Estimate of Static Track Modulus Using Elastic Foundation Models

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A new method for calculating the track modulus by using elastic properties of the individual track support components is developed. The properties include (*a*) the stiffness of the rail pad, (*b*) the vertical stiffness and bending rigidity of the tie treated as a beam on both a Winkler foundation and a two-parameter foundation, and (*c*) the Young's moduli and Poisson's ratio values of the soil layers underlying the track. Semi-infinite elastic foundation theories are used to estimate the equivalent ballast/subgrade stiffness distributed along the length of the tie. Track modulus charts and application examples are given for typical track support structures used in North America. A practical benefit of the new approach proposed is that it eliminates the need to carry out relatively expensive field tests. An overview of some existing methods to determine track modulus is also presented. The validation of the proposed method is currently in progress.

Since the early days of railway engineering, the beam-on-elastic-foundation (BOEF) method, or the Winkler method, has been used to analyze track mechanics, as illustrated in Figure 1. In this method, the rail is considered as an infinitely long beam resting on an elastic foundation comprising continuous linear springs. The physical effects of all the structural components underlying the rail are thus represented by a single parameter, k, termed the track modulus. Although the Winkler method does not consider the individual structural features of all the track components, it involves simple calculations and has become widely accepted by the railway industry for use in track design (*1*). A survey of the development of track mechanics was presented by Kerr (*2*), and a detailed analysis of track responses under wheel loads has been given by Raymond (*3*).

The Winkler method of analysis is based on the following differential equation for the rail deflection under load:

$$EI \frac{d^4w(x)}{dx^4} + kw(x) = q(x) \qquad (1)$$

where

- $w(x)$ = vertical deflection of the rail at point x from the applied wheel load,
- EI = flexural rigidity of the rail beam,
- $q(x)$ = distributed load equivalent to the wheel loads, and
- k = track modulus, which is the coefficient of proportionality between rail deflection and vertical contact pressure acting at the interface between the rail base and the track foundation.

For a single concentrated wheel load P applied vertically on the rail, Equation 1 yields the following rail deflection curve:

$$w(x) = \frac{P\beta}{2k} e^{-\beta x} (\cos \beta x + \sin \beta x) = \frac{P\beta}{2k} \eta(x) \qquad (2)$$

and the corresponding bending moment is given by

$$M(x) = \frac{P}{4\beta} e^{-\beta x} (\cos \beta x - \sin \beta x) \qquad (3)$$

where

$$\beta = \left(\frac{k}{4\,EI}\right)^{\frac{1}{4}} \qquad (4)$$

The flexural rigidity of the rail, EI, is an easily determined quantity, and P is the specified wheel load. For any chosen value of k, the desired track response can be readily calculated.

Determining a track modulus representative of the combined structural effects of the track support components is not easy, although the concept involved is relatively simple. The track modulus is likely to vary substantially at different locations along the track due to variations in ballast/subgrade properties, uneven construction effects, and track service life. Some ballast properties—such as hardness, toughness, durability, and specific gravity—can vary enormously from one ballast to another (e.g., a limestone versus a basalt or copper slag ballast). The track modulus can also be expected to vary seasonally, due to the effect of freeze-and-thaw cycles on ballast/subgrade.

Methods to determine the track modulus have traditionally been based on the BOEF theory and the use of large-scale tests involving prototype axle loads and rail deflection measurements. Commonly used experimental techniques and corresponding analytical methods have been described by Zarembski and Choros (*4*) and Kerr (*5*).

The purpose of this paper is to present an overview of the existing methods and propose a new method for calculating the track modulus, based on elastic foundation models. A practical benefit of the approach proposed in this paper is that it eliminates the need to carry out relatively expensive field tests. The validation of the proposed method is currently in progress.

EXISTING METHODS FOR DETERMINING TRACK MODULUS

Vertical Equilibrium (or Deflection-Area) Method

The vertical equilibrium method to calculate the track modulus k has been used by a number of researchers, including the ASCE-AREA Special Committee under the leadership of Talbot (*6*). The

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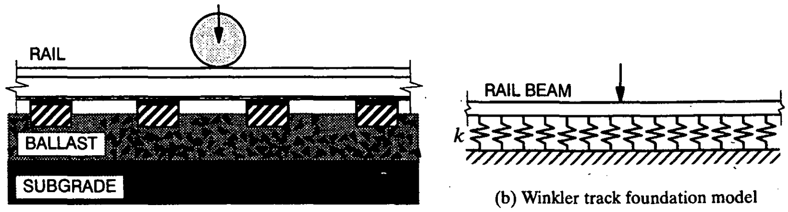


(a) Schematic of rail track

(b) Winkler track foundation model

FIGURE 1 Rail track formation modeled as a Winkler foundation.

concept is based on the vertical equilibrium of an infinitely long rail beam under an applied load P, that is,

$$P - \int_{-\infty}^{+\infty} kw(x)\,dx = 0 \tag{5}$$

If the deflection curve $w(x)$ is measured, then the track modulus k can be calculated as

$$k = \frac{P}{\int_{-\infty}^{+\infty} w(x)\,dx} \tag{6}$$

The integral denominator is the area formed by the deflection curve $w(x)$ along the rail axis. In practice, the following approximation is used:

$$k = \frac{P}{\sum_{i=1}^{n} s_i w_i} \tag{7}$$

where w_i is the measured deflection at the ith location along the rail and S_i is the distance between two adjacent measurement points. The number of deflection points will clearly influence the accuracy of k.

To reduce the effect of slack (or free play) existing in the track, the field test can be performed in two steps, the first with a light wheel load, P_l, and the second with a heavy wheel load, P_2. The parameter k is then calculated as follows:

$$k = \frac{P_1 - P_2}{\sum_{i=1}^{n} s_i(w_{i1} - w_{i2})} \tag{8}$$

where w_{i1} and w_{i2} are the corresponding measured rail deflections.

Beam-on-Elastic-Foundation Method

This method is derived directly from the solution of rail deflection using the Winkler's BOEF approach. If the deflection under a single wheel load is measured as w_0, then from Equation 2 with $x = 0$,

$$k = \frac{1}{4}\left(\frac{1}{EI}\right)^{\frac{1}{3}}\left(\frac{P}{w_0}\right)^{\frac{3}{4}} \tag{9}$$

The major advantage of this method over the vertical equilibrium method is that only one deflection measurement is required and the effect of rail-bending rigidity is considered explicitly. To apply this method, however, a special loading arrangement is required in the field, comprising a single load configuration.

Zarembski and Choros (*4*) and Kerr (*5*) described a similar approach to determine the track modulus under multiple wheel loads that uses the measured deflection under one wheel load, calculated by adding the contribution of all the wheel loads multiplied by their corresponding influence coefficients given by Equation 2, that is,

$$w_0 = \frac{\beta}{2k}\sum_{i=1}^{n} P_i \eta_i \tag{10}$$

Here the influence coefficient $\eta_i = e^{-\beta x_i}(\cos \beta x_i + \sin \beta_i x_i)$, in which x_i is the distance between the ith wheel load P_i and the measurement point. Thus, an iterative procedure is needed to obtain the value of the track modulus k since β and η_i are both a function of k. Zarembski and Choros (*4*) suggested the following procedure based on Equation 10:

$$\left| k - \frac{\beta}{2w_0}\sum_{i=1}^{n} P_i \eta_i \right| \leq \epsilon \tag{11}$$

where ϵ is a prescribed error of tolerance. Kerr (*5*) has proposed an alternative expression of Equation 10:

$$\frac{w_0}{P} = \frac{\beta}{2k}\sum_{i=1}^{n} a_i \eta_i \tag{12}$$

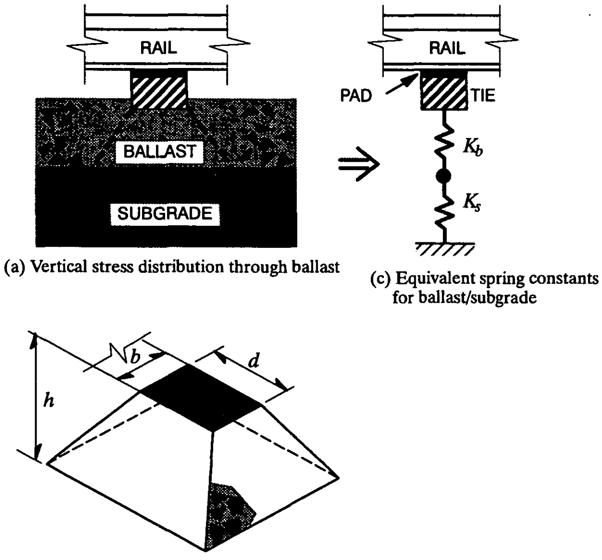
where $a_i = P_i/P$. Thus, the normalized deflection, w_0/P, can be plotted as a function of k. The above equation has been used to generate normalized deflection charts for three different rail sizes loaded by a standard freight car (*5*).

Recent field measurement results of track modulus using the BOEF approach against predictions using a FEM track program were described by Stewart (*7*). A method that considers track nonlinear effects on the track modulus has been given by Kerr and Shenton (*8*).

In the above methods, the calculation of k using the measured rail deflection(s) involves an averaging effect over the entire length of the depressed track. This implicitly takes into account soil particle interactions and the vertical elasticity and flexural rigidity of the ties. However, the individual effects of the track components on the track modulus are not distinguishable in the results of these testing methods. Furthermore, field tests on tracks are time consuming and costly, and cannot be performed for a track before design.

Pyramid Load Distribution (PLD) Method

The PLD method is illustrated in Figure 2, and assumes that the pressure beneath the rail seat is distributed uniformly at each depth across the area of an imaginary pyramid-shaped zone spreading



(a) Vertical stress distribution through ballast

(c) Equivalent spring constants for ballast/subgrade

(b) Pyramid zone of stress distribution

FIGURE 2 Pyramid load distribution method to calculate track modulus.

downward through the ballast layer (*9*). Thus, the Poisson's ratio effect is nonexistent and subsequently replaced by the "angle of internal friction." From the theory of elasticity, the effective stiffness of the ballast layer expressed as an equivalent elastic spring is given by the following:

$$K_b = \frac{2E_b(d - b)\tan\phi}{\log\left[\frac{d(b + 2h\tan\phi)}{b(d + 2h\tan\phi)}\right]} \tag{13}$$

where

E_b = Young's modulus of the ballast,
ϕ = angle of internal friction of the ballast,
d = length of loaded area,
b = width of loaded area, and
h = depth of the ballast layer.

The effective stiffness of the subgrade is calculated by multiplying the base area of the pyramid at the ballast/subgrade interface by the subgrade modulus k_o:

$$K_s = k_o(d + 2h\tan\phi)(b + 2h\tan\phi) \tag{14}$$

Thus, the effective stiffness of the entire rail support system can be calculated from the following:

$$\frac{1}{K_e} = \frac{1}{K_p} + \frac{1}{K_t} + \frac{1}{aK_{bs}} \tag{15}$$

where

K_e = effective spring stiffness of the rail support system,
K_p = stiffness of rail pad,
K_t = vertical stiffness of tie,
K_{bs} = combined stiffness of ballast and subgrade, and
a = experimental factor used to account for the continuity of the deflection of the track bed between adjacent ties.

Ahlbeck et al. (*9*) suggested, on the basis of experimental data, the use of $a = 1/2$.

Thus, the distributed track modulus is simply calculated by $k = K_e/s$, where s is the tie spacing.

PROPOSED NEW METHOD FOR CALCULATING TRACK MODULUS

As seen from the definition of the track modulus, the parameter k in the Winker beam-bending equation represents the gross compliance of the track subcomponents comprising the resilience of the rail pad, the vertical (or compressive) stiffness and bending rigidity of the tie, and the elasticity of the ballast/subgrade. The combined effect may be most properly determined from a loading test on field track. Field tests are time consuming and costly, and may not always be practical. The PLD method is one simple approach to obtaining a rapid estimate of the track modulus based on the physical properties of individual track components.

The following limitations, however, are inherent in the PLD method: (*a*) the flexural rigidity of the tie as a beam is not considered, (*b*) the vertical stresses are assumed to distribute through an imaginary pyramid zone, the accuracy of which is questionable, and (*c*) the shear effects of the soil media are not considered, although an empirical factor of 1/2 (see Equation 15) is suggested by Ahlbeck et al. (*9*) to compensate for that effect.

In the following sections, a new method is described for calculating the track modulus. It considers the tie as a flexible beam resting on an elastic medium described by both a Winkler foundation model and a two-parameter foundation model. The foundation modulus parameters of both models are related to the elastic properties of the foundation represented as an equivalent semi-infinite soil medium.

Tie as a Beam on Winkler Foundation

Formulation

In this method, the tie is described as a uniform beam resting on a Winkler-type foundation with a distributed across-track modulus, k_s, and subjected to two vertical loads Q, symmetrically applied at the rail seats, as illustrated in Figure 3. According to Hetenyi (*10*), the deflection of the tie at the rail seat is as follows:

$$z_s = \frac{Q\beta}{2k_s} H(\beta, D) \tag{16}$$

where

$$\beta = \left(\frac{k_s}{4\,EI_t}\right)^{\frac{1}{4}} \tag{17}$$

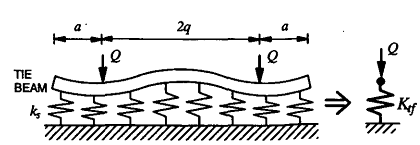


FIGURE 3 Equivalent spring stiffness of tie beam resting on elastic foundation.

in which k_s is distributed spring stiffness beneath the tie beam and EI_t is bending rigidity of the tie. $H(\beta, D)$ in Equation 16 is a dimensionless transcendental function of the parameter β and tie dimensions represented by the symbol D. Figure 4 shows $H(\beta, D)$ plotted against β for two ties with lengths of 2.5 m (8 ft, 6 in.) and 2.70 m (9 ft), respectively. For $\beta > 2$, $H(\beta, D) \approx 1$.

Thus, the equivalent spring stiffness (per rail) offered by a tie lying on the track foundation is given by the following equation:

$$K_{tf} = \frac{Q}{z_s} = \frac{2k_s}{\beta} \frac{1}{H(\beta, D)} \tag{18}$$

The only unknown in the above formula is the ballast/subgrade stiffness parameter, k_s. Selvadurai (*11*) has described a number of methods to estimate the value of k_s. An approximate method developed by Galin in 1934 and later confirmed by Sivashinsky in 1975 (*11*) treats the finite beam resting on an isotropic elastic continuum as equivalent to that of the Winkler foundation model. This yields the following expression for k_s:

$$k_s = \frac{\pi E_s}{2(1 - \mu_s^2)\, l_n(l/b)} \tag{19}$$

where E_s and μ_s are the effective elastic modulus and Poisson's ratio of the ballast/subgrade foundation, respectively, and b and l are, respectively, the width and length of the tie beam.

Vesic and Johnson (*12*) suggested the following expression for calculating k_s:

$$k_s = \frac{0.65\, E_s}{(1 - \mu_s^2)} \left(\frac{E_s b^4}{EI_t}\right)^{\frac{1}{2}} \tag{20}$$

Thus, the parameter k_s calculated as such is representative of the general elasticity of the track foundation. The major difficulty with Equations 19 and 20 lies in the determination of the effective modulus E_s of the layered ballast/subgrade foundation. A first approximation to estimate E_s may be to use Steinbrenner's settlement solution (*13*) for a flexible base resting on a layered elastic soil medium. This yields the following expression for the effective elastic modulus of the layered track foundation:

$$E_s = I_p \left[\sum_{i=1}^{n-1} \left(\frac{I_{n+1} - I_n}{E_i} \right) + \frac{I_{pn}}{E_n} \right] \tag{21}$$

where

E_s = effective elastic modulus of the equivalent elastic continuum;
E_i = elastic modulus of the ith soil layer;
I_p = displacement influence factor of the elastic half-space;
$I_{pi} = (I - \mu_{si}^2)\, F_{1i} + (1 - \mu_{si} - \mu_{si}^2) F_{2i}$ is the vertical displacement influence factor;
μ_{si} = Poisson's ratio for the ith soil layer; and
F_{1i} and F_{2i} = dimensionless factors related to the depth/width ratio (Z/b) of the soil layer and the length/width ratio (l/b) of the tie, as given in Figure 5.

Soil elastic properties can be conveniently obtained from laboratory tests (*14*). Tables 1 and 2 (*15*) give some typical ranges of values of soil elastic modulus and Poisson's ratio for different types of soils. Other methods that may be used to estimate the effective elastic modulus of a layered soil foundation are given elsewhere (*13,16,17*).

Thus, the equivalent track modulus is obtained from the following equation:

$$k = \frac{K_{tf}}{1 + \dfrac{K_{tf}}{K_v}} \frac{1}{s} \tag{22}$$

where K_{tf} is calculated from Equation 18 and K_v represents the combined vertical stiffness of the tie and the rail pad.

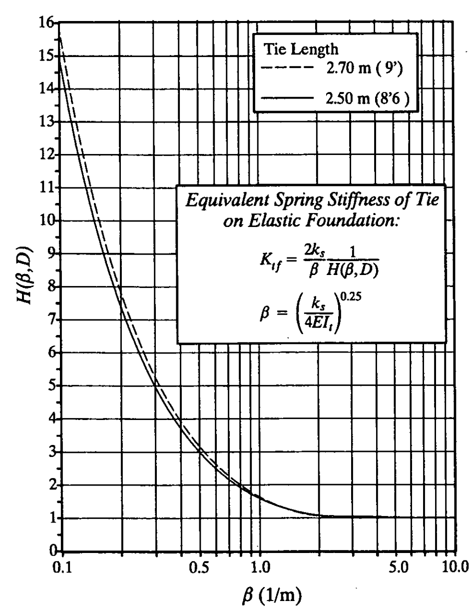


FIGURE 4 Relationship between H (β, D) and β for two example ties.

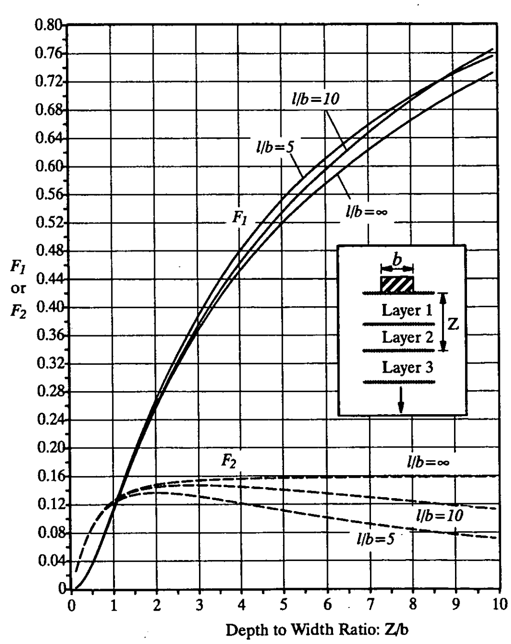


FIGURE 5 F_1 and F_2 values versus depth-to-width ratio.

TABLE 1 Typical Range of Values of E_s for Selected Soils (*15*)

Type of Soil	E_s (MN/m²)
Very soft clay	0.3 - 3
Soft clay	2 - 4
Medium clay	4.5 - 9
Hard clay	7 - 20
Sandy clay	30 - 42.5
Glacial till	10 - 16
Loess	6 - 15
Silt	2 - 20
Silty sand	5 - 20
Loose sand	10 - 25
Dense sand	50 - 100
Loose sand and gravel	50 - 140
Dense sand and gravel	80 - 200
Shale	140 - 1,400
Ballast	200 - 280 (after Ahlbeck et al, 1975) 105 - 245 (after Stewart, 1985)

Application Example

To demonstrate the use of the above method, an example track with wooden ties or concrete ties spaced at 0.6 m (24 in.) is assumed to rest on a foundation soil profile with elastic properties shown in Figure 6. The bottom clay layer is considered to extend to a depth of 5 times the tie width, where the displacement of the soil can be considered to be negligible.

Step 1. Calculate (*a*) I_p values for each layer according to the corresponding depth/width ratio and Poisson's ratio, (*b*) I_p for an infinite soil medium (assuming a Poisson's ratio of 0.1), and (*c*) E_s according to Equation 21 (in this example, E_s =98 MN/m²).

Step 2. Calculate (*a*) k_s using Equation 19 (k_s = 64 MN/m/m) and (*b*) β according to Equation 17 and H (β, D) from Figure 4. For the wooden tie, β = 1.91 and H(β, D) = 1.03. For the concrete tie, β = 1.17 and H(β, D) = 1.44.

Step 3. Calculate K_{tf} using Equation 18 and finally the track modulus k from Equation 22. For the wooden tie, K_{tf} = 65 MN/m, and assuming K_v = 100 MN/m, Equation 22 yields k = 65 MN/m/m. For the concrete tie, K_{tf} = 78 MN/m, and assuming K_v = 750 MN/m, Equation 22 yields k = 116 MN/m/m.

Tie as a Beam on Two-Parameter Foundation

The Winkler foundation model simplifies the soil medium to an assemblage of a series of independent elastic springs. In order to account for shear interaction effects within the soil mass, many two-parameter foundation models have been developed (*11,18,19*). In recent years, the use of two-parameter foundation models have become increasingly popular (*20–23*).

The governing differential equation for the deflection, $z(x)$, of the tie as a uniform beam on a two-parameter elastic foundation, is given by the following:

$$EI_t \frac{d^4z(x)}{dx^4} - k_g \frac{d^2z(x)}{dx^2} + k_s z(x) = q(x) \tag{23}$$

where k_s is the first parameter (Winkler's modulus) and k_g is the second parameter, which is a generalized quantity used to compensate for shear continuity in the foundation soil. A detailed explanation of k_g for various two-parameter models is given elsewhere (*11,19*).

To obtain the equivalent spring stiffness of the two-parameter tie/foundation system, the matrix method incorporating the exact stiffness of the tie is used to solve Equation 23. Many authors have derived in various forms the exact stiffness of a uniform beam on a two-parameter foundation (*19,22,24*). By using this approach, the nonuniformity of both the tie cross-section and the distributed foundation modulus can be taken into consideration. Note that if k_g = 0, Equation 23 reduces to the original Winkler model. Therefore, the following formulation also applies to a tie beam on a Winkler foundation.

Consider the half-tie shown in Figure 7. It is herein assumed that the tie consists of two different uniform segments resting on two

TABLE 2 Typical Range of Values of μ_s for Selected Soils (*15*)

Type of Soil	μ_s
Clay, saturated	0.4-0.5
Clay, unsaturated	0.1-0.3
Sandy clay	0.2-0.3
Silt	0.3-0.35
Sand (dense)	0.2-0.4
Coarse (e = 0.4-0.7)	0.15
Fine-grained (e = 0.4-0.7)	0.25
Rock	0.1-0.4

different sets of uniform springs. This is a reasonable assumption since most concrete ties comprise a uniform center segment and two approximately uniform shoulder segments, and ballast tamping is usually within the rail seat area resulting in a stiffer track foundation within that area. An empirical formula is given elsewhere (*1,3*) to calculate the length of tamped area.

The solution of Equation 23 at the four nodes of the three elements shown in Figure 7 can be obtained from the following matrix equation:

$$[K]\{\delta\} = \{P\} \quad (24)$$

where

$[K]$ = overall stiffness matrix (8 × 8) of the three elements,
$\{\delta\} = [z_1, \theta_1, z_2, \theta_2, z_3, \theta_3, z_4, q_4]^T$ is the displacement vector (in which $\theta_4 = 0$), and
$\{P\} = [0, 0, Q, 0, 0, 0, 0, M]^T$ is the nodal force vector.

Solving Equation 24 for the rail seat deflection, z_2, the following is obtained:

$$z_2 = Q\frac{\Delta_s}{S_{88}} \quad (25)$$

where $\Delta_s = S_{33} \times S_{88} - S_{38} \times S_{83}$. The terms S_{ij} (i, j = 3, 8) are the corresponding elements of the inverse matrix of the 8 × 8 general stiffness matrix $[K]$ formulated in Equation 24.

Thus, from Equation 25, the equivalent spring stiffness of the tie-foundation system is inferred to be the following:

$$K_{tf} = \frac{S_{88}}{\Delta_s} \quad (26)$$

The track modulus can be calculated by substituting this result into Equation 22. A Fortran program has been written to implement this solution procedure. Note that Equation 18 is a special case of Equation 26.

The number of nodes (or elements) used to discretize the tie can be increased to account for further variations in the tie cross-section and track bed stiffness. However, the two-segment beam approximation given in Figure 7 has been shown to yield adequate results for most concrete ties. For wooden ties, only variations in the track bed stiffness are considered.

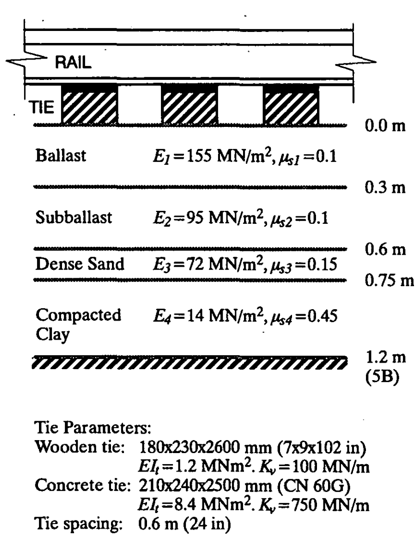


FIGURE 6 Example track and soil profile.

Application Example

Case 1—Winkler Foundation Model

Using the same ballast/subgrade soil profiles given in Figure 6, Equations 24, 26, and 22 are solved for the following:

1. A wooden tie having a vertical stiffness of k_v = 100 MN/m and a bending stiffness of EI_t = 1.2 MNm²; and
2. A nonuniform concrete tie (CN 55 A type) with a SYN pad (S-grooved synthetic rubber) of stiffness k_v = 180 MN/m, or an EVA pad of stiffness k_v = 750 MN/m.

For both ties, the distributed Winkler stiffness beneath the rail seat area is assumed to be 1.5 times that beneath the center portion of the tie to account for the more intense tamping applied to the rail seat area. The calculated track moduli for the above cases are as follows: for the wooden tie track, k = 63 MN/m/m; for the concrete tie track with SYN pads, k = 84 MN/m/m; and for the concrete tie track with EVA pads, k = 107 MN/m/m.

The modulus value of the concrete tie track with the EVA pads is nearly 30 percent higher than that with the softer SYN pads, and nearly 80 percent higher than that of the wooden tie track.

To consider variations in the ballast/subgrade stiffness, the track modulus k is plotted in Figure 8 as a function of k_s. Figure 8 clearly illustrates the marked effects on track flexibility of using different types of ties and pads. For example, a concrete tie track has a considerably stiffer track modulus than a wooden tie track lying on the same track foundation. The difference between the track modulus values for the three cases examined increases substantially as k_s increases. This is clearly a result of the interaction between the bending rigidity of the tie as a beam and the compliance of the soil foundation. For example, with a lower k_s value (for example, <20 MN/m/m), the track modulus values are not significantly different since the bending effect of the tie may not be appreciable when a soft subgrade is present. With a k_s value over 100 MN/m/m, however, the concrete tie track with EVA pads has a track modulus more than twice the track modulus of the wooden tie track, and is more than 1.4 times that of the concrete tie track with the softer SYN pads.

The steepness of the modulus curves illustrate the importance of the ballast/subgrade stiffness on the track modulus. The track mod-

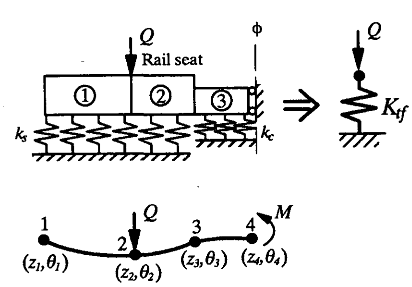


FIGURE 7 Discretization of half-tie beam for determination of equivalent spring stiffness.

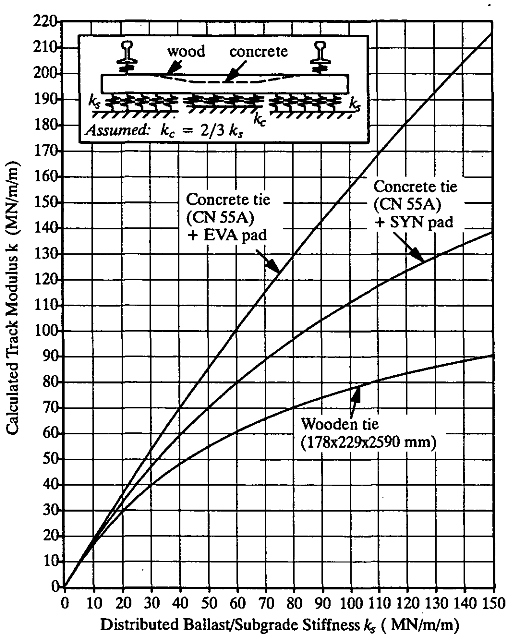


FIGURE 8 Track modulus versus ballast/subgrade stiffness for different types of track support.

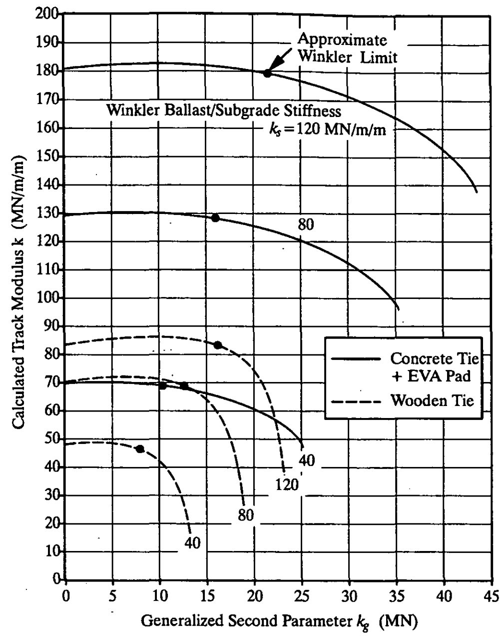


FIGURE 9 Effect of the generalized second parameter k_g on track modulus.

ulus of the concrete tie track increases more rapidly with the ballast/subgrade stiffness than the wooden tie track, especially when a harder pad (e.g., an EVA pad) is used. A practical consequence of this observation is that a concrete tie track is expected to experience higher increases in wheel/rail impact loads than a wooden tie track during the winter, when the ballast/subgrade is frozen.

Case 2—Two-Parameter Foundation Model

The principal difficulty involved in using the two-parameter model lies in the proper estimate of the two elastic parameters, k_s and k_g, particularly the second parameter. To demonstrate the general relationship between the track modulus and these two parameters, Equations 24, 26 and 22 are solved for the same wood and concrete ties (with EVA pad only) in Case 1. The results using three k_s values are presented in Figure 9.

For each case, it is observed that the variation in the track modulus with parameter k_g is insignificant up to a threshold value identified on each curve. Therefore, the track modulus within this range can be estimated with acceptable accuracy using the Winkler model ($k_g = 0$). Beyond that limit, however, the track modulus decreases substantially as k_g increases. This trend is more pronounced for the wooden tie track than for the concrete tie track.

The determination of the elastic parameters, k_s and k_g, has been addressed by many authors (11, 21, 25) and is not repeated here. For the problem given in Case 1, with $k_s = 64$ MN/m/m, and assuming $k_g = 13$ MN, the track modulus of the wooden tie track is calculated to be $k = 57.8$ MN/m/m, approximately an 8 percent change from the value (63 MN/m/m) obtained using the Winkler model. For the concrete tie track, with the same k_g value, the Winkler model solution is satisfactory.

In most situations, the second parameter is usually not large enough to introduce any appreciable deviations in the track modulus. However, this may not be the case for slab tracks or other types of track support where the shear interaction effect of the underlying track support medium is significant.

CONCLUSIONS

A new method has been presented to estimate the track modulus by using available elastic properties of the individual track support components. These include the stiffness of the rail pad, the compressive (or vertical) stiffness and bending rigidity of the tie as a beam on either a Winkler foundation or a two-parameter foundation, as well as the Young's moduli and Poisson's ratio values of the ballast/subgrade soil layers. Thus, the seasonal variations of these parameters can be taken into consideration when evaluating track performance.

Numerical examples have demonstrated that the modulus of a concrete tie track increases more rapidly with increasing ballast/subgrade stiffness than that of a wooden tie track. With the same ballast/subgrade stiffness, a concrete tie track can be twice as stiff as a wooden tie track. This results directly from the higher bending rigidity provided by the concrete tie, which is not explicitly considered in other methods. Soft rail pads are shown to provide an important means of increasing the compliance of concrete tie tracks.

For most applications, the Winkler theory is sufficient to model the tie as a beam on an elastic foundation. The method proposed in this paper can be used to establish the relationship between the track modulus and parameters of a two-parameter foundation model, and determine the threshold value of the second parameter above which the two-parameter model will yield a different estimate of the track modulus from the Winkler model.

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