Sensitivity Analysis of Input Parameters for Pavement Design and Reliability

BRIAN M. KILLINGSWORTH AND DAN G. ZOLLINGER

The reliability of a certain pavement design can be related directly to the variation of the input parameters and loading conditions the design process incorporates. To compare different designs using different materials that meet the same design criteria, it is important that the design process evaluate variability between designs in the same manner. It is under this premise that a rational, mechanistic pavement design process for different pavement types is required and hence introduced. These mechanistic pavement design approaches lend themselves well to probabilistic concepts, particularly in light of calibration procedures that can be used to significantly improve the accuracy of design results and desired levels of reliability. When design alternatives that evaluate reliability consistently are compared, it is important to know which variable inputs have the most impact on the range of any given level of reliability. A sensitivity analysis can be useful in assessing the effect on any given input parameter on the resulting design (i.e., mean value and increase in distress for a given level of reliability). This type of exercise will identify design values that must be carefully selected and that can have a signifcant impact on the design result. This will ensure that consistent levels of reliability are maintained in the design process and that reasonable judgments will be made with respect to the most cost-effective pavements.

The principal objective of any engineering design process is to produce a system that performs its intended function in a clear, concise, and accurate manner. To achieve reliability in design, the design process must correctly address, identify, and account for appropriate areas of variability. Because there is uncertainty associated with any engineering design process, an appropriate measure of reliability can be based on probability. This statement is corroborated by Ang and Tang (*1*), who state: "consistent levels of . . . reliability may be achieved if the criteria for design are based on such probabilistic measures of reliability."

A standard engineering definition of reliability is "the probability of an object (item or system) performing its required function adequately for a specified period of time under stated conditions" (*2*). Four essential elements of this definition are further identified, (*2*):

1. Reliability is expressed as a probability.
2. A quality of performance is expected.
3. The performance of the object is expected for a period time.
4. The object is expected to perform under specified conditions.

To compare different designs using different materials that meet the same design criteria, it is important that the design process evaluate variability between designs in the samer manner. This can be accomplished by consistently applying probabilistic concepts that will provide comparable levels of reliability in a format in which all design results are equitably accounted for in the system analysis.

Reliability-based design using probability concepts has been found to be useful in pavement design procedures. In the past, mechanistic design procedures were largely deterministic in that few design inputs were explicitly associated with a mean and a variance. A concept that has outlived its usefulness is the inclusion of some associated variance by applying a factor of safety to certain design inputs. This approach can account for some of the variance, but such empirical modifications only result in overly confounded estimates of design reliability. Consequently, there is no way to reasonably assess what level of reliability is achieved by such a factor.

It has been shown that the mechanistic pavement design approach lends itself well to probabilistic concepts, particularly in light of calibration procedures that can be used to significantly improve the accuracy of design results and desired levels of reliability. Because of this particular feature, a calibrated mechanistic-empirical design process allows the same criteria to be applied in any region, with any soil and climate condition, in the design of a suitable pavement structure. In addition, pavement designs for different pavement types can be compared because a consistent approach to reliability can be applied to the two pavement types.

Quantifying and analyzing variability of pavement materials and design inputs are fundamental concerns in developing a probabilistic-based design that evaluates reliability.

> Design reliability is an indelible aspect of the pavement design process and needs to be genuinely considered and weighted equally with other design factors included in the design procedure. Design reliability . . . positively reinforces and enhances every component of a design procedure in such a manner that the associated and inherent component variability is directly related to the overall probability of pavement failure. Design reliability is the key to realistically, mathematically, and logically accounting for the material and pavement design variabilities (*3*).

Reliability is important in pavement design because of the variance or uncertainty involved in every facet of the pavement process. Factors such as planning, design, construction, use, and maintenance are inherently variable in nature and affect the ability to predict what will happen.

If uncertainty is correctly accounted for and design criteria and inputs are comparable, equitable pavement designs for different pavement types can be achieved and can provide a basis for life-cycle cost analysis. Mechanistic-empirical pavement design models are tools by which this process can be accomplished on a total design systems basis. This type of approach can simultaneously consider paving materials, environment, and loading conditions, while also considering the associated variances for each.

B. M. Killingsworth, Brent Rauhut Engineering, Inc., 8240 North Mopac, Ste. 220, Austin, Tex. 78759. D. G. Zollinger, Texas A&M University, CE/TTI Tower, Room 402D, College Station, Tex. 77843.

FRAMEWORK FOR COMPARABLE PAVEMENT DESIGN

Reliability in mechanistic design approaches is based on engineering mechanics and probability theory formulated so that characteristic distribution parameters can be calibrated with information available in field and performance data bases. However, these parameters also are a function of an individual site or a regional characteristic that not only calibrates the mean level of distress but also the variance associated with the distress. The calibration process, therefore, fine tunes the reliability factors that are associated with the distress distributions and that can characterize the variability of unaccountable influences such as environment, material, and traffic effects (*4*).

Design factors usually incorporated in mechanistic designs include design life, environment, traffic prediction and loading, subgrade strength, and paving material characterization. If pavement designs are developed under the same premise and design reliability is applied identically, the designs can be considered consistent and can provide a basis for determining the most cost-effective pavement structure.

The following section describes one form of inputs and models that can be considered for each pavement type when developing the analytical framework for a mechanistic-empirical design procedure for pavements. Development of a flexible pavement design system is summarized in a Texas Transportation Institute (TTI) research report (*5*) that explains the theory behind a flexible pavement design program. Readers are encouraged to refer to this report for information and details not covered in this paper.

With the advent of high-speed personal computers, intricate mechanistic and empirical design models for various distresses can be incorporated into a single design framework that takes a systems approach to design and considers multiple modes of failure. Computer design algorithms, developed for different pavement types, that (*a*) consider appropriate design inputs and (*b*) incorporate design reliability consistently can be used as input to determine the most cost-effective pavement structure. Two pavement design frameworks that meet these criteria will demonstrate how the design of two pavement types—flexible and rigid—can be compared (*5,6*). The flexible pavement design procedure considers the following failure modes:

- Fatigue cracking (square meters per 1000 m^2 of pavement),
- Rutting, and
- Serviceability or roughness in terms of the present serviceability index (PSI).

The rigid pavement design procedure considers these failure conditions:

- Fatigue cracking (percent midslab cracks),
- Faulting (millimeters on the joint),
- Spalling (number of spalls/per km), and
- Serviceability or roughness in terms of PSI.

The cracking models for these pavement types incorporate reliability concepts using Miner's hypothesis (*7*) to accumulate fatigue damage to predict the mean level of pavement cracking under load and environment stresses. The mechanistic, load-induced cracking model for the flexible pavement system requires the following general inputs:

- Specific traffic loading,
- Resilient moduli of pavement materials,
- Fatigue law of asphalt materials, and
- Variances of resilient moduli, layer thicknesses, and fatigue law parameters.

The mechanistic, load-induced cracking model for the rigid pavement system requires the following inputs:

- Specific traffic loading,
- Modulus of rupture for the concrete,
- Subgrade strength,
- Layer thicknesses,
- Joint spacing,
- Fatigue law of the portland cement concrete mixture, and
- Associated variances of some of the listed variables.

The fatigue life of both pavement types is calculated in terms of the total number of load reptitions to failure, denoted N_f. The allowable loads to failure in asphalt pavements is a function of the maximum tensile stress at the bottom of the asphalt layer, and in concrete pavements it is a function of the total edge stress developed in the pavement slab. The form of the fatigue law used to predict fatigue cracking life in the flexible design program (*8*) is:

$$\log N_f = k_1 - k_2 \cdot \log \epsilon_t - k_3 \cdot \log E \qquad (1)$$

where:

ϵ_t = maximum tensile strain at bottom of asphalt layer,
E = resilient modulus (stiffness) of asphalt layer, and
k_i = parameters of fatigue law ($k_i = k_1$ or k_2; $i = 1, 2$, or 3).

The form of the fatigue law used to predict fatigue cracking in the rigid pavement design program (*9*) is:

$$\log N_f = k_1 - k_2 \cdot \frac{\sigma_{tot}}{MR} \qquad (2)$$

where

σ_{tot} = total pavement stress (combination of σ_{wls} and σ_{ELS}),
σ_{wls} = wheel load stress,
σ_{ELS} = environmental load stress,
MR = modulus of rupture for concrete, and
k_i = parameters of fatigue law.

The Miner law of cumulative damage has been adopted by both pavement design procedures to accumulate damage due to load and environmental effects. The general form of the law is

$$D_j = \sum_{i=1}^{j} \frac{n_i}{N_{fi}} \qquad (3)$$

where

D_j = relative accumulated damage caused to surface layer during j periods,
n_i = actual number of traffic repetitions applied to pavement during period i, and
N_{fi} = total number of traffic repetitions that can be applied to pavement during period i.

If traffic repetitions are in terms of axle load groups, damage can be accumulated over the summation of the load groups as well as over the period. Damage can be accumulated over the summation of a thermal gradient if one is used for curl analysis in the concrete pavement system.

The variance of damage (D_j) is calculated using Cornell's first-order, second-moment method based on the variabilities of the pertinent inputs for both pavement design processes. The computations assume that the mean values of D_j for the different periods are additive. This is equivalent to the assumption that the different damage accumulation periods are independent of each other.

After the expected (or mean) value and variance of the damage function are computed, the percent of cracked area or slabs cracked is computed as the probability that the damage function reaches or surpasses a critical value, which is normally assumed to be 1. This is shown by the following:

$$\%\overline{C} = 100 \cdot \text{prob}\,(D_j \geq D_c) \tag{4}$$

where

$\%\overline{C}$ = percent cracking, which can be converted into cracked area by multiplying by 10 for asphalt pavements or the percentage of slabs cracked in concrete pavements;
D_j = computed accumulated damage; and
D_c = critical damage level, normally equal to 1 according to Miner's hypothesis.

DEVELOPMENT OF STRESS-STRAIN CALCULATION MODELS

The fatigue life analysis for flexible and rigid pavement structures depends on the pavements' capability to sustain repeated loading for a specified period of time. As mentioned previously, this is the maximum tensile strain at the bottom of the asphalt layer for flexible pavements and the total stress on the edge of the concrete slab for rigid pavements.

The tensile strain at the bottom of the asphalt layer is expressed in a precise form based on analysis using the BISAR (*10*) program to develop regression algorithms for the data. The strain was computed for a dual wheel with a contact area radius equal to 114.3 mm and a center-to-center distance between wheels equal to 343 mm. The BISAR program was used to calculate the strains at two points in the upper layer and second layer, if one is present, of the pavement structure: under one wheel of the dual and between the wheels. The radial and tengential strains were computed for specific ranges of pavement layer thicknesses and moduli. The maximum strain between the radial and tangential strains was chosen for the regression analysis, which usually turns out to be the tangential strain. The regression algorithms developed from the BISAR output were used to adjust the wheel load strain calculated by the WESLAY model for different soil conditions and to adjust layer moduli as input by the design engineer.

The tensile strain at the bottom of the asphalt concrete layer is expressed as a Langrangian interpolation polynomial as follows:

$$\frac{E_{sg}\epsilon(x^k)}{P} = \sum_{i=\min}^{\min+d} L_i(x^k)\,\frac{E_{sg}\epsilon(x_i^k)}{P}\Bigg|_{x_l} \qquad k \neq 1 \tag{5}$$

where

$$L_i(x^k) = \prod_{\substack{j=\min \\ j\neq 1}}^{\min+d} \frac{(x^k - x_j^k)}{(x_i^k - x_j^k)}$$

and

i = min, min + 1, . . . , min + d (min = 1);
d = degree of polynomial;
E_{sg} = subgrade elastic modulus;
P = applied pressure;
x^k = variables that equal ratios of layer thicknesses to contact area and layer moduli to subgrade modulus; and
$\epsilon(x)$ = tensile strain at bottom of asphalt layer.

The TTI report (*5*) explains the process for the final strain calculation by stating that Equation 5

> . . . is an n[th] degree Langrangian interpolating polynomial which establishes the value of E_{sg}/P for all values within the allowable ranges of the layer thicknesses and modulus of elasticity. The tensile strain for a given pavement structure is interpolated over min + d values of strain associated with a single variable x_i^k in which the values of the x^l variables are held constant at each strain.

The report (*5*) continues by stating that Equation 5

> is repetitively solved for each combination of x^k variables in accordance to the pavement structure and layer moduli. Following this process, the design program develops a smaller table of strain values generated from interpolation over the layer thickness and layer moduli. The final strain value is determined when all the variables (x^k) for a given pavement structure have been accounted for in the interpolation process. The interpolation polynomial is regenerated for each new pavement structure or when the pavement structure is modified.

The total edge stress calculated for a specified concrete pavement structure is based on the summation of the wheel load stress, curl stress, and stress due to erosion beneath the slab. Wheel load stress is calculated by regression equations developed from numerous Illi-Slab (*11*) runs. Each regression equation is in the form of

$$s = a_1 + a_2 \cdot \ell + a_3 \cdot \ell^2 \tag{6}$$

where

s = dimension stress in form of $\frac{\sigma_{\text{wls}} \cdot h^2}{P}$

σ_{wls} = wheel load stress;
h = slab thickness;
P = wheel load;
ℓ = radius of relative stiffness, which is equal to

$$\sqrt[4]{\frac{E_c \cdot h^3}{12 \cdot (1 - \nu^2) \cdot k_{sg}}}$$

E_c = elastic modulus of concrete slab;
ν = Poisson's ratio for concrete slab; and
k_{sg} = foundation modulus of subgrate reaction.

The regression equations developed for dimensionless stress are affected by three design factors input by the design engineer: the degree of bond between the subbase and the surface slab, the shoulder type used in the design, and the axle configuration. An equation in the form of Equation 6 has been developed for every combination of these design factors.

Because all design factors are directly or indirectly predetermined from model inputs, the wheel load stress can be determined by rearranging the dimensionless stress equation to (*12*)

$$\sigma_{\text{wls}} = \frac{P \cdot s}{h^2} \tag{7}$$

Curl stress is calculated by considering the temperature differential that occurs within the slab due to daytime heating and nighttime cooling. The initial curl stress is calculated by Westergaard's analysis, and the stress is then corrected for slab length by the inclusion of a coefficient that considers the effect of L/ℓ. The following formulas mathematically describe total curl stress (*13*):

$$\sigma_{\text{curl}} = c_1 \cdot \sigma_0 \tag{8}$$

where

$$\sigma_0 = \frac{E_c \alpha \Delta t}{2(1 - \nu)}$$

and

$$c_1 = 1 - \frac{2 \cos \lambda \cosh \lambda}{\sin 2\lambda + \sinh 2\lambda} \cdot (\tan \lambda + \tanh \lambda)$$

where

$$\lambda = \frac{L}{\ell \sqrt{8}}$$

where

- σ_{curl} = curl stress corrected for curvature,
- σ_0 = Westergaard's uncorrected curl stress,
- c_1 = curl correction for $L/1$,
- α = coefficient of thermal contraction/expansion
- Δt = temperature differential between top and bottom of slab, and
- L = slab length.

The passing of axle loads over a concrete slab may sometimes cause erosion, which is the loss of underside slab support. Erosion can have a substantial effect on the total stress that develops on the slab edge. Erosion in the rigid pavement design program is incorporated by multiplying the total stress by an erosion factor, which is equal to 1 if there is no erosion, or a factor greater than 1 when erosion occurs. The erosion factor is based on the following equation:

$$\beta = 1.000 + 0.109 \cdot \left(\frac{\rho_s}{\ell}\right) + 0.034 \cdot \left(\frac{\rho_s}{\ell}\right)^2 \tag{9}$$

where β is the correction for erosion, and ρ_s is the rate of erosion.

The total stress can now be shown mathematically (*14*) as

$$\sigma_{\text{tot}} = (\sigma_{\text{wls}} + \alpha \sigma_{\text{curl}}) \cdot \beta \tag{10}$$

where α is a curl correction factor introduced to allow combination of the wls and the curl stress (*14*). However, it was set equal to 1 for this analysis because of favorable comparisons to other calculated total pavement stresses.

The methods described herein predict a single value for fatigue life in terms of N_f. This does not mean, however, that only one value of N_f exists for each specific pavement structure. Instead, N_f is probabilistically distributed. Because N_f is probabilistic, there exists for each pavement an expected value of N_f and an associated variance that describes the distribution N_f will follow. The variation affiliated with N_f results from the fact that the values used to calculate N_f are not exact values but are distributed over a range of values. As noted previously, most distribution parameters can be calibrated with field data. N_f is a parameter that fits into this category; however, N_f is sensitive to each specific pavement site, and the calibration can be different depending on the site.

LOAD-INDUCED CRACKING SUBSYSTEM FOR ASPHALT CONCRETE PAVEMENTS

As noted previously, the mean area of cracking is based on the damage function that relates allowable traffic to the tensile strain at the bottom of the asphalt layer. The cracked area is obtained from Equation 4, which states that the cracked area is given by the probability that the damage reaches or exceeds a value of 1. If a normal distribution for damage is assumed, the probability can be found from the following:

$$\text{prob}\,(D_j \geq 1) = \frac{1}{\sqrt{2\pi}\,\sigma_j} \int_1^{\infty} \exp\left[-\frac{1}{2}\left(\frac{t - \overline{D}_j}{\sigma_j}\right)^2\right] dt \tag{11}$$

where

- $D_j = \sum n_i/N_{fi}$, damage function accumulated up to the jth period;
- n_j = number of 18 Kip SAL repetitions during period i (ESAL_i);
- N_{fi} = fatigue number of 18 Kip SAL repetitions that material can withstand to failure, given by Equation 1;
- t = dummy variable of integration; and
- σ_j = standard deviation of damage function.;

By using Cornell's first-order, second-order moment theory, the average damage function and its variance can be determined for the damage function. The TTI report assumes that the periods are independent one of the other, and thus the covariance terms in Equation 14 are nil (*5*). The mean damage or the expected value then can be computed using

$$\overline{D}_j = \sum_{i=1}^{j} \frac{\overline{n}_i}{\overline{N}_{fi}} = \sum_{i=1}^{j} \overline{\Delta D_i} \tag{12}$$

and the variance from

$$\text{Var}(D_j) = \text{Var}\left(\sum_{i=1}^{j} \Delta D_j\right) = \sum_{i=1}^{j} \text{Var}(\Delta D_i) \tag{13}$$

where

$$\text{Var}(\Delta D_i) = \sum_k \left(\frac{\partial \Delta D_i}{\partial X_k}\,|\bar{x}_k\,\sigma_{x_k}\right)^2 + \left(\sum_l \sum_{k \neq l} \sum_k \frac{\partial \Delta D_i}{\partial X_k}\,|\bar{x}_l\,\sigma_{x_l}\right)\rho_{kl} \tag{14}$$

where ρ_{kl} is the correlation coefficient between X_k and X_l.

The variables denoted X_k are all the variables of which ΔD_i is a function, with the exception of n_i (or the load repetitions), because the variance of the cracked area with respect to traffic is calculated separately from the variance of the cracked area because of pavement material parameters. Therefore, the computed cracked area as explained by the TTI report will be the area which corresponds to a given number of load repetitions (5). The derivative of ΔD_i with respect to X_k can then be shown as

$$\frac{\partial \Delta D_i}{\partial X_k} = \frac{\partial\left(\frac{1}{N_{fi}}\right)}{\partial X_k} \tag{15}$$

showing ϵ_i and N_{fi} as

$$\epsilon_i = \frac{p}{E_{sg}} \cdot F$$

$$-\ln\left(\frac{1}{N_{fi}}\right) = k_1\text{Ln}10 + k_2\text{Ln}\left(\frac{\epsilon_i}{10^{-6}}\right) + k_3\text{Ln}\left(\frac{E_i}{1000}\right)$$

where F is a function of layer thicknesses and moduli, and k_i is parameter of fatigue law.

The derivatives then can be shown by the following expressions:

$$\frac{\partial\left(\frac{1}{N_{fi}}\right)}{\partial k_1} = -\frac{1}{N_{fi}}\text{Ln}10 \tag{16}$$

$$\frac{\partial\left(\frac{1}{N_{fi}}\right)}{\partial k_2} = -\frac{1}{N_{fi}} \cdot \text{Ln}\left(\frac{\epsilon_i}{10^{-6}}\right)$$

$$\frac{\partial\left(\frac{1}{N_{fi}}\right)}{\partial k_3} = -\frac{1}{N_{fi}} \cdot \text{Ln}\left(\frac{E_i}{1000}\right)$$

$$\frac{\partial\left(\frac{1}{N_{fi}}\right)}{\partial T_l} = -\frac{1}{N_{fi}} \cdot \frac{k_2}{\epsilon_i} \cdot \frac{p}{E_{sg}a} \cdot \frac{\partial F}{\partial\left(\frac{T_l}{a}\right)} \qquad l = 1, 2, 3$$

$$\frac{\partial\left(\frac{1}{N_{fi}}\right)}{\partial E_l} = -\frac{1}{N_{fi}} \cdot \frac{k_2}{\epsilon_i} \cdot \frac{p}{E_{sg}^2} \cdot \frac{\partial F}{\partial\left(\frac{E_l}{E_{sg}}\right)} - \frac{1}{N_{fi}}\frac{k_3}{E}\delta_{ml}$$

$$l = 1, 2, 3;\ m = 1 \text{ or } 2$$

$$\frac{\partial\left(\frac{1}{N_{fi}}\right)}{\partial E_{sg}} = -\frac{1}{N_{fi}} \cdot \frac{k_2}{\epsilon_i} \cdot \frac{p}{E_{sg}} \cdot \left[-\frac{F}{E_{sg}} + \sum_{l=1}^{3} \frac{\partial F}{\partial\left(\frac{E_l}{e_{sg}}\right)} \cdot \left(-\frac{E_l}{E_{sg}^2}\right)\right]$$

where δ_{ml} is Kronecker's Delta.

The derivatives of F are calculated by the Langrangian interpolation regression equations that are discussed in the TTI report and shown by Equation 5 in this paper.

In the TTI report it is stated that by knowing the average value of the damage D_j and its variance at the end of the period j, the average cracked area for a given number of repetitions can be computed (5).

Variance of Cracked Area with Respect to Number of Repetitions

The determination of the variance of the cracked area for the flexible pavement design system is divided into two components: (*a*) the variance with respect to the number of load applications up to and including the present period and (*b*) the variance with respect to the pavement parameters. The TTI report proceeds through the derivation of the variance of the cracked area with respect to the number of load applications by expressing the cracked area as

$$C = \frac{1000}{\sqrt{2\pi}} \int_1^\infty \frac{1}{\sigma_j} \exp\left[-\frac{1}{2}\left(\frac{t-\bar{D}_j}{\sigma_j}\right)\right] dt \tag{17}$$

The variance of cracking due to the variability in the number of load repetitions is shown by using the Taylor series approximation.

$$\text{Var}(C) = \left(\frac{\partial C}{\partial N}\Big|_{\bar{N}}\right)^2 \cdot \text{Var}(N) \tag{18}$$

where

$$D_j = \sum_{i=1}^{j} \frac{n_i}{N_{fi}}$$

and the variance is expressed as

$$\sigma_j = \sigma = \sum_{i=1}^{j} \sum_{k} \left[\frac{\partial\left(\frac{1}{N_{fi}}\right)}{\partial x_k}\Big|_{\bar{x}k}\right]^2 \cdot \text{Var}(X_k) \cdot n_i^2 = \sum_{i=1}^{j} c_i n_i^2$$

The TTI report continues with the rest of the derivation by the following equations and using the transformation

$$\frac{t - D}{\sigma} = T \qquad dt = \sigma dT$$

with the limits

$$t = \infty \qquad T = \infty$$

$$t = 1 \qquad T = T_1 = \frac{1 - D}{\sigma}$$

This gives the error function

$$C = 1000\left[\frac{1}{\sqrt{2\pi}} \int_1^\infty \exp\left(-\frac{1}{2}T^2\right) dT\right] \tag{19}$$

Then taking the derivative with respect to load repetitions gives

$$\frac{\partial C}{\partial N} = \frac{1000}{\sqrt{2\pi}} \int_{T_1}^\infty -T\exp\left(-\frac{1}{2}T^2\right) \frac{\partial T}{\partial N}\, dT \tag{20}$$

Using these above definitions,

$$\frac{\partial T}{\partial N} = -\frac{1}{\sigma}\frac{\partial D}{\partial N} - \frac{T}{\sigma}\frac{\partial \sigma}{\partial N} = -DN - T \cdot SN$$

$$\frac{\partial D}{\partial N} = \sum_{i=1}^{j} \frac{1}{N_{fi}}\frac{\partial n_i}{\partial N} = \sigma \cdot DN$$

$$\frac{\partial \sigma}{\partial N} = \frac{1}{\sigma}\sum_{i=1}^{j} c_i n_i \frac{\partial n_i}{\partial N} = \sigma \cdot SN$$

$$N = N_j = \sum_{i=1}^{j} n_i$$

where N is N_j, the number of load repetitions (ESALs) accumulated during j periods.

Substituting $\frac{\partial T}{\partial N}$ Equation (20) gives

$$\frac{\partial C}{\partial N} = \frac{1000}{\sqrt{2\pi}}\int_{T_1}^{\infty} (DN \cdot T + SN \cdot T^2) \exp\left(-\frac{1}{2}T^2\right) dT$$

This function is integrated by parts to give

$$\frac{\partial C}{\partial N} = \frac{1000}{\sqrt{2\pi}} (DN + SN \cdot T_1) \exp\left(-\frac{1}{2}T_1^2\right) + SN \cdot C$$

The TTI report details the derivative $\frac{\partial n_i}{\partial N}$ as shown in these definitions as evaluated from the traffic growth as follows (Figure 1):

$$\frac{\partial n_j}{\partial N_j} = \frac{\frac{\partial n_j}{\partial y}}{\frac{\partial N_j}{\partial y}} = \frac{\left(\frac{BL-AL}{20}\right)(y_j - y_{j-1})}{AL + \left(\frac{BL-AL}{20}\right) y_j} \qquad (21)$$

The report also discusses how the variance of traffic affects the variance of the cracked area,

> Since the cracked area increases sharply with the number of repetitions after cracking has initiated, the Var[C] with respect to N may be very large. The flexible pavement design model implements a Var[log N] = 0.0355. After transformation, one gets Var[N] = 0.188^2 (or a standard deviation of $0.43N$).
>
> The variance of N is due to the variation of several factors such as design ADT, the percent of trucks in the traffic mix, axle load equivalency factors, and the distribution of the axle loads. Variation in ADT may be considered to be a between project variation and should not be included in the Var[N] term. However, the remaining factors do have a contribution to within project variation and should be included in the Var[N] (5).

Variance of Cracked Area with Respect to Pavement Parameters

The second portion of the variance of the cracked area deals with the pavement parameters. The initial portion of the derivation is similar to that shown in the derivation of the variance of the cracked area with respect to the number of load applications. The variance of the cracked area with respect to the pavement parameters is shown mathematically as the following:

$$\mathrm{Var}(C) = \sum \left(\frac{\partial C}{\partial x_k}\Big|\bar{x}_k\right)^2 \mathrm{Var}(x_k) + \left(\sum_{\ell}\sum_{k \atop k\neq\ell} \frac{\partial C}{\partial x_k}\Big|\bar{x}_k\, \sigma x_k \frac{\partial C}{\partial X_l}\Big|\bar{x}_\ell \sigma x_\ell\right) \rho_{k\ell} \qquad (22)$$

By taking the derivative of Equation 19 with respect to the pavement parameters, the following is obtained:

$$\frac{\partial C}{\partial x_k} = \frac{1000}{\sqrt{2\pi}}\int_{T_1}^{\infty} -T \exp\left(-\frac{1}{2}T^2\right)\frac{\partial T}{\partial x_k}\, dT$$

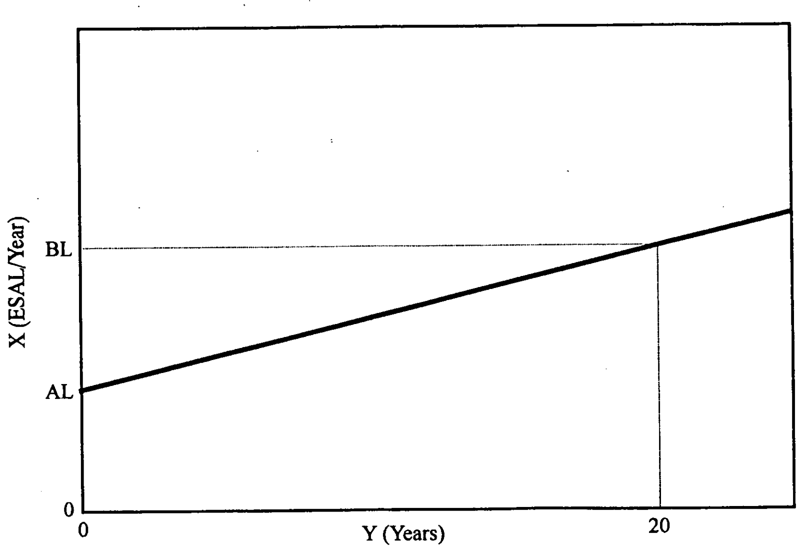

FIGURE 1 Traffic characterization in flexible pavement design system.

with

$$\frac{\partial T}{\partial x_k} = -\frac{1}{\sigma}\frac{\partial D}{\partial x_k} - \frac{T}{\sigma}\frac{\partial \sigma}{\partial x_k} = -DD - DS \cdot T$$

By following the same steps as before, the following is obtained:

$$\frac{\partial C}{\partial x_k} = \frac{1000}{\sqrt{2\pi}}\int_{T_1}^{\infty}(DD \cdot T + DS \cdot T^2)\exp\left(-\frac{1}{2}T^2\right)dT$$

Integration by parts gives

$$\frac{\partial C}{\partial X_k} = \frac{1000}{\sqrt{2\pi}} \cdot \frac{1}{\sigma}\frac{\partial D}{\partial X_k}\exp\left(-\frac{1}{2}T_1^2\right) + \frac{1}{\sigma}\frac{\partial \sigma}{\partial X_k}\left[\frac{1000}{\sqrt{2\pi}}T_1\exp\left(-\frac{1}{2}T_1^2\right) + C\right] \quad (23)$$

Recalling the damage function

$$D = \sum_{i=1}^{j}\frac{n_i}{N_{fi}}$$

the variance of the damage with respect to the pavement parameters can be obtained as

$$\sigma^2 = \sum_{i=1}^{j}\left[\sum_{\ell}\left(\frac{\partial\left\{\frac{1}{N_{fi}}\right\}}{\partial X_\ell}\Big|_{\bar{x}_\ell}\right)^2 \mathrm{Var}(X_\ell)\right]n_i^2$$

$$\frac{\partial D}{\partial X_k} = \sum_{i=1}\frac{\partial\left(\frac{1}{N_{fi}}\right)}{\partial X_k}n_i = \sum_i \frac{\partial \Delta D_i}{\partial X_k}$$

$$\frac{1}{\sigma}\frac{\partial \sigma}{\partial X_k} = \frac{1}{2\sigma^2}\sum_{i=1}^{j}\left\{\sum_{\ell}2 \cdot \left(\frac{\partial\left\{\frac{1}{N_{fi}}\right\}}{\partial X_\ell}\Big|\bar{x}l\right) \cdot \left(\frac{\partial^2\left(\frac{1}{N_{fi}}\right)}{\partial X_k \partial X_\ell}\Big|\bar{x}_k, \bar{x}_\ell\right)\mathrm{Var}(X_\ell)n_i^2\right\} \quad (24)$$

The determination of σ is explained in the TTI report by the following statements:

> It seems that the variance of C is due to two causes: (a) the variation of D with respect to the value of one (shifting of the density distribution curve), and (b) the variation of σ (flatness of the bell shape of the curve). The contribution of σ is very complex to evaluate because it involves the evaluation of mixed derivatives. Therefore, the term with σ was dropped, assuming that its contribution is negligible (*5*).

Substituting Equations 23 and 24 into 22 and assuming the contribution of σ is negligible,

$$\mathrm{Var}(C) = \sum_{i=1}^{j}\frac{10^6}{2\pi}\frac{1}{\sigma^2}\left\{\exp\left[-\frac{1}{2}\left(\frac{1-D_i}{\sigma_i}\right)^2\right]^2\right\} * \sum_k\left(\frac{\partial \Delta D_i}{\partial X_k}\Big|\bar{x}_k\right)\mathrm{Var}(X_k) + \left(\sum_{\ell}\sum_{\substack{k \\ k\neq\ell}}\frac{\partial \Delta D_i}{\partial X_k}\Big|\bar{x}_k\sigma x_k\frac{\partial \Delta D_i}{\partial X_l}\Big|\bar{x}_\ell\sigma x_\ell\right)\rho_{k\ell} \quad (25)$$

> The variance of cracking with respect to pavement parameters and the variance of cracking with respect to number of repetitions is combined into an overall variance of cracking (Var[C]) leading to the determination of the level of cracking corresponding to various levels of reliability . . . (*5*).

LOAD-INDUCED CRACKING SUBSYSTEM FOR PORTLAND CEMENT CONCRETE PAVEMENTS

The development of a fatigue crack in a concrete slab can be defined as the probability that the accumulated fatigue damage exceeds a critical level of fatigue damage ($D_c = 1$), as stated previously. Hence, the variance of cracking and mean level of cracking are related to variance and mean level of fatigue damage, as is the case with the asphalt concrete system. Accordingly, the percent mean cracking level is the probability that some critical level of damage has been surpassed, as shown by Equation 4.

The average damage function and its variance are computed in the same manner as in the asphalt concrete system in that Cornell's first-order, second-moment method is used and the assumptions are identical. The variance of damage is computed using the following equations:

$$\mathrm{Var}(D_i) = \sum \mathrm{Var}(\Delta D_i)$$

$$\mathrm{Var}(\Delta D_i) = \sum\left(\frac{\partial \Delta D_i}{\partial x_k}\right)^2 \mathrm{Var}(X_k)$$

where i corresponds to each time period.

The variance of the incremental damage for the asphalt concrete system is shown by Equation 14, and the only changes that would occur for the portland cement concrete (PCC) system are the variables X_k, which are involved with the computation of ΔD_i. The fatigue law for the PCC system is stated in Equation 2. Equation 10 shows that the total stress is an accumulation of the wheel load stress, curl stress, and stress induced from the loss of underside support. Accordingly, the percentage of midslab cracks computed will be a direct function of the number of load repetitions to which the pavement is subjected. The derivative of ΔD_i with respect to X_k is given by Equation (15) and is the same for the PCC pavement design system as it is for the asphalt concrete pavement design system. The respective PCC pavement derivatives of $1/(N_f)$ with respect to X_k are given as the following:

$$\frac{\delta\left(\frac{1}{N_f}\right)}{\delta k_1} = -\frac{b}{N_f}$$

$$\frac{\delta\left(\frac{1}{N_f}\right)}{\delta k_2} = \frac{br}{N_f}$$

$$\frac{\delta\left(\frac{1}{N_f}\right)}{\delta Mr} = -\frac{bk_2\sigma}{N_f Mr^2}$$

$$\frac{\delta\left(\frac{1}{N_f}\right)}{X_k} = \frac{bk_2}{N_f \cdot Mr} \cdot \frac{\delta\sigma}{\delta X_k}$$

where $X_k = h$, k_{sg}, E_c, ν, and a. The derivatives $\delta\sigma/\delta X_k$ equal the following:

$$\frac{\delta\sigma}{\delta X_k} = \frac{\delta\beta}{\delta X_k}(\sigma_{\text{wls}} + \sigma_{\text{curl}}) + \frac{\delta(\sigma_{\text{wls}} + \sigma_{\text{curl}})}{\delta X_k}\beta \quad (26)$$

where the derivatives of $\delta\beta/\delta X_k$ can be developed from the equations given previously.

Field data for damage suggest that their density function almost matches the Weibull distribution. Therefore, in the concrete pavement design procedure, the general form of damage distribution (or probability density function) can be approximated by

$$\text{pdf}(D_i) = \frac{\beta}{\alpha}\left(\frac{D_i}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{D_i}{\alpha}\right)^{\beta}\right] \quad \text{for } D_i > v_s$$

where

v_s = 0 for damage calculations,
D_i = accumulated fatigue damage up to period i, and
β, α = shape and scale parameters for the Weibull distribution, respectively.

If $\xi = (D_i/\alpha$ and $d\xi = 1/\alpha$ are substituted with $\xi^\beta = v$ and $\beta\xi^{l-l} = dv$,

$$\text{pdf}(D_i) = \frac{\beta}{\alpha}\xi^{\beta-l}e^{\xi\beta}$$

The probability that the accumulated damage (D_i) is less than the critical damage (D_c) is

$$\text{prob}(D_i < D_c) = \int_0^{D_c} f(D_i)\,dD_i$$

$$= \frac{\beta}{\alpha}\int_0^{\xi_c}\xi^{\beta-l}e^{\xi\beta}\alpha\,d\xi$$

$$= \int_0^{v_c} e^{-v}\,dv$$

with $v_c = \xi_c^\beta = \left(\frac{D_c}{\alpha}\right)^\beta$ and the previous substitutions. However, recall that

$$\%C = \text{prob}(D_i > D_c)$$

Therefore,

$$\%C = 100 - 100\int_0^{v_c} e^{-v}\,dv = \frac{100}{e^{v_c}} \quad (27)$$

The variance of cracking can be developed from Equation 28 and comprises the same two components as shown in the asphaltic concrete system: (a) variance of cracking due to variance in material parameters (X_k) and (b) variance of cracking due to variance in traffic (N). Mathematically, the total variance is expressed as

$$\text{Var}(C) = \text{Var}(C)_{x_k} + \text{Var}(C)_N$$

Variance of Cracking Due to Material Parameters

The material parameters (X_k) affecting the variance of slab cracking are identified as pavement thickness (h), subgrade k value (k_{sg}), concrete modulus of elasticity (E_c), concrete modulus of rupture (Mr), Poisson ratio (ν), and the fatigue parameters k_1 and k_2. Another term that can be included but is not a material property is the radius of the load area (a). The variance of cracking due to the material parameters is

$$\text{Var}(C)_{x_k} = \sum\left(\frac{\partial C}{\partial X_k}\right)^2\text{Var}(X_k) + \sum\sum\left(\frac{\partial C}{\partial X_k}\sigma_{x_k}\frac{\partial C}{\partial X_l}\sigma_{x_l}\right)\rho_{kl} \quad (28)$$

where ρ_{kl} is the correlation coefficient between X_k and X_l.

The partial differentiation of cracking with respect to material parameters, X_k, ($\{\partial C\}/\{\partial X_k\}$) is determinated by differentiating Equation 27:

$$\frac{\partial C}{\partial X_k} = 100\int_0^{v_c}\frac{\partial v}{\partial X_k}e^{-v}\,dv$$

$$\text{where } \frac{\partial v}{\partial X_k} = \frac{\partial[(\xi)^\beta]}{\partial X_k} = \beta(\xi)^{\beta-1}\frac{1}{\alpha}\left[\frac{\partial D_i}{\partial X_k}\right]$$

which can be further reduced to

$$\frac{\partial C}{\partial X_k} = -100\cdot\frac{\beta}{\alpha}\cdot\left(\frac{\partial D_i}{\partial X_k}\right)\int_0^{v_c}(\xi)^{\beta-1}e^{-v}dv$$

$$= -100\cdot\frac{e}{\alpha}\cdot\left(\frac{\partial D_i}{\partial X_k}\right)\int_0^{v_c}v^r e^{-v}\,dv$$

$$= -100\cdot\frac{\beta}{\alpha}e^{-v_c}(v_c^r)\left(\frac{\partial D_i}{\partial X_k}\right)$$

Therefore

$$\frac{\sigma C}{\sigma X_k} = -\%C\cdot\frac{\beta}{\alpha}v_c^r\left(\frac{\partial D_i}{\partial X_k}\right)$$

$$\text{where } r = \frac{\beta - 1}{\beta}$$

The derivative of cracking with respect to the material parameters can be substituted in Equation 28 to obtain the variance of cracking due to the material parameters. Note the following derivatives for damage with respect to the pavement parameters:

$$\frac{\sigma D_i}{\sigma X_k} = \frac{\partial\left(\frac{1}{N_f}\right)}{\partial X_k}n_i \quad (29)$$

The derivatives of Equation 29 are shown previously in this section.

Variance of Cracking Due to Traffic

The variance of cracking due to traffic (N) is expressed as the following:

$$\mathrm{Var}(C)_N = \left\{ \frac{\partial C}{\partial N} \mid_{\bar{N}} \right\}^2 \mathrm{Var}(N)$$

which is the same for asphalt concrete pavements. By using the probability density function for the Weibull distribution to express the percent of midslab cracks from Equation 27, the derivative $\delta C/\delta N$ can be evaluated as

$$\frac{\sigma C}{\sigma N} = 100 \cdot \int_0^{\nu_c} \frac{\delta \nu}{\delta N} e^{-\nu} \delta \nu$$

where $\frac{\delta \nu}{\delta N} = \frac{\beta}{\alpha} \left[\frac{D_i}{\alpha} \right]^{\beta-1} \frac{\delta D_i}{\delta N}$

and

$$\frac{\delta D_i}{\delta N} = \sum_{i=1}^{j} \frac{1}{N_f} \frac{\delta n_i}{\delta N}$$

The derivative $\delta D_i/\delta N$ is evaluated from the traffic growth, and the derivative $\delta n_i/\delta N$ is the same as in Equation 21 for the asphalt concrete pavements. Substituting and completing the integration for the evaluation of $\delta C/\delta N$ results in the following equation:

$$\frac{\delta C}{\delta N} = 100 \cdot \frac{\beta}{\alpha} e^{-\nu_c} \nu_c^r \frac{\delta D_i}{\delta N} = -\bar{C} \frac{\beta}{\alpha} \nu_c^r \frac{\delta D_i}{\delta N}$$

where

β = Weibull distribution shape parameter,
α = Weibull distribution scale parameter,
$\nu_c = (D_c/\alpha)^\beta$, and D_c is a calibration term and equals 1 for these purposes, and
$r = (\beta - 1)/\beta$.

Therefore, the derivatives for cracking with respect to the material parameters and with respect to traffic as well as the variance for each of the material parameters (by assuming a coefficient of variation for each parameter) have been defined. However, the variance due to traffic is yet to be determined.

The variance of traffic, or Var(N), is influenced by several factors previously discussed in the asphalt concrete pavements section on the variance of cracking due to traffic. These factors include, but are not exclusive to, the distribution of axle loads, axle equivalency factors, and the percentage of trucks in the design traffic. Design traffic repetitions do not necessarily have to be in terms of equivalent loads, ESALs, to compute appropriate distress, but can be expressed in terms of axle load groups as an alternative. The design engineer will determine which method is most appropriate for the project design. With the availability of high-speed personal computers, this type of traffic expression can be achieved without increasing computing time excessively.

SENSITIVITY ANALYSIS OF SELECTED DESIGN INPUTS FOR TWO KENTUCKY PAVEMENTS

When comparing design alternatives that evaluate reliability consistently, it is important to know which variable inputs have the most impact on the range of any given level of reliability. This stems from the notion that errors that do occur in the assessment of design reliablity can manifest themselves in two ways: (a) an incorrect prediction of the mean level of distress and (b) an incorrect prediction of the increase in the amount of distress for a given level of design reliability. If, for any reason, one or both of these incorrect predictions occur in the design process, a biased prediction of pavement life and reliability will cause one pavement type to have an advantage over or to be at a disadvantage to the other in terms of life-cycle cost predictions. A consistent comparison between the two designs, therefore, cannot be achieved.

A sensitivity analysis can be useful in assessing how any given input parameter affects the resulting design (i.e., mean value and the increase in distress for a given level of reliability). This type of exercise will identify design values that must be carefully selected and that can have a significant impact on the design result. In addition, the exercise will ensure that consistent levels of reliability are maintained in the design process and that reasonable judgments will be made with respect to the most cost-effective pavement.

Two pavements in Kentucky with different design criteria have been chosen as an example. A sensitivity study is concluded on various design inputs to determine which inputs have the greatest effect on the prediction of design life. A sensitivity study is conducted by varying the chosen design inputs by plus and minus the assigned coefficients of variation from the mean level of the design inputs. The design inputs used in the sensitivity study of these two pavements are the subgrade strength with a coefficient of variation of 30 percent, the traffic level in ESALs per year with a coefficient of variation of 20 percent, the surface layer modulus with a coefficient of variation of 20 percent, and the input surface thickness with a coefficient of variation of 10 percent. Change in the value of the coefficient of variation for a design variable does not necessarily have a significant effect on the overall calculated variance. Therefore, if a coefficient is assumed for a design value that is not known, the effect on the calculated variance should not be significant.

Pavement site A has a relatively weak subgrade (41,370 Kilopascals) and low traffic (3 million ESALs). Its design includes a granular base layer in both pavement systems. The sensitivity analysis for the flexible pavement indicated that as subgrade strength was varied by its coefficient of variation, a substantial percent change in thickness occurred (Figure 2). The percent change in thickness as traffic varied was slightly smaller but followed the same pattern. The pattern reversed somewhat for the surface modulus and surface thickness but stayed consistent. As surface thickness varied, the percent change was measured in allowable traffic. This analysis indicates that with low traffic and a weak subgrade, the flexible pavement design is moderately sensitive to changes in subgrade modulus, allowable traffic, and surface modulus; however, it is much less sensitive to changes in surface thickness.

The sensitivity analysis for the portland cement concrete pavement indicates that as subgrade modulus varied by its coefficient of variation, no difference was measured in the required thickness, and essentially the same was indicated as traffic varied by its coefficient (Figure 3). As the PCC surface modulus varied, some change in surface thickness was indicated; however as the surface thickness varied by its coefficient of variation, there was considerable change in

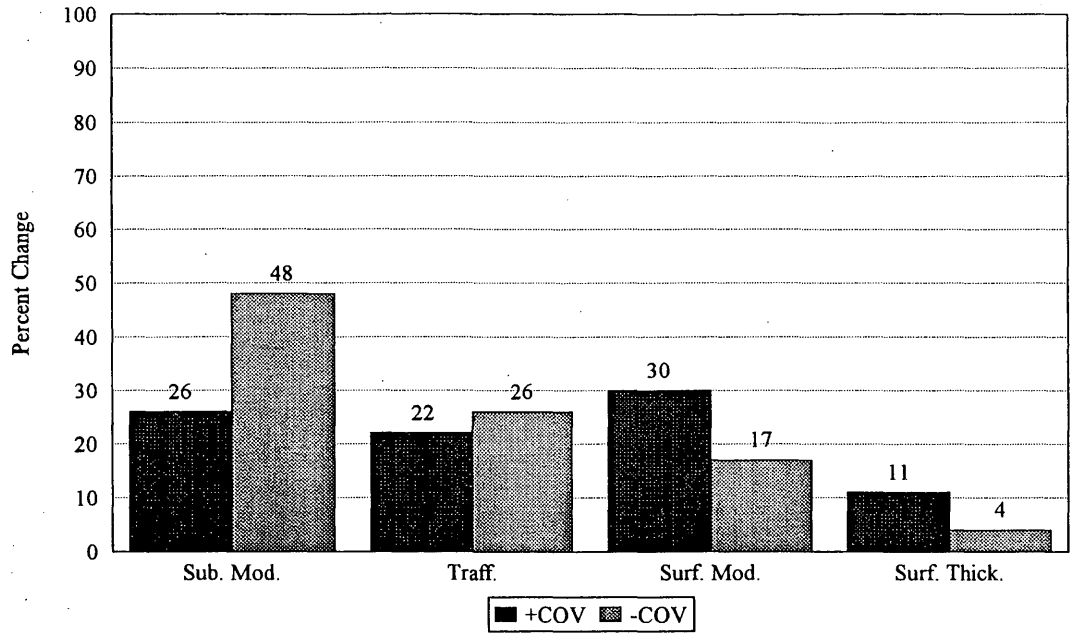

FIGURE 2 Percent change from mean input for asphalt concrete design, pavement site A.

the allowable traffic. This indicates that at lower traffic levels and subgrade strength, concrete pavements cannot be sensitive to subgrade modulus and allowable traffic, but are sensitive to the input surface thickness.

The sensitivity analysis for pavement site B, which consisted of a high traffic level (88 million ESALs) and a moderately strong subgrade (113,767 Kilopascals), generally followed the same type of trends as pavement site A for both pavement types.

For this design, the flexible pavement seemed to be less sensitive to variations in subgrade modulus, traffic, and surface modulus and more sensitive to variations in surface thickness (Figure 4). This indicates that as required thickness and allowable traffic increase, other design parameters become less of a factor.

The analysis of the PCC design for higher traffic indicates that all design parameters are somewhat sensitive to variations in their design values (Figure 5). The large differences between pavement

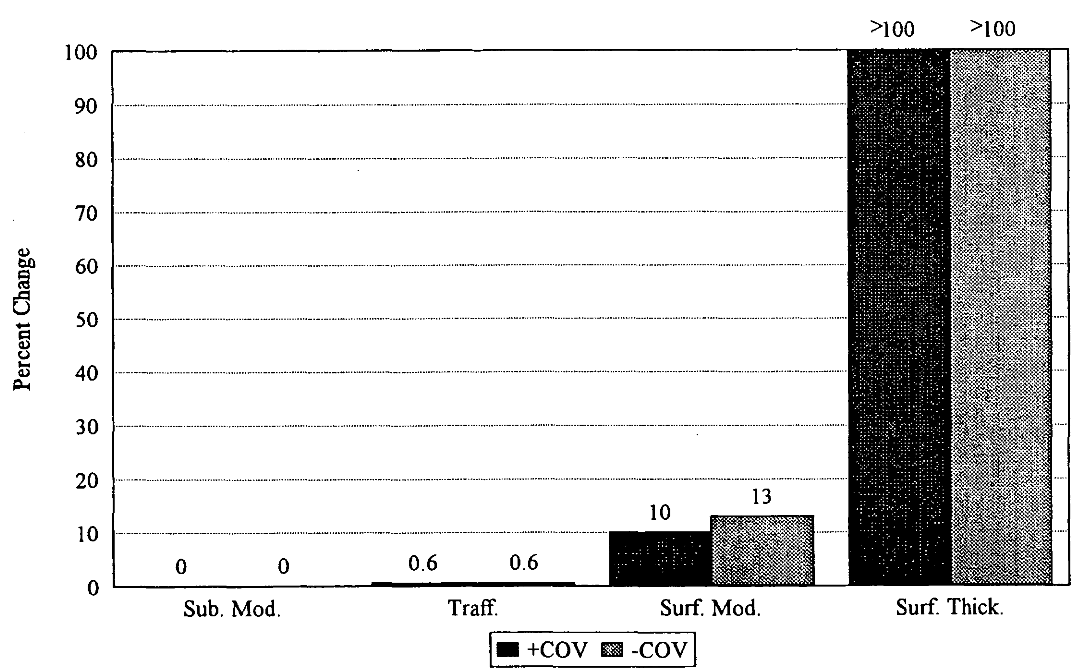

FIGURE 3 Percent change from mean input for PCC design, pavement site A.

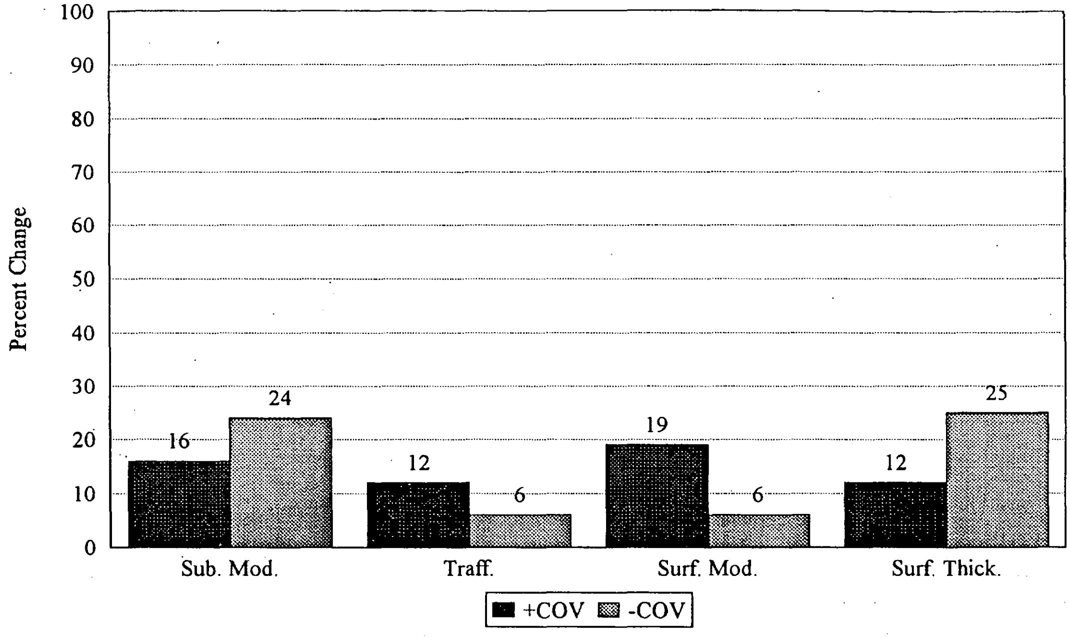

FIGURE 4 Percent change from mean input for asphalt concrete design, pavement site B.

sites A and B occurred as traffic and surface thickness varied. This indicates that as required thickness and allowable traffic increase, the rigid pavement design system becomes more sensitive to changes in subgrade modulus and input traffic and considerably less sensitive to variations in surface thickness. The two rigid pavement designs have approximately the same sensitivity to variations in surface modulus.

The conclusions that can be drawn from this sensitivity analysis follow:

- As traffic level and subgrade modulus differ, design parameter sensitivity changes in both pavement types, but to a lesser extent in PCC pavements.
- The trends in percent change for the flexible pavement design as predicted by the previously described design process are consistent with other studies on design parameter sensitivity.
- The trends in percent change for the rigid pavement design indicate that PCC pavements are not sensitive to small changes in subgrade strength or levels of traffic.

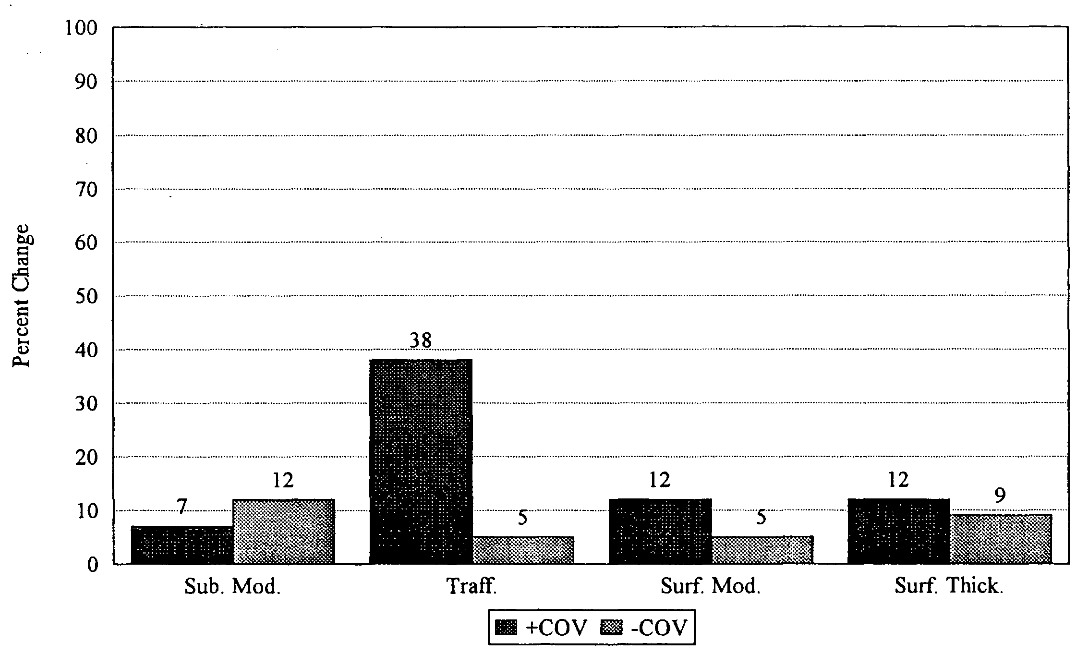

FIGURE 5 Percent change from mean input for PCC design, pavement site B.

CONCLUSIONS

The inclusion of reliability in pavement designs, or in any design of this type, is an important and required step to ensure safety and quality, while meeting economic considerations. Therefore, the design reliability must be quantifiable and based on proven mathematical concepts and statistics. Mechanistic/empirical pavement designs are tools with which sound reliability-based designs can be accomplished. One such approach that addresses many of the challenges facing pavement designers today has been presented. A rational attempt was made at applying reliability concepts consistently to designs of differing pavement types so that a fair and equitable judgment can be made between designs on a life-cycle cost basis.

Although many challenges have been addressed, there are still hurdles to overcome with these types of designs. One hurdle exists in the assignment of coefficients of variation to design inputs. Currently, there are limited data available for quantification of variations associated with construction activities, environment, traffic loadings, and materials testing. Therefore, previous experience, along with the available data, is used to assign coefficients of variation, which immediately introduces a possible bias when attempting to compare pavement designs.

A second challenge to overcome relates to the comparison between pavement types when designs have been completed. Every attempt was made to apply design concepts consistently between pavement types; however, no two pavement designs will be alike because of inherent differences in theory. For example, How is a designer to know whether the estimates of serviceability between two pavement types have equivalent variabilities? An even more complicated question arises: How is a designer to know whether the estimates of variability of rutting for the flexible pavement design are the same as estimates of variability of faulting and other distresses for the rigid pavement design? This question, which presents a very complex issue, currently has no answer.

Another bias along these same lines that can be introduced is in the assignment of failure criteria for condition measurements. Again, How is the design engineer supposed to know that the predicted performance between two pavement designs is equivalent? For example, if the flexible pavement design fails in rutting, which is specified at ½ in., and the rigid pavement design fails in fatigue cracking, which is specified at 20 percent failed slabs, how does the design engineer know that the predicted performance between the two pavement types is equivalent?

Even with these shortcomings, the mechanistic/empirical pavement designs introduced in this paper are a positive step toward improving today's pavement designs. With the help of personal computers and climatic effects models, mechanistic/empirical designs that incorporate consistent approaches to design reliability should produce suitable pavement systems for both pavement types, and reasonable judgments that show appropriate sensitivity to typical variations in pavement design parameters can be made.

REFERENCES

1. Ang, A. H. S., and W. H. Tang. Probability Concepts in Engineering Planning and Design. Vol. 1, *Basic Principles.* John Wiley & Sons, Inc., New York, 1975.
2. Harr, M. E. *Reliability Based Design in Civil Engineering.* McGraw-Hill, New York, 1987.
3. Zollinger, D. G., and R. L. Lytton. *Engineering Reliability in the Mechanistic/Empirical Design of Highway Pavements.* Unpublished report prepared for the National Coalition for Mechanistic Design. Nov. 1991.
4. Zollinger, D. G., and B. F. McCullough. Development of Reliability Factors and Analysis for the Calibration of Pavement Design Models Using Field Data. Presented at the Transportation Research Board 73rd Annual Meeting, Washington, D.C., 1993.
5. Uzan, J., D. G. Zollinger, and R. L. Lytton. *The Texas Flexible Pavement System (TFPS) Mechanistic/Empirical Model.* FHWA/Tx-91/455-1. Texas Transportation Institute, Research Report 2455-2, College Station, Nov. 1990.
6. Zollinger, D. G., and B. K. Killingsworth. *The Rigid Pavement Design System (RPDS) Mechanistic/Empirical Model.* Texas Transportation Institute, College Station, March 1992.
7. Miner, M. A. Cumulative Damage in Fatigue. *Transactions, American Society of Mechanical Engineers,* Vol. 67, 1945, pp. A159–A164.
8. Finn, F., C. Saraf, R. Kulkarni, K. Nair, W. Smith, and A. Abdullah. The Use of Distress Prediction Subsystems for the Design of Pavement Structures. *Proc., 4th International Conference on Structural Design of Asphalt Pavements,* Ann Arbor, Mich., Vol. 1, 1977, pp. 3–38.
9. Zollinger, D. G., and E. J. Barenberg. A Mechanistic Based Design Procedure for Jointed Concrete Pavements. Presented at 4th International Conference on Concrete Pavement Design and Rehabilitation, Purdue University, West Lafayette, Indiana, 1989, pp. 95–97.
10. *BISAR: Layered System Under Normal and Tangential Loads.* Shell-Koninilijke/Shell Laboritorium, Amsterdam, The Netherlands, 1972.
11. *A Computerized Two-Dimensional Finite Element Slab Analysis Model.* Illi-Slab, Update. University of Illinois, 1983.
12. Ioannides, A. M., and R. A. Salsilli-Murua. Temperature Curling in Rigid Pavements: An Application of Dimensional Analysis. In *Transportation Research Record 1127,* TRB, National Research Council, Washington, D.C., 1987, pp. 1–11.
13. Westergaard, H. M. Analysis of Stress in Concrete Pavement Due to Variations in Temperature. *Public Roads,* Vol. 8, No. 3, May 1927, pp. 54–60.
14. Darter, M. I. *Design of Zero-Maintenance Plan Jointed Concrete Pavement: Vol. I, Development of Design Procedures.* Technical report to FHWA. [University of Illinois, Champaign, June 1977].

Publication of this paper sponsored by Committee on Flexible Pavement Design.