# NCHRP 12-68

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# Rotational Limits for Elastomeric Bearings

**Final Report**

***Appendix I***

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# APPENDIX I Notation

| | |
|---|---|
| $a$ | = dimensionless coefficient in fatigue model |
| $A$ | = area of the bearing = $WL$ |
| $A_a$ | = dimensionless coefficient in axial stiffness |
| $A_{az}$ | = dimensionless coefficient in axial stiffness = $A_a$ (App. E) |
| $a_{ij}$ | = dimensionless coefficient in FEA error analysis |
| $A_{net}$ | = plan area of bearing based on net dimensions |
| $AR$ | = aspect ratio = the smaller of $L/W$ and $W/L$. |
| $A_r$ | = dimensionless coefficient in rotational stiffness |
| $A_{ry}$ | = dimensionless coefficient in rotational stiffness = $A_r$ (App. E) |
| $b$ | = dimensionless coefficient in fatigue model |
| $B_a$ | = dimensionless coefficient in axial stiffness |
| $B_{az}$ | = dimensionless coefficient in axial stiffness (App. E) |
| $B_r$ | = dimensionless coefficient in rotational stiffness for compressible layers |
| $B_{r0}$ | = dimensionless coefficient in rotational stiffness for incompressible layers |
| $B_{ry}$ | = dimensionless coefficient in rotational stiffness = $B_r$ |
| $\mathbf{C}$ | = Right Cauchy-Green strain tensor |
| $C_{10}$, $C_{20}$, $C_{30}$ | = Material parameters for Yeoh's model |
| $C_a$ | = dimensionless coefficient in shear strain due to axial load |
| $C_{azzx}$ | = dimensionless coefficient in shear strain due to axial load = $C_a$ (App. E) |
| $c_n$ | = dimensionless coefficient in fatigue model |
| $C_r$ | = dimensionless coefficient in shear strain due to rotational |
| $C_{ryzx}$ | = dimensionless coefficient in shear strain due to rotational = $C_r$ (App. E) |
| $c_S$ | = limiting permissible of $S^2/n$ for Method A design |
| $c_\sigma$ | = dimensionless stress coefficient (lift-off equations) |
| $D$ | = debonding level |
| $D$ | = diameter of the bearing (App. G) |
| $D_a$ | = dimensionless shear strain coefficient for axial load |
| $D_r$ | = dimensionless shear strain coefficient for rotation |
| e | = Euler's constant (basis of Napieran logarithm) |

| | |
|---|---|
| **E** | = Green-Lagrange strain tensor |
| $E$ | = Young's modulus |
| $E_{az}$ | = apparent Young's modulus for axial loading |
| $E_{ry}$ | = apparent Young's modulus for rotational loading |
| $F_r$ | = dimensionless coefficient for rotation (uplift equations) |
| $G$ | = shear modulus |
| $g_0$, $g_1$ | = dimensionless coefficients in fatigue model |
| $H_a$ | = dimensionless coefficient for axial load (uplift equations) |
| $H_r$ | = dimensionless coefficient for rotation (uplift equations) |
| $h_{ri}$ | = thickness of $i^{th}$ interior layer of elastomer |
| $h_{rt}$ | = total thickness of all interior layers of elastomer |
| $I$ | = moment of inertia (second moment of area) |
| $K$ | = bulk modulus |
| $K_a$ | = total axial stiffness |
| $K_r$ | = total rotational stiffness |
| $L$ | = length of bearing based on gross dimensions (= plan dimension of the bearing perpendicular to the axis of rotation under consideration) |
| $l$ | = span of a girder |
| $L_{net}$ | = net length of bearing (average of gross and shim dimensions) |
| $M$ | = moment on bearing |
| $m$ | = exponent in fatigue model |
| $N$ | = number of cycles |
| $n$ | = number of interior layers of elastomer |
| $N_{cr}$ | = characteristic number of cycles |
| $p$ | = force per unit length |
| $P$ | = total axial force |
| $P_{sd}$ | = minimum vertical force due to permanent loads |
| **S** | = 2nd Piola-Kirchhoff stress tensor |
| $S$ | = shape factor |
| $S_i$ | = shape factor of instantaneous compressed region (lift-off equations) |
| $t$ | = thickness of elastomeric layer |
| $W$ | = gross width of elastomeric layer (= plan dimension of the bearing parallel to the axis of rotation under consideration) |

| | |
|---|---|
| $W_{net}$ | = net width of elastomeric layer (average of gross and shim dimensions) |
| $x,y,z$ | = coordinates in Cartesian system |
| $\Delta_a$ | = axial deflection |
| $\Delta_s$ | = maximum total shear displacement of the bearing at the service limit state. |
| $I_1, (I_1^*)$ | = first invariant of **C** (specialized for uniaxial tension) |
| $\alpha$ | = dimensionless load combination parameter $=\varepsilon_a/S\theta_L$ |
| $\delta_{bottom}$ | = vertical displacement of bottom shim |
| $\delta_{top}$ | = vertical displacement of top shim |
| $\varepsilon_a$ | = average axial strain for bearing under axial load |
| $\varepsilon_{ai}$ | = axial strain at the middle of the instantaneous compressed region (lift-off equations) |
| $\varepsilon_{az}$ | = average axial strain = $\varepsilon_a$ |
| $\varepsilon_{zz}$ | = local vertical normal strain in rubber layer |
| $\gamma_a$ | = shear strain in *z-x* plane due to axial loading |
| $\gamma_{a,cy}$ | = cyclic portion of shear strain in *z-x* plane due to axial loading |
| $\gamma_{a,max}$ | = absolute maximum shear strain in *z-x* plane due to axial loading |
| $\gamma_{a,st}$ | = static portion of shear strain in *z-x* plane due to axial loading |
| $\gamma_{cap}$ | = shear strain capacity |
| $\gamma_r$ | = shear strain in *z-x* plane due to rotation loading |
| $\gamma_{r,cy}$ | = cyclic portion of shear strain in *z-x* plane due to rotation loading |
| $\gamma_{r,max}$ | = absolute maximum shear strain in *z-x* plane due to rotation loading |
| $\gamma_{r,st}$ | = static portion of shear strain in *z-x* plane due to rotation loading |
| $\gamma_{r0}$ | = shear strain constant in fatigue model |
| $\gamma_s$ | = shear strain in *z-x* plane due to shear displacement |
| $\gamma_{s,cy}$ | = cyclic portion of shear strain in *z-x* plane due to shear displacement |
| $\gamma_{s,st}$ | = static portion of shear strain in *z-x* plane due to shear displacement |
| $\gamma_{tot,max}$ | = maximum total shear strain in *z-x* plane |
| $\gamma_{zx}$ | = local shear strain in *z-x* plane |
| $\eta$ | = relative length of the instantaneous compressed region (lift-off equations) |
| $\lambda$ | = compressibility index = $S\sqrt{3G/K}$ |

| | |
|---|---|
| $\lambda_1, \lambda_2, \lambda_3$ | = principal stretches (App. E) |
| $\theta$ | = end rotation of a girder (rotation demand on bearing) |
| $\theta_c$ | = characteristic rotation for which the vertical displacement on the "tension" side becomes net upwards |
| $\theta_i$ | = rotation of the $i^{th}$ layer of elastomer |
| $\theta_L$ | = rotation per layer |
| $\theta_x$ , $\theta_y$ | = rotation of whole bearing about $x$ or $y$ axis |
| $\rho$ | = dimensionless rotation ratio (lift-off equations) |
| $\sigma_a$ | = average axial stress |
| $\sigma_{a0}$ | = fictitious average axial stress for entire bearing surface (lift-off equations) |
| $\sigma_{hyd}$ | = hydrostatic stress (mean direct stress) |
| $\sigma_{rupture}$ | = (hydrostatic) rupture strength of rubber |
| $\sigma_{zz}$ | = local vertical normal stress in rubber layer |
| $\tau_{zx}$ | = local shear stress in $z$-$x$ plane |
| $\xi$ | = dimensionless position parameter = $2x/L$ |