Hydraulics of Safety End Sections for Highway Culverts

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On highway reconstruction projects the Kansas Department of Transportation now fits pipe culverts with end sections designed specifically for collision safety. These safety end sections are long and narrow with steel bars over the top openings. Scale models of 10 safety end sections were tested in a large water flume in the hydraulics laboratory at the University of Kansas. The objectives were to determine the relationship between headwater depth and discharge (the rating curve) under inlet control, the entrance-loss coefficient for full flow, and the susceptibility to blockage by debris for each design. The hydraulic characteristics of the safety end sections compare favorably with those of standard end sections. The inlet-control rating curves are described by a simple dimensionless relationship. Entrance-loss coefficients for full flow range from 0.65 to 0.85. At headwater depths greater than 1.4 times the pipe diameter, a concrete culvert with safety end sections will always flow full. At headwater depths greater than 2.0 times the pipe diameter, a corrugated steel culvert with safety end sections will always flow full. Most safety end sections are not particularly susceptible to blockage by floating debris. Debris seldom obstructs more than about one-fourth of the area of the top opening at high flows.

The design of end sections for highway culverts may be governed by collision-safety criteria instead of hydraulic or structural considerations. On highway reconstruction projects the Kansas Department of Transportation (KDOT) now fits pipe culverts with end sections designed specifically for collision safety. Developed by J & J Drainage Products Co. of Hutchinson, Kansas, these safety end sections are long and narrow with steel bars over the top openings. Field tests have shown that automobiles can traverse these end sections safely.

The hydraulic characteristics of these safety end sections were investigated recently in model studies initiated and funded by KDOT. These model studies were motivated by concerns that culverts with the safety end section might have smaller hydraulic capacities than identical culverts with standard end sections. The concerns were based on the more constrictive appearance of the safety end sections. The model studies were conducted in the hydraulics laboratory at the University of Kansas.

The hydraulic studies had four specific objectives. The first objective was to determine the conditions that govern whether a culvert with the safety end section operates under inlet control or outlet control. The second was to determine the relationship between headwater depth and discharge (the rating curve) for each safety end section under inlet control. The third was to determine the entrance-loss coefficient for full flow through each safety end section. The fourth was to determine how partial blockage by debris affects the hydraulic performance of these structures.

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The safety end sections are manufactured for pipes from 61 cm (24 in.) to 152 cm (60 in.) in diameter. Safety end sections of different sizes are not geometrically similar. At the end that connects to the pipe, the sides of the end section extend to a level 10 cm (4 in.) below the soffit of the pipe, regardless of the pipe diameter. On an end section for a 152-cm (60-in.) pipe, the top opening is only 76 cm (30 in.) wide where it connects to the pipe. On end sections for smaller pipes, the width of the top opening is a larger fraction of the pipe diameter. The longitudinal slope of the top opening on a safety end section can be either 6:1 or 4:1. The longitudinal slope of the top opening must match the slope of the embankment. Most safety end sections installed by KDOT have longitudinal slopes of 6:1.

The top openings of the safety end sections are spanned by steel bars 7.6 cm (3 in.) in diameter. The spacing of the safety bars depends on the size and orientation of the culvert and the slope of the embankment. The safety bars are spaced no more than 76 cm (30 in.) apart in the most likely direction of collision impact. On end sections to be installed parallel to the highway (i.e., on culverts under crossroads at intersections with the highway), the normal spacing between safety bars is 61 cm (24 in.). End sections for cross-drainage installations (perpendicular to the highway) have transverse safety bars at 122-cm (48-in.) spacings. The larger cross-drainage structures also have longitudinal safety bars. Figure 1 shows the designs for the safety end sections for 122-cm (48-in.) pipe culverts and an embankment slope of 6:1. One bar arrangement is for parallel installations; the other is for cross-drainage installations.

EXPERIMENTAL PROCEDURES

Experimental Setup

Scale models of the 10 safety end sections were tested in a glass-walled water flume in the hydraulics laboratory at the University of Kansas. This recirculating flume is 75 cm (2.5 ft) wide, 91 cm (3.0 ft) deep, and 18 m (60 ft) long. The flume is fed from an elevated constant-head tank and drains to a sump pit.

The 10 end sections tested were the parallel and cross-drainage versions of the 61-, 91-, 122-, and 152-cm (24-, 36-, 48-, and 60-in.) end sections with 6:1 slopes and the 152-cm (60-in.) end section with a 4:1 slope. The models of these end sections were all scaled to fit pipes with inside diameters of 15 cm (6 in.). The bodies of the model end sections were made from light-gauge sheet metal, and the safety bars were made from copper tubing. All model end sections were tested on a smooth pipe of transparent acrylic that was 15 cm (6 in.) in diameter and 91 cm (3 ft) long. Selected end sections were also tested on a corrugated-steel pipe (1.5-in. × 0.25-in. helical corrugations) of the same size. In these tests the model end section was attached to the model culvert on the upstream side of the head wall that forced all the flow in the flume

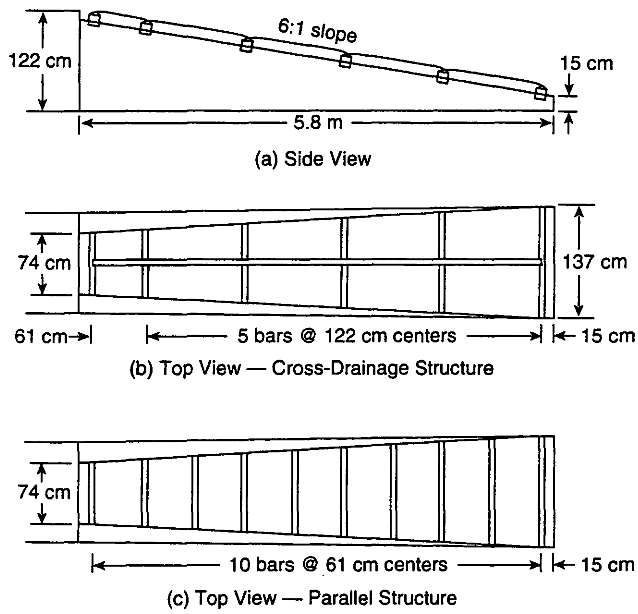

FIGURE 1 Safety end sections with 6:1 slopes for 122-cm (48-in.) pipe culverts.

through the pipe. The smooth acrylic pipe was installed on a slope of 0.02. The rough corrugated-steel pipe was installed on a slope of 0.12. These slopes were steep enough to ensure that, with models of conventional manufactured end sections attached, the pipes would not flow full.

The discharge was measured with a V-notch weir installed in the flume about 3 m (10 ft) upstream of the head wall. The bottom of the notch was 61 cm (2.0 ft) above the bottom of the flume. A honeycomb baffle between the weir and the test section distributed the flow fairly uniformly over the cross section of the flume. The tailwater level was controlled with a sluice gate at the downstream end of the flume. The head on the weir and the depths of flow upstream and downstream of the models were measured with point gauges.

Testing of Models

First, each end section was attached to the smooth acrylic pipe and tested under inlet control with no tailwater. The discharge in the flume was increased in steps. At each discharge the head on the weir and the headwater level at the culvert were measured and recorded, and the pattern of flow through the model was observed and noted. The headwater depth at the transition from inlet control to full flow was determined. Next, each model was tested under outlet control, with its outlet submerged by high tailwater. The headwater level, the tailwater level, and the head on the weir were measured at several different discharges. Entrance-loss coefficients were determined from these data. Selected models were also attached to the rough corrugated-steel pipe and tested with no tailwater. The objective of these tests was to determine the conditions at the transition from inlet control to full flow.

The hydraulic effects of debris were also investigated. In the initial test of each model with debris, loose straw was scattered on the bottom of the dry flume upstream of the end section. Then the valve on the inflow line was opened in small steps, so that the discharge in the flume increased gradually, with no tailwater downstream of the culvert. More straw was scattered on the water surface upstream of the end section as the discharge increased. The headwater was allowed to rise until the debris floating on the water surface pulled free of any debris retained on the end section. Then the valve on the inflow line was closed in small steps so that the discharge in the flume decreased gradually to zero. At various stages as the headwater rose and then fell the extent of any accumulation of debris on the end section was recorded.

Additional experiments were conducted to quantify the hydraulic effects of partial blockage by debris. The model of the parallel version of the 122-cm (48-in.) end section was selected for these experiments because it tended to trap more debris than the other models. The original experiments for inlet-control flow and for full flow were repeated for three different degrees of blockage. In each experiment the upper part of the top opening was obstructed by a hand-placed mat of loose straw. The opening was obstructed from the top down instead of from the bottom up for two reasons: top-down plugging is more likely than bottom-up plugging because most debris floats, and top-down plugging is the worst case hydraulically. The rating curve for inlet control, the upper limit on inlet control, and the entrance-loss coefficient for full flow were determined for each degree of blockage.

Dimensional Analysis and Scaling Laws for Inlet Control

Under inlet control the headwater depth, HW, is determined by the discharge, Q, the density of the water, ρ, the specific weight of the water, γ, and the geometry of the end section. (In this formulation, the headwater depth is measured from the invert of the pipe, and the velocity head of the approach flow is assumed to be negligible.) The geometry of the end section is characterized by the diameter of the conduit, D, and several other dimensions represented by the variables x_1 through x_n. In mathematical form, the relationship is

$$HW = f(Q, \rho, \gamma, D, x_1, x_2, \ldots, x_n) \tag{1}$$

where $f(\ )$ is a function that must be determined experimentally. The viscosity of the water is not included in Equation 1 because the flow of water through a culvert inlet is always turbulent and the Reynolds number is generally large enough that viscosity has no effect on the flow pattern. Even at the model scale the effects of viscosity and surface tension have been found to be negligible for inlets with sharp edges (*1–5*).

Dimensional analysis leads to the dimensionless relationship

$$\frac{HW}{D} = f\left(\frac{Q}{\sqrt{gD^5}}, X_1, X_2, \ldots, X_n\right) \tag{2}$$

where X_1 through X_n are dimensionless variables that describe the geometry of the end section. For end sections of a particular design, the functional relationship is

$$\frac{HW}{D} = f\left(\frac{Q}{\sqrt{gD^5}}\right) \tag{3}$$

The variable $Q/(gD^5)^{1/2}$ is a form of the Froude number.

Equation 3 can be used to relate the hydraulic performances of geometrically similar end sections of different sizes. Consider a model culvert of diameter D_m that carries a discharge Q_m at a headwater depth HW_m. The equivalent headwater depth for the full-sized culvert is obtained by simple geometric scaling. The formula is

$$HW_p = HW_m \frac{D_p}{D_m} \qquad (4)$$

where HW_p is the headwater depth for the prototype, and D_p is the diameter of the prototype. Equation 3 indicates that geometrically similar culverts with equal values of HW/D will also have equal values of $Q/(gD^5)^{1/2}$; therefore

$$\frac{HW_p}{HW_m} = \left(\frac{Q_p}{Q_m}\right)^{5/2} \qquad (5)$$

Equations 4 and 5 were used to convert measured headwater depths and discharges from the model scale to the prototype scale.

Determination of Entrance-Loss Coefficients for Full Flow

The entrance-loss coefficient for a culvert is the head loss through the inlet expressed as a fraction of the velocity head in the pipe. Its value can be determined indirectly from an energy balance across a culvert with both ends submerged. The difference between the total heads (Bernoulli sums) upstream and downstream of the culvert, ΔH, is the sum of the entrance loss, the friction loss through the pipe, and the exit loss. Stated mathematically,

$$\Delta H = \left(K_{en} + f\frac{L}{D} + K_{ex}\right)\frac{Q^2}{2gA^2} \qquad (6)$$

where

- K_{en} = entrance-loss coefficient,
- K_{ex} = exit-loss coefficient,
- f = Darcy-Weisbach friction factor,
- L = length of the pipe, and
- A = cross-sectional area of the pipe.

Equation 6 can be solved for K_{en} as follows:

$$K_{en} = \frac{\Delta H}{\left(\frac{Q^2}{2gA^2}\right)} - f\frac{L}{D} - K_{ex} \qquad (7)$$

The entrance-loss coefficients for the models were determined from tests in which the outlet of the model culvert was submerged. In these tests the velocity heads in the flume upstream and downstream of the culvert were negligible; therefore, the change in the Bernoulli sum through the culvert was simply the difference between the headwater and tailwater elevations, and the entire velocity head in the culvert was dissipated just downstream of the outlet (K_{ex} = 1).

HYDRAULIC PERFORMANCE OF END SECTIONS UNOBSTRUCTED BY DEBRIS

The inlet-control rating curve for each cross-drainage end section was compared with the rating curve for the parallel end section of the same size. Likewise, the rating curves for the 152-cm (60-in.) end sections with 4:1 slopes were compared with those for the 152-cm (60-in.) end sections with 6:1 slopes. The measured inlet-control rating curves for end sections of the same size were virtually identical, regardless of the slope of the end section and the arrangement of the safety bars. These differences in the designs of the safety end sections did not affect their performances under inlet control.

To determine the hydraulic significance of the size-related differences in the designs of the end sections, the inlet-control rating curves for all 10 end sections were expressed in terms of the dimensionless variables

$$Q^* = \frac{Q}{\sqrt{gD^5}} \qquad (8)$$

and

$$HW^* = \frac{HW}{D} \qquad (9)$$

and were plotted on a single graph. Figure 2 shows the dimensionless results for the five models with safety bars for parallel alignments. (The results for the other five models with safety bars for cross-drainage alignments are omitted for clarity.) The dimensionless data for all models form a single curve over the region of inlet control. This indicates that the minor differences in the geometries of these end sections do not affect their performances under inlet control. The curve defined by the data shows an abrupt change in curvature near HW^* equal to 1, the level at which the inlet starts to become submerged. The smooth pipe began to flow full at values of HW^* of between 1.2 and 1.4 and always flowed full at HW^* values of >1.4. The rough corrugated-steel pipe began to flow full at values of HW^* of between 1.8 and 2.0 and always flowed full at HW^* values of >2.0.

The two-part formula

$$HW^* = \begin{cases} 1.69(Q^*)^{0.6} & Q^* \leq 0.42 \\ 1.11 - 1.93Q^* + 4(Q^*)^2 & 0.42 < Q^* \leq 0.77 \end{cases} \qquad (10)$$

fits the experimental data in Figure 2 over the entire range of inlet-control conditions. Over the range $0 \leq Q^* \leq 0.42$, which corresponds to $0 \leq HW^* \leq 1.00$, the inlet is not submerged. At a Q^* value of >0.42, which corresponds to an HW^* value of >1.00, the inlet is submerged. The curve defined by Equation 10 is continuous and smooth at the break point (Q^* = 0.42).

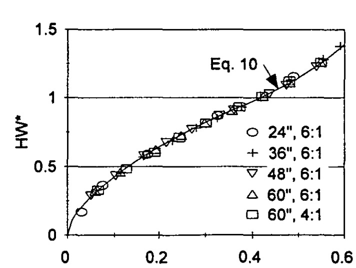

FIGURE 2 Dimensionless rating curve for inlet control with no blockage.

The experimental results indicate that a smooth (e.g., concrete) culvert with safety end sections will always flow full whenever the headwater depth exceeds 140 percent of the pipe diameter and that a rough (e.g., corrugated-steel) culvert with safety end sections will always flow full whenever the headwater depth exceeds 200 percent of the pipe diameter. A short smooth culvert with conventional manufactured end sections on a steep slope will not flow full unless the outlet is submerged.

Certain other types of inlets will cause a short smooth pipe on a steep slope to flow full whenever the inlet is submerged sufficiently (*6*). The hood inlet is the best-known example. This property of the hood inlet was first reported by Karr and Clayton (*7*) at Oregon State University. Blaisdell (*8*) of the Agricultural Research Service showed that a smooth culvert with a hood inlet flows full with no tailwater on slopes as steep as 20 percent provided that the inlet is slightly submerged. Blaisdell's original studies were conducted at the St. Anthony Falls Hydraulics Laboratory on small-scale models. His results were confirmed in studies of prototype pipe culverts up to 1829 mm (72 in.) in diameter (*1,2,4,5*). Rice (*5*) showed that corrugated-steel pipes with hood inlets behave the same as smooth pipes with hood inlets except that full flow starts at somewhat higher headwater depths. He found that smooth pipes with hood inlets start to flow full at an HW^* value of ≈ 1.2, whereas corrugated-steel pipes with hood inlets start to flow full at an HW^* value of ≈ 1.6.

The ability of the safety end section to force a culvert to flow full under adverse conditions is a consequence of its geometry. Water enters the end section over the entire length of its long, narrow top opening. At the pipe inlet (where the end section connects to the pipe) most of the flow is directed along the axis of the pipe. The downward momentum of the water that enters near the pipe inlet is insufficient to cause the flow to separate from the top of the pipe. With a standard end section, the flow at the pipe inlet has considerable momentum normal to the axis of the pipe, which causes the flow to separate from the top and sides of the pipe.

The hydraulic behavior of each end section under a falling headwater was also examined. The discharge in the flume was reduced gradually, starting from a level at which the culvert flowed full with no tailwater. In each case the culvert continued to flow full until the inlet was no longer submerged. A transition from full flow back to inlet control with the inlet submerged was never observed.

Figure 3 compares the hydraulic performances of a corrugated-steel pipe with KDOT safety end sections and a corrugated-steel pipe mitered to the slope of the embankment (*9*). In the unsubmerged condition ($Q^* < 0.48$), the safety end section is only slightly more efficient than the mitered pipe. In the submerged condition ($Q^* > 0.48$) the pipe with the safety inlet is considerably more efficient than the mitered pipe.

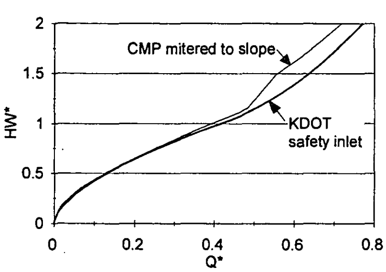

FIGURE 3 Comparison of inlet-control rating curves for KDOT safety inlet and corrugated-steel pipe mitered to slope.

The entrance-loss coefficients for the safety end sections range from 0.65 to 0.85. The larger end sections have larger entrance-loss coefficients than the smaller end sections because their top openings are narrower relative to their diameters. The arrangement of the safety bars has relatively little effect on the entrance-loss coefficient for full flow. On an end section of this type the safety bars do not cause much head loss because the area of the top opening is much larger than the cross-sectional area of the pipe, so the velocities around the bars are low.

EFFECTS OF DEBRIS

Most safety end sections are not particularly susceptible to blockage by floating debris. Any debris that accumulates on the crossbar at the water level tends to float free when the headwater rises. Once submerged, the top openings of most safety end sections are usually clear of debris. The exceptions are the end sections with crossbars near the pipe inlet. These end sections tend to trap debris between the top of the pipe inlet and the nearest crossbar. Once established, the blockage tends to grow in the upstream direction. In the worst case of unforced blockage observed in the laboratory, debris obstructed the upper one-third of the top opening of the submerged end section. This degree of blockage occurred after the culvert was subjected to a series of simulated floods, each carrying considerable debris, with no removal of accumulated debris between floods. In the worst case for a single simulated flood with no blockage initially, debris obstructed about one-quarter of the top opening of the submerged end section. When the headwater falls additional floating debris settles over the end section. After a flood the top opening of a safety end section may be covered entirely by debris. However, this debris might not have obstructed the end section during the flood.

Partial blockage by debris can delay or prevent the onset of full flow. The smooth pipe with the parallel version of the 122-cm (48-in.) end section began to flow full at HW^* equal to 1.30 with no blockage, at HW^* equal to 1.64 with the upper 18 percent of the top opening blocked, and at HW^* equal to 3.00 with 37 percent blockage. With 69 percent blockage the smooth pipe did not flow full at HW^* equal to 4, the maximum submergence in the flume used in the present study. These experimental results for the smooth pipe are fitted by the formula

$$Z = 1.30 + 16.5\,B^{2.3} \qquad (11)$$

where Z is the upper limit on HW^* for inlet control, and B is the fraction of the area of the top opening blocked by debris.

Figure 4 shows measured and fitted rating curves in dimensionless form for the parallel version of the 122-cm (48-in.) end section with three degrees of forced blockage. In the experiments the actual degree of blockage actually decreased slightly as the discharge increased because of compression of the straw cover by the flow. This is why the measured headwater levels tend to be smaller than the headwater levels on the fitted curves at the higher discharges. The fitted curves are of the form

$$HW^* = \begin{cases} 1.69(Q^*)^{0.6} & Q^* \le Q^*_1 \\ a + bQ^* + 4(Q^*)^2 & Q^*_1 < Q^* \le Q^*_2 \end{cases} \qquad (12)$$

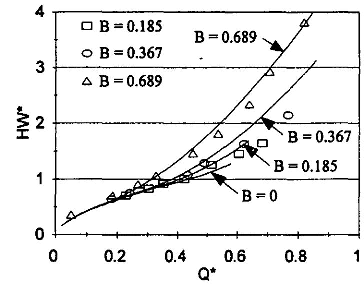

FIGURE 4 Dimensionless rating curves for inlet control with blockage.

Q_1^* is the dimensionless discharge at which the blockage begins to affect the flow, and Q_2^* is the dimensionless discharge at the upper limit on inlet control. The relationship between Q_1^* and B is approximated by the formula

$$Q_1^* = 0.42 - 0.39\,B \qquad (13)$$

The rating curve for any degree of blockage is approximated by Equation 12, with Q_1^* determined from Equation 13 and the constants a and b in Equation 12 chosen so that the fitted curve is continuous and smooth at Q^* equal to Q_1^*. Table 1 shows the values of the constants in Equation 12 for several degrees of blockage. The values of Q_2^* in Table 1 are for a smooth pipe. The values of Q_2^* for a rough pipe would be somewhat larger. Based on observations in the laboratory, one would not expect the degree of blockage in the field to exceed about 25 percent. Therefore, the use of a 25 percent degree of blockage as a basis for design should be sufficiently conservative.

The degree of blockage also affects the entrance-loss coefficient for full flow. The experimentally determined entrance-loss coefficients for the parallel version of the 48-in. end section are 0.78 for no blockage, 1.26 for 18 percent blockage, 1.64 for 37 percent blockage, and 2.49 for 93 percent blockage. These data are fitted satisfactorily by the equation

$$\frac{K_{en,b}}{K_{en,u}} = 1 + 2.4B^{0.8} \qquad (14)$$

where $K_{en,b}$ is the entrance-loss coefficient for a partially blocked end section, and $K_{en,u}$ is the entrance-loss coefficient for the same end section with no blockage.

TABLE 1 Values of Constants in Equation 12 for Partially Blocked End Section

B	Q_1*	Q_2*	a	b
0	0.42	0.57	1.11	-1.93
0.185	0.35	0.63	0.84	-1.24
0.250	0.32	0.69	0.76	-0.99
0.367	0.28	0.83	0.62	-0.52
0.500	0.23	1.02	0.48	+0.04
0.689	0.15	1.30	0.31	+0.95

Although Equations 11 through 14 were fitted to experimental results for one particular end section with a specific type of blockage, they provide a rough estimate of the hydraulic characteristics of any safety end section that is partially blocked.

CONCLUSIONS

The hydraulic characteristics of safety end sections compare favorably with those of standard end sections. Under inlet control a culvert with safety end sections performs about the same as a concrete pipe with a head wall inlet and slightly better than a mitered corrugated-steel pipe. The inlet-control rating curves for safety end sections are described by a simple dimensionless relationship (Figure 2; Equation 10). This relationship applies to all safety end sections, regardless of size, longitudinal slope (6:1 or 4:1), or safety bar arrangement (cross-drainage or parallel). A concrete pipe culvert with unobstructed safety end sections can operate under inlet control only at headwater depths of less than about 1.4 times the pipe diameter. At greater headwater depths the culvert will always flow full, even if the pipe is short and steep and the tailwater is low. A corrugated-steel pipe culvert with unobstructed safety end sections will always flow full at headwater depths greater than 2.0 times the pipe diameter. In contrast a short culvert with standard manufactured end sections on a steep slope normally will not flow full unless its outlet is submerged. For these reasons safety end sections may be more efficient than standard manufactured end sections for a short steep culvert with low tailwater. On the other hand, safety end sections may be slightly less efficient than standard manufactured end sections for a culvert with a submerged outlet, which would flow full in any case. The entrance-loss coefficients for safety end sections, although not excessive, are somewhat higher than those for other manufactured end sections. The entrance-loss coefficients for safety end sections range from about 0.65 for a 61-cm (24-in.) end section to about 0.85 for a 152-cm (60-in.) end section. For comparison, K_{en} is ≈ 0.2 for a concrete head wall inlet with rounded edges, and K_{en} is ≈ 0.5 for the common type of manufactured steel end section (*9*). Floating debris may partially obstruct the top opening of a safety end section. However, at high flows debris seldom blocks more than about one-quarter of the area of the top opening. This degree of blockage reduces the hydraulic capacity of the culvert only slightly.

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