Turning Vehicle Simulation: Interactive Computer-Aided Design and Drafting Application

MILTON S. E. CARRASCO

The use of computer-aided design and drafting (CADD) systems in transportation engineering is having a profound effect on the design process being undertaken by the designers of transportation facilities. The spatial requirements and turning capabilities of vehicles are important factors in the design of such facilities. Until now, designers have depended for the most part on the use of plastic turning vehicle templates to guide them in the design process. This paper describes a CADD application that provides an interactive means of simulating turning vehicles and determining their tire and swept paths in a flexible, efficient, and accurate way. As a result vehicle simulation and design development are maintained in the same graphics environment. The paper describes the use of the program and demonstrates its accuracy for design purposes. Several advantages of the program are noted, including the increased efficiency that can result from working in a single CADD environment, the ability to review alternative paths and vehicle types quickly, the ability to work in either metric or English units of measurement, and the ability to analyze the turning maneuvers of vehicles still in the development stage.

The design of a transportation facility such as a roadway intersection or truck terminal is influenced by the turning capability of one or more design vehicles and their swept paths.

Until the late 1970s, most highway design projects were completed "on the drawing board," that is, the designs were generated either manually or through a combination of stand-alone computer programs and manual drafting. The typical design process included the use of plastic turning vehicle templates to determine manually the swept paths of design vehicles. Designers manually placed the plastic templates or other generated plots onto the design to determine the location of appropriate design features such as curbs and gutters, traffic islands, lane widths and lane markings, pavement edges, retaining walls, and other physical features. Several design iterations were typically required before the design could be finalized.

The advent of the personal computer (PC) and computer-aided design and drafting (CADD) software has had a significant impact on the design and drafting procedures in all areas of engineering. These systems allow for an integrated design and drafting process. An interactive program that can simulate vehicle turning maneuvers and operates seamlessly in a CADD environment can enhance the efficiency of the design of transportation facilities and the preparation of design drawings.

In this paper the algorithm that is the basis of the CADD model presented is described. The extension of the basic algorithm for determining tracking and swept paths is also described.

Finally, a model known as AutoTURN, which is based on the principles and algorithms described above, is described. An overview of the model's operation is described demonstrating its flexibility and ease of use. Output from the CADD program is compared with other methods including turning templates, analytical methods, and field measurements.

EXISTING SIMULATION PROGRAMS

Despite the various mathematical and graphical methods that have been developed over the years, only a few computer programs have been developed in North America to simulate vehicle offtracking and swept paths. The programs are all based on the "constant pursuit method" (also known as the "incremental method of analysis" or "bicycle" model) and, as such, are kinematic models.

A program developed by Sayers (*1*) of the University of Michigan Transportation Research Institute (UMTRI) is one of the first computer simulations of turning vehicles to be documented. The program uses Cartesian coordinate geometry to calculate a vehicle's position as it traverses a given geometric path and was originally written for the Apple II computer. It provides a relatively quick way to produce swept paths for most vehicle combinations. The developers of the program note, however, that a major disadvantage of the program is the long execution time (from several minutes to about 20 mins) to produce plots of the desired path.

Fong and Chenu (*2*) have adapted the simulation portion of the UMTRI program and have developed a mainframe version of the program (known as "TOM"), which is being used at the California Department of Transportation (Caltrans) to improve the processing speed and plot quality. A major advantage of TOM (and by association, the UMTRI program) over other methods described previously is its flexibility in path and vehicle inputs. Virtually any simple and compound circular curves can be used as the vehicle path, and virtually any vehicle type and tractor-trailer combination can be modeled.

The Ministry of Transportation and Highways of British Columbia (*3*) has developed a PC DOS-based program known as "TRACKER," which is similar to TOM but has an added advantage in that the program output can be exported in DXF format to CADD software such as AutoCAD or MicroStation.

A major disadvantage of these computer programs is that they do not work in an interactive manner. As a result the designer must complete a simulation by inputting detailed vehicle path geometry on a separate computer or outside the CADD platform, then import the output into CADD or manually check the design.

PBK Engineering Ltd., 2150 West Broadway, Vancouver, British Columbia, V6Y 1J5 Canada.

To overcome this disadvantage an interactive turning vehicle simulation program that works directly within the CADD environment is required. Such a program must allow the designer to work directly within the design base (i.e., the drawing file), input a steering path, select or define the design vehicle's dimensions, and determine its swept path. The resulting output will allow the designer to evaluate or set design details, such as edge of pavement and traffic islands, and prepare detailed drawings all within the same working environment—the CADD environment.

In the following section the principles on which the CADD model presented in this paper is based are discussed. The term swept path refers to the aerial space between the outer and inner extremities of a vehicle, which may consist of vehicle units that are significantly wider than the axle widths. The term tracking path represents the path resulting from the tracing of the outer tire walls of outermost tires of a vehicle's steering and rear axles (or the geometric center of the rear axle group in the case of a multiaxle rear bogey).

CADD-BASED MODEL

The CADD model in this paper utilizes an algorithm based on the graphical method developed by Vaughan and Sims (*4*) and on Sayers' (*1*) model, that is, the incremental method of analysis or the constant pursuit method. The algorithm consists essentially of an analytical means of duplicating the operation of the tractrix integrator, and uses CADD routines to simplify input for analysis and to facilitate plotting of the desired vehicle paths. A geometry-based approach was selected because tire mechanics, in most practical situations, do not significantly affect offtracking.

Figure 1 (*a*) and (*b*) illustrate the basis of the algorithm as it applies to a single-unit vehicle and a multiple-unit vehicle, respectively. These vehicle units are represented by a "bicycle" model, with multiple axles represented by the geometric center of the axle group.

In coordinate geometry terms, the analysis for a single-unit vehicle reduces to the solution of the following equations to obtain the coordinates of the center of the rear axle:

$$x_{i2} = (x_{i1} - x_{(i-1)2})/[(1_{i(i-1)} - 1_{wb})/(1_{i(i-1)})] + x_{(i-1)2} \quad (1)$$

$$y_{i2} = (y_{i1} - y_{(i-1)2})/[(1_{i(i-1)} - 1_{wb})/(1_{i(i-1)})] + y_{(i-1)2} \quad (2)$$

where

i = the incremental location for which
1 = the geometric center of the front axle or towing point,

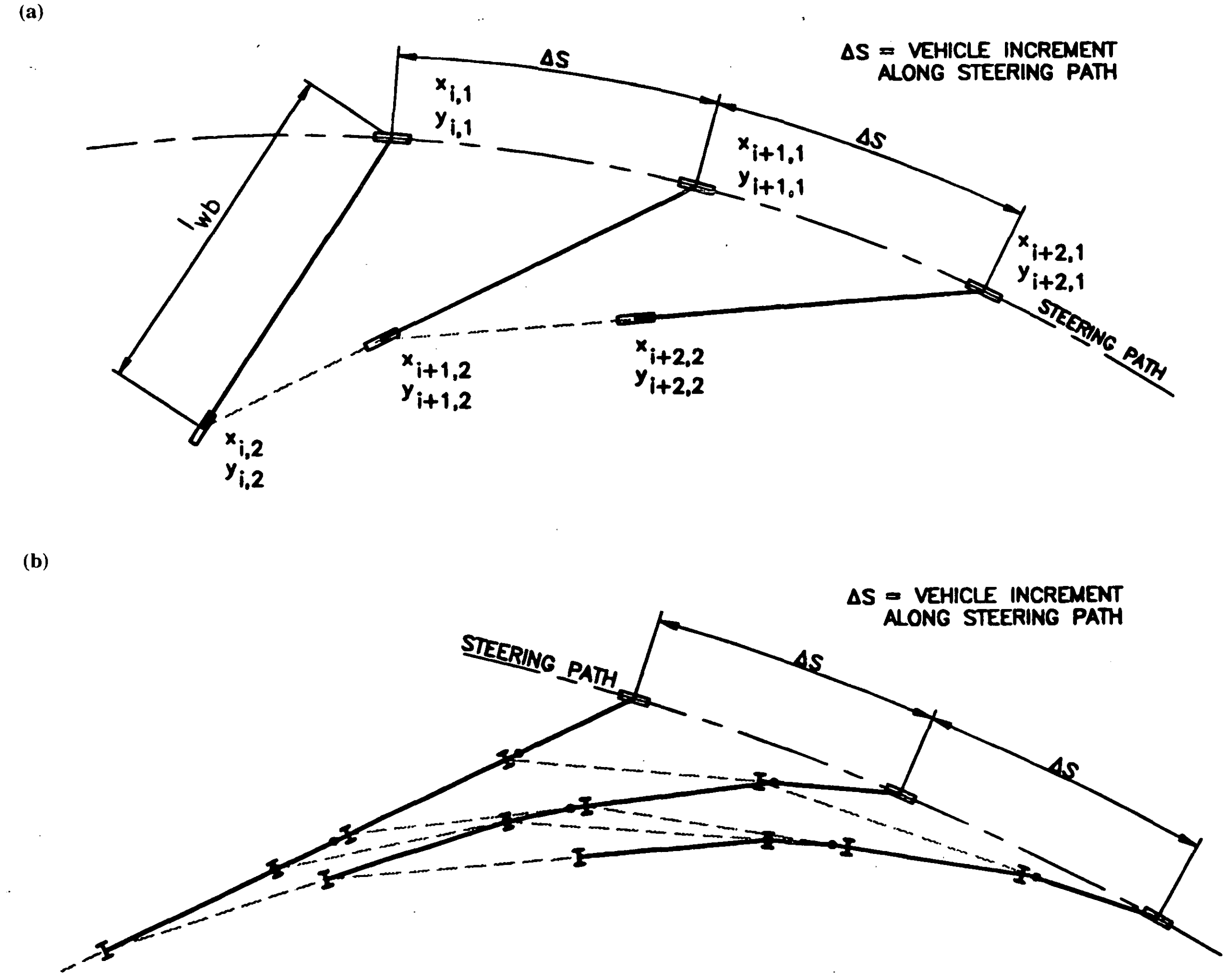


FIGURE 1 Application of algorithm to (*a*) single-unit vehicles and (*b*) multiple-unit vehicles (tractor semitrailer plus fulltrailer).

2 = the geometric center of the rear axle,

$l_{i(i-1)} = [(x_{i1} - x_{(i-1)2})^2 + (y_{i1} - y_{(i-1)2})^2]^{1/2}$ and is the distance from the steering or towing point at the ith location to the rear axle at the $(i - 1)$th location, and

l_{wb} = simple wheelbase of vehicle unit.

Derivation of this and other key equations pertaining to the CADD model is presented elsewhere (*5*).

The incremental position of the steering or towing point is set at a fixed distance equal to some small number relative to real-world vehicle and turning dimensions. In this model the incremental distance is set at 1/100th of a vehicle width, equaling about 25 mm. This incremental distance is selected because it represents the point at which smaller increments provide no appreciable change in offtracking values. Through coordinate geometry calculations, the coordinates of the incremental points and the rear axle point are obtained by solving Equations 1 and 2.

The same principle can be used for a vehicle consisting of several units, because there will always be two similar equations to solve to obtain the rear axle coordinates. Towing points such as a fifth wheel or a hinge point can be obtained once the position of the towing vehicle has been determined.

Application of Algorithm to Tracking and Swept Paths

The algorithm applies to the bicycle model concept and, in this configuration, is only good for obtaining offtracking values for a particular vehicle. For the model to be useful in determining tracking and swept paths, it must be enhanced. The bicycle model is used because it has been found to be reasonably accurate for design purposes, and it simplifies the analysis of the tracking and swept paths. Consequently it provides a shorter execution time. The following sections describe the process used to determine vehicle tracking paths and vehicle swept paths.

Determination of Vehicle Tracking Path

Figure 2 illustrates the concept used to develop the vehicle tracking path from the base bicycle model. The following briefly describes the step-by-step process that is built into the model.

1. For each incremental step, after the new bicycle position has been determined, the axle configurations of the vehicle in its new position are located using the vehicle body and axle dimensions.
2. The coordinates of the outer extremities of the axles are then calculated based on the axle width and are stored in a data base.
3. These calculations are repeated for each incremental step producing coordinates for the outer edges of the tires of the first and the last axles (or geometric center of the axle groups) of the vehicle. The data are stored in the data base until the analysis of the entire path is complete.
4. The coordinates of the stored tracking path points are then plotted and lines are drawn connecting the points. In this manner complex curves representing the path of the steering and the rearmost axles are plotted. These curves represent the tracking path of the vehicle.

Determination of Vehicle Swept Path

Using the basic bicycle model to determine the swept path of a vehicle requires a slightly different approach. Figure 3 illustrates the method used in this model and the following describes the procedure that is incorporated in the model for developing the swept path:

1. As in the development of tracking paths, coordinates for the body of the vehicle are calculated for each incremental step. The term "body" includes any oversize loads that may be carried on the vehicle.
2. Screen lines are established along the travel path at close intervals (the model uses an interval of 1/10 of the vehicle width—

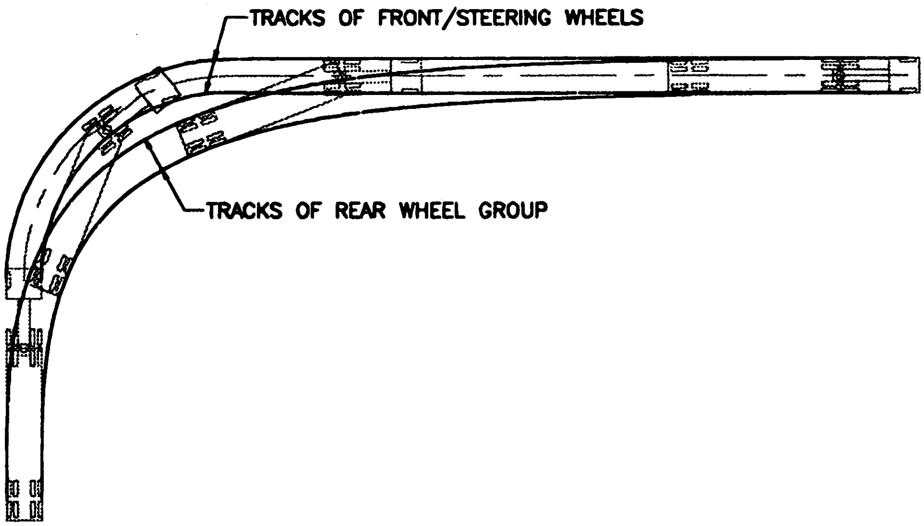


FIGURE 2 Vehicle tracking path.

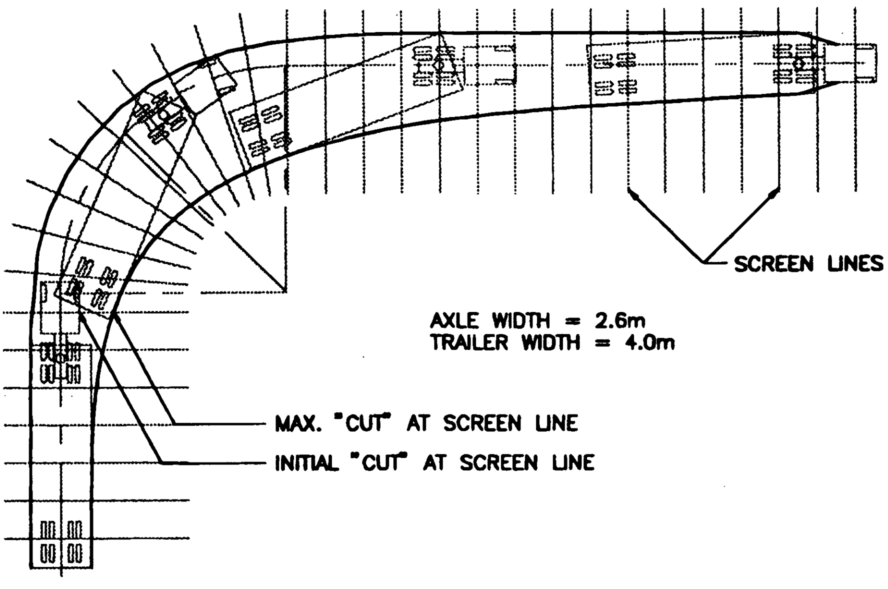


FIGURE 3 Vehicle swept path model.

approximately 250-mm intervals). The screen lines are typically set normal to the travel path.

3. As the vehicle crosses the screen lines with each incremental calculation, the coordinates at the intersection of the vehicle body with the screen line are calculated and stored. As the vehicle progresses along the travel path with each successive increment, the position of the extremities of the vehicle at each of the screen lines is compared with the previous vehicle position. In the process the maximum cut is determined and stored into a data base.
4. The coordinates of the maximum cut at each screen line are then plotted and a curve is drawn between the points to obtain the swept path of the vehicle.

Incorporating Vehicle Turning Limitations

This model and the tracking and swept path enhancements by themselves are insufficient to simulate fully "real world" turning conditions. Because the CADD model is essentially a kinematic model, the only way to incorporate the effects of turn limitation is through empirical means. Field tests were conducted using a tractor with a semitrailer to verify model results and to obtain an understanding of turn limitations.

Field tests showed that in a tight cornering situation, with steering wheels at their maximum turn position, a tractor with a semitrailer reaches a position where side frictional forces acting on the rear tandem axles become so large that the vehicle cannot proceed any further. Field measurements for the test vehicle showed the maximum steering angle to be approximately 27 degrees and the angle between the tractor and the semitrailer, under tight turn conditions, to be approximately 76 degrees.

A review of vehicle specifications of newer tractors shows that the maximum steering angle has steadily increased from previous values of 27 degrees to 30 degrees to values of as much as 40 degrees, making it possible for the new vehicles to negotiate sharper radii turns than older tractors.

With each incremental step in the CADD model, the steering angle and tractor-to-trailer or trailer-to-trailer angle (for multitrailer-unit vehicles) is calculated using the vehicle and path dimensions provided. The calculated values can then be compared with the limitations of the specified vehicle. If the calculated values exceed the specified turn limitations, it is possible to flag such an occurrence, warning the user of a problem with the simulation.

AUTOTURN—A CADD APPLICATION FOR SIMULATING TURNING VEHICLES

This section describes a working CADD model, known as AutoTURN, that is based on the model algorithm described previously. Enhancements pertaining to the determination of tracking and swept paths have also been included in this CADD model. The AutoTURN program is a CADD-based program and is written in the C programming language. It is a menu-driven graphics program that operates on a Cartesian coordinate system. This capability makes it ideal for undertaking the design and drafting of transportation facilities, because the determination of tracking and swept paths is an important element in setting or evaluating the design of such facilities. In turn, relating these parameters to the real world involves establishing coordinates of the design geometry utilizing either a local or universal system of coordinates and elevation data. CADD is an ideal environment for completing coordinate geometry calculations and for preparing design drawings to exact coordinates. As a result it permits total automation of the design and drafting process.

AutoTURN allows the user to establish a vehicle's turn requirements or check a vehicle's turning maneuver, and to finalize design and produce final drawings in a single working environment. This advantage is further enhanced by its interactive capability, whereby the user can quickly refine or review other alignments and compare the paths of other vehicles. The program will trace the path of specific points on the vehicle as it traverses a defined path and can even be used to determine turning needs of vehicles in the development stage.

Program Logic

The logic of the program as it applies within an AutoCAD environment is illustrated in Figure 4. Data entry, execution, and output are achieved through three distinct processes.

Data is entered via the CADD environment. Vehicle dimensions and the steering path are input by simply editing predefined vehicle dimensions and selecting predefined path entities. After being entered, data files are prepared for use in the execution phase.

Execution of the program is undertaken using C language to perform the coordinate geometry calculations; coordinates are calculated for the incremental points on the steering path and for the vehicle's axle and tire points. The resulting coordinates are then written to a Path.dxb file. The Path.dxb file is the data base file that is imported into CADD for drawing preparation.

CADD software provides a suitable environment in which not only is the data entry considerably simplified, but also the user is allowed to receive directly plots of the resulting paths. Program execution typically takes between 15 and 30 sec (using a 486 PC) from the time selection of vehicle and path configuration is completed to the time an "on-screen" plot of the desired path is produced.

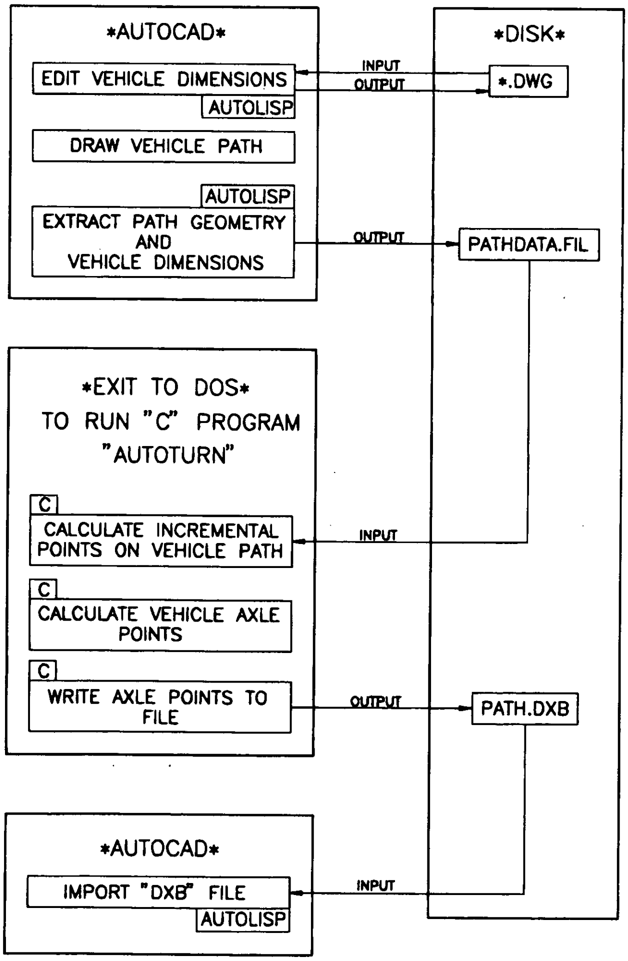


FIGURE 4 Program logic.

Ease of Use

The purpose of this section is to illustrate the ease of using the program. Figure 5 provides an overview of the steps involved, and a service road access design, shown in Figure 6(*a*), is used to demonstrate a practical application of the program. AutoTURN will be used to evaluate the design for accommodation of a WB-12 standard Road and Transportation Association of Canada (*6*) design vehicle, which is equivalent to a WB-40 AASHTO (*7*) design vehicle.

Step 1—Select Design Base

The designer selects the design base in AutoCAD. This may involve a drawing showing the layout of a proposed or existing facility such as a roadway intersection, parking lot, or a transit terminal. In the case of frontage road, the design base consists of the road layout prepared using AutoCAD.

Step 2—Draw Vehicle Path

The program requires as input the vehicle's steering path, which is the path to be followed by the center of the front axle of the vehicle. In the case of a roadway this path will typically be the centerline of the lane (Figure 6) and consists of a contiguous series of line and arc entities. Setting the steering path generally requires a knowledge of the start and end positions of the vehicle. Two vehicle paths, the major and minor left turns, will be tested in the example.

Step 3—Load AutoTURN

After the drawing containing the steering path is loaded into AutoCAD, the AutoTURN program is loaded by selecting the Load AutoTURN command from the AutoTURN pull-down menu.

Step 4—Set Configuration Menu

The next step is to set simulation parameters using the configuration menu of the program. This menu contains parameters relating to the type of vehicle being studied, the vehicle's start position, the type of simulation (i.e., tracking or swept path), the type of output files desired, drawing layering, and dimension units desired. The first three parameters affect the simulation output, and hence the design. The remaining parameters aid the drawing preparation process.

Step 5—Run Simulation

After the configuration menu parameters have been selected, the program is executed by selecting the Run Simulation option from the pull-down menu. The user is then prompted to select the steering path entities in the order and direction of travel.

Step 6—Evaluate Vehicle Path

Once the drawing of the required vehicle path is produced, the proposed steering path can be evaluated against available maneuvering

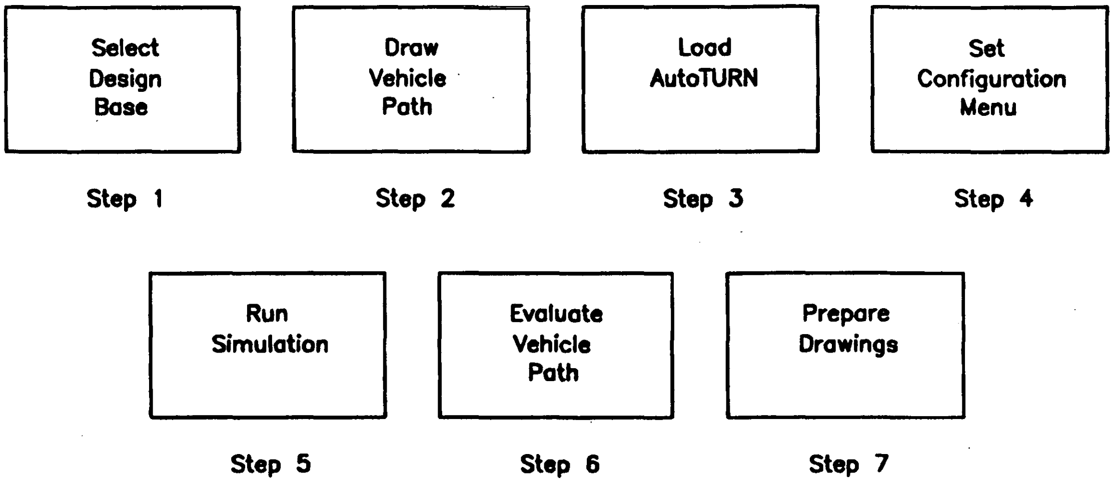


FIGURE 5 Program use.

space, taking into account conflicts with curbs, utility poles, buildings, or structural elements. If conflicts occur, the user can modify the path, if feasible, or consider the possibility of relocating the conflicting elements so that they lie outside the vehicle's swept path. Figure 6*b* shows the resulting swept paths for the vehicle paths selected. The design appears to accommodate the WB-12 design vehicle adequately except for the service road curb return radius. The adjusted radius is set using AutoCAD and is shown by a broken line.

Step 7—Prepare Drawings

Design details and drawings can be completed without leaving the AutoCAD graphics environment once an acceptable swept path and alignment has been obtained.

Program Validation

Validation of the AutoTURN outputs was undertaken through comparison with several offtracking methods and turning templates, as well as with field measurements. The following comparisons were made:

1. AASHTO templates;
2. Caltran's offtracking software (TOM);
3. Jindra's (*8*) and Woodroofe's (*9*) equations;
4. Jensen's (*10*) tire mechanics model; and
5. Field measurements.

AASHTO's turning templates and Caltran's TOM program are both used for design purposes. AutoTURN outputs matched AASHTO templates well with no noticeable differences in the swept paths. TOM and AutoTURN produced identical tire tracking outputs. Fong and Chenu (*2*) note that the results from TOM were found to be accurate to within 2 percent when compared with field measurements.

Jindra's (*8*) and Woodroofe's (*9*) equations are numerical approximations for estimating transient and steady-state values of offtracking. In addition the equations use an equivalent wheelbase to represent articulated vehicles. A comparison of the results of the equations with the outputs from AutoTURN is illustrated in Figure 7. The comparison shows that Jindra's (*8*) equations and AutoTURN produce results similar to within 14 percent when actual vehicle dimensions are used in the AutoTURN program. Results from Woodroofe's (*9*) equations match AutoTURN output (based on actual vehicle dimensions) to within 8 percent. Neither of these comparisons can be considered conclusive evidence of the absolute accuracy of the AutoTURN program, because the author is not aware of the level of validation of these equations with field measurement or the extent of any such testing.

Comparison with Jensen's (*10*) tire mechanics model (Figure 8) shows that offtracking values generated by AutoTURN agree to within 6 percent; the AutoTURN results tending to be larger in value and therefore more conservative.

Of these methods, only conducting field measurements provides a measure of the absolute accuracy of the AutoTURN program. All of the other methods provide a measure of the relative accuracy of the program. Unfortunately, not only are extensive field measurements impractical but conducting field measurements requires considerable planning and careful execution to obtain valid measurements, as Morrison (*11*) noted. In a field test conducted by the author, AutoTURN produced results accurate to within 3.5 percent of the maximum offtracking width, with a maximum difference of 165 mm (Figure 9).

Discussion

A factor that must be considered in determining an acceptable degree of accuracy for the program is the design consideration in accommodating vehicle turning maneuvers. The designer of transportation facilities, in particular, highways, must be considerate of the skills and abilities of the drivers and vehicle types and configurations that will use those facilities. To account for these conditions, good design practice suggests allowing for a forgiving clearance on either side of the swept path. The author suggests that a forgiving clearance of a minimum of 0.6 m on each side of a vehicle's actual swept path (for a total widening of 1.2 m) be provided to establish the minimum travel path for the design turn. In absolute terms, the

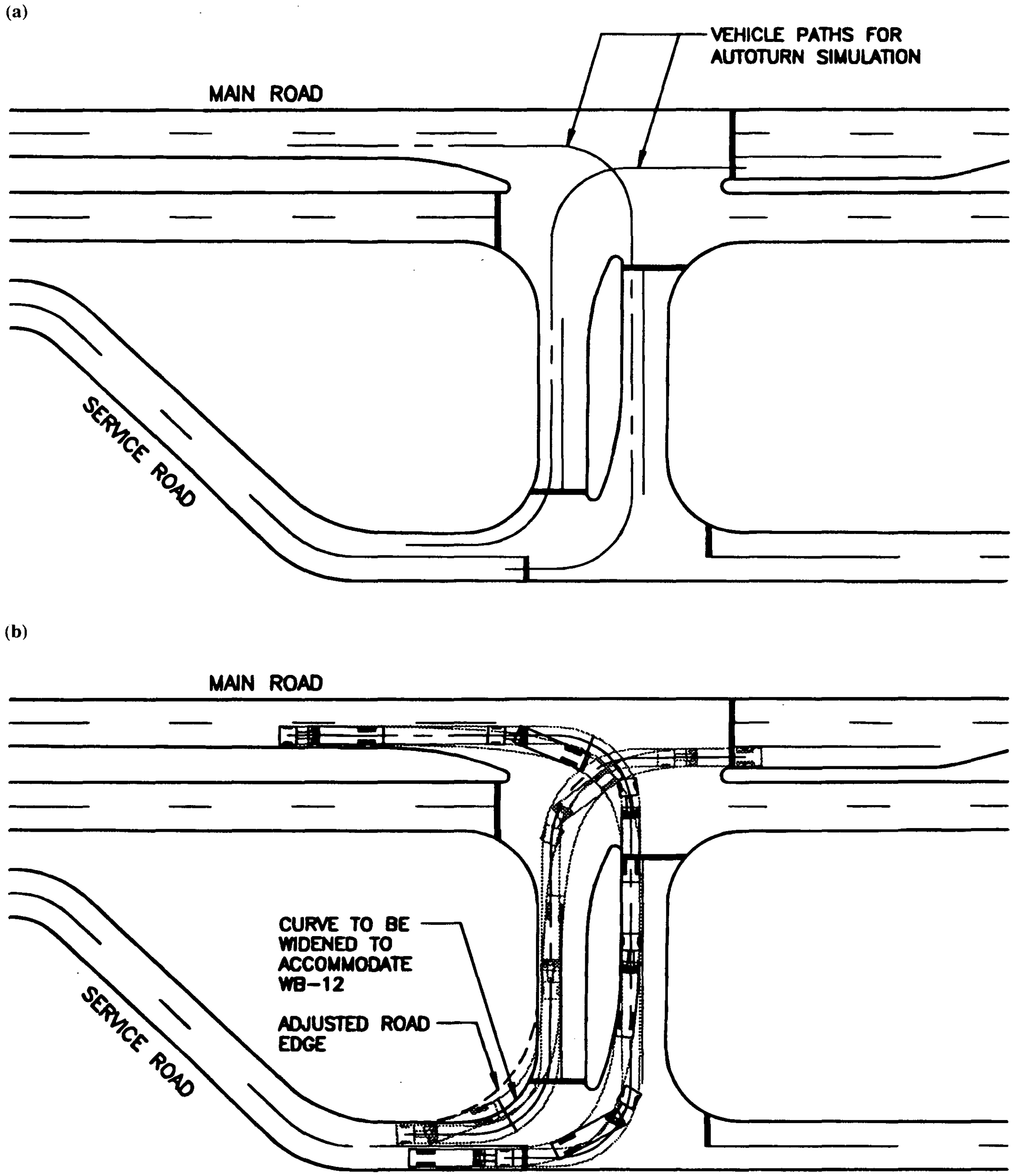


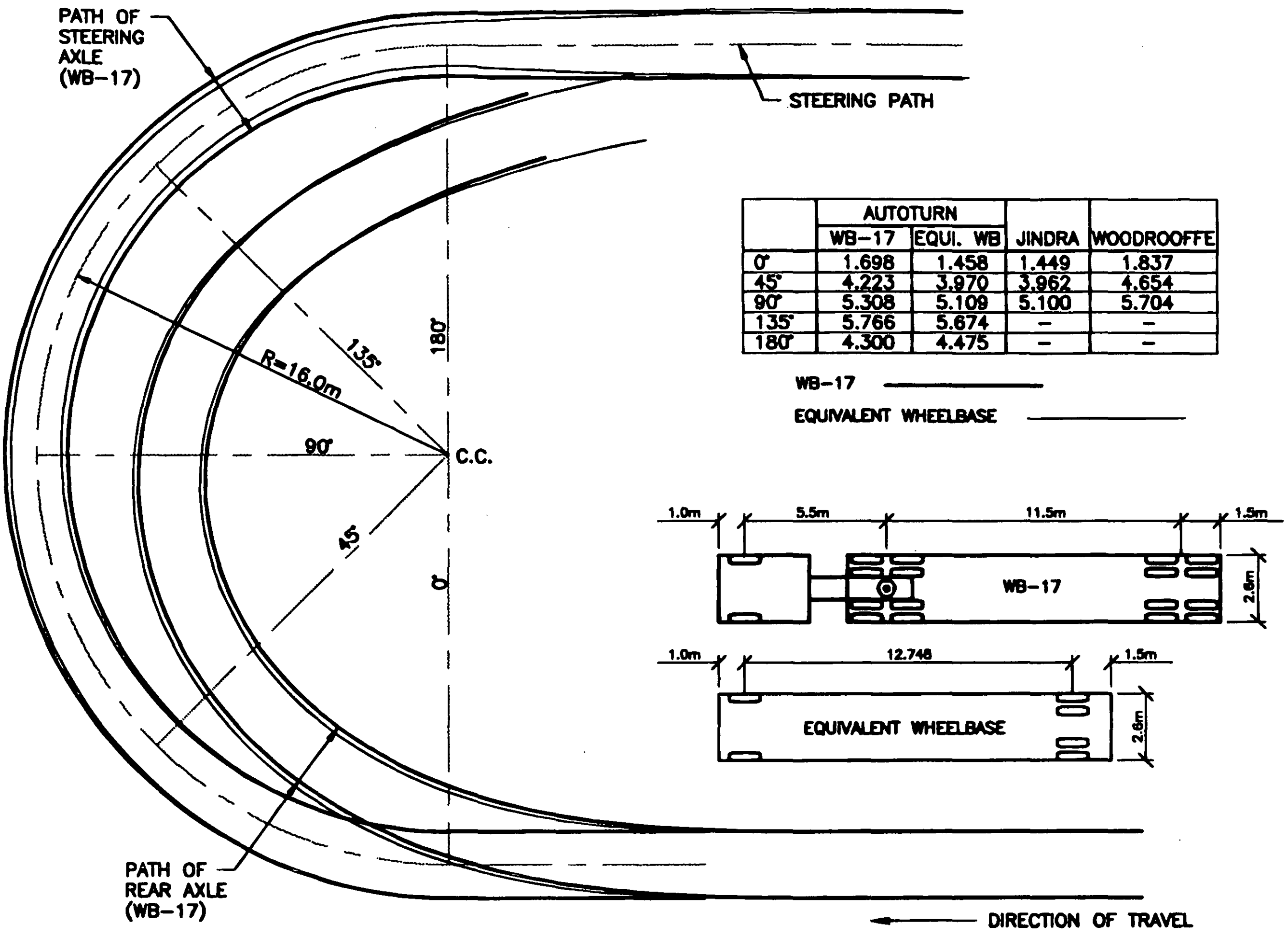
FIGURE 6 Service road access (*a*) base design and (*b*) base design with AutoTURN run.

author suggests that the calculated path widths should not encroach into this forgiving space by more than 25 percent. The CADD model described here appears to provide an accuracy that is well within this suggested limit.

Jensen's research (*10*), however, shows that there are potential limitations in using a kinematic model such as AutoTURN to simulate nonstandard vehicles, for example, those having multiple, widely spaced fixed-axle bogeys with short wheelbases, steerable or freely castering rear bogeys, or other vehicle or axle configuration in which frictional effects on the wheels can have a significant effect on offtracking dimensions. The program is also unable to model slippery or icy pavement surfaces, which can affect offtracking. A further limitation of the program is the effect of vehicle speed and the resulting centrifugal forces that act on a high-speed turning vehicle. This effect tends to produce lower offtracking values than lower speed maneuvers.

CONCLUSIONS

The AutoTURN program, which is based on the CADD model discussed in this paper, is an interactive CADD application that provides the designer with an alternative means of simulating turning vehicles. The advantages of this program include the following:



	AUTOTURN		JINDRA	WOODROOFFE
	WB-17	EQUI. WB		
0°	1.698	1.458	1.449	1.837
45°	4.223	3.970	3.962	4.654
90°	5.308	5.109	5.100	5.704
135°	5.766	5.674	–	–
180°	4.300	4.475	–	–

FIGURE 7 **AutoTURN versus Jindra's (*8*) and Woodroofe's (*9*) calculations.**

1. By operating directly within a CADD environment, it eliminates the need for importing simulation plots generated externally as most other programs do, and it eliminates the need for manual verification of vehicle swept paths.
2. The program enhances the automation capability of CADD in that it works directly within the design base (i.e., design drawings), produces vehicle paths that can be evaluated, and allows changes to be made to the design as required.
3. It can be used to simulate virtually any vehicle type with nonstandard dimensions.
4. It can be used to evaluate the turning maneuvers of vehicles still in the development phase.
5. Unlike turning templates, which are generally provided for a single radius of turn, AutoTURN can simulate any steering path geometry consisting of circular curves and tangent sections.
6. It is easy to operate and produces outputs relatively quickly.
7. It can determine both tracking and swept paths.
8. The program can identify maneuvers in which turn limitations are likely to be exceeded.
9. It can operate with any unit of measurement, that is, English or metric.

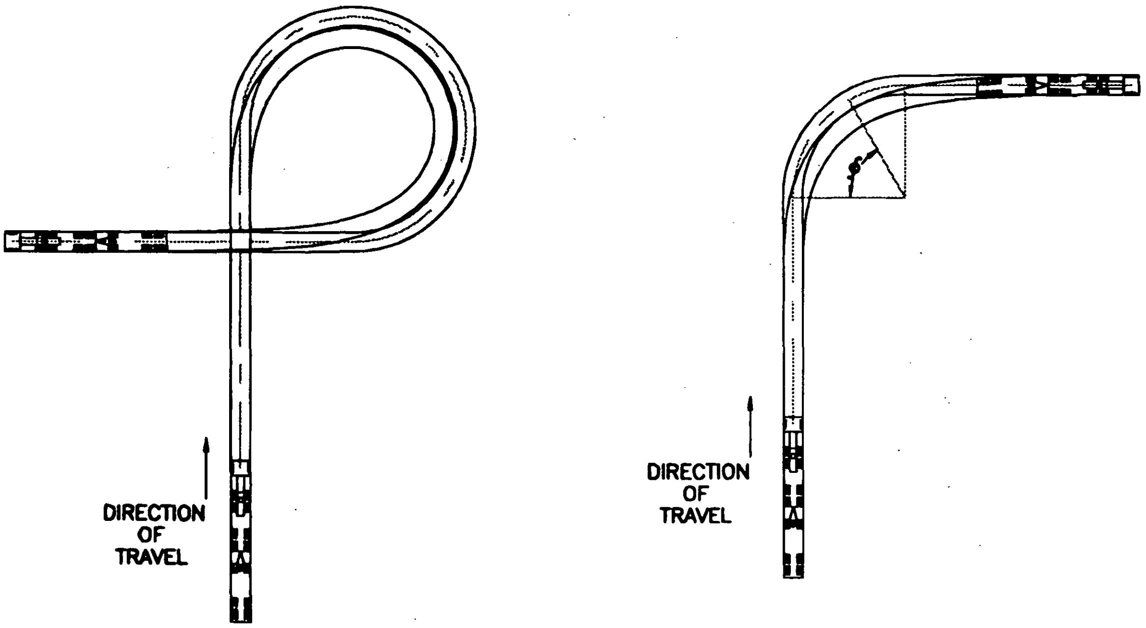
The AutoTURN program has been validated through comparisons with other methods of analysis, computer programs, and field measurements. The comparisons show that offtracking values generated by AutoTURN agree to within 6 percent with similar values generated by Jensen's (*10*) tire mechanics model, with the AutoTURN results tending to be larger in value. In absolute terms the maximum difference is approximately 200 mm. AutoTURN results also compare well against field measurements, producing a maximum error of 165 mm or 3.5 percent of offtracking values.

These comparisons and comparisons with other commonly used design methods confirm the acceptability of AutoTURN as a design tool for establishing vehicle tracking and swept paths for the design of roadways and other transportation facilities.

Although the program appears to produce acceptable results for most standard vehicle types, further research is required to establish its suitability or limitations when simulating certain nonstandard vehicles.

RECOMMENDATIONS

As noted above, a CADD model such as the AutoTURN program clearly has several advantages when compared with other non-CADD programs or any other methods that currently exist. The program can be improved further if the following features can be included:



BASED ON A-TRAIN WITH 3.0m TRACKWIDTH

	STEADY-STATE TURN				90° DEGREE TURN		
	SAE MODEL	SIMPLE MODEL	JENSEN'S MODEL	AUTOTURN	JENSEN'S MODEL	AUTOTURN	∮
A-TRAIN	–	–	3.063	2.919	2.651	2.566	60°
C-TRAIN (FIXED AXLE)	2.295	3.120	3.161	3.300	2.795	2.820	60°
C-TRAIN (FREE AXLE)	1.637	2.234	3.336	3.437	2.207	2.340	65°

NOTES:

1. OFFTRACKING VALUES SHOWN IN METRES. ∮ REPRESENTS APPROXIMATE LOCATION OF MAX. OFFTRACKING.
2. SAE AND SIMPLE MODEL OFFTRACKING VALUES OBTAINED FROM JENSEN'S THESIS
3. JENSEN'S VALUES BASED ON DRY ROAD CONDITIONS
4. JENSEN REFERS TO C-TRAIN CONFIGURATION AS B-DOLLY

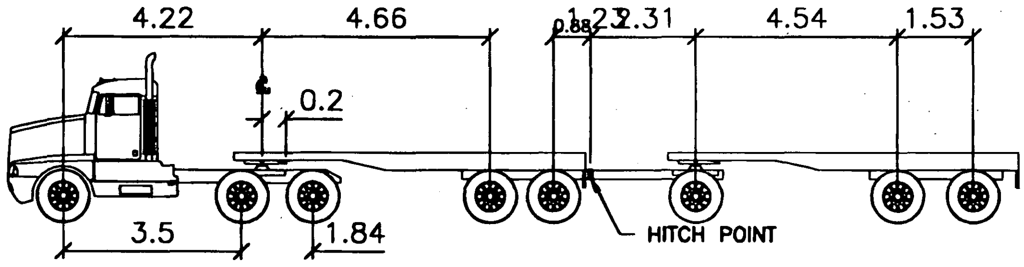


FIGURE 8 AutoTURN versus Jensen's model (*10*).

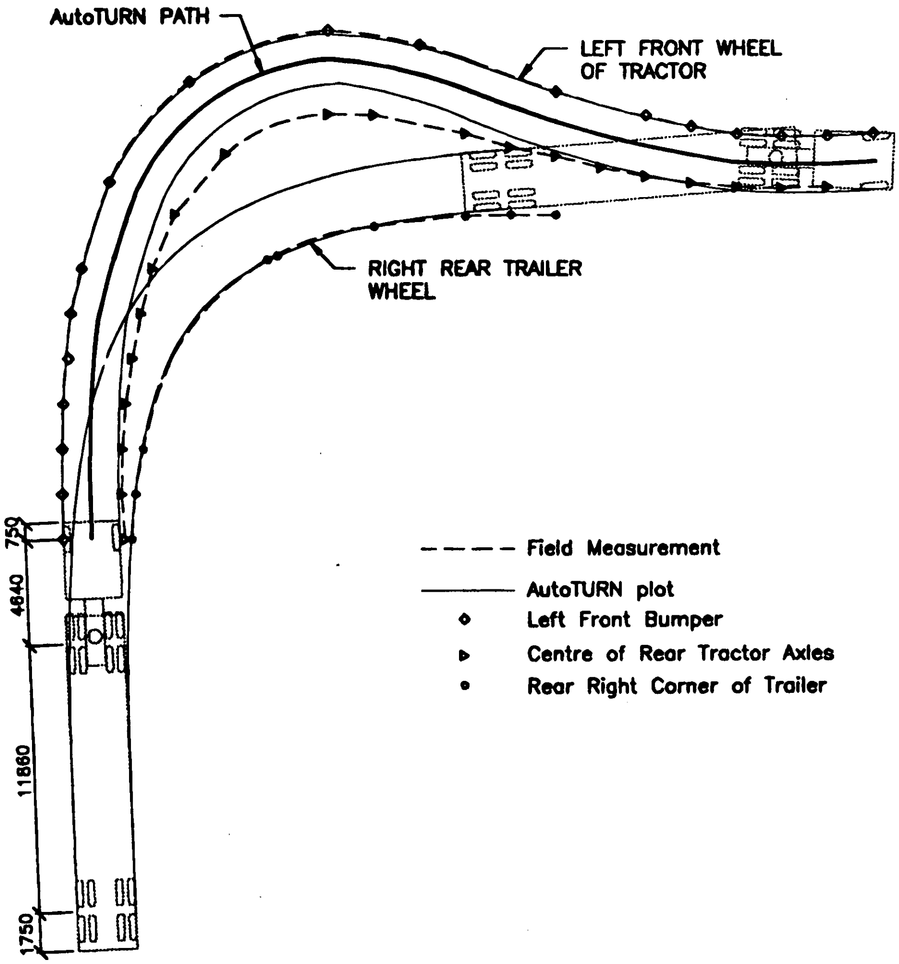


FIGURE 9 AutoTurn versus field measurements.

1. The effects of tire mechanics;
2. Reverse maneuvers of articulated vehicles;
3. Simulation of other types of vehicles such as aircrafts, trains, street cars and light rail transit, and trolley buses;
4. Steerable or freely castering rear bogeys; and
5. Interactive user-controlled steering simulations.

It is recommended that further research and development be undertaken to develop a program that includes these enhancements to make a more complete and versatile turning vehicle simulation program.

ACKNOWLEDGEMENTS

The author wishes to acknowledge Dr. John Morrall of the University of Calgary for his encouragement and support in undertaking this project. A very special note of appreciation is extended to Tim Evans, whose programming skills resulted in the development of the AutoTURN program. The assistance of Canadian Freightways Ltd., which provided the vehicle, driver, and facilities for conducting field tests, is also acknowledged.

REFERENCES

1. Sayers, M. W. The University of Michigan Transportation Research Institute. Vehicle Offtracking Models. In *Transportation Research Record 1054,* TRB, National Research Council, Washington, D.C., 1985.
2. Fong, K. T., and D. C. Chenu. *Simulation of Truck Turns with a Computer Model.* Report 86-1. Division of Transportation Planning, California Department of Transportation, Jan. 1986.
3. Garlick, G., D. N. Kanga, and G. G. Miller. TRACKER: A Program to Generate Vehicle Offtracking Templates. *Compendium of Technical Papers,* ITE District 7—1991 Annual Conference, Victoria, British Columbia, Canada, Vol. II, pp. 283–302.
4. Vaughan, R. G., and A. G. Sims. *Determination of Swept Paths of Vehicles.* Traffic Accident Research Unit, Department of Motor Transport, New South Wales, Australia, July 1970.
5. Carrasco, M. S. E. *Computerized Vehicle-Turning Simulation—An Interactive Application.* M. E. Thesis. University of Calgary, Alberta, 1992.

6. *Manual of Geometric Design Standards for Canadian Roads, 1986 Metric Edition.* Road and Transportation Association of Canada, Ottawa, Canada, 1986.
7. *A Policy on Geometric Design of Highways and Streets.* AASHTO, Washington, D.C., 1990.
8. Jindra, F. Offtracking of Tractor-Trailer Combinations. *Automotive Engineer,* Vol. 53, March 1966, pp. 96–101.
9. Woodroofe, J. H. F., C. A. M. Smith, and L. E. Morisset. A Generalized Solution of Non-Steady State Vehicle Offtracking in Constant Radius Curves. *SAE Technical Paper Series,* Truck and Bus Meeting and Exposition, Chicago, Illinois, Dec. 1985.
10. Jensen, L. G. A Mathematical Model for the Low-Speed Offtracking of Articulated Vehicles That Includes Tire Mechanics. M. S. Thesis, University of Calgary, Oct. 1987.
11. Morrison, W. R. B. A Swept Path Model Which Includes Tyre Mechanics. *Australian Road Research Board Proceedings,* Vol. 6, Part 1, 1972.

Publication of this paper sponsored by Committee on Geometric Design.