Use of Fuzzy Relations To Manage Decisions in Preserving Civil Infrastructure

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The use of fuzzy relations to manage uncertain information in civil infrastructure preservation is addressed. Subjectivity and imprecision of uncertain information and ambiguity of terminological knowledge in preserving infrastructure facilities are the primary motivation for the employment of fuzzy sets and fuzzy relations. Formal decision processes use the concept of knowledge graphs that enable the establishment of general-specific or cause-effect relations between condition factors, as well as relations between condition and symptom factors. Fuzzy relations allow the characterization of the uncertainty associated with these relations. Fuzzy graphs assimilating the degree of certainty involved in decision processes are used to illustrate the connection strength of the elements in the associated fuzzy subsets. A case study in pavement preservation applied to the New York State Thruway is presented. This research makes contributions to synthesizing uncertain information involved in the decision processes of preserving civil infrastructure. In particular, the case study exemplifies the benefits of using fuzzy relations to identify feasible treatment options.

This study is concerned with the use of fuzzy relations to structure decision processes in preserving infrastructure facilities such as pavements and bridges. Feasible preservation methods aim at improving or strengthening infrastructure facilities that are in deficient condition. The complex behavior of infrastructure components renders the preservation decision into an environment of uncertainty. The uncertainty associated with the decisions is concerned mostly with personal preference and judgment, which involves graded or qualified statements that are not strictly true or false.

The qualified statements of engineering judgment are mostly expressed in the form of conditional relation between different quantities, e.g., the relation between climate and cracking of a structural component such as pavement. Frequently the relationship between two mutually dependent classes or variables is neither exact nor inexact. In other words, the linkage between objects in the two classes, or values taken by the two variables, varies gradually from a condition of weak to strong. Such situations can arise basically from two sources: (a) fuzziness in the definition of classes or variables, and (b) ambiguity of the conditional relations.

Fuzzy relations enable the characterization of the uncertainty involved in conditional statements. In civil engineering, Blockley (*1*) illustrated fuzzy relations in the uncertainty analysis of structural safety. Among other applications, this approach was exemplified in a fuzzy relation between compressive stress and longitudinal slenderness for a steel column. Brown and Yao (*2*) examined the fundamental theory of fuzzy sets and illustrated engineering decisions in estimating the strength of concrete using fuzzy conditional relations. Kikuchi and Perinchery (*3*) introduced the concept of fuzzy sets and fuzzy measure for representing two types of uncertainty: vagueness and ambiguity, in engineering planning problems.

In this paper the uses of fuzzy relations to structure decision processes concerned with condition diagnosis and treatment identification in preserving civil infrastructure are addressed. The concept of knowledge graphs (*4*) is used to identify the relations. The uncertainties involved in the relations are manipulated with fuzzy set theory and illustrated with knowledge graphs. A computational framework is formulated to calculate the strength of belief of the diagnosed conditions and identified treatments. A case study on the preservation of pavements is presented. The strengths and weaknesses of using fuzzy relations to structure decisions involving uncertainties are discussed.

INFORMATION AND UNCERTAINTY

Several types of uncertain information, defined by Klir and Folger (*5*) as the amount of uncertainty associated with the system, are present in infrastructure preservation decisions, each of which occurs under its own distinct conditions. The most significant types of uncertainties in the present study are subjectiveness, imprecision, and statistical uncertainty. Information about condition assessment of infrastructure facilities is generally presented in linguistic form, which has meaning that is inherently vague or subjective. This vagueness reflects the uncertainty represented and manipulated with fuzzy sets. The second type of uncertain information involved in preservation is a measurement or test with an instrument, or the uncertainty of imprecision. Imprecision is represented and calculated using fuzzy set theory. Also, statistical uncertainty is involved in quantitative information such as traffic volume, climate, and others. Quantitative information is represented with probability density functions that address statistical uncertainty. In this study the focus is on the uncertainties that are represented with fuzzy sets and manipulated with fuzzy calculus (*6*).

The uncertainty concerned with the reliability of descriptive information, defined by Klir and Folger (*5*) as the shortest description of the system in some standard language, is a result of ill-defined concepts (fuzziness) involved in the problem domain. Within the category of descriptive information, uncertainty (ambiguity) may occur as a result of weak implication, when an engineer is unable to establish a strong correlation between premise and conclusion of conditional statements in preservation decisions.

PRESERVATION DECISIONS

The decision-making process for preserving civil infrastructure generally consists of problem identification, determination of potential solutions, and selection of the preferred solution. In prob-

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lem identification, existing infrastructure conditions are identified through data collection, data evaluation, and project constraints. On the basis of this information, feasible rehabilitation methods are analyzed and recommended. The preferred solution is selected by analyzing costs and by considering project constraints and non-monetary factors. Shen and Grivas (*4*) proposed a decision framework for pavement preservation that consists of symptom observation, condition diagnosis, and treatment identification.

For infrastructure in general, symptom observation is a process of gathering data and facts required to identify existing infrastructure condition. Condition diagnosis includes the knowledge required to evaluate infrastructure condition. This part of the decision task represents actual diagnostic processes for detecting the causes of deterioration of an infrastructure component. Treatment identification recommends several potential methods that are feasible to remedy infrastructure deterioration. Condition diagnosis and treatment identification are ill-structured decision problems. There are no definitive procedures for the evaluation of infrastructure condition and the identification of treatment options

Using the concept of knowledge graphs (*4*) to formalize the decision problems allows clear identification of the relation of contexts and the structure of knowledge.

KNOWLEDGE GRAPHS

A knowledge graph is a graphical representation of a decision process that attempts to mimic the knowledge of domain experts. Figure 1 shows an example of a knowledge graph for condition diagnosis. The structure of knowledge graphs is established by formalizing the decision processes with a three-step procedure: (a) problem decomposition, (b) term interpretation, and (c) heuristics organization. Formalization of the decision process was presented by Shen and Grivas (*4*) in a study of pavement preservation.

The sample knowledge graph indicates that the terms used by maintenance engineers have different interpretations in describing structural conditions. For example, the problem of insufficient support (Node 3) is recognized by most engineers. Some emphasize that the cause of insufficient support is overloaded traffic (Node 8),

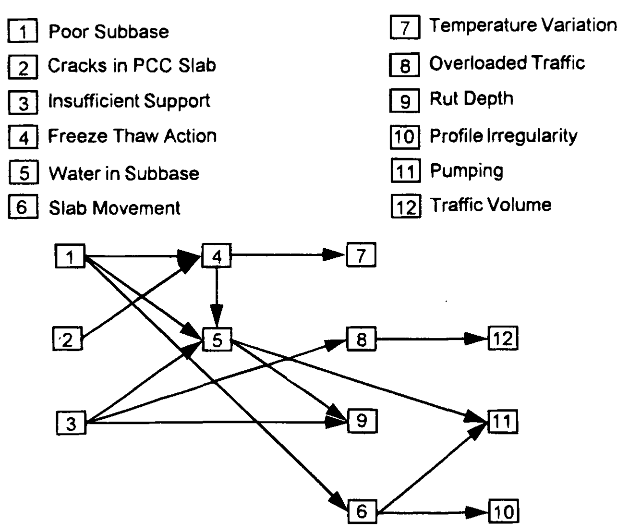

FIGURE 1 A sample of knowledge graph.

while others consider that the problem is evidenced by rut depth (Node 9), which, in turn, indicates the problem of water in the subbase (Node 5).

Understanding the precise meaning of each term was used to establish the knowledge graphs. Some terms are abstract and are defined informally and implicitly by the Thruway engineers because a standard vocabulary has not been designated. The meanings of such terms were derived mostly from engineers' experience in the maintenance actions. Therefore, the terms that experts use are generally vague because the degree of certainty (truth) of the term varies from case to case. The facts, relations, and rules of thumb (conditional statement) contained within the knowledge graphs usually manifest varying degrees of uncertainty. These uncertainties indicate either vagueness of a concept or ambiguity of a relation or both. The use of exact satisfaction of the premise of a conditional statement seems unnatural in the context of infrastructure preservation. Therefore, fuzzy relations from the derived knowledge graphs more realistically represent the decision process and, as well, more clearly account for the uncertainties involved.

FUZZY RELATIONS

The relations established in the knowledge graphs for condition diagnosis and treatment identification can be expressed as a conditional statement: If *A*, then *B*; *A* and *B* are fuzzy predicates represented by membership functions rather than the propositional variables defined in the classical propositional calculus. In essence, the conditional statement describes a fuzzy relation (*7*) between two fuzzy variables. In this study, fuzzy sets are used to account for the uncertainty (vagueness) of linguistic terms, while fuzzy relations help to clarify the confusion (ambiguity) in interpreting the terms.

Membership Matrix

Fuzzy sets help establish inexact relationships between different quantities or classes of objects presented in a membership matrix. (See Table 1.) Two classes of objects, e.g., temperature variation and slab cracking, form a cause-effect relation that describes the condition of infrastructure facilities, and can be denoted by the Cartesian product of two fuzzy sets, *X* and *Y*. In *X*, the elements of fuzzy sets are various degrees of temperature changes in a day that may affect deterioration (cracking) of slabs. In Y, the fuzzy elements are a six-level severity of cracking.

The Cartesian product of *X* and *Y*, $X \times Y$, forms a fuzzy relation, *R*, that constitutes a new universe with the ordered pairs as its elements, characterized by a membership function $\mu_R(x, y)$. A typical operation for the Cartesian product is represented as

$$\mu_R(x, y) = \mu_{X \times Y}(x, y) = \min [\mu_X(x), \mu_Y(y)] \quad (1)$$

A fuzzy relation is a subset of the Cartesian product, X × Y. An example of a fuzzy relation is established from the conditional statement: If temperature variation is high then slab cracking is moderate general. The linguistic value "high" is represented by fuzzy subset *A* and "moderate general" is *B* (*8*), as follows:

$$A = \frac{0.1}{x_1} + \frac{0.4}{x_2} + \frac{0.7}{x_3} + \frac{0.9}{x_4} + \frac{1}{x_5} \quad (2)$$

TABLE 1 Typical Two-Dimensional Membership Matrix of $X \times Y$.

X = Temperature Variation (°F)	Y = Slab Cracking					
	y_1 (Tight Cracks)	y_2 (Tight Cracks in 3 or more slabs)	y_3 (Open Cracks)	y_4 (Open Cracks in 3 or more slabs)	y_5 (Spalled Cracks)	y_6 (Spalled Cracks in 3 or more slabs)
$x_1 = 20$	$\mu_R(x_1,y_1)$	$\mu_R(x_1,y_2)$	$\mu_R(x_1,y_3)$	$\mu_R(x_1,y_4)$	$\mu_R(x_1,y_5)$	$\mu_R(x_1,y_6)$
$x_2 = 30$	$\mu_R(x_2,y_1)$	$\mu_R(x_2,y_2)$	$\mu_R(x_2,y_3)$	$\mu_R(x_2,y_4)$	$\mu_R(x_2,y_5)$	$\mu_R(x_2,y_6)$
$x_3 = 40$	$\mu_R(x_3,y_1)$	$\mu_R(x_3,y_2)$	$\mu_R(x_3,y_3)$	$\mu_R(x_3,y_4)$	$\mu_R(x_3,y_5)$	$\mu_R(x_3,y_6)$
$x_4 = 50$	$\mu_R(x_4,y_1)$	$\mu_R(x_4,y_2)$	$\mu_R(x_4,y_3)$	$\mu_R(x_4,y_4)$	$\mu_R(x_4,y_5)$	$\mu_R(x_4,y_6)$
$x_5 = 60$	$\mu_R(x_5,y_1)$	$\mu_R(x_5,y_2)$	$\mu_R(x_5,y_3)$	$\mu_R(x_5,y_4)$	$\mu_R(x_5,y_5)$	$\mu_R(x_5,y_6)$

$$B = \frac{0.2}{y_1} + \frac{0.5}{y_2} + \frac{0.9}{y_3} + \frac{1}{y_4} + \frac{0.6}{y_5} + \frac{0.1}{y_6} \quad (3)$$

Specifically, A is a fuzzy subset of the universe of discourse, X, and B is a fuzzy subset of Y. The fuzzy relation $R(A, B)$ is characterized by a membership function $\mu_R(x, y)$ and is expressed:

$$R = A \times B = \{\mu_R(x, y)/(x, y) \mid x \in A \text{ and } y \in B \quad (4)$$

This expression is a special fuzzy relation of $X \times Y$, and the relation combines all $x \in A$ and $y \in B$ in the form of ordered pairs represented as a membership matrix:

$$A \times B = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.4 & 0.4 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.7 & 0.7 & 0.6 & 0.1 \\ 0.2 & 0.5 & 0.9 & 0.9 & 0.6 & 0.1 \\ 0.2 & 0.5 & 0.9 & 1 & 0.6 & 0.1 \end{bmatrix} \quad (5)$$

The membership matrix represented for a fuzzy relation can be derived from the maintenance engineers about their strength of confidence on related fuzzy elements. In the case of preserving civil infrastructure, interviews with engineering experts would be necessary to identify membership values. Typical questionnaires for the interview are:

1. Does symptom i of concrete pavements *always* indicate a problem of condition j?
2. Is symptom p of a steel-girder bridge *often* caused by the condition q?
3. Is condition r of an overlaid pavement very likely a specific case of condition s?

The relations between condition factors as well as the relations between symptoms and conditions presented in the questionnaire are associated with implicit uncertainty. A membership grade [0,1] is assigned for the linguistic terms, always, often, may-not likely, very likely, and others. For example, the connection strength between symptom i and condition j is 0.9, which represents the uncertainty value "always." Furthermore, concentration and dilation operation are modeled with the linguistic modifiers: $\mu_{\text{very } A}(x) = \mu_A^{1/2}(x)$ and $\mu_{\text{may-not } A}(x) = \mu_A^2(x)$, respectively. The created membership matrix for a fuzzy relation can also be interpreted using a fuzzy graph (*9*).

Fuzzy Graph

Elements of the matrix with nonzero membership grades are represented in the diagram by lines connecting the respective nodes. Figure 2 presents the fuzzy graph of the conditional relation "If temperature variation is high then slab cracking is moderate general." The nodes of the fuzzy graph are considered as the elements of the

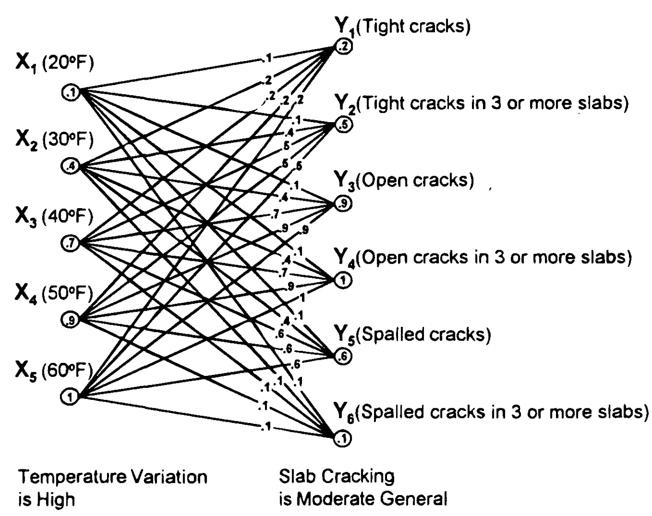

FIGURE 2 The fuzzy graph of the conditional relation "If temperature variation high *Then* cracking is moderate general."

fuzzy relation. These nodes are labeled with the membership grade of related fuzzy subsets. For a fuzzy relation, the connection strength of a path is defined as the minimum strength of related elements. As shown in Figure 2, temperature variation of 60°F (15.5°C) has the greatest influence (membership grades are 0.9 and 1.0) to open cracks on slabs. However, the 60°F (15.5°C) temperature variation does not cause a major problem to spalled cracks, where the membership grades of the relations are 0.6 and 0.1. In other words, spalled cracks are a phenomenon that temperature changes would not strongly affect.

Composition

Two fuzzy relations can be composed to a new relation. For example, a new relation can be established from a composition between symptom-condition relation and condition-treatment relation. A typical example of composition can be established from the following two conditional relations:

1. If temperature variation is high (A) then slab cracking is moderate general (B).
2. If slab cracking is moderate general (B) then seal the cracks (C).

In C, the fuzzy elements are the four generic types of treatments: preventive maintenance, minor rehabilitation, major rehabilitation, and reconstruction. The membership function of seal the cracks in this condition relation is defined as

$$C = \frac{0.8}{z_1} + \frac{1}{z_2} + \frac{0.5}{z_3} + \frac{0.1}{z_4} \tag{6}$$

Suppose that R is a relation on $A \times B$ (A and B are represented in Equation 2 and 3, respectively) and S is a relation on $B \times C$ (B and C are represented in Equation 3 and 6, respectively). One might want to know the fuzzy relation from A to C. An operator can be defined to establish the relation between A and C via B. This study follows max-min composition, denoted by $\circ$, that is:

$$R \circ S = \bigvee_{y \in Y} (\mu_R(x,y) \wedge \mu_s(y,z)). \tag{7}$$

The composition is formulated as follows:

$$R \circ S = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.4 & 0.4 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.7 & 0.7 & 0.6 & 0.1 \\ 0.2 & 0.5 & 0.9 & 0.9 & 0.6 & 0.1 \\ 0.2 & 0.5 & 0.9 & 1 & 0.6 & 0.1 \end{bmatrix} \circ \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.5 & 0.5 & 0.1 \\ 0.8 & 0.9 & 0.5 & 0.1 \\ 0.8 & 1 & 0.5 & 0.1 \\ 0.6 & 0.6 & 0.5 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \\ 0.4 & 0.4 & 0.4 & 0.1 \\ 0.7 & 0.7 & 0.5 & 0.1 \\ 0.8 & 0.9 & 0.5 & 0.1 \\ 0.8 & 1 & 0.5 & 0.1 \end{bmatrix}$$

Figure 3 shows the fuzzy graph of the composition. The connection strength between x_5 and z_2 via y_4 is the strongest among all the paths between them. Minor rehabilitation is recommended, which would require more work than just filling the sealer in the cracks. On the other hand, if temperature variation is at 50°F (10.0°C) and the cracks appear to be open on three or more slabs, then apply the treatment, and seal the cracks, as preventive maintenance.

Fuzzy Mapping

Mapping is a fuzzy transformation when uncertainties are involved in a system. When information is passed through a fuzzy system, an extra fuzziness will be added because of the fuzziness of the system

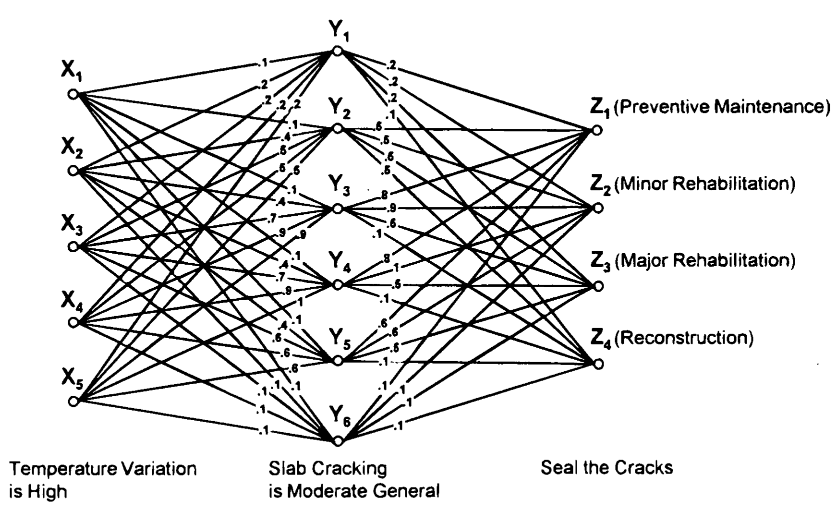

FIGURE 3 The fuzzy graph of the composition between temperature variation high and seal the cracks.

itself. Even if the input is crisp, the output will be fuzzy; if the input is fuzzy, the output will be fuzzier.

For a fuzzy transformation, a relation R between two fuzzy sets, X and Y is expressed by the membership function, $\mu_R(x,y)$. The image of A on X under this transformation, using a matrix expression, is given by

$$B = A \circ R \tag{9}$$

with the membership function

$$\mu_B(y) = \bigvee_{x \in X} (\mu_A(x) \wedge \mu_R(x, y)) \tag{10}$$

This transformation enables identification of treatments from the observed symptom once the fuzzy relations between symptoms and treatments are established.

CASE STUDY

The area covered in this study is a section of the New York State Thruway that was originally constructed in 1955. The 9-in. (22.5-cm) reinforced cement concrete pavement section was constructed over a 12-in. (30-cm) granular subbase. Later there were a 2½-in. (6.5-cm) asphalt concrete overlay, a 1-in. (2.5-cm) asphalt concrete overlay, and a 3-in. (7.5-cm) asphalt concrete overlay until 1993. The occurrence of cracking, rutting, spalling, and other conditions has affected the load-carrying of the pavement structure. This case study places emphasis on the identification of treatment options based on rut depth data collected from a roughness survey and engineering judgment about the preservation of pavement structures.

The decision process of treatment identification follows the symptom and/or data collected from the field. In accordance with the established knowledge graph (as shown in Figure 1), rut depth (Node 9) may be the effect of water in subbase (Node 5), which, in turn, indicates poor subbase (Node 1), or rut depth may be an indication of insufficient support (Node 3) of the pavement structure to carry overloaded traffic (Node 8). However, the rut depth may also be simply treated without rectifying any structural problem of the pavement.

In this study fuzzy relations are used to pursue three different decision processes: direct symptom-treatment relation explored in Method A, symptom-condition-treatment relation in Method B, and symptom-condition-condition-treatment relation in Method C. A fuzzy relation between symptom and treatment will be composed from all the parameters involved in each decision process. Fuzzy ordering enables a comparison of the strength of belief among the treatment options derived from each method. Thus, the three methods aim at establishing fuzzy relations for the conditional statement: If rut depth is large general then what kind of treatment is considered to be the most appropriate one.

Method A

The first method applies a direct symptom-treatment relation that enables identifying the strength of belief to the treatment, milling wheel rut, based on the severity of rut depth. In X (the symptom) the elements of fuzzy sets are defined as four levels of rut depth, and the five elements of fuzzy sets for Z (the treatment) are "do nothing" and four generic treatments.

X = 5 Rut Depth	*Z = Treatments*
$x_1 < 1\text{cm}$	z_1 = Do Nothing
$2\text{cm} > x_2 \geq 1\text{cm}$	z_2 = Preventive Maintenance
$3\text{cm} > x_3 \geq 2\text{cm}$	z_3 = Minor Rehabilitation
$x_4 \geq 3\text{cm}$	z_4 = Major Rehabilitation
	z_5 = Reconstruction

The membership matrix of the relation between rut depth and treatments is established from interviewing experts about their judgments on determining potential treatments for a certain range of rut depth, and represented as

$$R = X \times Z = \begin{bmatrix} 0.8 & 0.6 & 0.2 & 0 & 0 \\ 0.4 & 0.8 & 0.3 & 0 & 0 \\ 0.2 & 1 & 0.6 & 0.1 & 0 \\ 0.1 & 0.7 & 1 & 0.5 & 0.2 \end{bmatrix}$$

A fuzzy relation is applied to the conditional statement: If rut depth is large general (A), then treatment is mill wheel ruts (C) to derive the fuzzy subset of the treatment. In the premise of the conditional statement, linguistic value "large general" is represented by fuzzy subset A as

$$A = \frac{0.1}{x_1} + \frac{0.5}{x_2} + \frac{1}{x_3} + \frac{0.8}{x_4} \tag{12}$$

The membership function (C) is obtained from mapping the "rut depth is large general" to M_A, $C = A \circ M_A$:

$$C = \frac{0.4}{z_1} + \frac{1}{z_2} + \frac{0.8}{z_3} + \frac{0.5}{z_4} + \frac{0.2}{z_5} \tag{13}$$

The membership grade for the treatment, mill wheel ruts (C), is expressed in terms of the generic treatment, Z_1. The operation of mapping the symptom to treatment is illustrated using the fuzzy graph shown in Figure 4. Membership values of the fuzzy relation are marked on the lines connecting the symptom and the treatment. Among them, the connection strength between x_3 and z_2 is the highest.

Method B

The decision path of the second method, according to the knowledge graph in Figure 1, is from symptom rut depth to a structural

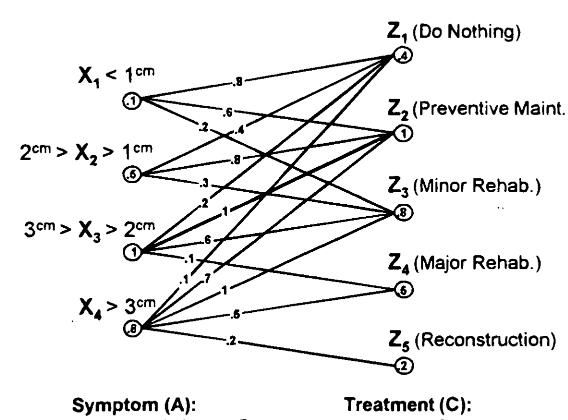

FIGURE 4 A fuzzy graph for Method A.

condition, insufficient support. The latter can be rectified by extended maintenance overlay (*4*). Thus, the conditional statements for the second method include:

1. If rut depth is large general (A), then insufficient support (B).
2. If insufficient support (B), then extended maintenance overlay (C).

The level of severity of insufficient support can be estimated from a falling weight deflectometer test. The deflection of pavement structure obtained from the test may be classified into none, small, moderate, and large, which become the elements of the fuzzy sets of defining the structural condition.

X = Rut Depth	*Y = Insufficient Support*	*Z = Treatments*
$x_1 < 1$cm	y_1 = None	z_1 = Do Nothing
2cm $> x_2 \geq 1$cm	y_2 = Small	z_2 = Preventive Maintenance
3cm $> x_3 \geq 2$cm	y_3 = Moderate	z_3 = Minor Rehabilitation
$x_4 \geq 3$cm	y_4 = Large	z_4 = Major Rehabilitation
		z_5 = Reconstruction

The fuzzy relations of symptom (*X*) and condition (*Y*), *R*, are established from interviewing maintenance engineers by identifying their confidence on a severity of insufficient support that causes a range of rut depth; and the fuzzy relations of condition (*Y*) and treatment (*Z*), *S*, are the confidence on the type of treatment for a severity of insufficient support. The relations, *R* and *S*, are represented as following:

$$R = \begin{bmatrix} 0.5 & 0.8 & 0.2 & 0 \\ 0.3 & 0.7 & 0.5 & 0.2 \\ 0.1 & 0.4 & 0.9 & 0.5 \\ 0.0 & 0.3 & 1 & 0.8 \end{bmatrix}$$

The composition of *R* and *S* forms a relation between symptom and treatment for the Method B.

$$S = \begin{bmatrix} 1 & 0.6 & 0.2 & 0 \\ 0.8 & 0.8 & 0.3 & 0.1 \\ 0.7 & 0.9 & 0.8 & 0.4 \\ 0.4 & 0.7 & 1 & 0.8 \end{bmatrix}$$

A fuzzy set representation of structural condition is obtained by mapping the membership function of "rut depth is large general" to *R*. Similarly, a fuzzy set representation of treatment (C) is a mapping to M_B. For the case of rut depth is large general a fuzzy subset describing the degree of truth of insufficient support (B) is

$$B = \frac{0.3}{y_1} + \frac{0.5}{y_2} + \frac{0.9}{y_3} + \frac{0.8}{y_4} \tag{17}$$

In addition, the extended maintenance overlay is given a fuzzy set representation as

$$C = \frac{0.7}{z_1} + \frac{0.9}{z_2} + \frac{0.9}{z_3} + \frac{0.8}{z_4} + \frac{0.8}{z_5} \tag{18}$$

Figure 5 presents a fuzzy graph showing the mapping from the symptom, rut depth, to the condition, insufficient support, and from the condition to the treatment. The highest connection strength among the decision paths is $X_3 - Y_3 - Z_2$ or $X_3 - Y_3 - Z_3$. This method indicates a lower certainty value on the suggested treatment, extended maintenance overlay. In addition, it shows greater uncertainty on selecting a type of treatment option.

Method C

The decision process follows a different path of the knowledge graph in Figure 1. The conditional statements for the third method include:

1. If rut depth is large general (A), then water in subbase (B^1),
2. If water in subbase (B^1), then poor subbase (B^2),
3. If poor subbase (B^2), then maintenance overlay (C).

The level of severity of water in subbase is classified into none, low, medium, and high, four levels. The certainty values for the fuzzy predicate poor subbase are defined as: (a) impossible, (b) very_low_chance, (c) it_may, (d) most_likely, and (e) certain. The elements of the fuzzy sets of each parameter used in Method C are given as follows:

X = Ruth Depth	*Y^1 = Water in Subbase*	*Y^2 = Poor Subbase*	*Z = Treatments*
$x_1 < 1$ cm	y_1 = None	y_1 = Impossible	Z_1 = Do Nothing
2cm $> x_2 \geq 1$cm	y_2 = Low	y_2 = Very_low _chance	z_2 = Preventive Maintenance
3cm $> x_3 \geq 2$cm	y_3 = Medium	y_3 = It_may	z_3 = Minor Rehabilitation
$x_4 \geq 3$cm	y_4 = High	y_4 = Most_likely	Z_4 = Major Rehabilitation
		y_5 = Certain	z_5 = Reconstruction

Applying the same interviewing procedures, fuzzy relations of X and Y^1, Y^1 and Y^2, and Y^2, and Z are shown in the following matrices, R, S^1, and S^2, respectively:

$$R = \begin{bmatrix} 0.9 & 0.3 & 0.2 & 0 \\ 0.8 & 0.4 & 0.2 & 0 \\ 0.7 & 1 & 0.6 & 0.2 \\ 0.4 & 0.8 & 0.9 & 0.5 \end{bmatrix} \tag{19}$$

$$S^1 = \begin{bmatrix} 1 & 0.9 & 0.2 & 0 \\ 0.9 & 1 & 0.3 & 0.1 \\ 0.8 & 0.9 & 0.4 & 0.2 \\ 0.6 & 0.9 & 0.7 & 0.5 \end{bmatrix} \tag{20}$$

$$S^2 = \begin{bmatrix} 1 & 0.5 & 0.2 & 0 & 0 \\ 0.8 & 1 & 0.7 & 0.4 & 0.1 \\ 0.6 & 0.9 & 0.8 & 0.6 & 0.2 \\ 0.5 & 0.8 & 0.9 & 0.7 & 0.4 \\ 0.3 & 0.7 & 0.9 & 1 & 0.8 \end{bmatrix} \tag{21}$$

The composition of the three relations in equations 19, 20 and 21 is as follows:

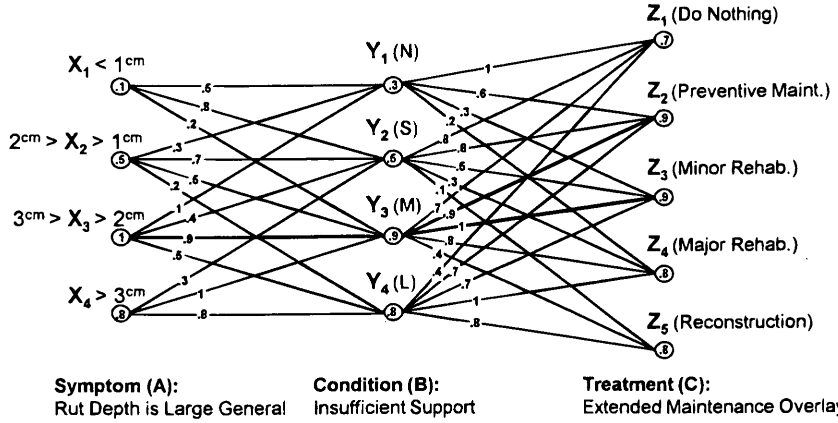

FIGURE 5 A fuzzy graph for Method B.

$$M_C = \begin{bmatrix} 0.9 & 0.9 & 0.7 & 0.6 & 0.3 \\ 0.8 & 0.8 & 0.7 & 0.6 & 0.3 \\ 0.9 & 1 & 0.7 & 0.6 & 0.4 \\ 0.8 & 0.9 & 0.8 & 0.6 & 0.5 \end{bmatrix} \qquad (22)$$

A fuzzy set representation of water in subbase and poor subbase can be obtained by mapping the membership function of "rut depth is large general" to R and S^1. Similarly, a fuzzy set representation of treatment (C) is a mapping to M_C. For the case of rut depth is large general a fuzzy subset describing the degree of truth of maintenance overlay (C) is derived as

$$C = \frac{0.9}{z_1} + \frac{1}{z_2} + \frac{0.8}{z_3} + \frac{0.6}{z_4} + \frac{0.5}{z_5} \qquad (23)$$

Figure 6 is a fuzzy graph showing a transformation of a fuzzy subset from symptom to treatment. The membership values obtained from maintenance expertise and represented in the fuzzy relation are marked on the lines connecting symptom, condition, and treatment. The highest strength of decision path in this graph is $X_3 - (Y^1)_2 - (Y^2)_2 - Z_2$. Although the identified treatment option is preventive maintenance, maintenance overlay, which is suggested in this case, is in the category of minor rehabilitation.

Fuzzy Ordering

Ranking the identified treatments is a major decision-making issue in preserving civil infrastructure. It is usually involved with uncertainties and fuzziness. Ordering the types of treatment is based on the strength of belief calculated from each method, and ordering the suggested treatments is based on the degree of certainty of the derived fuzzy subsets. The symptom-treatment relations established in the three methods are applied to identify a treatment for a symptom such as rut depth is large general.

Mapping the symptom to the fuzzy relations of Methods A, B, and C generates the fuzzy subsets of suggested treatments shown in equations 13, 18, and 23, respectively. The order of treatment types for Method A is $z_2 > z_3 > z_4 > z_1 > z_5$ in which preventive maintenance, mill wheel ruts, is recommended. Similarly, the order of treatment types for Methods B and C are $z_2 = z_3 > z_4 = z_5 > z_1$ and $z_2 > z_1 > z_3 > z_4 > z_5$ respectively. Preventive maintenance is also the identified treatment type for both Methods B and C. Although, extended maintenance overlay (a major rehabilitation) is recommended for Method B and maintenance overlay (a minor rehabilitation) for Method C. It appears that the associated treatments are neither totally committed for Method A nor for Method C, in accordance with the computed membership grades.

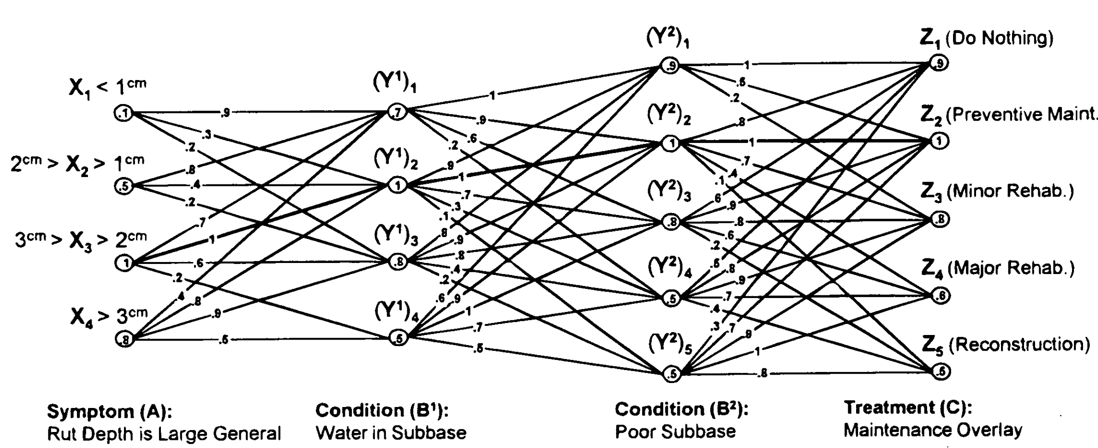

FIGURE 6 A fuzzy graph for Method C.

Ranking the suggested treatments (mill wheel ruts, maintenance overlay, and extended maintenance overlay) can be achieved from a fuzzy measure to estimate the degree of certainty.

DISCUSSION

The three different decision methods of identifying treatment options for a given rut depth information show different results. In this section, two interesting points are discussed that contribute to the investigation of this study.

First, the fuzzy relation matrices shown in Equations 11, 16, and 22 can be compared using an α-cut (*10*). α-cut is a method of converting a fuzzy set into a crisp set which provides a criterion of measuring the fuzziness of a set. A membership value of one-half has the highest degree of difficulty of deciding whether it is a member of the set or not. Membership grades close to one are closer to being in the set, membership grades close to zero are closer to being out of the set. In the present study, applying an α value to the three membership matrices, M_A, M_B, and M_C is to clarify the ambiguity of selecting a treatment among the three methods.

Setting $\alpha = 0.9$, the symptom-treatment relation obtained from Method B identifies preventive maintenance and minor rehabilitation as the potential treatment options.

$$(M_B)_{0.9} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \tag{24}$$

However, as the α value decreases to 0.7, this method essentially has no way of identifying a preferred treatment. This is evident from Equation 18 where it shows almost the same strength of belief of applying different maintenance strategies for the given symptom.

After decreasing the α-cut value to 0.6, Method C shows a higher strength of belief on the four treatment options.

$$(M_C)_{0.6} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \tag{25}$$

In comparison with Method B, Method C is less fuzzy in identifying treatment for a given symptom. On the contrary, Method C is fuzzier than Method A, because the membership matrix, M_A, shows a higher focus on preventive maintenance and minor rehabilitation.

$$(M_A)_{0.6} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \tag{26}$$

This result is consistent with the specific case study obtained from Equation 12. This higher focus means that the result obtained from Method A is less fuzzy than the results of Methods B and C.

Second, the condition fuzzy variables, insufficient support in Method B, and water in subbase and poor subbase in Method C, indicate some influence on the decisions of identifying treatment options because of the uncertainty factors of these variables involved in the processes. However, the results of Methods B and C provide more information about potential treatment availability and the degree of certainty assigned to each treatment.

The decision processes can be refined by interviewing maintenance experts who can present a more reasonable preference function (membership function) in describing the uncertain information. The membership function resulting from interviewing processes is a quantification of design preference that the engineers have the greatest confidence in using, or desire to use with respect to treatment options.

The fuzzy sets representation for general-specific or cause-effect relations has no distinguished differences. The cause-effect relation between water in subbase and rut depth is part of the decision path in Method C. The membership matrix of equation 19 establishes a fuzzy relation that allows evaluation of the cause (water in subbase) of the problem using fuzzy mapping for a given symptom (rut depth). Both water in subbase and poor subbase are the causes of rutting. However, there is a general-specific relation between poor subbase and water in subbase. Structuring expertise with the approach of problem decomposition is well suited to infrastructure condition reasoning that is involved in several levels of abstraction. Higher-level knowledge sources are employed to deal with more general concepts, while lower-level knowledge sources are used to deal with much more detailed operations applied to more specific domains.

In the case study the three decision paths are represented with three different membership matrices for a given symptom. Each membership matrix is established for a relation between the symptom, rut depth, and the treatments. The matrices allow us to explore the degree of confidence on the identified treatment type, which, in turn, clarify the ambiguity of the terms interpreted in the knowledge graphs. Uncertainties involved in the knowledge graph can be assimilated using fuzzy graphs. Fuzzy graphs provide a graphical presentation for the process of fuzzy mapping and the composition of fuzzy relations. The graphs allow the connection strength between the related fuzzy elements to be identified.

SUMMARY AND CONCLUSIONS

In this study the use of fuzzy relations to manage uncertain information in civil infrastructure preservation was investigated. Emphasis was placed on structuring decision processes concerned with condition diagnosis and treatment identification in preserving infrastructure. The concept of knowledge graphs was employed to identify the relations. The uncertainties involved in the relations are manipulated with fuzzy set theory and illustrated with fuzzy graphs. A computational framework was formulated to calculate the strength of belief of the diagnosed conditions and identified treatments.

Three different paths of a decision process were examined based on the fuzzy relations for the conditional statement: If rut depth is large general, then what kind of treatment is most appropriate? Following a series of compositions of fuzzy relations in association with the observed symptoms, the degree of confidence (certainty) in the identified treatments can be obtained. The composition of membership matrices was further illustrated using fuzzy graphs for the decision processes involved in the case study. On the basis of the findings from the use of fuzzy relations in the present study, the following conclusions may be drawn:

1. Fuzzy relations allow us to synthesize the uncertain information involved in civil infrastructure preservation decision processes.

2. Fuzzy graphs provide an efficient tool to explore the strength of confidence of the related parameters represented in fuzzy sets.

3. Fuzzy set representation has the advantages of ordering decision parameters in prioritizing identified treatments.

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