conditions		
site	u - MPa (ksi)	Layer no.	E_r - MPa (ksi)	Thickness - m (in.)
Leeds	(Tamping)	1	9 (1.3)	2.24 (88)
	before: 15 (2.2)	2	45 (6.6)	1.19 (47)
	after: 20 (2.9)	3	36 (5.2)	infinite
Broken Bow	17 (2.5)	1	34 (4.9)	infinite
Leona	19 (2.8)	1	19 (2.8)	2.82 (111)
		2	39 (5.7)	infinite
Crowley	28 (4.0)	1	30 (4.4)	4.32 (170)
		2	117 (17)	0.91 (36)
		3	52 (7.6)	infinite
FAST	36 (5.2)	1	90 (13)	0.38 (15)
		2	131 (19)	0.56 (22)
		3	55 (8)	0.99 (39)
		4	124 (18)	infinite

First of all, as demonstrated by these two tables, the conditions of subgrade are quite variable from place to place, including both resilient modulus and subgrade layer thicknesses. Track modulus changes from place to place, approximately consistent with a trend corresponding to the change of subgrade conditions. The track modulus at the Leeds site presents evidence of this. The track moduli for the wood tie track are consistently much lower than the track moduli for the concrete tie tracks, both before and after the maintenance. According to the previous parametric computer study, a wood tie track possesses a lower track modulus than a concrete tie track mainly because of a difference in "fastener" stiffness. The relative difference in track modulus caused by track type is relatively small for a soft-to-medium subgrade. Therefore, there must be other factors leading to the greater difference in the measured track modulus for the two types of track in Leeds before tamping. When the subgrade conditions of the concrete tie track and the wood tie track for this test site are compared, it becomes obvious that the subgrade for the wood tie track is softer than that for the concrete tie track. Therefore, these much lower track moduli for the wood tie track are reasonable as a result of the effect of the subgrade condition on track modulus.

METHODS FOR ALTERING TRACK MODULUS

The factor affecting the track modulus most is the character of the subgrade layers. The influence of subgrade condition on track modulus is further enhanced by the fact that the subgrade resilient modulus is the most variable quantity among all the track parameters, subject to change of soil type, environmental conditions, and stress state (*9*). Therefore, a change of track modulus in the field is primarily an indication of a change of subgrade condition. Since the subgrade condition is subject to weather, extremes of temperature and moisture, the track modulus may vary with seasonal changes.

The next most important factors affecting the track modulus are the thickness of the granular layer and the fastener stiffness. An increase in either quantity can lead to a significant increase in track modulus.

A higher track modulus is generally considered to provide better track performance. However there is probably a limit above which track modulus is too high for satisfactory performance. This upper limit has yet to be defined. Studies are needed to provide a better understanding of the effect of track modulus on track performance in order to determine desirable value.

If the track modulus is assumed to be too low, then the following changes will help to increase it, in descending order of effectiveness:

1. Increase composite subgrade resilient modulus.
2. Increase granular layer thickness.
3. Increase fastener stiffness.

In addition, use of an asphalt layer under ballast and use of the slab track can also improve track modulus.

In general, altering the subgrade stiffness is the most difficult task of the three alternatives. Examples of techniques used in the past to increase the subgrade resilient modulus are: removal and replacement, admixture stabilization, subsurface drainage, lime slurry injection, electroosmosis and grouting.

Changing the rail stiffness will not alter track modulus because track modulus represents the rail support stiffness. However the rail stiffness will affect track stiffness, *k*.

TRACK MODULUS AS A MEASURE FOR ASSESSING TRACK PERFORMANCE

Track modulus has been used for representing track quality. In general, a higher track modulus is considered to represent better supporting capacity, and therefore is associated with better track performance. Hay (*11*) and the AREA manual (*12*) suggested that a minimum value of 14 MPa (2,000 psi) be required to ensure a satisfactory performance of railway track. Based on the field observations, Ahlf (*13*) concluded that a track with a track modulus less than 14 MPa (2,000 psi) was poor, a track with a track modulus between 14 MPa (2,000 psi) and 28 MPa (4,000 psi) was average, and a track with a track modulus above 28 MPa (4,000 psi) was good. Raymond (*14*) suggested that the optimum track modulus is 34 to 69 MPa (5,000 to 10,000 psi). None of these references suggest that the track modulus can be too high.

In order to understand the correlation between track modulus and track performance, calculations were performed using GEOTRACK to relate track structure response parameters caused by traffic loading to track modulus. For calculation of track structure response parameters, the loading was chosen to represent two 2-axle bogies adjacent to a coupling as in a typical freight car. The axles were assumed to be 1.83 m (72 in.) apart. In GEOTRACK only 3 axles were used with the responses taken under the center axle, because the fourth axle was far enough away to permit neglecting it. The track modulus was varied by changing the subgrade layer thickness and subgrade resilient modulus.

Figure 7 shows the range of track modulus obtained from the GEOTRACK analysis corresponding to the different subgrade conditions. Subgrade resilient modulus is varied from 14 to 124 MPa (2,000 to 18,000 psi) for subgrade soil thicknesses of 1.22 m (4 ft) underlain by bedrock and infinity. Again, the values of the parameters that are not varied are given in Table 1. The corresponding range of track modulus is from 8.3 to 103 MPa (1,200 to 15,000 psi). As this figure illustrates, the same track modulus may be a result of two different combinations of subgrade resilient modulus and subgrade thickness. This indicates that although subgrade resilient modulus affects track modulus significantly, it is not related to track modulus uniquely, but is dependent on the subgrade layer thickness.

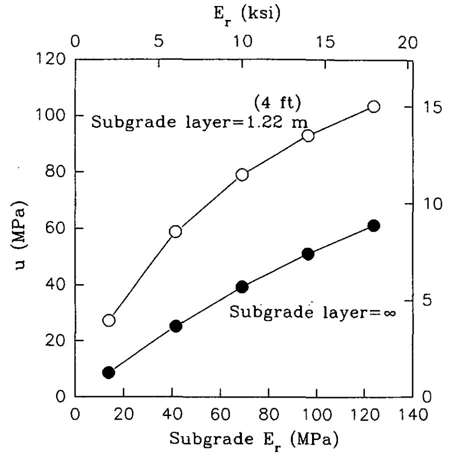


FIGURE 7 Correlations between track modulus and subgrade resilient modulus.

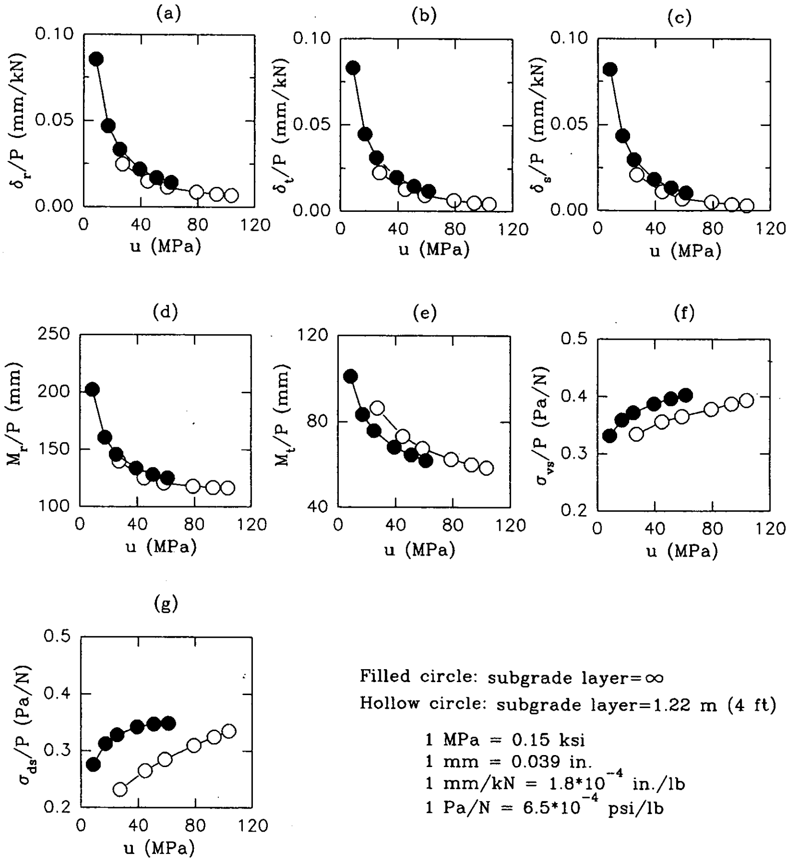
GEOTRACK analysis was performed to relate the track modulus corresponding to the different subgrade conditions to concrete tie track structure response parameters under the axle loadings. The responses taken under the rail seat are rail deflection, tie deflection, subgrade surface deflection, rail bending moment, and tie bending moment. In addition, the maximum vertical stress at the subgrade surface and the maximum deviator stress at the subgrade surface were also calculated. In general, these maximum stresses occur under the outer end of the tie for the concrete tie track. Assuming linear elastic behavior, all these responses are proportional to the magnitude of wheel load. Thus all these responses are normalized by the magnitude of wheel load. Unless specified in the figures, the nominal values listed in Table 1 for the concrete tie track are used for analysis.

Figures 8a, 8b and 8c show the correlations between track modulus and rail (δ_r), tie (δ_t), and subgrade surface (δ_s) vertical deflections. As expected, an increase in track modulus results in a decrease in all deflections. However, as all three figures show, when track modulus is approximately below 28 MPa (4,000 psi), the effect of track modulus on deflections is more dramatic. When the track modulus is higher than 28 MPa, the variation of deflections with changing track modulus becomes more gradual.

Another observation from Figures 8a, 8b, and 8c is that the correlation between track modulus and deflection is independent of the subgrade condition (different combinations of resilient modulus and thickness) as long as the track modulus is the same. This result is shown by the overlapping of two curves from two different subgrade layer thicknesses. This is expected, according to Equation 9, when the load, *P*, and the rail bending stiffness, *EI*, are constant.

Figures 8d and 8e show the influence of track modulus on the rail bending moment (M_r) and the tie bending moment (M_t). An increase in track modulus leads to a reduction of bending moments in both the rail and the tie. This means that an increase in track modulus results in lower bending stresses in the rails and the ties. Again, a track modulus of approximately 28 MPa (4,000 psi) can be considered as a dividing point below which the increase in the moments for both the rail and the tie with decreasing track modulus is more rapid. The rail bending moment is independent of the subgrade condition for the same track modulus (Figure 8d) and the tie bending moment nearly so (Figure 8e).

Figures 8f and 8g show the influence of track modulus on the maximum vertical stress (σ_{VS}) and the maximum deviator stress (σ_{ds}) at the subgrade surface. They occur under the outer end of the



Filled circle: subgrade layer=∞
Hollow circle: subgrade layer=1.22 m (4 ft)

1 MPa = 0.15 ksi
1 mm = 0.039 in.
1 mm/kN = $1.8*10^{-4}$ in./lb
1 Pa/N = $6.5*10^{-4}$ psi/lb

FIGURE 8 Influence of track modulus on track performance.

tie. An increase in track modulus results in an increase in stresses in the subgrade. The magnitude of the effect is dependent on the subgrade condition.

The trend in Figures 8f and 8g is potentially of concern. A higher track modulus leads to a higher stressing in the subgrade, thus seemingly would lead to a more detrimental situation in the subgrade. However, this is not actually true because the track performance depends not only on the generated stresses, but also on the soil strength. The track modulus is also an indication of subgrade soil strength. A higher track modulus represents a stiffer subgrade, and a stiffer subgrade generally has a higher soil strength. An approximate correlation between subgrade resilient modulus and soil compressive strength is given in the following table, which demonstrates that the strength of soil increases rapidly with an increase in soil-resilient modulus.

Soil Condition	*Resilient Modulus*		*Compressive Strength*	
	(MPa)	*(ksi)*	*(kPa)*	*(psi)*
Soft	7–28	(1–4)	34–103	(5–15)
Medium	28–69	(4–10)	103–207	(15–30)
Stiff	69–138	(10–20)	207–345	(30–50)

Therefore, although higher soil stresses will be generated for a higher track modulus, the subgrade soil will have a higher strength to resist the higher stresses. In general, it is expected that the increase in strength for a stiffer subgrade is more rapid than the increase of stresses in the subgrade for a stiffer subgrade.

The value of track modulus that should be required to ensure no excessive stresses and deformation depends on many factors. However, to avoid a rapid change of track responses caused by the change of track modulus under any magnitude of axle loads for the case studied, a track modulus above approximately 28 MPa (4,000 psi) may be considered favorable.

SUMMARY AND CONCLUSIONS

Track stiffness is defined as the ratio of the vertical force applied to the top of the rail to the corresponding vertical deflection. Track modulus is the rail foundation stiffness representing the fasteners, the ties, and the substructure. It is a parameter used in the beam-on-elastic-foundation track model. Track stiffness and track modulus are uniquely related for a given rail bending stiffness.

Several methods are given for determining track modulus and track stiffness in the field. Because the relationship between vertical load and vertical deflection is nonlinear, measurements at several load levels are generally required.

A computer model of the track structure was used to evaluate the factors influencing track modulus and track stiffness. The biggest factor is the subgrade condition as represented by the subgrade resilient modulus and layer thickness. Thus a change in track modulus or stiffness along the track is primarily an indication of the variation in the subgrade condition. The two next most important factors are granular layer thickness and fastener stiffness. Significantly altering track modulus or stiffness at a given location, therefore, requires a change in one of these factors.

Track modulus or track stiffness is a measure of the structural condition of the track and should provide an indication of expected track performance. The computer analysis indicated that an increase in track modulus or stiffness caused a decrease in rail, tie and subgrade vertical deflections; a decrease in rail and tie bending moments; and an increase in subgrade stresses. The theoretical results suggest that above a track modulus of 28 MPa (4,000 psi), changes in track modulus do not substantially alter the performance of the track, while below this level a decrease in track modulus greatly reduces the performance. A track modulus of approximately 28 MPa (4,000 psi) may be considered a minimum to ensure a consistently good track performance under traffic loading. However, the optimum or desirable values of track modulus for ensuring good track performance under different conditions should be further evaluated by studying track in service.

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