# Modelling the Safety Effect of Advisory Speed Signs: A Bivariate Multiplicative Factor on Number of Crashes based on the Speed Differential and the Side Friction Demand

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**ABSTRACT**

Posting advisory speed signs at sharp horizontal curve sites is a practice well established in the United States. The purpose of these signs is to provide the driving public with a safe speed to negotiate such curves; however, the link between these signs and safety performance has not yet been clearly established. A number of research projects have investigated the operational effectiveness of these signs by focusing on the direct response of drivers to the signs in terms of speed selection and reduction. This paper presents a recent Oregon effort to model the safety performance as it relates to these curve advisory speed signs. The authors developed a Generalized Linear Model that parameterizes the crash frequency at 2-lane rural highways in the state of Oregon in terms of curve advisory speed signs and other factors. Though ultimately the paper presents an analysis based on the Poisson model as this model provided the most appropriate fit to the data, the authors also tested an alternative Negative Binomial (NB) model. Because the data for this analysis was not over-dispersed, the model interpretation is valid for both the Poisson and NB model specifications. This research found that a bi-linear interpolant polynomial, contained in the selected statistical model, convincingly establishes a link between the presence of advisory speed signs and the expected numbers of crashes at these sites. Such a link also proved meaningful from the engineering and human factors perspectives. The authors anticipate that the alluded bivariate function should find applications in safety assessment and future speed posting practices. By using the developed sub-model, the authors estimated the safety effectiveness of advisory speeds. This research estimates that, for the state of Oregon, these signs are linked to an approximate reduction of 27% in the expected number of crashes. In general, this research found that advisory speed signs tend to enhance safety. However, the authors also determined that, under certain conditions, advisory speed signs may not be displaying the value that offer the greatest potential for safety enhancement. Furthermore, some advisory speeds can actually be negatively associated with safety performance. Based on the findings of this research, this negative relation can occur at sites with either excessively prohibitive or excessively permissive advisory speeds.

**Keywords:** advisory speed, safety, Side Friction Demand

## INTRODUCTION

Curve advisory speed signs are companions to curve warning signs. Their purpose is to recommend a safe speed for vehicles to negotiate horizontal curves. Although the practice of posting these signs is well established, a convincing linkage between these signs and their hypothesised long term safety benefit has not been clearly established. Current literature includes repeated documentation of poor adherence to these signs, but the authors of this paper believe that such lack of operational compliance may not directly translate into similar safety expectations. This paper presents a statistical analysis in pursuit of quantifying a potential safety benefit of advisory speed signs.

The research effort as summarized in this paper includes six general sections: (1) past and current advisory speed posting practices, (2) data characteristics and filtering, (3) statistical analysis, (4) the effects of advisory speed signs, (5) an evaluation of the resulting model adequacy, and (6) conclusions and recommendations.

## PAST AND CURRENT ADVISORY SPEED POSTED PRACTICES

Advisory speed signs in the United States have been in use since the 1930s. The standardized practice of posting these signs dates back to the 1948 *Manual on Uniform Traffic Control Devices* (MUTCD), where the use of the ball bank indicator is recommended to determine safe speeds for horizontal curves. In its latest edition, the 2009 MUTCD recognizes the potential use of alternative methods to establish advisory speeds. According to this document, advisory speeds shall be determined by an "engineering study that follows established engineering practices" (FHWA, 2009, p. Section 2C.08). This version of the MUTCD indicates that using the ball bank indicator, the geometric design equation, or an accelerometer are examples of such advisory speed engineering assessment practices. The most widely implemented assessment technique is the ball bank indicator. The thresholds for this method have been continually updated through subsequent editions of the MUTCD (FHWA, 2009).

Available literature consistently indicates that advisory speed values, developed using the standardized ball bank indicator procedure, have a large variation in recommended values (Chowdhury, et al., 1998; Courage, et al., 1978). Furthermore, a recent study by Dixon and Rohani (2008) found that a large proportion of curve sites in the state of Oregon do not comply with the state policy. Various authors argue that such lack of consistency results in poor adherence to advisory speeds (Lyles, 1983; Bonneson, et al, 2009).

Surprisingly, there does not appear to be any available literature that quantifies if advisory speed signs actually play a role in enhancing safety performance at horizontal curve locations. This paper investigates the possibility that these signs, despite potentially poor operational compliance, are conveying additional and meaningful information to the public about the severity of downstream horizontal alignments. Accordingly, drivers may respond by adjusting, in some way, their driving behaviour at posted horizontal curves, and this heightened awareness at these locations may result in fewer crashes.

## DATA CHARACTERISTICS AND FILTERING

This research is based upon the data from a probability sample of 105 curve sites located at 2-lane rural highways in Oregon. The sites were selected from the road inventory database of state maintained highways in Oregon. Dixon and Rohani (2008) collected geometric data at these sites to assess the consistency of posting practice in the state. The researchers used probability sampling to ensure representativeness of their results to the state of Oregon. A detailed quantification of the underlying probability structure of a subset of these sites is documented in detail in further work by Avelar (2010). The data collected on site include: curve length, number of lanes, lane and shoulder width, superelevation, vertical grade and vertical signage. Dixon and Rohani determined the corresponding horizontal radii by analyzing aerial images. Additionally, they also collected the Average Annual Daly Traffic, available from the Oregon Department of Transportation (ODOT).

Subsequently, the authors of the current paper compiled the crash records for the sampled sites using ODOT's State-wide Crash Data System. This research used crashes from the period of 2000 to 2004, which closely preceded the site data collection so as to ensure the crash records were appropriately linked to the physical site characteristics at the time of the crash.

Before performing the statistical analysis, the authors filtered the crash data to exclude those crashes that were likely associated with intersections, driveways, and other features not typical to segment locations where the horizontal curve-related crashes could be located. In order to draw meaningful comparisons, the authors of this paper compiled all the crashes that occurred along the 2-mile study corridors and linked them to curve locations, where present. Isolated crash records where mile point locations were recorded to a whole number were suspicious due to potential rounding errors, and therefore were noted and then excluded from further analysis. More detail regarding the data characteristics, sites selected, distribution of crashes, and data filtering is available in Dixon and Avelar (2011).

## STATISTICAL ANALYSIS

In an effort to assess the associated safety effects of advisory speed signs, the authors determined that a univariate statistical test with a simplified direct comparison between crashes at sites with and without advisory speed signs would not suffice due to the large number of potential factors associated with horizontal curve locations at the rural two-lane study sites. For instance, the associated horizontal radius is a natural choice to compare the crashes that occurred within horizontal curves with and without advisory speeds posted, but the wide range of candidate horizontal curve radii within the study sample prohibited a meaningful comparison. The analysis should simultaneously incorporate the effect of radii and other relevant factors, and any associated assumptions should be verifiable. The following sections of this paper review the types of statistical models available, the model selection process, and the results of the modelling procedures.

### Overview of Statistical Models

Traditionally, Poisson regression models have been used for regressing count responses to a vector of potential explanatory variables; however, overdispersion with respect to the Poisson distribution is commonly encountered in crash data. The use of negative binomial regression models (NB) is an attractive alternative to cope with this issue as such models represent Poisson-overdispersion using an additional parameter in the conditional variance of the Poisson model, while still preserving the conditional expectation of the mean as the regressed parameter. In fact, when the dispersion parameter is equal to one, the Poisson model emerges as a particular case of the NB model.

The two simpler forms of NB models are known as NB1 and NB2. The difference between these two specifications is represented by the conditional variance function, particularly with relation to how the dispersion parameter is specified. The conditional variance for the NB1 model is a simple linear function of the conditional mean, while the conditional variance for the NB2 model is a quadratic function of the conditional mean. Naturally, NB models with more complex parameterizations are also available, but were not incorporated as part of this analysis.

The authors of this paper, therefore, explored the use of Poisson and NB2 models only. It is the NB2 model and not the NB1 that may be formulated as a Generalized Linear Model (GLM), and thus, model evaluation metrics are easily obtainable. A quick and more direct comparison with the significance and fit of Poisson models is therefore easily attained. As previously indicated, as long as the data is not over-dispersed, the Poisson model results are equally valid to the NB2 model results.

Although the Poisson and NB2 regression models have a relatively simple structure, some complexity arises in this case because the authors chose to explicitly account for interactions among the explanatory variables. Explicitly modelling variable interactions creates departures from both simple linearity of the mathematical form and an independent-like covariance structure among predictors (both typical assumptions of non-interacting linear regression models). The authors paid special attention to the fact that using interrelated variables as predictors increases the risk of encountering multicollinearity and its derived issues. These issues, however, when assessed and well accounted for, do not invalidate the procedure; they simply require additional computational efforts and further interpretation of the results.

## Model Selection

Crash occurrences may be understood as a Poisson process. This Poisson process may be homogeneous, in which case a Poisson regression model would be appropriate, or heterogeneous, in which case the NB2 specification would be a more appropriate choice to develop the corresponding GLM (assuming a Gamma distribution as the mix-function for the Poisson parameter).

It is important to mention that since over-dispersion issues were not present in the data, both the Poisson and NB model specifications could be used interchangeably in this case. The magnitudes and p-values are essentially the same for the resulting parameterization. The authors selected the Poisson model for this analysis, in spite of the availability of a dual but comparably well fitted NB model, for the following reasons: (1) the principle of parsimony, and (2) the straightforward implications that derive from the simpler structure and well known statistical properties of the Poisson Model. These properties enabled testing the model goodness of fit beyond the statistical software output, by performing a Convoluted Poisson Distribution test. The authors performed an extended assessment of the selected model to dispel any doubts regarding the adequacy of the Poisson GLM. This assessment is presented following the model results and interpretation. Beyond the preferred statistical distribution, the authors deem one contribution of this paper lies upon the parameterization of the mean itself, as the interpretation of selected Poisson specifications are equivalent to the more general NB model for this particular case.

The authors performed the statistical procedures summarized in this paper with the statistical computing language R (The R Development Core Team, 2009).

**Model Results**

The resulting safety effects model is depicted in Table 1. The functional form of the expected number of crashes is provided by Equation 1.

Table 1 Selected Poisson Regression Model for Crash Data

| Term | Estimate | Standard Error | z-value | p-value | Significance[1] |
|---|---|---|---|---|---|
| (Intercept) | -1.862 | 2.259 | -0.824 | 0.410 | |
| LnAADT | 0.931 | 0.108 | 8.635 | < 2e-16 | *** |
| LnCurveLength | -0.956 | 0.246 | -3.886 | 1.02E-04 | *** |
| LaneWidth | -0.282 | 0.129 | -2.182 | 0.029 | * |
| Radius | 0.001 | 0.000 | 1.868 | 0.062 | º |
| Angle | 0.892 | 0.686 | 1.299 | 0.194 | |
| Radius:Angle | 0.002 | 0.001 | 2.791 | 0.005 | ** |
| Radius:Adv.SpdPresent | -0.004 | 0.001 | -4.439 | 9.03E-06 | *** |
| Adv.SpdPresent:Angle | -1.211 | 0.538 | -2.250 | 0.024 | * |
| Adv.SpdPresent | 4.026 | 0.724 | 5.563 | 2.65E-08 | *** |
| ASD | 0.024 | 0.023 | 1.048 | 0.295 | |
| SFD | 5.799 | 2.275 | 2.549 | 0.011 | * |
| ASD:SFD | -0.553 | 0.151 | -3.668 | 2.44E-04 | *** |

[1]*Significance values are as follows:*
º p<0.1; * p < 0.05; ** p < 0.01; and *** p < 0.001

Equation 1: Functional Form of Selected Model

$$\begin{aligned}\#Crashes = exp[&-1.862 + 0.931Ln(AADT) - 0.931Ln(CurveLength) - 0.282(LaneWidth)\\ &+ 0.892(Angle) + 0.001(Radius) + 0.002(Angle \times Radius)\\ &- 0.004(AdvSpdPresent \times Radius) - 1.211(AdvSpdPresent \times Angle)\\ &+ 4.026(AdvSpdPresent) + \{5.799(SFD) + 0.024(ASD) - 0.553(ASD \times SFD)\}]\end{aligned}$$

Where:
AADT = Annual Average Daily Traffic (vpd);
CurveLength = Length of the Curve (ft);
LaneWidth=Width of travl lane (ft)
Radius = Horizontal Radius (ft);
Angle=Horizontal Curve Central Angle (Radians)
SFD = Side Friction Demand at Advisory Speed (no units);
ASD = Advisory Speed Differential, defined as speed limit minus posted advisory speed (mph); and
AdvSpdPresent = Indicator variable equals to one when advisory speed signs are present, otherwise the value is zero.

The authors assessed the option of removing the Angle and ASD constituent terms from the model since they appear statistically insignificant as shown in Table 1; however, each variable is associated with significant interactions and so their effects cannot be considered independent of these associated interacting variables. As a result, the coefficients of the constituent terms should be interpreted in conjunction with these identified interactions. This model includes three variables associated with advisory speeds: ASD (the difference between the speed limit and the posted advisory speed), SFD (the Side Friction Demand that a vehicle would experience if it navigates the curve at the advisory speed), and AdvSpdPresent (a binary variable indicating the presence of posted advisory speeds). Based on the statistic AIC used for model selection, these variables, although interrelated, improved the information quality of the model. The authors also tested and discarded other relevant variables based on the model selection algorithm that ultimately converged and stabilized to the model shown. The authors monitored this algorithm to avoid simultaneity of variables that could destabilize the convergence of the algorithm to attain maximum likelihood of estimates, or extreme increments in the Variance Inflation Factors (VIFs) as these are clear indicators of extreme multicollinearity. For instance, once the working model had significantly increased the AIC value by including the ASD variable with an interaction term, the inclusion of variables for the advisory speed and the speed limit created convergence issues to the fitting algorithm. For these cases, the authors explored two separate branches of the step-wise model selection and chose the model with a better AIC value.

## THE EFFECT OF ADVISORY SPEED SIGNS

It is important to evaluate the influence of relevant geometric design and posting practice concepts as they relate to the rather complex vector of predictors generated as a result of the structure of the model. Due to the presence of interaction terms in the regression model, it is not possible to gather from the model a simple "independent" effect for some of the variables. Instead, the effect of a set of interacting variables is interpreted jointly as a composite multivariate entity affecting the number of crashes. Before presenting a formal multivariate assessment, however, the authors deem appropriate to present an interpretation of the model variables and their perceived influence on safety performance.

### Model Interpretation

Figure 1 represents conceptually how relevant variables fall into the three influential categories of geometric design, signage, and operations. This diagram only includes the significant variables indicated as a result of the statistical analysis. As the figure shows, many of the variables do not perform independently and some overlap can be expected as a result. For instance, the SFD can be understood as both an operational and a geometric variable, since this variable is a function of speed, radius, and superelevation.

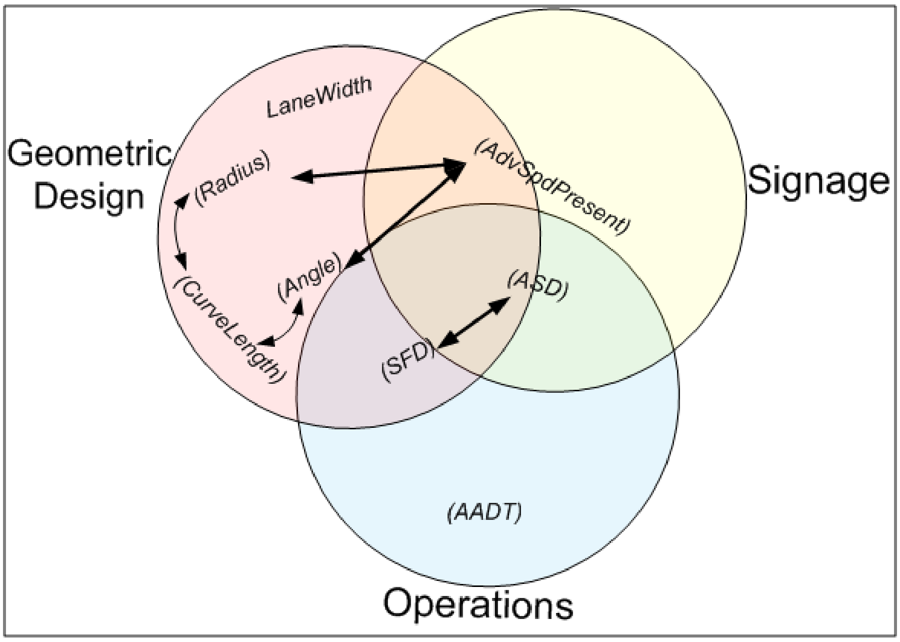

Figure 1  Model Variables Schematic

In addition to the variables shown in the diagram, the step-wise model selection procedure included additional interaction terms as indicated by the two-headed arrows in Figure 1. The researchers did not find these interactions surprising, given that the established geometric design methods and MUTCD posting procedures ensure the interrelation between the three depicted categories of variables.

It is important to note that an interaction term between two variables may be seen as the conditioning of the marginal effect of one variable to a particular value of the other. Additional information regarding the statistical interpretation of this type of model can be found in Brambor et al. (2006). Since a purely statistical interpretation may tend to disregard known engineering relationships, however, the authors felt that it would be helpful to further articulate an interpretation of the model interactions based on a transportation engineering perspective.

Based on the three categories of variables depicted in Figure 1, the authors hypothesize that safety performance emerges from the model in the following way: geometry and signage impact safety by changing road operations, which will result in higher or lower crash frequencies over an extended period of time. According to this premise, interactions in the model should translate into a shift in the short-term operations, ultimately impacting the long-term likelihood of crashes. As an example, the authors believe that the mere presence of the speed plaques (located at warranted curve locations) and the information the drivers may gather from the displayed values may trigger a change in behaviour, which would translate into a shift in operations.  A mix of pre-existing road geometry factors, such as radius and cross-slope, in combination with the expected operations upstream (roughly captured by the speed limit) ultimately dictate the advisory speed plaque message which then can influence the likelihood of a crash.

Based on this interpretation, if a variable capturing an aspect related to advisory speeds was part of an interaction with another model variable, regardless of the direct link one may draw from the mathematical form, it may make more sense to think of the signage variable shifting the effect of the other, more influential variable. This is a relevant observation, since the model contains two such interactions: the presence of advisory speeds interacting with a geometric characteristic (Radius and Angle). A shift towards fewer crashes in the effects of these geometric variables is indicated by corresponding negative coefficients of the interaction terms. However, to quantify the total joint effect and draw meaningful conclusions, each advisory speed variable and the corresponding interactions should be explored in order to draw a holistic interpretation of the effect of advisory speeds in safety performance.

**The Marginal Effects of Advisory Speed Model Variables**

Although the presence of advisory speed plaques seems to affect the likelihood of crashes by shifting the effects of geometric variables, the two other advisory speed variables appear to directly contribute to the overall safety of the studied sites. These variables are the SFD and the ASD (previously defined in Equation 1). The SFD can be computed using Equation 2, an equation available from any highway design book.

$$SFD = \frac{V^2}{15R} - 0.01e \qquad (2)$$

Where:

| | | |
|---|---|---|
| SFD | = | Side Friction Demand; |
| V | = | Advisory Speed (mph); |
| R | = | Horizontal Radius (ft); and |
| e | = | Superelevation (%). |

The SFD variable emerges from the known associations of the geometric and operations categories as depicted in Figure 1. Since this value is a function of vehicle dynamics as well as road geometrics, it plays an important role in establishing the appropriate advisory speed at curve locations. The authors hypothesize that the SFD variable implicitly captures the drivers' expected discomfort associated with negotiating the curve safely. Although the actual SFD would vary among drivers (e.g. varying vehicle capabilities, driving aggressiveness, etc.), research shows that ultimately drivers would tend to respond similarly to a higher degree of discomfort [(Bonneson, Pratt, & Miles, 2009), (Chowdhury, Warren, Bissell, & Taori, 1998), (Avelar, 2010)]. On the other hand, the drivers may judge the severity of the approaching curve based on how small the posted advisory speed is, or relative to the speed limit, how large they perceive the Advisory Speed Differential (defined as the speed limit minus the advisory speed value). This value would provide information supplemental to their individual visual assessment (based on perceiving the curvature and length of the curve as the driver approaches the curve). For this reason, the ASD spans the signage category as well as the operations and geometric categories in Figure 1.

As previously indicated, the resulting statistical model included an interaction between the SFD and ASD. A simple description of this effect may prove challenging; however, the authors speculate that the underlying relationship captured by this bivariate function is as follows: The long-term safety benefit of advisory speeds would emerge as drivers adjusting their behaviour after combining the information these variables carry jointly (how much slower should they be taking the curve as suggested by the ASD, and how severe the associated discomfort can be expected as represented by the SFD variable).

From a mathematical stand point, this bivariate function may be seen marginally for each of the involved variables. This perspective implies, however, that both the marginal effect and the statistical significance of one variable will depend on the particular values of the other variable. The authors judge that a brief review of both marginal effects may prove helpful.

Marginal Effect of ASD and SFD

A simplified approach to understanding the effect derived from the interaction of the ASD and SFD is to look at the marginal effect of the involved variables. Figure 2 displays the marginal Effect of ASD.

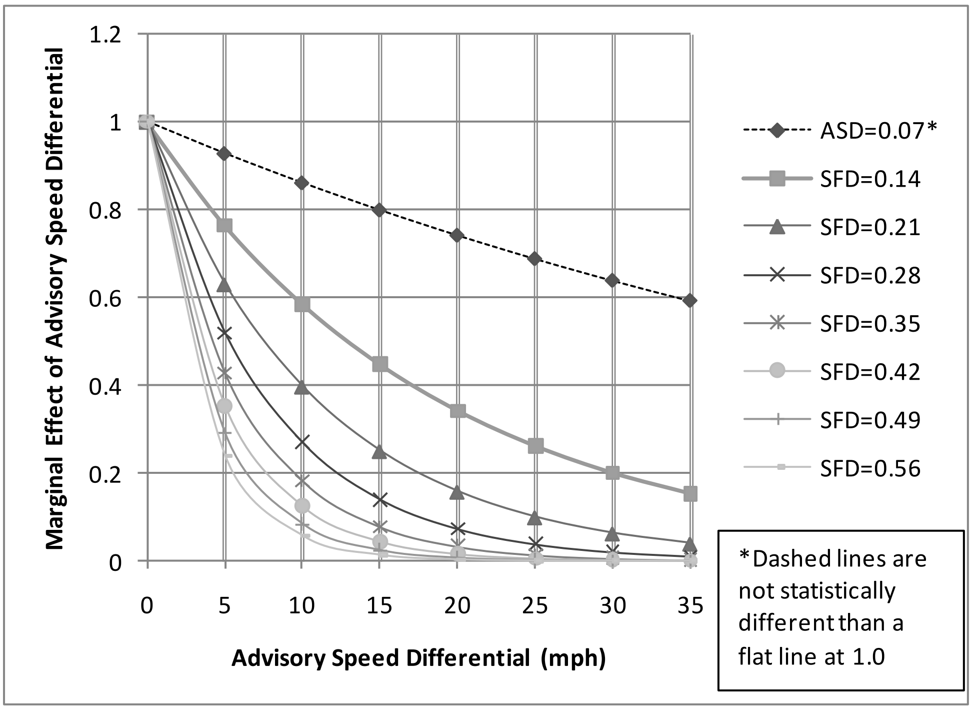


Figure 2 Marginal Effect of ASD at Different SFD Levels

Three items are worth noting regarding the marginal effect of the ASD: (1) all factors are smaller than one, which means that this effect is beneficial; (2) as the SFD increases the marginal effect improves; and (3) the model does not exhibit a statistical significance for the marginal effect at SFD values smaller than 0.14. Complementary, Figure 3 shows the marginal effect of Side Friction Demand at different levels of the ASD.

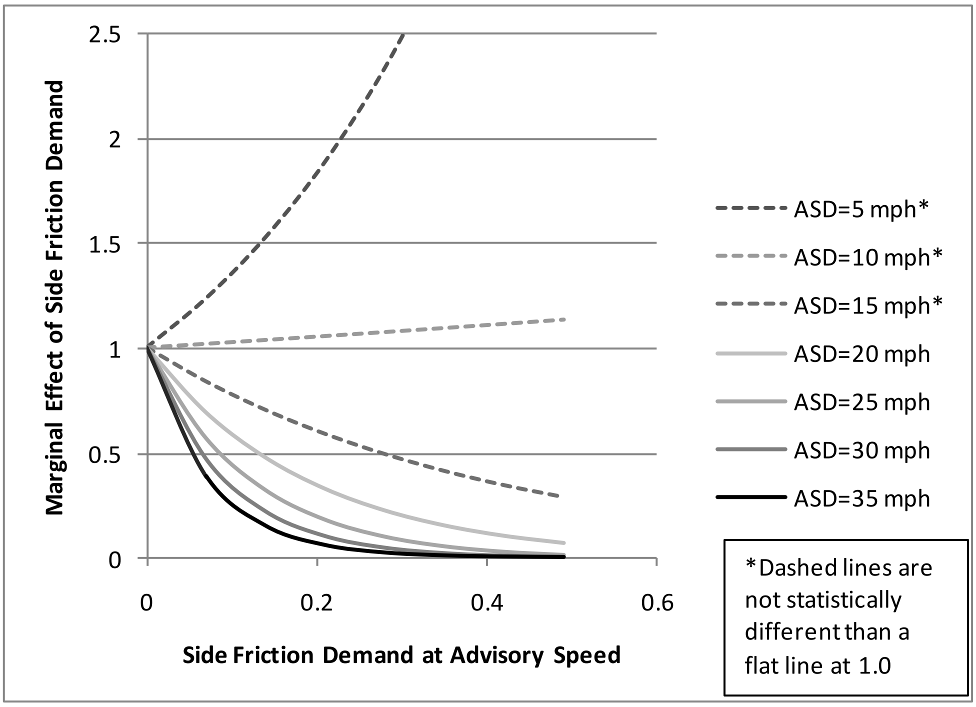


Figure 3 Marginal Effect of SFD at Different ASD Values

There are two features worth noting in the case of the marginal effect of the SFD. First, this marginal effect appears severely adverse for an ASD of 5 mph, and mildly adverse for ASDs of 10 mph. It is worth noting that these marginal effects are not statistically significant, given the data set available. An ASD value of 5 mph means that the advisory speed is 5 mph below the speed limit. However, this marginal effect should be interpreted in a different way. In Oregon the standard posting procedures do not require an advisory speed sign if the recommended advisory speed is only 5 mph below the speed limit. Since the drivers are not presented with an advisory speed plaque, it would be expected that the effect is null. This is suggested by the lack of statistical significance. Similarly, the marginal effect of SFD is not significant for ASD values of 10 and 15 mph. That is not surprising for the case of ASD=10mph, since this value falls very close to a flat line of 1.0. In general, the authors speculate that these two marginal effects may have actually proven to be statistically significant for a larger data set with more observations in these boundary regions.

By examining the marginal effects for both the SFD and ASD, the authors emphasize the following points: (1) advisory speeds tend to be beneficial (both marginal effects are smaller than one when advisory speed signs are present); (2) advisory speed signs provide more safety benefits as their values tend to differ from the regulatory speed limit (larger SFD marginal effect for larger ASDs); and (3) advisory speed signs are more beneficial when greater driver discomfort results from driving at the suggested speeds (larger ASD marginal effect for larger SFDs).

The use of marginal effect trends, as those shown in Figures 2 and 3, is most useful when the purpose is to isolate the effect of a single variable and its interaction with a less critical variable in the model. However, the authors recognize that a disjoint interpretation of the marginal effects in this particular case may appear contradictory from the traffic engineering stand point: to reap the safety benefit of posting advisory speeds, one needs to increase both the ASD and the SFD, but to increase the ASD one needs to post low advisory speeds, which in turn have small SFD associated. These marginal effects are closely intertwined, and their isolated view, as discussed here, is merely informative. Both the SFD and the ASD should be considered jointly. Thus, the authors recognize that the global effect of advisory speeds may be more informative by interpreting the complete bi-linear interpolant polynomial of ASD and SFD as a single entity.

## The Advisory Speed Crash Factor

In a Poisson regression model, the effect of a non-interacting variable is a multiplicative factor to the average expectancy of the response variable. The corresponding multiplicative factor emerging from the unconditional bi-linear interpolant polynomial may be computed by disregarding the marginal effects and evaluating directly the ASD and SFD values in the polynomial. An additional benefit of this procedure is that the mathematical form is simple and clear enough to provide a direct interpretation in the scale of the response.

This section focuses on deriving and describing the corresponding multiplicative factor of the bi-linear interpolant polynomial of ASD and SFD. This newly developed multiplicative factor is denoted as the Advisory Speed Crash Factor (ASCF) from this point forward. Equation 3 depicts the mathematical form of the ASCF. Notice that this value is derived directly from Equation 1. As a result, the ASCF functions as a sub-model contained in the full Poisson regression model.

$$ASCF = exp[5.799(SFD) - 0.5528(ASD \times SFD) + 0.0237(ASD)] \qquad (3)$$

The authors explored the mathematical properties of the ASCF to determine if this sub-model is meaningful in describing the safety effect of advisory speeds. Table 2 shows the numeric values of the ASCF for an extended range of ASD and SFD. These values vary from extremely detrimental to safety (9.421 for ASD of 5 mph and SFD of 0.70) to extremely beneficial ($1.75 \times 10^{-4}$ for ASD of 35 mph and SFD of 0.70). The matrix of values presented in Table 2 extends beyond the typical range of combined values. For example, current posting procedures would prevent any advisory speeds from being associated with SFD values larger than 0.3, so both the above examples do not correspond to any expected physical site. The values shown in this table, however, are useful as they proved a clearer exploration of the properties of the ASCF as it relates to the ASD and SFD variables.

Table 2 Advisory Speed Crash Factor Values

| | | ASD (mph) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| SFD | 0.07 | 1.392 | 1.292 | 1.199 | 1.113 | 1.032 | 0.958 | 0.889 |
| | 0.14 | 1.722 | 1.317 | 1.007 | 0.770 | 0.589 | 0.450 | 0.344 |
| | 0.21 | 2.130 | 1.342 | 0.846 | 0.533 | 0.336 | 0.212 | 0.133 |
| | 0.28 | 2.634 | 1.368 | 0.710 | 0.369 | 0.192 | 0.099 | 0.052 |
| | 0.35 | 3.257 | 1.394 | 0.597 | 0.255 | 0.109 | 0.047 | 0.020 |
| | 0.42 | 4.028 | 1.421 | 0.501 | 0.177 | 0.062 | 0.022 | 0.008 |
| | 0.49 | 4.981 | 1.448 | 0.421 | 0.122 | 0.036 | 0.010 | 0.003 |
| | 0.56 | 6.160 | 1.475 | 0.353 | 0.085 | 0.020 | 0.005 | 0.001 |
| | 0.63 | 7.618 | 1.504 | 0.297 | 0.059 | 0.012 | 0.002 | 4.51E-04 |
| | 0.70 | 9.421 | 1.532 | 0.249 | 0.041 | 0.007 | 0.001 | 1.75E-04 |

Table 2 demonstrates that the ASCF values are beneficial as both ASD and SFD increase. This observation corresponds to the interpretation of the marginal effects previously provided. Because the ASCF is the ordinate of two explanatory variables, it can be represented as response surface or by a contour map. Figure 4 is the contour map representation of the ASCF. The dotted line corresponds to a multiplicative crash factor equal to 1.0. This is the level at which there is no effect on the expected number of crashes. The region to the left and below this dotted line corresponds to ASCF values larger than one, indicating more crashes. Finally, the region to the right and above the dotted line corresponds to ASCF values less than one, suggesting fewer expected crashes. This mathematical representation of the ASCF corresponds with the marginal view of its two components, the ASD and the SFD.

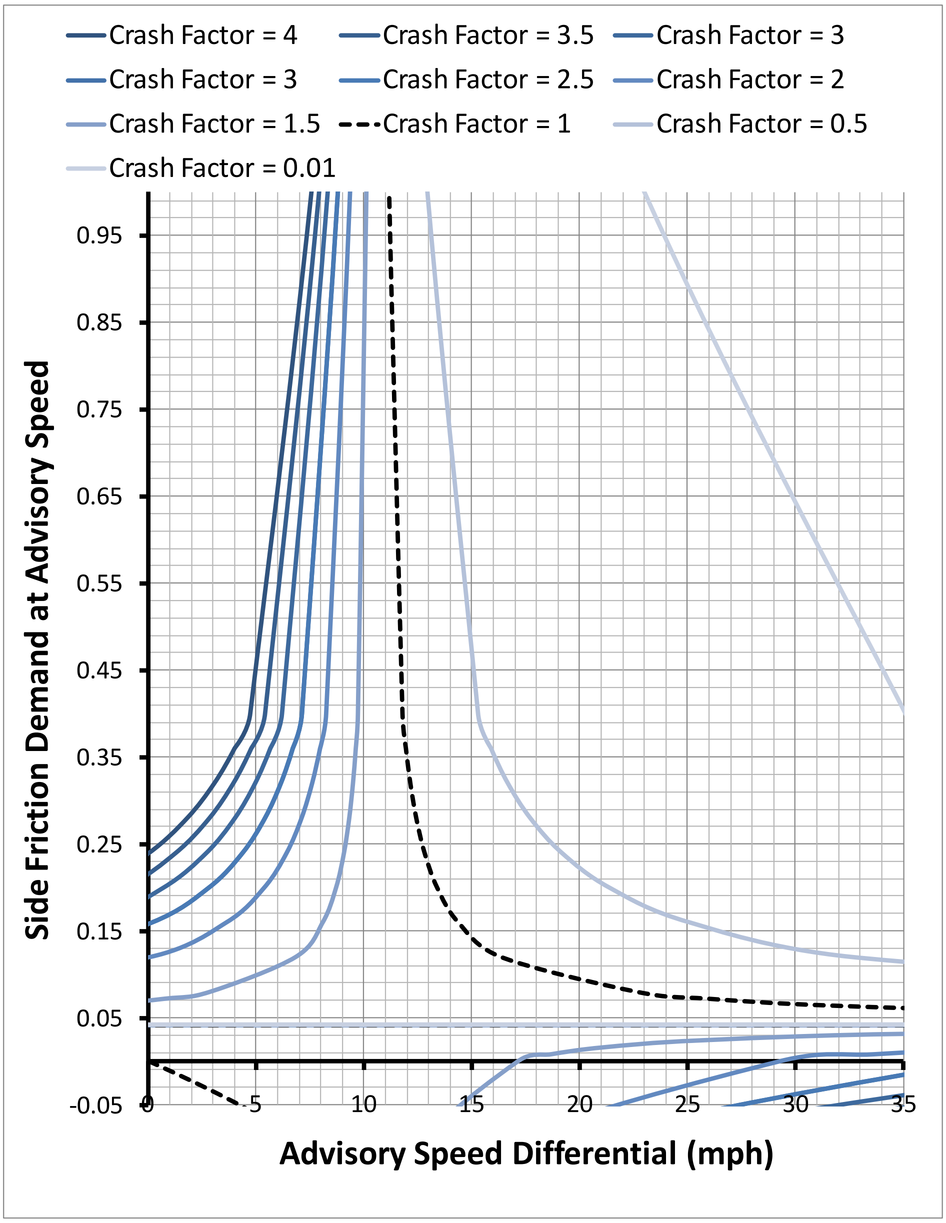


Figure 4 Contour Map Representation of the ASCF

**Effectiveness of Advisory Speeds in Oregon**

Given the complexities of the model structure, it is not simple to draw a generalized conclusion about the effectiveness of advisory speeds. According to the model, such effectiveness depends jointly on how much the advisory speed differs from the speed limit and on the degree of discomfort associated with navigating the curve at such an advisory speed. Furthermore, it appears from the model that advisory speeds may have a detrimental effect if they are either too low for the associated SFD or too high in general.

In order to preliminarily assess the effectiveness of current posting practices, the authors used the available probability sample of advisory speeds from the state of Oregon. This assessment consists of a theoretical exercise of "virtually removing" advisory speed signs and observing the expected safety effect, as predicted by the model. The actual effect for this hypothetical scenario would likely be very different: informational campaigns about the change would result in an immediate rise in familiar drivers' awareness of the altered signage and would initially reduce the likelihood of crashes. Eventually, drivers would reach a new generalized perception of the driving environment, at which point the ASCF surface would be completely unrepresentative of the new operations and associated safety. However, the authors consider this exercise of some use, in order to extract the extent of the safety benefit of advisory speed plaques.

The computed measure of effectiveness is the ratio of ASCF before the hypothetical removal of advisory speed plaques to the ASCF after the removal. This quantity is referred to as absolute ASCF, or AASCF. Figure 5 shows a graphical display of AASCF versus ASD. This trend has an AASCF overall average of 0.728. This value suggests that advisory speed plaques may reduce crash frequency, on average, by 27.2% in the state of Oregon. It is reasonable to consider, however, that the AASCF values ranging from 0.951 to 1.05 correspond to sites with virtually no benefit associated with advisory speeds. Interestingly, 99 out of 210 sites exhibit AACSF values within this range. Additionally, there are 3 sites in the sample for which the model predicts an adverse effect of advisory speeds (i.e. AASCF larger than 1.0).

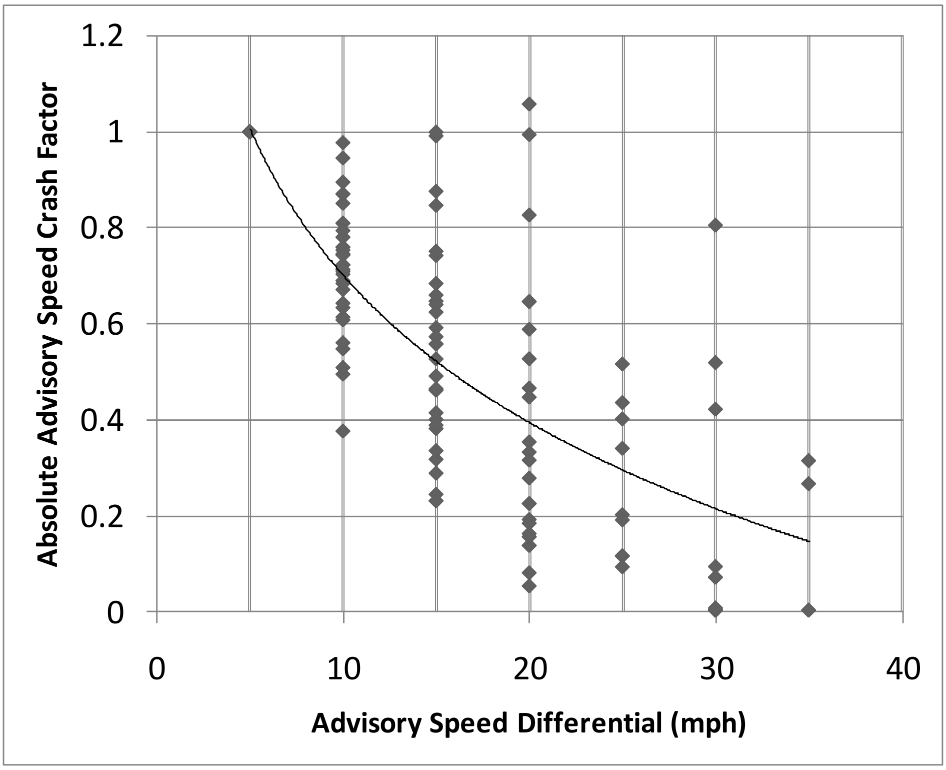

Figure 5 AASCF vs. Advisory Speed Differential

The authors recognized some important elements from this assessment: (1) 91 out of 99 sites with values of AASCFs close to 1.0 have an associated ASD of 5 mph. This ASD value corresponds to sites without advisory speeds signs; (2) the absolute effect on crashes appears beneficial for most of the remaining sites (i.e. AASCFs smaller than 1); (3) the AASCF diminishes systematically as the ASD increases, which in general indicates a good balance of ASD and SFD values underlying current posting practices in Oregon; (4) the range of AASCF values roughly remains the same as the ASD increases; and (5) despite the observed general benefits of advisory speed signs, there is one site in the sample with AASCF slightly larger than 1.0 (suggesting an advisory speed value that mislead rather than guide drivers). This preliminary evaluation suggests that cost-effective measures, such as changing the advisory speed displayed at such sites or even removing the plaque, have a potential to improve safety.

Table 3 demonstrates the impact of modifying advisory speeds at 5 of the sites with different advisory speed values.

Table 3 Effect of Modifying Advisory Speeds at Selected Sites

| Site | Speed Limit (mph) | Radius (ft) | Superelevation (%) | Current Advisory Speed (mph) | Current AASCF | Modified Advisory Speed (mph) | Modified AASCF |
|---|---|---|---|---|---|---|---|
| 1 | 55 | 1770 | 11 | NA | 1.000 | NA | 1.000 |
| 2 | 55 | 900 | 11 | NA | 1.000 | 45 | 0.906 |
| 3 | 55 | 575 | 14.5 | 45 | 0.743 | 40 | 0.738 |
| 4 | 55 | 700 | 12.5 | 35 | 1.058 | 45 | 0.814 |
| 5 | 55 | 520 | 11 | 35 | 0.588 | 40 | 0.528 |
| 6 | 55 | 300 | 14 | 25 | 0.519 | 35 | 0.202 |

Except for site 3, it is expected that all of the sites already displaying advisory speed plaques would benefit by increasing their posted advisory speed (sites 4, 5 and 6). Incidentally, these three sites display 25 and 35 mph advisory speeds. This observation is not surprising, given that previous research indicates that the posting policy in Oregon is among the most conservative in the United States (Dixon & Avelar, 2011). An extreme case is site 4, which would require an increment of 10 mph. This site is also abnormal in that it has the only AASCF larger than one in the whole sample. Site 3, though, would benefit from lowering its advisory speed, which suggests that the plaque may be too permissive. Finally, while site 1 would still not require an advisory speed plaque, site 2 would improve its safety performance by displaying a new one. Conditions at all sites except site 1 are such that an AASCF value smaller than 1.0 is achievable. According to this research, therefore, current advisory speeds in Oregon may not be exploiting all of their potential safety benefit. Though this simplistic example demonstrates how the AASCF can be used as an indicator of expected safety performance, it is clear that the use of this AASCF method (similar to the common crash modification factor) can help engineers assess the potential for safety improvements as one consideration in advisory speed selection.

## EVALUATION OF MODEL ADEQUACY

Prior to developing concluding comments, this section addresses concerns that may arise regarding the adequacy of the selected model. Specifically, this section explores the following three associated issues: structural correlation in the response data, multicollinearity for both potentially correlated covariates and the statistical structure of interactive models, and general goodness of fit to the data.

### Assessing the Structural Correlation in the Response Variable

For the rural 2-lane 2-way study corridors used for this analysis, every "curve site" in the study comprises two directions of travel, and each pair contains relevant common factors (e.g. driving population, traffic volume, and horizontal radius). As a result, the authors expect a high correlation between the numbers of crashes from each pair of directions of travel. It would be problematic to use both directions of travel in fitting the regression if such a correlation is substantial and beyond the explanatory power of the statistical model. Doing so would be equivalent to artificially duplicating the number of data points; however, assessing both directions of travel as one site is also problematic since issues such as direction of curve and relative cross slope would differ (be exactly opposite for most locations).

It is reasonable to expect that many similarities exist for the pairs of directions of travel and that these characteristics are, to some extent, explicitly accounted for by the corresponding regression variables. The horizontal radius, AADT and curve length, for instance, are the same for the two directions of travel at each location. Rather than simply eliminating 50% of the candidate sites and given this potential correlation, the authors then assessed how the correlation in the paired data compared to the predicted (based on regression) correlation from pairs of independent Poisson variables stemming from the fitted univariate model.

The authors paired the data by site and computed the correlation for the total sample of 105 pairs of crash counts and found a correlation value of 0.698. The authors compared the correlation in the sample to the distribution of correlations that arise from repeated realizations of the theoretically independent Poisson distributions.

The authors developed a synthetic sample of the paired-sites correlation distribution by using the technique of static simulation of paired but independent Poisson distributions, so that the observed distribution of correlations emerges only from the pairing of similar independent, univariate realizations such as from the regression model. The synthetic sample consisted of 200 replications of the overall correlation.

Simulation results suggest that a normal curve could approximate this distribution (simulated data have very small $3^{rd}$ and $4^{th}$ moments; a -0.10 skewness indicating rough symmetry and a normalized kurtosis of -0.623 indicating a peakness that is close to the normal distribution). The authors used the mean and the standard deviation of the simulated data to assess the statistical significance of the correlation from the crash data. The actual correlation of 0.698 compares very closely to the mean simulated correlation (0.581). Using the simulated standard deviation (0.088), a 0.184 two-sided p-value may be obtained from the standard normal distribution. Comparably, an empirical one-sided p-value of 0.09 may be computed from the raw synthetic sample as the proportion of simulated correlations that resulted in values larger than 0.698, the sample statistic.

From the results, the authors conclude that the correlation observed between the pairs of directions of travel in the sample is not atypical, and that it is reasonable to expect such a degree of correlation from pairs of truly independent Poisson variables with similar parameters such as those associated with the regression model.

### Discussion on Multicollinearity among Regressors

It is worthwhile to notice that a certain degree of multicollinearity was unavoidable in the model, despite the variable selection procedure that included strategies to minimize multicollinearity. One such strategy, for instance, was to avoid including two highly correlated variables simultaneously as predictors. However, the authors included interaction terms to contribute to improving the quality of information in the model (i.e. significant drops in AIC), but also included these terms because the joint interpretation with their constituent terms explain reasonably expected transportation engineering safety behaviour. The mathematical structure of the ASCF, the main sub-model developed in this paper, similarly rests upon a bi-linear interpolant polynomial emerging from two interacting variables. The only drawback of choosing an interacting model, as of this paper, is the requirement of slightly more complex procedures for joint interpretation of the co-dependent terms.

It is recognized that the degree of multicollinearity increases when the covariates are no longer independent. If the severity and the effects of multicollinearity among predictors are properly treated in the modelling process (mainly monitoring VIFs and algorithm convergence issues) and with adequate interpretation of the results, the authors advocate for the use of interactive models, especially because of their ability to represent complex interrelationships. Furthermore, the explicit account of multicollinear predictors may become attractive because of the need to account for factors that are not entirely independent. Such interdependency may transcend into explaining the response variable, and if that is so, interactions between variables are a useful tool to explicitly model such joint effects. However, a model structure that includes interactions implies a potential source of multicollinearity due to the model structure itself in addition to that resulting from the use of co-dependent covariates.

Multicollinearity manifests itself as an inflation of the standard errors from the regression. This circumstance, in turn, results in convergence issues in the fitting algorithm. The authors observed convergence issues in the early and intermediate stages of the step-wise procedure, for both the NB and Poisson models. Some judicious decisions were necessary in order to manually exclude some of the correlated variables as a requirement of the step-wise procedure. One such decision was to exclude advisory speed related variables in favour of keeping horizontal geometry covariates in the early models. Later the model selection procedure allowed advisory speed and synthesized variables, such as the ASD and SFD. Ultimately, some of these variables were included in the model because of their significant contribution to the quality of the information in the model (i.e. significant drops in AIC).

After the adjustments described in this section, the fitting algorithm did not indicate convergence issues, nor did the VIFs exhibit extreme values. Additionally, almost all of the coefficients in the model present small enough standard errors to indicate statistically significant results. Only two terms are not statistically significant, but each of them is of prime importance to derive statistically significant marginal or joint effects, as shown in previous sections of this paper. After this assessment, the authors believe that no serious multicollinearity issues required further attention.

## Goodness of Fit

It is important that a representative statistical model have an overall good fit to the data. To establish the appropriateness of the Poisson model in describing the data, the authors tested the goodness-of-fit at three different conceptual levels: residual deviance, dispersion, and Poisson distribution suitability.

### Approximate Chi-Squared Test on Model Residual Deviance

The authors used an approximate chi-squared test to assess the residual deviance. This quantity, obtained from the Maximum Likelihood Estimation (MLE) algorithm, is expected to converge in distribution to the chi-squared function as the sample size increases (i.e. by virtue of the Central Limit Theorem). This test resulted in a p-value of 0.6413 from a 184.35 chi-squared statistic (i.e. the residual deviance) for 197 degrees of freedom suggesting a lack of evidence against the appropriateness of the model fit to the data.

### Approximate Dispersion Parameter

A good fit to the Poisson distribution can be evaluated when the ratio of the variance to the mean of the response variable is approximately equal to one. This expected mean-variance relationship can be estimated using the ratio of the residual deviance to its degrees of freedom from the regression algorithm. This ratio is referred to as the dispersion parameter in some literature. In this case, a value of 0.961 indicates that there is no significant over-dispersion present in the data. Since the expected value of a chi-squared distribution is its associated degrees of freedom, a corresponding p-value for this statistic assumes a null hypothesis that the expected ratio parameter was 1.0. This value corresponds to the p-value of the residual deviance statistic of 0.6413 as previously shown. This result also suggests that if a NB regression were used instead, the magnitude and statistical significance of the coefficients would have been virtually the same.

Convoluted Poisson Distribution Test on Total Number of Crashes

The discrete convolution theorem applied to Poisson distributions (Samaniego, 1976) states that the distribution of a sum of independent Poisson variables is also a Poisson variable with a scale parameter equal to the sum of the scale parameters for each data point. This evaluation is depicted by Equation 3.

$$P\left(\sum Y_i = z\right) = e^{-\left(\sum \lambda_i\right)} \times \frac{\left(\sum \lambda_i\right)^z}{z!} \tag{3}$$

Where:

$Y_i$ = Observed number of crashes at site i;

z = Arbitrary value from the domain of Y; and

$\lambda_i$ = Predicted number of crashes at site i per the regression model.

The test statistic is the total number of crashes and the associated p-value is obtained from the convoluted Poisson distribution. Since the statistic percentile is rather large (one tailed p-value of 0.4914 from a convoluted Poisson random variable of 180 with an expected value of 180.67), the test clearly failed to reject the hypothesis that the sample is a realization of the convoluted distribution emerging from the model fitted values.

Given the results of these tests, the researchers are confident that the Poisson model is appropriate to describe the available crash data. Since the Poisson is a particular case of the NB2 distribution, these tests also mean that an NB2 model with dispersion parameter approximately equal to 1.0 also describes the data satisfactorily.

## CONCLUSIONS AND RECOMMENDATIONS

The authors of this paper developed a mathematical model to describe the safety impact of advisory speed signs. The purpose of this paper is to quantifiably link the displayed value of advisory speeds to the safety performance of the sites.

The basis of this mathematical model is a statistical analysis involving 105 randomly selected two-directional sites located in the state of Oregon. The functional form of the model included a bi-linear interpolant polynomial of two quantities linked to advisory speeds: the advisory speed differential (ASD) and the side friction demand (SFD). This effect was named the advisory speed crash factor (ASCF). Because the Poisson regression model did not suffer from overdispersion when fitting the data, either the Poisson or the NB2 specifications can be used interchangeably when accounting for the ASCF. This is convenient, as the NB2 specification is the naturally assumed posterior distribution (i.e. Safety Performance Function) for widely accepted Empirical Bayes and Full Bayes analyses for before-after studies.

The ASCF consists of a multiplicative value that directly affects the expected number of crashes for a curve rural 2-lane road location. The concept of the ASCF is analogous to the crash modification factor (CMF). Currently, the most closely associated reference work (Elvik & Vaa, 2004) proposed the use of a CMF that suggests a single value that ranges from 0.71 to 0.87 depending on crash severity. The ASCF resulting from the work outline in this paper, however, is more suitably referred to as a crash modification function as it varies based on the specific advisory speed value and site conditions.

This paper introduced a new element referred to as the absolute ASCF (AASCF) that helps to assess the notional impact of advisory speed signs as opposed to a theoretical scenario where the plaques are not displayed. The values proposed by Elvik and Vaa (2004) are aligned with the derived AASCF average value of 0.728 which functions as a measure of the overall effectiveness of advisory speeds in Oregon.

Although most of the sites included in this study appeared to benefit from the practice of posting advisory speeds, there was one instance in which the posted advisory speed seemed detrimental to safety. The ASCF further provides a computational tool to assess the safety effect of particular values of advisory speeds. Therefore, the authors expect that the concept developed in this paper is a useful function to evaluate safety performance.

Additionally, the authors recognize that the AASCF is a detailed functional form that results in a value comparable to the crash modification factor for advisory speeds similar to that recommended by Elvik and Vaa (2004). As such, the authors anticipate that the AASCF may be used as a crash modification function to improve the accuracy of current HSM procedures.

The authors believe that the ASCF may also be used as the criterion for an improved safety-based posting procedure. Recent work by Dixon and Avelar (2011) proposed such a procedure as a computational alternative to the currently wide-spread ball-bank indicator method. The authors recognize that such a method allows for further improvement, particularly with the potential combination of instrumentation based procedures, such as those developed by Pratt, Bonneson and Miles (2011). To enhance this method for transferable posting procedures, the authors recommend further research in order to field validate the concept of ASCF in Oregon and other states.

Finally, the authors also recommend future work to explore and strengthen the link of the ASCF to field operational data, since this type of data would closely contribute to the overall validation of the ASCF concept. Specifically future research should explore how the operating speed relates to the components of the ASCF bivariate function.

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