A NEW FLEXIBLE MULTIPLE DISCRETE-CONTINUOUS EXTREME VALUE (MDCEV) CHOICE MODEL

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Introduction

- Multiple discreteness
  - Choice of multiple alternatives simultaneously
- Modeling methodologies of multiple discrete situations
  - Traditional random utility-based (RUM) single discrete choice
    - Number of composite alternatives explodes
  - Multivariate probit (logit) methods
    - Not based on underlying utility-maximizing framework
  - Other issues with these methods
    - Cannot easily accommodate diminishing marginal returns

Joint Activities Example

- Conventional discrete choice frameworks **need to generate mutually exclusive alternatives** → results in an explosion in the number of alternatives
- MDC allows us to tackle the problem by considering **activity participation as a household decision**
- MDC offers **substantial computational and behavioral advantages**
  - Employ one model to generate activities
  - Accommodate substitution/complementarity in activity participation and household member dimensions

MDC Model

Each box represents an alternative

P1	P2	P1 P2
None	None	None
A1	None	None
A2	None	None
A1 A2	None	None

P1	P2	P1 P2
None	None	A1
A1	None	A1
A2	None	A1
A1 A2	None	A1

P1	P2	P1 P2
None	None	A2
A1	None	A2
A2	None	A2
A1 A2	None	A2

P1	P2	P1 P2
None	None	A1 A2
A1	None	A1 A2
A2	None	A1 A2
A1 A2	None	A1 A2

P1	P2	P1 P2
None	A1	None
A1	A1	None
A2	A1	None
A1 A2	A1	None

P1	P2	P1 P2
None	A1	A1
A1	A1	A1
A2	A1	A1
A1 A2	A1	A1

P1	P2	P1 P2
None	A1	A2
A1	A1	A2
A2	A1	A2
A1 A2	A1	A2

P1	P2	P1 P2
None	A1	A1 A2
A1	A1	A1 A2
A2	A1	A1 A2
A1 A2	A1	A1 A2

P1	P2	P1 P2
None	A2	None
A1	A2	None
A2	A2	None
A1 A2	A2	None

P1	P2	P1 P2
None	A2	A1
A1	A2	A1
A2	A2	A1
A1 A2	A2	A1

P1	P2	P1 P2
None	A2	A2
A1	A2	A2
A2	A2	A2
A1 A2	A2	A2

P1	P2	P1 P2
None	A2	A1 A2
A1	A2	A1 A2
A2	A2	A1 A2
A1 A2	A2	A1 A2

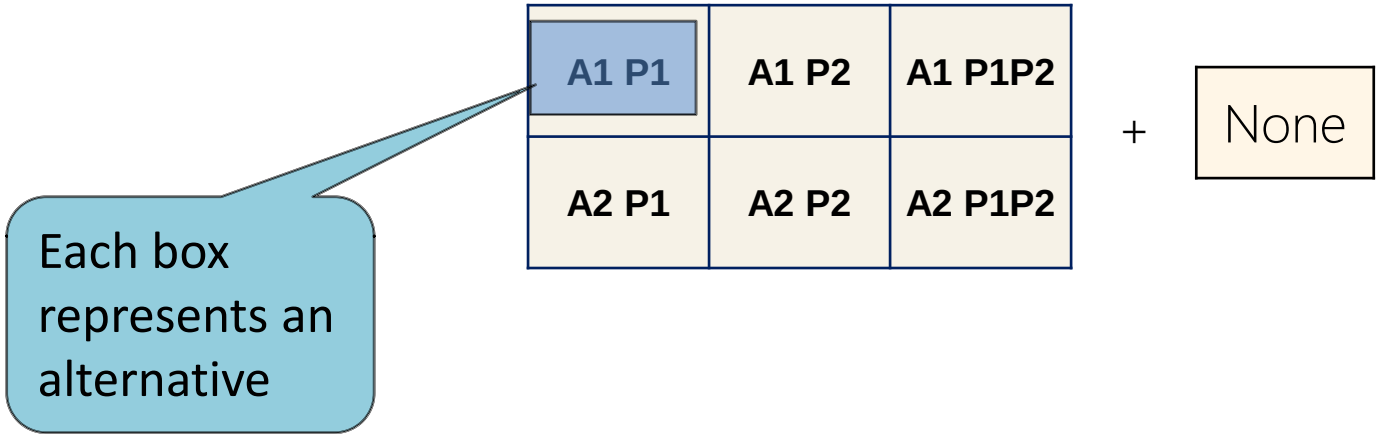
P1	P2	P1 P2
None	A1 A2	None
A1	A1 A2	None
A2	A1 A2	None
A1 A2	A1 A2	None

P1	P2	P1 P2
None	A1 A2	A1
A1	A1 A2	A1
A2	A1 A2	A1
A1 A2	A1 A2	A1

P1	P2	P1 P2
None	A1 A2	A2
A1	A1 A2	A2
A2	A1 A2	A2
A1 A2	A1 A2	A2

P1	P2	P1 P2
None	A1 A2	A1 A2
A1	A1 A2	A1 A2
A2	A1 A2	A1 A2
A1 A2	A1 A2	A1 A2

MDC Model



Alternatives - Total 7 alternatives versus 64 in traditional case

Total choice set size comparison for 3 activity purposes

Household Size	Single Discrete Model (MNL)	MDCEV
1	8	3
2	512	9
3	2097152	21
4	3.52×10^{13}	45
5	9.9×10^{27}	93
Total	9.9×10^{27}	171

Once the number of activities increases, the difference will be even more stark!

MDC Models

Karush-Kuhn-Tucker (KKT) approach

- Wales and Woodland (1983)
- Consistent with utility theory
- Allows incorporation of a combination of corner solutions (zero consumption) for some goods and interior solutions (strictly positive consumption) for other goods
- Random utility distribution assumptions lead to a complicated likelihood function that entails multi-dimensional integration

Kim et al. (2002)

- Used the GHK simulator to evaluate the multivariate normal integral appearing in the likelihood function in the KKT approach
- Used a generalized variant of the well-known translated constant elasticity of substitution (CES) direct utility function

Bhat (2005)

- Based on the generalized variant of the translated CES utility function but with a multiplicative log-extreme value error term
- Labeled as the multiple discrete-continuous extreme value (MDCEV) model
- MDCEV model collapses to the MNL

- The basic approach in a utility maximization framework for multiple discreteness hinges upon the use of a non-linear utility structure with decreasing marginal utility (or satiation).

- Bhat (2008) proposed a Box-Cox utility function form that is quite general and subsumes earlier utility specifications

- Earlier formulations assume that the same baseline utility preference influences both the choice of making a positive consumption of a good (the discrete choice) as well as constitutes the starting point for satiation effects (impacting the continuous choice).

- Do not consider….

  - Variety seeking that operates at the pure discrete level of consumption

  - Branding effects (that is, prestige/image effects) operating at the discrete level, but do necessarily carry over to the continuous consumption decision.

  - Need to consume an “adequate” amount of a good to gain value

Our paper

- A new MDC model that breaks the tight linkage between the discrete and continuous choice dimensions.
- Allows utility determining discrete decision to be different from the baseline preference utility determining the continuous choice.

A New Flexible MDC Model Structure

$$U(\boldsymbol{x}) = \psi_1 x_1 + \sum_{k=2}^{K} \gamma_k \left(\left[\psi_{kd} \right]^{1(x_k=0)} \times \left[\psi_{kc} \right]^{1(x_k>0)} \right) \ln\left\{ \left(\frac{x_k}{\gamma_k} + 1 \right) \right\}$$

An Application: Time Use of Individuals

- Data and Sample: Puget Sound household travel survey conducted during the spring (April–June) of 2014

- Activity Purpose Classification
  - In-home (non-work, non-education, and non-sleep)
  - Personal Business
  - Shopping (including buying goods and food)
  - Recreation
  - Eating out
  - Social activities
  - Serve passenger

$\Phi[-39.72] << 0.0001$

Estimation Results - Traditional MDCEV Model

Independent Variables/ Data Fit Measures	TFIXED-MDCEV	T-MDCEV
Scale parameter	Fixed to the value of 1.0	0.429 (t-stat of 62.08 wr.t 1.0)
Fit measures for continuous consumptions		
LL at convergence	-4163.88	-3529.26
LL at constants	-4534.94	-3940.22
# parameters	19	20
Adjusted likelihood ratio index	0.0776	0.2174
Nested likelihood ratio test	1269.24 > Chi-Squared statistic with 1 degree of freedom; T-MDCEV model preferred	
Fit measures for discrete consumptions		
Predictive log-likelihood at convergence	-13143.75	-13931.32
Predictive log-likelihood at constants	-13540.06	-14319.56
Number of parameters	13	14
Predictive adjusted likelihood ratio index	0.0821	0.0261
Predictive non-nested test	$\Phi[-39.72] << 0.0001$; TFIXED-MDCEV model is preferred	

Percentage of individuals participating in in-home and…	Actual percentage of individuals participating	Traditional Gamma-profile MDCEV prediction	Traditional LGamma-profile MDCEV prediction	Proposed Flexible MDCEV model prediction
Shopping (S) only	9.76	6.39	7.24	7.06
Personal Business (PB) only	5.94	5.16	6.54	6.54
Eat Out only (EO)	5.58	3.03	3.44	3.56
S and PB	5.88	3.63	4.62	4.43
S and EO	2.89	1.99	2.19	2.16
PB and EO	2.42	1.55	1.90	1.93
S and PB and EO	2.69	2.36	2.68	2.60
Weighted Mean Absolute Percentage Error	-	31.40	22.06	22.96
Weighted Mean Absolute Percentage Error for number of inside alternatives picked	-	39.55	22.59	22.42

Percentage of overall time-budget spent in …	Actual percentage	Traditional Gamma-profile MDCEV prediction	Traditional LGamma-profile MDCEV prediction	Proposed Flexible MDCEV model prediction
Shopping	2.76	3.12	2.41	3.07
Personal Business	8.92	6.91	8.35	8.58
Recreation	3.65	3.18	3.10	3.94
Eat Out	2.44	2.26	1.95	2.39
Social	3.26	2.42	2.72	3.48
Serve Passenger	0.97	0.72	0.70	0.70
Weighted Mean Absolute Percentage Error	-	18.69	12.61	6.75

Conclusions

- Proposed a new flexible closed-form MDCEV formulation

- Framework can form the basis for more general MDC models, such as introducing flexible forms of stochasticity through
  - multivariate error structures (such as a multivariate normal error structure),
  - correlations across error terms in the D-preference and C-preference
  - mixing error structures to accommodate unobserved heterogeneity

- The traditional MDCEV may perform almost as well as the proposed model in other empirical contexts where the discrete and continuous marginal utility functions are not very different.

Thank you!

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