Seismic Pushover Analysis Using AASHTO *Guide Specifications for LRFD Seismic Bridge Design*

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Learning Objective

- Given the plans for a conventional bridge pier, calculate the lateral force-displacement (*pushover*) response of the system necessary for the seismic design of bridges in accordance with the **AASHTO Guide Specifications for LRFD Seismic Bridge Design** (SGS).

- Focus is *pushover analysis* not modeling, seismic displacement demand or capacity design

Overview

- Flexural mechanics refresher
- Generation of Moment-Curvature (M - ϕ)
- Material models and failure strains
- Hand checking M - ϕ
- Analytical plastic hinge length, L_p
- Force-Displacement *pushover* example
- Hand checking force-displacement
- Design Example – Two circular column bent

Why Pushover?

- SGS is primarily a displacement-based approach
- More rational approach than force-based method of AASHTO LRFD
- Seismic Design Category D (SDC D) requires pushover analysis

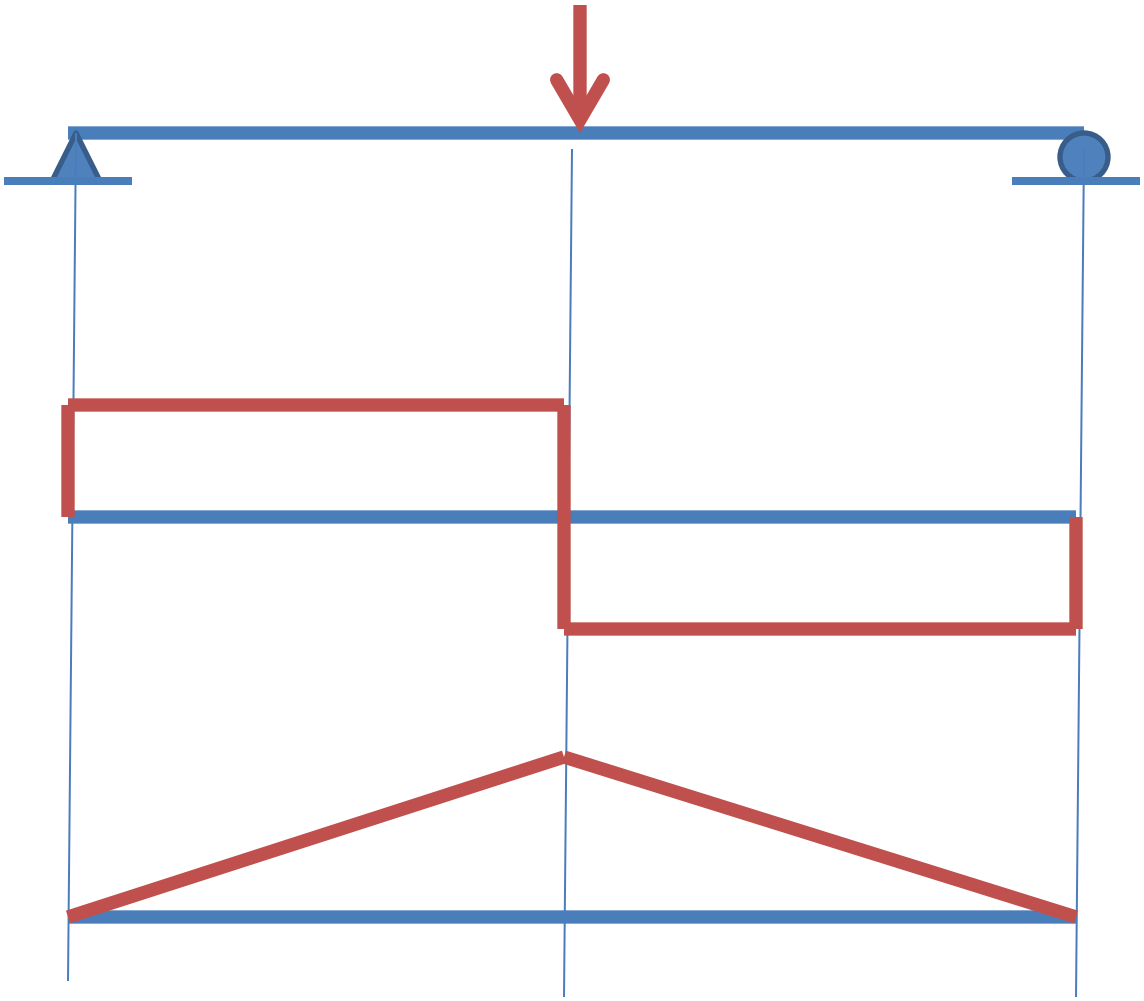
Why Pushover?

- Need to verify that the seismic displacement demand, Δ^L_D, exceed the displacement capacity, Δ^L_C
- Results can be used to help create the seismic model and subsequent displacement demands
- Provides for a better understanding of bridge response and behavior

Flexural Mechanics

- Relationship between force, shear, moment, curvature, slope and deflection
  - *Integration*
  - Moment area
  - Energy methods
  - Stiffness methods

Force – Displacement Relationship

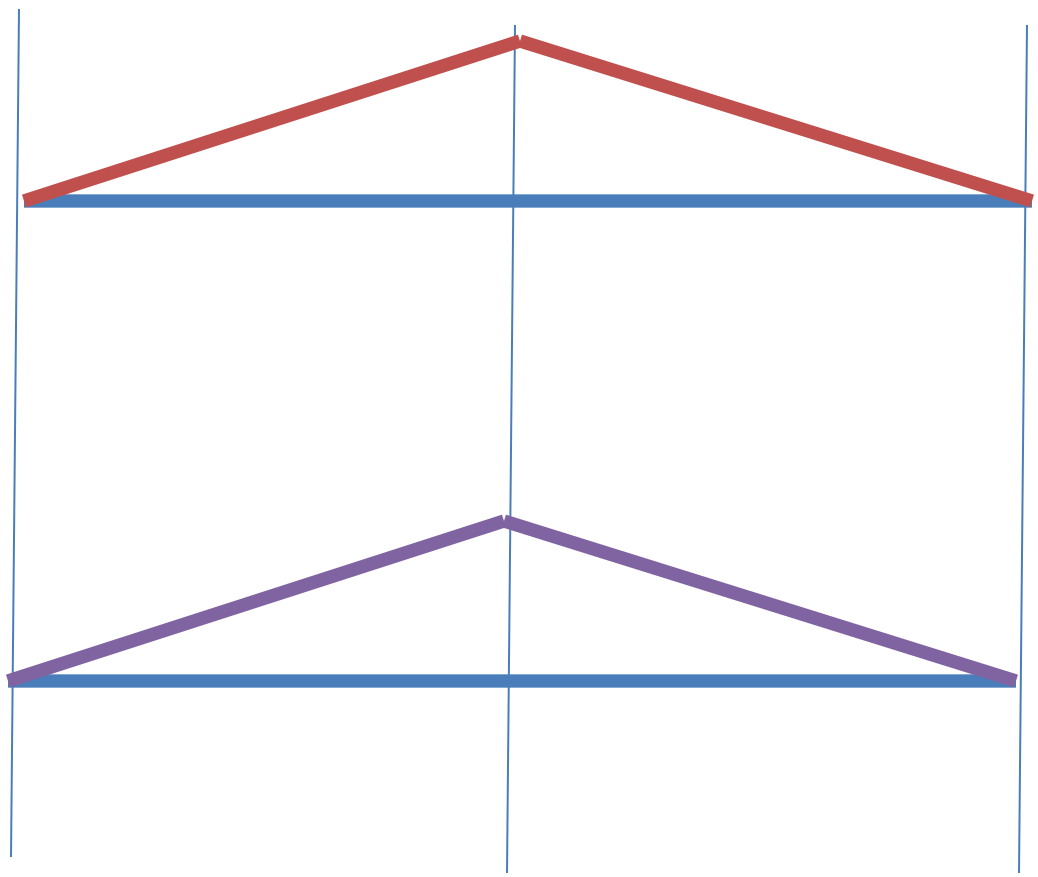


$$P = load$$

$$V = \int P \cdot dx$$

$$M = \int V \cdot dx$$

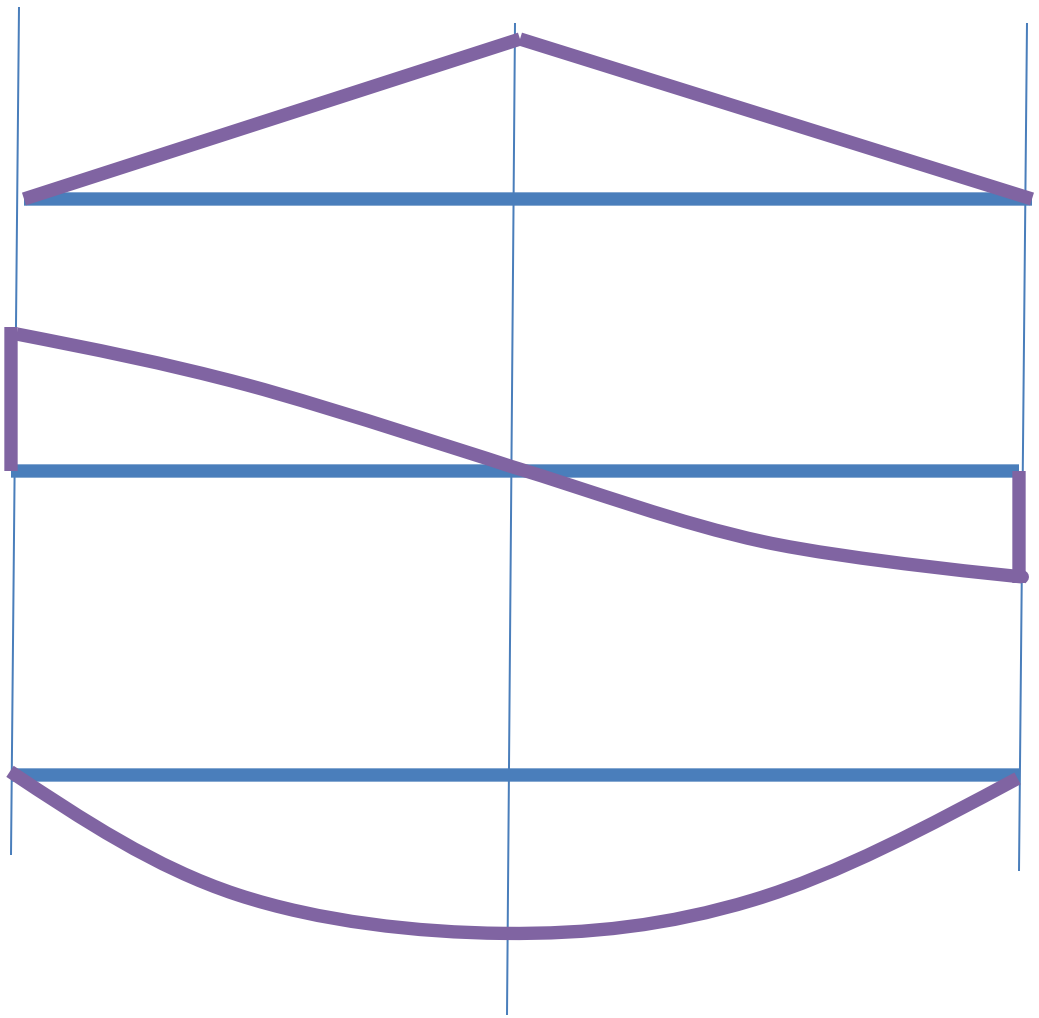
Force – Displacement Relationship



$$M = \int V \cdot dx$$

$$\varphi = \frac{M}{EI}$$

Force – Displacement Relationship



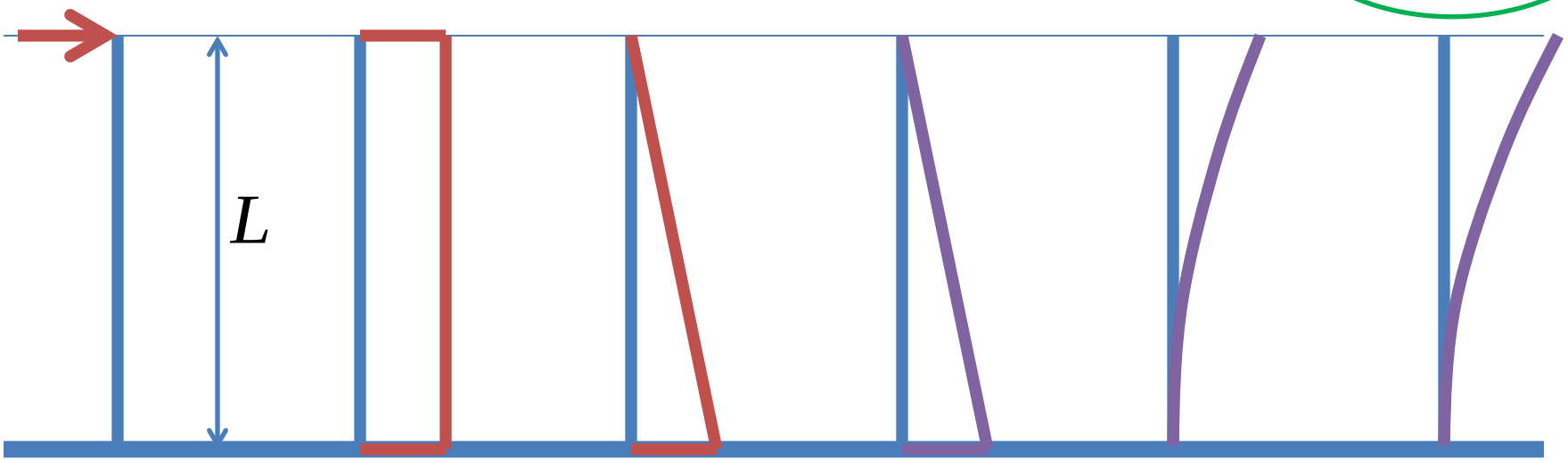
$$\varphi = \frac{M}{E \cdot I}$$

$$\theta = \int \varphi \cdot dx$$

$$\Delta = \int \theta \cdot dx$$

Example - Cantilever

$$\theta_L = \frac{\varphi_o \cdot L}{2} \qquad \Delta_L = \frac{\varphi_o \cdot L^2}{3}$$



$$M_o = P \cdot L \qquad \varphi_o = \frac{M_o}{E \cdot I} = \frac{P \cdot L}{E \cdot I}$$

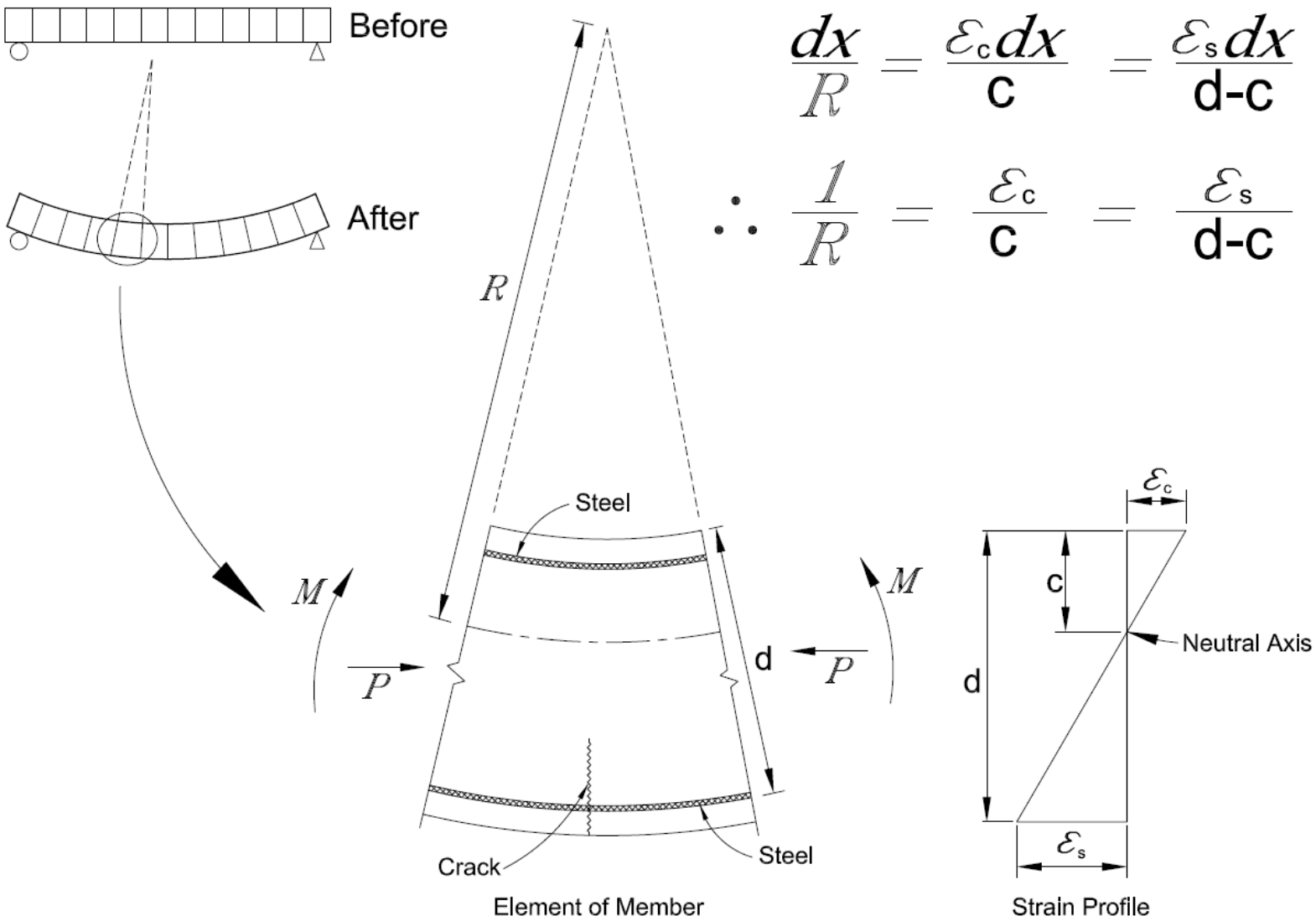
Compatibility

- Bernoulli-Euler assumption – sections that are plane (linear) before bending remain plane after bending

- Perfect bond between the steel reinforcing bars and surrounding concrete

- Material (constitutive σ-ε) models are representative of actual material response (confinement, strain hardening, buckling, spalling, cracking, creep, strain-rate, shear, etc.)

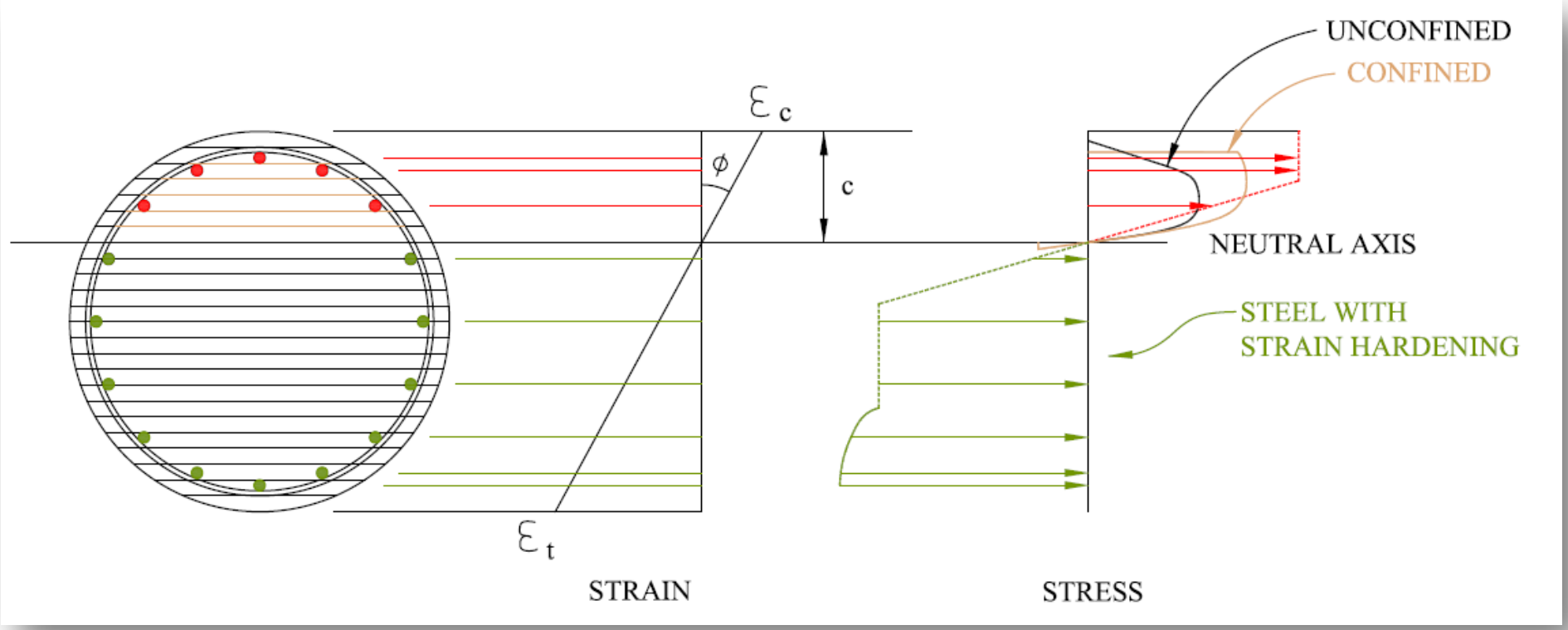
Moment – Curvature (M - ϕ)



$$\frac{dx}{R} = \frac{\varepsilon_c dx}{c} = \frac{\varepsilon_s dx}{d-c}$$

$$\therefore \frac{1}{R} = \frac{\varepsilon_c}{c} = \frac{\varepsilon_s}{d-c}$$

Moment – Curvature (M - ϕ)

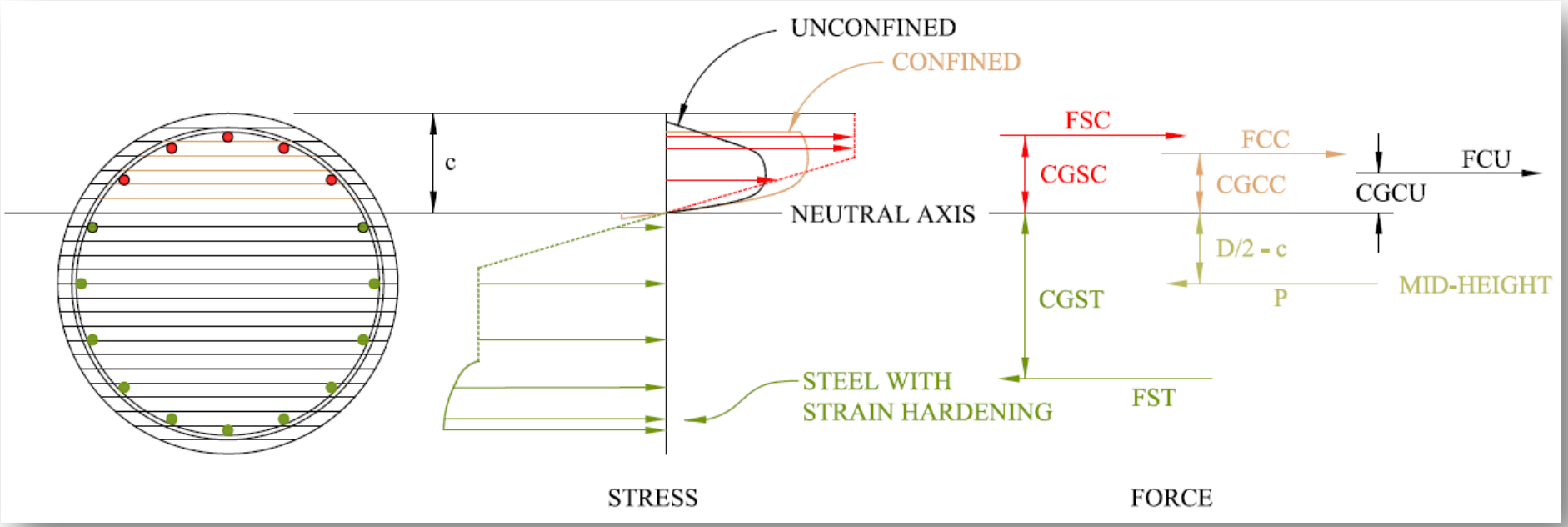


- For small angles, curvature (ϕ) may be calculated as:

$$\phi = \varepsilon_c / c = \varepsilon_t / (D - c)$$

$$\phi = \textit{strain / distance from neutral axis}$$

Moment – Curvature (M - ϕ)

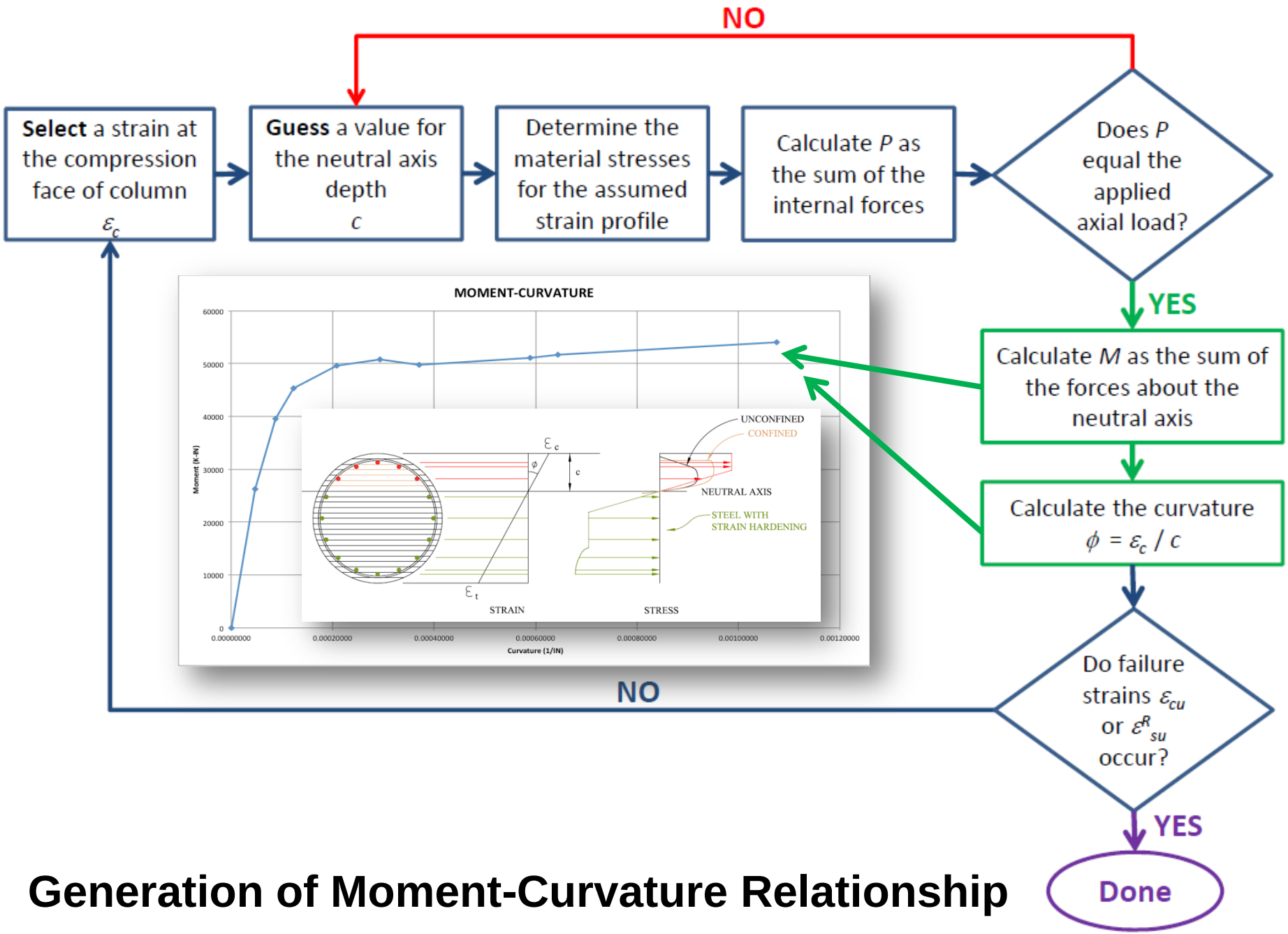


- From equilibrium:

$$P = F_{SC} + F_{CC} + F_{CU} - F_{ST}$$

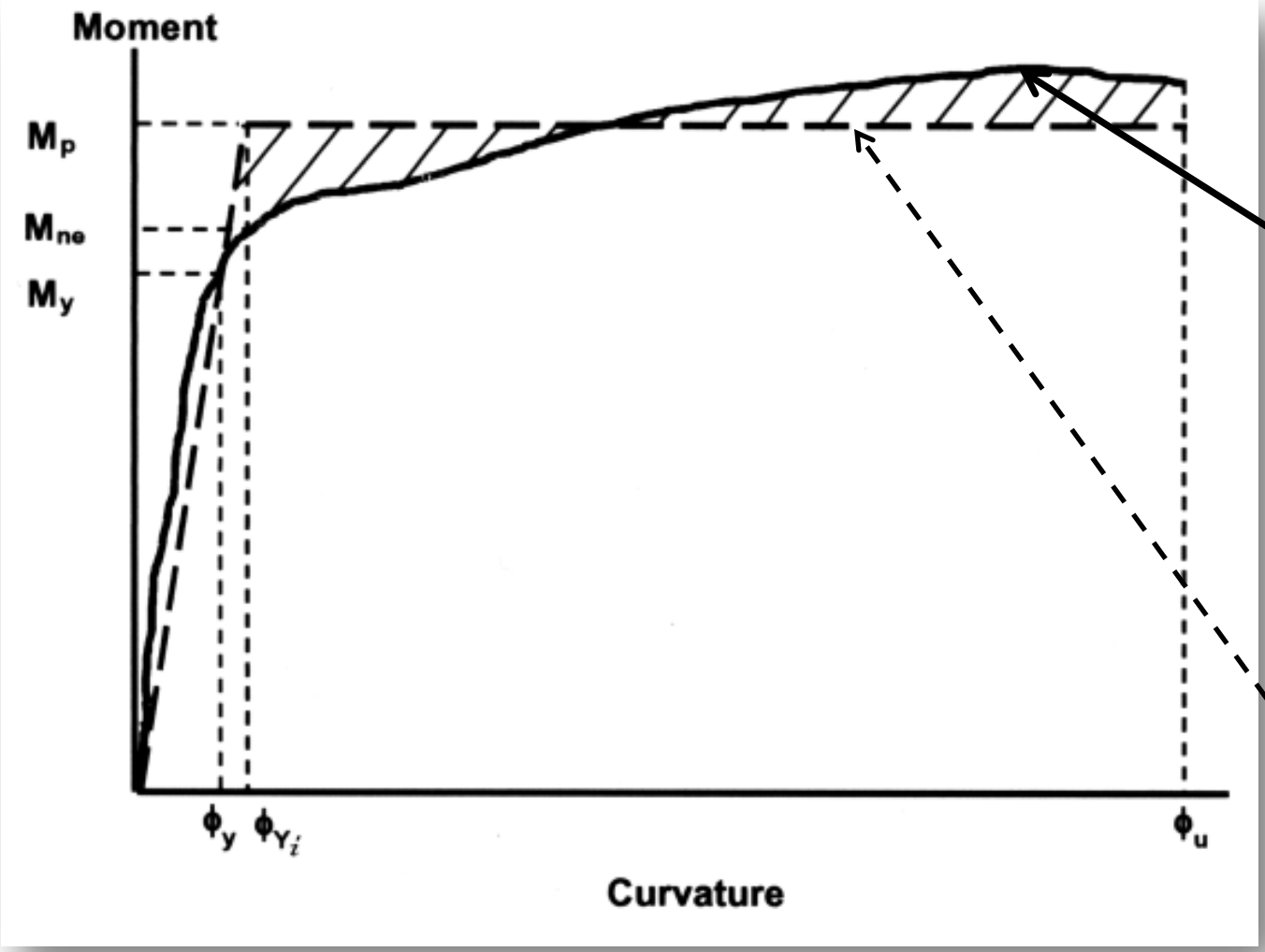
- Summing moments about the neutral axis

$$M = CG_{SC}*F_{SC}+CG_{CC}*F_{CC}+CG_{CU}*F_{CU}+CG_{ST}*F_{ST}+P*(D/2 - c)$$



Generation of Moment-Curvature Relationship

Moment – Curvature (M - ϕ)



Predicted Response

Idealized Bilinear Response used in SGS

Idealized $M - \phi$ Response

- Used for design purposes
- *First yield* Moment and Curvature, M_y and ϕ_y
- Effective stiffness, $E_{ce} * I_{eff} = M_y / \phi_y$
- *Idealized yield* Moment and Curvature, M_p and ϕ_{yi}
- Expected nominal moment, M_{ne} at $\varepsilon_c = 0.003$
- Balance the area above and below the curve

SGS Material Models

- *Concrete*: Mander et al model for unconfined and confined condition
- *Reinforcing steel*: elastic - perfectly plastic with strain hardening
- *Other*: prestressing steel (see SGS)
- Any function / model that accurately captures the material σ-ε relationship is acceptable

SGS Material Models - Concrete

- Stress-strain relationship for concrete

$$f_c = \frac{f_{cc}^{'} x r}{r - 1 + x^r}$$

where:

$$f_{cc}^{'} = f_{ce}^{'}\left(2.254\sqrt{1 + \frac{7.94 f_l^{'}}{f_{ce}^{'}}} - \frac{2 f_l^{'}}{f_{ce}^{'}} - 1.254\right)$$

$$r = \frac{E_{ce}}{E_{ce} - E_{\sec}}$$

$$x = \frac{\varepsilon_c}{\varepsilon_{cc}}$$

Mander, Priestley, Park 1988

SGS Material Models - Concrete

(continue)

$$\varepsilon_{cc} = 0.002\left[1 + 5\left(\frac{f'_{cc}}{f'_{ce}} - 1\right)\right]$$

$$E_{ce} = 1900\sqrt{f'_{ce}}$$

$$E_{\sec} = \frac{f'_{cc}}{\varepsilon_{cc}}$$

Mander, Priestley, Park 1988

SGS Material Models - Concrete

- Confinement of **circular** columns

$$\rho_s = \frac{4A_{sp}}{D's}$$

$$f_l' = K_e f_l \approx 0.95 f_l$$

$$f_l = \frac{2A_{sp} f_{yh}}{D's} = \frac{\rho_s f_{yh}}{2}$$

Mander, Priestley, Park 1988

SGS Material Models - Concrete

- Confinement of **rectangular** columns

$$f_l^{'} \approx \frac{K_e\left(\rho_x + \rho_y\right) f_{yh}}{2} \approx \frac{0.8(\rho_x + \rho_y)\, f_{yh}}{2}$$

$$\rho_x = \frac{A_{sx}}{sH_c}$$

$$\rho_y = \frac{A_{sy}}{sB_c}$$

Mander, Priestley, Park 1988

SGS Material Models - Concrete

ε_c = strain in concrete (IN/IN)

f_c = stress in concrete corresponding to strain ε_c (KSI)

f'_{ce} = expected nominal compressive strength (KSI)

f_{yh} = nominal yield stress of transverse reinforcing (KSI)

ε^R_{su} = reduced ultimate tensile strain of transverse bars (IN/IN

A_{sx} = total area of transverse bars in the "x" axis (IN^2)

A_{sy} = total area of transverse bars in the "y" axis (IN^2)

A_{sp} = area of hoop /spiral bar for circular column (IN^2)

H_c = confined core dimension in the "y" axis (IN)

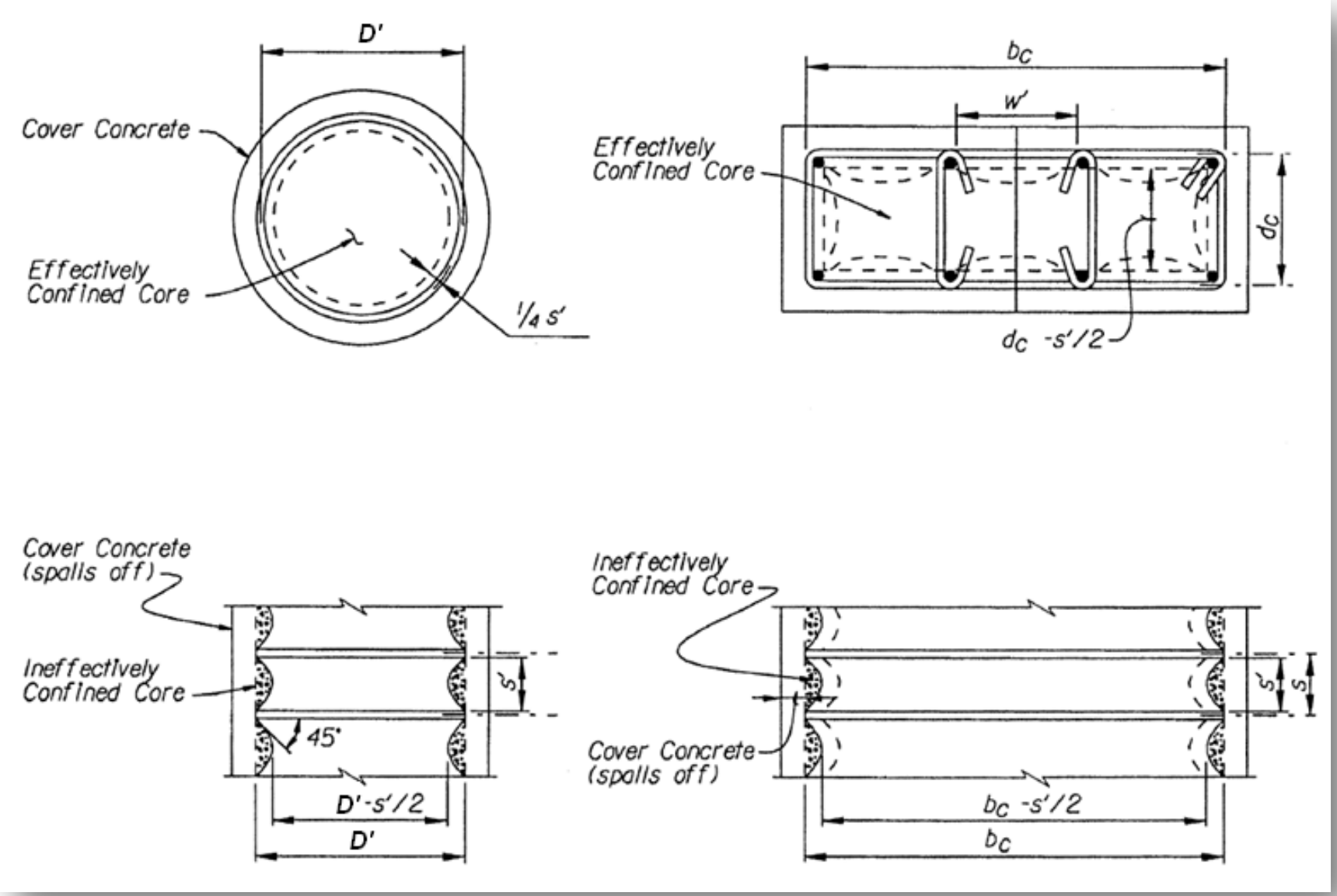
B_c = confined core dimension in the "x" axis (IN)

D' = diameter of spiral or hoop for circular column (IN)

s = pitch of spiral or spacing of hoops or ties (IN)

Mander, Priestley, Park 1988

SGS Material Models - Concrete



Chen 2003

SGS Material Models - Concrete

- Effective confinement factor for **circular** columns

$$K_e = \frac{\left(1 - \frac{s'}{2D'}\right)^n}{1 - \rho_{cc}} \sim 0.95$$

- Effective confinement factor for **rectangular** columns

$$K_e = \frac{\left[1 - \sum_{i=1}^{N} \frac{(w_i')^2}{6 b_d d_c}\right]\left(1 - \frac{s'}{2b_c}\right)\left(1 - \frac{s'}{2d_c}\right)}{1 - \rho_{cc}} \sim 0.80$$

Chen 2003

SGS Material Models - Concrete

s' = clear distance between transverse bars = $s - d_{bh}$ (IN)

d_{bh} = diameter of transverse reinforcing bar (IN)

D' = centerline diameter of hoop or spiral (IN)

$\rho_{cc} = A_{st} / A_{cc}$

A_{st} = total area of longitudinal reinforcement (IN^2)

A_{cc} = area of confined concrete core (IN^2)

n = 1 for continuous spiral

n = 2 for individual hoops

w_i' = clear distance between adjacent tied longitudinal bars(IN)

b_d = confined core dimension in the longer direction(IN)

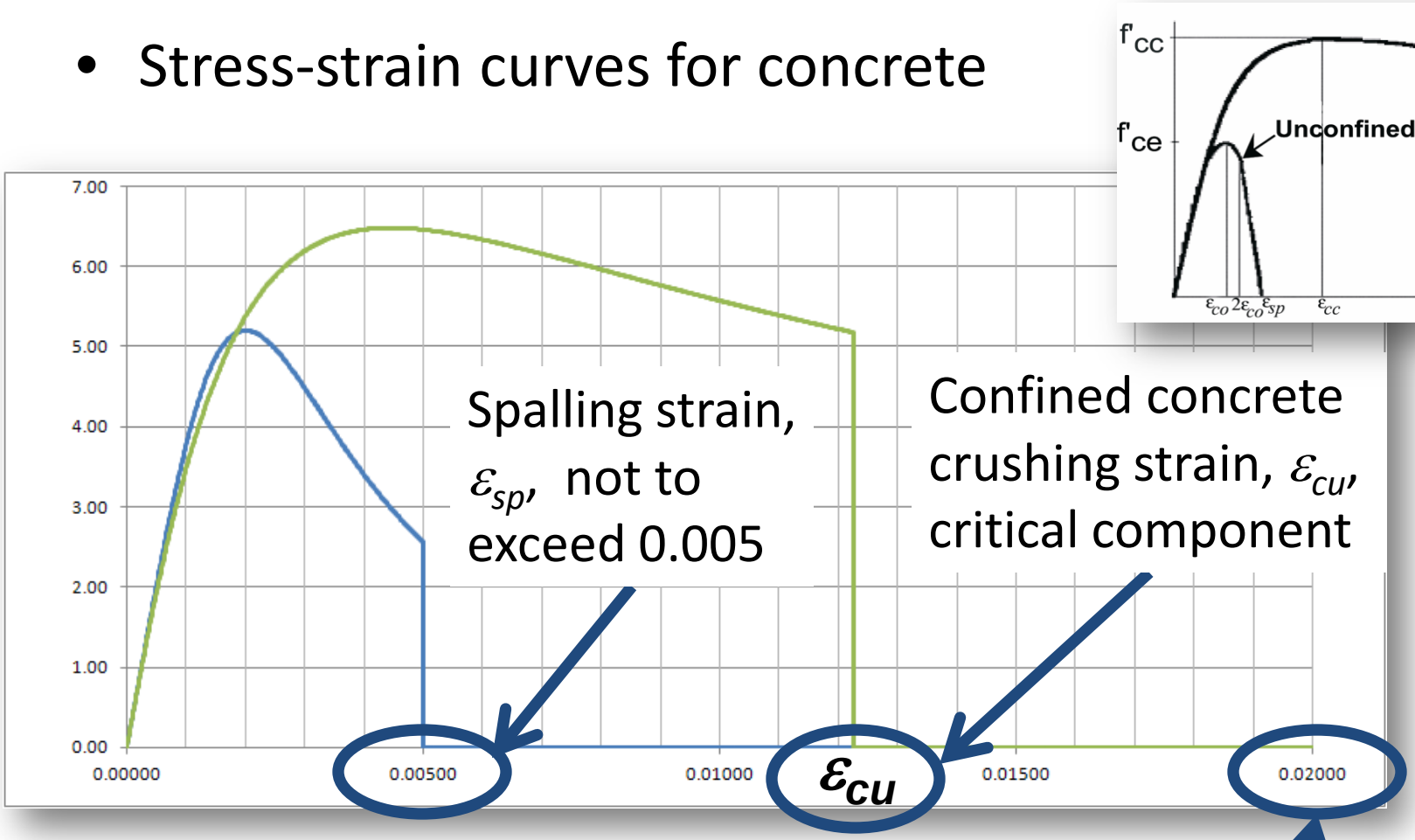
d_c = confined core dimension in the shorter direction (IN)

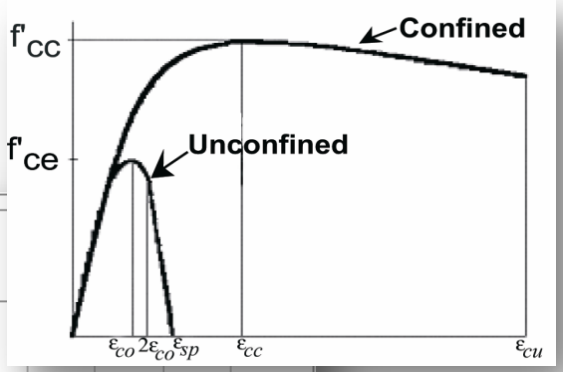
N = number of spaces between longitudinal bars

Chen 2003

SGS Material Models - Concrete

- Stress-strain curves for concrete





- Recommend $\varepsilon_{cu} < 0.02$ for design purposes

SGS Failure Strain - Concrete

- Confined concrete crushing strain limit, ε_{cu}

important

$$\varepsilon_{cu} = 0.004 + \frac{1.4\rho_s f_{yh}\varepsilon_{su}}{f'_{cc}} < 0.025$$

where:

ρ_s = transverse reinforcement ratio (ρ_x+ρ_y for rect.)

f_{yh} = nominal yield stress of transverse steel

ε_{su} = rupture strain of transverse steel **~ 0.09 to 0.12**

f'_{cc} = confined concrete compressive stress

Mander, Priestley, Park 1988

SGS Material Models - Steel

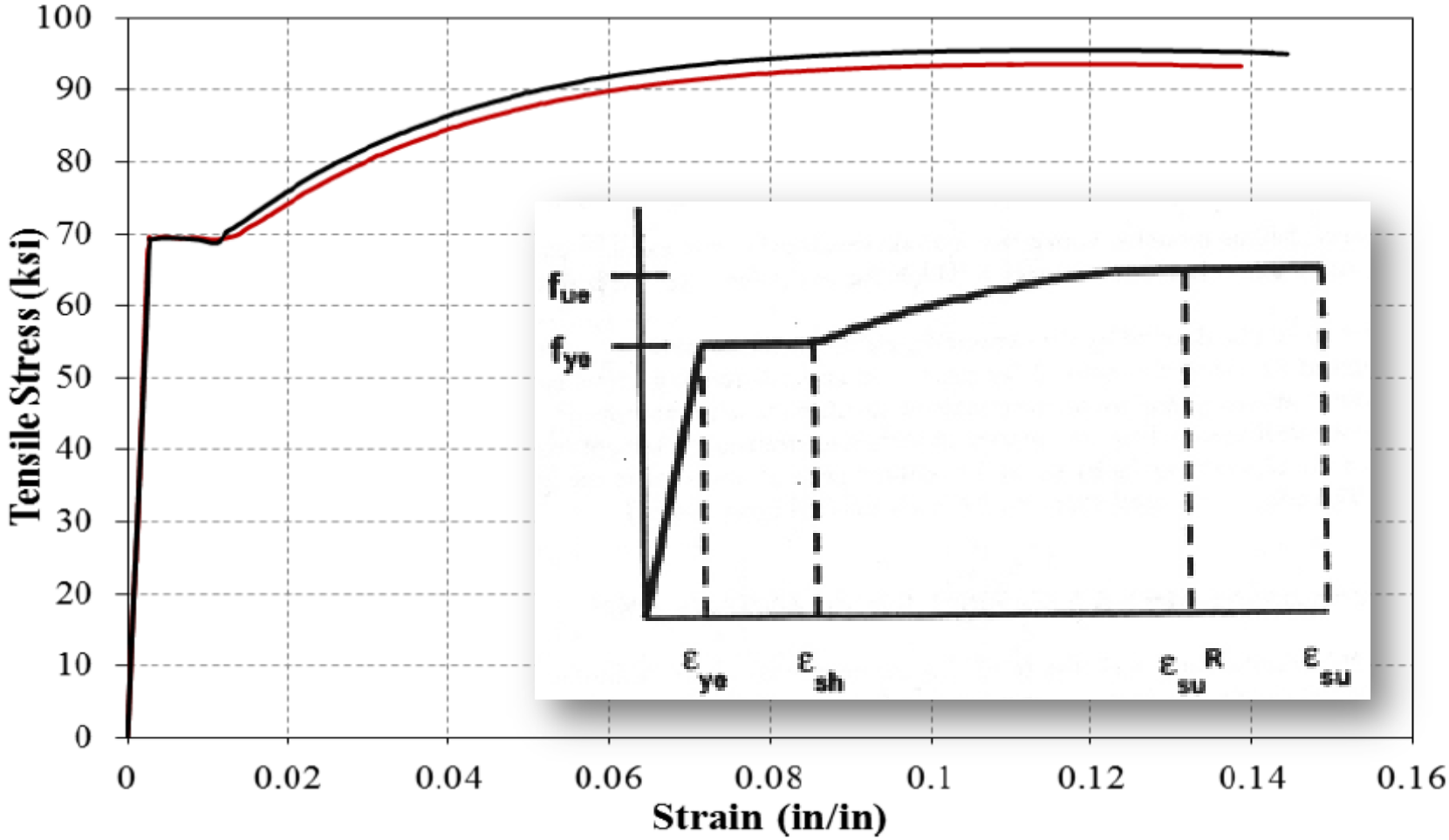


Figure 4. Longitudinal Steel (#6 and #7 Bars) Sample Tensile Test Results

Kowalsky 2013 and SGS

SGS Material Models - Steel

- Stress-strain relationship for reinforcing steel

Elastic $\varepsilon_s \le \varepsilon_{ye}$ $\quad f_s = E_s \varepsilon_s$

Plastic $\varepsilon_{ye} \le \varepsilon_s \le \varepsilon_{sh}$ $\quad f_s = f_{ye}$

Strain hardening $\varepsilon_{sh} \le \varepsilon_s \le \varepsilon_{su}^R$ $\quad f_s = f_{ue}\left[1-(f_{ue}-f_{ye})\left(\frac{\varepsilon_{su}-\varepsilon_s}{\varepsilon_{su}-\varepsilon_{sh}}\right)^2\right]$

Priestley, Calvi, Kowalsky 2007

SGS Failure Strains - Steel

- Reinforcing steel failure strain, ε^R_{su}

Property	Notation	Bar Size	ASTM A706	ASTM A615 Grade 60
Specified minimum yield stress (ksi)	f_y	#3 - #18	60	60
Expected yield stress (ksi)	f_{ye}	#3 - #18	68	68
Expected tensile strength (ksi)	f_{ue}	#3 - #18	95	95
Expected yield strain	ε_{ye}	#3 - #18	0.0023	0.0023
Onset of strain hardening	ε_{sh}	#3 - #8	0.0150	0.0150
		#9	0.0125	0.0125
		#10 - #11	0.0115	0.0115
		#14	0.0075	0.0075
		#18	0.0050	0.0050
Reduced ultimate tensile strain	ε^R_{su}	***#4 - #10***	***0.090***	***0.060***
		#11 - #18	***0.060***	***0.040***
Ultimate tensile strain	ε_{su}	#4 - #10	0.120	0.090
		#11 - #18	0.090	0.060

SGS Material Models - Steel

- ASTM A 706 Grade 60 v. ASTM A 615 Grade 60

	Grade 60 [420]
Tensile strength, min, psi [MPa]	80 000 [550][A]
Yield strength, min, psi [MPa]	60 000 [420]
Yield strength, max, psi [MPa]	78 000 [540]
Elongation in 8 in. [200 mm], min, %	
Bar Designation Nos.	
3, 4, 5, 6 [10, 13, 16, 19]	14
7, 8, 9, 10, 11 [22, 25, 29, 32, 36]	12
14, 18 [43, 57]	10

[A] Tensile strength shall not be less than 1.25 times the actual yield strength.

	Grade 60 [420]
Tensile strength, min, psi [MPa]	90 000 [620]
Yield strength, min, psi [MPa]	60 000 [420]
Elongation in 8 in. [200 mm], min, %	
Bar Designation No.	
3 [10]	9
4, 5 [13, 16]	9
6 [19]	9
7, 8 [22, 25]	8
9, 10, 11 [29, 32, 36]	7
14, 18 [43, 57]	7

ASTM 2011, 2012

Quasi-static v. Cyclic Discussion

- Quasi-static constitutive models for cyclic response

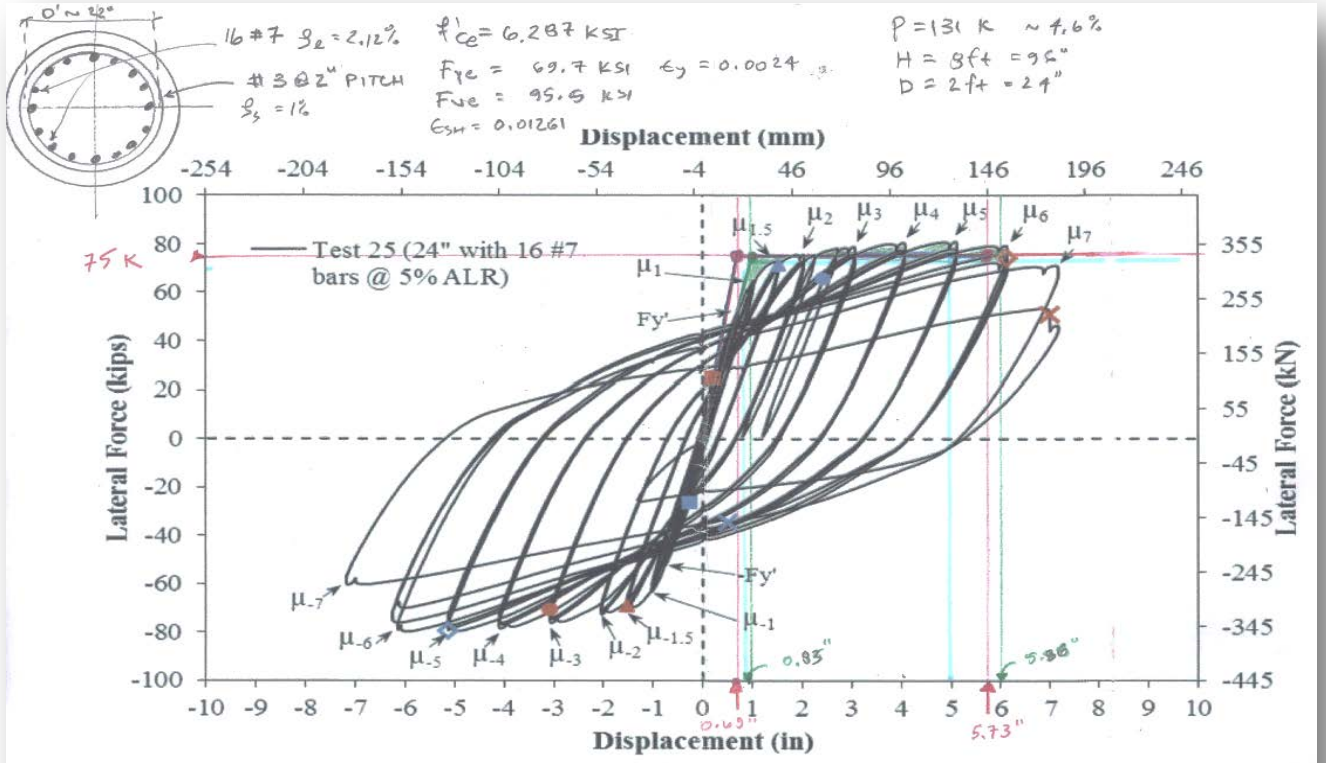
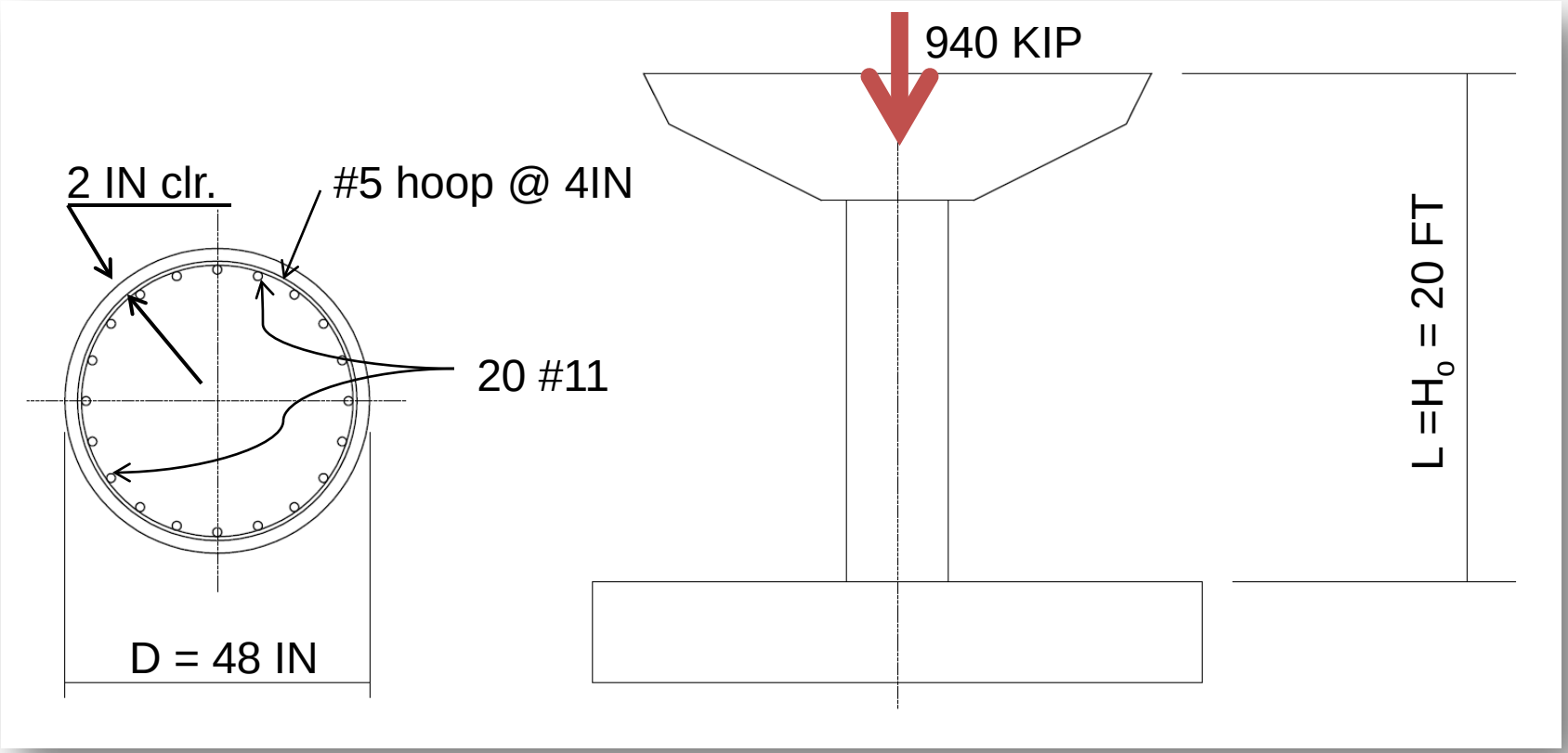


Figure 6. Lateral Force vs. Top Column Displacement Response

Kowalsky et al. 2010

Moment – Curvature Example

- Find the M-ϕ for the column shown below



- ASTM A 706 Grade 60 f'_c = 4 KSI

Moment – Curvature Example

- Diameter, D = 48 IN
- Gross Area, $A_g = \pi * D^2/4$ = 1810 IN^2
- 20 #11 ➔ A_{st} = 20*1.56 = 31.2 IN^2
- $\rho_l = A_{st} / A_g$ = 0.01724 = 1.724%
- #5 hoop (d_{sp} = 0.625 IN and A_{sp} = 0.31 IN^2)
- Hoop spacing = 4 IN pitch (s = 4 IN)
- Clear cover, cov = 2 IN over transverse bars
- Core diameter, $D' = D - 2*cov - d_{sp}$ = 43.375 IN
- $\rho_s = 4*A_{sp} / (D' * s)$ = 0.00715 = 0.715%
- ASTM A 706 Grade 60 so f_{ye} = 68 KSI, f_{ue} = 95 KSI
- f'_c = 4 KSI so f'_{ce} = 5.2 KSI
- P = 940 K Axial Load Ratio, ALR = $P / (f'_{ce} * A_g)$ = 0.1

Moment – Curvature Example

- Confined concrete crushing strain limit, ε_{cu}

$$\varepsilon_{cu} = 0.004 + \frac{1.4\rho_s f_{yh}\varepsilon_{su}}{f_{cc}^{'}} = 0.01232$$

where:

$\rho_s = 4*A_{sp}/(D'*s) = 0.00715$

$f'_l = K_e*\rho_s*f_{yh}/2 \sim 0.95*0.00715*60/2 = 0.204$ KSI

$f_{yh} = 60$ KSI (use nominal for horizontal steel)

$\varepsilon_{su} = \varepsilon^R_{su} = 0.09$ for #5 hoops

$$f_{cc}^{'} = f_{ce}^{'}\left(2.254\sqrt{1+\frac{7.94 f_l^{'}}{f_{ce}^{'}}} - \frac{2 f_l^{'}}{f_{ce}^{'}} - 1.254\right) = 6.49$$

- *Spreadsheet demo then check with commercial*

$M - \phi$ Spreadsheet Example

Slice Number	Distance from Top	Dist to kd	Strain -0.014899	Distance from D/2	Width Steel Out	Width Steel In	Area of Steel	Width Unconfined	Width Confined	Area Unconfined	Area Confined	Unconfined Stress	Confined Stress	Steel Stress	Unconfined Force	Confined Force	Steel Force	Unconfined Moment	Confined Moment	Steel Moment	Compression Total	Tension Total	Compression Moment	Tension Moment
0	0.00	13.87	-0.01490	24.00	0.00	0.00	0.00	0.00	0.00	0.813	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.24	13.63	-0.01464	23.76	0.00	0.00	0.00	6.77	0.00	1.959	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.48	13.39	-0.01438	23.52	0.00	0.00	0.00	9.55	0.00	2.547	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.72	13.15	-0.01413	23.28	0.00	0.00	0.00	11.67	0.00	3.013	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.96	12.91	-0.01387	23.04	0.00	0.00	0.00	13.44	0.00	3.411	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	1.20	12.67	-0.01361	22.80	0.00	0.00	0.00	14.99	0.00	3.764	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	1.44	12.43	-0.01335	22.56	0.00	0.00	0.00	16.38	0.00	4.082	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	1.68	12.19	-0.01309	22.32	0.00	0.00	0.00	17.64	0.00	4.375	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	1.92	11.95	-0.01284	22.08	0.00	0.00	0.00	18.81	0.00	4.646	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
195	46.80	-32.93	0.03539	-22.80	0.00	0.00	0.00	14.99	0.00	3.411	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
196	47.04	-33.17	0.03564	-23.04	0.00	0.00	0.00	13.44	0.00	3.013	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
197	47.28	-33.41	0.03590	-23.28	0.00	0.00	0.00	11.67	0.00	2.547	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
198	47.52	-33.65	0.03616	-23.52	0.00	0.00	0.00	9.55	0.00	1.959	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
199	47.76	-33.89	0.03642	-23.76	0.00	0.00	0.00	6.77	0.00	0.813	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
200	48.00	-34.13	0.03667	-24.00	0.00	0.00		0.00	0.00			0.00	0.00	0.00										
TOTAL							integrated area =	31.20		integrated area =	330.96	1477.92			-80.1	-1729.6	869.7	165.6	8891.4	35505.8	-2482.3	1542.4	-13949.7	-30821.8

P*(D/2-kd) = 9525.4

Comp CG above NA= 5.62 -19.98

Comp CG above D/2 = 15.75 -9.85

Approx. M ~ 54297 99.6%

actual rebar area = 31.20 total integrated area = 1808.88

integrated/actual adjustement = 100.0% actual column area = 1809.56

integrated/actual adjustement = 100.0%

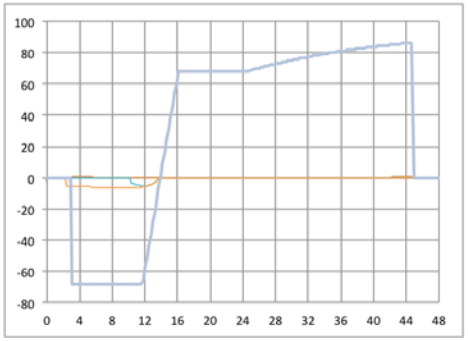
STRAIN SUMMARY (hand adjust core and tension locations)

strain at extreme compression fiber =	-0.0149
CONCRETE CORE STRAIN	
strain at extreme confined core =	-0.0123
strain at extreme confined core X 100 =	-1.2320
Confine concrete crushing strain X 100, ecu =	-1.2323
TENSION STEEL STRAIN	
strain at extreme tension steel =	0.0331
strain at extreme tension steel X 100=	3.3064
Reduced ultimate tensile strain X 100, esur =	6.0000

SUMMARY OF NUMERICAL ANALYSIS

Neutral axis depth, kd =	13.87	IN		
Equilibrium force sum total, P =	**-940.0**	**K**		
P/Ag*f'ce =	-0.10			
Mid height, H/2 =	24	IN		
Mid height - kd, eccentricity of P =	10.1	IN		
Compressive resultant, C =	-2482.3	K	+ P/2 =	-2012.4
Tensile resultant, T =	1542.36	K	+ P/2 =	2012.4
Lever arm, jd = dv =	25.60	IN		
Total Moment =	54088	K-IN		
Total Moment =	**4507**	**K-FT**		
Curvature, phi =	**0.0010744**	**1/IN**	0.0010744	
Curvature, phi =	0.0128933	1/FT	0.0128933	

curvature	moment
1/IN	K-IN
0.00107444	54088

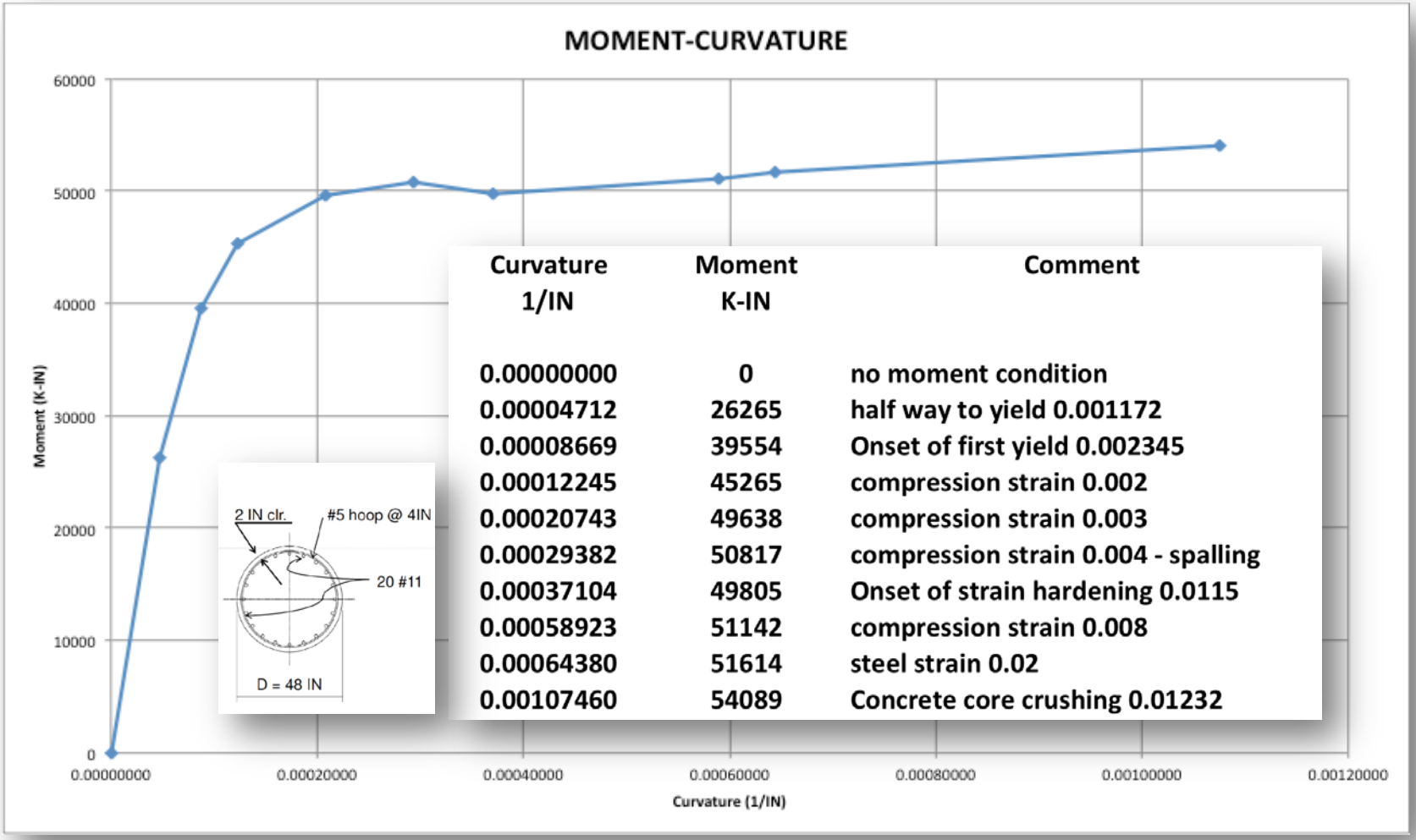


VERIFICATION USING APPROXIMATE METHODS

D =	48	IN		
Ast =	31.20	IN^2		
Ag =	1810	IN^2		
rho_l =	1.7242	%		
(E*I)eff =	50340891	K-IN^2	check at first yield	
Ieff =M/(phi *Ece) =	11619	IN^4	check at first yield	
Ig =	260576	IN^4		
Ieff/Ig =	0.045			
Ieff ~ Ig*(0.2+0.1*rho_l+0.5*P/(Ag*f'ce)) ~	110059	IN^4		
Ieff v. Ieff-approx ~	11%			
Mp ~ D^3*(0.05+.2*rho_l+P/(Ag*f'ce)) ~	4559	K-FT	54713	K-IN
Approx idealized yield curvature ~	0.0001099	1/IN		
Approx ultimate curvature ~	0.0012500	1/IN		
T ~ 0.7*Ast*Fye =	1485.1	K	104%	

Moment – Curvature Example

- Spreadsheet summary results for ALR = 0.1



Curvature 1/IN	Moment K-IN	Comment
0.00000000	0	no moment condition
0.00004712	26265	half way to yield 0.001172
0.00008669	39554	Onset of first yield 0.002345
0.00012245	45265	compression strain 0.002
0.00020743	49638	compression strain 0.003
0.00029382	50817	compression strain 0.004 - spalling
0.00037104	49805	Onset of strain hardening 0.0115
0.00058923	51142	compression strain 0.008
0.00064380	51614	steel strain 0.02
0.00107460	54089	Concrete core crushing 0.01232

Idealized Moment – Curvature

- Elastic-perfectly plastic relationship is defined as:

  M_i = idealized moment values = $E_{ce}I_{eff} * \phi < M_p$
  M_p = idealized plastic moment (*solving for this*)
  ϕ = actual calculated curvature from M-ϕ results
  $E_{ce}I_{eff}$ = effective stiffness at first yield = M_y / ϕ_y
  $\Delta(M\phi)$ = actual - idealized = $(\Delta M)*(\Delta\phi)$

  $\Sigma\ \Delta(M\phi)$ = sum of difference = 0 by changing M_p

- *Spreadsheet demo then check with commercial*

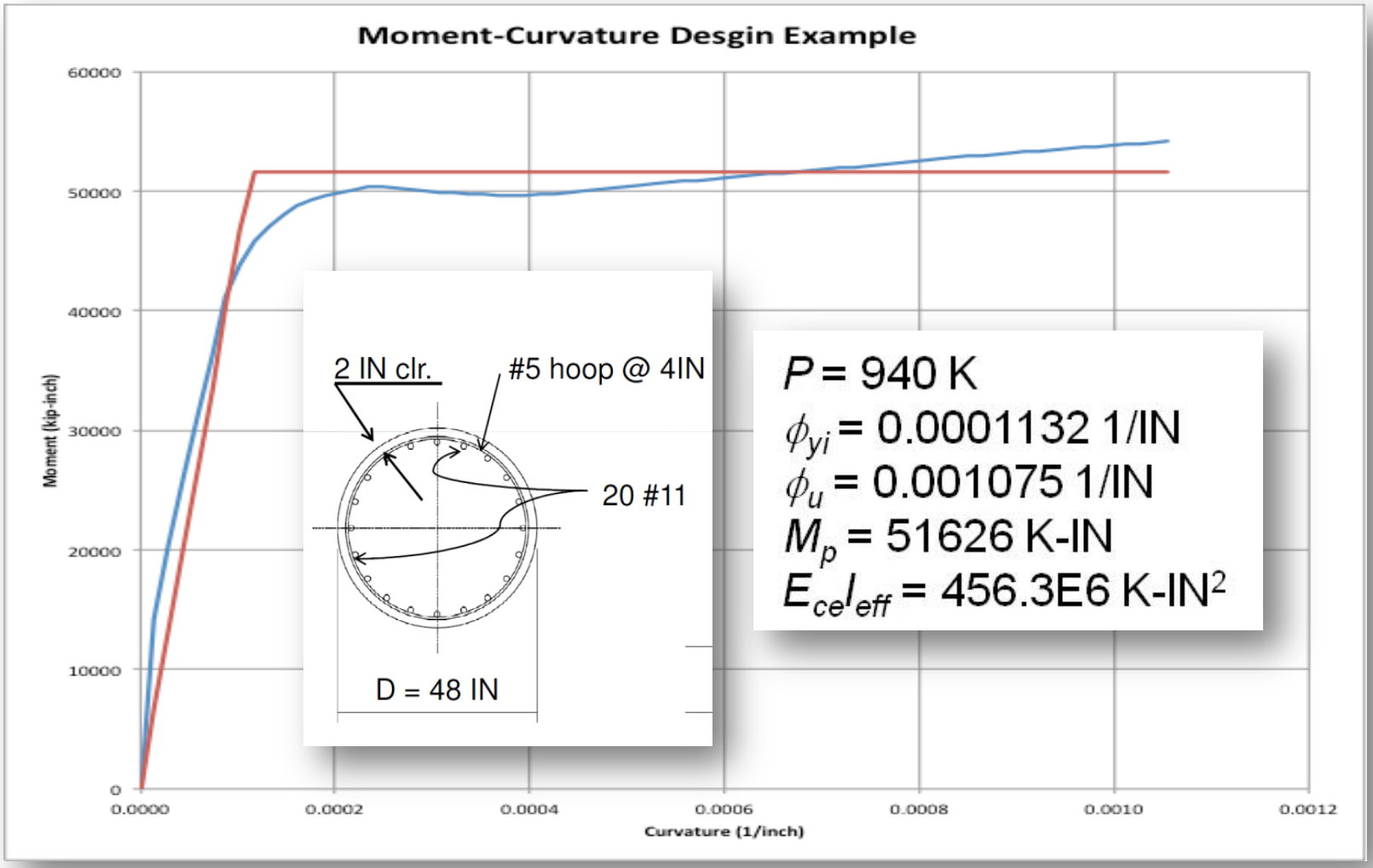
Idealized Moment – Curvature

Predicted Curvature 1/IN	Predicted Moment K-IN	Idealized Moment K-IN	Difference between M-phi curves	Comment
0.00000000	0	0	0.1123	no moment condition
0.00004712	26265	21498	0.0943	half way to yield 0.001172
0.00008669	39554	39554	-0.1097	Onset of first yield 0.002345
0.00012245	45265	51398	-0.3354	compression strain 0.002
0.00020743	49638	51398	-0.1011	compression strain 0.003
0.00029382	50817	51398	-0.0839	compression strain 0.004 - spalling
0.00037104	49805	51398	-0.2017	Onset of strain hardening 0.0115
0.00058923	51142	51398	-0.0011	compression strain 0.008
0.00064380	51614	51398	0.6262	steel strain 0.02
0.00107460	54089	51398		Concrete core crushing 0.01232
		Sum Total Difference under curve =	0.0000	

(EI)eff = My/phi-y=	456,259,058	K-IN^2	defined at onset of first yielding of extreme tension bar
Mp =	51398	K-IN	alter until area differential area is zero
phi-yi =Mp/(EI)eff =	0.0001127	1/IN	idealized curvature

Moment – Curvature Example

- Idealized M - ϕ relationship using more data points



Moment – Curvature Example

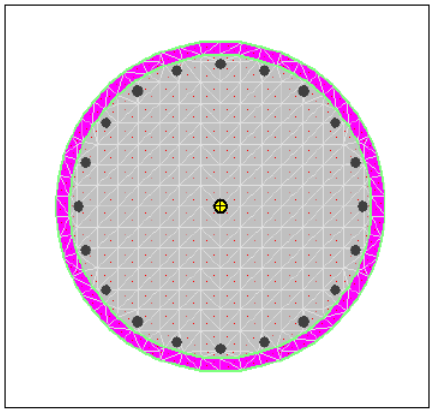
XTRACT Section Report

Section Name: Section1

Alaska DOT
Elmer Marx
10/27/2013
TRB Example
TRB column example
Page __ of __

Section Details:

X Centroid:	-.4973E-6 in
Y Centroid:	.1229E-3 in
Section Area:	1805 in^2
EI gross about X:	1.228E+9 kip-in^2
EI gross about Y:	1.228E+9 kip-in^2
I trans (Confined1) about X:	298.8E+3 in^4
I trans (Confined1) about Y:	298.8E+3 in^4
Reinforcing Bar Area:	31.23 in^2
Percent Longitudinal Steel:	1.730 %
Overall Width:	47.91 in
Overall Height:	48.00 in
Number of Fibers:	518
Number of Bars:	20
Number of Materials:	3



Material Types and Names:

Unconfined Concrete:	Unconfined1
Confined Concrete:	Confined1
Strain Hardening Steel:	Steel1

Comments:

Section Type: Circular Column
Type of Reinforcing: Spiral Reinforcing
Transverse Reinforcing Bar Size: #5
Spacing of Transverse Steel: 4 in
Outside Diameter: 48 in
Cover Thickness: 2 in
Number of Longitudinal Bars: 20
Longitudinal Bar Size: #11
Cover Concrete: Unconfined1
Column Core Concrete: Confined1
Longitudinal Steel: Steel1

XTRACT Analysis Report

Section Name: Section1
Loading Name: M-PHI-1
Analysis Type: Moment Curvature

Alaska DOT
Elmer Marx
11/29/2013
TRB
problem
Page __ of __

Section Details:

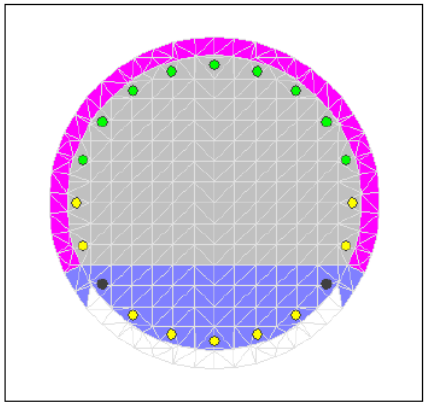
X Centroid:	-.2764E-6 in
Y Centroid:	.1071E-3 in
Section Area:	1805 in^2

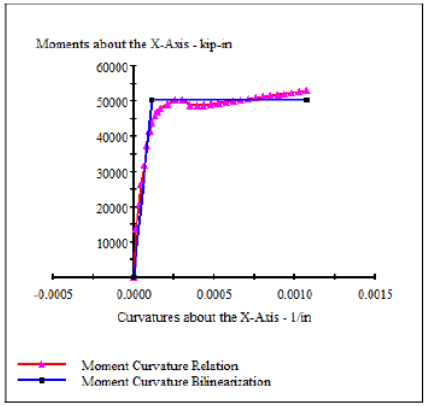
Loading Details:

Constant Load - P:	940.0 kips
Incrementing Loads:	Mxx Only
Number of Points:	31
Analysis Strategy:	Displacement Control

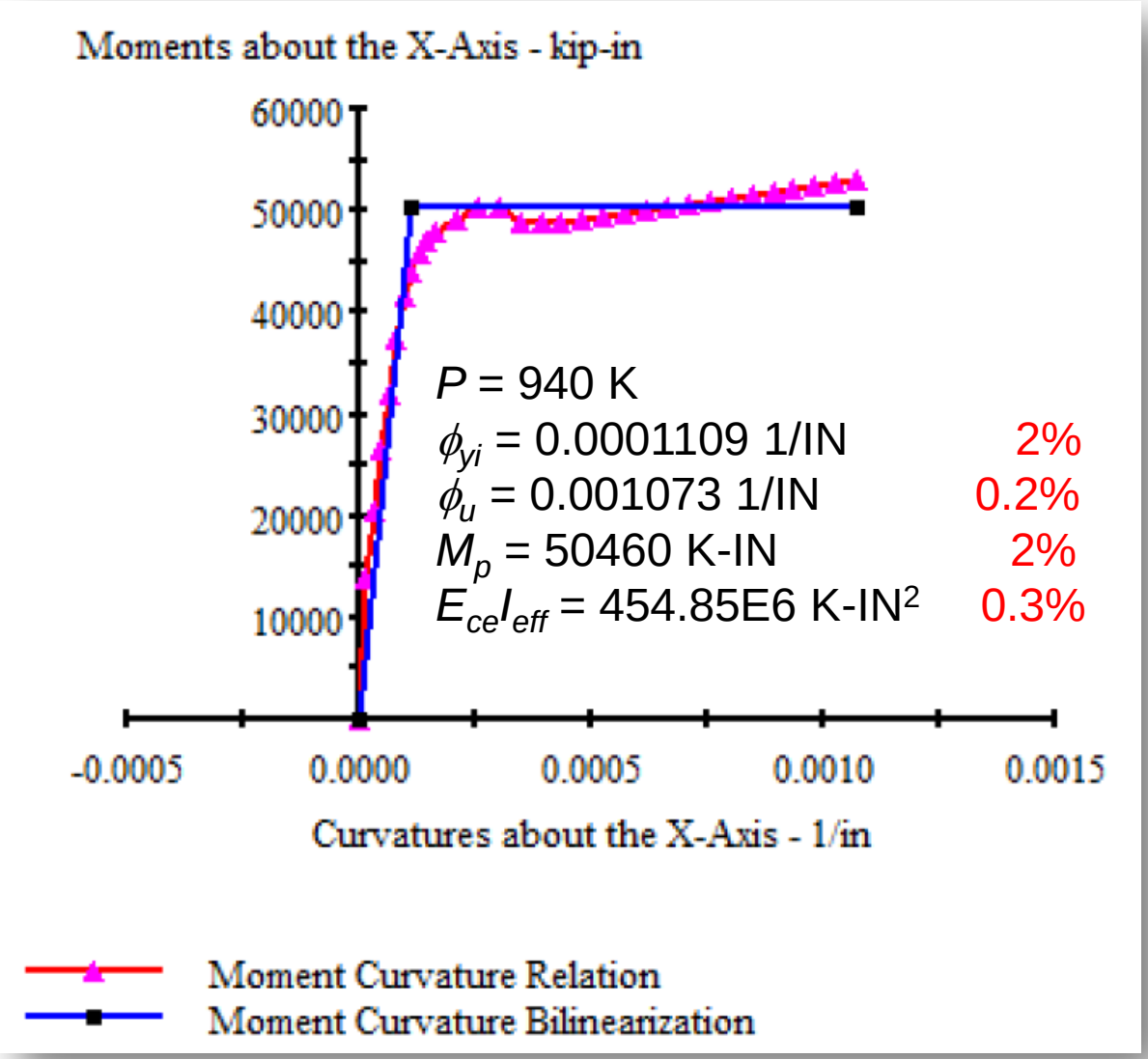
Analysis Results:

Failing Material:	Confined1
Failure Strain:	12.27E-3 Compression
Curvature at Initial Load:	-1.83E-13 1/in
Curvature at First Yield:	82.23E-6 1/in
Ultimate Curvature:	1.073E-3 1/in
Moment at First Yield:	37.40E+3 kip-in
Ultimate Moment:	52.99E+3 kip-in
Centroid Strain at Yield:	.4963E-3 Ten
Centroid Strain at Ultimate:	10.48E-3 Ten
N.A. at First Yield:	6.035 in
N.A. at Ultimate:	9.760 in
Energy per Length:	51.37 kips
Effective Yield Curvature:	.1109E-3 1/in
Effective Yield Moment:	50.46E+3 kip-in
Over Strength Factor:	1.0000
EI Effective:	454.85E+6 kip-in^2
Yield EI Effective:	0 kip-in^2
Bilinear Harding Slope:	0 %
Curvature Ductility:	9.676





Moment – Curvature Example



Spreadsheet Comparison

ϕ_{yi} = 0.0001132 1/IN

ϕ_u = 0.001075 1/IN

M_p = 51626 K-IN

$E_{ce}I_{eff}$ = 456.3E6 K-IN^2

Approximate Methods

- the use of curvature (ϕ) in design is uncommon
- don't have a good "feel" for curvature values
- simple method to check the computer results
- can help with iterative processes
- *use these approximations with due skepticism*

Approximate Methods - ϕ_{yi}

- For a rough check of conventional **circular** reinforced concrete column sections:

$$\phi_{yi} \sim 2.25 * \varepsilon_{ye}/12B_o \sim 1/2300B_o$$

$$\phi_{yi} \sim 2.25 * \varepsilon_{ye}/D \sim 1/190D$$

where:

ϕ_{yi} = idealized yield curvature (1/IN)

B_o = column diameter (FT)

D = column diameter (IN)

ε_{ye} = expected yield strain ~ 0.002345 (IN/IN)

Priestley, Calvi, Kowalsky 2007

Approximate Methods - ϕ_{yi}

- For a rough check of conventional **rectangular** reinforced concrete column sections:

$$\phi_{yi} \sim 2.1 * \varepsilon_{ye}/12B_o \sim 1/2500B_o$$

where:

ϕ_{yi} = idealized yield curvature (1/IN)

B_o = column width in loaded direction (FT)

ε_{ye} = expected yield strain ~ 0.002345 (IN/IN)

Priestley, Calvi, Kowalsky 2007

Approximate Methods - ϕ_u

- And for a *very rough check* of conventional **circular** or **rectangular** reinforced concrete column sections with good confinement:

$$\phi_u = \min\left(\varepsilon_{cu}/c_c \ , \ \varepsilon_{su}^R/d\text{-}c\right) \sim \varepsilon_{su}^R/12B_o$$

where:

ϕ_u = ultimate curvature (1/IN)

ε_{cu} = ultimate confined concrete strain (1/IN)

ε_{su}^R = reduced ultimate tensile strain (IN/IN)

c_c = neutral axis to edge of confined core (IN)

$d\text{-}c$ = neutral axis to extreme tension bar (IN)

B_o = column diameter / width (FT)

Effective Stiffness - $E_{ce}*I_{eff}$

$$E_{ce}*I_{eff} = M_y / \phi_y$$

where:

E_{ce} = Expected modulus of elasticity for concrete

M_y = moment at first yield (expected materials at first yield)

ϕ_y = curvature at first yield (expected materials at first yield)

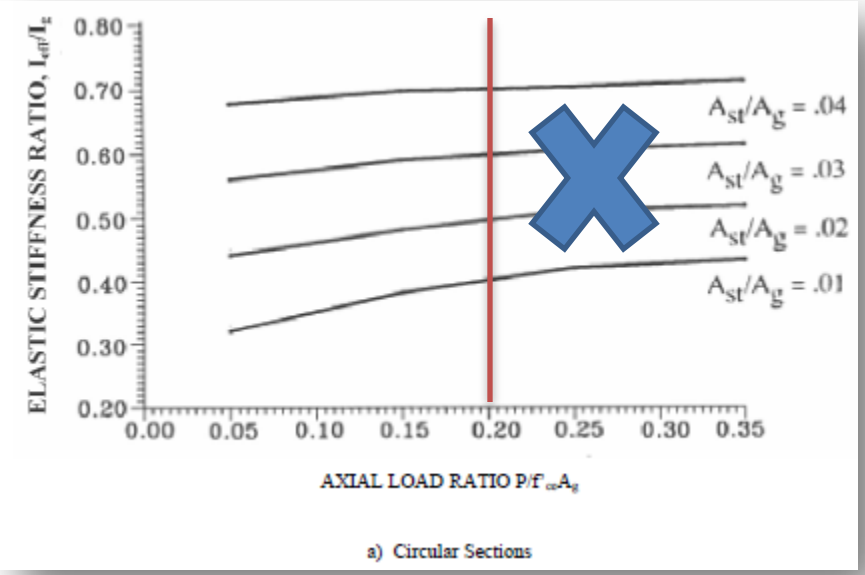
So,

$$I_{eff} = M_y /(\phi_y *E_{ce})$$

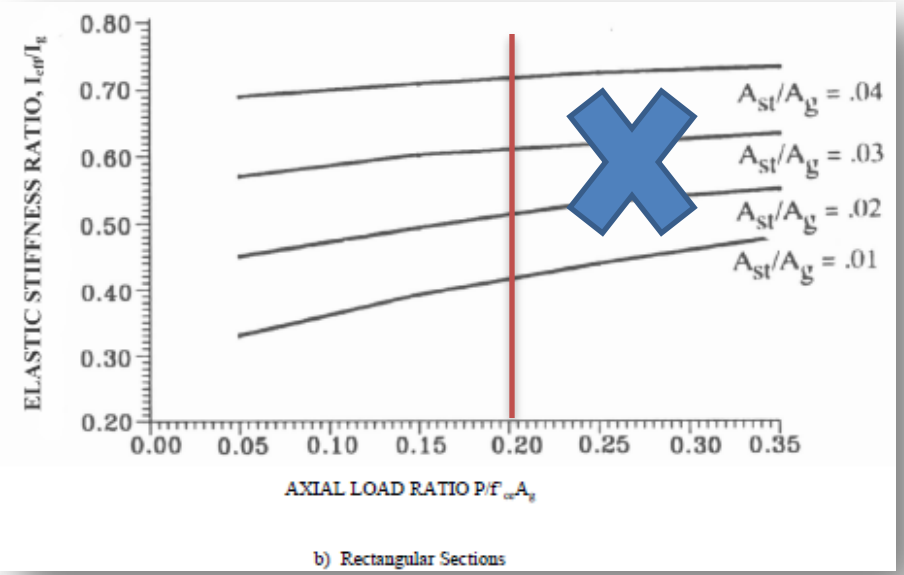
And typically,

$$0.7 < I_{eff} / I_g < 0.3$$

Approximate Methods – I_{eff} / I_g



a) Circular Sections



b) Rectangular Sections

$$I_{eff}/I_g \sim 0.2 + 0.1*\rho_l + 0.5*(P/f'_{ce}*A_g)$$

where:

ρ_l = longitudinal reinforcement ratio in %

$= A_{st} / A_g * 100 < 3\%$ practical limit

P = axial load – **keep below $0.2*A_g*f'_{ce}$**

Approximate Methods - M_p

$$M_p \sim D^3 * [0.05 + 0.2*\rho_l + P/(f'_{ce}*A_g)]$$

where:

M_p = idealized plastic moment for **circular** section (K-IN)

D = column diameter (IN) > 30 IN

ρ_l = longitudinal reinforcement ratio in %

= $A_{st} / A_g * 100$ < 4% but 3% is practical limit

P = axial load on column (K) < $0.2 * f'_{ce} * A_g$

f'_{ce} = expected concrete strength (KSI)

A_g = gross area of column (IN^2)

- Assumes ASTM A 706 Grade 60 reinforcing steel and f'_{ce} = 5.2

Approximate Methods - M_p

$$M_p \sim b*d^2 * [0.15 + 0.25*\rho_l + 1.5*P/(f'_{ce}*A_g)]$$

where:

M_p = idealized plastic moment for **rectangular** section (K-IN)

d = column depth in direction of loading (IN) > 30 IN

b = column width (IN) > 30 IN

ρ_l = $A_{st} / A_g * 100$ = longitudinal reinforcement ratio in %

P = axial load on column (K) < $0.2 * f'_{ce} * A_g$

f'_{ce} = expected concrete strength (KSI)

A_g = $b * d$ = gross area of column (IN^2)

- Assumes ASTM A 706 Grade 60 reinforcing steel and f'_{ce} = 5.2

Moment – Curvature Check

- Diameter, D = 48 IN
- $I_g = D^4\pi/64 = 260576$ IN^4
- $\rho_l = A_{st} / A_g = 0.01724 = 1.724\%$
- Axial Load Ratio, ALR = $P / (f'_{ce} * A_g) = 0.1$ [P ~ 940 K]
- $\varepsilon_{ye} = f_{ye}/E_s = 68/29000 = 0.002345$ $\varepsilon^R_{su} = 0.06$ (#11)

ϕ_{yi} ~2.25*0.00234/48 = 0.000110 1/IN v. *0.0001132 1/IN*

ϕ_u ~ 0.06/48 = 0.00125 1/IN v. *0.001075 1/IN*

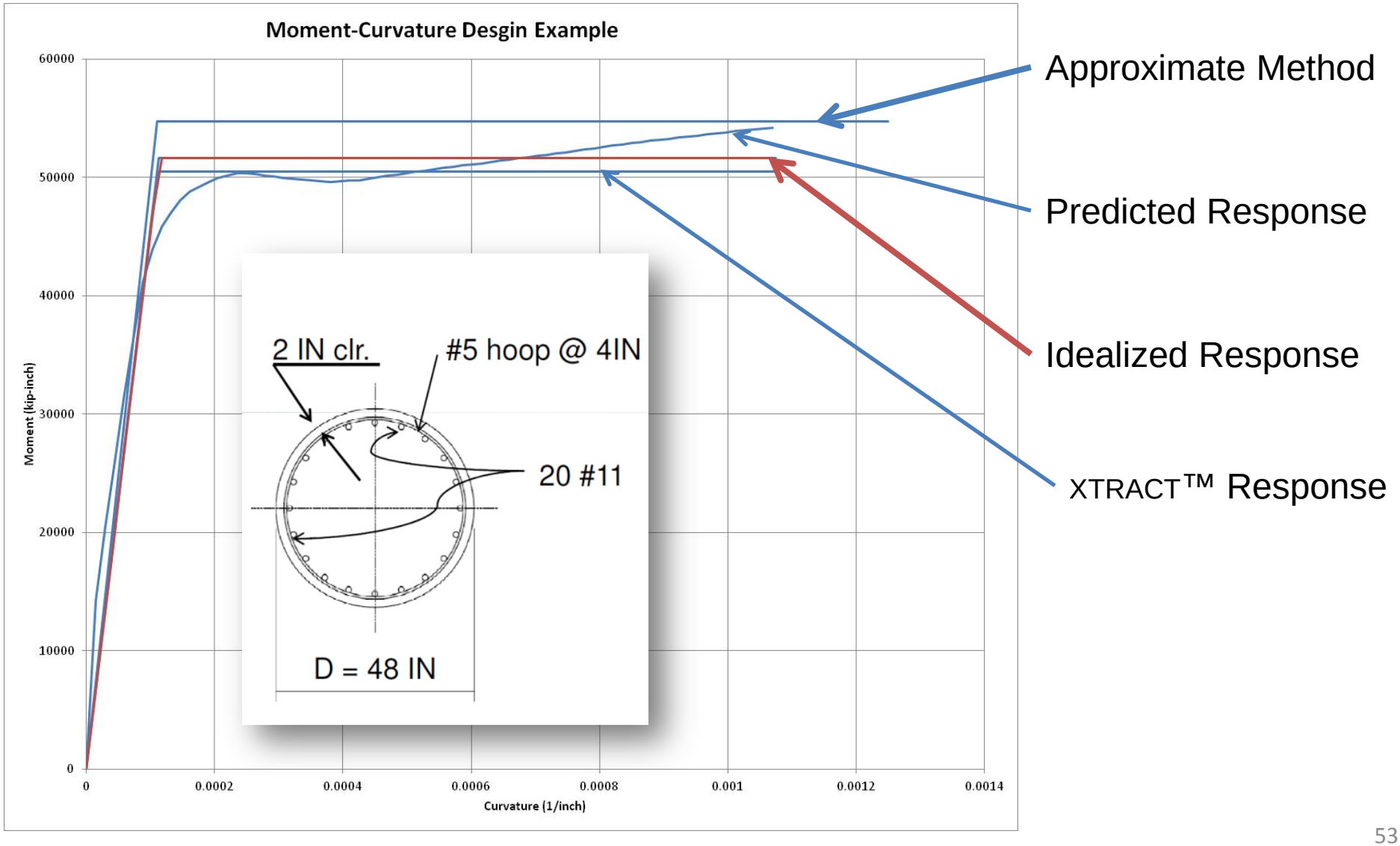
I_{eff} ~ (0.2+.1*1.724+0.5*0.1)*260576 = 0.42*260576

I_{eff} ~ 109963 IN^4 v. *105307 IN^4* *4%*

M_p ~ 48^3(0.05+0.2*1.724+0.1)

M_p ~ 54713 K-IN v. *51626 K-IN* *6%*

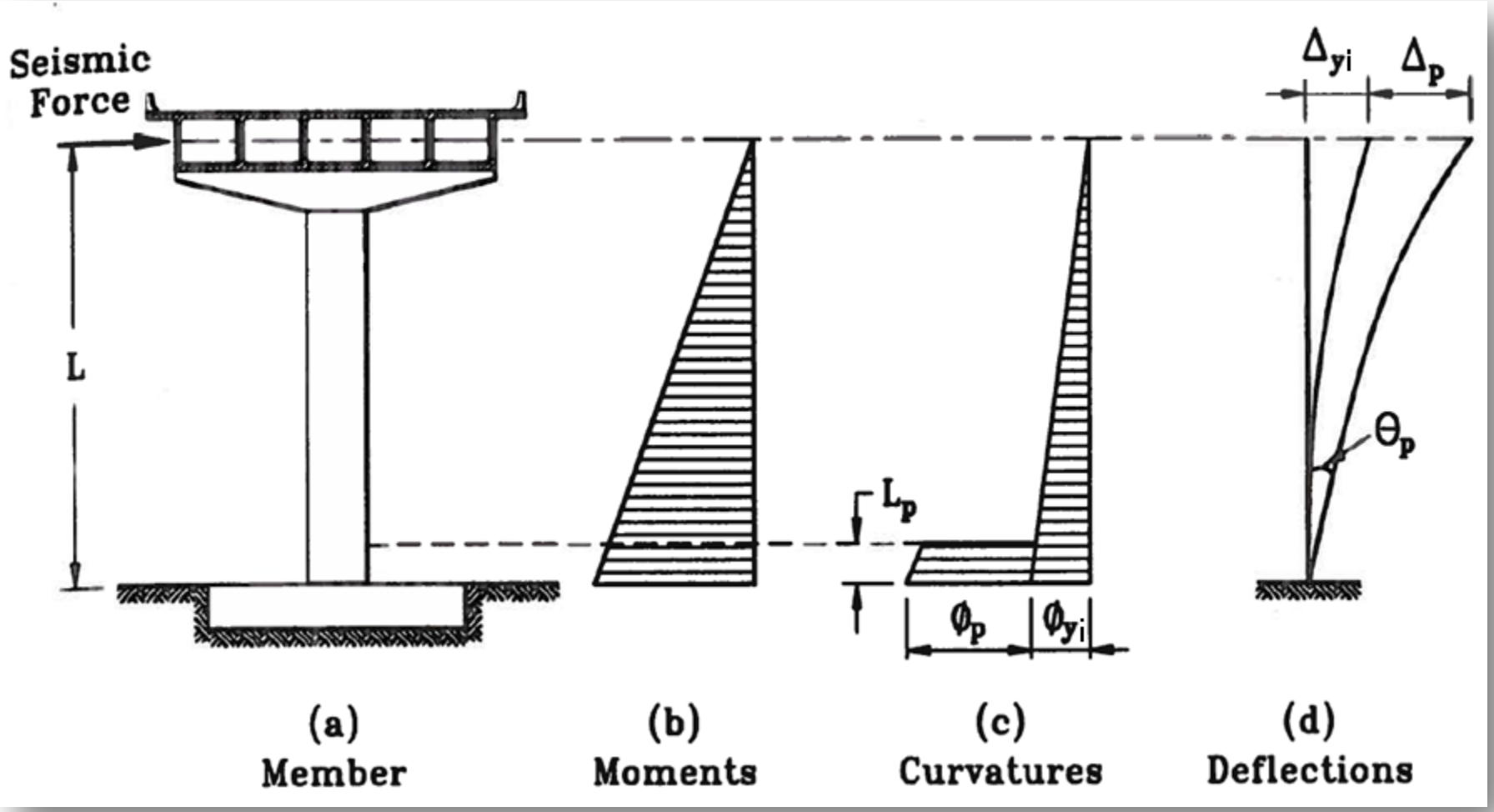
Moment – Curvature Check



Force - Displacement

- Calculate deflections using M-ϕ result
- Elastic deformation component, Δ_{yi}
- Plastic deformation component, Δ_p
- *Foundation deformation component – important but not specifically addressed in this workshop*
- Conservative to neglect shear deformations

Force - Displacement



Analytical Plastic Hinge Length - L_p

- Approximation used to simplify analysis
- Converts (integrates) curvature to rotation
- Includes a moment gradient part (integration), tension shift (diagonal cracking) and a strain penetration part (yielding into cap, footing or shaft)
- Calibrated to the failure condition only and modification may be needed for full strain-displacement response

Analytical Plastic Hinge Length - L_p

$$L_p = k * L + L_{sp} > 2 * L_{sp}$$

$$L_p = 0.08 * L + 0.15 * f_{ye} * d_{bl} > 0.3 * f_{ye} * d_{bl}$$

where:

$k = 0.2*(f_{ue}/f_{ye} - 1) \leq 0.08$

L = length of column from point of maximum moment to the point of moment contraflexure (IN)

L_{sp} = strain penetration component = $0.15*f_{ye}*d_{bl}$ (IN)

f_{ye} = expected yield stress of longitudinal bars (KSI)

f_{ue} = expected tensile strength (KSI)

d_{bl} = diameter of longitudinal column bars (IN)

Predicted Force – Displacement

$$\Delta_y \sim 1/3 * \phi_y * (L + L_{sp})^2$$

$$\Delta(M, \phi) \sim \Delta_y * (M/M_y) + (\phi - \phi_y) * L_p * (L - L_p/2)$$

where:

ϕ_y	= curvature at first yield
L	= column height
L_p	= analytical plastic hinge length
L_{sp}	= strain penetration
ϕ	= curvature at point of interest
M_y	= moment at first yield
M	= moment associated with ϕ
F	= force associated with $\Delta = M / L$

Priestley, Calvi, Kowalsky 2007

Idealized Force – Displacement

$$\Delta_{yi} \sim 1/3 * \phi_{yi} * (L + L_{sp})^2$$

$$\Delta^L{}_C = \Delta_{yi} + \Delta_p \sim \Delta_{yi} + (\phi_u - \phi_{yi}) * L_p * (L - L_p/2)$$

where:

ϕ_{yi}	= idealized yield curvature
Δ_{yi}	= idealized yield displacement
Δ_p	= plastic displacement capacity
L	= column height
L_p	= analytical plastic hinge length
L_{sp}	= strain penetration
ϕ_u	= ultimate curvature
$\Delta^L{}_C$	= ultimate displacement
F_p	= plastic force = M_p / L

Priestley, Calvi, Kowalsky 2007

Idealized Force - Displacement

- So the deformation values for the example problem are:

$$\Delta_{yi} = 1/3*0.0001132*(240+14.38)^2 = 2.44 \text{ IN}$$

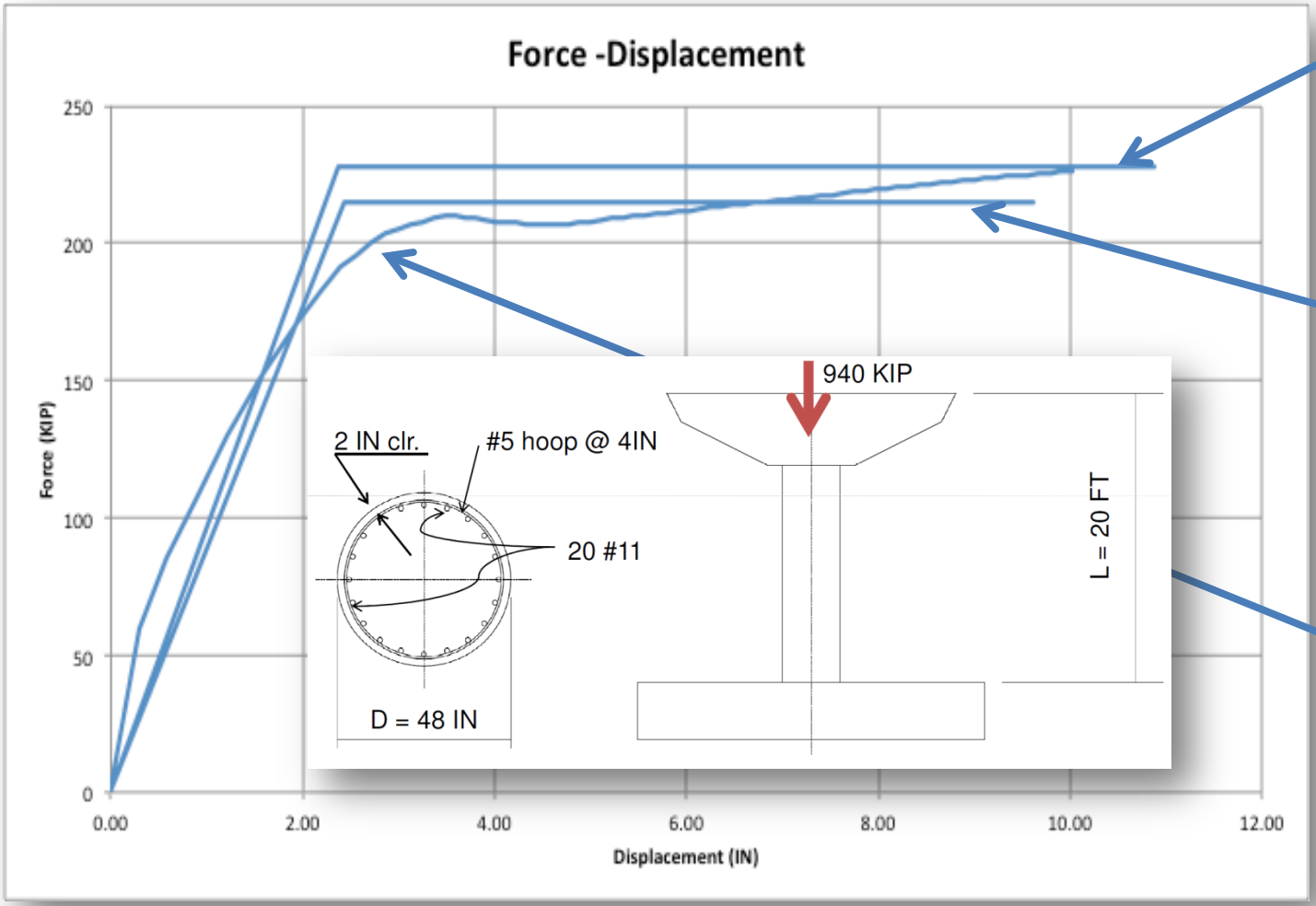
$$\Delta^L_C = 2.44 + (0.001075 - 0.0001132)*33.58*(240-33.58/2) = 9.62 \text{ IN}$$

where:

ϕ_{yi} = 0.0001132 1/IN
L = 20 FT = 240 IN
L_{sp} = 0.15*68*1.41 = 14.38 IN
L_p = 0.08*240 + 14.38 = 33.58 IN > 28.8 IN
ϕ_u = 0.001075 1/IN
M_p = M_{yi} = M_u = 51626 K-IN
F_p = M_p / L = 51626 / 240 = 215 KIP
μ_D = Δ^L_C / Δ_{yi} = 9.62 / 2.44 = 3.94

Force - Displacement

- The idealized response for the single column example



Approximate M_p, ϕ_{yi} and ϕ_u

Idealized Spreadsheet M_p, ϕ_{yi} and ϕ_u

Predicted Response from Spreadsheet calculated M-ϕ

Approximate Methods

- As with *M*-ϕ, we prefer a simplified method to check the computer results
- simple method to check the computer results
- could use the closed-form AASHTO equations to start but they are developed for specific target ductility / strain limits
- *use these approximations with due skepticism*

Approximate Methods - Δ_{yi}

- For a rough check of conventional **circular** reinforced concrete column sections:

$$\Delta_{yi} \sim 1/3*\phi_{yi}*(12*L+0.15*f_{ye}*d_b)^2 \sim L^2 / 42B_o$$

where:

Δ_{yi} = idealized yield displacement (IN)

L = contraflexure to plastic hinge distance (FT)

d_b = diameter of longitudinal column bar (IN)

ϕ_{yi} = idealized yield curvature ~ $2.25*\varepsilon_{ye}/B_o$ (1/IN)

B_o = column diameter (FT)

f_{ye} = expected yield stress (KSI)

Approximate Methods - Δ_{yi}

- For a rough check of conventional **rectangular** reinforced concrete column sections:

$$\Delta_{yi} \sim 1/3*\phi_{yi}*(12*L+0.15*f_{ye}*d_b)^2 \sim L^2 / 45B_o$$

where:

Δ_{yi} = idealized yield displacement (IN)

L = contraflexure to plastic hinge distance (FT)

d_b = diameter of longitudinal column bar (IN)

ϕ_{yi} = idealized yield curvature ~ $2.1*\varepsilon_{ye}/12B_o$ (1/IN)

B_o = column width in direction of loading (FT)

f_{ye} = expected yield stress (KSI)

Approximate Methods - Δ_c^L

- And for a *very rough check* of conventional reinforced concrete columns with $L/B_o > 4$:

$$\Delta^L_C \sim \Delta_{yi} + (\phi_u - \phi_{yi})*L_p*(12*L-L_p/2) \sim L^2 / 10B_o$$

where:

Δ^L_C = local displacement capacity (IN)

L = contraflexure to plastic hinge distance (FT)

L_p = analytical plastic hinge length (IN)

ϕ_u = ultimate curvature ~ ε_{su}^R /12B_o (1/IN)

ϕ_{yi} = idealized yield curvature (1/IN)

B_o = column diameter / width (FT)

Force-Displacement Check

- Diameter, D = 48 IN => B_o = 4 FT
- Height, $L = H_o$ = 20 FT
- M_p ~ 54713K-IN/12 = 4560 K-FT (see previous check)

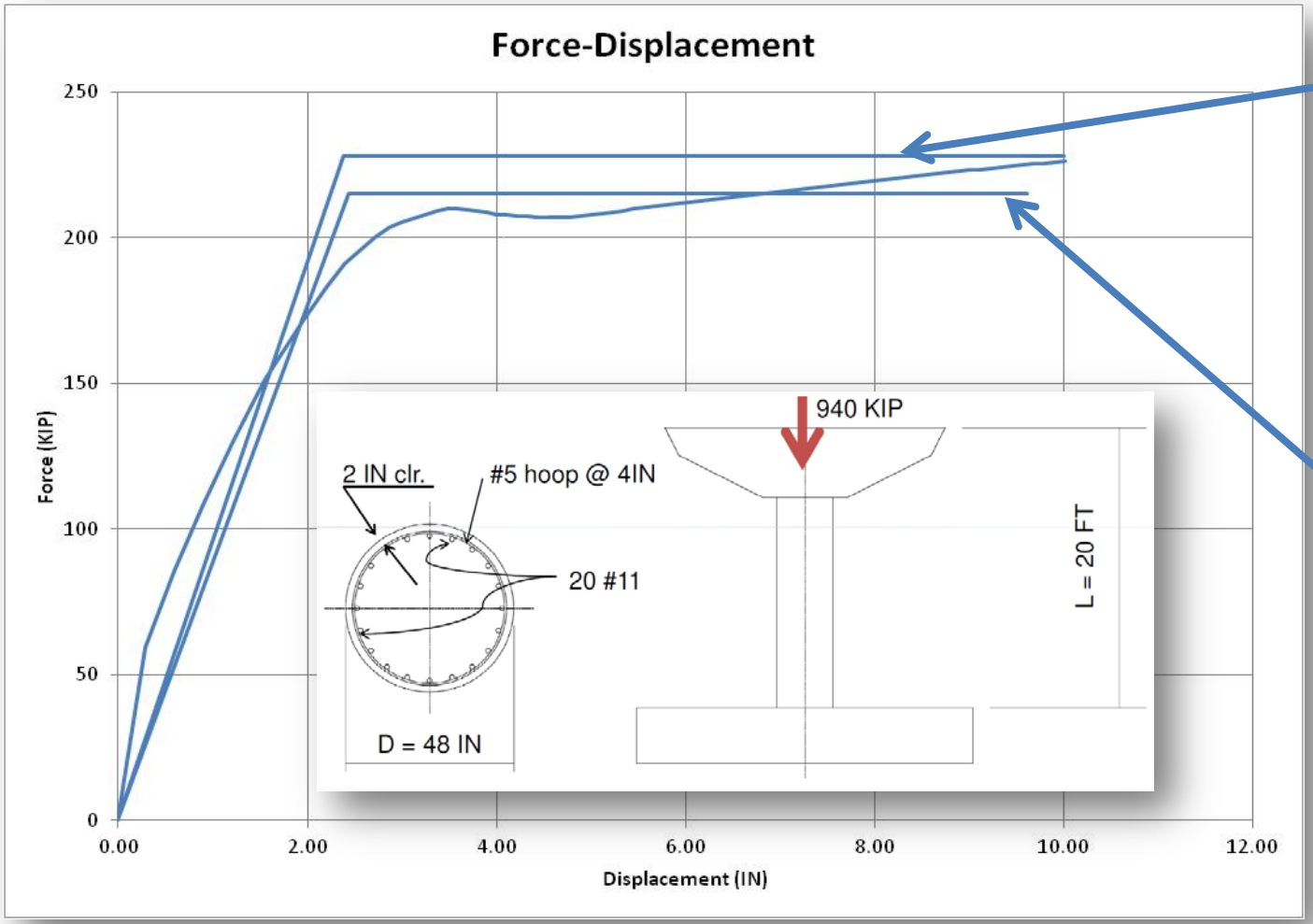
$$\Delta_{yi} \sim L^2 / 42B_o = 20^2/(42*4) = 2.4 \text{ IN v. } \textit{2.4 IN}$$

$$\Delta^L_C \sim L^2 / 10B_o = 20^2/(10*4) = 10 \text{ IN v. } \textit{9.6 IN}$$

$$F_p = M_p / L = 4560/20 = 228 \text{ KIP v. } \textit{215 KIP}$$

Force - Displacement

- Idealized response for the single column example

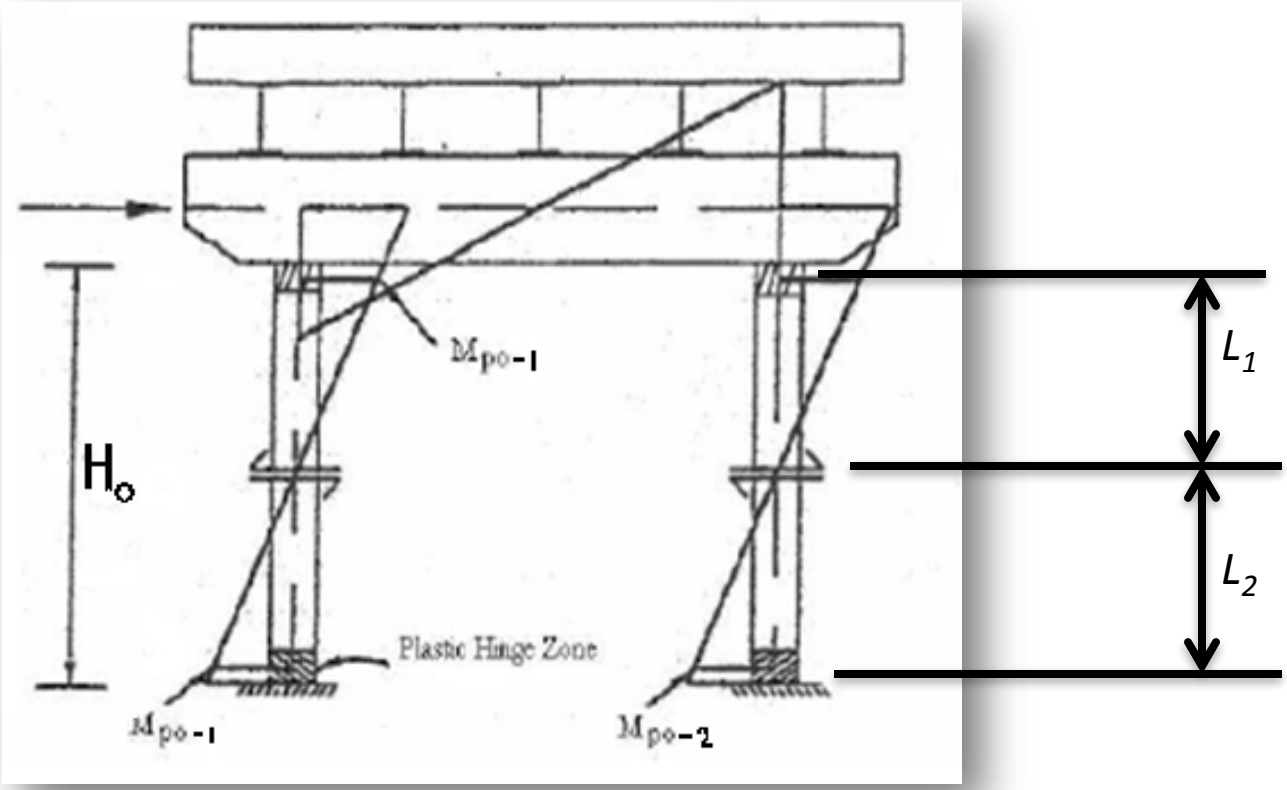


Approximate M_p, Δ_{yi} and $\Delta^L{}_C$

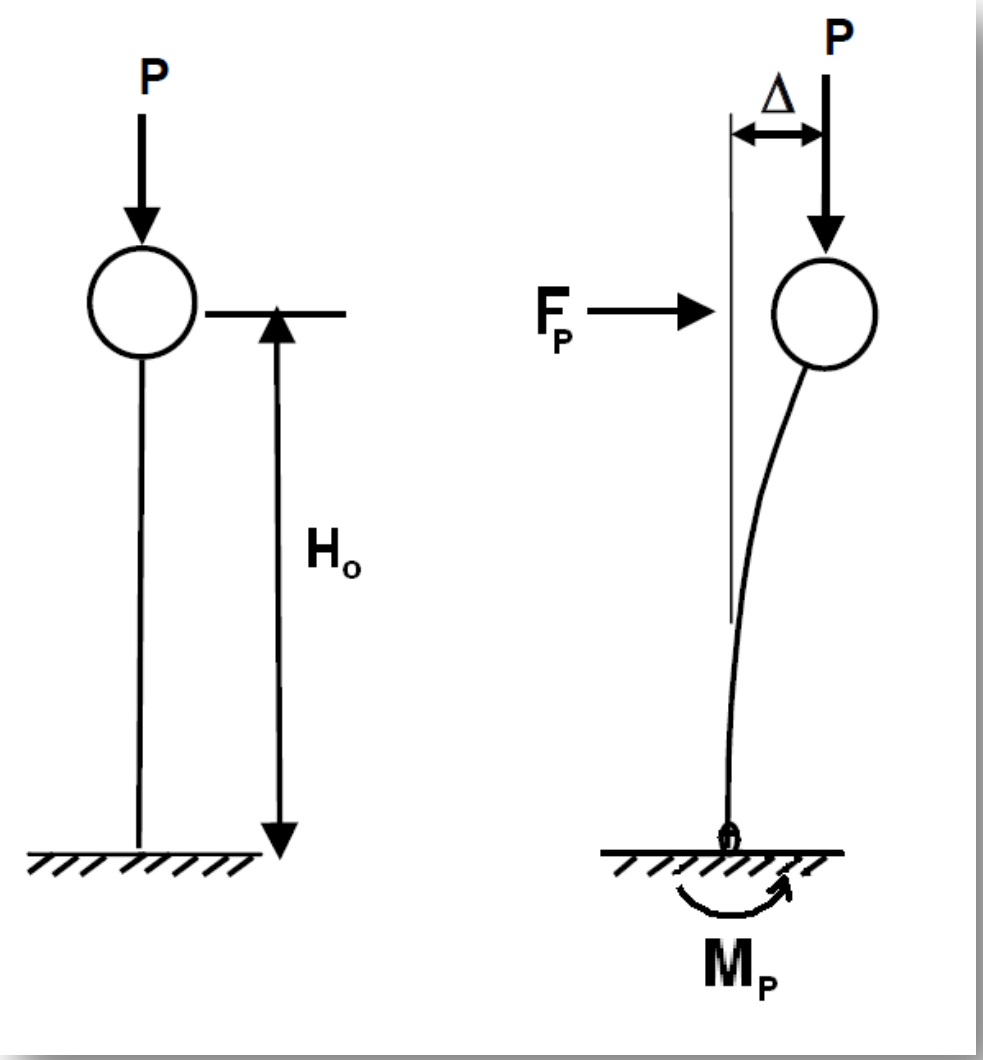
Spreadsheet calculated M_p, ϕ_{yi} and ϕ_u

What about Double Curvature?

- Effective column height, *L*, is taken from the maximum moment location to the contraflexure point
- Then add the displacement results for each part



What about P-Δ Effects?



From equilibrium and summing moments about the base of the column:

$$M_p = F_p * H_o + P * \Delta$$

What about P-Δ Effects?

- The moment left to resist lateral forces becomes,

$$M'_p = M_p - P * \Delta$$

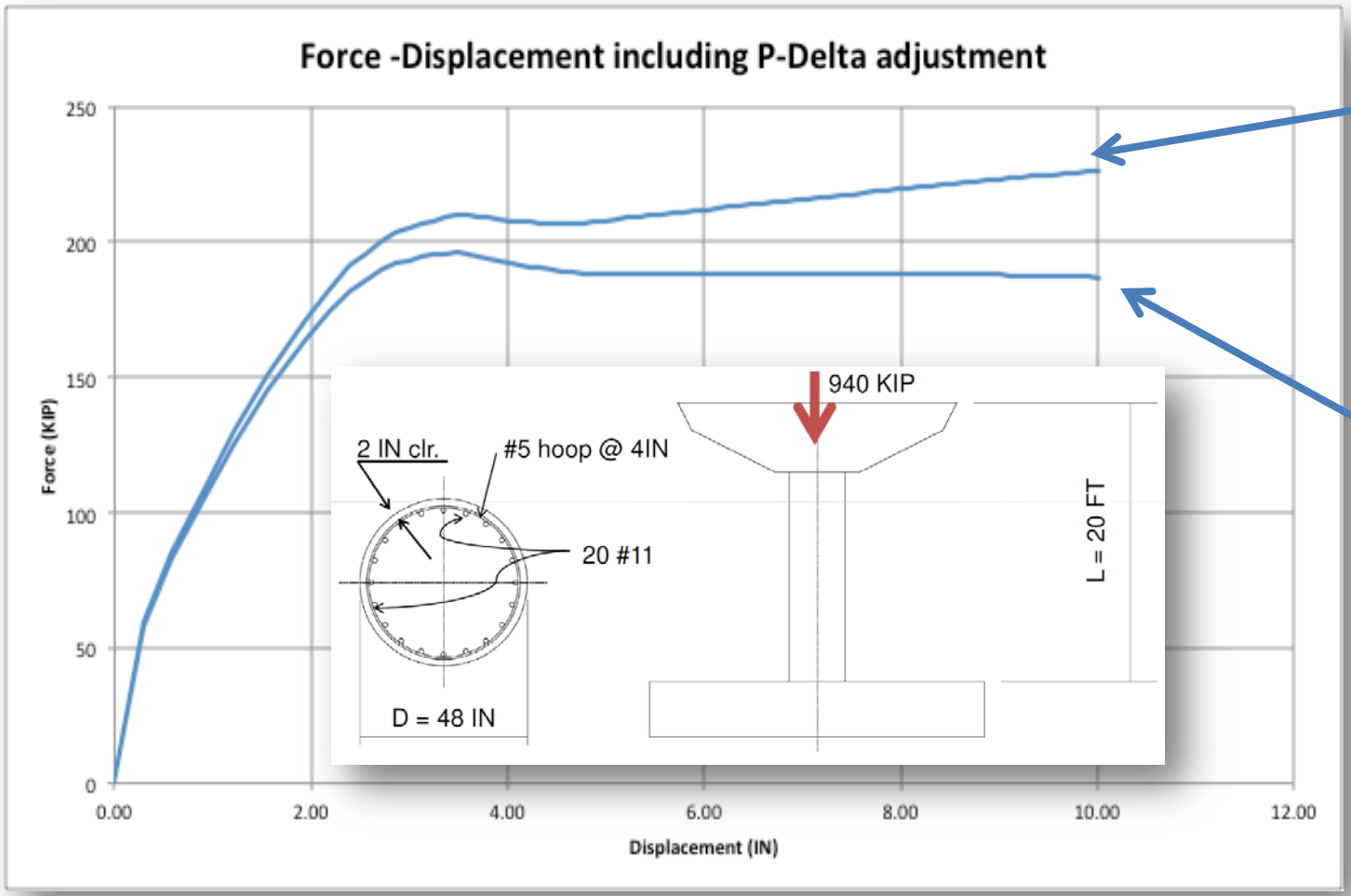
$$F'_P = M'_P / H_o$$

- But when using an idealized elastic perfectly-plastic force-displacement relationship check (SGS method)

$$P_{dl} * \Delta_r \leq 0.25 * M_p$$

What about P-Δ Effects?

- Adjusted response for the single column example



Unadjusted for *P-Δ*

Adjusted for *P-Δ*

What about P-Δ Effects?

- Rearranging the *P-Δ* limit the maximum permissible deflection for the single column example

$$\Delta_r \leq 0.25 * M_p / P_{dl}$$

$$\Delta_r \leq 0.25 * 51626 \text{ K-IN} / 940 \text{ K} = 13.7 \text{ IN}$$

- In this case, the *P-Δ* limit is greater than the calculated Δ^L_C value

- With more confinement, the *P-Δ* limit *may* govern

Local Ductility v. Global Ductility

- Use *local* member displacements

$$\Delta_D^L < \Delta_C^L$$

$$\mu_D = \Delta_D^L \ / \ \Delta_{yi}$$

where:

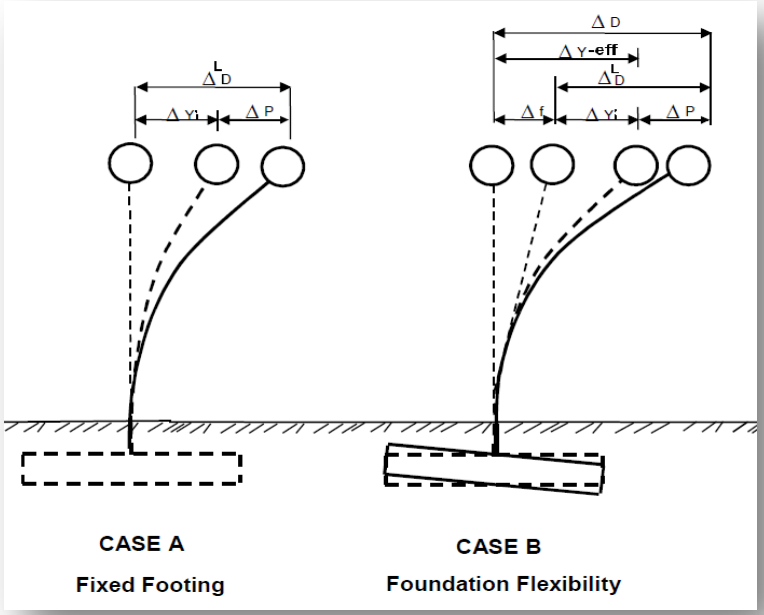
Δ_D^L = Local member deformation demand

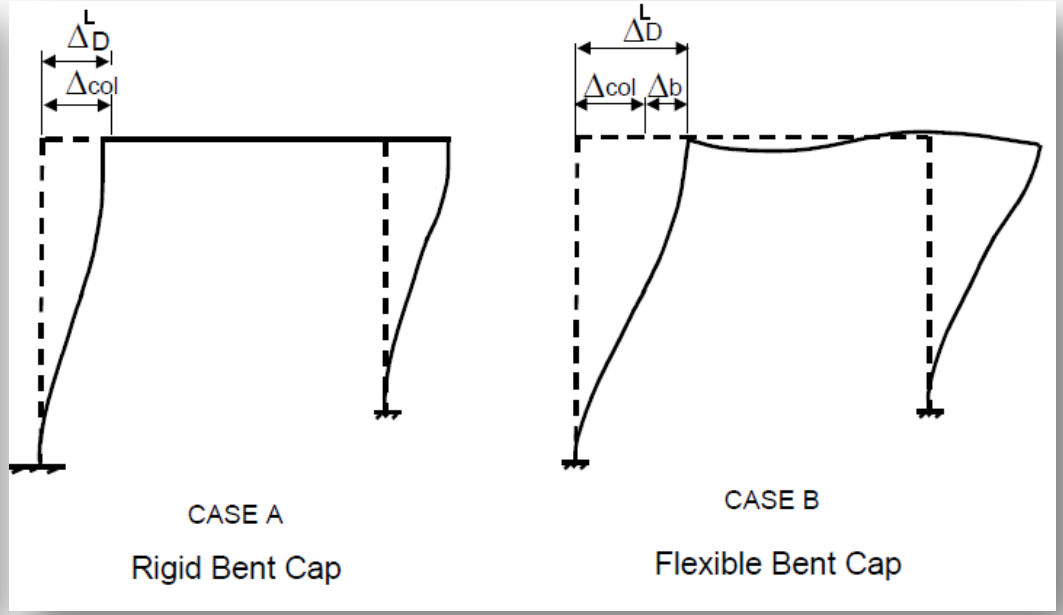
Δ_C^L = Local member deformation capacity

μ_D = local member displacement ductility demand

Δ_{yi} = idealized yield deformation

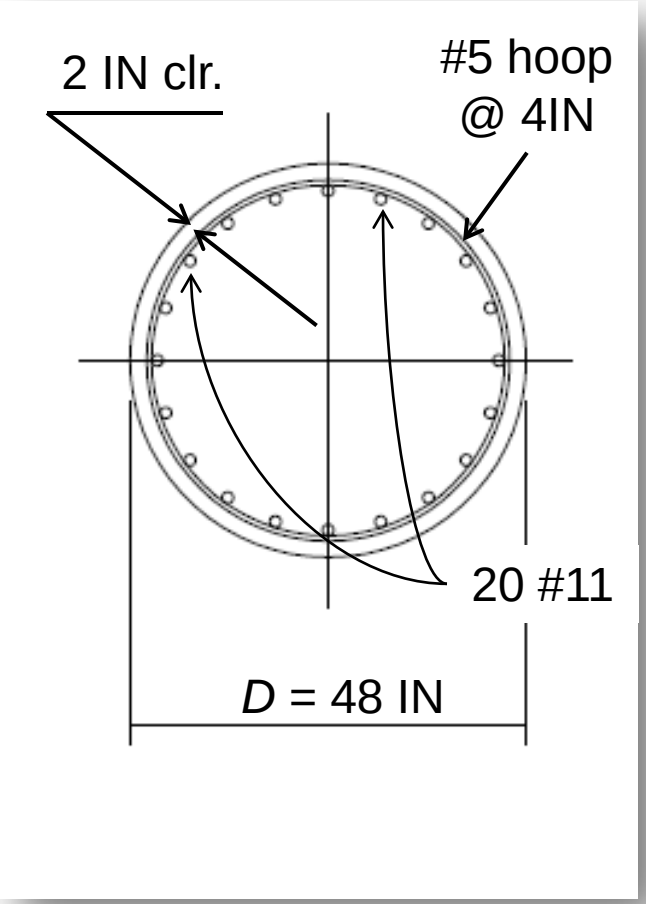
Local Ductility v. Global Ductility

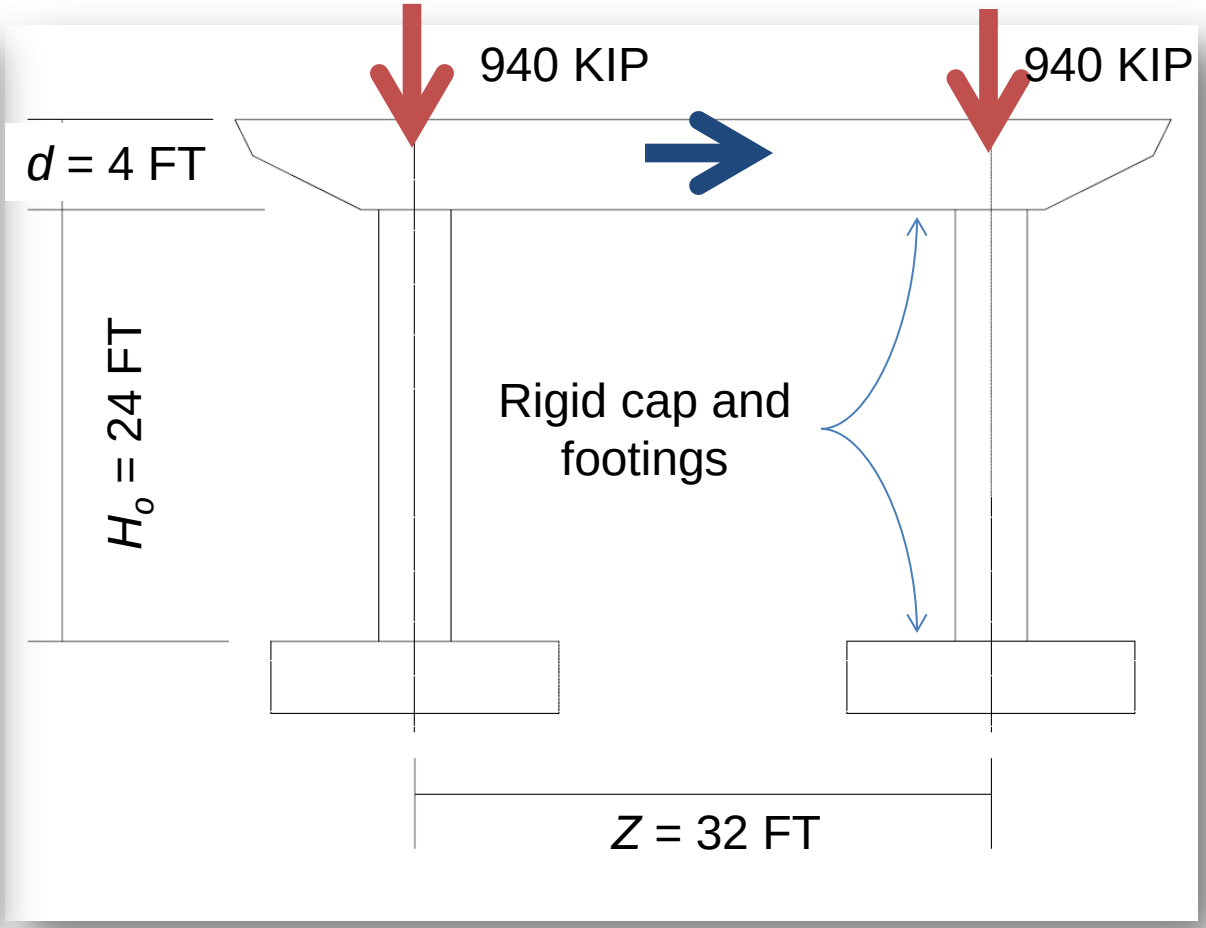




- Must remove the deformation components associated with non-column deformation demands

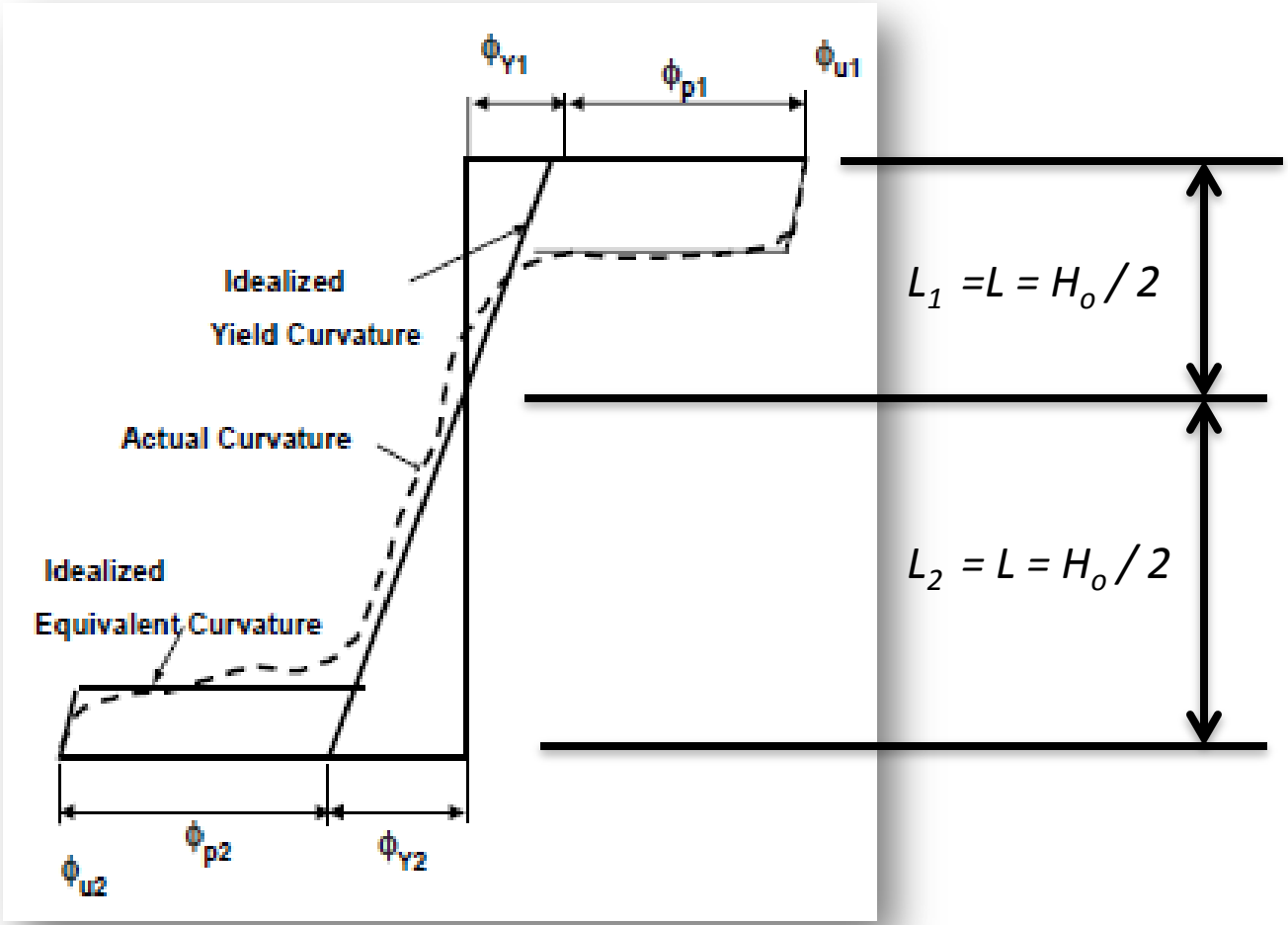
Design Example





Design Example

- Assume that the point of contraflexure is at column mid-height



Design Example – Predict Response

- Use approximate methods to predict expected response
- Diameter, D = 48 IN => B_o = 4 FT
- Height, H_o = 24 FT so for the transverse direction, L = 12 FT
- $P = P_{DL} \pm P_{EQ}$
- $P_{EQ} = (M_{p\text{-}L} + M_{p\text{-}R}) * (1 + d / H_o) / Z$ ← *why?*
- *U*se $P = P_{DL}$ = 940 K for first iteration

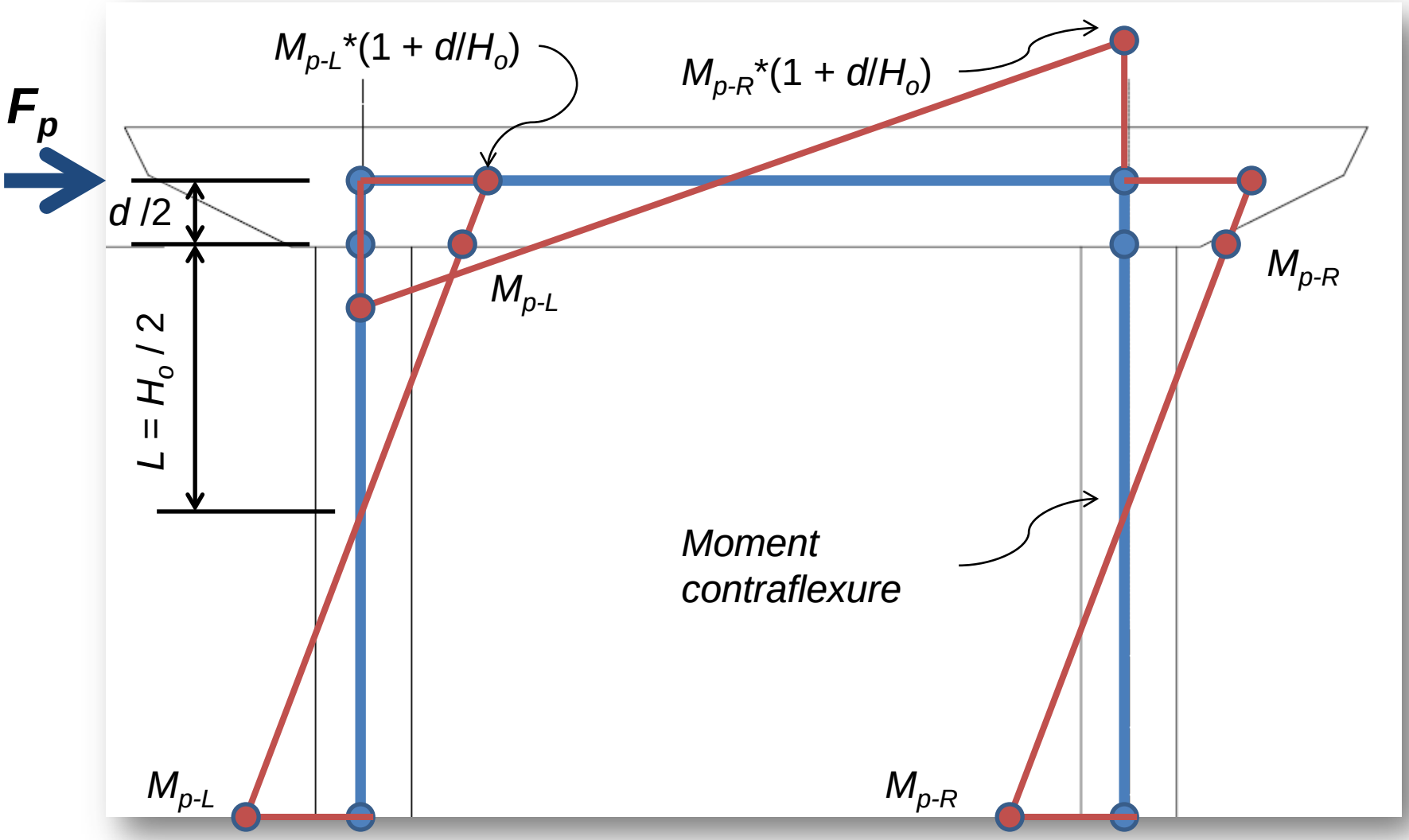
- M_p ~ 54720K-IN/12 = 4560 K-FT (see previous calculations)

$$\Delta_{yi} \sim 2 * L^2/42B_o = 2 * 12^2/(42*4) = 1.71 \text{ IN}$$

$$\Delta^L_C \sim 2 * L^2/10B_o = 2 * 12^2/(10*4) = 7.2 \text{ IN}$$

$$F_p = (2*M_{p\text{-}L} + 2*M_{p\text{-}R}) / H_o \sim 4 * M_p /24 = 760 \text{ KIP}$$ ← *why?*

Design Example - FBD



Moment Diagram – slope of line is the shear – shear in cap beam is axial force in column

Design Example – Predict Response

- Perform a second iteration to verify initial assumptions

- $P_{EQ} = (M_{p\text{-}L}+M_{p\text{-}R})*(1+d/H_o)/ Z$ ~ 2*54720*1.167 /12/32 ~ 332 K

- P = $P_{DL} \pm P_{EQ}$ = 940 -/+ 332 = 608K and 1272K [compression]

- $M_{p\text{-}L}$ ~ (0.05+0.2*1.724+0.0643) * 48^3 = 50808 K-IN

- $M_{p\text{-}R}$ ~ (0.05+0.2*1.724+0.130) * 48^3 = 58612 K-IN

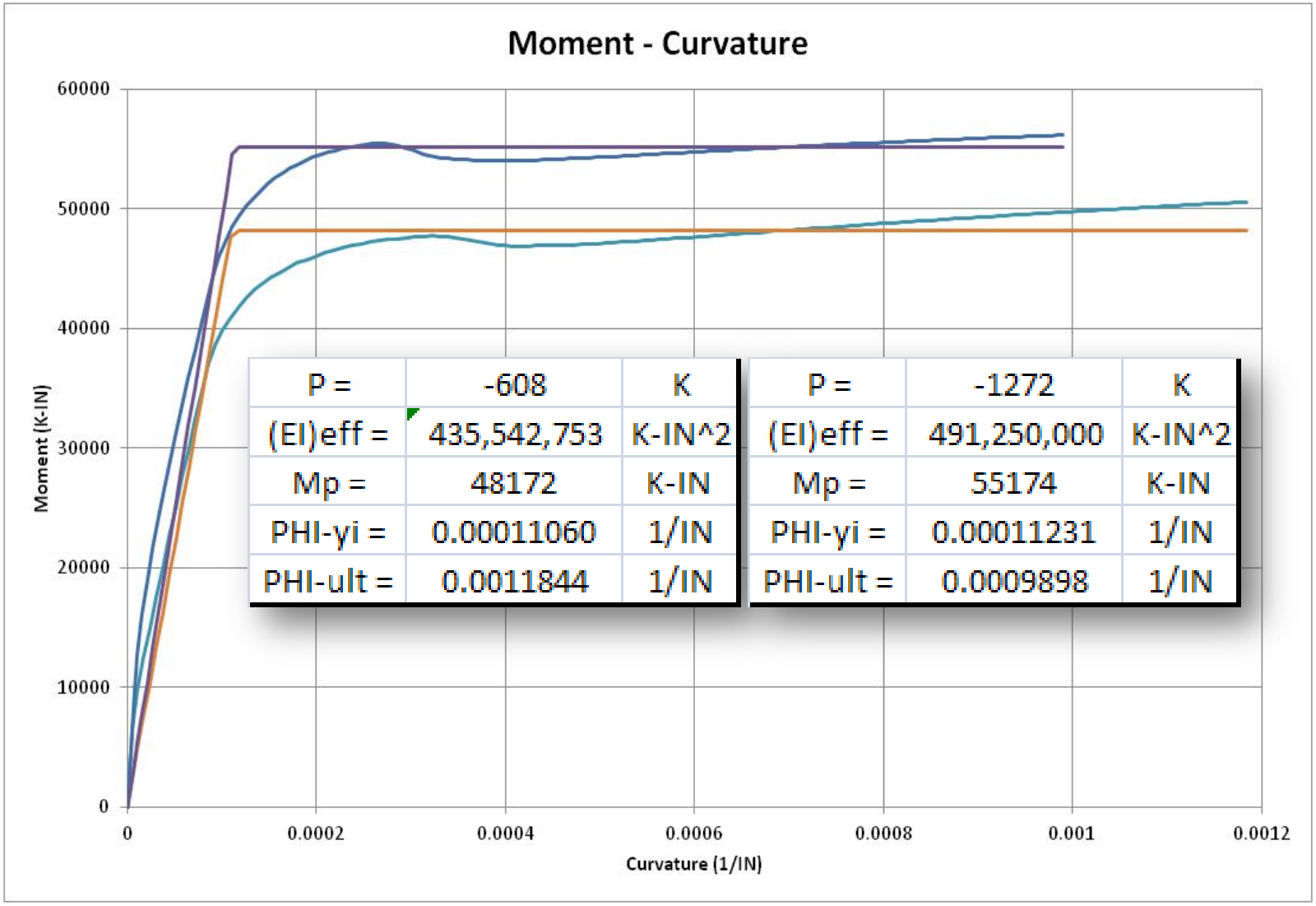
- $P_{EQ} = (M_{p\text{-}L} + M_{p\text{-}R})*(1 + d / H_o) / Z$
- P_{EQ} = (50808+58612)(1.167)/32/12 = 332 K ✓

- $F_p = (2 * M_{p\text{-}L} + 2 * M_{p\text{-}R}) / H_o$ = 760 KIP ✓

Design Example - Process

- Now use the refined analysis to determine the force-displacement response of the pier
- First calculation idealized M-ϕ
- Then calculate L_{sp} and L_p
- Then calculate the force-displacement for each column
- Then add the results of each column to find the total pier response

Design Example – Refined Analysis



P =	-608	K
(EI)eff =	435,542,753	K-IN^2
Mp =	48172	K-IN
PHI-yi =	0.00011060	1/IN
PHI-ult =	0.0011844	1/IN

P =	-1272	K
(EI)eff =	491,250,000	K-IN^2
Mp =	55174	K-IN
PHI-yi =	0.00011231	1/IN
PHI-ult =	0.0009898	1/IN

Design Example – Refined Analysis

- Verify that the axial forces are reasonably close to the initial estimate

$P_{EQ} = (M_{p\text{-}L} + M_{p\text{-}R}) * (1 + d / H_o) / Z$

P_{EQ} = (48172+55174)*1.167 /(12*32) ~ 314 K v. 332k

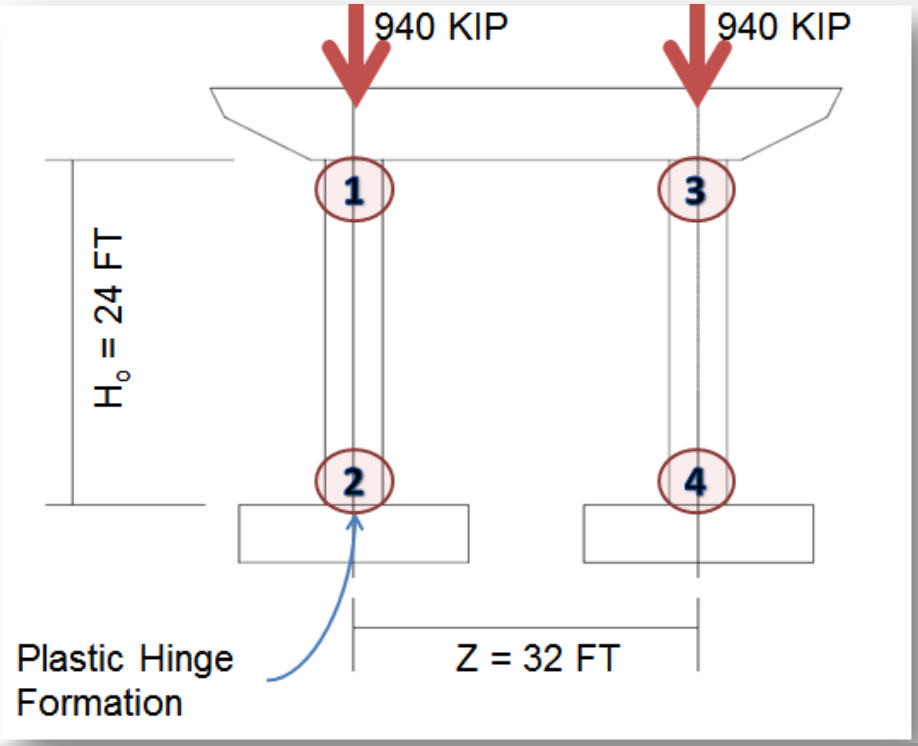
$P = P_{DL} - P_{EQ}$ = 940 - 314 = 626K v. 608K – close enough @ 3%

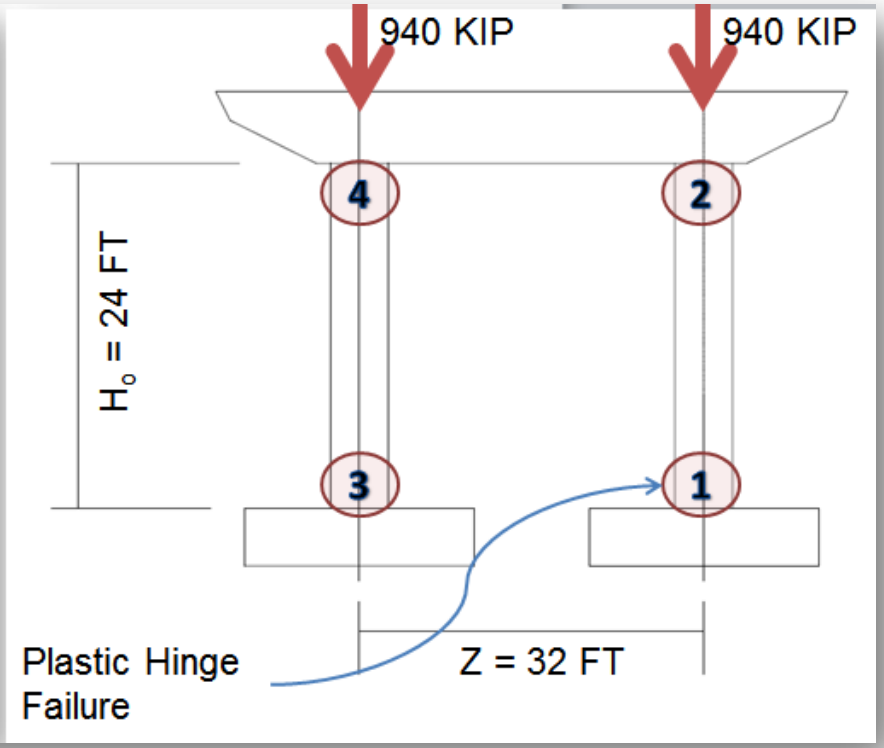
$P = P_{DL} + P_{EQ}$ = 940 + 314 = 1254K v. 1272K – close enough @ 2%

- Use the initial values since they are close

Design Example – Hinge Formation

- Order of hinge formation and hinge failure





Design Example – Displacement

- For the left / trailing side column:

$$\Delta_{yi\text{-}L} = 1/3*0.0001106*(144+14.38)^2 = 0.92 \text{ IN}$$

$$\Delta^L_{C\text{-}L} = 0.92 +(0.0011844 – 0.0001106)*28.8*(144-28.8/2) = 4.92 \text{ IN}$$

where:

ϕ_{yi} = 0.0001106/IN (from M-ϕ analysis)
L = H_o / 2 = 24 FT / 2 * 12 = 144 IN
L_{sp} = 0.15*68*1.41 = 14.38 IN
L_p = 0.08*144 + 14.38 = 25.9 IN < 2*L_{sp} = 28.8 IN
ϕ_u = 0.0011844 1/IN (from M-ϕ analysis)
M_p = 48172 K-IN (from M-ϕ analysis)
$F_{p\text{-}L}$ = M_p / L = 48172 / 144 = 335 KIP

Design Example – Displacement

- For the right / right side column:

$$\Delta_{yi\text{-}R} = 1/3*0.00011231*(144+14.38)^2 = 0.94 \text{ IN}$$

$$\Delta^L_{C\text{-}R} = 0.94 + (0.0009898 - 0.00011231)*28.8*(144-28.8/2) = 4.21 \text{ IN}$$

where:

ϕ_{yi} = 0.00011231/IN (from M-ϕ analysis)
L = H_o / 2 = 24 FT / 2 * 12 = 144 IN
L_{sp} = 0.15*68*1.41 = 14.38 IN
L_p = 0.08*144 + 14.38 = 25.9 IN < 2*L_{sp} = 28.8 IN
ϕ_u = 0.0009898 1/IN (from M-ϕ analysis)
M_p = 55174 K-IN (from M-ϕ analysis)
$F_{p\text{-}R}$ = M_p / L = 55174 / 144 = 383 KIP

Design Example

	Left Column	Right Column
Axial compression, P	608	1272
Effective stiffness, $(EI)_{eff}$	4.36E9	4.91E9
Plastic moment, M_p	48172	55174
Idealized yield curvature, ϕ_{yi}	0.00011060	0.00011231
Ultimate curvature, ϕ_u	0.0011844	0.0009898
Plastic curvature, ϕ_p	0.001074	0.000877
Analytical Plastic Hinge Length, L_p	28.8	28.8
Δ_{yi} (per half of column height)	0.92	0.94
Δ_p (per half of column height)	4.00	3.27
Δ_u (per half of column height)	4.92	4.21
Plastic shear, F_p	335	383

Design Example – Combined Response

- Total effective lateral force

$$F_p = F_{p\text{-}L} + F_{p\text{-}R} = 335\text{K} + 383\text{K} = 718\text{ K}$$

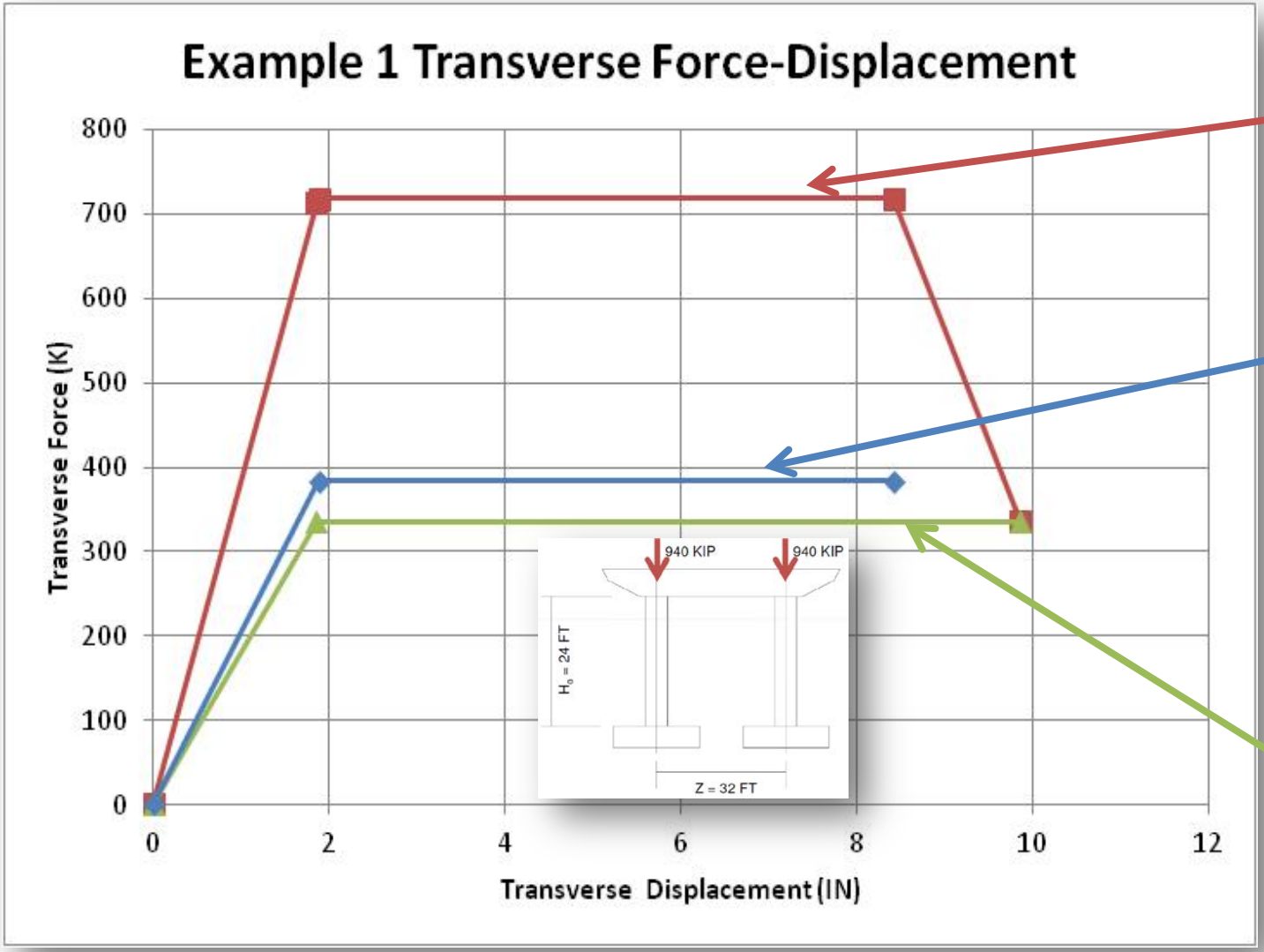
- Idealized yield displacement

$$\Delta_{yi} \sim (2*\Delta_{yi\text{-}L} + 2*\Delta_{yi\text{-}R}) / 2 = 0.92 + 0.94 = 1.86\text{ IN}$$

- Ultimate displacement (first hinge failure on right)

$$\Delta^L_c = 2 * \Delta^L_{c\text{-}R} = 2 * 4.21\text{ IN} = 8.42\text{ IN}$$

Design Example – Idealized Response



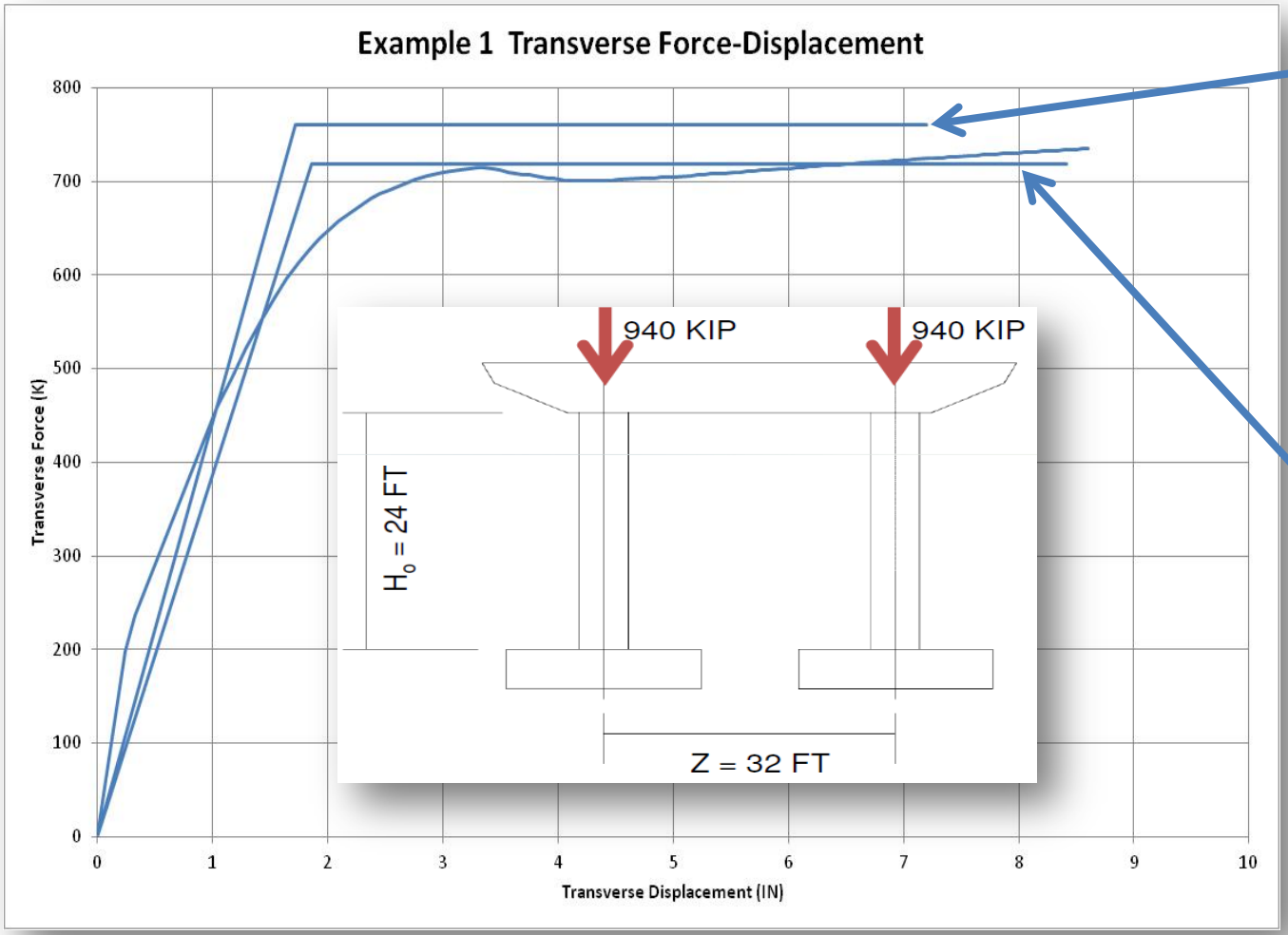
Combined

Right / Leading Column

Left / Trailing Column

Design Example – Refined Response

- Idealized force displacement for the two column pier



Approximate M_p, Δ_{yi} and $\Delta^L{}_C$

Spreadsheet Idealized M_p, ϕ_{yi} and ϕ_u

Design Example – Verification

- The maximum deflection based on *P-Δ* limits

$$\Delta_r \leq 0.25 * 48172 \text{ K-IN} / 940 \text{ K} = 12.8 \text{ IN} \checkmark \text{ (per half)}$$

- And the displacement ductility *capacity*

$$\mu_D < \Delta^L_c / \Delta_{yi} = 8.42 / 1.86 = 4.5$$

Capacity Design

- Designer dictates the bridge response (e.g., column hinge response mechanism – Type I)
- Preclude all failure modes that would prevent the formation of the predicted force-deflection response including:
  - Shear failure inside and outside the plastic hinge region (not the same as analytical plastic hinge length, L_p)
  - Column-cap and column-footing joint failure
  - Cap beam, footing or shaft moment, shear, and axial overload / hinging
  - Footing overturning, sliding, uplift and rocking failure
  - Any other failure that occurs prior to reaching Δ^L_C

Thank you - Questions

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