Reliability-Based Processing of Markov Chains for Modeling Pavement Network Deterioration

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Accurate prediction of pavement deterioration is the most important factor in the determination of pavement repair years and optimization programming of highway network maintenance. The Nonhomogeneous Markov Probabilistic Modeling Program, developed to determine pavement deterioration rates in different stages, is described. In this program the transition probability matrices (TPMs) are considered as a time-related transition process. Each element of the TPMs is determined on the basis of a reliability analysis and a Monte Carlo simulation technique. This avoids the use of the existing conventional methods, which involve taking an average subjective opinion of pavement engineers or observing a large number of multiyear pavement performance data and conducting a number of statistical calculations. As a result a series of TPMs for an individual pavement section for different stages can be determined by running the program. Furthermore, the pavement condition state in terms of a probability vector at each stage (year) is calculated. In applying the models both the predicted actual traffic (in terms of equivalent single axle loads) at each stage and the maximum traffic that the pavement can withstand at each defined pavement condition state interval are considered to be random variables. In addition, the sensitivities of pavement deterioration rates to pavement design parameters, such as traffic growth rate, subgrade strength, and material properties, are studied. Finally, an example of calculating the TPMs for a pavement section located in southeastern Ontario, Canada, is demonstrated. It shows that the sensitivities of the TPMs to traffic growth rate, subgrade deflection, and pavement thickness are significant.

The development of probabilistic models for the prediction of network deterioration has been a key technical challenge to pavement engineers. Other models used in network-level pavement management are mainly dependent on the reliability of the prediction of deterioration for each individual pavement section. In other words, pavement deterioration prediction influences the quality of many other components of pavement management, such as determination of the years that rehabilitation is needed and the corresponding treatment alternatives, improving the existing road network to a required service level, and selecting the optimal cost-effective rehabilitation and maintenance alternatives. Over the last two decades, although considerable progress has been made toward the achievement of effective management systems, there is still a need for probabilistic modeling of network pavement deterioration (*1*).

OVERVIEW OF EXISTING PREDICTION MODELS

Since the concepts of pavement management were initiated in the 1960s, many prediction models have been developed in North America and elsewhere. Basically, the current prediction models can be divided into two categories: deterministic and probabilistic (*2*). These two types of prediction models have been used by many highway agencies at both the project and the network levels. Deterministic models can be further broken down into purely mechanistic, mechanistic-empirical, and regression models. A detailed description, with examples, of each type of deterministic model is presented elsewhere (*3*). However, it is inadequate to apply deterministic models to all situations of pavement management because of (a) the uncertainties in pavement behavior under changeable traffic load and environmental conditions, (b) the difficulties in quantifying the factors or parameters that substantially affect pavement deterioration, and (c) the errors associated with measuring pavement condition and bias from a subjective evaluation of pavement condition.

The principle of applying probabilistic performance models to flexible pavement design was introduced by Darter and Hudson (*4*). Consequently, the probabilistic modeling of future pavement condition states has recently received considerable attention in pavement management (*5*). Probabilistic modeling of pavement deterioration has been further applied in many other areas, such as dynamic programming of pavement maintenance with pavement deterioration modeled as a Markov transition process, pavement network budget planning (*6*), and cost-effectiveness analysis for financial planning of pavement network management (*7*). Furthermore, probabilistic models have been used to minimize the total expected cost and to keep all pavement sections in the network above a required service level (*8*).

Although considerable effort has been devoted to improve the quality of the probabilistic modeling of pavement deterioration, the applicability of the existing transition probability matrix (TPM) building method is limited to only several widely spaced pavement categories, which are classified on the basis of traffic level, subgrade condition, and pavement thickness. For example, Karan (*7*) considered only 18 flexible pavement classes by roughly defining the flexible pavement types on the basis of two levels of subgrade soil (strong and weak), three levels of thickness, and three levels of traffic volume (low, medium, and high). Thus, the total number of combinations is 18; corresponding to these pavement classes, 18 different TPMs have been established by processing a large quantity of information and questionnaires from many individual pavement engineers.

Similarly, Wang et al. (*9*) recently carried out an extensive analysis of the probabilistic behaviors of the Markovian prediction models modified for the pavement network optimization system of the Arizona Department of Transportation. In the road system 15 highway categories were defined on the basis of traffic volume,

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regional factor, and functional class; 45 pavement condition states were classified for the purpose of defining the transition process. Each element of the TPMs in the system for each road category was obtained from more than 10 years of observed pavement performance data.

Essentially, the existing Markov chain TPM building method has two major technical problems that make it difficult to predict pavement deterioration appropriately. One is the assumptions for dealing with the effects of traffic and environmental conditions on the pavement. Another one is the techniques applied in building the TPMs for pavements.

A description of the existing TPM method and assessment-associated quality can be summarized as follows.

1. According to the definition of the Markov process, there can be two different transition consequences: homogeneous and nonhomogeneous, depending mainly on the assumptions or parameters defined for a system. When applying the homogeneous Markov process in modeling pavement deterioration, it has been assumed that the variables such as traffic (including volume, growth rate, truck percentage, etc.) and environmental conditions (including strength of subgrade soil, annual average temperature and precipitation, etc.) are constant throughout the analysis period, which is not correct in many real situations. For a multistep transition the existing method uses the Chapman-Kolmogorov equation to calculate the future condition states.
2. Each element of the TPM is quantified either by using the average of subjective opinions of experienced engineers, through individual interviews and questionnaires, or by observing the performance of a large number of pavements in the same category under different initial pavement conditions over a long period of time. For the former it is known that each individual has his or her own bias and different response to or assessment of the same question; therefore, it is difficult for a pavement manager to deal with the variety of data and to transform the information into a TPM of pavement deterioration. Furthermore, it normally takes considerable time and expense to perform subjective information collection and processing. For the latter situation a large amount of soundly measured performance data for all road categories is required, which is time-consuming and costly.
3. Since the existing TPM building methods have the forgoing disadvantages, it is impossible to establish a set of TPMs for each individual pavement section in a network. In fact, the existing TPM building methods can construct only a few TPMs for several roughly classified pavement categories, as described previously. The effects on pavement deterioration of many other important factors, such as pavement thickness, construction methods, traffic volume, and growth rate, are neglected. Each pavement is assigned to one of the categories so that the established TPMs can be applied. This process will cause large variations in the prediction of pavement network deterioration. In addition, for a defined pavement section, the existing methods are not able to establish a set of time-related TPMs for use in different stages.

These drawbacks will, in turn, influence the quality or the correctness of many other decisions, ranging from determination of the years when rehabilitation is needed to programming of optimal pavement rehabilitation and maintenance strategies. Consequently, the general purpose of this paper is to minimize these drawbacks on predicting the outcome of pavement deterioration.

OBJECTIVES AND SCOPE

As shown in Figure 1 the main components of the Nonhomogeneous Markov Probabilistic Modeling Program (NHMPMP) are data input and simulation, pavement design subsystem, reliability/performance analysis model, generation of TPMs and condition state vectors, and Bayesian calibration of TPMs. The data input model generates pavement design parameters and actual traffic loads in the form of normal distributions through Monte Carlo simulation. The pavement design subsystem selects an appropriate design equation for each individual pavement section and determines the layer thickness. The reliability/performance analysis model determines the pavement condition states at each stage under the applied traffic loads. The generation and calibration of TPMs are specifically described in a subsequent part of this paper. More specifically, the research described in this paper is targeted at performing the following:

1. To select pavement design models for a specific pavement section and to generate normally distributed design parameters, such as material modulus of each pavement layer, thickness, subgrade soil coefficient, and predicted actual traffic.
2. To determine each element of the time-related TPMs of pavement deterioration through a data input simulation program and reliability evaluation. These TPMs form a nonhomogeneous Markov chain with respect to yearly increased traffic loads and changeable environment.
3. To test the sensitivities of the established TPMs to the major pavement deterioration-related factors or parameters such as traffic characteristics and subgrade strength.
4. To compare the TPMs generated by NHMPMP with those established by the two existing traditional techniques, that is, subjective opinion-related approach and the method based on the collection of long-term performance data.
5. To apply the NHMPMP to dynamic optimization of a pavement rehabilitation program at the network level. The program can be universally used to establish the TPMs for any actually constructed pavement if all of the design parameters are given. In other words a set of TPMs may be established for any individually designed pavement section without being treated as in one of the several roughly classified road categories.

CONCEPT OF NONHOMOGENEOUS MARKOV PROCESSES

A nonhomogeneous Markov process can be characterized by states, stages, and a sequence of TPMs, which are defined as follows.

Stages and States in Pavements

In the present study stages are considered to be a series of consecutive equal periods of time. The time interval is decided according to the pavement deterioration rate of each individual pavement section and the characteristics of the changeable variables. In pavement management a stage is normally defined as 1 year since seasonal climate change is cycled in 1 year and the traffic variable is usually estimated on an annual variation basis. Therefore, in this research one stage becomes 1 year of the life-cycle analysis period.

Deterioration of a pavement is measured in pavement condition states (PCSs), each of which is specifically ranked to a certain level

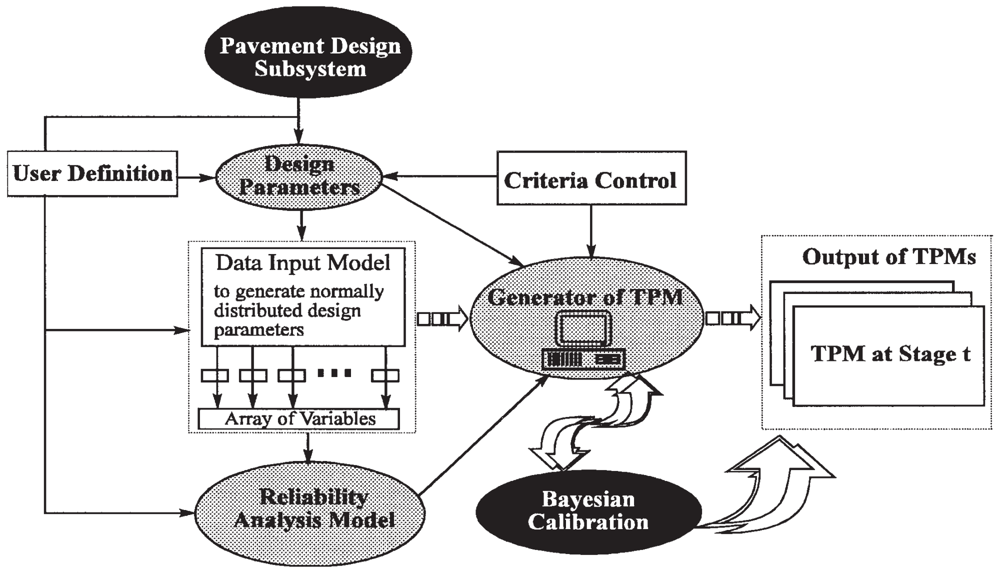


FIGURE 1 Major components of NHMPMP.

according to pavement ride quality, roughness, structural adequacy, surface distresses, and safety. For practical purposes it is convenient to divide PCSs into 10 states, each of which can be further divided if necessary. On the basis of the prediction of pavement deterioration (roughness, etc.) and the selected pavement design model, PCS may refer to such widely used measures as riding comfort index, present serviceability index, pavement quality index, and so on. It should be noted that a higher PCS value represents a better pavement condition, that is, 10 means perfect and 1 means extremely poor. The probability vector indicates the proportions of the pavement section in each of the possible condition states. For example, at a given stage, say stage 5, the probability vector $\boldsymbol{p}(5)$ or $\boldsymbol{PCS}(5) = (0, 0, 0, 0.4, 0.4, 0.2, 0, 0, 0, 0)$ means that there are 10 defined pavement condition states and, after 5 years of opening to traffic, 40 percent of the pavement section will be in State 7, 40 percent will be in State 6, and 20 percent will be in State 5.

TPMs

In most situations the pavement deterioration rate varies over stages (time) because of variations in traffic characteristics (truck volume, growth rate, and traffic configuration) and environmental conditions (precipitation, temperature, freeze-thaw effects, etc.). It is therefore practical to model pavement deterioration by using a Markov probability transition process. If a set of TPMs for a pavement section is provided, the future condition state vector, $\boldsymbol{PCS}(t)$, of the pavement at any stage (year) t can be calculated by the following procedure:

$$\begin{aligned}&\boldsymbol{PCS}(1) = \boldsymbol{P}_1\boldsymbol{PCS}(0),\\&\boldsymbol{PCS}(2) = \boldsymbol{P}_2\boldsymbol{PCS}(1) = \boldsymbol{P}_2\boldsymbol{P}_1\boldsymbol{PCS}(0), \ldots\\&\boldsymbol{PCS}(t) = \boldsymbol{P}_1\boldsymbol{PCS}(t-1) = \boldsymbol{P}_t\boldsymbol{P}_{t-1}\ldots\boldsymbol{P}_1\boldsymbol{PCS}(0)\end{aligned} \tag{1}$$

where $\boldsymbol{Pt}$ is the transition probability matrix at stage (year) t. In a homogeneous Markov transition process the TPM is assumed to be a constant. In other words, the TPMs used at any two different stages are identical, that is, $\boldsymbol{P}_1 = \boldsymbol{P}_2 = \ldots = \boldsymbol{Pt} = \boldsymbol{P}$. This time-independent TPM $\boldsymbol{P}$ can be used to calculate the future multistage transition of pavement condition state by means of the Chapman-Kolmogorov equation. The matrix of n-step transition probabilities, $\boldsymbol{P}^{(n)}$, can be achieved by multiplying the one-step transition probability matrix n times. For example, the n-step transition of a PCS matrix will be

$$P^{(n)} = P\,P\,P\ldots P$$

$$= \begin{bmatrix} p_{10,10}^{(n)} & \cdots & p_{10,j}^{(n)} & \cdots & p_{10,10}^{(n)} \\ \vdots & & \vdots & & \vdots \\ p_{i,10}^{(n)} & \ddots & p_{i,j}^{(n)} & \ddots & p_{i,0}^{(n)} \\ \vdots & & \vdots & & \vdots \\ p_{0,10}^{(n)} & \ddots & p_{0,j}^{(n)} & \ddots & p_{0,0}^{(n)} \end{bmatrix} \tag{2}$$

where $p_{i,j}^{(n)}$ is the probability that the pavement condition state will change from the current state i to state j after n steps (stages) of the transition process.

However, in most cases the TPM at one stage is different from that at another stage; the transition process is then called a nonhomogeneous Markov transition process. Consequently, the multistage transition of pavement condition state is determined by a sequence of transition matrices $\boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_3, \ldots$. Each of these transition probability matrices $\boldsymbol{P}_t$ contains the conditional transition probabilities that hold at time t, given the status at time $t - 1$. The length of each equally divided stage t is decided on the basis of traffic characteristics (traffic volume and growth rate) and environmental factors. It is common practice that the length of each stage t is conveniently defined as 1 year. To analyze the change in transition probabilities from one stage to the next in a nonhomogeneous Markov chain, an accompanying sequence of matrices, $\boldsymbol{C}_1, \boldsymbol{C}_2, \boldsymbol{C}_3, \ldots$, which are called causative matrices, is introduced:

$$\boldsymbol{P}_1\boldsymbol{C}_1 = \boldsymbol{P}_2,\ \boldsymbol{P}_2\boldsymbol{C}_2 = \boldsymbol{P}_3, \ldots \boldsymbol{P}_t\boldsymbol{C}_t = \boldsymbol{P}_{t+1}, \ldots \tag{3}$$

Each causative matrix can therefore be obtained from the following equation:

$$\boldsymbol{C}_t = \boldsymbol{P}_t^{-1}\boldsymbol{P}_{t+1}, \qquad t = 1, 2, \ldots \tag{4}$$

Thus, the causative matrices are analogous to derivatives in calculus as an indication of the rate of change. From these causative matrices the change between the transition matrix at one stage and the transition matrix at the next stage can be determined. It is obvious that a homogeneous Markov process is the special case with $\boldsymbol{C}_t = \boldsymbol{I}$, the identity matrix of dimension $n \times n$, where n is the number of states. When all the transition matrices are different, none of the causative matrices will be the identity matrix.

When all of the causative matrices are equal, that is $\boldsymbol{C}_1 = \boldsymbol{C}_2 = \ldots = \boldsymbol{C}$, the nonhomogeneous Markov chain is called constant causative. It can be verified that

$$\boldsymbol{P}_{t+s} = \boldsymbol{P}_t\boldsymbol{C}^s, \qquad s = 0, 1, \ldots \tag{5}$$

Since $\boldsymbol{C} = \boldsymbol{P}_1^{-1}\boldsymbol{P}_2 = \boldsymbol{P}_2^{-1}\boldsymbol{P}_3$, $\boldsymbol{P}_3$ can be expressed in terms of $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ by the equation $\boldsymbol{P}_3 = \boldsymbol{P}_2\boldsymbol{P}_1^{-1}\boldsymbol{P}_2$, and in general,

$$\boldsymbol{P}_{t+1} = \boldsymbol{P}_t\,\boldsymbol{P}_{t-1}^{-1}\boldsymbol{P}_t \tag{6}$$

Therefore, every transition matrix $\boldsymbol{P}_t$ of a constant causative chain may be expressed in terms of $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ by the equation

$$\boldsymbol{P}_t = (\boldsymbol{P}_2\boldsymbol{P}_1)^{t-2}\,\boldsymbol{P}_2 = \boldsymbol{P}_2(\boldsymbol{P}_1^{-1}\,\boldsymbol{P}_2)^{t-2}, t = 2, 3, \ldots \tag{7}$$

Using a causative matrix $\boldsymbol{C}_t$, the relationship and change involved between two consecutive transition matrices $\boldsymbol{P}_t$ and $\boldsymbol{P}_{t+1}$ can be described.

GENERATION OF TPMs

In the present research the deterioration of each pavement section in a road network is handled individually in terms of assigning a set of TPMs. Each of these sets of matrices is used to model the transition of the pavement condition state corresponding to each of the stages within the life cycle. A unit of scale defining pavement condition state should be small enough so that even a slight deterioration in the condition state can be predicted. For example, if only 10 pavement condition states are defined for a road network, a possible minimum drop within one stage of say 0.2 unit will not be detected. Small deterioration is possible because of a strong pavement structure, very light traffic, or slight environmental influence. In this case the number of condition states should be taken as 50, with a unit of the scale being 0.2.

The basic elements for generating the TPMs include (a) data input simulation, (b) reliability/performance model, and (c) determination of staged TPMs.

Data Input Simulation

Typical input data for a flexible pavement design equation include an initial PCS immediately after construction or rehabilitation, initial annual traffic characteristics (including traffic volume and growth rate, percent trucks, and truck equivalency factor), number of traffic lanes in each direction, subgrade deflection or resilient modulus (M_s), and equivalent granular thickness or structural number. By applying a Monte Carlo simulation technique, each design parameter is generated randomly with a defined distribution.

In the present study each pavement design parameter is generated independently by running a random number-generating subroutine. The main steps of the subroutine can be summarized as follows:

1. Generate U_1 and U_2 as two independent, standard uniformly distributed random numbers. Let $V_i = 2U_i - 1$ for $i = 1$ and 2 and let $W = V_1^2 + V_2^2$.
2. Convert the generated, uniformly distributed numbers into standard normal random numbers. If W is >1, go back to Step 1. Otherwise, let

$$Y = \sqrt{(-2\ln W)/W}$$

$$Z_1 = V_1 Y, \text{and } Z_2 = V_2 Y.$$

Then Z_1 and Z_2 are independent standard normal random numbers.

3. A normally distributed design parameter $X \sim N(\mu, \sigma)$ may be generated as $X = \mu + \sigma Z$, where Z is a standard normal random number generated in Step 2.

Probability Calculation of Pavement Condition Deterioration

Traffic is generally considered a major factor associated with pavement deterioration. Reliability analysis of PCSs at each stage (year) can be performed by comparing the potential traffic loading in equivalent single axle loads (ESALs) that the pavement structure can withstand before its condition state drops to a defined level and the actual predicted annual traffic accumulation. According to the reliability definitions (*4*), the number of ESALs [$N_{\text{pcs}(i)}$] that the pavement can withstand before its PCS drops from its initial state to state i can be calculated. On the other hand, the actual number of ESALs (N_t) that will be applied to the pavement may be estimated from a traffic prediction model, which is based on the existing traffic volume and estimated traffic growth rate. By comparing N_t with $N_{\text{pcs}(i)}$ the reliability R_i at any stage (year) can be calculated by the following equations:

$$R_i = P[(\log N_{\text{pcs(i)}} - \log N_t) > 0]$$

$$= \Phi\left[\frac{\overline{\log N_{\text{pcs}(i)}} - \overline{\log N_t}}{\sqrt{S^2_{\log N_{\text{pcs}(i)}} + S^2_{\log N_t}}}\right] = \Phi(z) \tag{8}$$

where

$\Phi(z)$ = probability distribution function for standard normal random variable,
$\overline{\log N_{\text{pcs}(i)}}$ = mean value of $\log N_{\text{pcs}(i)}$,
$\overline{\log N_t}$ = mean value of $\log N_t$, and
$S_{\log N_{\text{pcs}(i)}}$ and $S_{\log N_t}$ = standard deviations of $\overline{\log N_{\text{pcs}(i)}}$ and $\overline{\log N_t}$, respectively.

Thus, the pavement condition state vector $\mathbf{PCS}(t)$ at stage t is determined. Furthermore, by applying these formulas to calculate the probability $P\,[(\overline{\log N_{\text{pcs}(i)}} - \overline{\log N_{\text{pcs}(j)}}) - (\overline{\log N_{t+1}} - \overline{\log N_t})]$, the transition probability at stage t can be established, where i and j vary from 10 to 0 with an interval of any defined value, for example, 0.2.

In reality, many uncertain factors are involved in all aspects of pavement management systems. According to the results of previous studies and statistical analysis of a large amount of observed pavement performance data (*10*), the actual number of ESALs that cause a pavement to deteriorate from condition state i to state j cannot be calculated without error. Similarly, the predicted actual traffic in terms of ESALs in future years cannot be determined precisely. Consequently, they can be treated as random variables with certain probability distributions, as shown in Figure 2. In Figure 2 a scale of only 10 units of PCSs is defined; $p_{N_t}(N)$ is the probability density function of the predicted actual traffic (ESALs) accumulated in t years, $\bar{N}_{\text{pcs}(i)}$ is the mean value of the traffic (ESALs) that drives the pavement condition state to deteriorate from the initial state to state i, and $p_{N_{\text{pcs}(i)}}$ is the probability density function of $N_{\text{pcs}(i)}$, which is the traffic (ESALs) that forces the pavement to deteriorate from the initial condition state to condition state i.

Determination of Staged TPMs

If a pavement section serves a higher traffic volume with a certain growth rate and the traffic is the major factor affecting pavement deterioration in each stage (year), then the nonhomogeneous Markov TPMs of pavement deterioration may be determined analytically, as follows.

1. Let N_{ij}^s be the maximum number of ESALs that pavement section s can withstand before it drops from condition state i to state j, and let $p_{N_t}^s$ be the probability density function of the predicted actual number of ESALs N_t accumulated in stage t. If N_{ij}^s are deterministic numbers or constants, then the transition probabilities from state i to state j [$p_{ij}^s(t)$], as shown in the upper part of Figure 3, are given by

$$p_{ij}^s(t) = P(N_{i,j+1}^s < N_t < N_{ij}^s)$$

$$= P(N_t < N_{ij}^s) - P(N_t < N_{i,j+1}^s)$$

$$= \int_0^{N_{ij}^s} p_{N_t}^s(N)dN - \sum_{k=1}^{j+1} p_{ik}^s(t),\ j < i \tag{9}$$

where $p_{ij}^s(t)$ is transition probability that pavement section s transitions from condition state i to state j during the period of the year t.

2. However, N_{ij}^s is in general a random variable with probability density function $p_{N_{ij}^s}(N)$, as shown in the lower part of Figure 3. Thus, the following equations can be established:

$$p_{ij}^s(t) = 0,\ j > i,$$

$$p_{ii}^s(t) = P(N_t < N_{ii}^s) = \int_0^\infty \left[\int_0^y p_{N_t,N_{ii}^s}^s(x,y)dx\right]dy$$

$$= \int_0^\infty \left[\int_0^y p_{Ni}^s(x)dx\right]dy,$$

$$p_{ij}^s(t) = P(N_{i,j+1}^s < N_t < N_{ij}^s) = P(N_t < N_{ij}^s) - P(N_t < N_{i,j+1}^s)$$

$$= \int_0^\infty \left[\int_0^y p_{N_t}^s(x)dx\right] p_{N_{ij}^s}(y)dy - \sum_{k=1}^{j+1} p_{ik}^s(t),\quad j < i \tag{10}$$

By applying these equations to each specific section of pavement in a road network, the nonhomogeneous Markov TPM for pavement section s at stage t is given as follows:

$$\boldsymbol{P}^s(t) = \begin{bmatrix} p_{10,10}^s(t) & p_{10,9}^s(t) & \cdots & p_{10,0}^s(t) \\ p_{9,10}^s(t) & p_{9,9}^s(t) & \cdots & p_{9,0}^s(t) \\ \vdots & \vdots & \cdots & \vdots \\ p_{0,10}^s(t) & p_{0,9}^s(t) & \cdots & p_{0,0}^s(t) \end{bmatrix} \tag{11}$$

BAYESIAN UPDATE OF TPMS

As described earlier the nonhomogeneous Markov TPMs for pavement deterioration may be efficiently determined if the pavement

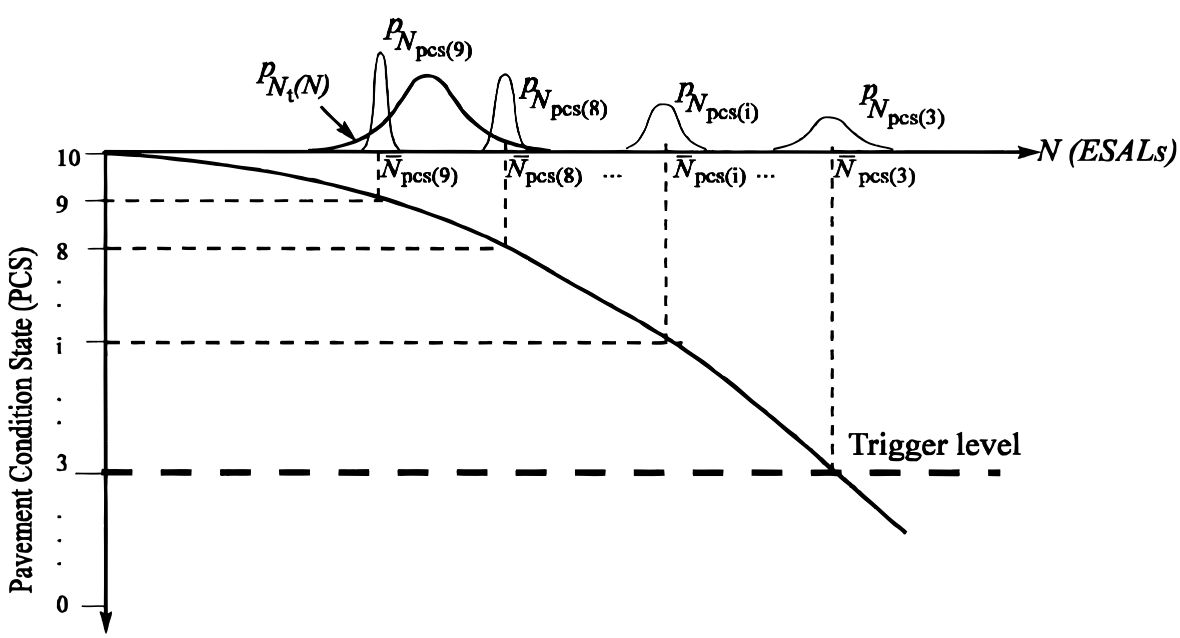


FIGURE 2 Distribution of predicted actual traffic and pavement performance curves.

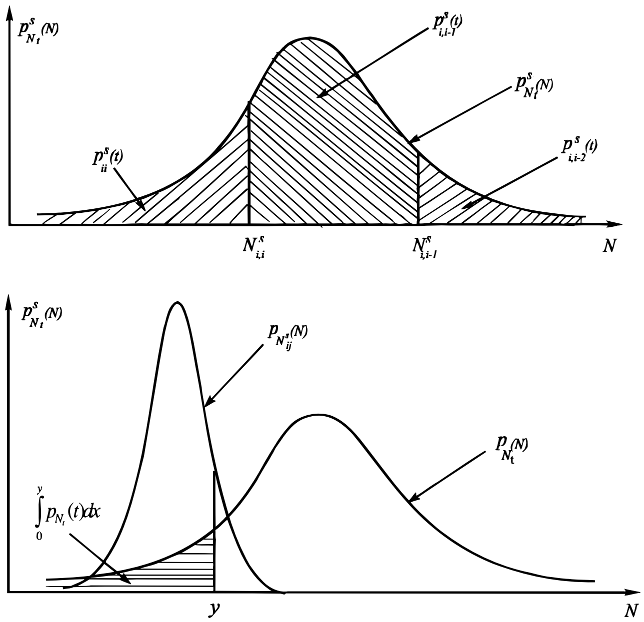


FIGURE 3 Probability calculation of pavement condition transition from state$_i$ to state$_j$ at year$_t$.

design model and the corresponding design parameters are given. In addition, if a set of actually observed pavement performance data for a specific pavement section is available, then the Bayesian posterior probability approach can be applied to update the established TPMs. For this research the collection of multiple years of pavement performance data for the Ontario highway network has been planned. The main purpose is to use the actually observed data in the calibration of the established TPMs through the Bayesian technique. When the data are collected and properly processed, they will be used in the Bayesian posterior probability calibration model to update the TPMs established by NHMPMP. The model and procedure in developing the Bayesian updated TPMs are explained in the following discussion.

The basic theoretical foundation that connects a Markov process and the Bayesian posterior probability approach has been summarized previously (*11*). It is based on the assumption that the prior distribution function of the matrix of the transition probabilities belongs to a family of distributions that is closed under consecutive sampling. The matrix beta density function is the natural conjugate distribution for the likelihood function of the consecutive sampling rule. Moreover, many of the properties of arbitrary families of distributions that are closely related to the consecutive sampling rule are related to characteristics of the matrix beta distribution. In the following discussion, some of the main results of matrix beta distribution are summarized.

Concept of Matrix Beta Distribution

A $K \times N$ matrix $\boldsymbol{P} = [p_{ij}^k]$ is called a generalized stochastic matrix if each element of the matrix is nonnegative and the sum of each row is unit 1, where p_{ij}^k is the probability that the system makes a transition to state j, given that it is currently in state i and the kth alternative is used. The $K \times N$ random generalized stochastic matrix $\boldsymbol{P}$ is said to have the matrix beta distribution with parameter $\boldsymbol{M} = [m_{ij}^k]$ if $\boldsymbol{P}$ has the joint density function

$$f_{M\beta}^{K,N}(P|M) = \begin{cases} k(M)\prod_{\substack{i=1\\j=1}}^{N}\prod_{k=1}^{K_i}(p_{ij}^k)^{m_{ij}^k-1}, P\in S_{KN}, \\ 0, \qquad \text{elsewhere} \end{cases} \tag{12}$$

where S_{KN} is the set of all $K \times N$ generalized stochastic matrices. The normalizing constant $k(\boldsymbol{M})$ is given by

$$k(M) = \prod_{i=1}^{N}\prod_{k=1}^{K_i}\frac{\Gamma(M_i^k)}{\prod_{j=1}^{N}\Gamma(M_{ij}^k)} \tag{13}$$

where

$$M_i^k = \sum_{j=1}^{N} m_{ij}^k, k = 1, 2, \ldots K_i, \; i, j = 1, 2, \ldots N \tag{14}$$

K_i is the number of alternatives that the decision maker can choose when the system is in state i, and N is the number of states that the system can occupy. The parameter $\boldsymbol{M}$ is a $K \times N$ matrix such that

$$m_{ij}^k > 0, k = 1, 2, \ldots K_i, \; i, j = 1, 2, \ldots N$$

For $k = 1, 2, \ldots, K_i$, and $i, j = 1, 2, \ldots, N$, the means and variances of the elements of $\boldsymbol{P}$ are given by the following formulas

$$E[\tilde{p}_{ij}^k] = \frac{m_{ij}^k}{M_i^k} = \bar{p}_{ij}^k \tag{15}$$

and

$$Var[\tilde{p}_{ij}^k] = \frac{m_{ij}^k(M_i^k - m_{ij}^k)}{(M_i^k)^2(M_i^k+1)} = \frac{\bar{p}_{ij}^k(1-\bar{p}_{ij}^k)}{M_i^k+1} \tag{16}$$

Let $\boldsymbol{x}_n = (x_0, x_1, \ldots, x_n)$ be a sample of n transitions observed under the consecutive sampling rule in which x_0 is the initial state, which is known in advance of sampling. Denote f_{ij}^k the number of transitions in $\boldsymbol{x}_n$ from state i to state j under the kth alternative in state i ($k = 1, 2, \ldots, k_i$; $i, j = 1, 2, \ldots, N$) and define the transition count of the sample as the $K \times N$ matrix $\boldsymbol{F} = [f_{ij}^k]$, $K = \sum_{i=1}^{N} K_i$.

Theorem: Let $\boldsymbol{P}$ have the matrix beta distribution with parameter $\boldsymbol{M}'$ and suppose that a sample with transition count $\boldsymbol{F}$ is observed under the consecutive sampling rule with noninformative stopping. Then the posterior distribution of $\boldsymbol{P}$ is matrix beta with parameter

$$\boldsymbol{M}'' = \boldsymbol{M}' + \boldsymbol{F} \tag{17}$$

Application of Theorem in Probabilistic Modeling of Pavement Deterioration

For the transition probability matrix $\boldsymbol{P} = [p_{ij}^{n,k}]$, it is known that

$$E[\tilde{p}_{ij}^{n,k}] = \bar{p}_{ij}^{n,k}, \quad Var[p_{ij}^{n,k}] = (\sigma_{ij}^{n,k})^2, i, j = 1, 2, \ldots N \tag{18}$$

From Equations 14 and 15,

$$\frac{m_{ij}^{n,k}}{M_i^{n,k}} = \bar{p}_{ij}^{n,k} \tag{19}$$

$$\frac{\bar{p}_{ij}^{n,k}(1-\bar{p}_{ij}^{n,k})}{M_i^{n,k}+1}=(\sigma_{ij}^{n,k})^2 \tag{20}$$

where $M_i^{n,k} = \sum_{j=1}^{N} m_{ij}^{n,k}$. Summing up Equation 20 for j from 1 to N yields

$$\frac{1-\sum_{j=1}^{N}(\bar{p}_{ij}^{n,k})^2}{M_i^{n,k}+1}=\sum_{j=1}^{N}(\sigma_{ij}^{n,k})^2 \tag{21}$$

That is,

$$M_i^{n,k}=\frac{1-\sum_{j=1}^{N}(\bar{p}_{ij}^{n,k})^2}{\sum_{j=1}^{N}(\sigma_{ij}^{n,k})^2}-1 \tag{22}$$

Letting $\delta_{ij}^{n,k} = \sigma_{ij}^{n,k}/\bar{p}_{ij}^{n,k}$ be the coefficient of variance, then $\sigma_{ij}^{n,k} = \delta_{ij}^{n,k}\bar{p}_{ij}^{n,k}$. From Equation 19

$$m_{ij}^{n,k}=M_i^{n,k}\bar{p}_{ij}^{n,k}. \tag{23}$$

By using the method presented in the previous section, that is the reliability performance concepts and a Monte Carlo simulation approach, the TPMs of pavement performance $\boldsymbol{P}^{(n)} = [\bar{p}_{ij}^{n,k}]$, $n = 1, 2, \ldots, N_y$, have been obtained. Knowing the mean values $\bar{p}_{ij}^{n,k}$ and the given coefficient of variations $\delta_{ij}^{n,k}$ of the transition probabilities $\bar{p}_{ij}^{n,k}$, the element $m_{ij}^{n,k}$ of the parameter $\boldsymbol{M}$ of the matrix beta distribution can be determined by using Equations 22 and 23. The parameter $\boldsymbol{M}$ obtained is the parameter of the prior distribution, that is, $\boldsymbol{M}'$.

The coefficients of variation $\delta_{ij}^{n,k}$ may be used as control parameters of the confidence level of a pavement manager on the observed data. If the coefficients of variation are small, the values of the elements of $\boldsymbol{M}'$ are large and the relative effect of the observed data (transition count $\boldsymbol{F}$) is small. A pavement manager may select a smaller $\delta_{ij}^{n,k}$ when the observed data are scattered or are not very accurate. On the other hand, if the coefficients of variation are large, the entries of $\boldsymbol{M}'$ are small and the relative effect of the observed data (transition count $\boldsymbol{F}$) is large. A pavement manager may select a larger $\delta_{ij}^{n,k}$ when the conditions of a pavement are significantly different from those specified in the code or a large data base of pavement performance for pavements with similar characteristics is available.

SENSITIVITY OF GENERATED TPMs TO DESIGN PARAMETERS

It is important to know that variations in each element of the TPMs will affect the predicted values of future pavement condition states. Therefore, it is meaningful to perform a sensitivity analysis to investigate the variation in the TPMs generated if the design parameters of a pavement take on other possible values.

A case study of Highway 402 is used as an illustration. Highway 402 is a 102-km, four-lane rural expressway built in southern Ontario during the early 1980s. The Ontario Pavement Analysis of Cost (OPAC) flexible pavement design method (*12*) has been selected to perform the sensitivity analysis. The OPAC design model considers both the traffic effects (P_T) and the environmental effects (P_E) on pavement deterioration. Traffic- and environment-related deterioration, P_T and P_E (on a scale of 0 to 10), respectively, are calculated by the following equations:

$$P_T = 2.445\Psi + 8.805\Psi^3 \tag{24}$$

$$P_E = \left(P_0 - \frac{P_0}{1+Bw}\right)(1-e^{-\alpha Y}) \tag{25}$$

where $\Psi = 3.7238 \times 10^{-6} w^6 N$; w is the subgrade deflection (in millimeters) and is determined by

$$w=\frac{9{,}000\times 25.4}{2M_s\left(0.9He\sqrt[3]{\dfrac{M_2}{M_s}}\right)\sqrt{1+\dfrac{6.4}{0.9He\sqrt[3]{\dfrac{M_2}{M_s}}}}} \tag{26}$$

where

- P_0 = as-built riding comfort index (scale of 0 to 10);
- He = total pavement equivalent granular base thickness;
- N = the number of ESALs that changes the pavement by an amount P_T;
- M_2 = modulus of granular base layer;
- M_s = modulus of subgrade soil;
- B = regional factor 1, $B = 60$ in southern Ontario;
- α = regional factor 2, $\alpha = 0.006$ in southern Ontario; and
- Y = number of years.

The design parameters used in the sensitivity analysis are subgrade deflection or soil modulus, traffic growth rate, and pavement thickness (total equivalent granular base thickness). Some of the major parameters used in the pavement design of this highway are provided in Table 1. Estimation of ESAL applications of 80 000 N is based on the OPAC traffic input model, that is,

$$N=\frac{N_f}{A_p}\left[\frac{2AADT_i}{(AADT_i+AADT_f)}Y+\frac{AADT_f-AADT_i}{A_p(AADT_i+AADT_f)}Y^2\right] \tag{27}$$

$$N_f=\frac{A_p}{2}\left[\left(\frac{AADT_i}{2}\times \text{days}\times T_i\times LDF_i\times TF_i\right)+\left(\frac{AADT_f}{2}\times \text{days}\times T_f\times LDF_f\times TF_f\right)\right] \tag{28}$$

where

- T = truck fraction;
- $AADT$ = one-directional average annual daily traffic;
- LDF = lane distribution factor (0.8 for four-lane highways);
- TF = track factor;
- days = number of days per year for truck traffic (generally 300); and
- i and f = initial and final, respectively.

On the basis of the input design parameters, the TPMs of the pavement at different stages (years) can be established by running the NHMPMP. The generated TPM of the pavement deterioration at Stage 1 is provided in Table 2. It should be pointed out that because of space limitations of the paper, only parts of the TPM elements are listed in Table 2. Some of the TPMs (Stages 1, 5, and 10), which indicate the nonhomogeneous Markov chain of pavement deterioration when annual traffic growth rate is 8.0 percent, are presented in Table 2.

TABLE 1 Input Design Parameters for Calculating the TPMs on Pavement Section on Highway 402

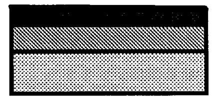
Design Parameters	Description
Structure	90 mm asphalt surface, $M_1 = 400{,}000$ 150 mm granular base, $M_2 = 50{,}000$ 300 mm granular subbase, $M_3 = 15{,}000$ $He = 90 \times 2 + 150 \times 1 + 300 \times 0.67 = 530$ mm Subgrade layer $M_s = 5{,}000$ 
AADT, two directions	$AADT_1 = 6500$, $AADT_{20} = 9012$, traffic growth rate is 2.5% per year, analysis period $A_p = 20$, $LDF_i = LDF_f$ (lane distribution factor) = 0.8
Truck factor	Truck % is 20 at initial year and 35 at the end of analysis year; truck factor is 0.91 at the initial year and 1.14 at the end of analysis year.
ESALs (thousands)	Accumulated ESALs in one year and 20 years are 251 and 6519, respectively, coefficient of variance is 0.1.

TABLE 2 Tests of Sensitivity of TPMs to Time-Related Traffic Volume

(a) TPM of the Pavement Deterioration at Stage 1

	10	9.6	9.2	8.8	8.4	8.0	7.6	7.2	6.8	6.4	6.0	5.6	5.2	4.8	4.4	4.0	3.6
10	0.000	0.981	0.019														
9.6		0.002	0.985	0.013													
9.2			0.006	0.982	0.012												
8.8				0.024	0.966	0.010											
8.4					0.043	0.949	0.008										
8.0						0.070	0.926	0.004									
7.6							0.109	0.888	0.003								
7.2								0.164	0.833	0.003							
6.8									0.235	0.763	0.002						
6.4										0.320	0.679	0.001					
6.0											0.403	0.594	0.003				
5.6												0.473	0.525	0.002			
5.2													0.543	0.456	0.001		

(b) TPM of the Pavement Deterioration at Stage 5

	10	9.6	9.2	8.8	8.4	8.0	7.6	7.2	6.8	6.4	6.0	5.6	5.2	4.8	4.4	4.0	3.6
10	0.001	0.794	0.190	0.016													
9.6		0.002	0.808	0.176	0.015												
9.2			0.004	0.818	0.167	0.012											
8.8				0.009	0.825	0.155	0.012										
8.4					0.016	0.832	0.141	0.012									
8.0						0.028	0.828	0.133	0.012								
7.6							0.036	0.831	0.124	0.010							
7.2								0.043	0.836	0.112	0.010						
6.8									0.062	0.828	0.101	0.010					
6.4										0.073	0.826	0.093	0.009				
6.0											0.090	0.818	0.084	0.009			
5.6												0.109	0.801	0.083	0.008		
5.2													0.132	0.786	0.075	0.008	

(c) TPM of the Pavement Deterioration at Stage 10

	10	9.6	9.2	8.8	8.4	8.0	7.6	7.2	6.8	6.4	6.0	5.6	5.2	4.8	4.4	4.0	3.6
10	0.000	0.436	0.400	0.120	0.029	0.017											
9.6		0.000	0.446	0.394	0.115	0.029	0.017										
9.2			0.002	0.456	0.385	0.112	0.029	0.017									
8.8				0.002	0.470	0.377	0.107	0.028	0.017								
8.4					0.002	0.477	0.376	0.102	0.027	0.017							
8.0						0.002	0.492	0.362	0.101	0.028	0.016						
7.6							0.005	0.499	0.355	0.099	0.027	0.016					
7.2								0.007	0.508	0.349	0.096	0.025	0.016				
6.8									0.010	0.513	0.343	0.095	0.024	0.016			
6.4										0.013	0.519	0.336	0.093	0.026	0.014		
6.0											0.017	0.528	0.327	0.090	0.026	0.013	
5.6												0.024	0.533	0.319	0.089	0.023	0.013
5.2													0.028	0.535	0.318	0.085	0.035

Tests of Sensitivity to Subgrade Deflection

The sensitivities of the TPMs to subgrade soil modulus or deflection (*w*) is studied by varying *w* from 0.6604 mm (0.026 in.) to 0.7336 mm (0.029 in.) with an increment of 0.0254 mm, whereas all other design parameters remain the same as in Table 1. The TPMs determined by the NHMPMP are provided in Table 3. This result implies that the strength of subgrade soil is most critical to pavement deterioration, as might be expected.

Tests of Sensitivity to Pavement Thickness

If the subgrade coefficient and the traffic growth rate are 5,000 and 2.5 percent, respectively, the other design parameters in Table 1 remain unchanged, and the sensitivities of the TPMs to different total pavement equivalent thickness are given in Table 4. The results in Table 4 indicate that the thicker pavement has a high probability of retaining its state in a 1-year transition period, whereas the thinner pavement tends to deteriorate mostly to the next lower condition state.

In addition, the sensitivities of the TPMs to different traffic growth rates (2, 5, and 8 percent) were studied. It was concluded that the greater the traffic growth rate, the greater the difference in any two consecutive TPMs or the more nonhomogeneous the Markov chain is for parameter deterioration.

SUMMARY AND CONCLUSIONS

In this paper application of the reliability concepts has been extended so that they can be used at the network level of pavement management. The condition state of a pavement section at each year (stage) can be expressed in the form of a probability vector. The use

TABLE 3 Tests of Sensitivity of TPMs to Different Subgrade Strengths

(a) TPM at Stage 5 with Subgrade Deflection = 0.6604 mm

	10	9.6	9.2	8.8	8.4	8.0	7.6	7.2	6.8	6.4	6.0	5.6	5.2	4.8	4.4	4.0	3.6
10	0.2053	0.6996	0.0775	0.0130	0.0045												
9.6		0.236	0.675	0.073	0.012	0.004											
9.2			0.272	0.644	0.069	0.012	0.004										
8.8				0.311	0.609	0.064	0.011	0.004									
8.4					0.344	0.581	0.060	0.010	0.004								
8.0						0.380	0.550	0.057	0.010	0.004							
7.6							0.414	0.520	0.053	0.009	0.004						
7.2								0.447	0.490	0.050	0.009	0.004					
6.8									0.478	0.462	0.048	0.008	0.004				
6.4										0.509	0.434	0.045	0.008	0.003			
6.0											0.539	0.408	0.0429	0.008	0.003		
5.6												0.567	0.383	0.040	0.008	0.003	
5.2													0.592	0.359	0.039	0.007	0.002

(b) TPM at Stage 5 with Subgrade Deflection = 0.6858 mm

	10	9.6	9.2	8.8	8.4	8.0	7.6	7.2	6.8	6.4	6.0	5.6	5.2	4.8	4.4	4.0	3.6
10	0.1321	0.6707	0.1381	0.0400	0.019												
9.6		0.155	0.661	0.127	0.038	0.019											
9.2			0.176	0.647	0.122	0.036	0.019										
8.8				0.211	0.617	0.120	0.033	0.019									
8.4					0.235	0.600	0.116	0.030	0.021								
8.0						0.260	0.581	0.113	0.027	0.019							
7.6							0.292	0.557	0.106	0.026	0.019						
7.2								0.325	0.531	0.100	0.025	0.019					
6.8									0.356	0.503	0.098	0.025	0.018				
6.4										0.387	0.478	0.092	0.025	0.018			
6.0											0.411	0.458	0.089	0.025	0.017	0.018	
5.6												0.432	0.441	0.088	0.022	0.018	
5.2													0.457	0.425	0.079	0.022	0.018

(c) TPM at Stage 5 with Subgrade Deflection = 0.7112 mm

	10	9.6	9.2	8.8	8.4	8.0	7.6	7.2	6.8	6.4	6.0	5.6	5.2	4.8	4.4	4.0	3.6
10	0.078	0.601	0.210	0.056	0.027	0.011	0.018										
9.6		0.094	0.599	0.200	0.053	0.026	0.011	0.018									
9.2			0.107	0.597	0.194	0.051	0.023	0.011	0.018								
8.8				0.126	0.585	0.188	0.052	0.022	0.010	0.018							
8.4					0.147	0.571	0.185	0.051	0.020	0.009	0.018						
8.0						0.164	0.565	0.178	0.047	0.021	0.008	0.018					
7.6							0.185	0.557	0.167	0.046	0.020	0.009	0.017				
7.2								0.212	0.539	0.158	0.046	0.020	0.009	0.017			
6.8									0.230	0.532	0.147	0.048	0.018	0.009	0.017		
6.4										0.252	0.518	0.139	0.048	0.018	0.009	0.017	
6.0											0.276	0.505	0.130	0.046	0.019	0.009	0.016
5.6												0.305	0.480	0.131	0.041	0.019	0.025
5.2													0.329	0.462	0.126	0.041	0.033

TABLE 4 Tests of Sensitivity of TPMs to Different Pavement Thicknesses

(a) TPM at Stage 5 with He = 380 mm

	10	9.6	9.2	8.8	8.4	8.0	7.6	7.2	6.8	6.4	6.0	5.6	5.2	4.8	4.4	4.0	3.6
10	0.052	0.360	0.213	0.124	0.075	0.042											
9.6		0.059	0.359	0.208	0.125	0.074	0.041										
9.2			0.064	0.361	0.202	0.126	0.073	0.041									
8.8				0.067	0.360	0.200	0.129	0.071	0.040								
8.4					0.070	0.365	0.196	0.128	0.068	0.041							
8.0						0.077	0.367	0.189	0.130	0.066	0.039						
7.6							0.080	0.366	0.191	0.128	0.065	0.038					
7.2								0.092	0.364	0.184	0.126	0.065	0.037				
6.8									0.095	0.363	0.186	0.124	0.065	0.036			
6.4										0.101	0.364	0.182	0.124	0.063	0.036		
6.0											0.108	0.362	0.182	0.120	0.062	0.037	
5.6												0.111	0.362	0.180	0.121	0.061	0.036

(b) TPM at Stage 5 with He = 430 mm

	10	9.6	9.2	8.8	8.4	8.0	7.6	7.2	6.8	6.4	6.0	5.6	5.2	4.8	4.4	4.0	3.6
10	0.259	0.562	0.117	0.035	0.012	0.016											
9.6		0.283	0.541	0.114	0.036	0.011	0.016										
9.2			0.3063	0.5195	0.1131	0.0360	0.0100	0.016									
8.8				0.326	0.504	0.111	0.034	0.010	0.016								
8.4					0.341	0.494	0.106	0.034	0.010	0.016							
8.0						0.365	0.475	0.101	0.034	0.010	0.016						
7.6							0.389	0.454	0.099	0.033	0.010	0.016					
7.2								0.403	0.446	0.095	0.032	0.009	0.016				
6.8									0.423	0.429	0.091	0.032	0.010	0.015			
6.4										0.435	0.420	0.090	0.031	0.011	0.013		
6.0											0.450	0.409	0.089	0.030	0.010	0.013	
5.6												0.467	0.396	0.089	0.027	0.011	0.012

(c) TPM at Stage 5 with He = 510 mm

	10	9.6	9.2	8.8	8.4	8.0	7.6	7.2	6.8	6.4	6.0	5.6	5.2	4.8	4.4	4.0	3.6
10	0.806	0.191	0.004														
9.6		0.823	0.175	0.003													
9.2			0.836	0.162	0.003												
8.8				0.853	0.145	0.003											
8.4					0.862	0.136	0.003										
8.0						0.872	0.126	0.003									
7.6							0.886	0.112	0.003								
7.2								0.899	0.099	0.003							
6.8									0.909	0.089	0.003						
6.4										0.910	0.088	0.003					
6.0											0.918	0.081	0.002				
5.6												0.928	0.071	0.002			
5.2													0.934	0.065	0.002		

of a nonhomogeneous Markov transition process in the modeling of pavement deterioration has taken both the actual traffic and the environmental conditions into consideration. This allows the random nature of pavement behavior and different deterioration rates with time to be included in the modeling of the pavement deterioration process.

Probability vectors of future condition states for a pavement section in a road network may be interpreted as the probability that the pavement will be in each of the possible condition states or the percentage of the pavement in each category of condition states in terms of length. Probability vector is an important concept in pavement dynamic programming and in maintenance and rehabilitation optimization at the network level.

This new technique could be used in pavement management to perform four important functions: (a) to simulate the probabilistic behavior of pavement deterioration in predicting the pavement serviceability level at different stages, (b) to establish the nonhomogeneous Markov TPMs by considering actually changeable traffic and environmental effects on pavements, (c) to determine the year(s) when rehabilitation is needed and a rehabilitation priority program for pavement management at the network level, and (4) to provide a foundation for optimization and dynamic programming of pavement rehabilitation and maintenance alternatives.

One of the advantages of this newly developed probabilistic transition prediction methodology over the existing ones is that it avoids the problems of either processing many individual, possibly biased, subjective opinions or requiring the observation of long-term performance condition data.

REFERENCES

1. Li, N., W. Xie, and R. Haas. A New Application of Markov Modeling and Dynamic Programming in Pavement Management. *Proc., Second International Conference on Road and Airfield Pavement Technology*, Singapore, Vol. 2, Sept. 1995, pp. 683–691.
2. Lytton, R. Concepts of Pavement Performance Prediction and Modeling. *Proc., Second North American Conference on Managing Pavements*, Canada, Vol. 2, Nov. 1987, pp. 2.3–2.20.
3. Haas, R., R. Hudson, and J. Zaniewski. *Modern Pavement Management*. Krieger Publishing Co., Malabar, Fl., 1994.

4. Darter, M. I., and W. R. Hudson. *Probabilistic Design Concepts Applied to Flexible Pavement System Design*. Report No. 123-18. Center for Highway Research, University of Texas at Austin, 1973.
5. Alsherri, A., and K. P. George. Reliability Model for Pavement Performance. *ASCE Journal of Transportation Engineering*, Vol. 114, No. 5, 1988, pp. 294–360.
6. Kher, R. and W. D. Cook. PARS: The MTC Model for Program and Financial Planning in Pavement Rehabilitation. *Proc., First North American Conference on Managing Pavements*, Vol. 3, Sept. 1985, pp. 6.23–6.40.
7. Karan, M. *Municipal Pavement Management System*. Ph.D. thesis. Department of Civil Engineering, University of Waterloo, Waterloo, Ontario, Canada, 1977.
8. Feighan, K. J., M. Y. Shahin, and K. C. Sinha. A Dynamic Programming Approach to Optimization for Pavement Management Systems. *Proc., Second North American Conference on Managing Pavements*, Toronto, Ontario, Canada, Vol. 1, Nov. 1987, pp. 195–206.
9. Wang, K. C. P., J. Zaniewski, and G. Way. Probabilistic Behavior of Pavements. *ASCE Journal of Transportation Engineering*, Vol. 120, No. 3, 1994, pp. 358–375.
10. Hudson, W. R. *State-of-the-Art in Predicting Pavement Reliability from Input Variability*. Report No. FAA-RD-75-207. U.S. Army Engineer Waterways Experiment Station, Vicksburg, Miss. 1975.
11. Martin, J. J. *Bayesian Decision Problems and Markov Chains*. John Wiley and Sons, Inc., New York, 1967.
12. Ontario Ministry of Transportation. *Pavement Design and Rehabilitation Manual*. Ministry of Transportation of Ontario, Downsview, Ontario, Canada, 1990.

Publication of this paper sponsored by Committee on Pavement Management Systems.