Uniform and Variable Bandwidth Arterial Progression Schemes

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Compared with conventional uniform bandwidth progressions, variable bandwidth progression schemes offer considerable advantages for arterial traffic signal control. The variable schemes have a traffic-dependent capability that the conventional schemes lack, providing additional design flexibility and superior traffic performance. A simplified and efficient method for calculating variable-bandwidth progressions given optimized uniform bandwidth progressions is presented. Little's half-integer optimization algorithm is used first for uniform bandwidth maximization, coupled with a combinatorial phase-sequence optimization procedure. The method is then extended to calculate variable-bandwidth progressions using the multiband optimization criterion. A principal feature of this method is that it can be applied to any arterial synchronization scheme after the uniform bandwidth has been maximized.

Coordinating traffic signals on arterial streets is vital for transportation systems management. The principal objective of signal coordination is to promote the smooth and efficient flow of traffic throughout the network. Traffic signals tend to group traffic into platoons with more uniform headways than those that would otherwise occur. This platooning effect is more evident on major arterial streets, which have more signalized intersections. Under such circumstances, the uninterrupted movement of vehicle platoons through successive traffic signals can be obtained by synchronizing the signals according to the green bandwidth maximization criterion.

Models that maximize the green bandwidth have been developed by several researchers. The first computer model of the bandwidth maximization problem was developed by Little et al. (*1*). Their model is a search procedure that determines the offsets resulting in the largest two-directional bands at the given green progression speeds and cycle times. Subsequent models developed by Brooks (*2*), Bleyl (*3*), and Leuthardt (*4*) had a similar theoretical basis and similar computational results. All these models maximize the green bandwidth progression on arterial streets with two-phase signal settings. Messer et al. (*5*) developed the PASSER-II model, which is based on Little's and Brooks' algorithms and enhanced by a phase sequence optimization procedure. A mathematical programming formulation of the problem was introduced by Little et al. (*6*) in the MAXBAND model, in which bandwidth, cycle length, phase sequence, and progression speeds are optimized. This approach, later extended to network optimization by Chang et al. (*7*), is based on mixed-integer linear programming and requires a mathematical programming package.

A basic limitation of those early bandwidth maximization models is that the progression schemes that result are based on the total directional arterial traffic volume. Thus, signal settings are not sensitive to the actual traffic flows on the links of the arterial, which can vary significantly due to variations in turn-in and turn-out amounts of traffic at the different intersections of the arterial. Therefore, in a uniform bandwidth progression scheme, the green band may either be wasted at intersections with lower through moving traffic, or deprived from other intersections with higher through moving traffic. An attempt to remedy this problem was made by Tsay and Lin (*8*), who developed an "inverted funnel" progression scheme. However, the band could only grow wider along the arterial, but not be reduced, and hence could not be adequately tailored to variable flows. A more effective variable bandwidth progression scheme was developed by Gartner et al. (*9*) in the MULTIBAND model. MULTIBAND is an extension of the MAXBAND model, which calculates an individual bandwidth for each directional link of the arterial while maintaining main street platoon progression. The individual bandwidth depends on the actual traffic the link carries. By introducing a traffic-dependent capability, which the conventional schemes lack, the model provides additional design flexibility to the traffic engineer as well as improved traffic performance. Because MULTIBAND is based on mixed-integer linear programming, it requires a mathematical programming package similar to the MAXBAND model. Mathematical programming is a formidable optimization tool; however, it is a general purpose tool that can be applied to any mathematical model that has been cast in the required format. Therefore it is not particularly effective for solving the traffic signal synchronization problem per se. The optimization procedure that uses a branch-and-bound algorithm is cumbersome and may take a long time to reach an optimal solution; sometimes the calculation does not converge at all. Recently, attempts have been made to develop heuristic procedures that will speed up the solution process at the expense of achieving sub-optimal solutions (*10,11*). As a consequence, one of the principal strengths of the math programming methodology, that of obtaining globally optimal solutions, is being relinquished.

To remedy these limitations, a different approach was used for this study. Instead of using mathematical programming, special-purpose search procedures were developed that are specifically tailored to the arterial synchronization problem and therefore can solve it much more efficiently. Two new simplified models were developed, referred to as U-BAND and V-BAND, to calculate optimal uniform solution and variable bandwidth progression solution, respectively. U-BAND [stands for Uniform Band; i.e., bands of uniform width throughout both directions of the arterial (see Figures 4 and 5)] is based on Little's half-integer optimization algorithm, enhanced by a search procedure for phase-sequence optimization. V-BAND [stands for Variable Band; i.e., continuous bands of variable width along each direction of the arterial (see Figures 6 and 7)] is a further extension of the U-BAND model to calculate variable bandwidth progressions based on the different flow patterns experienced on the individual directional sections of the arterial. The result is a simple and efficient method that is sensitive enough to tai-

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lor the progression scheme to varying traffic conditions along the arterial street. The development and performance of these models are described in the next section.

UNIFORM BANDWIDTH: THE U-BAND MODEL

The model developed by Little, Martin, and Morgan (*1*) is used as the basic procedure for obtaining a uniform bandwidth. The model finds the optimal offsets that will produce the maximum bandwidth for the simple case of two-phase traffic signals (this restriction is later abandoned). The algorithm uses the half-integer synchronization procedure, in which the middle point of all the intersections' red times are synchronized. Cycle length, signal time splits, traveling speeds on the arterial links, and distances between the intersections are assumed to be known. The middle point of the red time at each intersection is placed in a position to maximize the equal bandwidth in both directions. This position depends mainly on the traveling speed on the links in both directions. If the speeds are different, the position of the middle point of the red time may be placed at any point between zero and the cycle length. For the typical case of equal speeds in both directions, maximum bandwidth will be obtained when the middle point of the red time at each intersection is placed either at the beginning or the middle of the cycle length (hence the term, half-integer synchronization). Thus, the traffic signals of the arterial are synchronized for maximum total bandwidth in both directions by selecting one of the two possibilities. This algorithm will calculate the optimal offsets for equal bandwidths in both directions. If the ratio of the inbound volume to the outbound volume is equal to one, the algorithm will simply give inbound bandwidth equal to outbound bandwidth and the calculated offsets for that bandwidth. If the ratio is different than one, then the total bandwidth is split in proportion to the directional arterial volumes, and the new corresponding offset are calculated.

The preceding algorithm was used as the basis for the U-BAND model, which is further extended to include green split calculations, phase sequence optimization for multiple-phase signalized intersections, arterial progression speed adjustment, and cycle time optimization.

Green Splits

The green splits at each intersection can be calculated as follows. From the input volumes and capacities for the different movements at each intersection, the volume-to-capacity ratios (v/c) for all the movements are calculated. The (v/c) ratio of the main street through inbound movement ($(v/c)_{\mathrm{OML}}$) are added. This value is compared with the value obtained by adding the ((v/c) ratio of the main street left outbound movement ($(v/c)_{\mathrm{OMT}}$) and left inbound movement ($(v/c)_{\mathrm{IML}}$). The maximum of the two values, $(v/c)_M$, is the value that will be used in the green split calculation:

$$(v/c)_M = \max\{[(v/c)_{\mathrm{IMT}} + (v/c)_{\mathrm{OML}}], [(v/c)_{\mathrm{OMT}} + (v/c)_{\mathrm{IML}}]\} \tag{1}$$

Similarly, for the cross street $(v/c)_c$ is obtained as the maximum of the (v/c) ratios of the cross street through outbound movement ($(v/c)_{OCT}$) added to the cross street left inbound movement ((v/c_{ICL}), and the cross street through inbound movement (($v/c)_{\mathrm{ICT}}$ added to the cross street left outbound movement ($(V/c)_{\mathrm{OCL}}$):

$$(v/c)_c = \max\{[(v/c)_{\mathrm{ICT}} + (v/c)_{\mathrm{OCL}}], [(v/c)_{\mathrm{OCT}} + (v/c)_{\mathrm{ICL}}]\} \tag{2}$$

The available cycle length is divided between the main street and cross street in proportion to the values of $(v/c)_M$, and $(v/c)_C$. The main street and cross street green times are subsequently divided into through movement and opposing left turning movement proportionally to their (v/c) ratios, and thus the green times for the different movements at each intersection are calculated. The red times for the main street are calculated based on the green times allocated to the cross street and the left turning movement in the opposing direction.

Multi-Phase Sequence Optimization

The four possible phase sequences considered in the model are shown in Figure 1: (*a*) out bound left leads, inbound left lags; (*b*) out bound left lags, inbound left leads; (*c*) out bound left leads, inbound left leads; (*d*) outbound left lags, inbound left lags.

The phase sequence selection is performed after an initial set of offsets and bandwidths has been calculated. This preliminary set is found by Little's half-integer synchronization procedure. The algorithm assumes that inbound red times are equal to outbound red times, which is not applicable in the case of multiple phase signals; therefore, it is used only as the means for establishing the initial settings. Subsequently, the offsets and the bandwidth in one of the two directions (i.e., the outbound direction) are kept constant, whereas offset and bandwidth in the other direction are allowed to vary as the different phase sequences are examined.

In the case of multiple phase sequences, the offsets in the inbound direction vary with respect to the outbound direction offsets, depending only on the green times of the left turning movements. For each phase sequence, the offset in the inbound direction can be calculated based on the outbound offset (Figure 1). Therefore, at each intersection there are four known possible offsets, corresponding to the four different phase sequences:

(*a*) For the first phase sequence, the inbound offset (θ_{IN}) is greater than the outbound offset (θ_{OUT}) by the amount of the left turn green time of the outbound direction (g_{OL}):

$$\theta_{IN} = \theta_{OUT} + R = \theta_{OUT} + g_{OL} \tag{3}$$

(*b*) For the second phase sequence, the inbound offset is less than the outbound offset by an amount equal to the left turning green time of the inbound direction (g_{IL}):

$$\theta_{IN} = \theta_{OUT} + R = \theta_{OUT} - g_{IL} \tag{4}$$

(*c*) For the third phase sequence, the inbound offset is different from the outbound offset by an amount equal to the difference between the green times of the outbound and the inbound directions:

$$\theta_{IN} = \theta_{OUT} + R = \theta_{OUT} + (g_{OL} - g_{IL}) \tag{5}$$

(*d*) For the fourth phase sequence, the inbound offset is equal to the outbound offset:

$$\theta_{IN} = \theta_{OUT} + R = \theta_{OUT} \tag{6}$$

Because there are four possible phase sequences at each intersection, there are 4^n possible combinations of phase sequences on an artery with n intersections. The optimal phase sequence combi-

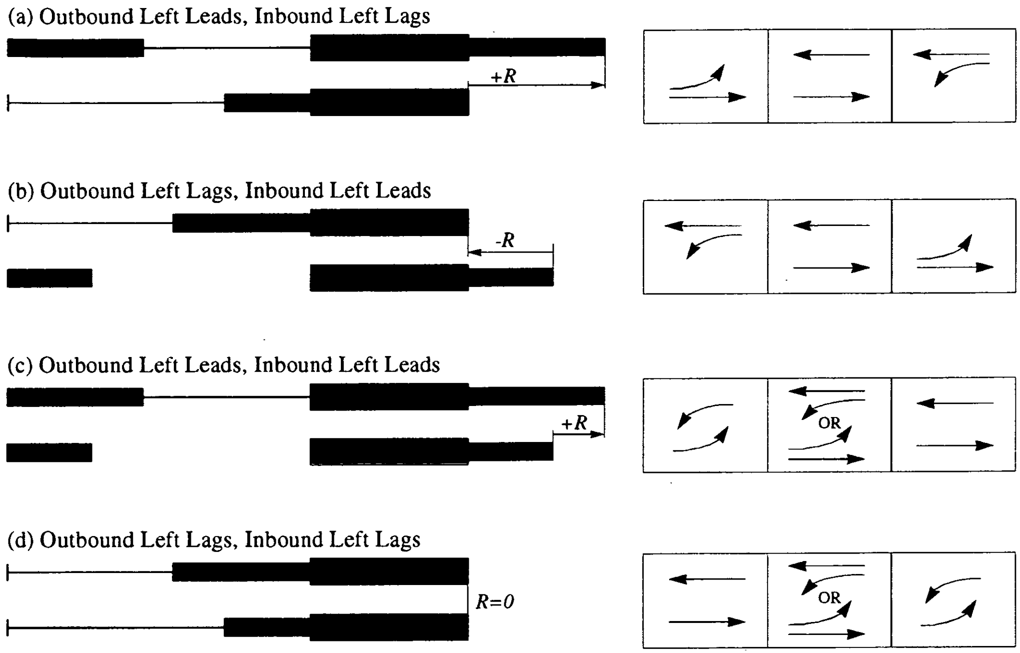

FIGURE 1 Inbound offsets relative to outbound offset for the four different phase sequences.

nation is selected so as to maximize the bandwidth in the inbound direction, through an exhaustive search procedure (Figure 2). Each time a particular phase sequence combination is selected, the inbound bandwidth is recalculated by the NO-OBSTRUCTION technique, described in the next paragraph. This bandwidth is compared with the largest bandwidth found so far from the combinations already examined. If the new bandwidth is larger than the previously largest bandwidth, it replaces the latter; otherwise it is discarded. This procedure is repeated until all phase sequence combinations have been examined.

Inbound-Outbound Bandwidth Calculation

For each phase sequence that is examined, the inbound bandwidth is recalculated with the NO-OBSTRUCTION procedure. Because the offsets and green times are set for each intersection, the procedure involves a simple subtraction of the obstructions to the band at each intersection (Figure 3). The band cannot be greater than the minimum green time, thus the procedure starts from the intersection with the least green time and proceeds in both directions. At this intersection, it is initially assumed that the inbound bandwidth is equal to the green time available for the inbound through movement. The edges of this band are projected to the adjacent intersections with time lags equal to the travel times between the intersections. Travel times between intersections are calculated from the given speeds and distances for each link. If the projected edge does not intersect the red time at the next intersection, the "obstruction" is zero; otherwise, the value of the "obstruction" is calculated as the difference between the point of intersection and the offset at that intersection. The obstruction is then subtracted from the bandwidth and a new bandwidth is calculated, which will be projected to the next intersection. This procedure is repeated until all the intersections are considered, resulting in the adjusted value of the inbound bandwidth.

An adjustment of the previously obtained outbound bandwidth is required to obtain the maximum band for this direction. This is done by adjusting the offsets to the left or right based on the interference values. Since the changes in the offsets should not reduce the inbound bandwidth, the offsets are shifted to the right or left only up to the minimum value of the interferences at that intersection.

Optimization of Cycle Length and Travel Speeds

In the U-BAND model, optimization procedure is repeated for different cycle time values within some specified range. The cycle length is increased by a given increment, the whole procedure is repeated, and the new total bandwidth is recalculated and compared with the best bandwidth from the cycle times examined so far.

Another enhancement to the model is that design speeds on the arterial are allowed to vary slightly. This is achieved by modifying all speeds on the links of the arterial by -1, 0, and $+1$ mph for each cycle time value and selecting the speeds that result in the largest total band.

VARIABLE BANDWIDTHS: THE V-BAND MODEL

The U-BAND model described in the previous section furnishes a uniform bandwidth for the entire arterial for each direction. Therefore, variations in the volumes along the arterial are not taken into

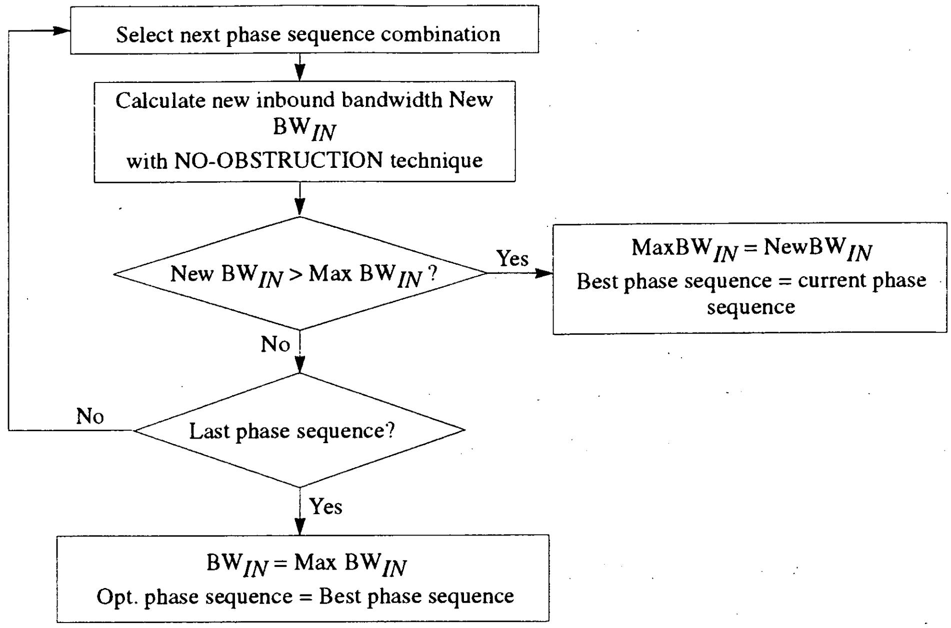

FIGURE 2 Logic for phase sequence optimization procedure.

consideration, which, if significant, may diminish the effectiveness of the bandwidth maximization approach. Because of turn-in and turn-out traffic, such variations in the directional volumes typically exist and must be considered in the model. This is accomplished in the V-BAND model, in which the offset at each intersection is adjusted with the hill-climb search technique to maximize the opportunity for traffic to cross this intersection using the directional green progressions in both directions of the arterial.

In the V-BAND model the total available bandwidth on each link is apportioned to the inbound and outbound directions by giving link-specific weights to the bands that depend on the directional volumes of the link. The link-specific weights that are considered are

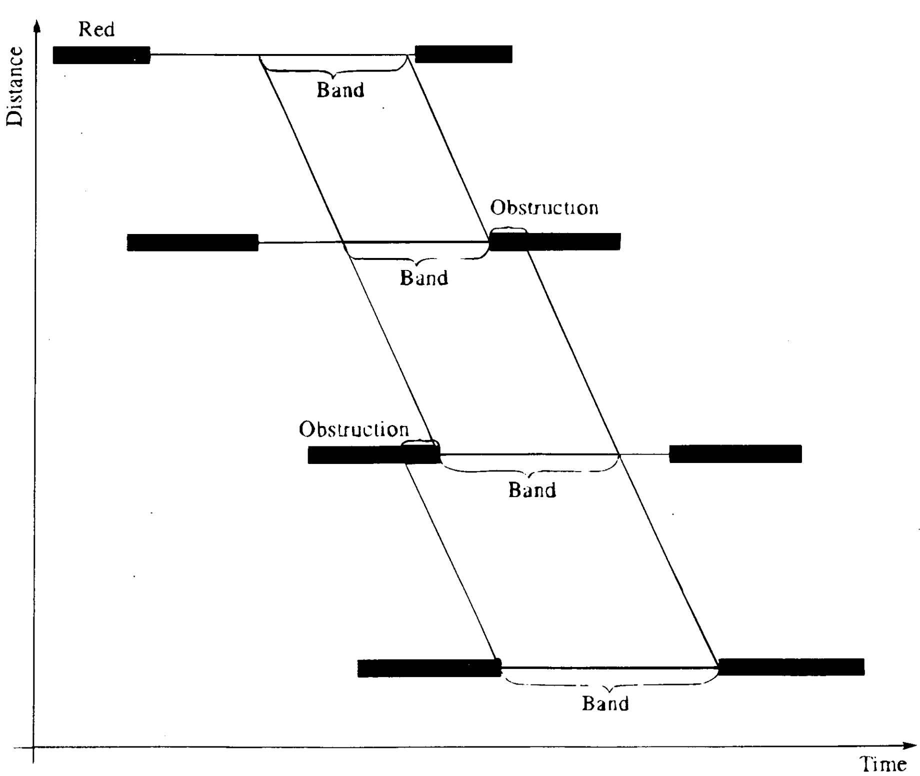

FIGURE 3 NO-OBSTRUCTION procedure for calculating inbound bandwidth.

the volume-to-saturation flow rate ratios of the link (v/s). Therefore, at each intersection the function that must be maximized takes the form:

$$\text{Maximize } Z = b_i(v/s)_i + b_j(v/s)_j + \bar{b}_i\,\overline{(v/s)}_i + \bar{b}_j\,\overline{(v/s)}_j \qquad (7)$$

where

b_l = outbound bandwidth of link l,
$\bar{b}_l$ = inbound bandwidth of link l,
i = upstream link,
j = downstream link,
$(v/s)_l$ = (v/s) ratio of the outbound direction of link l, and
$\overline{(v/s)}_l$ = (v/s) ratio of the inbound direction of link l.

Other coefficients also may be used in function Z; that is, on some occasions the (v/s) ratio raised to the fourth power, or exclusion of turning traffic from the (v/s) ratio, has given better results (*9*). The algorithmic approach taken to achieve the objective function described in Equation 7 is to expand or shrink the uniform bands obtained from the U-BAND model on each link symmetrically about the center line of the band. This is called the multiband optimization criterion (*9*).

For this purpose, at each intersection the interference values are calculated. For each link l the interferences of both adjacent intersections are considered, and the minimum value is taken. If this value is equal to 0, the band on link l remains the same; if the value is greater than 0, the band on link l will be increased by up to double the value of the minimum interference to accommodate the increase on both sides of the center line of the band.

At each intersection the offset is then adjusted and the effect of this adjustment on the objective function Z is examined. The initial adjustment depends on the new interference values. If the values of the left interference of the inbound band (w_{il}) or right interference of outbound band (w_{or}) are greater than 0, the offset is shifted to the right by 1 sec. This will increase the outbound bandwidth by 2 sec and reduce the inbound bandwidth by 2 sec. Similarly, if the values of the right interference of the inbound band (w_{ir}) or left interference of outbound band (w_{ol}) are equal to 0, then the offset is shifted to the right by 1 sec, which will increase the inbound bandwidth by 2 sec and reduce the outbound bandwidth by the same amount.

If the new value of Z is smaller than its previous value, the offsets are shifted in the opposite direction; otherwise the offset is shifted in the same direction by one additional second. This process continues until the objective function value decreases, at which point it stops. A constraint that must be satisfied each time an offset adjustment is performed is that the total shift of the offset cannot be greater than the minimum interference in the direction of increase in the bandwidth. This is because the reduction in bandwidth in one direction must be equal to the increase in bandwidth in the other direction. If the offset is moved beyond the minimum interference, the reduction in one direction will not result in increase in the bandwidth in the other direction.

COMPARISON AND EVALUATION

In this section the simplified models previously discussed are compared with their more rigorous brethren. The criterion used in the comparison is the value of the optimization objective. Afterward, simulation is used to evaluate the performance for realistic traffic conditions. To evaluate the effectiveness of the U-BAND model, it is compared with MAXBAND, which can achieve global optimum solutions thus establishing a reliable benchmark. Because the objective of both models is to obtain the maximum bandwidth for a given set of traffic and geometric conditions, the comparison is made in terms of the width of the green band. The test arterials considered in this evaluation include:

1. Canal Street, New Orleans, Louisiana: an arterial street with nine signalized intersections. All intersections have only two phase signal settings;
2. Main Street, Waltham, Massachusetts: an arterial street with nine multiple-phase signalized intersections; and
3. Massachusetts Avenue, Boston, Massachusetts: an arterial street with eight multiple-phase signalized intersections.

MAXBAND can optimize each of the link traveling speeds independently; cycle length is treated as a continuous variable. These features are not currently available in the U-BAND model. Therefore, the U-BAND model was run first, and the optimum traveling speeds and cycle length obtained from these runs were used to set the speeds and the cycle time in MAXBAND. A cycle time of 70 sec and a progression speed of 25 mph were used for all arterials in both models. For all data sets the U-BAND model gave optimal solutions in terms of bandwidth, which are almost identical to the ones obtained from MAXBAND. The phase sequences obtained from the two models for the two arterials with multiple-phase signal settings were not always identical, but it is known that an optimal solution for this type of problem is not unique, and there is a multiplicity of optimal points. The time-space diagrams for Canal and Main streets produced by U-BAND are shown in Figures 4 and 5, respectively.

To evaluate its performance, the V-BAND model was compared with U-BAND, MAXBAND, and MULTIBAND. MULTIBAND (*9*) is an extension of MAXBAND to give link-volume-dependent variable bands, symmetric about their center line. Therefore, like MAXBAND for uniform bandwidth solutions, it can serve as a dependable benchmark for the performance of the U-BAND model. The arterial data sets that were used for this experiment were the ones from Canal and Main streets. For the same reasons as in the previous experiment, the cycle time was set at 70 sec and the progression speeds on all the arterial links were set at 25 mph for all models. The signal settings obtained from the different models were simulated using NETSIM, a microscopic simulation program of traffic in a signalized network. Statistics obtained from NETSIM include average delay per vehicle, average number of stops, average stopped delay per vehicle, and average speed.

The simulation results for these performance measures are shown in Tables 1–4. Table 4 and Table 2 show the performance measures on the arterial streets without taking into consideration the effect of traffic on the side streets, and Table 3 and Table 1 show the same measures with traffic on the side streets. In both cases the advantages of variable bandwidth progression schemes are evident. The V-BAND and MULTIBAND models give better results than the U-BAND and MAXBAND models for both arterials in average delay per vehicle and average stopped delay per vehicle. For example, average delay is reduced by 10 and 11 percent by V-BAND and MULTIBAND, respectively, compared with the MAXBAND results for Canal Street, and average stopped delay is reduced by as much as 13 percent by both models for the same arterial street example.

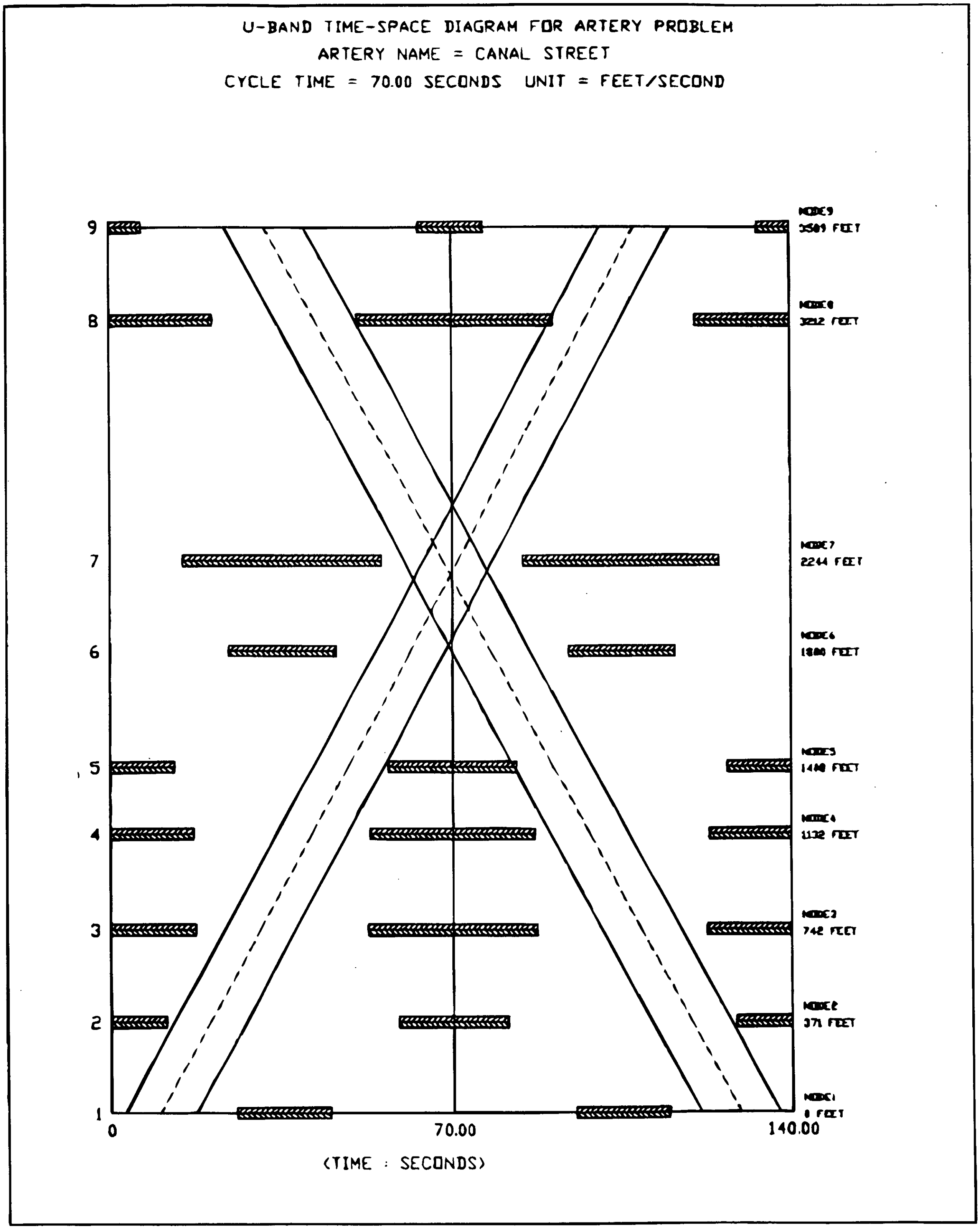

FIGURE 4 Time-space diagram for Canal Street; U-BAND model.

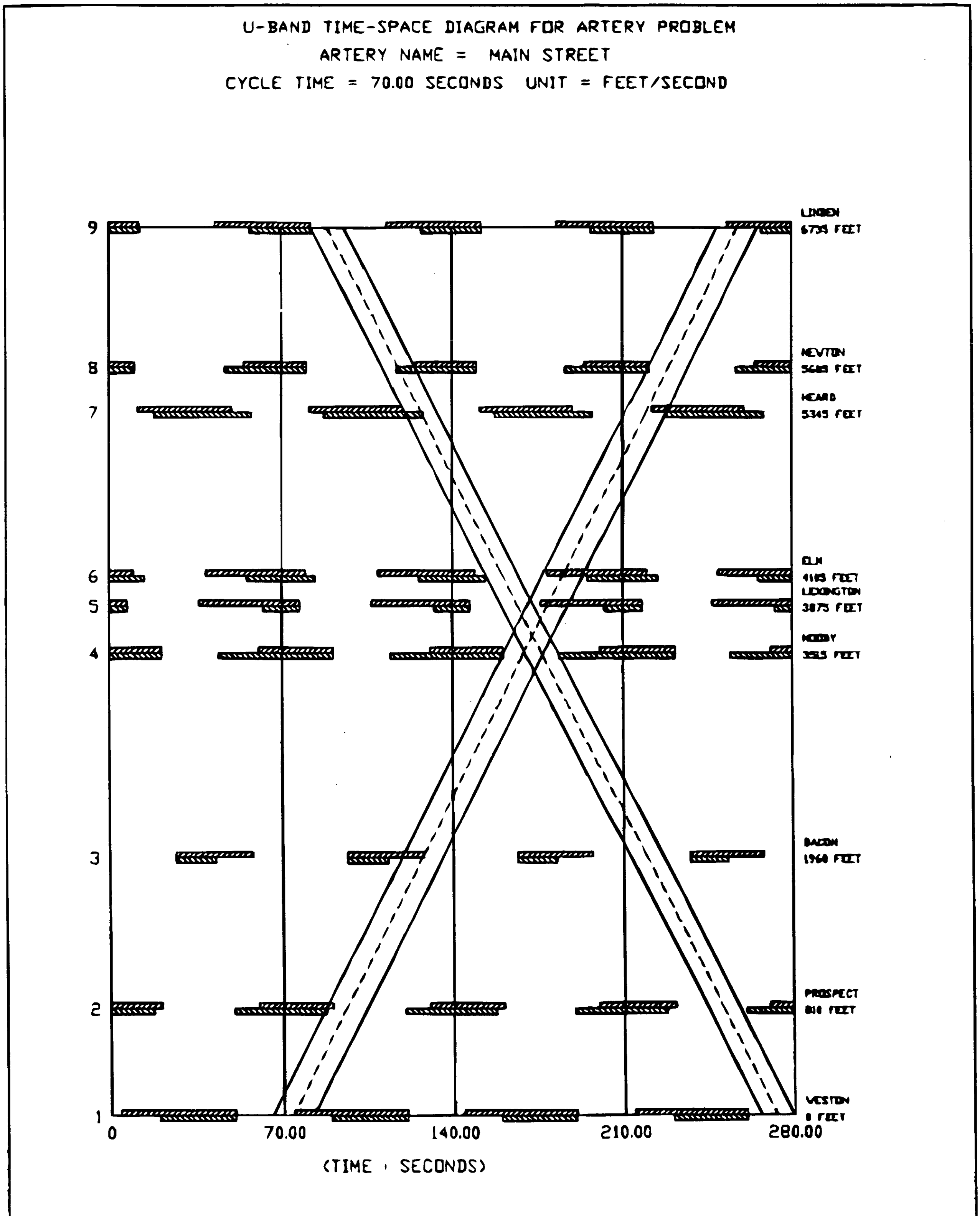

FIGURE 5 Time-space diagram for Main Street; U-BAND model.

TABLE 1 NETSIM Simulation Results: Main Street (With Side Streets)

	Avg. Delay (sec./veh.)	Avg. Stopped Delay (sec./veh.)	Avg. % of Stops	Avg. Speed (mph)
U-BAND	30.46	21.83	69.45	11.08
MAXBAND	29.42	20.85	69.2	11.29
V-BAND	27.75	19.16	68.97	11.22
MULTIBAND	27.3	18.72	67.10	11.38

TABLE 2 NETSIM Simulation Results: Main Street (Without Side Streets)

	Avg. Delay (sec./veh.)	Avg. Stopped Delay (sec./veh.)	Avg.% of Stops	Avg. Speed (mph)
U-BAND	33.09	24.08	69.09	10.58
MAXBAND	31.79	22.84	68.77	10.83
V-BAND	29.57	20.61	68.53	10.77
MULTIBAND	29.51	20.49	66.7	10.84

TABLE 3 NETSIM Simulation Results: Canal Street (With Side Streets)

	Avg. Delay (sec./veh.)	Avg. Stopped Delay (sec./veh.)	Avg.% of Stops	Avg. Speed (mph)
U-BAND	26.07	14.08	55.13	10.64
MAXBAND	23.28	14.53	57.91	10.34
V-BAND	21.23	12.85	55.84	10.51
MULTIBAND	20.89	12.75	55.29	10.63

TABLE 4 NETSIM Simulation Results: Canal Street (Without Side Streets)

	Avg. Delay (sec./veh.)	Avg. Stopped Delay (sec./veh.)	Avg.% of Stops	Avg. Speed (mph)
U-BAND	27.75	13.90	50.48	11.54
MAXBAND	23.82	14.57	54.08	11.11
V-BAND	20.80	12.16	51.66	11.39
MULTIBAND	20.58	12.21	50.51	11.48

The improvements in delay are even more pronounced when only the main street traffic is considered. For example, for Canal Street the average delay is improved by 14.5 and 16 percent by V-BAND and MULTIBAND, respectively, over the MAXBAND results, and average stopped delay is reduced by as much as 19 percent by both models. There are also some improvements in the average number of stops when the average traveling speed is approximately the same for all models.

The results also show that the new simplified V-BAND model performs in a way similar to the more sophisticated MULTIBAND model in terms of delays, number of stops, and average speed. Hence, it may be concluded that V-BAND obtains results that are virtually identical to MULTIBAND. The time-space diagrams for Canal and Main streets produced by the V-BAND model are shown in Figures 6 and 7, respectively.

CONCLUSIONS

A simplified and efficient method to calculate variable-bandwidth progressions given optimized uniform bandwidth progressions is presented. Little's half-integer optimization algorithm was used as the basic tool for the uniform bandwidth maximization, coupled with a combinatorial phase-sequence optimization procedure to develop the U-BAND model. The method was then extended to the V-BAND model to calculate variable-bandwidth progressions

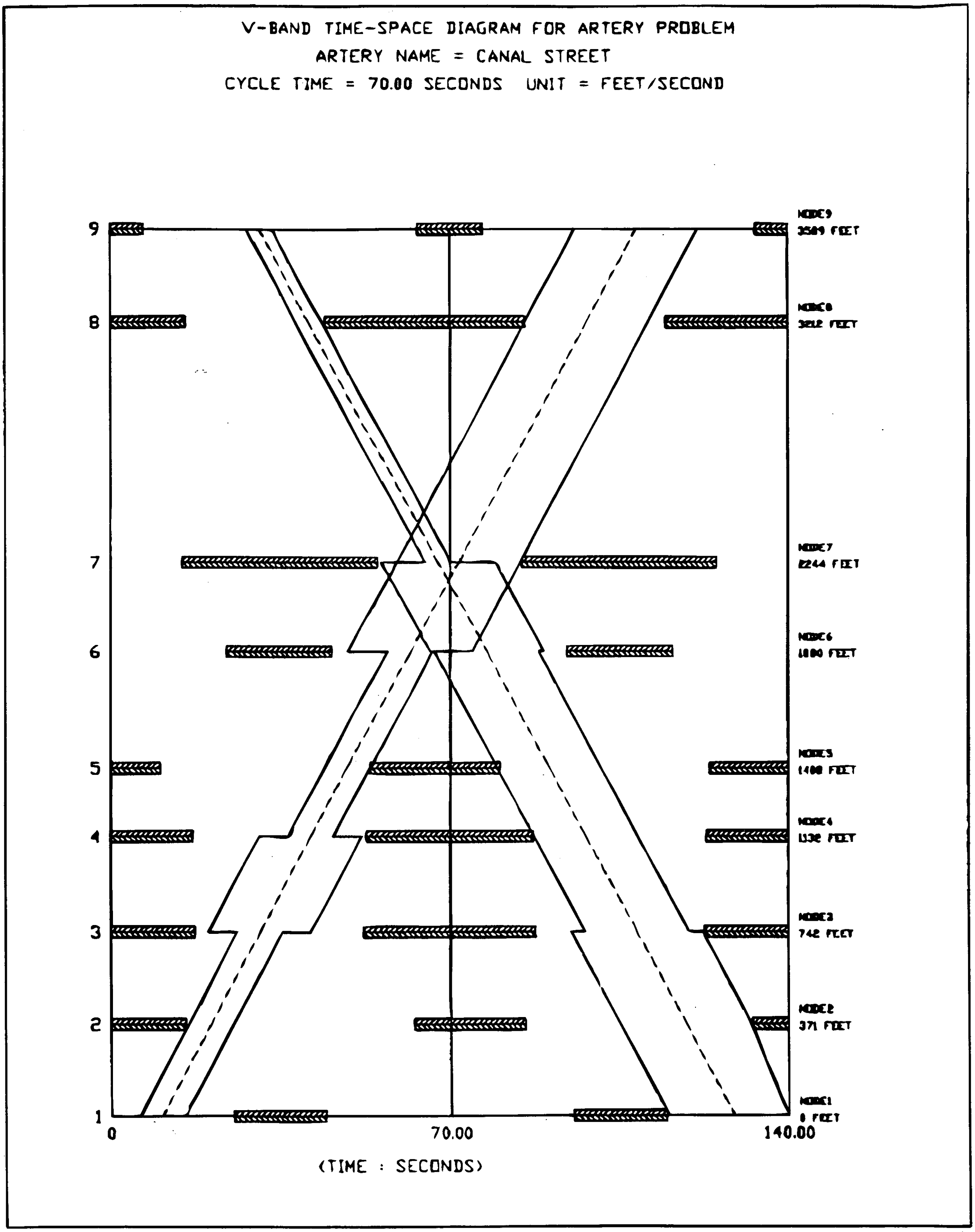

FIGURE 6 Time-space diagram for Canal Street; V-BAND model.

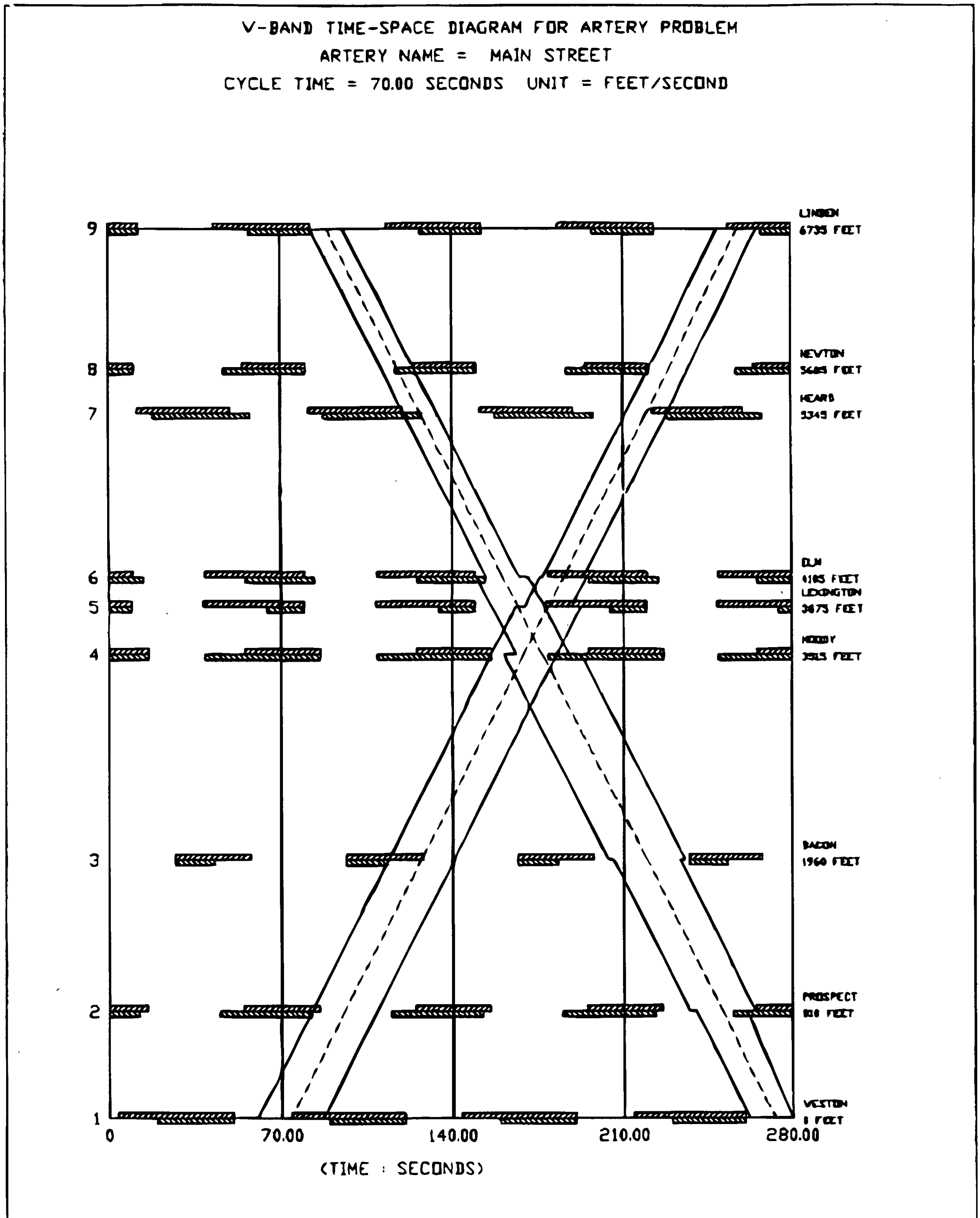

FIGURE 7 Time-space diagram for Main Street; V-BAND model.

using the multiband optimization criterion. An important feature of this method is that it can be applied to any arterial synchronization scheme after the uniform bandwidth progression has been optimized.

Several arterial examples were given to illustrate the effectiveness of the U-BAND and V-BAND models. The results from the V-BAND model were simulated with NETSIM and it was demonstrated that significant benefits can be obtained in traffic performance. An important aspect of this approach is that near-optimal bandwidth progressions can be obtained without sophisticated and cumbersome mathematical programming tools. Further research is under way to extend the approach described in this study to networks of arterials where it is likely to have comparable beneficial effects. Finally, this approach is expected to (*a*) provide advantages compared with established models such as PASSER-II and (*b*) compare favorably with recent versions of TRANSYT.

REFERENCES

1. Little, J. D. C., B. V. Martin, and J. T. Morgan. Synchronizing Traffic Signals For Maximal Bandwidth. *Highway Research Record 118,* 1967, pp. 21–47.
2. Brooks, W. D. Vehicular Traffic Control—Designing Arterial Progressions Using a Digital Computer IBM (undated).
3. Bleyl, R. L. A Practical Computer Program for Designing traffic Signal Timing Plans. *Highway Research Record 211,* 1967, pp. 19–33.
4. Leuthardt, H. R. Design of a Progressively Timed Signal System. *Traffic Engineering,* No. 45, 1974.
5. Messer, C. J., R. H. Whitson, C: L. Dudek, and E. J. Romano. A Variable-Sequence Multi Phase Progression Optimization Program. *Highway Research Record 445,* 1973, pp. 24–33.
6. Little, J. D. C., M. D. Kelson, and N. H. Gartner. MAXBAND: A Program for Setting Signals on Arteries and Triangular Networks. *Transportation Research Record 795,* 1981, pp. 40–46.
7. Chang, E. C-P, S. L. Cohen, C. Liu, N. A. Chaudhary, and C. Messer. MAXBAND-86: A Program for Optimizing Left-Turn Phase Sequence in Multiarterial Closed Networks. *Transportation Research Record 1181,* 1988, pp. 61–67.
8. Tsay, H. S., and L. T. Lin. New Algorithm for Solving the Maximum Progression Bandwidth. *Transportation Research Record 1194,* 1988, pp. 15–30.
9. Gartner, N. H., S. F. Assmann, F. Lasaga, and D. L. Hou. MULTIBAND—A Variable-Bandwidth Arterial Progression Scheme. *Transportation Research Record 1287,* 1990, pp. 212–222.
10. Chaudhary, N. A., A. Pinnoi, and C. J. Messer. Proposed Enhancements to MAXBAND-86 Program. *Transportation Research Record 1324,* 1991, pp. 98–104.
11. Solanki, R. S., and R. S. Pillai, and A. K. Rathi. *A Fast Heuristic for Maximizing Bandwidth in Traffic Network,* Oak Ridge National Lab Report, 1993.

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