# Effect of Varying Foundation Stiffness on Seismically Induced Loads in Bridge Bents: A Sensitivity Study

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Research was undertaken to assist the Tennessee Department of Transportation (TnDOT) in its analysis of bridges subjected to seismic loading. Specifically, consideration was given to the modeling of the soil-pile interface where friction piles are used in loessial soil. TnDOT uses the SEISAB bridge analysis program, developed by Imbsen and Associates, for the seismic analysis of its bridges. This program was used for all analysis of the research. Current TnDOT modeling practice is to consider that the bridge piers are fixed at the top of the pile cap rather than to assign values of stiffness to springs used to model the resistance of the soil to foundation movement. Elastic spring coefficients, developed using methods presented previously, and the traditional beam-on-elastic-foundation theory are used in the modeling of the bridges. The primary focus of this study is the sensitivity of the calculated axial loads and moments to variations in these spring coefficients. The results from this study underscore the need for more experimental data that could lead to more realistic and reliable values for spring stiffness.

In the development of any structural model, particular attention should always be given to the selection of the model's boundary conditions. In a dynamic analysis the selection of proper boundary conditions becomes increasingly more important because member forces can change by several orders of magnitude, depending on the characteristics of the model. Traditionally, engineers have had little information about techniques available for modeling a pilesupported foundation. Because the testing of a full-scale pile or pile group is both difficult and expensive, there have been limited data to supplement design assumptions, and most structural models are based largely on theoretical information. However, in recent years the destructive effects of several earthquakes on both building and highway structures have increased awareness and provided motivation to achieve a better understanding of the behavior of pilesupported foundations. As a result, several dynamic lateral testing programs on single piles and pile groups have been conducted in many different types of soils (1-5). However, essentially no information is available on the dynamic response of piles located in the soil type known as loess, the soil that exists in the western portion of Tennessee. Because West Tennessee is in Seismic Zone 3, this response is of considerable interest.

The Tennessee Department of Transportation (TnDOT) uses the bridge analysis program SEISAB (6), developed by Imbsen and Associates, for seismic analysis of bridges. Because of the lack of

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information on the properties of loess needed to quantify the resistance provided by the soil to the pile caps, the piers are modeled as fixed at the tops of the pile caps. Recognizing that this condition is not precisely representative of actual conditions, TnDOT is sponsoring research to determine more realistic values of stiffness to model the soil-pier interface. As a part of that research, an analytical study was done to evaluate the sensitivity of pier column moments and axial loads to the values assigned to the stiffnesses of springs used to model the soil-pier interface.

The purpose of the sensitivity study reported herein was to determine the effect of large variations in interface stiffness on the axial loads and moments in the pier columns of bridges subjected to the seismic loading used in the design of bridges in West Tennessee. Although some discussion of typical methods used to obtain stiffness coefficients is included, the emphasis is not the accurate determination of stiffness but rather the effect that variations in assumed stiffness have on the results. To accomplish the stated purpose, two actual West Tennessee bridges were modeled and analyzed using SEISAB, and the effects of varying stiffness coefficients at the soil-pier interface were studied.

# BRIDGE STRUCTURES TO BE MODELED

A bridge in Madison County, Tennessee, consisting of two continuous spans (Figure 1) and a bridge in Haywood County, Tennessee, consisting of three continuous spans (Figure 2) were selected for this study because of their geographical location in an area with loess and because the structures represent the typical types of bridges used in West Tennessee by TnDOT.

The Madison County bridge consists of a two-span continuous structure with six prestressed concrete girders supporting two lanes of traffic; the girders are spaced 2.51 m (8 ft 3 in.) apart. The bridge is constructed with a 68-degree angle of skew (measured from the direction of traffic). For this analysis a 90-degree skew was considered as shown in Figure 1; however, further investigations into the effect of skew on the dynamic response were conducted by Cook (7) with results similar to those reported here. The foundation of the bridge is composed of two monolithic reinforced concrete abutments on either end of the road deck and a central bent pier with three columns resting on embedded footings. Each abutment is supported by a single row of 13 piles along the endwall and a single pile under each wingwall, as indicated in Figure 3. With respect to Figure 1, the piles at Abutments 1 and 2 are embedded to depths of 14 and 17 m (45 and 55 ft), respectively. The central bent pier foun-

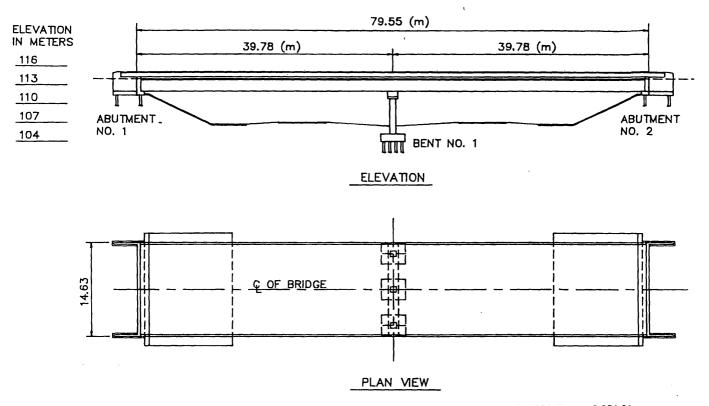


FIGURE 1 Plan and elevation of Madison County bridge (SR-233 over I-40) constructed by TnDOT in 1992 (1 m = 3.281 ft).

dation consists of a pile cap under each column supported by 12 piles embedded to a depth of 7.6 m (25 ft). Piles under both the abutments and central bent pier are precast concrete piles  $356\times356$  mm (14  $\times$  14 in.). All piles used for foundation support of the Madison County bridge are floating or friction piles and are considered to be long and flexible.

The Haywood County bridge structure consists of three continuous spans with five prestressed concrete box beams supporting two lanes of traffic; girders are spaced 2.29 m (7 ft 5 in.) apart. The abutments and bent piers were constructed with a 90-degree skew and were analyzed as such. The abutments at either end of the bridge serve primarily to provide a bearing point for the prestressed box

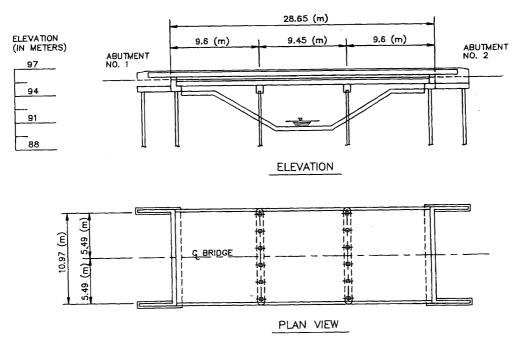


FIGURE 2 Plan and elevation of Haywood County bridge (SR-54 over Nixion Creek) constructed by TnDOT in 1992 (1 m = 3.281 ft).

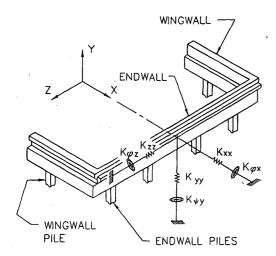


FIGURE 3 Madison County bridge abutment modeled with equivalent rotational and translational springs.

beams and retain little backfill. Each abutment is supported by six embedded piles—four along the endwall and one under each wingwall. The two bent piers are constructed with six driven piles with a portion of the pile free-standing with a cap beam at the top to allow for a bearing surface for the prestressed box beams. The piles at the abutments and bent piers are precast concrete piles  $356 \times 356$  mm  $(14 \times 14 \text{ in.})$  embedded to a depth of 16.8 m (55 ft) at the abutments and approximately 12.2 m (40 ft) at each bent pier. The piles supporting the Haywood County bridge are also friction piles, and for this analysis the piles are considered long and flexible.

## SOIL PROPERTIES

No dynamic soil properties were measured at the sites of either bridge; however, geotechnical subsurface investigations were performed at both bridge locations. For the Madison County bridge site, three boreholes were drilled 15.4 m (50.5 ft) deep near the base of the pier and at each of the abutment locations. The log of the test borings indicates that the soil below the bridge is composed mostly of silt or clay terminating in dense sand or stiff clay.

An existing steel and timber bridge structure was removed completely from the site of the Haywood County bridge and replaced with the current concrete structure. The subsurface investigation consisted of two borings that were drilled 50 ft deep at each abutment location. Generally, the borings encountered interbedded layers of silts, sands, and clays with various combinations of the three soil types.

Because no dynamic soil data are available for the sites of the bridges to be modeled, data from research by Chang et al. (8) conducted at Memphis State University (now the University of Memphis) on dynamic soil properties for loess deposits are used in this analysis. For purposes of this study, the soil is assumed to be homogeneous along the full length of the pile.

# PROPOSED MODEL FOR BRIDGE AND FOUNDATIONS

The bridge analysis program SEISAB was used to conduct response spectrum analysis on both of the aforementioned bridges. The input seismic loading in SEISAB consisted of ATC 3-06 (9) response spectra scaled to the maximum ground acceleration at the particular bridge location, adjusted for the prevalent soil conditions. The acceleration scaling factors used were 0.12 and 0.18 for the Madison County and Haywood County bridges, respectively. A damping value of 5 percent of critical damping was assumed for both bridges. ATC 3-06 Soil Type II was used for both bridges. Four seismic loading cases were considered for each bridge. Two cases consisted of ground motion in the longitudinal and transverse directions with respect to the bridge centerline. The other two loading cases consisted of combining 100 percent of the ground motion from one of the first two cases with 30 percent of the ground motion from the other case.

The SEISAB program allows the input of six stiffness values for modeling of the foundation elements: three for translation and three for rotation for both the bridge pier and abutments. The individual spring coefficients are defined in Figure 4 for the pile cap and pile group for the foundation elements of the Madison County bridge model. The elements of the bent pier and abutment foundations to be replaced by spring coefficients for modeling of the Madison County bridge are indicated in Figures 3 and 5 for the abutments and bent piers, respectively. The modeling of the Haywood County bridge foundation elements was similar to that of the Madison County bridge with the exception of the modeling of the bents, which are composed of free-standing piles. Figures 6 and 7 indicate how the foundations for the Haywood County bridge are modeled in this analysis.

In modeling the bridge superstructure, SEISAB "lumps" the combined cross-sectional properties of the girders and deck at the bridge centerline. The spans for both bridges were modeled as being continuous over all interior supports, with pin connections at each bent and abutment (no relative movement between superstructure and supports). Each span of the superstructure was broken into four elements by three nodes along its length, and each column was modeled with two nodes (i.e., three elements) along its height. Figure 8 shows how the model for the Madison County bridge was

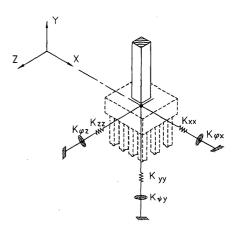


FIGURE 4 Pile group and pile cap equivalent spring model  $K_{xx}$  = translational spring along X-axis,  $K_{\varphi x}$  = rotational spring about X-axis,  $K_{zz}$  = translational spring along Z-axis,  $K_{\varphi z}$  = rotational spring about Z-axis,  $K_{yy}$  = vertical spring along Y-axis, and  $K_{\psi y}$  = torsional spring about Y-axis.

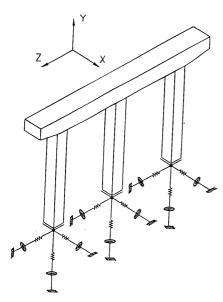


FIGURE 5 Madison County bridge central bent with pile cap and pile group modeled with rotational and translational springs.

interpreted by SEISAB. The model for the Haywood County bridge was similar to that shown in Figure 8.

# DEVELOPMENT OF FOUNDATION SPRING COEFFICIENTS

A detailed account of the application of various methods to determine foundation spring coefficients is presented by Cook (7). A condensed description of the methodology is presented here with

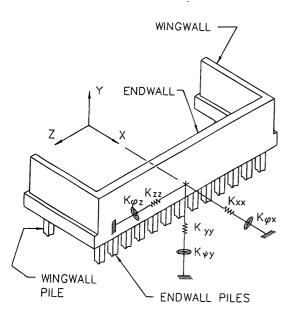


FIGURE 6 Haywood County bridge abutment modeled with equivalent rotational and translational springs.

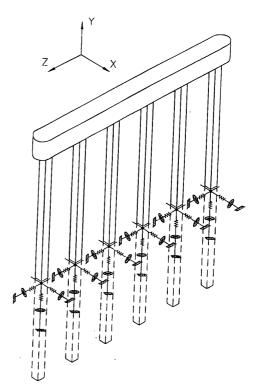


FIGURE 7 Haywood County bridge central bent modeled with rotational and translational springs.

the note that this paper is not concerned with the precise determination of stiffness but with the effect of variations in stiffness.

To estimate values for the foundation spring coefficients, two methods were investigated in considerable detail. A method developed by Novak (10) and revised by Novak and El Sharnouby (11), based on the theoretical behavior of an embedded pile in an elastic medium, was used to model the stiffness of single piles. The individual spring stiffnesses were combined by methods developed by Poulos (12,13) to account for pile group interaction. In addition to the modeling techniques of Novak, a second method was also studied. An approach using beam-on-elastic-foundation analysis was utilized in which the pile is considered to act as a beam on an elastic half-space. The equations used in development of the elastic spring coefficients were derived from methods suggested by Scott (14). The individual spring stiffnesses were again combined by methods suggested by Poulos to account for pile group interaction.

The development of elastic spring coefficients to account for the stiffnesses of the abutments was derived from techniques suggested by Wilson (15). Wilson's model accounts for the stiffness supplied by both the abutment walls and embedded piles incorporated into elastic spring coefficients. To reduce the amount of data to be generated in this analysis, only the methods suggested by Novak and Wilson were used to model the pile foundations and abutments of the Madison County bridge. The stiffness of the pile cap, modeled as a foundation on an elastic half-space, was added to the stiffness of the pile. The beam-on-elastic-foundation theory, in addition to Wilson's techniques, was used to model the free-standing pile foundations and abutments of the Haywood County bridge. From previous modeling by Cook (7), the individual spring coefficients from

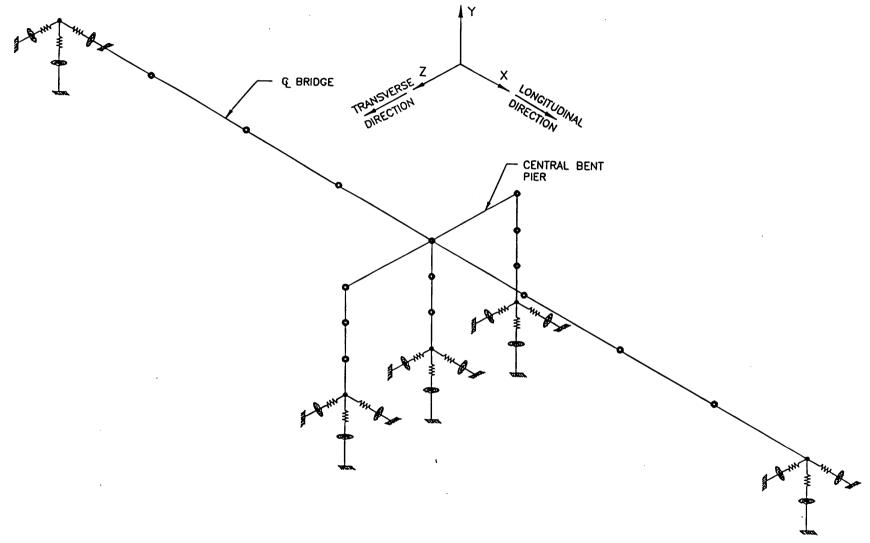


FIGURE 8 Equivalent SEISAB model for Madison County bridge structure.

the beam-on-elastic-foundation theory and Novak's method did not correlate very well, although the response of the model was not significantly affected by the use of one method versus the other.

Because the Madison County bridge is a symmetrical structure (i.e., two equal spans), the model was reanalyzed with one of the span lengths increased by 20 percent. With this model any additional effects on member forces in the substructure caused by unequal span lengths could be investigated.

The spring coefficients used for the Madison County bridge pier are shown in Table 1. These are examples of the coefficients used in the sensitivity analysis that follows.

#### SENSITIVITY ANALYSIS

In modeling the Madison County bridge, initially all the supports were considered "fixed" (i.e., no translation or rotation of the foundation elements). Spring coefficients were then applied at both the abutments and bent columns. From Figure 9 it is evident that modeling of the foundations with the Novak spring coefficients caused

the forces in the bent columns (with the exception of the longitudinal moment) to decrease somewhat as stiffness was taken out of the foundation system. To account for the possibility of variations in the spring stiffnesses, all the spring coefficients were reduced by a factor of 10. Dividing the spring coefficients by a factor of 10 reflects a lack of confidence in the calculated values caused by a limited amount of knowledge concerning the dynamic characteristics of loess. This lack of confidence is somewhat less acute for the abutment foundation springs because the makeup and placement of backfill surrounding the abutments is controlled. For this reason the spring coefficients at the abutments were held constant as the bent column spring coefficients were reduced by a factor of 10. Finally, all spring coefficients at both the bents and abutments were reduced by 10.

The SEISAB program performs a dynamic analysis for loading in both the transverse and longitudinal directions, as well as a combination of loading in both directions simultaneously. In Figures 9 through 12, only absolute maximum moments are plotted without regard to the direction of loading, because these maximum moments are the ones used for design purposes. Figure 13 indicates

TABLE 1 Spring Coefficients for Modeling Central Bent Foundation for Madison County Bridge

| DIR. OF<br>MOTION                                     | SINGLE PILE             | PILE GROUP              | PILE CAP                | TOTAL                   |
|---|-------------------------|-------------------------|-------------------------|-------------------------|
| (K <sub>xx</sub> )<br>X-DIR.<br>HORIZ.<br>(kN/m)      | 2.703 x 10 <sup>5</sup> | 1.229 x 10 <sup>6</sup> | 4.024 x 10 <sup>5</sup> | 1.631 x 10 <sup>6</sup> |
| (K <sub>zz</sub> )<br>Z-DIR.<br>HORIZ.<br>(KN/m )     | 2.703 x 10 <sup>5</sup> | 1.127 x 10 <sup>6</sup> | 4.024 x 10 <sup>6</sup> | 1.529 x 10 <sup>6</sup> |
| (K <sub>yy</sub> )<br>Y-DIR.<br>VERTICAL<br>(kN/m)    | 7.831 x 10 <sup>5</sup> | 1.973 x 10 <sup>7</sup> | 3.539 x 10 <sup>5</sup> | 2.327 x 10 <sup>6</sup> |
| $(K_{\phi x})$ ROT. ABT. X-AXIS $(kN-m/rad)$          | 9.190 x 10 <sup>5</sup> | 1.402 x 10 <sup>7</sup> | 1.014 × 10 <sup>6</sup> | 1.504 x 10 <sup>7</sup> |
| $(K_{\phi z})$ ROT. ABT. Z-AXIS $(kN-m/rad)$          | 9.190 x 10⁴             | 1.818 x 10 <sup>7</sup> | 1.014 x 10 <sup>6</sup> | 1.919 x 10 <sup>7</sup> |
| (K <sub>oy</sub> )<br>TORSION<br>Y-AXIS<br>(kN-m/rad) | 3.332 x 10 <sup>4</sup> | 6.120 x 10 <sup>6</sup> | 5.225 x 10 <sup>6</sup> | 1.135 x 10 <sup>7</sup> |

NOTE:

To convert from kN/m to k/ft., multiply by 0.0685. To convert from kN-m/rad to k-ft./rad, multiply by 0.735.

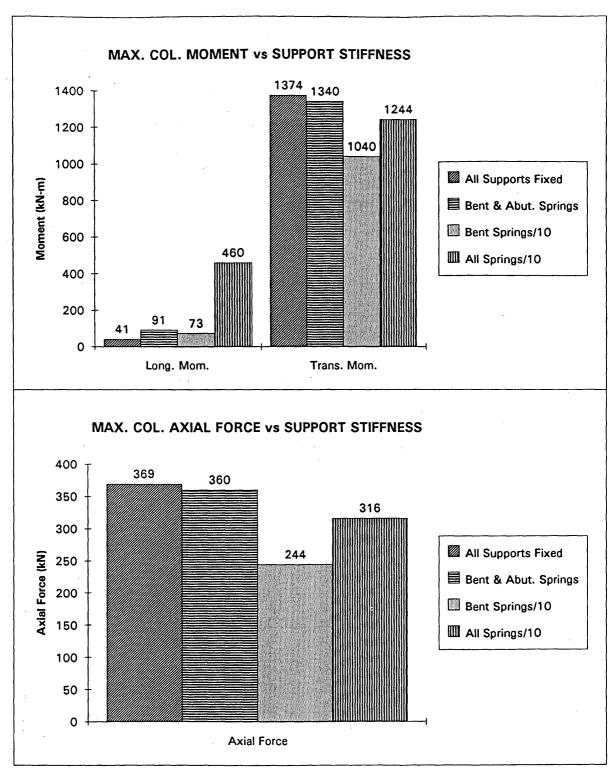


FIGURE 9 Effect of support stiffness on column moments and forces for Madison County bridge with unequal spans (1 kN = 0.225 kip; 1 kN-m = 0.735 kip-ft).

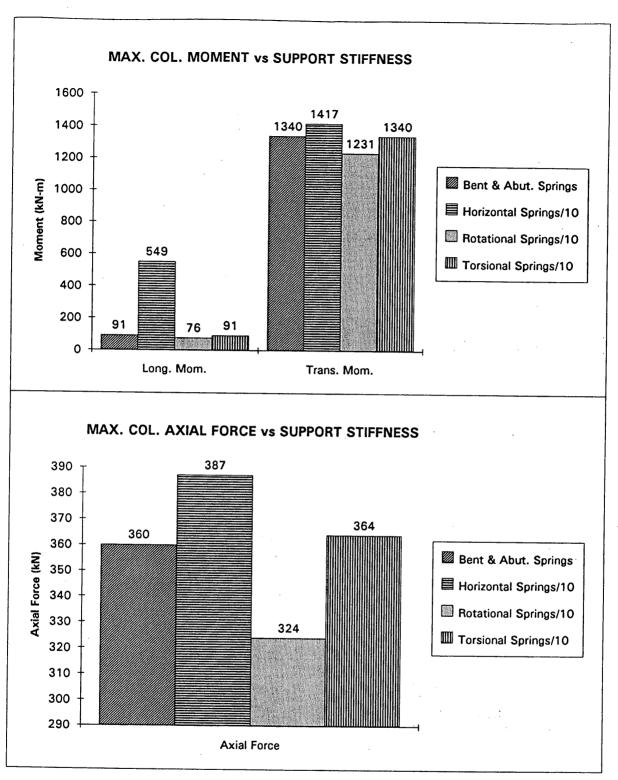


FIGURE 10 Relative effects of variations in horizontal versus rotational and torsional springs for Madison County bridge with unequal spans (1 kN = 0.225 kip; 1 kN-m = 0.735 kip-ft).

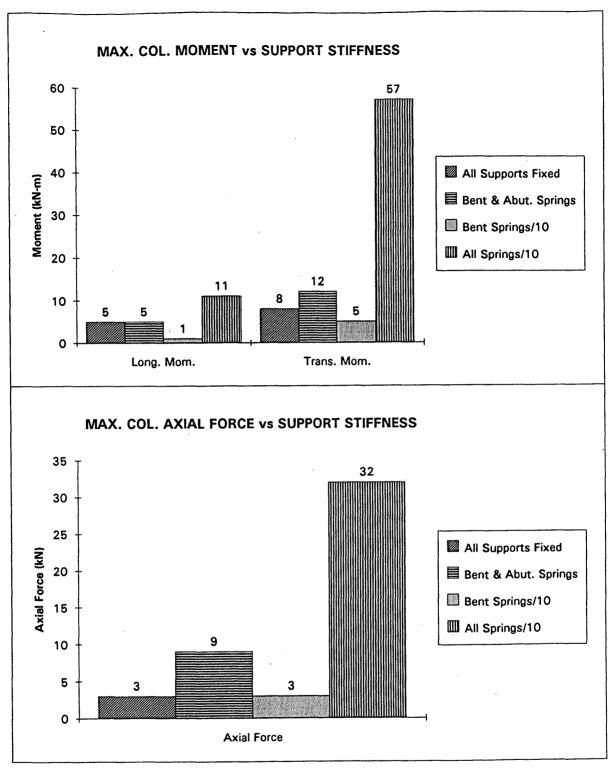


FIGURE 11 Effect of support stiffness on column moments and forces for Haywood County bridge (1 kN = 0.225 kip; 1 kN-m = 0.735 kip-ft).

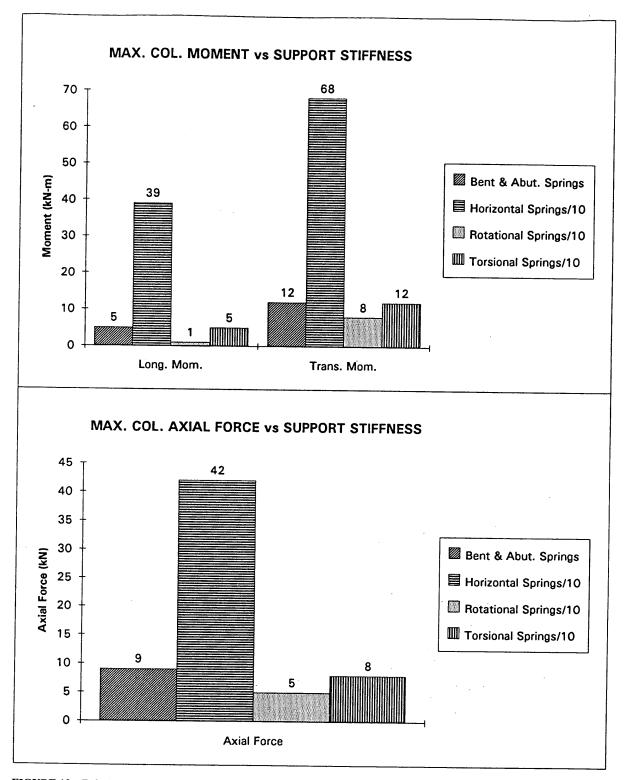


FIGURE 12 Relative effects of variations in horizontal versus rotational and torsional springs for Haywood County bridge (1 kN = 0.225 kip; 1 kN-m = 0.735 kip-ft).

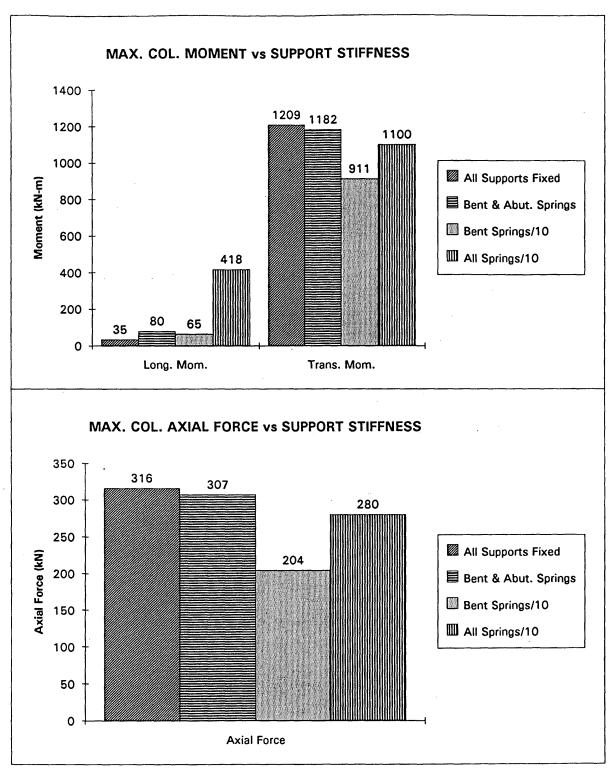


FIGURE 13 Effect of support stiffness on column moments and forces for Madison County bridge with equal spans (1 kN = 0.225 kip; 1 kN-m = 0.735 kip-ft).

that the reduction in all spring coefficients by a factor of 10 caused the transverse and axial forces to decrease somewhat; however, the longitudinal moment increased by a factor of approximately 12 from the case of a "fixed" support condition. This large increase occurred only when the structure was analyzed with loading in the longitudinal direction. As the foundation springs were reduced, the transverse moments in the bent columns were not significantly increased since the short span structure essentially acted as a deep beam laterally and any increase in dynamic effects were probably redistributed to the superstructure. When the structure was analyzed for loading in the longitudinal direction with a reduction in all spring stiffnesses, the central bent pier attracted more moment as stiffness was taken out of the foundations. This increase in column moment was not observed for the case when only the bent springs were reduced and abutment springs were held constant, indicating that the abutment stiffnesses are critical in determining bent column forces for the longitudinal direction.

To investigate the contributions of the lateral stiffness coefficients, the lateral springs were reduced by a factor of 10. From Figure 14, it appears that the most significant terms affecting an overall increase in member forces come from the contributions of the lateral (horizontal) spring coefficients. Thus, it appears that a reduction in the lateral stiffness coefficients can cause significant increases in column forces.

To confirm that the contributions of the rotational and torsional spring coefficients are minimal for both abutments and bent piers, these coefficients were investigated separately. The rotational and torsional spring coefficients were divided by a factor of 10. The torsional spring coefficient is probably the least understood and least investigated component of a pile foundation system. Although an attempt was made in the analysis described earlier to determine a value for the torsional spring coefficient, the results indicated in Figure 10 show that varying magnitudes of the torsional spring coefficient has an insignificant effect on the overall response of the foundation system. The reduction in the rotational spring coefficient from a fixed condition did not produce any increase in member forces. In all cases analyzed the member forces tended to decrease by small amounts as rotational stiffness was reduced in the foundation system.

Figures 9 and 10 reflect the results of the analysis of the unequal span modeling of the Madison County bridge. The same general trends observed in the modeling of the two-span symmetrical structure appear to be evident again. The results of the analysis with reduced lateral, rotational, and torsional stiffness coefficients are shown in Figure 10. These results again confirm that, as the lateral stiffness is reduced, member forces in the bridge substructure are increased. From the plot of the magnitude of the member forces from the reduced rotational and torsional spring coefficients in Figure 10, it is apparent that the changes in column moments and axial forces are minimal. Therefore, for the unequal-span model of the Madison County bridge, the response patterns generally follow those of the two-equal-span model with no significant changes because of geometry.

The Haywood County bridge structure was modeled using the same procedures as those used for the Madison County bridge structure. The bents and abutments were initially held fixed; then spring coefficients were applied at the abutments and bent columns. The bent spring coefficients were reduced by a factor of 10, whereas the abutment spring coefficients were held constant. Finally, all the spring coefficients were reduced by a factor of 10. As was the case

for the two-span Madison County bridge structure, the longitudinal moment increased when all spring coefficients were reduced, but only by a factor of approximately 2 from the case of fixed supports, as indicated by Figure 11. The magnitude of increase in column moments for the longitudinal direction is somewhat less for the case of the three-span structure than for the case of the two-span bridge because an additional bent column is available to absorb moment.

The lateral spring coefficients were again reduced by a factor of 10 to study their effect on substructure forces. The results shown on the graphs in Figure 12 indicate that the column forces for the threespan structure were influenced by the variation of spring coefficients in a way similar to that for the two-span Madison County bridge. As the lateral spring coefficients were reduced, a significant increase in the magnitude of both column moments and axial forces was observed.

Rotational and torsional spring coefficients were also investigated for the Haywood County bridge to evaluate their significance to the overall structural response. The results shown in Figure 12 indicate that the magnitudes of the rotational and torsional spring coefficients do not significantly affect the structural response of the model.

#### SUMMARY AND CONCLUSIONS

No attempt was made in the study reported here to evaluate current methods of calculating foundation stiffness. Instead, the study evaluated the sensitivity of the moments and axial loads in bent columns to variations in foundation stiffness.

From the results of the sensitivity analysis, it appears that the moments and forces in the bent columns are not sensitive to small variations in spring stiffnesses. The effects of creating an unsymmetrical structure by varying a span length were minimal, with results consistent with those for a symmetrical two-span bridge. By reducing the bent spring stiffness and holding the abutment stiffness coefficient values constant, moments and forces in the substructure were somewhat reduced from the case with the bents fixed. But if lower abutment stiffness is assumed to exist concurrently with low bent stiffness, larger moments result in the longitudinal direction. As the lateral spring coefficients were reduced, significant increases in longitudinal, transverse, and axial forces were observed for all three bridge geometries. On the other hand, variations in rotational and torsional stiffnesses appear to be relatively unimportant. Therefore, it appears that, for relatively short-span structures, the forces and moments in the bent columns are most affected by variations in the lateral (horizontal) spring coefficients.

Because of the lack of information on the behavior of single piles and pile groups in loessial soil deposits, further investigation in the form of dynamic field testing is clearly needed to define a more realistic analysis model. Whether or not it is conservative to model a pile foundation simply as being a fixed support depends on the magnitude of the stiffness of the bent and abutment springs. Full-scale testing is also needed to investigate the effect of pile group interaction, especially for loess.

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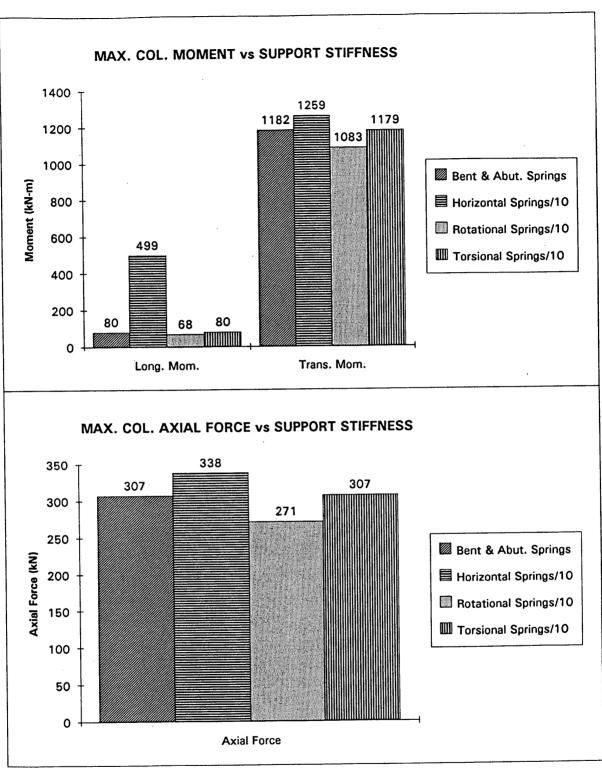


FIGURE 14 Relative effects of variations in horizontal versus rotational and torsional springs for Madison County bridge with equal spans (1 kN = 0.225 kip; 1 kN-m = 0.735 kip-ft).

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