NCHRP 20-7 / Task 262(M2)

FINAL REPORT

SEISMIC ISOLATION DESIGN EXAMPLES OF HIGHWAY BRIDGES

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EXECUTIVE SUMMARY

Today about 200 bridges have been designed and constructed in the U.S. using the AASHTO *Guide Specifications for Seismic Isolation Design* (AASHTO, 2010) but this figure is a fraction of the potential number of applications and falls far short of the number of isolated bridges in other countries.

One of the major barriers to implementation is that fact that isolation is a significant departure from conventional seismic design and one that is not routinely taught in university degree courses. Furthermore, very few text books on this topic have been published and those that are available focus on applications to buildings rather than bridges. The absence of formal instruction and a lack of reference material, means that many designers are not familiar with the approach and uncomfortable using the technique, despite the potential for significant benefits.

In an effort to address this need, this NCHRP 20-7 project was funded to develop and publish of a number of design examples to illustrate the design process of an isolated bridge and related hardware in accordance with the recently revised AASHTO *Guide Specifications* (AASHTO, 2010).

Fourteen examples have been developed illustrating the application of seismic isolation to a range of bridges for varying seismic hazard, site classification, isolator type, and bridge type. In general, each example illustrates the suitability of the bridge for isolation (or otherwise), and presents calculations for preliminary design using the Simplified Method, preliminary and final isolator design, detailed analysis using a Multi-Modal Spectral Analysis procedure, and non-seismic requirements. Detailed designs of the superstructure, substructure (piers) and foundations are not included. Likewise, the testing requirements for these isolators (as required in the AASHTO *Guide Specifications*) are not covered.

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INTRODUCTION

1.1 BACKGROUND

Today about 200 bridges have been designed and constructed in the U.S. using the AASHTO *Guide Specifications for Seismic Isolation Design* (AASHTO, 2010) but this figure is a fraction of the potential number of applications and falls far short of the number of isolated bridges in other countries (Buckle et. al., 2006).

One of the major barriers to implementation is that fact that isolation is a significant departure from conventional seismic design and one that is not routinely taught in university degree courses. Furthermore, very few text books on this topic have been published and those that are available focus on applications to buildings rather than bridges. The absence of formal instruction and a lack of reference material, means that many designers are not familiar with the approach and uncomfortable using the technique, despite the potential for significant benefits.

In an effort to address this need, this NCHRP 20-7 project was funded to develop and publish of a number of design examples to illustrate the design process of an isolated bridge and related hardware in accordance with the recently revised AASHTO *Guide Specifications* (AASHTO, 2010).

Fourteen examples have therefore been developed illustrating application to a range of bridges for varying seismic hazard, site classification, isolator type, and bridge type. In general, each example illustrates the suitability of the bridge for isolation (or otherwise), and presents calculations for preliminary design using the Simplified Method (Art 7.1, AASHTO, 2010), preliminary and final isolator design, detailed analysis using a Multi-Modal Spectral Analysis procedure (Art 7.3, AASHTO 2010), and non-seismic requirements. Detailed designs of the superstructure, substructure (piers) and foundations are not included. Likewise, the testing requirements for these isolators (as required in the AASHTO *Guide Specifications*) are not covered.

1.2 DESIGN EXAMPLES

The fourteen examples are summarized in Table 1. It will be seen they fall into two sets: one based on a PC-girder bridge with short spans and multiple column piers (Benchmark Bridge #1) and the other on a steel plate-girder bridge with long spans and single column piers (Benchmark Bridge #2). For each bridge there are six variations as shown in Table 1. Both benchmark bridges have the following attributes:

- Seismic Hazard: Spectral Acceleration at 1.0 sec (S_1) = 0.2g
- Site class: B (rock)
- Pier heights: Uniform
- Skew: None
- Isolator: Lead-Rubber Bearings (LRB)

These five attributes are varied (one at a time) to give 12 additional examples as shown in Table 1. Variations covered include S_1 = 0.6g, Site Class D, unequal pier heights, 45^0 skew, Spherical Friction Bearing (SFB) and Eradiquake (EQS) isolators.

Brief descriptions of the two benchmark bridges are given in the following sections.

Table 1. Seismic Isolation Design Examples.

Example	S_1	Site class	Spans	Girders	Column size and heights	Skew	Isolator
EXAMPLE SET 1: PC Girder Bridge, short spans, multi-column concrete piers							
1.0 Benchmark Bridge #1	0.2g Zone 2	B	3 25-50-25 ft	6 PC girders (AASHTO Type II)	2 x 3-col piers	0^0	LRB
1.1	Zone 3	D	3 25-50-25 ft	6 PC girders (AASHTO Type II)	2 x 3-col piers	0^0	LRB
1.2	0.6g Zone 4	B	3 25-50-25 ft	6 PC girders (AASHTO Type II)	2 x 3-col piers	0^0	LRB
1.3	0.2g Zone 2	B	3 25-50-25 ft	6 PC girders (AASHTO Type II)	2 x 3-col piers	0^0	SFB
1.4	0.2g Zone 2	B	3 25-50-25 ft	6 PC girders (AASHTO Type II)	2 x 3-col piers	0^0	EQS
1.5	0.2g Zone 2	B	3 25-50-25 ft	6 PC girders (AASHTO Type II)	2 x 3-col piers unequal height	0^0	LRB
1.6	0.2g Zone 2	B	3 25-50-25 ft	6 PC girders (AASHTO Type II)	2 x 3-col piers	45^0	LRB
EXAMPLE SET 2: Steel Plate Girder Bridge, long spans, single-column concrete piers							
2.0 Benchmark Bridge #2	0.2g Zone 2	B	3 105-152.5-105 ft	3 steel plate girders with slab	2 x single-col piers.	0^0	LRB
2.1	Zone 3	D	3 105-152.5-105 ft	3 steel plate girders with slab	2 x single-col piers	0^0	LRB
2.2	0.6g Zone 4	B	3 105-152.5-105 ft	3 steel plate girders with slab	2 x single-col piers	0^0	LRB
2.3	0.2g Zone 2	B	3 105-152.5-105 ft	3 steel plate girders with slab	2 x single-col piers	0^0	SFB
2.4	0.2g Zone 2	B	3 105-152.5-105 ft	3 steel plate girders with slab	2 x single-col piers	0^0	EQS
2.5	0.2g Zone 2	B	3 105-152.5-105 ft	3 steel plate girders with slab	2 x single-col piers with unequal height	0^0	LRB
2.6	0.2g Zone 2	B	3 105-152.5-105 ft	3 steel plate girders with slab	2 x single-col piers	45^0	LRB

1.2.1 Benchmark Bridge No. 1

Benchmark Bridge No. 1 is a straight, 3-span, slab-and-girder structure with three columns at each pier and seat-type abutments. The spans are continuous over the piers with span lengths of 25 ft, 50 ft, and 25 ft for a total length of 100 ft (Figure 1.1). The superstructure comprises six AASHTO Type II girders spaced at 7.17 ft with 3.1 ft overhangs for a total width of 42.5 ft. The total weight of the superstructure is 651 k.

The two piers each consist of three circular columns spaced at 14 ft., longitudinal steel ratio of 1%, and a transverse steel ratio of 1%. The plastic shear capacity of each column (in single curvature) is 25 k. The height of the superstructure is approximately 20 ft above ground.

The bridge is located on a rock site where the *PGA* = 0.4g, S_S = 0.75g and S_1 = 0.20g.

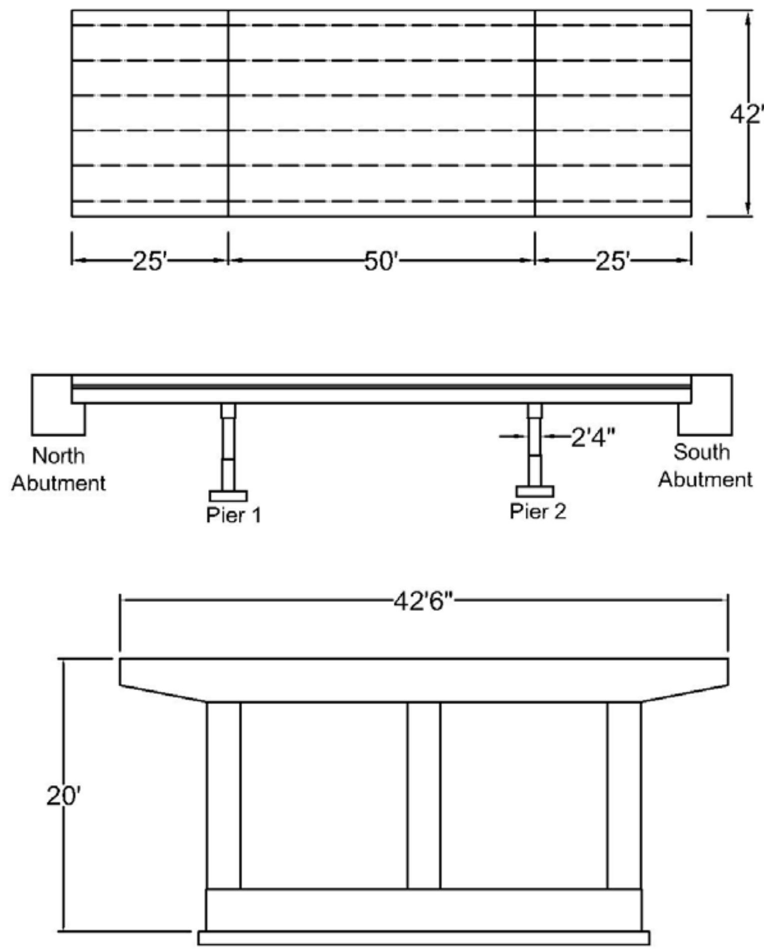


Figure 1.1. Plan, Side View, and Pier Elevation for 3-Span Benchmark Bridge No. 1.

1.2.2 Benchmark Bridge No. 2

Benchmark bridge No. 2 is a straight, 3-span, steel plate-girder structure with single column piers and seat-type abutments. The spans are continuous over the piers with span lengths of 105 ft, 152.5 ft, and 105 ft for a total length of 362.5 ft (Figure 1.2). The girders are spaced 11.25 ft apart with 3.75 ft overhangs for a total width of 30 ft. The built-up girders are composed of 1.625 in by 22.5 in top and bottom flange plates and 0.9375 in. by 65 in. web plate. The reinforced concrete deck slab is 8.125 in thick with 1.875 in. haunch. The support and intermediate cross-frames are of V-type configuration as shown in Figure 1.3. Cross-frame spacing is about 15 ft throughout the bridge length. The total weight of superstructure is 1,651 kips.

All the piers are single concrete columns with a longitudinal steel ratio of 1%, and transverse steel ratio of 1%. The plastic shear capacity (in single curvature) is 128k. The height of the superstructure is approximately 24 ft above the ground.

The bridge is located on a rock site where the *PGA* = 0.4g, S_S = 0.75g and S_1 = 0.20g.

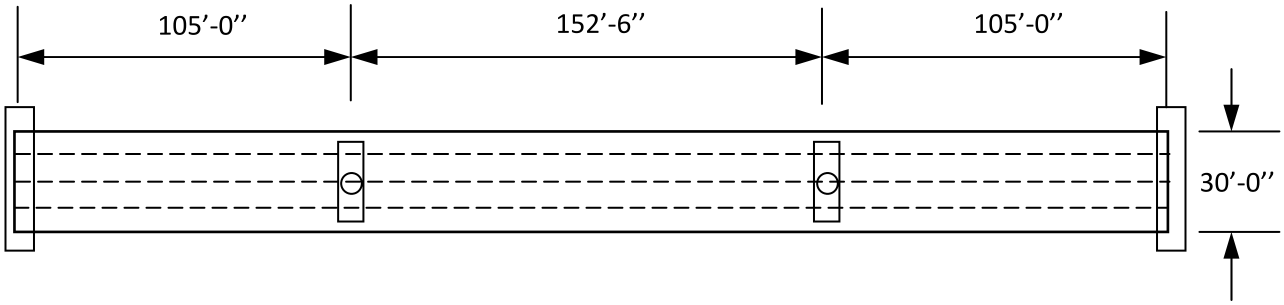


Figure 1.2. Plan of 3-Span Benchmark Bridge No. 2.

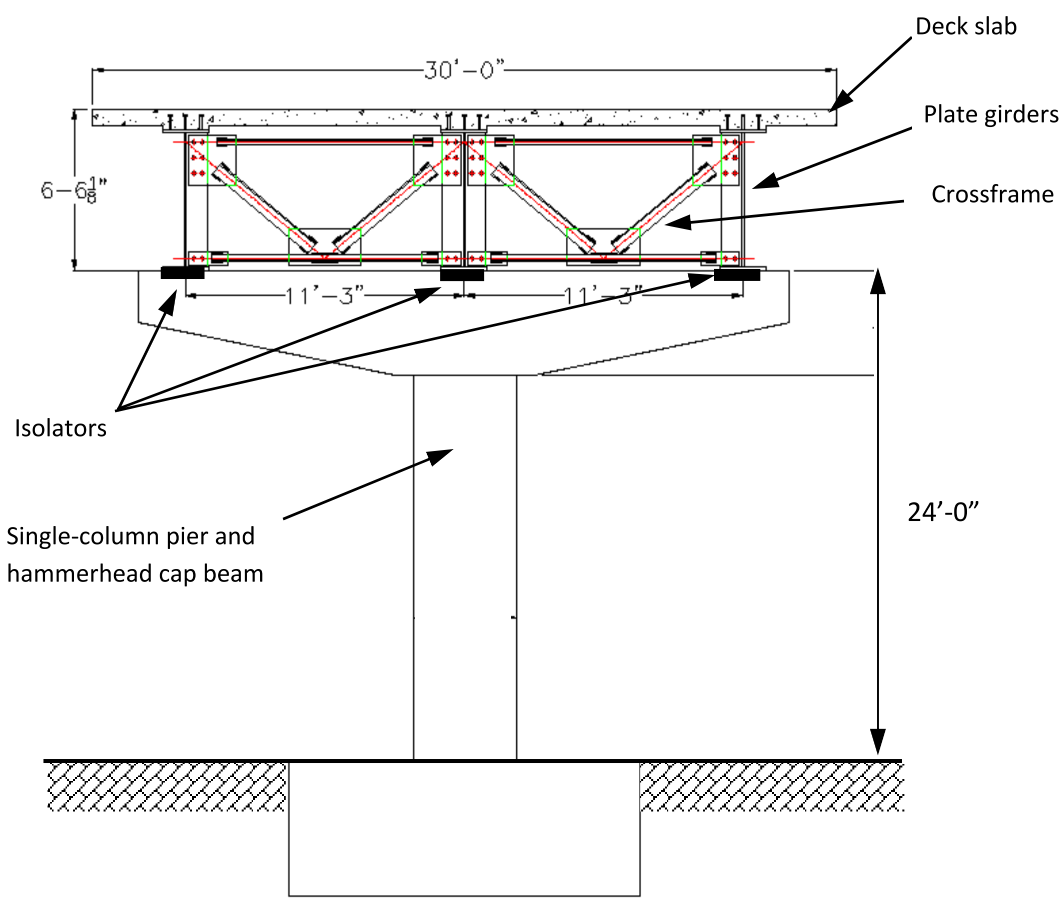


Figure 1.3. Typical Section of Superstructure and Elevation at Pier of Benchmark Bridge No. 2.

1.3 DESIGN METHODOLOGY

All of the isolation systems used in the above design examples have nonlinear properties in order to be 'near-rigid' for non-seismic loads but soften for earthquake loads. To avoid having to use nonlinear methods of analysis, equivalent linear springs and viscous damping is assumed to represent the nonlinear properties of isolators. But since these equivalent properties are dependent on displacement, an iterative approach is required to obtain a solution. This approach is sometimes known as the 'Direct Displacement Method'.

Each of the 14 examples has been designed using the same assumptions and design methodology. This methodology has five basic steps as below:

- Step A. Determine bridge and site data including required performance criteria
- Step B. Analyze bridge for earthquake in longitudinal direction using the Simplified Method to obtain initial estimates for use in Multi-modal Spectral Analysis
- Step C. Analyze bridge for earthquake in transverse direction using the Simplified Method to obtain initial estimates for use in Multi-modal Spectral Analysis
- Step D. Combine results from Steps B and C and obtain design values for displacements and forces
- Step E. Design isolators

Further detail for each step is given in Table 2.

1.4 PRESENTATION OF DESIGN EXAMPLES

The same 2-column format is used for each design example. A step-by-step design procedure, based on the methodology in the previous section, is given in the left-hand column and the application of this procedure to the example in hand is given in the right- hand column. The left hand column is therefore the same for all examples. The right hand column changes from example to example. Each example is presented as a stand-alone exercise to improve readability. However, in these circumstances, repetition of some material is unavoidable.

References to provisions in the AASHTO Specifications are made throughout the examples using the following notation:

- GSID refers to *Guide Specifications Seismic Isolation Design*, AASHTO 2010
- LRFD refers to *LRFD Bridge Design Specifications*, AASHTO 2008

1.5 SUMMARY OF RESULTS

Table 3 summarizes the results of the 14 designs. The basic dimensions of each isolator required to achieve (or almost achieve) the desired performance are given in this Table. Column shear forces and superstructure displacements are also given for each bridge.

Table 2. Methodology and Steps in Design of Seismically Isolated Bridge.

METHODOLOGY

Assume equivalent linear springs and viscous damping can be used to represent the nonlinear hysteretic properties of isolators, so that linear methods of analysis methods may be used to determine response. Since equivalent properties are dependent on displacement, an iterative approach is required to obtain a solution. The methodology below uses the Simplified Method to obtain initial estimates of displacement for use in an iterative solution using the Multi-Modal Spectral Analysis Method.

STEP A. BRIDGE AND SITE DATA

A1. Obtain bridge properties: weight, geometry, substructure stiffnesses and capacities, isolators, soil conditions
A2. Determine seismic hazard at site (acceleration coefficients and soil factors); plot response spectrum
A3. Determine required performance of isolated bridge (e.g. elastic columns for design earthquake)

↓

STEP B. ANALYZE BRIDGE IN LONGITUDINAL DIRECTION.

B1. Apply response spectrum in longitudinal direction of bridge and use Simplified Method to analyze a single degree-of-freedom model of bridge to obtain first estimate of superstructure displacement and required properties of each isolator necessary to obtain desired performance (i.e. find d, characteristic strength, Qdj, and post elastic stiffness, Kdj for each isolator j)

B2. Apply response spectrum in longitudinal direction of bridge and use Multimodal Spectral Analysis Method to analyze 3-dimensional, multi-degree-of-freedom model of bridge and obtain final estimates of superstructure displacement and required properties of each isolator to obtain desired performance. [Use the results from the Simplified Method to determine equivalent spring elements to represent the isolators in the 3-D model used in this analysis.]

Obtain longitudinal and transverse displacements (u_L, v_L) for each isolator
Obtain longitudinal and transverse displacements for superstructure
Obtain biaxial column moments and shears at critical locations

↓

STEP C. ANALYZE BRIDGE IN TRANSVERSE DIRECTION

Repeat B1 and B2 above for response spectrum applied in transverse direction.
Obtain longitudinal and transverse displacements (u_T, v_T) for each isolator
Obtain longitudinal and transverse displacements for superstructure
Obtain biaxial column moments and shears at critical locations

↓

STEP D. COMBINE RESULTS AND OBTAIN DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (100L+30T) & (30L+100T) rule to obtain design values for isolator and superstructure displacement, moment and shear.
Check that required performance is satisfied.

↓

STEP E. DESIGN ISOLATORS

Select isolator type (e.g. lead-rubber isolator, spherical friction isolator, Eradiquake isolator)
Design isolators to have the required characteristic strengths (Q_{dj}) and post elastic stiffnesses (K_{dj}) calculated above. If actual values of Q_{dj} and K_{dj} differ significantly from above values, reanalyze bridge. Check design for strain limit state, and vertical stability, Conduct upper and lower analyses using minimum and maximum properties to account for isolator aging, temperature effects, scragging, contamination and wear. Revise design if required performance objective is not satisfied.

Table 3. Summary of Seismic Isolator Designs.

Ex.	ID	Isolator size including mounting plates (in)	Isolator size without mounting plates (in)	Diam. lead core (in)	Rubber Shear modulus (psi)	Column shear (k)	Super-structure resultant displace-ment (in)
EXAMPLE SET 1: PC GIRDER BRIDGE (Column yield shear force = 25.0 k)							
1.0	Benchmark 1	17.00 x 17.00 x 11.50 (H)	13.00 dia. x 10.00(H)	1.61	60	18.03	1.72
1.1	Site Class D	17.25 x 17.25 x 11.875(H)	13.25 dia. x 10.375(H)	1.97	60	25.55*	3.96
1.2	S_1=0.6g	20.25 x 20.25 x 16.75(H)	16.25 dia. x 15.25(H)	1.97	60	29.15*	7.32
1.3	SFB isolator	16.25 x 16.25 x 4.50(H)	12.25 dia. x 4.50(H)	R (in) 39.0	PTFE 15GF	18.03	1.72
1.4	EQS isolator	32.0 x 18.0 x 4.00(H)	18.0 x 18.0 x 4.00(H)	Polyurethane springs 4	 1.25 dia.	18.03	1.72
1.5	H_1=0.5H_2	17.00 x 17.00 x 11.50(H)	13.00 dia. x 10.00(H)	1.61	60	19.56 (P1) 2.56 (P2)	2.32
1.6	45^0 skew	16.00 x 16.00 x 10.00(H)	12.00 dia. x 8.50(H)	1.63	60	28.32*	1.61
EXAMPLE SET 2: STEEL PLATE GIRDER BRIDGE (Column yield shear force) = 128 k)							
2.0	Benchmark 2	17.50 x 17.50 x 5.50(H)	13.50 dia. x 4.00(H)	3.49	100	71.74	1.82
2.1	Class D	21.25 x 21.25 x 8.125(H)	17.25 dia. x 6.625(H)	4.13	60	121.0	3.79
2.2	S_1=0.6g	24.0 x 24.0 x 12.625(H)	20.0 dia. x 11.125(H)	4.68	60	175.0*	8.21
2.3	SFB isolator	17.75 x 17.75 x 9.00(H)	13.75 dia. x 7.00(H)	R (in) 27.75	PTFE 25GF	71.74	1.82
2.4	EQS isolator	36.0 x 23.0 6.20(H)	23.0 x 23.0 x 6.20(H)	Polyurethane springs 4	 2.75 dia.	71.74	1.82
2.5	H_1=0.5H_2	17.50 x 17.50 x 5.875(H)	13.50 dia. x 4.375(H)	3.49	100	87.56 (P1) 47.53 (P2)	2.05
2.6	45^0 skew	17.50 x 17.50 x 5.50(H)	13.50 dia. x 4.00(H)	3.49	100	106.8	1.69

Note: * exceeds column yield shear force.

For the PC Girder Bridge, the elastic performance criterion is satisfied in 4 of the 7 cases. But in three cases (soft soils, higher hazard, and extreme skew), it is not possible for the LRB system to keep the column shear forces below yield. However the excess is small (less than 16%) and ‘essentially’ elastic behavior is to be expected. It is noted that these three cases use LRB isolators which, for this bridge, are governed by vertical stability requirements. It is possible that the SFB and EQS systems might be able to achieve fully elastic behavior in these cases, since they are not as sensitive to stability requirements.

For the Steel Plate Girder Bridge, the elastic criterion is satisfied in 6 of the 7 cases. The exception is the case where S_1 =0.6g (Example 2.2)and it is clear that for this level of seismicity either some level of yield must be accepted, or the column increased in size to increase its elastic strength. As noted in Example 2.2, a pushover analysis of this column will quickly determine the ductility demand during this earthquake and a judgment can them be made whether it is acceptably small. It is noted that the value of this demand will be significantly less than if isolation had not been used in the design.

For the other six cases it is shown that isolation designs may be found (using LRB, SFB and EQS systems), for softer soils (Site Class D) and asymmetric geometry (unequal column heights and high skew), and still keep the columns elastic. This improved performance compared to the PC Girder Bridge is due to the Steel Plate Girder Bridge being heavier with fewer isolators (12 vs 24), a fact that favors most isolation systems.

It is interesting to note in Table 3 that, although both bridges are significantly different in weight and length, they have similar displacements for the same hazard, soil conditions and geometry. This is because, when these bridges are isolated, they have similar fundamental periods and therefore respond to the same hazard in similar ways.

1.6 REFERENCES

AASHTO, 2008. *LRFD Bridge Design Specifications*, American Association State Highway and Transportation Officials, Washington DC

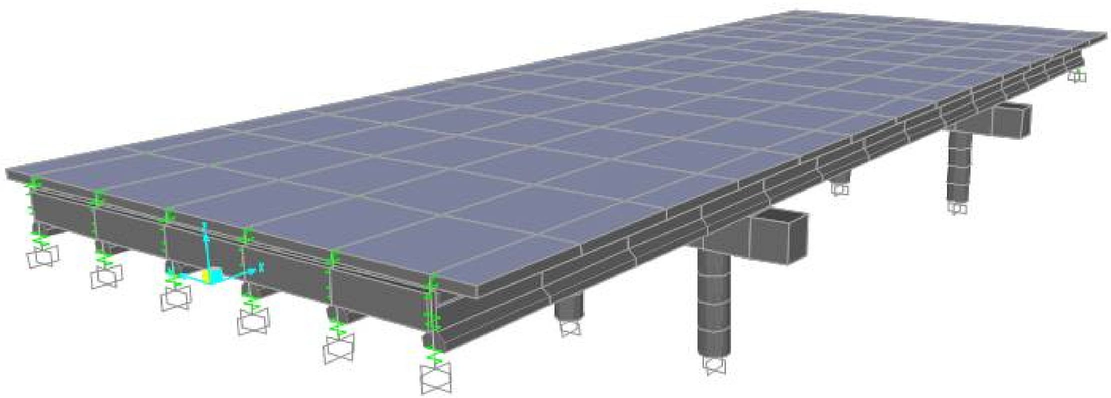
AASHTO, 2010. *Guide Specifications for Seismic Isolation Design*, Third Edition, American Association State Highway and Transportation Officials, Washington DC

Buckle, I.G., Constantinou, M., Dicleli, M., Ghasemi, H., 2006. *Seismic Isolation of Highway Bridges*, Special Report MCEER-06-SP07, Multidisciplinary Center Earthquake Engineering Research, University at Buffalo, NY

SECTION 1

PC Girder Bridge, Short Spans, Multi-Column Piers

DESIGN EXAMPLE 1.0: Benchmark Bridge #1



Design Examples in Section 1

ID	Description	S_1	Site Class	Pier height	Skew	Isolator type
1.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
1.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
1.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
1.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
1.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
1.5	Change column height	0.2g	B	H_1=0.5 H_2	0	Lead rubber bearing
1.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

DESIGN PROCEDURE	DESIGN EXAMPLE 1.0 (Benchmark #1)
STEP A: BRIDGE AND SITE DATA	

A1. Bridge Properties
Determine properties of the bridge:
• number of supports, m
• number of girders per support, n
• angle of skew
• weight of superstructure including railings, curbs, barriers and to the permanent loads, W_{SS}
• weight of piers participating with superstructure in dynamic response, W_{PP}
• weight of superstructure, W_j, at each support
• stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
• column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
• allowable movement at expansion joints
• isolator type if known, otherwise to be determined.
	A1. Bridge Properties, Example 1.0
	• Number of supports, $m = 4$
	  o North Abutment ($m = 1$)
	  o Pier 1 ($m = 2$)
	  o Pier 2 ($m = 3$)
	  o South Abutment ($m = 4$)
	• Number of girders per support, $n = 6$
	• Number of columns per support = 3
	• Angle of skew = 0^0
	• Weight of superstructure including permanent loads, $W_{SS} = 650.52$ k
	• Weight of superstructure at each support:
	  o $W_1 = 44.95$ k
	  o $W_2 = 280.31$ k
	  o $W_3 = 280.31$ k
	  o $W_4 = 44.95$ k
	• Participating weight of piers, $W_{PP} = 107.16$ k
	• Effective weight (for calculation of period), $W_{eff} = W_{SS} + W_{PP} = 757.68$ k
	• Stiffness of each pier in the longitudinal direction:
	  o $K_{sub,pier1,long} = 172.0$ k/in
	  o $K_{sub,pier2,long} = 172.0$ k/in
	• Stiffness of each pier in the transverse direction:
	  o $K_{sub,pier1,trans} = 687.0$ k/in
	  o $K_{sub,pier2,trans} = 687.0$ k/in
	• Minimum column shear strength based on flexural yield capacity of column = 25 k
	• Displacement capacity of expansion joints (longitudinal) = 2.0 in for thermal and other movements.
	• Lead rubber isolators

A2. Seismic Hazard Determine seismic hazard at site: • acceleration coefficients • site class and site factors • seismic zone Plot response spectrum. Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of PGA, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD). Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for	**A2. Seismic Hazard, Example 1.0** Acceleration coefficients for bridge site are given in design statement as follows: • $PGA = 0.40$ • $S_1 = 0.20$ • $S_S = 0.75$ Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec. Table 3.10.3.1-1 LRFD gives Site Class as B. Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors: • $F_{pga} = 1.0$ • $F_a = 1.0$ • $F_v = 1.0$

conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD.

Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows:

- $A_s = F_{pga}\, PGA$
- $S_{DS} = F_a\, S_S$
- $S_{D1} = F_v\, S_1$

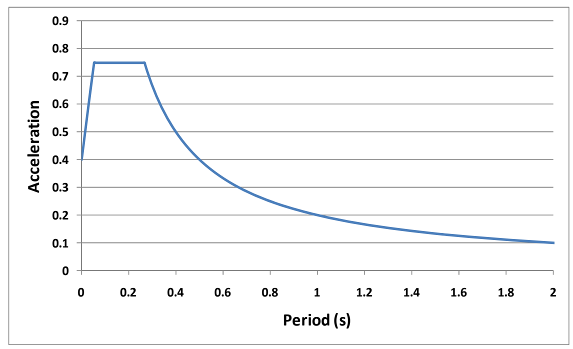
Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table5-1 GSID.

These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.

- $A_s = F_{pga}\, PGA = 1.0(0.40) = 0.40$
- $S_{DS} = F_a\, S_S = 1.0(0.75) = 0.75$
- $S_{D1} = F_v\, S_1 = 1.0(0.20) = 0.20$

Since $0.15 < S_{D1} \leq 0.30$, bridge is located in Seismic Zone 2.

Design Response Spectrum is as below:



A3. Performance Requirements

Determine required performance of isolated bridge during Design Earthquake (1000-yr return period).

Examples of performance that might be specified by the Owner include:

- Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake.
- Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake.
- For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements)
- Reduced substructure forces for bridges on weak soils to reduce foundation costs.

A3. Performance Requirements, Example 1.0

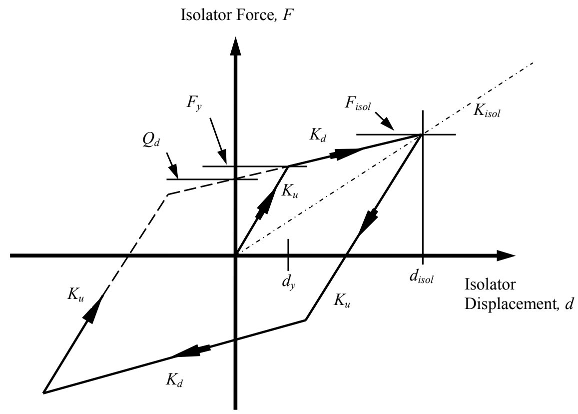
In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield).

To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (25 k).

The maximum shear in the column must therefore be less than 25 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

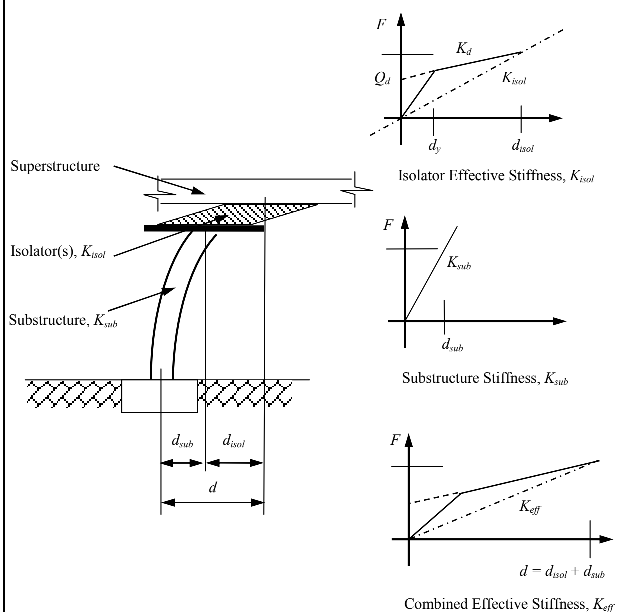
Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties	**B1.1 Initial System Displacement and Properties, Example 1.0**
To begin the iterative solution, an estimate is required of : (1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)	
Art C7.1 GSID $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \qquad \text{(B-1)}$$	$$d \cong 10 S_{D1} = 10(0.20) \cong 2.0\ in$$
(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not occur under non-seismic loads (e.g. wind) but	

low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e. $Q_d = 0.05W$ (B-2) (3) Post-yield stiffness, K_d Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3. Art. 12.2 GSID $K_{d,min} \geq \frac{0.025W}{d}$ (B-3) Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$		$Q_d = 0.05W = 0.05(650.52) = 32.53\ k$ $K_d = 0.05\frac{W}{d} = 0.05\frac{650.52}{2.0} = 16.26\ k/in$
B1.2 Initial Isolator Properties at Supports Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support: $Q_{d,j} = Q_d \frac{W_j}{W}$ (B-4) and $K_{d,j} = K_d \frac{W_j}{W}$ (B-5)		**B1.2 Initial Isolator Properties at Supports, Example 1.0** $Q_{d,j} = Q_d \frac{W_j}{W}$ o $Q_{d,1}$ = 2.25 k o $Q_{d,2}$ = 14.02 k o $Q_{d,3}$ = 14.02 k o $Q_{d,4}$ = 2.25 k and $K_{d,j} = K_d \frac{W_j}{W}$ o $K_{d,1}$ = 1.12 k/in o $K_{d,2}$ = 7.01 k/in o $K_{d,3}$ = 7.01 k/in o $K_{d,4}$ = 1.12 k/in
B1.3 Effective Stiffness of Combined Pier and Isolator System Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID). An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula as follows (MCEER 2006): $K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$ (B-6) where $\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$ (B-7) and $K_{sub,j}$ for the piers are given in Step A1. For the abutments, take $K_{sub,j}$ to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high		**B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 1.0** $\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$ o $\alpha_1 = 2.25\text{x}10^{-4}$ o $\alpha_2 = 8.49\text{x}10^{-2}$ o $\alpha_3 = 8.49\text{x}10^{-2}$ o $\alpha_4 = 2.25\text{x}10^{-4}$ $K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$ o $K_{eff,1}$ = 2.25 k/in o $K_{eff,2}$ = 13.47 k/in o $K_{eff,3}$ = 13.47 k/in o $K_{eff,4}$ = 2.25 k/in

values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.  		
B1.4 Total Effective Stiffness Calculate the total effective stiffness, K_{eff}, of the bridge: Eq. 7.1-6 GSID $\quad K_{eff} = \sum_{j=1}^{m} K_{eff,j} \quad$ (B-8)		**B1.4 Total Effective Stiffness, Example 1.0** $K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 31.43\ k/in$
B1.5 Isolation System Displacement at Each Support Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports: $d_{isol,j} = \dfrac{d}{1+\alpha_j} \quad$ (B-9)		**B1.5 Isolation System Displacement at Each Support, Example 1.0** $d_{isol,j} = \dfrac{d}{1+\alpha_j}$ o $d_{isol,1}$ = 2.00 in o $d_{isol,2}$ = 1.84 in o $d_{isol,3}$ = 1.84 in o $d_{isol,4}$ = 2.00 in
B1.6 Isolation System Stiffness at Each Support Calculate the effective stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports: $K_{isol,j} = \dfrac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \quad$ (B-10)		**B1.6 Isolation System Stiffness at Each Support, Example 1.0** $K_{isol,j} = \dfrac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$ o $K_{isol,1}$ = 2.25 k/in o $K_{isol,2}$ = 14.61 k/in o $K_{isol,3}$ = 14.61 k/in o $K_{isol,4}$ = 2.25 k/in

B1.7 Substructure Displacement at Each Support
Calculate the displacement of substructure ‘j’, $d_{sub,j}$, for all supports:

$$d_{sub,j} = d - d_{isol,j} \qquad \text{(B-11)}$$

B1.7 Substructure Displacement at Each Support, Example 1.0

$$d_{sub,j} = d - d_{isol,j}$$

- $d_{sub,1}$ = 4.49x10^{-4} in
- $d_{sub,2}$ = 1.57x10^{-1} in
- $d_{sub,3}$ = 1.57x10^{-1} in
- $d_{sub,4}$ = 4.49x10^{-4} in

B1.8 Lateral Load in Each Substructure Support
Calculate the shear at support ‘j’, $F_{sub,j}$, for all supports:

$$F_{sub,j} = K_{sub,j} d_{sub,j} \qquad \text{(B-12)}$$

where values for $K_{sub,j}$ are given in Step A1.

B1.8 Lateral Load in Each Substructure, Example 1.0

$$F_{sub,j} = K_{sub,j} d_{sub,j}$$

- $F_{sub,1}$ = 4.49 k
- $F_{sub,2}$ = 26.93 k
- $F_{sub,3}$ = 26.93 k
- $F_{sub,4}$ = 4.49 k

B1.9 Column Shear Force at Each Support
Calculate the shear in column ‘k’ at support ‘j’, $F_{col,j,k}$, assuming equal distribution of shear for all columns at support ‘j’:

$$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j} \qquad \text{(B-13)}$$

Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.

B1.9 Column Shear Force at Each Support, Example 1.0

$$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$$

- $F_{col,2,1\text{-}3} \approx$ 8.98 k
- $F_{col,3,1\text{-}3} \approx$ 8.98 k

These column shears are less than the yield capacity of each column (25 k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.

B1.10 Effective Period and Damping Ratio
Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge:

Eq. 7.1-5 GSID

$$T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} \qquad \text{(B-14)}$$

and

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi\sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} \qquad \text{(B-15)}$$

where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j}$ = 0. For other types of isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for the Simplified Method, set $d_{y,j}$ = 0 for all isolator

B1.10 Effective Period and Damping Ratio, Example 1.0

$$T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{757.68}{386.4(31.43)}}$$

= 1.57 sec

and taking $d_{y,j}$ = 0:

$$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - 0)}{\pi\sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.31$$

<table>
<tr><td>types. See Step B2.2 where the value of $d_{y,j}$ is revisited for the Multimode Spectral Analysis Method.</td><td></td><td></td></tr>
<tr><td>B1.11 Damping Factor
Calculate the damping factor, B_L, and the displacement, d, of the bridge:

Eq. 7.1-3 GSID $$B_L = \begin{cases} \left(\frac{\xi}{0.05}\right)^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases} \quad \text{(B-16)}$$
Eq. 7.1-4 GSID $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \quad \text{(B-17)}$$</td><td></td><td>B1.11 Damping Factor, Example 1.0
Since $\xi = 0.31 \geq 0.3$
$$B_L = 1.70$$
and
$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.2)1.57}{1.70} = 1.81\ in$$</td></tr>
<tr><td>B1.12 Convergence Check
Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement.

This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5).

After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both.

Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)</td><td></td><td>B1.12 Convergence Check, Example 1.0
Since the calculated value for displacement, d (=1.81 in) is not close to that assumed at the beginning of the cycle (Step B1.1, d = 2.0), use the value of 1.81 in as the new assumed displacement and repeat from Step B1.3.

After one iteration, convergence is reached at a superstructure displacement of 1.76 in, with an effective period of 1.52 seconds, and a damping factor of 1.70 (33% damping ratio). The displacement in the isolators at Pier 1 is 1.61 in and the effective stiffness of the same isolators is 15.69 k/in.

See spreadsheet in Table B1.12-1 for results of final iteration.

Ignoring the weight of the cap beams, the sum of the column shears at a pier must equal the total isolator shear force. Hence, approximate column shear (per column) = 15.69(1.61)/3 = 8.42 k which is less than the maximum allowable (25k) if elastic behavior is to be achieved (as required in Step A3).

Also the superstructure displacement = 1.76 in, which is less than the available clearance of 2.0 in.

Therefore the above solution is acceptable and go to Step B2.

Note that available clearance (2.0 in) is greater than minimum required which is given by:
$$= \frac{8\, S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.52}{1.7} = 1.43in$$</td></tr>
</table>

Table B1.12-1 Simplified Method Solution for Design Example 1.0 – Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	650.52	107.16	757.68	0.2	6							
Step B1.1	d	1.72	Assumed displacement									
	Q_d	32.53	Characteristic strength									
	K_d	16.26	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	44.95	2.25	1.12	10000	2.40E-04	2.40	1.76	2.40	4.23E-04	4.23	3.958	7.444
Pier 1	280.31	14.02	7.01	172.0	9.12E-02	14.38	1.61	15.69	1.47E-01	25.33	22.623	44.611
Pier 2	280.31	14.02	7.01	172.0	9.12E-02	14.38	1.61	15.69	1.47E-01	25.33	22.623	44.611
Abut 2	44.95	2.25	1.12	10000	2.40E-04	2.40	1.76	2.40	4.23E-04	4.23	3.958	7.444
Total	650.52	32.526	16.263		$\Sigma K_{eff,j}$	33.557				59.107	53.161	104.109
					Step	**B1.4**						
Step B1.10	T_{eff}	1.52	Effective period									
	ξ	0.33	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.75										
	B_L	1.70	Damping Factor									
	d	1.75	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	0.375	0.187	0.400		1.66	0.413						
Pier 1	2.336	1.168	2.615		1.47	2.757						
Pier 2	2.336	1.168	2.615		1.47	2.757						
Abut 2	0.375	0.187	0.400		1.66	0.413						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength

Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \quad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \quad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 1.0

Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-1) by the number of isolators at each support (n = 6), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,1}$ = 2.25/6 = 0.37 k
- $Q_{d,2}$ = 14.02/6= 2.34 k
- $Q_{d,3}$ = 14.02/6= 2.34 k
- $Q_{d,4}$ = 2.25/6 = 0.37 k

and

- $K_{d,1}$ = 1.12/6 = 0.19 k/in
- $K_{d,2}$ = 7.01/6 = 1.17 k/in
- $K_{d,3}$ = 7.01/6 = 1.17 k/in
- $K_{d,4}$ = 1.12/6 = 0.19 k/in

B2.2 Initial Stiffness and Yield Displacement

Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

(1) For friction-based isolators: $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \quad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \quad \text{(B-22)}$$

B2.2 Initial Stiffness and Yield Displacement, Example 1.0

Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(1.17) = 11.7\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{2.34}{(11.7 - 1.17)} = 0.22\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2 in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i} = 0$.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$

Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

$$k_{isol,i} = \frac{K_{isol,j}}{n} \quad \text{(B -23)}$$

B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 1.0

Dividing the results for K_{isol} (Step B1.12) among the 6 isolators at each support, the following values for K_{isol} /isolator are obtained:

- $K_{isol,1}$ = 2.40/6 = 0.40 k/in
- $K_{isol,2}$ = 15.69/6 = 2.62 k/in
- $K_{isol,3}$ = 15.69/6 = 2.62 k/in
- $K_{isol,4}$ = 2.40/6 = 0.40 k/in

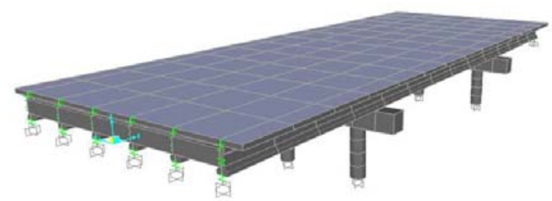
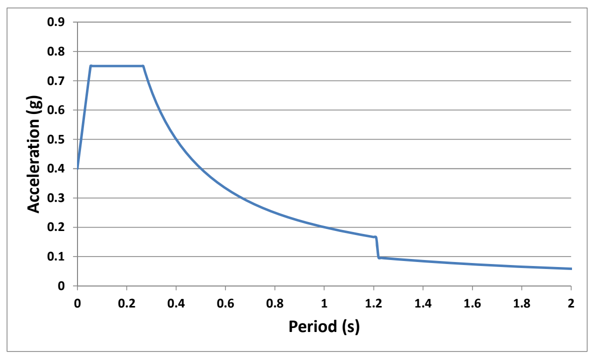
B2.4 Three-Dimensional Bridge Model Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is justified.		**B2.4 Three-Dimensional Bridge Model, Example 1.0** Although the bridge in this Design Example is regular and is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below. 
B2.5 Composite Design Response Spectrum Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, $B_{L.}$		**B.2.5 Composite Design Response Spectrum, Example 1.0** From the final results of Simplified Method (Step B1.12), B_L = 1.70 and T_{eff} = 1.52 sec. Hence the transition in the composite spectrum from 5% to 33% damping occurs at 0.8 T_{eff} = 0.8 (1.46) = 1.22 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods $\geq$ 1.22 sec by 1.70. 
B2.6 Multimodal Analysis of Finite Element Model Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.		**B2.6 Multimodal Analysis of Finite Element Model, Example 1.0** Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first two modes are the principal longitudinal and transverse modes with periods of 1.41 and 1.35 sec respectively. The period of the longitudinal mode (1.41 sec) is close to that calculated in the Simplified Method.

Table B2.6-1 Modal Properties of Bridge Example 1.0 – First Iteration

Mode	Period	Modal Participating Mass Ratios					
No.	Sec	UX	UY	UZ	RX	RY	RZ
1	1.410	0.761	0.000	0.000	0.000	0.003	0.000
2	1.346	0.000	0.738	0.031	0.059	0.000	0.534
3	1.325	0.000	0.000	0.000	0.000	0.000	0.217
4	0.187	0.000	0.000	0.000	0.000	0.000	0.000
5	0.186	0.125	0.000	0.000	0.000	0.000	0.000
6	0.121	0.000	0.000	0.000	0.000	0.000	0.000
7	0.121	0.000	0.000	0.000	0.000	0.000	0.006
8	0.104	0.000	0.034	0.183	0.107	0.000	0.064
9	0.101	0.000	0.000	0.000	0.000	0.226	0.000
10	0.095	0.000	0.121	0.081	0.184	0.000	0.041
11	0.094	0.000	0.000	0.000	0.000	0.000	0.000
12	0.074	0.000	0.002	0.000	0.003	0.000	0.000

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 1.65 (1.76) in
- $d_{isol,2}$ = 1.47 (1.61) in
- $d_{isol,3}$ = 1.47 (1.61) in
- $d_{isol,4}$ = 1.65 (1.76) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 1.0

The new superstructure displacement is 1.65 in, more than a 5% difference from the displacement assumed at the start of the Multimode Spectral Analysis.

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

$$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \qquad \text{(B-24)}$$

Recalculate $K_{eff,j}$:

Eq. 7.1-6 GSID

$$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \qquad \text{(B-25)}$$

Recalculate system damping ratio, ξ :

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \qquad \text{(B-26)}$$

Recalculate system damping factor, B_L:

Eq. 7.1-3

$$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \qquad \text{(B-27)}$$

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 1.0

Updated values for $K_{isol,i}$ are given below (previous values are in parentheses):

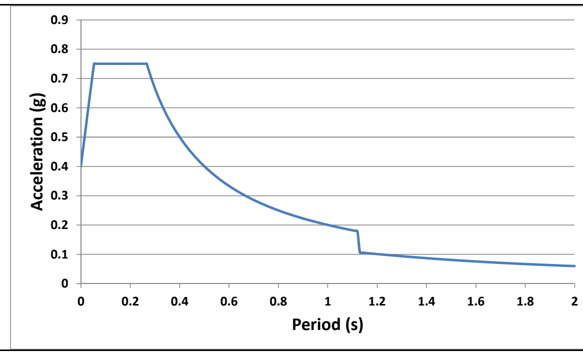
- $K_{isol,1}$ = 0.41 (0.40) k/in
- $K_{isol,2}$ = 2.76 (2.62) k/in
- $K_{isol,3}$ = 2.76 (2.62) k/in
- $K_{isol,4}$ = 0.41 (0.40) k/in

Updated values for $K_{eff,j}$, ξ, B_L and T_{eff} are given below (previous values are in parentheses):

- $K_{eff,1}$ = 2.48 (2.40) k/in
- $K_{eff,2}$ = 15.48 (14.38) k/in
- $K_{eff,3}$ = 15.48 (14.38) k/in
- $K_{eff,4}$ = 2.48 (2.40) k/in

- ξ = 27% (33%)
- B_L = 1.66 (1.70)
- T_{eff} = 1.41 (1.52) sec

The updated composite response spectrum is shown below:

GSID Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.		
		B2.6 Multimodal Analysis Second Iteration, Example 1.0 Reanalysis gives the following values for the isolator displacements (numbers in parentheses are from the previous cycle): o $d_{isol,1}$ = 1.66 (1.65) in o $d_{isol,2}$ = 1.47 (1.47) in o $d_{isol,3}$ = 1.47 (1.47) in o $d_{isol,4}$ = 1.66 (1.65) in
B2.7 Convergence Check Compare the resulting displacements at the superstructure level (*d*) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.		**B2.7 Convergence Check, Example 1.0** The new superstructure displacement is 1.66 in, less than a 1% difference from the displacement assumed at the start of the second cycle of Multimode Spectral Analysis.
B2.9 Superstructure and Isolator Displacements Once convergence has been reached, obtain o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.		**B2.9 Superstructure and Isolator Displacements, Example 1.0** From the above analysis: o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are: x_L= 1.66 in y_L= 0.0 in o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are: o Abutments: u_L = 1.66 in, v_L = 0.00 in o Piers: u_L = 1.47 in, v_L = 0.00 in All isolators at same support have the same displacements.
B2.10 Pier Bending Moments and Shear Forces Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.		**B2.10 Pier Bending Moments and Shear Forces, Example 1.0** Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are: Exterior Columns: o M_{PLL}= 0 kft o M_{PTL}= 240 kft

- V_{PLL}= 15.81 k
- V_{PTL}= 0 k

Interior Columns:

- M_{PLL}= 0 kft
- M_{PTL}= 235 kft
- V_{PLL}= 15.24 k
- V_{PTL}= 0 k

Both piers have the same distribution of bending moments and shear forces among the columns.

B2.11 Isolator Shear and Axial Forces

Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 1.0

Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abutment	1	0.69	0.00	0.36
	2	0.69	0.00	0.39
	3	0.69	0.00	0.39
	4	0.69	0.00	0.39
	5	0.69	0.00	0.39
	6	0.69	0.00	0.36
Pier	1	4.04	0.00	0.13
	2	4.05	0.00	0.19
	3	4.05	0.00	0.22
	4	4.05	0.00	0.22
	5	4.05	0.00	0.19
	6	4.04	0.00	0.13

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 1.0

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.43 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows: x_T = 0 and y_T = 1.53 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions are as follows:
  - Abutments u_T = 0.00 in, v_T = 1.53 in
  - Piers u_T = 0.00 in, v_T = 1.49 in
- Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are:

  Exterior Columns:
  - M_{PLT}= 108 kft
  - M_{PTT}= 1 kft
  - V_{PLT}= 0.06 k
  - V_{PTT}= 14.87 k

  Interior Columns:
  - M_{PLT}= 120 kft
  - M_{PTT}= 0 kft
  - V_{PLT}= 0 k
  - V_{PTT}= 17.29 k

  Both piers have the same distribution of bending moments and shear forces among the columns.

- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abutment	1	0.00	0.65	3.50
	2	0.00	0.65	1.93
	3	0.00	0.65	0.68
	4	0.00	0.65	0.68
	5	0.00	0.65	1.93
	6	0.00	0.65	3.50
Pier	1	0.02	3.98	12.56
	2	0.01	4.00	1.14
	3	0.00	4.01	2.29
	4	0.00	4.01	2.29
	5	0.01	4.00	1.14
	6	0.02	3.98	12.56

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements
Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)
- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)
- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements at Pier 1, Example 1.0
To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

Load Case 1:
$u_1 = u_L + 0.3u_T$ = 1.0(1.47) + 0.3(0) = 1.47 in
$v_1 = v_L + 0.3v_T$ = 1.0(0) + 0.3(1.49) = 0.45 in
$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.47^2 + 0.45^2}$ = 1.54 in

Load Case 2:
$u_2 = 0.3u_L + u_T$ = 0.3(1.47) + 1.0(0) = 0.44 in
$v_2 = 0.3v_L + v_T$ = 0.3(0) + 1.0(1.49) = 1.49in
$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{0.44^2 + 1.49^2}$ = 1.55in

Governing Case:
Total design displacement, d_t = max(R_1, R_2)
= 1.55 in

D2. Design Moments and Shears
Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

D2. Design Moments and Shears in Pier 1, Example 1.0
Design moments and shear forces are calculated for Pier 1, Column 1, below to illustrate the process.

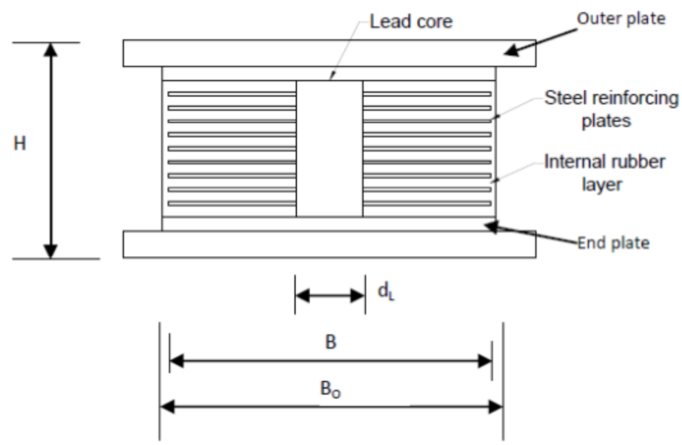
Load Case 1:
$V_{PL1} = V_{PLL} + 0.3V_{PLT}$ = 1.0(15.81) + 0.3(0.06)
= 15.83 k
$V_{PT1} = V_{PTL} + 0.3V_{PTT}$ = 1.0(0.02) + 0.3(14.87)
= 4.48 k
$R_1 = \sqrt{V_{PL1}^2 + V_{PT1}^2} = \sqrt{15.83^2 + 4.48^2}$ = 16.45 k

Load Case 2:
$V_{PL2} = 0.3V_{PLL} + V_{PLT}$ = 0.3(15.81) + 1.0(0.06)
= 4.80 k
$V_{PT2} = 0.3V_{PTL} + V_{PTT}$ = 0.3(0.02) + 1.0(14.87)
= 14.88 k
$R_2 = \sqrt{V_{PL2}^2 + V_{PT2}^2} = \sqrt{4.80^2 + 14.88^2}$ = 15.63 k

Governing Case:
Design column shear = max ($R1$, $R2$)
= 16.45 k

STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS

A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness (K_u).With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness K_d is given by the rubber alone. More details are given in MCEER 2006.



While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary. When sizing the physical dimensions of the bearing, plan dimensions (B, d_L) should be rounded up to the next $^1/_4$" increment, while the total thickness of elastomer, T_r, is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are $^1/_4$" and $^3/_8$". High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in MCEER 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See MCEER 2006.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1). • required characteristic strength, Q_d / isolator • required post-elastic stiffness, K_d / isolator • total design displacement, d_t, for each isolator • maximum applied dead and live load (P_{DL}, P_{LL}) and seismic load (P_{SL}) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator, and • maximum wind load, P_{WL}		**E1. Required Properties, Example 1.0** The design of one of the exterior isolators on a pier is given below to illustrate the design process for lead-rubber isolators. From previous work • Q_d / isolator = 2.34 k • K_d / isolator = 1.17 k/in • Total design displacement, d_t = 1.55 in • P_{DL} = 45.52 k • P_{LL} = 15.50 k • P_{SL} = 12.56 k • P_{WL} = 1.76 k < Q_d OK
E2. Isolator Sizing		
E2.1 Lead Core Diameter Determine the required diameter of the lead plug, d_L, using: $$d_L = \sqrt{\frac{Q_d}{0.9}} \quad \text{(E-1)}$$ See Step E2.5 for limitations on d_L		**E2.1 Lead Core Diameter, Example 1.0** $$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{2.34}{0.9}} = 1.61\ in$$

<table>
<tr><td></td><td></td><td></td></tr>
<tr><td>E2.2 Plan Area and Isolator Diameter
Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.0 ksi.

Then the bonded area of the isolator is given by:
$$A_b = \frac{P_{DL} + P_{LL}}{1.0}\ in^2 \quad \text{(E-2)}$$
and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by:
$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} \quad \text{(E-3)}$$
Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using
$$A_b = \frac{\pi}{4}(B^2 - d_L^2) \quad \text{(E-4)}$$
Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, B_o is given by:
$$B_o = B + 1.0 \quad \text{(E-5)}$$</td><td></td><td>E2.2 Plan Area and Isolator Diameter, Example 1.0

$$A_b = \frac{P_{DL} + P_{LL}}{1.0}\ in^2 = \frac{45.52 + 15.50}{1.0} = 61.02\ in^2$$
$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} = \sqrt{\frac{4\,(61.02)}{\pi} + 1.61^2}$$
= 8.96 in

Round B up to 9.0 in and the actual bonded area is:
$$A_b = \frac{\pi}{4}(9.0^2 - 1.61^2) = 61.57\ in^2$$
B_o = 9.0 + 2(0.5) = 10.0 in</td></tr>
<tr><td>E2.3 Elastomer Thickness and Number of Layers
Since the shear stiffness of the elastomeric bearing is given by:
$$K_d = \frac{GA_b}{T_r} \quad \text{(E-6)}$$
where G = shear modulus of the rubber, and
T_r = the total thickness of elastomer,
it follows Eq. E-5 may be used to obtain T_r given a required value for K_d
$$T_r = \frac{GA_b}{K_d} \quad \text{(E-7)}$$
A typical range for shear modulus, G, is 60-120 psi. Higher and lower values are available and are used in special applications.

If the layer thickness is t_r, the number of layers, n, is given by:
$$n = \frac{T_r}{t_r} \quad \text{(E-8)}$$
rounded up to the nearest integer.</td><td></td><td>E2.3 Elastomer Thickness and Number of Layers, Example 1.0

Select G, shear modulus of rubber, = 100 psi (0.1 ksi)

Then
$$T_r = \frac{GA_b}{K_d} = \frac{0.1(61.57)}{1.17} = 5.27\ in$$
$$n = \frac{5.27}{0.25} = 21.09$$
Round to nearest integer, i.e. n = 22</td></tr>
</table>

Note that because of rounding the plan dimensions and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be necessary if the differences are large.	
E2.4 Overall Height The overall height of the isolator, H, is given by: $$H = n\,t_r + (n-1)t_s + 2t_c \qquad \text{(E-9)}$$ where t_s = thickness of an internal shim (usually about 1/8 in), and t_c = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in)	**E2.4 Overall Height, Example 1.0** $$H = 22(0.25) + 21(0.125) + 2 * 1.5 = 11.125\ in$$
E2.5 Size Checks Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows: $$\frac{B}{3} \geq d_L \geq \frac{B}{6} \qquad \text{(E-10)}$$ Art. 12.2 GSID requires that the isolation system provides a lateral restoring force at d_t greater than the restoring force at $0.5d_t$ by not less than $W/80$. This equates to a minimum K_d of $0.025W/d$. $$K_{d,min} = \frac{0.025W}{d}$$	**E2.5 Size Checks, Example 1.0** Since B=9.0 check $$\frac{9.0}{3} \geq d_L \geq \frac{9.0}{6}$$ i.e., $3.0 \geq d_L \geq 1.5$ Since d_L = 1.6, lead core size is acceptable. $$K_{d,min} = \frac{0.025W}{d} = \frac{0.025(45.52)}{1.73} = 0.66\ k/in$$ As $$K_d = \frac{GA_b}{T_r} = \frac{0.1(64.15)}{5.5} = 1.17k/in\ > K_{d,min}$$
E3. Strain Limit Check Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e., $$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5 \qquad \text{(E-11)}$$ where γ_c, $\gamma_{s,eq}$, and γ_r are defined below. (a) γ_c is the maximum shear strain in the layer due to compression and is given by: $$\gamma_c = \frac{D_c\sigma_S}{GS} \qquad \text{(E-12)}$$ where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = {P_{DL}}/{A_b}$, G is shear modulus, and S is the layer shape factor given by: $$S = \frac{A_b}{\pi B t_r} \qquad \text{(E-13)}$$ (b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by:	**E3. Strain Limit Check, Example 1.0** Since $$\sigma_S = \frac{45.52}{61.57} = 0.739\ ksi$$ G = 0.1 ksi and $$S = \frac{61.57}{\pi 9.0(0.25)} = 8.71$$ then $$\gamma_c = \frac{1.0(0.739)}{0.1(8.71)} = 0.849$$

$\gamma_{s,eq} = \dfrac{d_t}{T_r}$ (E-14) (c) γ_r is the shear strain due to rotation and is given by: $\gamma_r = \dfrac{D_r B^2 \theta}{t_r T_r}$ (E-15) where *Dr* is shape coefficient for rotation in circular bearings = 0.375, and θ is design rotation due to DL, LL and construction effects Actual value for θ may not be known at this time and value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1)		$\gamma_{s,eq} = \dfrac{1.58}{4.5} = 0.282$ $\gamma_r = \dfrac{0.375(9.0^2)(0.01)}{0.25(5.5)} = 0.221$ Substitution in Eq E-11 gives $\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.85 + 0.28 + 0.5(0.22) = 1.24 \leq 5.5\ OK$
E4. Vertical Load Stability Check Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state. Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either 2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or 1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4.		**E4. Vertical Load Stability Check, Example 1.0**
E4.1 Vertical Load Stability in Undeformed State The critical load capacity of an elastomeric isolator at zero shear displacement is given by $P_{cr(\Delta=0)} = \dfrac{K_d H_{eff}}{2}\left[\sqrt{(1 + \dfrac{4\pi^2 K_\theta}{K_d {H_{eff}}^2})} - 1\right]$ (E-16) where $H_{eff} = T_r + T_s$ T_s = total shim thickness $K_\theta = {E_b I}/{T_r}$ $E_b = E(1 + 0.67S^2)$ E = elastic modulus of elastomer = 3G $I = {\pi B^4}/{64}$ It is noted that typical elastomeric isolators have high shape factors, *S*, in which case: $\dfrac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1$ (E-17) and Eq. E-16 reduces to: $P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta}$ (E-18) Check that:		**E4.1 Vertical Load Stability in Undeformed State, Example 1.0** $E = 3G = 3(0.1) = 0.3\ ksi$ $E_b = 0.3(1 + 0.67(8.71^2)) = 15.55\ ksi$ $I = \pi\dfrac{9.0^4}{64} = 322.1\ in^4$ $K_\theta = \dfrac{15.48(322.1)}{5.5} = 910.89\ kin/rad$ $K_d = \dfrac{GA_b}{T_r} = \dfrac{0.1(61.57)}{4.5} = 1.12\ k/in$ $P_{cr(\Delta=0)} = \pi\sqrt{1.12(910.89)} = 100.33\ k$

$\dfrac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} \geq 3$ (E-19)		$\dfrac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \dfrac{100.33}{(45.52+15.5)} = 1.64 \ngeq 3 \quad NOK$
E4.2 Vertical Load Stability in Deformed State The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by: $P_{cr(\Delta)} = \dfrac{A_r}{A_{gross}} P_{cr(\Delta=0)}$ (E-20) where A_r = overlap area between top and bottom plates of isolator at displacement Δ (Fig. 2.2-1 GSID) = $B^2(\delta - sin\delta)/4$ δ = $2cos^{-1}(\Delta/B)$ A_{gross} = $\pi B^2/4$ It follows that: $\dfrac{A_r}{A_{gross}} = \dfrac{(\delta - sin\delta)}{\pi}$ (E-21) Check that: $\dfrac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} \geq 1$ (E-22)		**E4.2 Vertical Load Stability in Deformed State, Example 1.0** Since bridge is in Zone 2, $\Delta = 2d_t = 2(1.55) = 3.10$ $\delta = 2cos^{-1}\left(\dfrac{3.10}{9.0}\right) = 2.44$ $\dfrac{A_r}{A_{gross}} = \dfrac{(2.44 - sin2.44)}{\pi} = 0.570$ $P_{cr(\Delta)} = 0.570(100.33) = 57.16\ k$ $\dfrac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} = \dfrac{57.16}{1.2(45.52)+12.56} = 0.85 \ngeq 1 \quad NOK$
E5. Design Review		**E5. Design Review, Example 1.0** The basic dimensions of the isolator designed above are as follows: 10.00 in (od) x 11.125 in (high) x 1.61 in dia. lead core and the volume, excluding steel end and cover plates, is 638 in^3. This design satisfies the shear strain limit criteria, but not the vertical load stability ratio in the undeformed and deformed states. A redesign is therefore required and the easiest way to increase the P_{cr} is to increase the shape factor, *S*, since the bending stiffness of an isolator is a function of the shape factor squared. See equations in Step E4.1. To increase *S*, increase the bonded area A_b while keeping t_r constant (Eq. E-13). But to keep K_d constant while increasing A_b and T_r is constant, decrease the shear modulus, *G* (Eq. E-6). This redesign is outlined below. After repeating the calculation for diameter of lead core, the process begins by reducing the shear modulus to 60 psi (0.06 ksi) and

increasing the bonded diameter to 12 in.

E2.1

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{2.34}{0.9}} = 1.61\ in$$

E2.2

$$A_b = \frac{T_r K_d}{G}\ in^2 = \frac{5.5(1.17)}{0.06} = 107.25\ in^2$$

$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} = \sqrt{\frac{4\,(107.25)}{\pi} + 1.61^2} = 11.80\ in$$

Round *B* to 12 in and the actual bonded area becomes:

$$A_b = \frac{\pi}{4}(12^2 - 1.61^2) = 111.06\ in^2$$

$$B_o = 12 + 2(0.5) = 13 \text{ in}$$

E2.3

$$T_r = \frac{GA_b}{K_d} = \frac{0.06(111.06)}{1.17} = 5.71\ in$$

$$n = \frac{5.71}{0.25} = 22.82$$

Round to nearest integer, i.e. $n = 23$.

E2.4

$$H = 23(0.25) + 22(0.125) + 2 * 1.5 = 11.5\ in$$

E2.5

Since B=12 check

$$\frac{12}{3} \geq d_L \geq \frac{12}{6}$$

$$\text{i.e., } 4 \geq d_L \geq 2$$

Since d_L = 1.61, the size of lead core is too small, and there are 2 options: (1) Accept the undersize and check for adequate performance during the Quality Control Tests required by GSID Art. 15.2.2; or (2) Only have lead cores in every second isolator, in which case the core diameter, in those isolators with cores, will be √2 x 1.61 = 2.27 in (which satisfies above criterion).

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(111.06)}{5.75} = 1.16k/in\ > K_{d,min}$$

E3.

$$\sigma_S = \frac{45.52}{111.06} = 0.41\ ksi$$

$$S = \frac{111.06}{\pi 12(0.25)} = 11.78$$

$$\gamma_c = \frac{1.0(0.41)}{0.06(11.27)} = 0.580$$

$$\gamma_{s,eq} = \frac{1.55}{5.75} = 0.270$$

$$\gamma_r = \frac{0.375(12^2)(0.01)}{0.25(5.75)} = 0.376$$

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.580 + 0.270 + 0.5(0.376)$$
$$= 1.04 \leq 5.5\ OK$$

E4.1

$$E = 3G = 3(0.06) = 0.18\ ksi$$

$$E_b = 0.18(1 + 0.67(11.78^2)) = 16.93\ ksi$$

$$I = \pi \frac{12^4}{64} = 1017.88\ in^4$$

$$K_\theta = \frac{16.93(1017.88)}{5.75} = 2996.42\ kin/rad$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(111.06)}{5.75} = 1.159\ k/in$$

$$P_{cr(\Delta=0)} = \pi\sqrt{1.159(2996.42)} = 185.13\ k$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} = \frac{185.13}{(45.52 + 15.50)} = 3.03 \geq 3 \quad OK$$

E4.2

$$\delta = 2cos^{-1}\left(\frac{3.10}{12}\right) = 2.62$$

$$\frac{A_r}{A_{gross}} = \frac{(2.62 - sin2.62)}{\pi} = 0.674$$

$$P_{cr(\Delta)} = 0.674(185.13) = 124.84\ k$$

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{124.84}{1.2(45.52) + 12.56} = 1.86 \geq 1\ \ OK$$

E5.

The basic dimensions of the redesigned isolator are as follows:

13.0 in (od) x 11.5 in (high) x 1.61 in dia. lead core and its volume (excluding steel end and cover plates) is 1128 in^3.

This design meets all the design criteria but is about 75% larger by volume than the previous design. This increase in size is driven by the need to satisfy the vertical load stability ratio of 3.0 in the undeformed state.

E6. Minimum and Maximum Performance Check

Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E6-1.

Table E6-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-23)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-24)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-25)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-26)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E6-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})\,(\lambda_{min,a,Kd})\,(\lambda_{min,v,Kd})\,(\lambda_{min,tr,Kd})\,(\lambda_{min,c,Kd})\,(\lambda_{min,scrag,Kd})$	(E-27)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})\,(\lambda_{max,a,Kd})\,(\lambda_{max,v,Kd})\,(\lambda_{max,tr,Kd})\,(\lambda_{max,c,Kd})\,(\lambda_{max,scrag,Kd})$	(E-28)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})\,(\lambda_{min,a,Qd})\,(\lambda_{min,v,Qd})\,(\lambda_{min,tr,Qd})\,(\lambda_{min,c,Qd})\,(\lambda_{min,scrag,Qd})$	(E-29)
Eq. 8.2.1-4	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})\,(\lambda_{max,a,Qd})\,(\lambda_{max,v,Qd})\,(\lambda_{max,tr,Qd})\,(\lambda_{max,c,Qd})$	(E-30)

E6. Minimum and Maximum Performance Check, Example 1.0

Minimum Property Modification factors are:

$\lambda_{min,Kd} = 1.0$
$\lambda_{min,Qd} = 1.0$

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are:

$\lambda_{max,a,Kd} = 1.1$
$\lambda_{max,a,Qd} = 1.1$

$\lambda_{max,t,Kd} = 1.1$
$\lambda_{max,t,Qd} = 1.4$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,a,Qd} = 1.0 + 0.1(0.66) = 1.066$

$\lambda_{max,t,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,t,Qd} = 1.0 + 0.4(0.66) = 1.264$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Therefore the maximum overall modification factors

$\lambda_{max,Kd} = 1.066(1.066)1.0 = 1.14$
$\lambda_{max,Qd} = 1.066(1.264)1.0 = 1.35$

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:

$Q_{d,max} = 1.35\ (2.34) = 3.16$ k

and

$K_{d,max} = 1.14(1.16) = 1.32$ k/in

GSID	$(\lambda_{max,scrag,Qd})$	

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

1.00 for critical bridges
0.75 for essential bridges
0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

E7. Design and Performance Summary

E7. Design and Performance Summary, Example 1.0

E7.1 Isolator dimensions

Summarize final dimensions of isolators:

- Overall diameter (includes cover layer)
- Overall height
- Diameter lead core
- Bonded diameter
- Number of rubber layers
- Thickness of rubber layers
- Total rubber thickness
- Thickness of steel shims
- Shear modulus of elastomer

Check all dimensions with manufacturer.

E7.1 Isolator dimensions, Example 1.0

Isolator dimensions are summarized in Table E7.1-1.

Table E7.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Diam. lead core (in)
Under edge girder on Pier 1	17.0 x 17.0 x 11.5 (H)	13.0 dia. x 10.0 (H)	1.61

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in)	Total rubber thick-ness (in)	Steel shim thick-ness (in)
Under edge girder on Pier 1	23	0.25	5.75	0.125

Shear modulus of elastomer = 60 psi

E7.2 Bridge Performance

Summarize bridge performance

- Maximum superstructure displacement (longitudinal)
- Maximum superstructure displacement

E7.2 Bridge Performance, Example 1.0

Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 18.03 k. This less than the column plastic shear (25 k) and therefore the required performance criterion is

(transverse)

- Maximum superstructure displacement (resultant)
- Maximum column shear (resultant)
- Maximum column moment (about transverse axis)
- Maximum column moment (about longitudinal axis)
- Maximum column torque

Check required performance as determined in Step A3, is satisfied.

satisfied (fully elastic behavior). Furthermore the maximum longitudinal displacement is 1.66 in which is less than the 2.0 in available at the abutment expansion joints and is therefore acceptable.

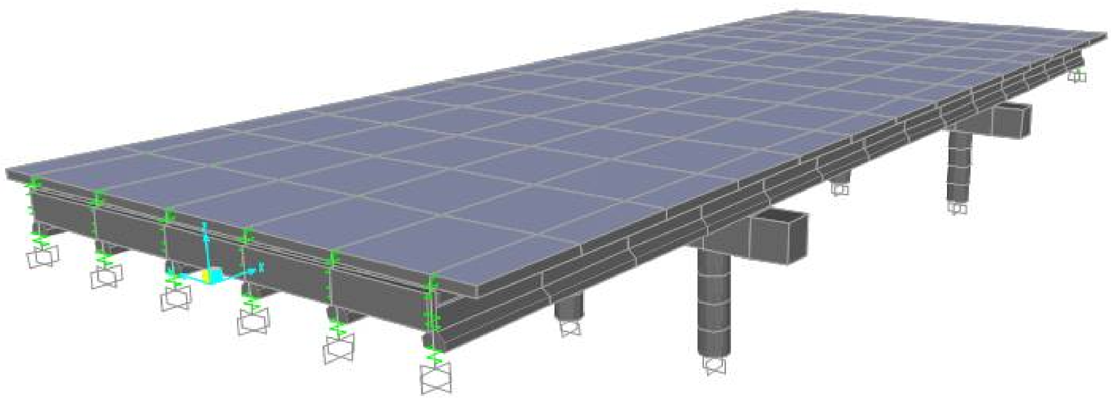
Table E7.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	1.66 in
Maximum superstructure displacement (transverse)	1.54 in
Maximum superstructure displacement (resultant)	1.72 in
Maximum column shear (resultant)	18.03 k
Maximum column moment about transverse axis	242 kft
Maximum column moment about longitudinal axis	121 kft
Maximum column torque	1.82 kft

SECTION 1

PC Girder Bridge, Short Spans, Multi-Column Piers

DESIGN EXAMPLE 1.1: Site Class D



Design Examples in Section 1

ID	Description	S_1	Site Class	Pier height	Skew	Isolator type
1.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
1.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
1.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
1.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
1.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
1.5	Change column height	0.2g	B	H_1=0.5 H_2	0	Lead rubber bearing
1.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

<table>
<tr><th>DESIGN PROCEDURE</th><th></th><th>DESIGN EXAMPLE 1.1 (Site Class D)</th></tr>
<tr><td colspan="3">STEP A: BRIDGE AND SITE DATA</td></tr>
<tr><td>A1. Bridge Properties
Determine properties of the bridge:

number of supports, m
number of girders per support, n
angle of skew
weight of superstructure including railings, curbs, barriers and to the permanent loads, W_{SS}
weight of piers participating with superstructure in dynamic response, W_{PP}
weight of superstructure, W_j, at each support
stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
allowable movement at expansion joints
isolator type if known, otherwise to be determined
</td><td></td><td>A1. Bridge Properties, Example 1.1

Number of supports, m = 4

North Abutment (m = 1)
Pier 1 (m = 2)
Pier 2 (m = 3)
South Abutment (m =4)

Number of girders per support, n = 6
Number of columns per support = 3
Angle of skew = 0^0
Weight of superstructure including permanent loads, W_{SS} = 650.52 k
Weight of superstructure at each support:

W_1 = 44.95 k
W_2 = 280.31 k
W_3 = 280.31 k
W_4 = 44.95 k

Participating weight of piers, W_{PP} = 107.16 k
Effective weight (for calculation of period), $W_{eff} = W_{SS} + W_{PP}$ = 757.68 k
Stiffness of each pier in the longitudinal direction:

$K_{sub,pier1,long}$ = 172.0 k/in
$K_{sub,pier2,long}$ = 172.0 k/in

Stiffness of each pier in the transverse direction:

$K_{sub,pier1,trans}$ = 687.0 k/in
$K_{sub,pier2,trans}$ = 687.0 k/in

Minimum column shear strength based on flexural yield capacity of column = 25 k
Displacement capacity of expansion joints (longitudinal) = 2.0 in for thermal and other movements.
Lead rubber isolators
</td></tr>
<tr><td>A2. Seismic Hazard
Determine seismic hazard at site:

acceleration coefficients
site class and site factors
seismic zone

Plot response spectrum.

Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of PGA, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD).

Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for</td><td></td><td>A2. Seismic Hazard, Example 1.1
Acceleration coefficients for bridge site are given in design statement as follows:

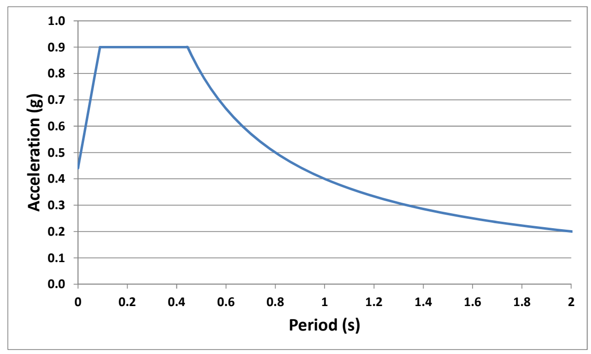
PGA = 0.40
S_1 = 0.20
S_S = 0.75

Bridge is on a stiff soil site with shear wave velocity in upper 100 ft of soil = 1,000 ft/sec.

Table 3.10.3.1-1 LRFD gives Site Class as D.

Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors:

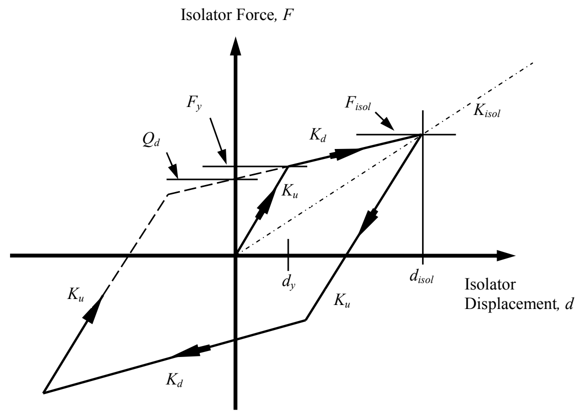
F_{pga} = 1.1
F_a = 1.2
F_v = 2.0
</td></tr>
</table>

conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD. Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows: • $A_s = F_{pga}\ PGA$ • $S_{DS} = F_a\ S_S$ • $S_{D1} = F_v\ S_1$ Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table5-1 GSID. These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.		• $A_s = F_{pga}\ PGA = 1.1(0.40) = 0.44$ • $S_{DS} = F_a\ S_S = 1.2(0.75) = 0.9$ • $S_{D1} = F_v\ S_1 = 2.0(0.20) = 0.40$ Since $0.30 < S_{D1} \le 0.50$, bridge is located in Seismic Zone 3. Design Response Spectrum is as below: 
A3. Performance Requirements Determine required performance of isolated bridge during Design Earthquake (1000-yr return period). Examples of performance that might be specified by the Owner include: • Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake. • Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake. • For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements) • Reduced substructure forces for bridges on weak soils to reduce foundation costs.		**A3. Performance Requirements, Example 1.1** In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield). To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (25 k). The maximum shear in the column must therefore be less than 25 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties

To begin the iterative solution, an estimate is required of :

(1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)

Art C7.1 GSID

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \qquad \text{(B-1)}$$

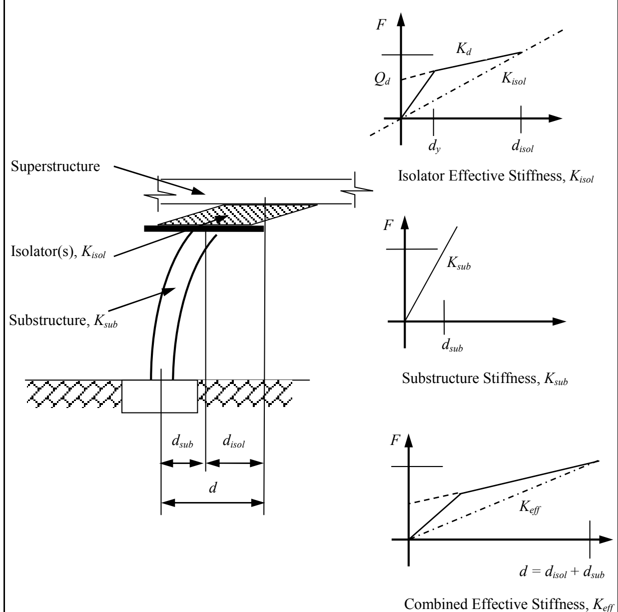
(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not occur under non-seismic loads (e.g. wind) but

B1.1 Initial System Displacement and Properties, Example 1.1

$$d \cong 10 S_{D1} = 10(0.40) \cong 4.0\ in$$

low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e. $$Q_d = 0.05W \qquad \text{(B-2)}$$ (3) Post-yield stiffness, K_d Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3. Art. 12.2 GSID $$K_{d,min} \geq \frac{0.025W}{d} \qquad \text{(B-3)}$$ Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$		Due to larger estimated displacements (Eq B-1) than for the benchmark bridge, Q_d is increased to 7.5% of the bridge weight to introduce additional damping and reduce these displacements as much as possible, i.e., $$Q_d = 0.075W = 0.075(650.52) = 48.79\ k$$ Also, in view of these larger displacements, the post yield stiffness is increased to $0.1W/d$, to give essentially the same value for K_d found to be satisfactory in Example 1.0. $$K_d = 0.1\frac{W}{d} = 0.1\frac{650.52}{4.0} = 16.26\ k/in$$
B1.2 Initial Isolator Properties at Supports Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support: $$Q_{d,j} = Q_d\frac{W_j}{W} \qquad \text{(B-4)}$$ and $$K_{d,j} = K_d\frac{W_j}{W} \qquad \text{(B-5)}$$		**B1.2 Initial Isolator Properties at Supports, Example 1.1** $$Q_{d,j} = Q_d\frac{W_j}{W}$$ o $Q_{d,1}$ = 3.37 k o $Q_{d,2}$ = 21.02 k o $Q_{d,3}$ = 21.02 k o $Q_{d,4}$ = 3.37 k and $$K_{d,j} = K_d\frac{W_j}{W}$$ o $K_{d,1}$ = 1.12 k/in o $K_{d,2}$ = 7.01 k/in o $K_{d,3}$ = 7.01 k/in o $K_{d,4}$ = 1.12 k/in
B1.3 Effective Stiffness of Combined Pier and Isolator System Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID). An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula as follows (MCEER 2006): $$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j} \qquad \text{(B-6)}$$ where $$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}} \qquad \text{(B-7)}$$ and $K_{sub,j}$ for the piers are given in Step A1. For the abutments, take $K_{sub,j}$ to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high		**B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 1.1** $$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$ o $\alpha_1 = 1.97\text{x}10^{-4}$ o $\alpha_2 = 7.35\text{x}10^{-2}$ o $\alpha_3 = 7.35\text{x}10^{-2}$ o $\alpha_4 = 1.97\text{x}10^{-4}$ $$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$$ o $K_{eff,1}$ = 1.97 k/in o $K_{eff,2}$ = 11.78 k/in o $K_{eff,3}$ = 11.78 k/in o $K_{eff,4}$ = 1.97 k/in

values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.



B1.4 Total Effective Stiffness

Calculate the total effective stiffness, K_{eff}, of the bridge:

Eq. 7.1-6 GSID

$$K_{eff} = \sum_{j=1}^{m} K_{eff,j} \qquad \text{(B-8)}$$

B1.4 Total Effective Stiffness, Example 1.1

$$K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 27.50\ k/in$$

B1.5 Isolation System Displacement at Each Support

Calculate the displacement of the isolation system at support ‘j’, $d_{isol,j}$, for all supports:

$$d_{isol,j} = \frac{d}{1+\alpha_j} \qquad \text{(B-9)}$$

B1.5 Isolation System Displacement at Each Support, Example 1.1

$$d_{isol,j} = \frac{d}{1+\alpha_j}$$

- $d_{isol,1}$ = 4.00 in
- $d_{isol,2}$ = 3.73 in
- $d_{isol,3}$ = 3.73 in
- $d_{isol,4}$ = 4.00 in

B1.6 Isolation System Stiffness at Each Support

Calculate the effective stiffness of the isolation system at support ‘j’, $K_{isol,j}$, for all supports:

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \qquad \text{(B-10)}$$

B1.6 Isolation System Stiffness at Each Support, Example 1.1

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$

- $K_{isol,1}$ = 1.97 k/in
- $K_{isol,2}$ = 12.65 k/in
- $K_{isol,3}$ = 12.65 k/in
- $K_{isol,4}$ = 1.97 k/in

B1.7 Substructure Displacement at Each Support Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports: $$d_{sub,j} = d - d_{isol,j} \quad \text{(B-11)}$$		**B1.7 Substructure Displacement at Each Support, Example 1.1** $$d_{sub,j} = d - d_{isol,j}$$ o $d_{sub,1}$ = 7.86x10^{-4} in o $d_{sub,2}$ = 2.74x10^{-1} in o $d_{sub,3}$ = 2.74x10^{-1} in o $d_{sub,4}$ = 7.86x10^{-4} in
B1.8 Lateral Load in Each Substructure Support Calculate the shear at support 'j', $F_{sub,j}$, for all supports: $$F_{sub,j} = K_{sub,j} d_{sub,j} \quad \text{(B-12)}$$ where values for $K_{sub,j}$ are given in Step A1.		**B1.8 Lateral Load in Each Substructure, Example 1.1** $$F_{sub,j} = K_{sub,j} d_{sub,j}$$ o $F_{sub,1}$ = 7.86 k o $F_{sub,2}$ = 47.13 k o $F_{sub,3}$ = 47.13 k o $F_{sub,4}$ = 7.86 k
B1.9 Column Shear Force at Each Support Calculate the shear in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j': $$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j} \quad \text{(B-13)}$$ Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.		**B1.9 Column Shear Force at Each Support, Example 1.1** $$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$$ o $F_{col,2,1\text{-}3} \approx$ 15.71 k o $F_{col,3,1\text{-}3} \approx$ 15.71 k These column shears are less than the yield capacity of each column (25 k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.
B1.10 Effective Period and Damping Ratio Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge: Eq. 7.1-5 GSID $$T_{eff} = 2\pi \sqrt{\frac{W_{eff}}{gK_{eff}}} \quad \text{(B-14)}$$ and Eq. 7.1-10 GSID $$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} \quad \text{(B-15)}$$ where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j}$ = 0. For other types of isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for the Simplified Method, set $d_{y,j}$ = 0 for all isolator		**B1.10 Effective Period and Damping Ratio, Example 1.1** $$T_{eff} = 2\pi \sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi \sqrt{\frac{757.68}{386.4(27.50)}}$$ = 1.68 sec and taking $d_{y,j}$ = 0: $$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - 0)}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.27$$

types. See Step B2.2 where the value of $d_{y,j}$ is revisited for the Multimode Spectral Analysis Method.

B1.11 Damping Factor

Calculate the damping factor, B_L, and the displacement, d, of the bridge:

Eq. 7.1-3 GSID

$$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases} \qquad \text{(B-16)}$$

Eq. 7.1-4 GSID

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \qquad \text{(B-17)}$$

B1.11 Damping Factor, Example 1.1

Since $\xi = 0.27 \ \leq 0.3$

$$B_L = 1.65$$

and

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.4)1.68}{1.65} = 3.98\ in$$

B1.12 Convergence Check

Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement.

This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5).

After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both.

Note that Art 9 GSID requires that a minimum clearance be provided equal to $8\ S_{D1}\ T_{eff}/\ B_L$. (B-18)

B1.12 Convergence Check, Example 1.1

Since the calculated value for displacement, d (=3.98 in) is very close to that assumed at the beginning of the cycle (Step B1.1, d = 4.0), use the value of 3.98 in as the displacement for the start of the Multimode Spectral Analysis.

The final values to be used for the Multimode Spectral Analysis are; an effective period of 1.68 seconds, a damping factor of 1.65 (27% damping ratio). The displacement in the isolators at Pier 1 is 3.71 in and the effective stiffness of the same isolators is 12.68 k/in.

See spreadsheet in Table B1.12-1 for results of final iteration.

Ignoring the weight of the cap beams, the sum of the column shears at a pier must equal the total isolator shear force. Hence, approximate column shear (per column) = 12.68(3.71)/3 = 15.68 k which is less than the maximum allowable (25k) if elastic behavior is to be achieved (as required in Step A3).

Also the superstructure displacement = 3.98 in, which is more than the available clearance of 2.0 in. It is therefore apparent that the clearance should be increased to 4.0 in, in which case the above solution is acceptable and go to Step B2.

Note that if the available clearance is increased to 4.0 in, it will satisfy the minimum requirement given by:

$$= \frac{8\ S_{D1} T_{eff}}{B_L} = \frac{8(0.40)1.68}{1.65} = 3.26 in$$

Table B1.12-1 Simplified Method Solution for Design Example 1.1 – Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	650.52	107.16	757.68	0.4	6							
Step B1.1	d	3.98	Assumed displacement									
	Q_d	48.79	Characteristic strength									
	K_d	16.26	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	44.95	3.37	1.12	10000	1.97E-04	1.97	3.98	1.97	7.84E-04	7.84	13.417	31.220
Pier 1	280.31	21.02	7.01	172.0	7.37E-02	11.81	3.71	12.68	2.73E-01	47.00	77.946	187.100
Pier 2	280.31	21.02	7.01	172.0	7.37E-02	11.81	3.71	12.68	2.73E-01	47.00	77.946	187.100
Abut 2	44.95	3.37	1.12	10000	1.97E-04	1.97	3.98	1.97	7.84E-04	7.84	13.417	31.220
Total	650.52	48.789	16.260		$\Sigma K_{eff,j}$	27.554				109.686	182.726	436.639
					Step	**B1.4**						
Step B1.10	T_{eff}	1.68	Effective period									
	ξ	0.27	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.65										
	B_L	1.65	Damping Factor									
	d	3.98	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	0.562	0.187	0.328		3.81	0.335						
Pier 1	3.504	1.168	2.113		3.41	2.195						
Pier 2	3.504	1.168	2.113		3.41	2.195						
Abut 2	0.562	0.187	0.328		3.81	0.335						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength
Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \qquad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \qquad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 1.1
Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-1) by the number of isolators at each support (n = 6), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,\,1}$ = 3.37/6 = 0.56 k
- $Q_{d,\,2}$ = 21.02/6= 3.50 k
- $Q_{d,\,3}$ = 21.02/6= 3.50 k
- $Q_{d,\,4}$ = 3.37/6 = 0.56 k

and

- $K_{d,1}$ = 1.12/6 = 0.19 k/in
- $K_{d,2}$ = 7.01/6 = 1.17 k/in
- $K_{d,3}$ = 7.01/6 = 1.17 k/in
- $K_{d,4}$ = 1.12/6 = 0.19 k/in

B2.2 Initial Stiffness and Yield Displacement
Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:
(1) For friction-based isolators: $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \qquad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \qquad \text{(B-22)}$$

B2.2 Initial Stiffness and Yield Displacement, Example 1.1
Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(1.17) = 11.7\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{3.50}{(11.7 - 1.17)} = 0.33\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~4 in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i} = 0$.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$
Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

$$k_{isol,i} = \frac{K_{isol,j}}{n} \qquad \text{(B -23)}$$

B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 1.1
Dividing the results for K_{isol} (Step B1.12) among the 6 isolators at each support, the following values for K_{isol} /isolator are obtained:

- $K_{isol,1}$ = 1.97/6 = 0.33 k/in
- $K_{isol,2}$ = 12.68/6 = 2.11 k/in
- $K_{isol,3}$ = 12.68/6 = 2.11 k/in

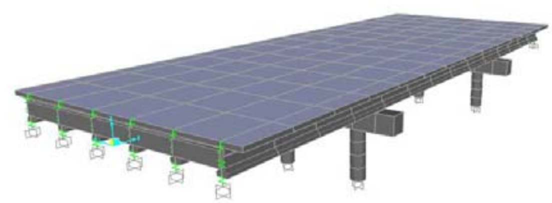
- $K_{isol,4} = 1.97/6 = 0.33$ k/in

B2.4 Three-Dimensional Bridge Model

Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is justified.

B2.4 Three-Dimensional Bridge Model, Example 1.1

Although the bridge in this Design Example is regular and is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below.



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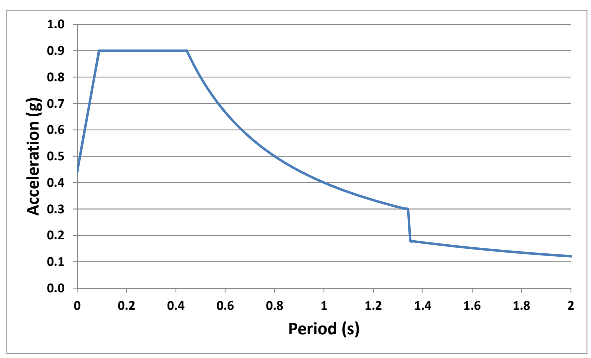
B2.5 Composite Design Response Spectrum

Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.

B.2.5 Composite Design Response Spectrum, Example 1.1

From the final results of Simplified Method (Step B1.12), $B_L = 1.65$ and $T_{eff} = 1.68$ sec. Hence the transition in the composite spectrum from 5% to 27% damping occurs at $0.8\ T_{eff} = 0.8\ (1.68) = 1.34$ sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods ≥ 1.34 sec by 1.65.



B2.6 Multimodal Analysis of Finite Element Model

Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

B2.6 Multimodal Analysis of Finite Element Model, Example 1.1

Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first two modes are the principal longitudinal and transverse modes with periods of 1.55 and 1.49 sec respectively.

Table B2.6-1 Modal Properties of Bridge Example 1.1 – First Iteration

Mode	Period	Modal Participating Mass Ratios					
No.	Sec	UX	UY	UZ	RX	RY	RZ
1	1.550	0.756	0.000	0.000	0.000	0.002	0.000
2	1.492	0.000	0.737	0.000	0.031	0.000	0.529
3	1.467	0.000	0.000	0.000	0.000	0.000	0.210
4	0.189	0.000	0.000	0.000	0.000	0.000	0.000
5	0.189	0.130	0.000	0.000	0.000	0.000	0.000
6	0.121	0.000	0.000	0.000	0.000	0.000	0.000
7	0.121	0.000	0.000	0.000	0.000	0.000	0.007
8	0.104	0.000	0.036	0.000	0.180	0.000	0.026
9	0.101	0.000	0.000	0.237	0.000	0.184	0.000
10	0.095	0.000	0.120	0.000	0.083	0.000	0.086
11	0.094	0.000	0.000	0.014	0.000	0.011	0.000
12	0.074	0.000	0.002	0.000	0.000	0.000	0.001

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 3.73 (3.98) in
- $d_{isol,2}$ = 3.36 (3.71) in
- $d_{isol,3}$ = 3.36 (3.71) in
- $d_{isol,4}$ = 3.36 (3.98) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 1.1

The new superstructure displacement is 3.73 in, more than a 5% difference from the displacement assumed at the start of the Multimode Spectral Analysis (3.98 in).

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

$$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \qquad \text{(B-24)}$$

Recalculate $K_{eff,j}$:

Eq. 7.1-6 GSID

$$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \qquad \text{(B-25)}$$

Recalculate system damping ratio, ξ :

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \qquad \text{(B-26)}$$

Recalculate system damping factor, B_L:

Eq. 7.1-3

$$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \qquad \text{(B-27)}$$

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 1.1

Updated values for $K_{isol,i}$ are given below (previous values are in parentheses):

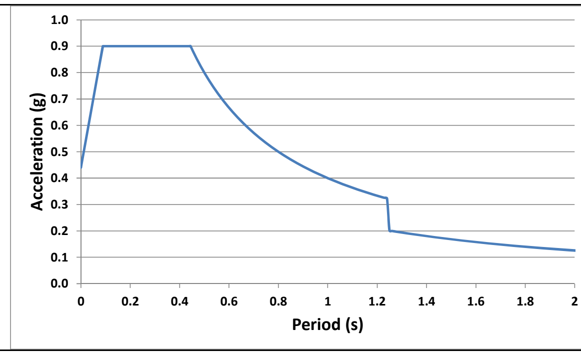
- $K_{isol,1}$ = 0.34 (0.33) k/in
- $K_{isol,2}$ = 2.21 (2.11) k/in
- $K_{isol,3}$ = 2.21 (2.11) k/in
- $K_{isol,4}$ = 0.34 (0.33) k/in

Updated values for $K_{eff,j}$, ξ, B_L and T_{eff} are given below (previous values are in parentheses):

- $K_{eff,1}$ = 2.03 (1.97) k/in
- $K_{eff,2}$ = 12.64 (12.68) k/in
- $K_{eff,3}$ = 12.64 (12.68) k/in
- $K_{eff,4}$ = 2.03 (1.97) k/in

- ξ = 23% (27%)
- B_L = 1.59 (1.65)
- T_{eff} = 1.55 (1.68) sec

The updated composite response spectrum is shown below:

GSID Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.	
	B2.6 Multimodal Analysis Second Iteration, Example 1.1 Reanalysis gives the following values for the isolator displacements (numbers in parentheses are from the previous cycle): o $d_{isol,1}$ = 3.81 (3.73) in o $d_{isol,2}$ = 3.41 (3.36) in o $d_{isol,3}$ = 3.41 (3.36) in o $d_{isol,4}$ = 3.81 (3.73) in o
B2.7 Convergence Check Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.	**B2.7 Convergence Check, Example 1.1** The new superstructure displacement is 3.81 in, a 2% difference from the displacement assumed at the start of the second cycle of Multimode Spectral Analysis (3.73 in).
B2.9 Superstructure and Isolator Displacements Once convergence has been reached, obtain o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.	**B2.9 Superstructure and Isolator Displacements, Example 1.1** From the above analysis: o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are: x_L= 3.81 in y_L= 0.0 in o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are: o Abutments: u_L = 3.81 in, v_L = 0.00 in o Piers: u_L = 3.41 in, v_L = 0.00 in All isolators at same support have the same displacements.
B2.10 Pier Bending Moments and Shear Forces Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.	**B2.10 Pier Bending Moments and Shear Forces, Example 1.1** Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are: Exterior Columns: o M_{PLL}= 0 kft o M_{PTL}= 360 kft o V_{PLL}= 22.91 k

- V_{PTL}= 0.02 k

Interior Columns:

- M_{PLL}= 0 kft
- M_{PTL}= 354 kft
- V_{PLL}= 22.08 k
- V_{PTL}= 0.00 k

Both piers have the same distribution of bending moments and shear forces among the columns.

B2.11 Isolator Shear and Axial Forces

Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 1.1

Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abutment	1	1.29	0.00	0.60
	2	1.29	0.00	0.61
	3	1.29	0.00	0.61
	4	1.29	0.00	0.61
	5	1.29	0.00	0.61
	6	1.29	0.00	0.60
Pier	1	7.53	0.00	0.20
	2	7.54	0.00	0.24
	3	7.55	0.00	0.25
	4	7.55	0.00	0.25
	5	7.54	0.00	0.24
	6	7.53	0.00	0.20

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake
Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 1.1
Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.56 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows: x_T = 0 and y_T = 3.53 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions are as follows:
  Abutments u_T = 0.00 in, v_T = 3.53 in
  Piers u_T = 0.00 in, v_T = 3.45 in
- Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are:
  Exterior Columns:
    - M_{PLT}= 153 kft
    - M_{PTT}= 0 kft
    - V_{PLT}= 0.06 k
    - V_{PTT}= 20.829 k

  Interior Columns:
    - M_{PLT}= 172 kft
    - M_{PTT}= 0 kft
    - V_{PLT}= 0.00 k
    - V_{PTT}= 24.68 k

  Both piers have the same distribution of bending moments and shear forces among the columns.

- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abutment	1	0.00	1.22	4.37
	2	0.00	1.22	2.41
	3	0.00	1.22	0.83
	4	0.00	1.22	0.83
	5	0.00	1.22	2.41
	6	0.00	1.22	4.37
Pier	1	0.02	7.50	15.56
	2	0.01	7.52	1.79
	3	0.00	7.53	2.72
	4	0.00	7.53	2.72
	5	0.01	7.52	1.79
	6	0.02	7.50	15.56

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements
Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)
- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)
- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements at Pier 1, Example 1.1
To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

Load Case 1:
$u_1 = u_L + 0.3u_T = 1.0(3.41) + 0.3(0.01) = 3.41$ in
$v_1 = v_L + 0.3v_T = 1.0(0) + 0.3(3.44) = 1.03$ in
$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{3.41^2 + 1.03^2} = 3.56$ in

Load Case 2:
$u_2 = 0.3u_L + u_T = 0.3(3.41) + 1.0(0.01) = 1.03$ in
$v_2 = 0.3v_L + v_T = 0.3(0) + 1.0(3.44) = 3.44$ in
$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{1.03^2 + 3.44^2} = 3.59$ in

Governing Case:
Total design displacement, $d_t = \max(R_1, R_2)$
$= 3.59$ in

D2. Design Moments and Shears
Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

D2. Design Moments and Shears in Pier 1, Example 1.1
Design moments and shear forces are calculated for Pier 1, Column 1, below to illustrate the process.

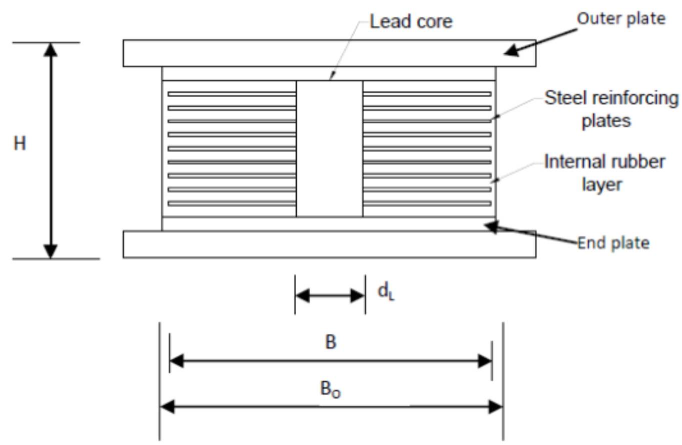
Load Case 1:
$V_{PL1} = V_{PLL} + 0.3V_{PLT} = 1.0(22.91) + 0.3(0.06)$
$= 22.93$ k
$V_{PT1} = V_{PTL} + 0.3V_{PTT} = 1.0(0.02) + 0.3(20.83)$
$= 6.27$ k
$R_1 = \sqrt{V_{PL1}^2 + V_{PT1}^2} = \sqrt{22.93^2 + 6.27^2} = 23.77$ k

Load Case 2:
$V_{PL2} = 0.3V_{PLL} + V_{PLT} = 0.3(22.91) + 1.0(0.06)$
$= 6.93$ k
$V_{PT2} = 0.3V_{PTL} + V_{PTT} = 0.3(0.02) + 1.0(20.83)$
$= 20.84$ k
$R_2 = \sqrt{V_{PL2}^2 + V_{PT2}^2} = \sqrt{6.93^2 + 20.84^2} = 21.96$ k

Governing Case:
Design column shear $= \max(R1, R2)$
$= 23.77$ k

STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS

A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness (K_u).With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness K_d is given by the rubber alone. More details are given in MCEER 2006.



While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary. When sizing the physical dimensions of the bearing, plan dimensions (B, d_L) should be rounded up to the next $^1/_4$" increment, while the total thickness of elastomer, T_r, is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are $^1/_4$" and $^3/_8$". High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in MCEER 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See MCEER 2006.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1). • required characteristic strength, Q_d / isolator • required post-elastic stiffness, K_d / isolator • total design displacement, d_t, for each isolator • maximum applied dead and live load (P_{DL}, P_{LL}) and seismic load (P_{SL}) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator, and • maximum wind load, P_{WL}		**E1. Required Properties, Example 1.1** The design of one of the exterior isolators on a pier is given below to illustrate the design process for lead-rubber isolators. From previous work • Q_d / isolator = 3.50 k • K_d / isolator = 1.17 k/in • Total design displacement, d_t = 3.59 in • P_{DL} = 45.52 k • P_{LL} = 15.50 k • P_{SL} = 15.56 k • P_{WL} = 1.76 k < Q_d OK
E2. Isolator Sizing		
E2.1 Lead Core Diameter Determine the required diameter of the lead plug, d_L, using: $$d_L = \sqrt{\frac{Q_d}{0.9}} \quad \text{(E-1)}$$ See Step E2.5 for limitations on d_L		**E2.1 Lead Core Diameter, Example 1.1** $$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{3.50}{0.9}} = 1.97\ in$$

E2.2 Plan Area and Isolator Diameter Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.0 ksi. Then the bonded area of the isolator is given by: $$A_b = \frac{P_{DL} + P_{LL}}{1.0}\ in^2 \qquad \text{(E-2)}$$ and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by: $$B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} \qquad \text{(E-3)}$$ Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using $$A_b = \frac{\pi}{4}(B^2 - d_L^2) \qquad \text{(E-4)}$$ Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, B_o is given by: $$B_o = B + 1.0 \qquad \text{(E-5)}$$		**E2.2 Plan Area and Isolator Diameter, Example 1.1** $$A_b = \frac{P_{DL} + P_{LL}}{1.0}\ in^2 = \frac{45.52 + 15.50}{1.0} = 61.02\ in^2$$ $$B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} = \sqrt{\frac{4\ (61.02)}{\pi} + 1.97^2}$$ = 9.03 in Round B up to 9.25 in and the actual bonded area is: $$A_b = \frac{\pi}{4}(9.25^2 - 1.97^2) = 64.15\ in^2$$ B_o = 9.25 + 2(0.5) = 10.25 in
E2.3 Elastomer Thickness and Number of Layers Since the shear stiffness of the elastomeric bearing is given by: $$K_d = \frac{GA_b}{T_r} \qquad \text{(E-6)}$$ where G = shear modulus of the rubber, and T_r = the total thickness of elastomer, it follows Eq. E-5 may be used to obtain T_r given a required value for K_d $$T_r = \frac{GA_b}{K_d} \qquad \text{(E-7)}$$ A typical range for shear modulus, G, is 60-120 psi. Higher and lower values are available and are used in special applications. If the layer thickness is t_r, the number of layers, n, is given by: $$n = \frac{T_r}{t_r} \qquad \text{(E-8)}$$ rounded up to the nearest integer.		**E2.3 Elastomer Thickness and Number of Layers, Example 1.1** Select G, shear modulus of rubber, = 100 psi (0.1 ksi) Then $$T_r = \frac{GA_b}{K_d} = \frac{0.1(64.15)}{1.17} = 5.49\ in$$ $$n = \frac{5.49}{0.25} = 21.97$$ Round to nearest integer, i.e. n = 22

Note that because of rounding the plan dimensions and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be necessary if the differences are large.	
E2.4 Overall Height The overall height of the isolator, H, is given by: $$H = n\,t_r + (n-1)t_s + 2t_c \qquad \text{(E-9)}$$ where t_s = thickness of an internal shim (usually about 1/8 in), and t_c = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in)	**E2.4 Overall Height, Example 1.1** $$H = 22(0.25) + 21(0.125) + 2*1.5 = 11.125\ in$$
E2.5 Size Checks Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows: $$\frac{B}{3} \geq d_L \geq \frac{B}{6} \qquad \text{(E-10)}$$ Art. 12.2 GSID requires that the isolation system provides a lateral restoring force at d_t greater than the restoring force at $0.5d_t$ by not less than $W/80$. This equates to a minimum K_d of $0.025W/d$. $$K_{d,min} = \frac{0.025W}{d}$$	**E2.5 Size Checks, Example 1.1** Since B=9.0 check $$\frac{9.25}{3} \geq d_L \geq \frac{9.25}{6}$$ i.e., $3.08 \geq d_L \geq 1.54$ Since d_L = 1.97, lead core size is acceptable. $$K_{d,min} = \frac{0.025W}{d} = \frac{0.025(45.52)}{3.96} = 0.29\ k/in$$ As $$K_d = \frac{GA_b}{T_r} = \frac{0.1(64.15)}{5.5} = 1.17k/in > K_{d,min}$$
E3. Strain Limit Check Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e., $$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5 \qquad \text{(E-11)}$$ where γ_c, $\gamma_{s,eq}$, and γ_r are defined below. (a) γ_c is the maximum shear strain in the layer due to compression and is given by: $$\gamma_c = \frac{D_c\sigma_S}{GS} \qquad \text{(E-12)}$$ where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = {P_{DL}}/{A_b}$, G is shear modulus, and S is the layer shape factor given by: $$S = \frac{A_b}{\pi B t_r} \qquad \text{(E-13)}$$ (b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by:	**E3. Strain Limit Check, Example 1.1** Since $$\sigma_S = \frac{45.52}{64.15} = 0.710\ ksi$$ G = 0.1 ksi and $$S = \frac{64.15}{\pi 9.25(0.25)} = 8.83$$ then $$\gamma_c = \frac{1.0(0.710)}{0.1(8.83)} = 0.804$$

$$\gamma_{s,eq} = \frac{d_t}{T_r} \quad \text{(E-14)}$$ (c) γ_r is the shear strain due to rotation and is given by: $$\gamma_r = \frac{D_r B^2 \theta}{t_r T_r} \quad \text{(E-15)}$$ where *Dr* is shape coefficient for rotation in circular bearings = 0.375, and θ is design rotation due to DL, LL and construction effects Actual value for θ may not be known at this time and value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1)	$$\gamma_{s,eq} = \frac{3.59}{5.5} = 0.653$$ $$\gamma_r = \frac{0.375(9.25^2)(0.01)}{0.25(5.5)} = 0.233$$ Substitution in Eq E-11 gives $$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.804 + 0.653 + 0.5(0.233) = 1.57 \leq 5.5\ OK$$
E4. Vertical Load Stability Check Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state. Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either 2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or 1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4.	**E4. Vertical Load Stability Check, Example 1.1**
E4.1 Vertical Load Stability in Undeformed State The critical load capacity of an elastomeric isolator at zero shear displacement is given by $$P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2}\left[\sqrt{(1+\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2})} - 1\right] \quad \text{(E-16)}$$ where $H_{eff} = T_r + T_s$ T_s = total shim thickness $K_\theta = {E_b I}/{T_r}$ $E_b = E(1 + 0.67S^2)$ E = elastic modulus of elastomer = $3G$ $I = {\pi B^4}/{64}$ It is noted that typical elastomeric isolators have high shape factors, *S*, in which case: $$\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1 \quad \text{(E-17)}$$ and Eq. E-16 reduces to: $$P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta} \quad \text{(E-18)}$$	**E4.1 Vertical Load Stability in Undeformed State, Example 1.1** $$E = 3G = 3(0.1) = 0.3\ ksi$$ $$E_b = 0.3(1 + 0.67(8.83^2)) = 15.97\ ksi$$ $$I = \pi\frac{9.25^4}{64} = 359.37\ in^4$$ $$K_\theta = \frac{15.97(359.37)}{5.5} = 1043.68\ kin/rad$$ $$K_d = \frac{GA_b}{T_r} = \frac{0.1(64.15)}{5.5} = 1.17\ k/in$$ $$P_{cr(\Delta=0)} = \pi\sqrt{1.17(1043.68)} = 109.61\ k$$

Check that: $$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} \geq 3 \qquad \text{(E-19)}$$	$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \frac{109.61}{(45.52+15.5)} = 1.80 \ngeq 3 \quad NOK$$
E4.2 Vertical Load Stability in Deformed State The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by: $$P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)} \qquad \text{(E-20)}$$ where A_r = overlap area between top and bottom plates of isolator at displacement Δ (Fig. 2.2-1 GSID) = $B^2(\delta - sin\delta)/4$ δ = $2cos^{-1}(\Delta/B)$ A_{gross} = $\pi B^2/4$ It follows that: $$\frac{A_r}{A_{gross}} = \frac{(\delta - sin\delta)}{\pi} \qquad \text{(E-21)}$$ Check that: $$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} \geq 1 \qquad \text{(E-22)}$$	**E4.2 Vertical Load Stability in Deformed State, Example 1.1** Since bridge is in Zone 3, Δ= $1.5d_t = 1.5(3.59) = 5.38$ $$\delta = 2cos^{-1}\left(\frac{5.38}{9.25}\right) = 1.90$$ $$\frac{A_r}{A_{gross}} = \frac{(1.90 - sin1.90)}{\pi} = 0.303$$ $$P_{cr(\Delta)} = 0.303(109.61) = 33.24\ k$$ $$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} = \frac{33.24}{1.2(45.52)+15.56} = 0.47 \ngeq 1 \quad NOK$$
E5. Design Review	**E5. Design Review, Example 1.1** The basic dimensions of the isolator designed above are as follows: 10.25 in (od) x 11.125 in (high) x 1.97 in dia. lead core and the volume, excluding steel end and cover plates, is 670 in^3. This design satisfies the shear strain limit criteria, but not the vertical load stability ratio in the undeformed and deformed states. A redesign is therefore required and the easiest way to increase the P_{cr} is to increase the shape factor, *S*, since the bending stiffness of an isolator is a function of the shape factor squared. See equations in Step E4.1. To increase *S*, increase the bonded area A_b while keeping t_r constant (Eq. E-13). But to keep K_d constant while increasing A_b and T_r is constant, decrease the shear modulus, *G* (Eq. E-6). This redesign is outlined below. After repeating the calculation for diameter of lead core, the process begins

by reducing the shear modulus to 60 psi (0.06 ksi) and increasing the bonded diameter to 12.25 in.

E2.1

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{3.50}{0.9}} = 1.97\ in$$

E2.2

$$A_b = \frac{\pi}{4}(12.25^2 - 1.97^2) = 114.81\ in^2$$

$B_o = 12.25 + 2(0.5) = 13.25$ in

E2.3

$$T_r = \frac{GA_b}{K_d} = \frac{0.06(114.81)}{1.17} = 5.90\ in$$

$$n = \frac{5.90}{0.25} = 23.60$$

Round to nearest integer, i.e. $n = 24$.

E2.4

$$H = 24(0.25) + 23(0.125) + 2 * 1.5 = 11.875\ in$$

E2.5

Since B=12 check

$$\frac{12.25}{3} \geq d_L \geq \frac{12.25}{6}$$

i.e., $4.08 \geq d_L \geq 2.04$

Since d_L = 1.97, the size of lead core is too small, and there are 2 options: (1) Accept the undersize and check for adequate performance during the Quality Control Tests required by GSID Art. 15.2.2; or (2) Only have lead cores in every second isolator, in which case the core diameter, in those isolators with cores, will be √2 x 1.97 = 2.79 in (which satisfies above criterion).

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(114.81)}{6.0} = 1.15k/in\ > K_{d,min}$$

E3.

$$\sigma_S = \frac{45.52}{114.81} = 0.396\ ksi$$

$$S = \frac{114.81}{\pi 12.25(0.25)} = 11.93$$

$$\gamma_c = \frac{1.0(0.396)}{0.06(11.93)} = 0.554$$

$$\gamma_{s,eq} = \frac{3.59}{6.00} = 0.598$$

$$\gamma_r = \frac{0.375(12.25^2)(0.01)}{0.25(6)} = 0.375$$

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.554 + 0.598 + 0.5(0.375)$$
$$= 1.34 \le 5.5\ OK$$

E4.1

$$E = 3G = 3(0.06) = 0.18\ ksi$$

$$E_b = 0.18(1 + 0.67(11.93^2)) = 17.35\ ksi$$

$$I = \pi \frac{12.25^4}{64} = 1105.39\ in^4$$

$$K_\theta = \frac{17.35(1105.39)}{6} = 3197.07\ kin/rad$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(114.81)}{6} = 1.148\ k/in$$

$$P_{cr(\Delta=0)} = \pi\sqrt{1.148(3197.07)} = 190.33\ k$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} = \frac{190.33}{(45.52 + 15.50)} = 3.12 \ge 3\quad OK$$

E4.2

$$\delta = 2cos^{-1}\left(\frac{5.38}{12.25}\right) = 2.23$$

$$\frac{A_r}{A_{gross}} = \frac{(2.23 - sin2.23)}{\pi} = 0.459$$

$$P_{cr(\Delta)} = 0.459(190.33) = 87.37\ k$$

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{87.37}{1.2(45.52) + 15.56} = 1.24 \ge 1\ \ OK$$

E5.

The basic dimensions of the redesigned isolator are as follows:

13.25 in (od) x 11.875 in (high) x 1.97 in dia. lead core and its volume (excluding steel end and cover plates) is 1224 in^3.

This design meets all the design criteria but is about 80% larger by volume than the previous design. This increase in size is driven by the need to satisfy the vertical load stability ratio of 3.0 in the undeformed state.

E6. Minimum and Maximum Performance Check

Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E6-1.

Table E6-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-23)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-24)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-25)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-26)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E6-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})\,(\lambda_{min,a,Kd})\,(\lambda_{min,v,Kd})\,(\lambda_{min,tr,Kd})\,(\lambda_{min,c,Kd})\,(\lambda_{min,scrag,Kd})$	(E-27)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})\,(\lambda_{max,a,Kd})\,(\lambda_{max,v,Kd})\,(\lambda_{max,tr,Kd})\,(\lambda_{max,c,Kd})\,(\lambda_{max,scrag,Kd})$	(E-28)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})\,(\lambda_{min,a,Qd})\,(\lambda_{min,v,Qd})\,(\lambda_{min,tr,Qd})\,(\lambda_{min,c,Qd})\,(\lambda_{min,scrag,Qd})$	(E-29)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})\,(\lambda_{max,a,Qd})\,(\lambda_{max,v,Qd})\,(\lambda_{max,tr,Qd})\,(\lambda_{max,c,Qd})\,(\lambda_{max,scrag,Qd})$	(E-30)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

E6. Minimum and Maximum Performance Check, Example 1.1

Minimum Property Modification factors are:

$\lambda_{min,Kd} = 1.0$
$\lambda_{min,Qd} = 1.0$

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are:

$\lambda_{max,a,Kd} = 1.1$
$\lambda_{max,a,Qd} = 1.1$

$\lambda_{max,t,Kd} = 1.1$
$\lambda_{max,t,Qd} = 1.4$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,a,Qd} = 1.0 + 0.1(0.66) = 1.066$

$\lambda_{max,t,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,t,Qd} = 1.0 + 0.4(0.66) = 1.264$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Therefore the maximum overall modification factors

$\lambda_{max,Kd} = 1.066(1.066)1.0 = 1.14$
$\lambda_{max,Qd} = 1.066(1.264)1.0 = 1.35$

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:

$Q_{d,max} = 1.35\ (3.50) = 4.73$ k
and
$K_{d,max} = 1.14(1.15) = 1.31$ k/in

1.00 for critical bridges
0.75 for essential bridges
0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

E7. Design and Performance Summary

E7.1 Isolator dimensions

Summarize final dimensions of isolators:

- Overall diameter (includes cover layer)
- Overall height
- Diameter lead core
- Bonded diameter
- Number of rubber layers
- Thickness of rubber layers
- Total rubber thickness
- Thickness of steel shims
- Shear modulus of elastomer

Check all dimensions with manufacturer.

E7.2 Bridge Performance

Summarize bridge performance

- Maximum superstructure displacement (longitudinal)
- Maximum superstructure displacement (transverse)
- Maximum superstructure displacement (resultant)
- Maximum column shear (resultant)
- Maximum column moment (about transverse axis)
- Maximum column moment (about longitudinal axis)
- Maximum column torque

E7. Design and Performance Summary, Example 1.1

E7.1 Isolator dimensions, Example 1.1

Isolator dimensions are summarized in Table E7.1-1.

Table E7.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Diam. lead core (in)
Under edge girder on Pier 1	17.25 x 17.25 x 11.875 (H)	13.25 dia. x 10.375(H)	1.97

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in)	Total rubber thick-ness (in)	Steel shim thick-ness (in)
Under edge girder on Pier 1	24	0.25	6	0.125

Shear modulus of elastomer = 60 psi

E7.2 Bridge Performance, Example 1.1

Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 25.55 k. This is slightly more than the column plastic shear strength (25 k), but is sufficiently close as to allow the column to remain ‘essentially’ elastic. Furthermore the maximum longitudinal displacement is 3.81 in which is less than the 4.0 in available at the abutment expansion joints and is therefore acceptable.

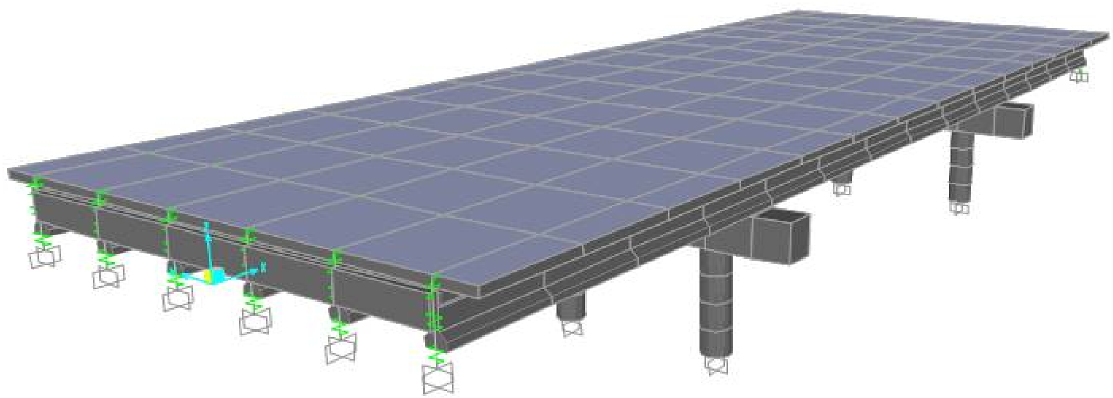
Table E7.2-1 Summary of Bridge Performance

Check required performance as determined in Step A3, is satisfied.		Maximum superstructure displacement (longitudinal)	3.81 in
		Maximum superstructure displacement (transverse)	3.53 in
		Maximum superstructure displacement (resultant)	3.96 in
		Maximum column shear (resultant)	25.55 k
		Maximum column moment about transverse axis	362 kft
		Maximum column moment about longitudinal axis	172 kft
		Maximum column torque	2.83 kft

SECTION 1

PC Girder Bridge, Short Spans, Multi-Column Piers

DESIGN EXAMPLE 1.2: S_1 = 0.6g



Design Examples in Section 1

ID	Description	S_1	Site Class	Pier height	Skew	Isolator type
1.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
1.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
1.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
1.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
1.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
1.5	Change column height	0.2g	B	H_1=0.5 H_2	0	Lead rubber bearing
1.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

DESIGN PROCEDURE		DESIGN EXAMPLE 1.2 (S_1 = 0.6g)

STEP A: BRIDGE AND SITE DATA

A1. Bridge Properties

Determine properties of the bridge:

- number of supports, m
- number of girders per support, n
- angle of skew
- weight of superstructure including railings, curbs, barriers and to the permanent loads, W_{SS}
- weight of piers participating with superstructure in dynamic response, W_{PP}
- weight of superstructure, W_j, at each support
- stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
- column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
- allowable movement at expansion joints
- isolator type if known, otherwise to be determined

A1. Bridge Properties, Example 1.2

- Number of supports, m = 4
  - North Abutment (m = 1)
  - Pier 1 (m = 2)
  - Pier 2 (m = 3)
  - South Abutment (m =4)
- Number of girders per support, n = 6
- Number of columns per support = 3
- Angle of skew = 0^0
- Weight of superstructure including permanent loads, W_{SS} = 650.52 k
- Weight of superstructure at each support:
  - W_1 = 44.95 k
  - W_2 = 280.31 k
  - W_3 = 280.31 k
  - W_4 = 44.95 k
- Participating weight of piers, W_{PP} = 107.16 k
- Effective weight (for calculation of period), $W_{eff} = W_{SS} + W_{PP}$ = 757.68 k
- Stiffness of each pier in the longitudinal direction:
  - $K_{sub,pier1,long}$ = 172.0 k/in
  - $K_{sub,pier2,long}$ = 172.0 k/in
- Stiffness of each pier in the transverse direction:
  - $K_{sub,pier1,trans}$ = 687.0 k/in
  - $K_{sub,pier2,trans}$ = 687.0 k/in
- Minimum column shear strength based on flexural yield capacity of column = 25 k
- Displacement capacity of expansion joints (longitudinal) = 2.0 in for thermal and other movements.
- Lead rubber isolators

A2. Seismic Hazard

Determine seismic hazard at site:

- acceleration coefficients
- site class and site factors
- seismic zone

Plot response spectrum.

Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of PGA, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD).

Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for

A2. Seismic Hazard, Example 1.2

Acceleration coefficients for bridge site are given in design statement as follows:

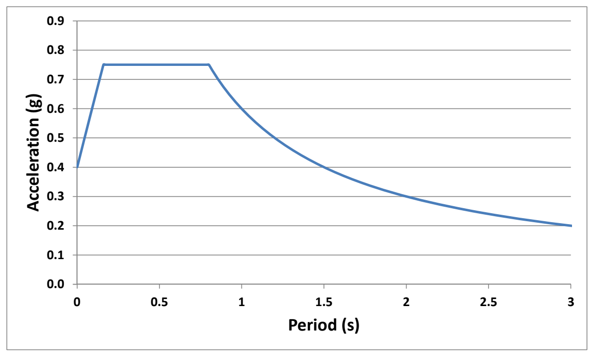
- PGA = 0.40
- S_1 = 0.60
- S_S = 0.75

Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec.

Table 3.10.3.1-1 LRFD gives Site Class as B.

Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors:

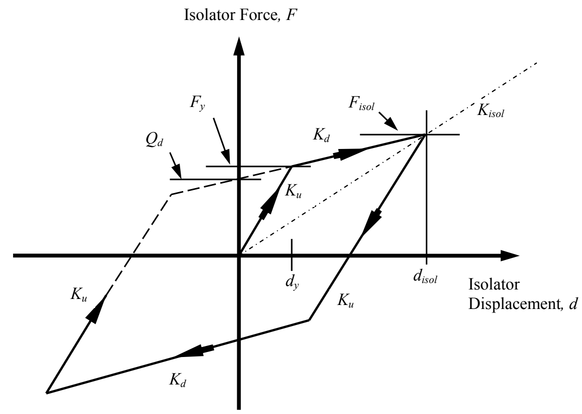
- F_{pga} = 1.0
- F_a = 1.0
- F_v = 1.0

conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD. Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows: • $A_s = F_{pga}\, PGA$ • $S_{DS} = F_a\, S_S$ • $S_{D1} = F_v\, S_1$ Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table5-1 GSID. These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.		• $A_s = F_{pga}\, PGA = 1.0(0.40) = 0.40$ • $S_{DS} = F_a\, S_S = 1.0(0.75) = 0.75$ • $S_{D1} = F_v\, S_1 = 1.0(0.60) = 0.60$ Since $0.50 < S_{D1}$, bridge is located in Seismic Zone 4. Design Response Spectrum is as below: 
A3. Performance Requirements Determine required performance of isolated bridge during Design Earthquake (1000-yr return period). Examples of performance that might be specified by the Owner include: • Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake. • Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake. • For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements) • Reduced substructure forces for bridges on weak soils to reduce foundation costs.		**A3. Performance Requirements, Example 1.2** In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield). To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (25 k). The maximum shear in the column must therefore be less than 25 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties

To begin the iterative solution, an estimate is required of :

(1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)

Art C7.1 GSID

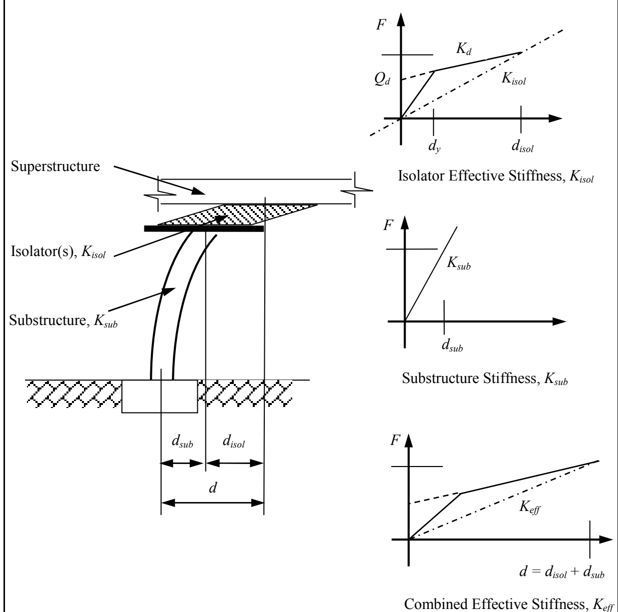
$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \qquad \text{(B-1)}$$

(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not occur under non-seismic loads (e.g. wind)

B1.1 Initial System Displacement and Properties, Example 1.2

$$d \cong 10 S_{D1} = 10(0.60) \cong 6.0\ in$$

but low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e. $Q_d = 0.05W$ (B-2) (3) Post-yield stiffness, K_d Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3. Art. 12.2 GSID $K_{d,min} \geq \frac{0.025W}{d}$ (B-3) Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$		Due to larger estimated displacements (Eq B-1) than for the benchmark bridge, Q_d is increased to 7.5% of the bridge weight to introduce additional damping and reduce these displacements as much as possible, i.e., $Q_d = 0.075W = 0.075(650.52) = 49.79\ k$ Also, in view of these larger displacements, the post yield stiffness is increased to $0.1W/d$, to give essentially the same value for K_d found to be satisfactory in Example 1.0. $K_d = 0.1\frac{W}{d} = 0.1\frac{650.52}{4.0} = 16.26\ k/in$
B1.2 Initial Isolator Properties at Supports Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support: $Q_{d,j} = Q_d\frac{W_j}{W}$ (B-4) and $K_{d,j} = K_d\frac{W_j}{W}$ (B-5)		**B1.2 Initial Isolator Properties at Supports, Example 1.2** $Q_{d,j} = Q_d\frac{W_j}{W}$ o $Q_{d,1} = 3.37$ k o $Q_{d,2} = 21.02$ k o $Q_{d,3} = 21.02$ k o $Q_{d,4} = 3.37$ k and $K_{d,j} = K_d\frac{W_j}{W}$ o $K_{d,1} = 1.12$ k/in o $K_{d,2} = 7.01$ k/in o $K_{d,3} = 7.01$ k/in o $K_{d,4} = 1.12$ k/in
B1.3 Effective Stiffness of Combined Pier and Isolator System Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID). An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula as follows (MCEER 2006): $K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$ (B-6) where $\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$ (B-7) and $K_{sub,j}$ for the piers are given in Step A1. For the abutments, take $K_{sub,j}$ to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high		**B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 1.2** $\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$ o $\alpha_1 = 1.69\text{x}10^{-4}$ o $\alpha_2 = 6.24\text{x}10^{-2}$ o $\alpha_3 = 6.24\text{x}10^{-2}$ o $\alpha_4 = 1.69\text{x}10^{-4}$ $K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$ o $K_{eff,1} = 1.69$ k/in o $K_{eff,2} = 10.10$ k/in o $K_{eff,3} = 10.10$ k/in o $K_{eff,4} = 1.69$ k/in

values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.  		
B1.4 Total Effective Stiffness Calculate the total effective stiffness, K_{eff}, of the bridge: Eq. 7.1-6 GSID $$K_{eff} = \sum_{j=1}^{m} K_{eff,j} \quad \text{(B-8)}$$		**B1.4 Total Effective Stiffness, Example 1.2** $$K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 23.57\ k/in$$
B1.5 Isolation System Displacement at Each Support Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports: $$d_{isol,j} = \frac{d}{1+\alpha_j} \quad \text{(B-9)}$$		**B1.5 Isolation System Displacement at Each Support, Example 1.2** $$d_{isol,j} = \frac{d}{1+\alpha_j}$$ o $d_{isol,1}$ = 6.00 in o $d_{isol,2}$ = 5.65 in o $d_{isol,3}$ = 5.65 in o $d_{isol,4}$ = 6.00 in
B1.6 Isolation System Stiffness at Each Support Calculate the effective stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports: $$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \quad \text{(B-10)}$$		**B1.6 Isolation System Stiffness at Each Support, Example 1.2** $$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$ o $K_{isol,1}$ = 1.69 k/in o $K_{isol,2}$ = 10.73 k/in o $K_{isol,3}$ = 10.73 k/in o $K_{isol,4}$ = 1.69 k/in

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B1.7 Substructure Displacement at Each Support
Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports:

$$d_{sub,j} = d - d_{isol,j} \qquad \text{(B-11)}$$

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B1.7 Substructure Displacement at Each Support, Example 1.2

$$d_{sub,j} = d - d_{isol,j}$$

- $d_{sub,1}$ = 1.01x10^{-3} in
- $d_{sub,2}$ = 3.52x10^{-1} in
- $d_{sub,3}$ = 3.52x10^{-1} in
- $d_{sub,4}$ = 1.01x10^{-3} in

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B1.8 Lateral Load in Each Substructure Support
Calculate the shear at support 'j', $F_{sub,j}$, for all supports:

$$F_{sub,j} = K_{sub,j} d_{sub,j} \qquad \text{(B-12)}$$

where values for $K_{sub,j}$ are given in Step A1.

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B1.8 Lateral Load in Each Substructure, Example 1.2

$$F_{sub,j} = K_{sub,j} d_{sub,j}$$

- $F_{sub,1}$ = 10.11 k
- $F_{sub,2}$ = 60.59 k
- $F_{sub,3}$ = 60.59 k
- $F_{sub,4}$ = 10.11 k

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B1.9 Column Shear Force at Each Support
Calculate the shear in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j':

$$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j} \qquad \text{(B-13)}$$

Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.

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B1.9 Column Shear Force at Each Support, Example 1.2

$$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$$

- $F_{col,2,1\text{-}3}$ ≈ 20.20 k
- $F_{col,3,1\text{-}3}$ ≈ 20.20 k

These column shears are less than the yield capacity of each column (25 k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.

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B1.10 Effective Period and Damping Ratio
Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge:

Eq. 7.1-5 GSID

$$T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} \qquad \text{(B-14)}$$

and

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} \qquad \text{(B-15)}$$

where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j}$ = 0. For other types of isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for the Simplified Method, set $d_{y,j}$ = 0 for all isolator

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B1.10 Effective Period and Damping Ratio, Example 1.2

$$T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{757.68}{386.4(23.57)}}$$

= 1.81 sec

and taking $d_{y,j}$ = 0:

$$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - 0)}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.21$$

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<tr><td>types. See Step B2.2 where the value of $d_{y,j}$ is revisited for the Multimode Spectral Analysis Method.</td><td></td><td></td></tr>
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B1.11 Damping Factor

Calculate the damping factor, B_L, and the displacement, d, of the bridge:

Eq. 7.1-3 GSID
$$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases} \quad \text{(B-16)}$$

Eq. 7.1-4 GSID
$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \quad \text{(B-17)}$$

</td><td></td><td>

B1.11 Damping Factor, Example 1.2

Since $\xi = 0.21 < 0.3$

$$B_L = 1.53$$

and

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.6)1.81}{1.53} = 6.94\ in$$

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B1.12 Convergence Check

Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement.

This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5).

After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both.

Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)

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B1.12 Convergence Check, Example 1.2

Since the calculated value for displacement, d (=6.94 in) is not close to that assumed at the beginning of the cycle (Step B1.1, d = 6.0), use the value of 6.94 in as the new assumed displacement and repeat from Step B1.3.

After three iterations, convergence is reached at a superstructure displacement of 7.44 in, with an effective period of 1.87 seconds, and a damping factor of 1.47 (18% damping ratio). The displacement in the isolators at Pier 1 is 7.03 in and the effective stiffness of the same isolators is 10.00 k/in.

See spreadsheet in Table B1.12-1 for results of final iteration.

Ignoring the weight of the cap beams, the sum of the column shears at a pier must equal the total isolator shear force. Hence, approximate column shear (per column) = 10(7.03)/3 = 23.43 k which is less than the maximum allowable (25k) if elastic behavior is to be achieved (as required in Step A3).

The superstructure displacement = 7.44 in, which is much greater than the available clearance of 2.0 in. There are two options; (1) increase the available clearance to allow for this displacement, or (2) accept that abutment pounding is likely to occur. However, the minimum required clearance is given by:

$$= \frac{8\ S_{D1} T_{eff}}{B_L} = \frac{8(0.60)1.87}{1.47} = 6.11 in$$

Therefore, the available clearance needs to be increased to above this minimum value, say 8.0 in.

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Table B1.12-1 Simplified Method Solution for Design Example 1.2 – Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	650.52	107.16	757.68	0.6	6							
Step B1.1	d	7.44	Assumed displacement									
	Q_d	48.79	Characteristic strength									
	K_d	16.26	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\, d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	44.95	3.37	1.12	10000	1.58E-04	1.58	7.44	1.58	1.17E-03	11.73	25.085	87.305
Pier 1	280.31	21.02	7.01	172.0	5.81E-02	9.45	7.03	10.00	4.09E-01	70.30	147.869	523.218
Pier 2	280.31	21.02	7.01	172.0	5.81E-02	9.45	7.03	10.00	4.09E-01	70.30	147.869	523.218
Abut 2	44.95	3.37	1.12	10000	1.58E-04	1.58	7.44	1.58	1.17E-03	11.73	25.085	87.305
Total	650.52	48.789	16.260		$\Sigma K_{eff,j}$	22.046				164.070	345.907	1,221.047
					Step	**B1.4**						
Step B1.10	T_{eff}	1.87	Effective period									
	ξ	0.18	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.47										
	B_L	1.47	Damping Factor									
	d	7.49	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	0.562	0.187	0.263		7.05	0.267						
Pier 1	3.504	1.168	1.666		6.45	1.711						
Pier 2	3.504	1.168	1.666		6.45	1.711						
Abut 2	0.562	0.187	0.263		7.05	0.267						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength
Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \tag{B-19}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \tag{B-20}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 1.2
Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-1) by the number of isolators at each support (n = 6), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,1}$ = 3.37/6 = 0.56 k
- $Q_{d,2}$ = 21.02/6= 3.50 k
- $Q_{d,3}$ = 21.02/6= 3.50 k
- $Q_{d,4}$ = 3.37/6 = 0.56 k

and

- $K_{d,1}$ = 1.12/6 = 0.19 k/in
- $K_{d,2}$ = 7.01/6 = 1.17 k/in
- $K_{d,3}$ = 7.01/6 = 1.17 k/in
- $K_{d,4}$ = 1.12/6 = 0.19 k/in

B2.2 Initial Stiffness and Yield Displacement
Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:
(1) For friction-based isolators: $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \tag{B-21}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \tag{B-22}$$

B2.2 Initial Stiffness and Yield Displacement, Example 1.2
Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(1.17) = 11.7\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{3.50}{(11.7 - 1.17)} = 0.33\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2 in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i}$ = 0.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$
Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

$$k_{isol,i} = \frac{K_{isol,j}}{n} \tag{B -23}$$

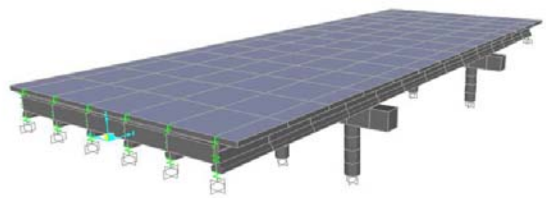
B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 1.2
Dividing the results for K_{isol} (Step B1.12) among the 6 isolators at each support, the following values for K_{isol} /isolator are obtained:

- $K_{isol,1}$ = 1.58/6 = 0.26 k/in
- $K_{isol,2}$ = 10.00/6 = 1.67 k/in
- $K_{isol,3}$ = 10.00/6 = 1.67 k/in

- $K_{isol,4}$ = 1.58/6 = 0.26 k/in

B2.4 Three-Dimensional Bridge Model
Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is justified.

B2.4 Three-Dimensional Bridge Model, Example 1.2
Although the bridge in this Design Example is regular and is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below.



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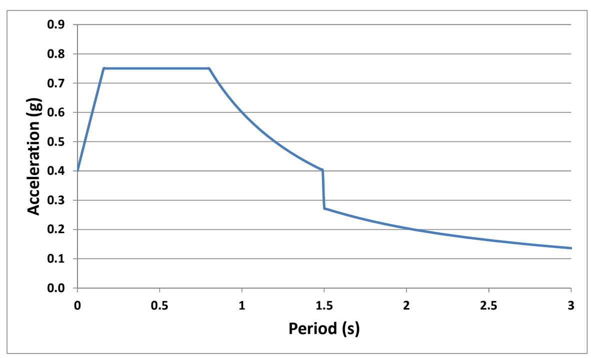
B2.5 Composite Design Response Spectrum
Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.

B.2.5 Composite Design Response Spectrum, Example 1.2
From the final results of Simplified Method (Step B1.12), B_L = 1.47 and T_{eff} = 1.87 sec. Hence the transition in the composite spectrum from 5% to 18% damping occurs at 0.8 T_{eff} = 0.8 (1.87) = 1.50 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods ≥ 1.50 sec by 1.47.



B2.6 Multimodal Analysis of Finite Element Model
Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

B2.6 Multimodal Analysis of Finite Element Model, Example 1.2
Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first two modes are the principal longitudinal and transverse modes with periods of 1.73 and 1.68 sec respectively.

Table B2.6-1 Modal Properties of Bridge Example 1.2 – First Iteration

Mode	Period	Modal Participating Mass Ratios					
No.	Sec	UX	UY	UZ	RX	RY	RZ
1	1.727	0.751	0.000	0.000	0.000	0.002	0.000
2	1.675	0.000	0.736	0.000	0.031	0.000	0.528
3	1.644	0.000	0.000	0.000	0.000	0.000	0.210
4	0.191	0.000	0.000	0.000	0.000	0.000	0.000
5	0.191	0.135	0.000	0.000	0.000	0.000	0.000
6	0.122	0.000	0.000	0.000	0.000	0.000	0.000
7	0.122	0.000	0.000	0.000	0.000	0.000	0.007
8	0.104	0.000	0.038	0.000	0.178	0.000	0.027
9	0.101	0.000	0.000	0.237	0.000	0.184	0.000
10	0.095	0.000	0.120	0.000	0.086	0.000	0.086
11	0.094	0.000	0.000	0.014	0.000	0.011	0.000
12	0.074	0.000	0.002	0.000	0.000	0.000	0.001

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 6.99 (7.44) in
- $d_{isol,2}$ = 6.41 (7.03) in
- $d_{isol,3}$ = 6.41 (7.03) in
- $d_{isol,4}$ = 6.99 (7.44) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 1.2

The new superstructure displacement is 6.99 in, more than a 5% difference from the displacement assumed at the start of the Multimode Spectral Analysis (7.44 in).

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

$$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \qquad \text{(B-24)}$$

Recalculate $K_{eff,j}$:

Eq. 7.1-6 GSID

$$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \qquad \text{(B-25)}$$

Recalculate system damping ratio, ξ :

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \qquad \text{(B-26)}$$

Recalculate system damping factor, B_L:

Eq. 7.1-3

$$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \le 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \qquad \text{(B-27)}$$

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 1.2

Updated values for $K_{isol,i}$ are given below (previous values are in parentheses):

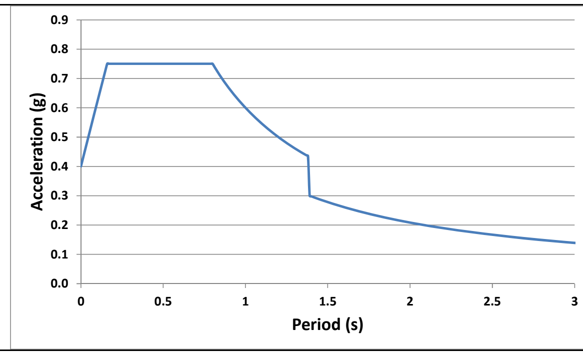
- $K_{isol,1}$ = 0.27 (0.26) k/in
- $K_{isol,2}$ = 1.72 (1.67) k/in
- $K_{isol,3}$ = 1.72 (1.67) k/in
- $K_{isol,4}$ = 0.27 (0.26) k/in

Updated values for $K_{eff,j}$, ξ, B_L and T_{eff} are given below (previous values are in parentheses):

- $K_{eff,1}$ = 1.61 (1.58) k/in
- $K_{eff,2}$ = 10.01 (9.45) k/in
- $K_{eff,3}$ = 10.01 (9.45) k/in
- $K_{eff,4}$ = 1.61 (1.58) k/in

- ξ = 17% (18%)
- B_L = 1.44 (1.47)
- T_{eff} = 1.73 (1.87) sec

The updated composite response spectrum is shown below:

GSID Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.		
		B2.6 Multimodal Analysis Second Iteration, Example 1.2 Reanalysis gives the following values for the isolator displacements (numbers in parentheses are from the previous cycle): o $d_{isol,1}$ = 7.05 (6.99) in o $d_{isol,2}$ = 6.45 (6.41) in o $d_{isol,3}$ = 6.45 (6.41) in o $d_{isol,4}$ = 7.05 (6.99) in
B2.7 Convergence Check Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.		**B2.7 Convergence Check, Example 1.2** The new superstructure displacement is 7.05 in, less than a 1% difference from the displacement assumed at the start of the second cycle of Multimode Spectral Analysis.
B2.9 Superstructure and Isolator Displacements Once convergence has been reached, obtain o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.		**B2.9 Superstructure and Isolator Displacements, Example 1.2** From the above analysis: o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are: x_L= 7.05 in y_L= 0.0 in o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are: o Abutments: u_L = 7.05 in, v_L = 0.00 in o Piers: u_L = 6.45 in, v_L = 0.00 in All isolators at same support have the same displacements.
B2.10 Pier Bending Moments and Shear Forces Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.		**B2.10 Pier Bending Moments and Shear Forces, Example 1.2** Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are: Exterior Columns: o M_{PLL}= 0 kft o M_{PTL}= 443 kft

- V_{PLL}= 27.03 k
- V_{PTL}= 0.02 k

Interior Columns:

- M_{PLL}= 0 kft
- M_{PTL}= 436 kft
- V_{PLL}= 26.05 k
- V_{PTL}= 0.00 k

Both piers have the same distribution of bending moments and shear forces among the columns.

B2.11 Isolator Shear and Axial Forces

Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 1.2

Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abutment	1	1.89	0.00	0.85
	2	1.89	0.00	0.84
	3	1.89	0.00	0.84
	4	1.89	0.00	0.84
	5	1.89	0.00	0.84
	6	1.89	0.00	0.85
Pier	1	11.05	0.00	0.27
	2	11.06	0.00	0.30
	3	11.06	0.00	0.28
	4	11.06	0.00	0.28
	5	11.06	0.00	0.30
	6	11.05	0.00	0.27

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 1.2

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.66 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows: x_T = 0 and y_T = 6.54 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions are as follows:
  - Abutments u_T = 0.00 in, v_T = 6.54 in
  - Piers u_T = 0.00 in, v_T = 6.42 in
- Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are:

  Exterior Columns:
  - M_{PLT}= 173 kft
  - M_{PTT}= 0 kft
  - V_{PLT}= 0.03 k
  - V_{PTT}= 22.99 k

  Interior Columns:
  - M_{PLT}= 198 kft
  - M_{PTT}= 0 kft
  - V_{PLT}= 0.00 k
  - V_{PTT}= 28.09 k

  Both piers have the same distribution of bending moments and shear forces among the columns.

- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abutment	1	0.00	1.79	3.48
	2	0.00	1.79	1.92
	3	0.00	1.79	0.61
	4	0.00	1.79	0.61
	5	0.00	1.79	1.92
	6	0.00	1.79	3.48
Pier	1	0.01	11.00	11.52
	2	0.01	11.03	2.16
	3	0.00	11.04	2.31
	4	0.00	11.04	2.31
	5	0.01	11.03	2.16
	6	0.01	11.00	11.52

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements

Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)
- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)
- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements at Pier 1, Example 1.2

To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

<u>Load Case 1</u>:

$u_1 = u_L + 0.3u_T$ = 1.0(6.45) + 0.3(0) = 6.45 in

$v_1 = v_L + 0.3v_T$ = 1.0(0) + 0.3(6.42) = 1.93 in

$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{6.45^2 + 1.93^2}$ = 6.73 in

<u>Load Case 2</u>:

$u_2 = 0.3u_L + u_T$ = 0.3(6.45) + 1.0(0) = 1.94 in

$v_2 = 0.3v_L + v_T$ = 0.3(0) + 1.0(6.42) = 6.42 in

$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{1.94^2 + 6.42^2}$ = 6.71 in

<u>Governing Case</u>:

Total design displacement, $d_t = \max(R_1, R_2)$
= 6.71 in

D2. Design Moments and Shears

Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

D2. Design Moments and Shears in Pier 1, Example 1.2

Design moments and shear forces are calculated for Pier 1, Column 1, below to illustrate the process.

<u>Load Case 1</u>:

$V_{PL1} = V_{PLL} + 0.3V_{PLT}$ = 1.0(27.03) + 0.3(0.03)
= 27.04 k

$V_{PT1} = V_{PTL} + 0.3V_{PTT}$ = 1.0(0.02) + 0.3(22.99)
= 6.92 k

$R_1 = \sqrt{V_{PL1}^2 + V_{PT1}^2} = \sqrt{27.04^2 + 6.92^2}$ = 27.91 k

<u>Load Case 2</u>:

$V_{PL2} = 0.3V_{PLL} + V_{PLT}$ = 0.3(27.03) + 1.0(0.03)
= 8.14 k

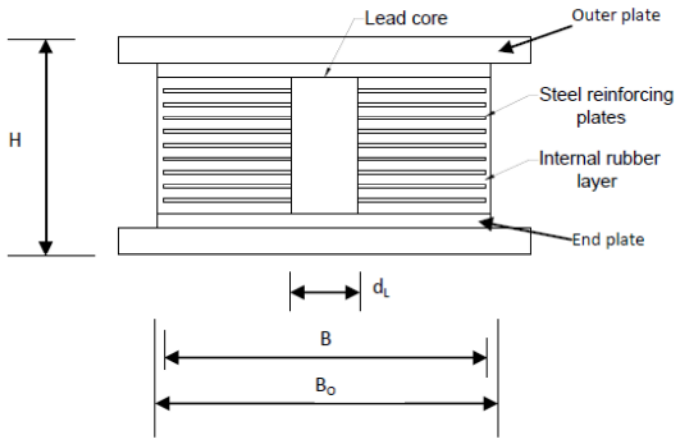
$V_{PT2} = 0.3V_{PTL} + V_{PTT}$ = 0.3(0.02) + 1.0(22.99)
= 23.00 k

$R_2 = \sqrt{V_{PL2}^2 + V_{PT2}^2} = \sqrt{8.14^2 + 23.00^2}$ = 24.40 k

<u>Governing Case</u>:

Design column shear = max ($R1$, $R2$)
= 27.91 k

STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS



A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness (K_u).With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness K_d is given by the rubber alone. More details are given in MCEER 2006.

While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary. When sizing the physical dimensions of the bearing, plan dimensions (B, d_L) should be rounded up to the next $^1/_4$" increment, while the total thickness of elastomer, T_r, is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are $^1/_4$" and $^3/_8$". High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in MCEER 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See MCEER 2006.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1). • required characteristic strength, Q_d / isolator • required post-elastic stiffness, K_d / isolator • total design displacement, d_t, for each isolator • maximum applied dead and live load (P_{DL}, P_{LL}) and seismic load (P_{SL}) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator, and • maximum wind load, P_{WL}		**E1. Required Properties, Example 1.2** The design of one of the exterior isolators on a pier is given below to illustrate the design process for lead-rubber isolators. From previous work • Q_d / isolator = 3.50 k • K_d / isolator = 1.17 k/in • Total design displacement, d_t = 6.71 in • P_{DL} = 45.52 k • P_{LL} = 15.50 k • P_{SL} = 11.52 k • P_{WL} = 1.76 k < Q_d OK
E2. Isolator Sizing		
E2.1 Lead Core Diameter Determine the required diameter of the lead plug, d_L, using: $$d_L = \sqrt{\frac{Q_d}{0.9}} \quad \text{(E-1)}$$ See Step E2.5 for limitations on d_L		**E2.1 Lead Core Diameter, Example 1.2** $$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{3.50}{0.9}} = 1.97\ in$$

E2.2 Plan Area and Isolator Diameter Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.0 ksi. Then the bonded area of the isolator is given by: $A_b = \frac{P_{DL} + P_{LL}}{1.0}\ in^2$ (E-2) and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by: $B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2}$ (E-3) Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using $A_b = \frac{\pi}{4}(B^2 - d_L^2)$ (E-4) Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, B_o is given by: $B_o = B + 1.0$ (E-5)		**E2.2 Plan Area and Isolator Diameter, Example 1.2** $A_b = \frac{P_{DL} + P_{LL}}{1.0}\ in^2 = \frac{45.52 + 15.50}{1.0} = 61.02\ in^2$ $B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} = \sqrt{\frac{4\ (61.02)}{\pi} + 1.61^2}$ = 9.03 in Round B up to 9.25 in and the actual bonded area is: $A_b = \frac{\pi}{4}(9.25^2 - 1.97^2) = 64.15\ in^2$ B_o = 9.25 + 2(0.5) = 10.25 in
E2.3 Elastomer Thickness and Number of Layers Since the shear stiffness of the elastomeric bearing is given by: $K_d = \frac{GA_b}{T_r}$ (E-6) where G = shear modulus of the rubber, and T_r = the total thickness of elastomer, it follows Eq. E-5 may be used to obtain T_r given a required value for K_d $T_r = \frac{GA_b}{K_d}$ (E-7) A typical range for shear modulus, G, is 60-120 psi. Higher and lower values are available and are used in special applications. If the layer thickness is t_r, the number of layers, n, is given by: $n = \frac{T_r}{t_r}$ (E-8) rounded up to the nearest integer. Note that because of rounding the plan dimensions		**E2.3 Elastomer Thickness and Number of Layers, Example 1.2** Select G, shear modulus of rubber, = 100 psi (0.1 ksi) Then $T_r = \frac{GA_b}{K_d} = \frac{0.1(64.15)}{1.17} = 5.49\ in$ $n = \frac{5.49}{0.25} = 21.97$ Round to nearest integer, i.e. $n = 22$

and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be necessary if the differences are large.

E2.4 Overall Height

The overall height of the isolator, H, is given by:

$$H = n\, t_r + (n-1)t_s + 2t_c \qquad \text{(E-9)}$$

where t_s = thickness of an internal shim (usually about 1/8 in), and

t_c = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in)

E2.4 Overall Height, Example 1.2

$$H = 22(0.25) + 21(0.125) + 2 * 1.5 = 11.125\ in$$

E2.5 Size Checks

Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows:

$$\frac{B}{3} \geq d_L \geq \frac{B}{6} \qquad \text{(E-10)}$$

Art. 12.2 GSID requires that the isolation system provides a lateral restoring force at d_t greater than the restoring force at $0.5d_t$ by not less than $W/80$. This equates to a minimum K_d of $0.025W/d$.

$$K_{d,min} = \frac{0.025W}{d}$$

E2.5 Size Checks, Example 1.2

Since B=9.25 check

$$\frac{9.25}{3} \geq d_L \geq \frac{9.25}{6}$$

i.e., $3.08 \geq d_L \geq 1.54$

Since d_L = 1.97, lead core size is acceptable.

$$K_{d,min} = \frac{0.025W}{d} = \frac{0.025(45.52)}{7.32} = 0.16\ k/in$$

As

$$K_d = \frac{GA_b}{T_r} = \frac{0.1(64.15)}{5.5} = 1.17k/in > K_{d,min}$$

E3. Strain Limit Check

Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e.,

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5 \qquad \text{(E-11)}$$

where $\gamma_c,\ \gamma_{s,eq}, and\ \gamma_r$ are defined below.

(a) γ_c is the maximum shear strain in the layer due to compression and is given by:

$$\gamma_c = \frac{D_c \sigma_S}{GS} \qquad \text{(E-12)}$$

where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = {P_{DL}}/{A_b}$, G is shear modulus, and S is the layer shape factor given by:

$$S = \frac{A_b}{\pi B t_r} \qquad \text{(E-13)}$$

(b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by:

E3. Strain Limit Check, Example 1.2

Since

$$\sigma_S = \frac{45.52}{64.15} = 0.710\ ksi$$

$$G = 0.1 \text{ ksi}$$

and

$$S = \frac{64.15}{\pi 9.25(0.25)} = 8.83$$

then

$$\gamma_c = \frac{1.0(0.710)}{0.1(8.83)} = 0.804$$

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$$\gamma_{s,eq} = \frac{d_t}{T_r} \qquad \text{(E-14)}$$

(c) γ_r is the shear strain due to rotation and is given by:

$$\gamma_r = \frac{D_r B^2 \theta}{t_r T_r} \qquad \text{(E-15)}$$

where *Dr* is shape coefficient for rotation in circular bearings = 0.375, and θ is design rotation due to DL, LL and construction effects Actual value for θ may not be known at this time and value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1)

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$$\gamma_{s,eq} = \frac{6.71}{5.5} = 1.222$$

$$\gamma_r = \frac{0.375(9.25^2)(0.01)}{0.25(5.5)} = 0.233$$

Substitution in Eq E-11 gives

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.804 + 1.222 + 0.5(0.233) = 2.14 \leq 5.5\ OK$$

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E4. Vertical Load Stability Check
Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state.

Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either
2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or
1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4.

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E4. Vertical Load Stability Check, Example 1.2

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E4.1 Vertical Load Stability in Undeformed State
The critical load capacity of an elastomeric isolator at zero shear displacement is given by

$$P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2}\left[\sqrt{(1 + \frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2})} - 1\right] \qquad \text{(E-16)}$$

where
$H_{eff} = T_r + T_s$
T_s = total shim thickness
$K_\theta = {E_b I}/{T_r}$
$E_b = E(1 + 0.67S^2)$
E = elastic modulus of elastomer = $3G$
$I = {\pi B^4}/{64}$

It is noted that typical elastomeric isolators have high shape factors, *S*, in which case:

$$\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1 \qquad \text{(E-17)}$$

and Eq. E-16 reduces to:

$$P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta} \qquad \text{(E-18)}$$

Check that:

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E4.1 Vertical Load Stability in Undeformed State, Example 1.2

$$E = 3G = 3(0.1) = 0.3\ ksi$$

$$E_b = 0.3(1 + 0.67(8.83^2)) = 15.97\ ksi$$

$$I = \pi\frac{9.25^4}{64} = 359.37\ in^4$$

$$K_\theta = \frac{15.97(359.37)}{5.5} = 1043.68\ kin/rad$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.1(64.15)}{5.5} = 1.17\ k/in$$

$$P_{cr(\Delta=0)} = \pi\sqrt{1.17(1043.68)} = 109.61\ k$$

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$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} \geq 3 \qquad \text{(E-19)}$$

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$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \frac{109.61}{(45.52+15.5)} = 1.80 \ngeq 3 \quad NOK$$

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E4.2 Vertical Load Stability in Deformed State
The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by:

$$P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)} \qquad \text{(E-20)}$$

where
A_r = overlap area between top and bottom plates of isolator at displacement Δ (Fig. 2.2-1 GSID)
= $B^2(\delta - sin\delta)/4$
δ = $2cos^{-1}(\Delta/B)$

A_{gross} = $\pi B^2/4$

It follows that:

$$\frac{A_r}{A_{gross}} = \frac{(\delta - sin\delta)}{\pi} \qquad \text{(E-21)}$$

Check that:

$$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} \geq 1 \qquad \text{(E-22)}$$

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E4.2 Vertical Load Stability in Deformed State, Example 1.2
Since bridge is in Zone 4, $\Delta = 1.5d_t = 1.5(6.71) = 10.07$ in

Since

$$\Delta > B$$

$$A_r = 0$$

Therefore, clearly the design displacement is too large for this initial design. A redesign should be undertaken with increased isolator dimensions. As a general rule, the minimum isolator diameter, *B*, should be of the order of 1.5Δ to ensure a sufficient overlap area.

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E5. Design Review

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E5. Design Review, Example 1.2
The redesign is outlined below. After repeating the calculation for diameter of lead core, the process begins by reducing the shear modulus to 60 psi (0.06 ksi), increasing the bonded diameter to 1.5Δ = 15.11 in, rounded up to 15.25 in.

E2.1

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{3.50}{0.9}} = 1.97\ in$$

E2.2

$$A_b = \frac{\pi}{4}(15.25^2 - 1.97^2) = 179.61\ in^2$$

B_o = 15.25 + 2(0.5) = 16.25 in

E2.3

$$T_r = \frac{GA_b}{K_d} = \frac{0.06(179.61)}{1.17} = 9.23\ in$$

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$$n = \frac{9.23}{0.25} = 36.91$$

Round to nearest integer, i.e. $n = 37$

E2.4

$$H = 37(0.25) + 36(0.125) + 2 * 1.5 = 16.75\ in$$

E2.5

Since B = 15.25 in, check

$$\frac{15.25}{3} \geq d_L \geq \frac{15.25}{6}$$

i.e., $5.08 \geq d_L \geq 2.54$

Since d_L = 1.97, the size of lead core is too small, and there are 2 options: (1) accept the undersize and check for adequate performance during the Quality Control Tests required by GSID Art. 15.2.2; or (2) only have lead cores in every second isolator, in which case the core diameter, in isolators with cores, will be √2 x 1.97 = 2.79 in (which satisfies above criterion).

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(179.61)}{9.25} = 1.17 k/in > K_{d,min}$$

E3.

$$\sigma_S = \frac{45.52}{179.61} = 0.25\ ksi$$

$$S = \frac{179.61}{\pi 15.25(0.25)} = 15.00$$

$$\gamma_c = \frac{1.0(0.25)}{0.06(15.00)} = 0.282$$

$$\gamma_{s,eq} = \frac{6.71}{9.25} = 0.727$$

$$\gamma_r = \frac{0.375(15.25^2)(0.01)}{0.25(9.25)} = 0.377$$

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.282 + 0.727 + 0.5(0.377) = 1.20 \leq 5.5\ OK$$

E4.1

$$E = 3G = 3(0.06) = 0.18\ ksi$$

$$E_b = 0.18(1 + 0.67(15.00^2)) = 27.30\ ksi$$

$$I = \pi \frac{15.25^4}{64} = 2654.91\ in^4$$

$$K_\theta = \frac{27.30(2654.91)}{9.25} = 7835.21\ kin/rad$$

		$$K_d = \frac{GA_b}{T_r} = \frac{0.06(179.61)}{9.25} = 1.165\ k/in$$ $$P_{cr(\Delta=0)} = \pi\sqrt{1.165(7835.21)} = 300.15\ k$$ $$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \frac{300.15}{(45.52+15.50)} = 4.92 \geq 3\ \ OK$$ **E4.2** $$\delta = 2cos^{-1}\left(\frac{10.07}{15.25}\right) = 1.70$$ $$\frac{A_r}{A_{gross}} = \frac{(1.70 - sin1.70)}{\pi} = 0.224$$ $$P_{cr(\Delta)} = 0.224(300.15) = 67.33\ k$$ $$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} = \frac{67.33}{1.2(45.52)+11.52} = 1.02 \geq 1\ \ OK$$ **E5.** The basic dimensions of the redesigned isolator are as follows: 16.25 in (od) x 16.75 in (high) x 1.97 in dia. lead core and its volume (excluding steel end and cover plates) is 2852 in^3.
E6. Minimum and Maximum Performance Check Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E6-1. **Table E6-1. Minimum and maximum values for K_d and Q_d.**		**E6. Minimum and Maximum Performance Check, Example 1.2** Minimum Property Modification factors are: $\lambda_{min,Kd} = 1.0$ $\lambda_{min,Qd} = 1.0$ which means there is no need to reanalyze the bridge with a set of minimum values. Maximum Property Modification factors are: $\lambda_{max,a,Kd} = 1.1$ $\lambda_{max,a,Qd} = 1.1$ $\lambda_{max,t,Kd} = 1.1$ $\lambda_{max,t,Qd} = 1.4$ $\lambda_{max,scrag,Kd} = 1.0$ $\lambda_{max,scrag,Qd} = 1.0$ Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become: $\lambda_{max,a,Kd} = 1.0 + 0.1(0.66) = 1.066$

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d\,\lambda_{max,Kd}$	(E-23)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d\,\lambda_{min,Kd}$	(E-24)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d\,\lambda_{max,Qd}$	(E-25)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d\,\lambda_{min,Qd}$	(E-26)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E6-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})(\lambda_{min,a,Kd})(\lambda_{min,v,Kd})(\lambda_{min,tr,Kd})(\lambda_{min,c,Kd})(\lambda_{min,scrag,Kd})$	(E-27)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})(\lambda_{max,a,Kd})(\lambda_{max,v,Kd})(\lambda_{max,tr,Kd})(\lambda_{max,c,Kd})(\lambda_{max,scrag,Kd})$	(E-28)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})(\lambda_{min,a,Qd})(\lambda_{min,v,Qd})(\lambda_{min,tr,Qd})(\lambda_{min,c,Qd})(\lambda_{min,scrag,Qd})$	(E-29)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})(\lambda_{max,a,Qd})(\lambda_{max,v,Qd})(\lambda_{max,tr,Qd})(\lambda_{max,c,Qd})(\lambda_{max,scrag,Qd})$	(E-30)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

1.00 for critical bridges
0.75 for essential bridges
0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

$\lambda_{max,a,Qd} = 1.0 + 0.1(0.66) = 1.066$

$\lambda_{max,t,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,t,Qd} = 1.0 + 0.4(0.66) = 1.264$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Therefore the maximum overall modification factors

$\lambda_{max,Kd} = 1.066(1.066)1.0 = 1.14$
$\lambda_{max,Qd} = 1.066(1.264)1.0 = 1.35$

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:
$Q_{d,max} = 1.35\ (3.50) = 4.73$ k
and
$K_{d,max} = 1.14(1.17) = 1.33$ k/in

E7. Design and Performance Summary		E7. Design and Performance Summary, Example 1.2
E7.1 Isolator dimensions Summarize final dimensions of isolators: • Overall diameter (includes cover layer) • Overall height • Diameter lead core • Bonded diameter • Number of rubber layers • Thickness of rubber layers • Total rubber thickness • Thickness of steel shims • Shear modulus of elastomer Check all dimensions with manufacturer.		**E7.1 Isolator dimensions, Example 1.2** Isolator dimensions are summarized in Table E7.1-1. **Table E7.1-1 Isolator Dimensions** (see tables below) Shear modulus of elastomer = 60 psi
E7.2 Bridge Performance Summarize bridge performance • Maximum superstructure displacement (longitudinal) • Maximum superstructure displacement (transverse) • Maximum superstructure displacement (resultant) • Maximum column shear (resultant) • Maximum column moment (about transverse axis) • Maximum column moment (about longitudinal axis) • Maximum column torque Check required performance as determined in Step A3, is satisfied.		**E7.2 Bridge Performance, Example 1.2** Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 29.15 k. This is more than the column plastic shear (25 k) and therefore the required performance criterion is not satisfied (fully elastic behavior). Clearly the seismic demand (S_1 = 0.6g) is too high and the column too small for isolation to give fully elastic response. The next step might be to conduct a pushover analysis of the pier to determine if the ductility demand at 7.32 in is acceptable. If not, and this is an existing bridge, jacket the column (as well as isolate the bridge). If a new design, increase the size of the column and thereby increase its strength. It is noted that the maximum longitudinal displacement (7.05 in) exceeds the available clearance, and as noted in Section B1.12, this gap needs to be increased to say 8.0 in to meet the requirements for isolation. An alternative is to accept pounding at the abutments. The consequential damage is not likely to be life-threatening and easily repaired.

Table E7.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Diam. lead core (in)
Under edge girder on Pier 1	20.25 x 20.25 x 16.75(H)	16.25 dia. x 15.25(H)	1.97

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in)	Total rubber thick-ness (in)	Steel shim thick-ness (in)
Under edge girder on Pier 1	37	0.25	9.25	0.125

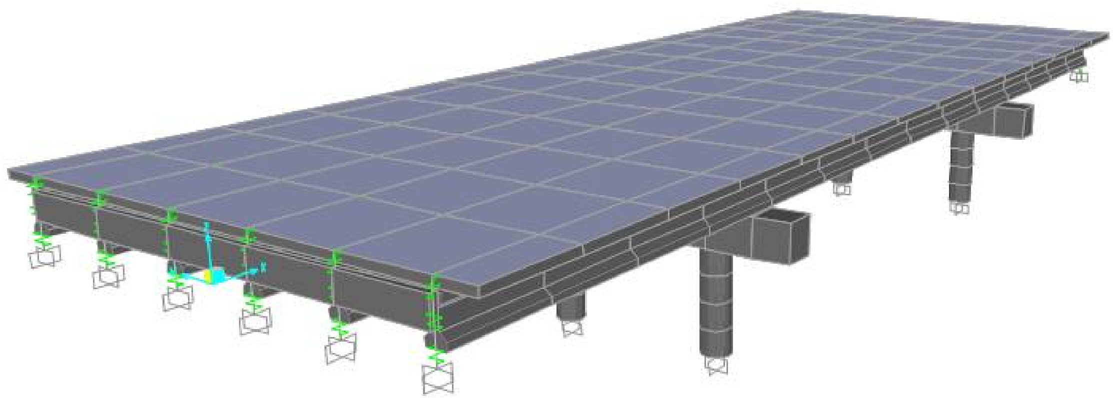
Table E7.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	7.05 in
Maximum superstructure displacement (transverse)	6.54 in
Maximum superstructure displacement (resultant)	7.32 in
Maximum column shear (resultant)	29.15 k
Maximum column moment about transverse axis	444 kft
Maximum column moment about longitudinal axis	199 kft
Maximum column torque	3.21 kft

SECTION 1

PC Girder Bridge, Short Spans, Multi-Column Piers

DESIGN EXAMPLE 1.3: Spherical Friction Isolators



Design Examples in Section 1

ID	Description	S_1	Site Class	Pier height	Skew	Isolator type
1.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
1.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
1.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
1.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
1.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
1.5	Change column height	0.2g	B	H_1=0.5 H_2	0	Lead rubber bearing
1.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

<table>
<tr><td>DESIGN PROCEDURE</td><td></td><td>DESIGN EXAMPLE 1.3 (Spherical Friction Isolator)</td></tr>
<tr><td colspan="3">STEP A: BRIDGE AND SITE DATA</td></tr>
<tr><td>A1. Bridge Properties
Determine properties of the bridge:
• number of supports, m
• number of girders per support, n
• angle of skew
• weight of superstructure including railings, curbs, barriers and to the permanent loads, W_{SS}
• weight of piers participating with superstructure in dynamic response, W_{PP}
• weight of superstructure, W_j, at each support
• piers heights (clear dimensions)
• stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge
• column flexural yield strength (minimum value)
• column shear strength (minimum value)
• allowable movement at expansion joints
• isolator type if known, otherwise to be determined</td><td></td><td>A1. Bridge Properties, Example 1.3
• Number of supports, m = 4
  o North Abutment (m = 1)
  o Pier 1 (m = 2)
  o Pier 2 (m = 3)
  o South Abutment (m =4)
• Number of girders per support, n = 6
• Number of columns per support = 3
• Angle of skew = 0^0
• Weight of superstructure including permanent loads, W_{SS} = 650.52 k
• Weight of superstructure at each support:
  o W_1 = 44.95 k
  o W_2 = 280.31 k
  o W_3 = 280.31 k
  o W_4 = 44.95 k
• Participating weight of piers, W_{PP} = 107.16 k
• Effective weight (for calculation of period), $W_{eff} = W_{SS} + W_{PP}$ = 757.68 k
• Stiffness of each pier in the longitudinal direction:
  o $K_{sub,pier1,long}$ = 172.0 k/in
  o $K_{sub,pier2,long}$ = 172.0 k/in
• Stiffness of each pier in the transverse direction:
  o $K_{sub,pier1,trans}$ = 687.0 k/in
  o $K_{sub,pier2,trans}$ = 687.0 k/in
• Minimum flexural yield strength of one column = 425 kft (plastic moment capacity)
• Displacement capacity of expansion joints (longitudinal) = 2.0 in for thermal and other movements.
• Spherical friction bearing isolators</td></tr>
<tr><td>A2. Seismic Hazard
Determine seismic hazard at site:
• acceleration coefficients
• site class and site factors
• seismic zone
Plot response spectrum.

Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of PGA, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD).

Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for</td><td></td><td>A2. Seismic Hazard, Example 1.3
Acceleration coefficients for bridge site are given in design statement as follows:
• PGA = 0.40
• S_1 = 0.20
• S_S = 0.75

Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec.

Table 3.10.3.1-1 LRFD gives Site Class as B.

Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors:
• F_{pga} = 1.0
• F_a = 1.0
• F_v = 1.0</td></tr>
</table>

conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD.

Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows:

- $A_s = F_{pga}\, PGA$
- $S_{DS} = F_a\, S_S$
- $S_{D1} = F_v\, S_1$

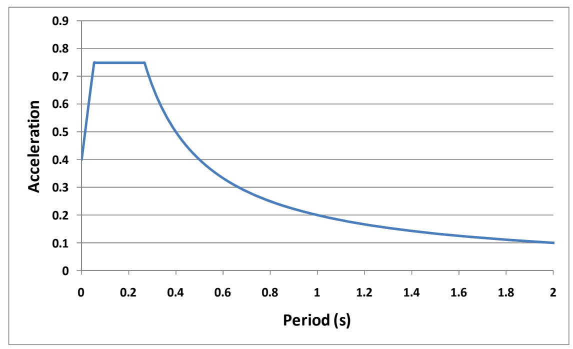
Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table5-1 GSID.

These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.

- $A_s = F_{pga}\, PGA = 1.0(0.40) = 0.40$
- $S_{DS} = F_a\, S_S = 1.0(0.75) = 0.75$
- $S_{D1} = F_v\, S_1 = 1.0(0.20) = 0.20$

Since $0.15 < S_{D1} \leq 0.30$, bridge is located in Seismic Zone 2.

Design Response Spectrum is as below:



A3. Performance Requirements
Determine required performance of isolated bridge during Design Earthquake (1000-yr return period).

Examples of performance that might be specified by the Owner include:

- Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake.
- Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake.
- For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements)
- Reduced substructure forces for bridges on weak soils to reduce foundation costs.

A3. Performance Requirements, Example 1.3
In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield).

To remain elastic the maximum lateral load on the pier must be less than the load to yield the column. This load is taken as the plastic moment capacity (strength) of the column (425 kft, see above) divided by the overall column height (17 ft). This calculation assumes the column is acting as a simple cantilever in single curvature in the longitudinal direction.

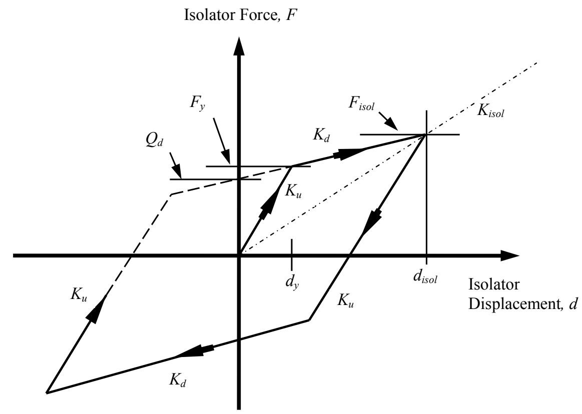
Hence load to yield column = 425 /17 = 25.0 k

The maximum shear in the column must therefore be less than 25 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties

To begin the iterative solution, an estimate is required of :

(1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)

Art C7.1 GSID

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \qquad \text{(B-1)}$$

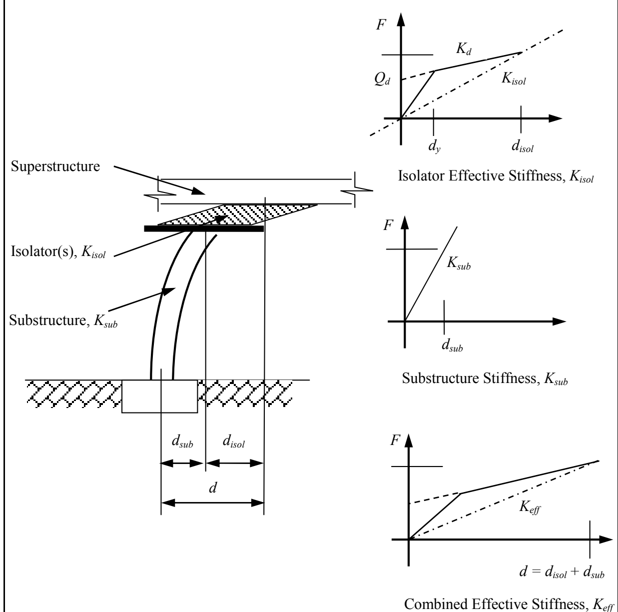
(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not occur under non-seismic loads (e.g. wind) but

B1.1 Initial System Displacement and Properties, Example 1.3

$$d \cong 10 S_{D1} = 10(0.20) \cong 2.0\ in$$

low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e. $Q_d = 0.05W$ (B-2) (3) Post-yield stiffness, K_d Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3. Art. 12.2 GSID $K_{d,min} \geq \frac{0.025W}{d}$ (B-3) Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$		$Q_d = 0.05W = 0.05(650.52) = 32.53\ k$ $K_d = 0.05\frac{W}{d} = 0.05\frac{650.52}{2.0} = 16.26\ k/in$
B1.2 Initial Isolator Properties at Supports Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support: $Q_{d,j} = Q_d\frac{W_j}{W}$ (B-4) and $K_{d,j} = K_d\frac{W_j}{W}$ (B-5)		**B1.2 Initial Isolator Properties at Supports, Example 1.3** $Q_{d,j} = Q_d\frac{W_j}{W}$ o $Q_{d,1}$ = 2.25 k o $Q_{d,2}$ = 14.02 k o $Q_{d,3}$ = 14.02 k o $Q_{d,4}$ = 2.25 k and $K_{d,j} = K_d\frac{W_j}{W}$ o $K_{d,1}$ = 1.12 k/in o $K_{d,2}$ = 7.01 k/in o $K_{d,3}$ = 7.01 k/in o $K_{d,4}$ = 1.12 k/in
B1.3 Effective Stiffness of Combined Pier and Isolator System Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID). An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula as follows (MCEER 2006): $K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$ (B-6) where $\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$ (B-7) and $K_{sub,j}$ for the piers are given in Step A1. For the abutments, take $K_{sub,j}$ to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high		**B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 1.3** $\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$ o $\alpha_1 = 2.25\text{x}10^{-4}$ o $\alpha_2 = 8.49\text{x}10^{-2}$ o $\alpha_3 = 8.49\text{x}10^{-2}$ o $\alpha_4 = 2.25\text{x}10^{-4}$ $K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$ o $K_{eff,1}$ = 2.25 k/in o $K_{eff,2}$ = 13.47 k/in o $K_{eff,3}$ = 13.47 k/in o $K_{eff,4}$ = 2.25 k/in

values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.



B1.4 Total Effective Stiffness
Calculate the total effective stiffness, K_{eff}, of the bridge:

Eq. 7.1-6 GSID

$$K_{eff} = \sum_{j=1}^{m} K_{eff,j} \quad \text{(B-8)}$$

B1.4 Total Effective Stiffness, Example 1.3

$$K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 31.43\ k/in$$

B1.5 Isolation System Displacement at Each Support
Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports:

$$d_{isol,j} = \frac{d}{1+\alpha_j} \quad \text{(B-9)}$$

B1.5 Isolation System Displacement at Each Support, Example 1.3

$$d_{isol,j} = \frac{d}{1+\alpha_j}$$

- $d_{isol,1}$ = 2.00 in
- $d_{isol,2}$ = 1.84 in
- $d_{isol,3}$ = 1.84 in
- $d_{isol,4}$ = 2.00 in

B1.6 Isolation System Stiffness at Each Support
Calculate the effective stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports:

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \quad \text{(B-10)}$$

B1.6 Isolation System Stiffness at Each Support, Example 1.3

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$

- $K_{isol,1}$ = 2.25 k/in
- $K_{isol,2}$ = 14.61 k/in
- $K_{isol,3}$ = 14.61 k/in
- $K_{isol,4}$ = 2.25 k/in

B1.7 Substructure Displacement at Each Support Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports: $$d_{sub,j} = d - d_{isol,j} \quad \text{(B-11)}$$		**B1.7 Substructure Displacement at Each Support, Example 1.3** $$d_{sub,j} = d - d_{isol,j}$$ o $d_{sub,1} = 4.49\text{x}10^{-4}$ in o $d_{sub,2} = 1.57\text{x}10^{-1}$ in o $d_{sub,3} = 1.57\text{x}10^{-1}$ in o $d_{sub,4} = 4.49\text{x}10^{-4}$ in
B1.8 Lateral Load in Each Substructure Support Calculate the shear at support 'j', $F_{sub,j}$, for all supports: $$F_{sub,j} = K_{sub,j} d_{sub,j} \quad \text{(B-12)}$$ where values for $K_{sub,j}$ are given in Step A1.		**B1.8 Lateral Load in Each Substructure, Example 1.3** $$F_{sub,j} = K_{sub,j} d_{sub,j}$$ o $F_{sub,1} =$ 4.49 k o $F_{sub,2} =$ 26.93 k o $F_{sub,3} =$ 26.93 k o $F_{sub,4} =$ 4.49 k
B1.9 Column Shear Force at Each Support Calculate the shear in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j': $$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j} \quad \text{(B-13)}$$ Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.		**B1.9 Column Shear Force at Each Support, Example 1.3** $$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$$ o $F_{col,2,1\text{-}3} \approx 8.98$ k o $F_{col,3,1\text{-}3} \approx 8.98$ k These column shears are less than the yield capacity of each column (25 k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.
B1.10 Effective Period and Damping Ratio Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge: Eq. 7.1-5 GSID $$T_{eff} = 2\pi \sqrt{\frac{W_{eff}}{gK_{eff}}} \quad \text{(B-14)}$$ and Eq. 7.1-10 GSID $$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} \quad \text{(B-15)}$$ where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j} = 0$. For other types of isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for the Simplified Method, set $d_{y,j} = 0$ for all isolator		**B1.10 Effective Period and Damping Ratio, Example 1.3** $$T_{eff} = 2\pi \sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi \sqrt{\frac{757.68}{386.4(31.43)}}$$ = 1.57 sec and taking $d_{y,j} = 0$: $$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - 0)}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.31$$

<table>
<tr><td>types. See Step B2.2 where the value of $d_{y,j}$ is revisited for the Multimode Spectral Analysis Method.</td><td></td><td></td></tr>
<tr><td>B1.11 Damping Factor
Calculate the damping factor, B_L, and the displacement, d, of the bridge:

Eq. 7.1-3 GSID $$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases} \quad \text{(B-16)}$$
Eq. 7.1-4 GSID $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \quad \text{(B-17)}$$</td><td></td><td>B1.11 Damping Factor, Example 1.3
Since $\xi = 0.31 \geq 0.3$
$$B_L = 1.70$$
and
$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.2)1.57}{1.70} = 1.81\ in$$</td></tr>
<tr><td>B1.12 Convergence Check
Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement.

This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5).

After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both.

Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)</td><td></td><td>B1.12 Convergence Check, Example 1.3
Since the calculated value for displacement, d (=1.81 in) is not close to that assumed at the beginning of the cycle (Step B1.1, d = 2.0), use the value of 1.81 in as the new assumed displacement and repeat from Step B1.3.

After one iteration, convergence is reached at a superstructure displacement of 1.76 in, with an effective period of 1.52 seconds, and a damping factor of 1.70 (33% damping ratio). The displacement in the isolators at Pier 1 is 1.61 in and the effective stiffness of the same isolators is 15.69 k/in.

See spreadsheet in Table B1.12-1 for results of final iteration.

Ignoring the weight of the cap beams, the sum of the column shears at a pier must equal the total isolator shear force. Hence, approximate column shear (per column) = 15.69(1.61)/3 = 8.42 k which is less than the maximum allowable (25k) if elastic behavior is to be achieved (as required in Step A3).

Also the superstructure displacement = 1.76 in, which is less than the available clearance of 2.0 in.

Therefore the above solution is acceptable and go to Step B2.

Note that available clearance (2.0 in) is greater than minimum required which is given by:
$$= \frac{8\, S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.52}{1.7} = 1.43 in$$</td></tr>
</table>

Table B1.12-1 Simplified Method Solution for Design Example 1.3 – Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	650.52	107.16	757.68	0.2	6							
Step B1.1	d	1.72	Assumed displacement									
	Q_d	32.53	Characteristic strength									
	K_d	16.26	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j}+d_{sub,j})^2$
Abut 1	44.95	2.25	1.12	10000	2.40E-04	2.40	1.76	2.40	4.23E-04	4.23	3.958	7.444
Pier 1	280.31	14.02	7.01	172.0	9.12E-02	14.38	1.61	15.69	1.47E-01	25.33	22.623	44.611
Pier 2	280.31	14.02	7.01	172.0	9.12E-02	14.38	1.61	15.69	1.47E-01	25.33	22.623	44.611
Abut 2	44.95	2.25	1.12	10000	2.40E-04	2.40	1.76	2.40	4.23E-04	4.23	3.958	7.444
Total	650.52	32.526	16.263		$\Sigma K_{eff,j}$	33.557				59.107	53.161	104.109
					Step	**B1.4**						
Step B1.10	T_{eff}	1.52	Effective period									
	ξ	0.33	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.75										
	B_L	1.70	Damping Factor									
	d	1.75	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	0.375	0.187	0.400		1.66	0.413						
Pier 1	2.336	1.168	2.615		1.47	2.757						
Pier 2	2.336	1.168	2.615		1.47	2.757						
Abut 2	0.375	0.187	0.400		1.66	0.413						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength
Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \quad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \quad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 1.3
Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-1) by the number of isolators at each support (n = 6), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,\,1}$ = 2.25/6 = 0.37 k
- $Q_{d,\,2}$ = 14.02/6= 2.34 k
- $Q_{d,\,3}$ = 14.02/6= 2.34 k
- $Q_{d,\,4}$ = 2.25/6 = 0.37 k

and

- $K_{d,1}$ = 1.12/6 = 0.19 k/in
- $K_{d,2}$ = 7.01/6 = 1.17 k/in
- $K_{d,3}$ = 7.01/6 = 1.17 k/in
- $K_{d,4}$ = 1.12/6 = 0.19 k/in

B2.2 Initial Stiffness and Yield Displacement
Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:
(1) For friction-based isolators: $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \quad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \quad \text{(B-22)}$$

B2.2 Initial Stiffness and Yield Displacement, Example 1.3
Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(1.17) = 11.7\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{2.34}{(11.7 - 1.17)} = 0.22\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2 in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i} = 0$.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$
Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

$$k_{isol,i} = \frac{K_{isol,j}}{n} \quad \text{(B -23)}$$

B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 1.3
Dividing the results for K_{isol} (Step B1.12) among the 6 isolators at each support, the following values for K_{isol} /isolator are obtained:

- $K_{isol,1}$ = 2.40/6 = 0.40 k/in
- $K_{isol,2}$ = 15.69/6 = 2.62 k/in
- $K_{isol,3}$ = 15.69/6 = 2.62 k/in

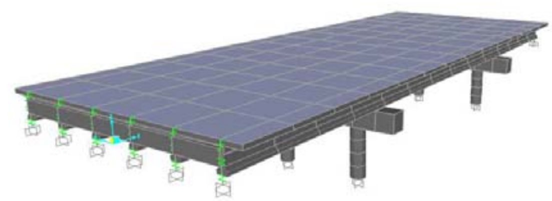
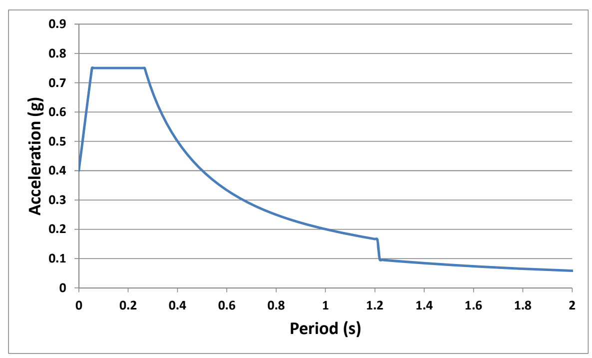
		o $K_{isol,4}$ = 2.40/6 = 0.40 k/in
B2.4 Three-Dimensional Bridge Model Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is justified.		**B2.4 Three-Dimensional Bridge Model, Example 1.3** Although the bridge in this Design Example is regular and is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below.  .
B2.5 Composite Design Response Spectrum Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, $B_{L.}$		**B.2.5 Composite Design Response Spectrum, Example 1.3** From the final results of Simplified Method (Step B1.12), B_L = 1.70 and T_{eff} = 1.52 sec. Hence the transition in the composite spectrum from 5% to 33% damping occurs at 0.8 T_{eff} = 0.8 (1.46) = 1.22 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods $\geq$ 1.22 sec by 1.70. 
B2.6 Multimodal Analysis of Finite Element Model Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.		**B2.6 Multimodal Analysis of Finite Element Model, Example 1.3** Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first two modes are the principal longitudinal and transverse modes with periods of 1.41 and 1.35 sec respectively. The period of the longitudinal mode (1.41 sec) is close to that calculated in the Simplified Method.

Table B2.6-1 Modal Properties of Bridge Example 1.3 – First Iteration

Mode	Period	Modal Participating Mass Ratios					
No.	Sec	UX	UY	UZ	RX	RY	RZ
1	1.410	0.761	0.000	0.000	0.000	0.003	0.000
2	1.346	0.000	0.738	0.031	0.059	0.000	0.534
3	1.325	0.000	0.000	0.000	0.000	0.000	0.217
4	0.187	0.000	0.000	0.000	0.000	0.000	0.000
5	0.186	0.125	0.000	0.000	0.000	0.000	0.000
6	0.121	0.000	0.000	0.000	0.000	0.000	0.000
7	0.121	0.000	0.000	0.000	0.000	0.000	0.006
8	0.104	0.000	0.034	0.183	0.107	0.000	0.064
9	0.101	0.000	0.000	0.000	0.000	0.226	0.000
10	0.095	0.000	0.121	0.081	0.184	0.000	0.041
11	0.094	0.000	0.000	0.000	0.000	0.000	0.000
12	0.074	0.000	0.002	0.000	0.003	0.000	0.000

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 1.65 (1.76) in
- $d_{isol,2}$ = 1.47 (1.61) in
- $d_{isol,3}$ = 1.47 (1.61) in
- $d_{isol,4}$ = 1.65 (1.76) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 1.3

The new superstructure displacement is 1.65 in, more than a 5% difference from the displacement assumed at the start of the Multimode Spectral Analysis.

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

$$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \qquad \text{(B-24)}$$

Recalculate $K_{eff,j}$:

Eq. 7.1-6 GSID

$$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \qquad \text{(B-25)}$$

Recalculate system damping ratio, ξ :

Eq. 7.1-10 GSID

$$\xi = \frac{2 \sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \qquad \text{(B-26)}$$

Recalculate system damping factor, B_L:

Eq. 7.1-3

$$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \qquad \text{(B-27)}$$

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 1.3

Updated values for $K_{isol,i}$ are given below (previous values are in parentheses):

- $K_{isol,1}$ = 0.41 (0.40) k/in
- $K_{isol,2}$ = 2.76 (2.62) k/in
- $K_{isol,3}$ = 2.76 (2.62) k/in
- $K_{isol,4}$ = 0.41 (0.40) k/in

Updated values for $K_{eff,j}$, ξ, B_L and T_{eff} are given below (previous values are in parentheses):

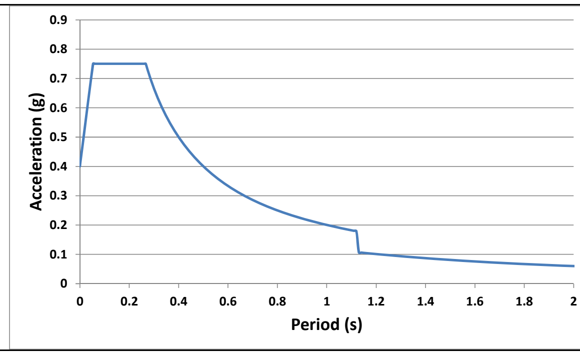
- $K_{eff,1}$ = 2.48 (2.40) k/in
- $K_{eff,2}$ = 15.48 (14.38) k/in
- $K_{eff,3}$ = 15.48 (14.38) k/in
- $K_{eff,4}$ = 2.48 (2.40) k/in

- ξ = 27% (33%)
- B_L = 1.66 (1.70)
- T_{eff} = 1.41 (1.52) sec

The updated composite response spectrum is shown below:

GSID

Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.



B2.6 Multimodal Analysis Second Iteration, Example 1.3

Reanalysis gives the following values for the isolator displacements (numbers in parentheses are from the previous cycle):

- $d_{isol,1}$ = 1.66 (1.65) in
- $d_{isol,2}$ = 1.47 (1.47) in
- $d_{isol,3}$ = 1.47 (1.47) in
- $d_{isol,4}$ = 1.66 (1.65) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (*d*) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 1.3

The new superstructure displacement is 1.66 in, less than a 1% difference from the displacement assumed at the start of the second cycle of Multimode Spectral Analysis.

B2.9 Superstructure and Isolator Displacements

Once convergence has been reached, obtain

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and
- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.

B2.9 Superstructure and Isolator Displacements, Example 1.3

From the above analysis:

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are:

  x_L= 1.66 in
  y_L= 0.0 in

- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are:
  - Abutments: u_L = 1.66 in, v_L = 0.00 in
  - Piers: u_L = 1.47 in, v_L = 0.00 in

  All isolators at same support have the same displacements.

B2.10 Pier Bending Moments and Shear Forces

Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.

B2.10 Pier Bending Moments and Shear Forces, Example 1.3

Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are:

Exterior Columns:

- M_{PLL}= 0 kft
- M_{PTL}= 240 kft

- V_{PLL}= 15.81 k
- V_{PTL}= 0 k

Interior Columns:

- M_{PLL}= 0 kft
- M_{PTL}= 235 kft
- V_{PLL}= 15.24 k
- V_{PTL}= 0 k

Both piers have the same distribution of bending moments and shear forces among the columns.

B2.11 Isolator Shear and Axial Forces

Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 1.3

Isolator shear and axial forces are summarized in Table B2.11-1.

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abutment	1	0.69	0.00	0.36
	2	0.69	0.00	0.39
	3	0.69	0.00	0.39
	4	0.69	0.00	0.39
	5	0.69	0.00	0.39
	6	0.69	0.00	0.36
Pier	1	4.04	0.00	0.13
	2	4.05	0.00	0.19
	3	4.05	0.00	0.22
	4	4.05	0.00	0.22
	5	4.05	0.00	0.19
	6	4.04	0.00	0.13

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 1.3

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.43 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows: x_T = 0 and y_T = 1.53 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions are as follows:
  - Abutments u_T = 0.00 in, v_T = 1.53 in
  - Piers u_T = 0.00 in, v_T = 1.49 in
- Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are:
  - Exterior Columns:
    - M_{PLT}= 108 kft
    - M_{PTT}= 1 kft
    - V_{PLT}= 0.06 k
    - V_{PTT}= 14.87 k
  - Interior Columns:
    - M_{PLT}= 120 kft
    - M_{PTT}= 0 kft
    - V_{PLT}= 0 k
    - V_{PTT}= 17.29 k

  Both piers have the same distribution of bending moments and shear forces among the columns.

- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abutment	1	0.00	0.65	3.50
	2	0.00	0.65	1.93
	3	0.00	0.65	0.68
	4	0.00	0.65	0.68
	5	0.00	0.65	1.93
	6	0.00	0.65	3.50
Pier	1	0.02	3.98	12.56
	2	0.01	4.00	1.14
	3	0.00	4.01	2.29
	4	0.00	4.01	2.29
	5	0.01	4.00	1.14
	6	0.02	3.98	12.56

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements
Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)

- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)

- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements at Pier 1, Example 1.3
To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

<u>Load Case 1</u>:
$u_1 = u_L + 0.3u_T$ = 1.0(1.47) + 0.3(0) = 1.47 in
$v_1 = v_L + 0.3v_T$ = 1.0(0) + 0.3(1.49) = 0.45 in
$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.47^2 + 0.45^2}$ = 1.54 in

<u>Load Case 2</u>:
$u_2 = 0.3u_L + u_T$ = 0.3(1.47) + 1.0(0) = 0.44 in
$v_2 = 0.3v_L + v_T$ = 0.3(0) + 1.0(1.49) = 1.49in
$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{0.44^2 + 1.49^2}$ = 1.55in

<u>Governing Case</u>:
Total design displacement, $d_t = \max(R_1, R_2)$
= 1.55 in

D2. Design Moments and Shears
Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

D2. Design Moments and Shears in Pier 1, Example 1.3
Design moments and shear forces are calculated for Pier 1, Column 1, below to illustrate the process.

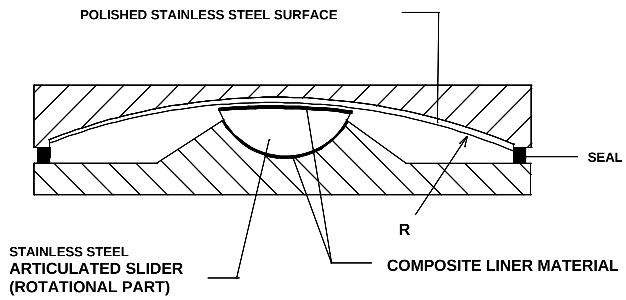
<u>Load Case 1</u>:
$V_{PL1} = V_{PLL} + 0.3V_{PLT}$ = 1.0(15.81) + 0.3(0.06)
= 15.83 k
$V_{PT1} = V_{PTL} + 0.3V_{PTT}$ = 1.0(0.02) + 0.3(14.87)
= 4.48 k
$R_1 = \sqrt{V_{PL1}^2 + V_{PT1}^2} = \sqrt{15.83^2 + 4.48^2}$ = 16.45 k

<u>Load Case 2</u>:
$V_{PL2} = 0.3V_{PLL} + V_{PLT}$ = 0.3(15.81) + 1.0(0.06)
= 4.80 k
$V_{PT2} = 0.3V_{PTL} + V_{PTT}$ = 0.3(0.02) + 1.0(14.87)
= 14.88 k
$R_2 = \sqrt{V_{PL2}^2 + V_{PT2}^2} = \sqrt{4.80^2 + 14.88^2}$ = 15.63 k

<u>Governing Case</u>:
Design column shear = max ($R1$, $R2$)
= 16.45 k

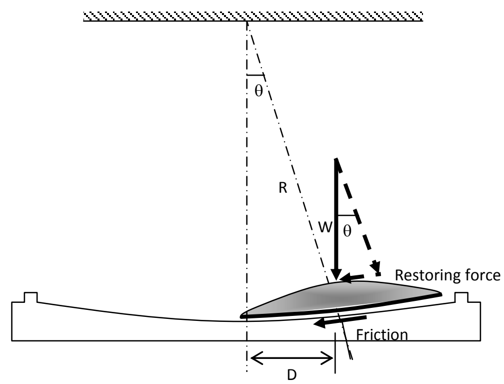
STEP E. DESIGN OF SPHERICAL FRICTION ISOLATORS

The Spherical Friction Bearing (SFB) isolator has an articulated slider to permit rotation, and a spherical sliding interface. It has lateral stiffness due to the curvature of this interface. These isolators are capable of carrying very large axial loads and can be designed to have long periods of vibration (5 seconds or longer).



The main components of an SFB isolator are a stainless steel concave spherical plate, an articulated slider and a housing plate as illustrated in figure above. In this figure, the concave spherical plate is facing down. The bearings may also be installed with this surface facing up as in the figure below. The side of the articulated slider in contact with the concave spherical surface is coated with a low-friction composite material, usually PTFE. The other side of the slider is also spherical but lined with stainless steel and sits in a spherical cavity coated with PTFE.

Spherical friction bearings are described by the same equation of motion as conventional pendulums. As a consequence their period of vibration is directly proportional to the radius of curvature of the concave surface. See figure at right. Long period shifts are therefore possible with surfaces that have large radii of curvature. Friction between the articulated slider and the concave surface dissipates energy and the weight of the bridge acts as a restoring force, due to the curvature of the sliding surface.



The required values for Q_d and K_d determine the coefficient of friction at the sliding interface and the radius of curvature.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties

Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1).

- required characteristic strength, Q_d / isolator
- required post-elastic stiffness, K_d / isolator
- the total design displacement, d_t, for each isolator
- maximum applied dead load, P_{DL}
- maximum live load, P_{LL} and
- maximum wind load, P_{WL}

E1. Required Properties, Example 1.3

The design of one of the pier isolators is given below to illustrate the design process for spherical friction isolators.

From previous work

- Q_d / isolator = 2.34 k
- K_d / isolator = 1.17 k/in
- Total design displacement, d_t = 1.55 in
- P_{DL} = 45.52 k
- P_{LL} = 15.50 k
- P_{WL} = 1.76 k < Q_d OK

E2. Isolator Dimensions

E2.1 Radius of Curvature

Determine the required radius of curvature, R, using:

$$R = \frac{P_{DL}}{K_d} \qquad \text{(E-1)}$$

E2.1 Radius of Curvature, Example 1.3

$$R = \frac{45.52}{1.17} = 38.91 \approx 39.0\ in$$

E2.2 Coefficient of Friction
Determine the required coefficient of friction, μ, using:

$$\mu = \frac{Q_d}{P_{DL}} \qquad \text{(E-2)}$$

E2.2 Coefficient of Friction, Example 1.3

$$\mu = \frac{2.34}{45.52} = 0.0514 = 5.14\%$$

E2.3 Material Selection
Based on the required coefficient of friction select an appropriate PTFE compound and contact pressure, σ_c, from Table E2.3-1.

Table E2.3-1 Material Properties

PTFE Compound (Filled and Unfilled Teflon)	Contact Pressure, σ_c (psi)	μ (%)
Unfilled (UF)	1,000	11.93
	2,000	8.70
	3,000	7.03
	6,500	5.72
Glass-filled 15% by weight (15GF)	1,000	14.61
	2,000	10.08
	3,000	8.49
	6,500	5.27
Glass-filled 25% by weight (25GF)	1,000	13.20
	2,000	11.20
	3,000	9.60
	6,500	5.89

E2.3 Material Selection, Example 1.3
Select 15GF Teflon and size disc to achieve required contact pressure of 6,500 psi (Step E2.4).

E2.4 Disk Diameter
Determine the required contact area, A_c, and disk diameter, d, using:

$$A_c = \frac{P_{DL}}{\sigma_c} \qquad \text{(E-3)}$$

and

$$d_d = \sqrt{\frac{4A_c}{\pi}} \qquad \text{(E-4)}$$

E2.4 Disk Diameter, Example 1.3

$$A_c = \frac{45.52}{6.5} = 7.00\ in^2$$

$$d_d = \sqrt{\frac{4(7.00)}{\pi}} = 2.99 \approx 3.00\ in$$

E2.5 Isolator Diameter
Determine the required diameter of the concave surface, L_{chord}, and overall isolator width, B, using:

$$L_{chord} = 2(\Delta + {}^{d_d}/_{2}) \qquad \text{(E-5)}$$

and

$$B = L_{chord} + 2s \qquad \text{(E-6)}$$

E2.5 Isolator Diameter, Example 1.3
As the bridge is in Seismic Zone 2,

$$\Delta = 2(d_i) = 2(1.55) = 3.10 \text{ in}$$

$$L_{chord} = 2(3.10 + 1.50) = 9.20\ in$$

Select s = 1.5 in:

where: Δ = 2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or 1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4. s = width of shoulder of concave plate.		$B = 9.20 + 2(1.5) = 12.20 \approx 12.25\ in$
E2.6 Isolator Height		**E2.6 Isolator Height, Example 1.3**
E2.6.1 Rise Determine the rise of the concave surface, h, using: $$h = \frac{(L_{chord})^2}{8R} \quad \text{(E-7)}$$		**E2.6.1 Rise, Example 1.3** $$h = \frac{(9.20)^2}{8(38.91)} = 0.27\ in$$
E2.6.2 Throat Thickness Determine the required throat thickness, t, based on the minimum required bearing area, A_b, such that the maximum allowable bearing stress, $\sigma_{bearing}$, is not exceeded on either the sole plate above or the masonry plate below, depending on whether the isolator is installed with concave surface facing up or down. $$A_b = \frac{P_{DL} + P_{LL}}{\sigma_{bearing}} \quad \text{(E-8)}$$ $$d_b = \sqrt{\frac{4A_b}{\pi}} \quad \text{(E-9)}$$ $$t = 0.5(d_b - d_d) \quad \text{(E-10)}$$ This assumes a 45° distribution of compressive stress through the throat to the support plates.		**E2.6.2 Throat Thickness, Example 1.3** Assume safe bearing stress below isolator: $\sigma_{bearing}$ = 2.0 ksi. $$A_b = \frac{45.52 + 15.50}{2.0} = 30.51\ in^2$$ $$d_b = \sqrt{\frac{4(30.51)}{\pi}} = 6.23\ in$$ $$t = 0.5(6.23 - 3.0) = 1.62 \approx 1.75\ in$$
E2.6.3 Total Height Determine the thickness of concave plate, T_1, using: $$T_1 = h + t \quad \text{(E-11)}$$ Thickness of slider plate (T_2) will vary with detail for socket that holds articulated slider and rotation requirement. Check with manufacturer for value. For estimating purposes take $T_2 = T_1$. Then total height of isolator: $$H = T_1 + T_2 \quad \text{(E-12)}$$		**E2.6.3 Total Height, Example 1.3** $$T_1 = 0.27 + 1.75 = 2.02 \approx 2.25\ in$$ $$T_2 = 2.25\ in\ (est)$$ $$H = 2.25 + 2.25 = 4.50\ in\ (est)$$
E3. Design Summary		**E3. Design Summary, Example 1.3** Overall diameter = 12.25 in Overall height = 4.50 in (est.) Radius concave surface = 39.0 in

PTFE is 15% GF; contact pressure = 6,500 psi
Diameter PTFE sliding disc = 3.00 in

E4. Minimum and Maximum Performance Check
Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E4-1.

Table E4-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-13)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-14)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-15)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-16)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E4-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E4-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})(\lambda_{min,a,Kd})(\lambda_{min,v,Kd})(\lambda_{min,tr,Kd})(\lambda_{min,c,Kd})(\lambda_{min,scrag,Kd})$	(E-17)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})(\lambda_{max,a,Kd})(\lambda_{max,v,Kd})(\lambda_{max,tr,Kd})(\lambda_{max,c,Kd})(\lambda_{max,scrag,Kd})$	(E-18)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})(\lambda_{min,a,Qd})(\lambda_{min,v,Qd})(\lambda_{min,tr,Qd})(\lambda_{min,c,Qd})(\lambda_{min,scrag,Qd})$	(E-19)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})(\lambda_{max,a,Qd})(\lambda_{max,v,Qd})(\lambda_{max,tr,Qd})(\lambda_{max,c,Qd})(\lambda_{max,scrag,Qd})$	(E-20)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the

E4. Minimum and Maximum Performance Check, Example 1.3
For a spherical friction isolator, property modification factors are applied to Q_d only.

Minimum Property Modification factors are:
λ_{min} = 1.0

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are (GSID Appendix A):

$\lambda_{max,a}$ = 1.1
$\lambda_{max,c}$ = 1.0
$\lambda_{max,tr}$ = 1.2
$\lambda_{max,t}$ = 1.2

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:
$\lambda_{max,a}$ = 1.0 + 0.1(0.66) = 1.066
$\lambda_{max,c}$ = 1.0

portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows: 1.00 for critical bridges 0.75 for essential bridges 0.66 for all other bridges As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).	$\lambda_{max,t}$ = 1.0 + 0.2(0.66) = 1.132 $\lambda_{max,a}$ = 1.0 + 0.2(0.66) = 1.132 Therefore the maximum overall modification factors λ_{max} = 1.066(1.0)(1.132)(1.132) = 1.37 Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties. The upper-bound properties are: $Q_{d,max}$ = 1.37 (2.34) = 3.21 k $K_{dmax} = K_d$ = 1.17 k/in
E5. Design and Performance Summary	**E5. Design and Performance Summary, Example 1.3**
E5.1 Isolator dimensions Summarize final dimensions of isolators: • Overall diameter of isolator • Overall height • Radius of curvature of concave plate • Diameter of PTFE disc • PTFE Compound • PTFE contact pressure	**E5.1 Isolator dimensions, Example 1.3** Isolator dimensions are summarized in Table E5.1-1. **Table E5.1-1 Isolator Dimensions** (see table below) PTFE is 15% Glass-filled; 6,500 psi contact pressure; 3.00 in diameter.
E5.2 Bridge Performance Summarize bridge performance • Maximum superstructure displacement (longitudinal) • Maximum superstructure displacement (transverse) • Maximum superstructure displacement (resultant) • Maximum column shear (resultant) • Maximum column moment (about transverse axis) • Maximum column moment (about longitudinal axis) • Maximum column torque Check required performance as determined in Step A3, is satisfied.	**E5.2 Bridge Performance, Example 1.3** Bridge performance is summarized in Table E5.2-1 where it is seen that the maximum column shear is 18.03 k. This less than the column plastic shear (25 k) and therefore the required performance criterion is satisfied (fully elastic behavior). Furthermore the maximum longitudinal displacement is 1.66 in which is less than the 2.0 in available at the abutment expansion joints and is therefore acceptable.

Table E5.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Radius (in)
Under edge girder on Pier 1	16.25x16.25 x4.5(H) (est)	12.25 dia. x 4.50(H) (est)	39.0

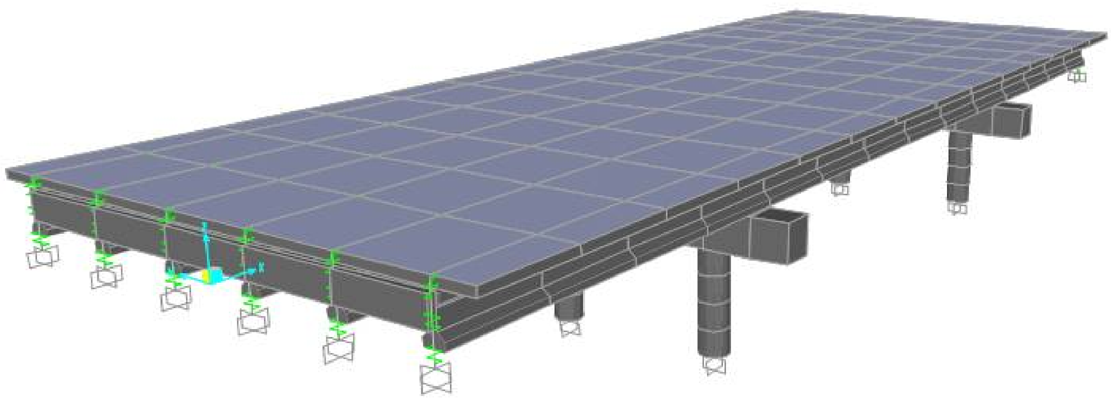
Table E5.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	1.66 in
Maximum superstructure displacement (transverse)	1.54 in
Maximum superstructure displacement (resultant)	1.72 in
Maximum column shear (resultant)	18.03 k
Maximum column moment about transverse axis	242 kft
Maximum column moment about longitudinal axis	121 kft
Maximum column torque	1.82 kft

SECTION 1

PC Girder Bridge, Short Spans, Multi-Column Piers

DESIGN EXAMPLE 1.4: Eradiquake Isolators



Design Examples in Section 1

ID	Description	S_1	Site Class	Pier height	Skew	Isolator type
1.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
1.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
1.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
1.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
1.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
1.5	Change column height	0.2g	B	H_1=0.5 H_2	0	Lead rubber bearing
1.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

DESIGN PROCEDURE		DESIGN EXAMPLE 1.4 (EQS Isolators)
STEP A: BRIDGE AND SITE DATA		

A1. Bridge Properties
Determine properties of the bridge:
• number of supports, m
• number of girders per support, n
• angle of skew
• weight of superstructure including railings, curbs, barriers and to the permanent loads, W_{SS}
• weight of piers participating with superstructure in dynamic response, W_{PP}
• weight of superstructure, W_j, at each support
• piers heights (clear dimensions)
• stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge
• column flexural yield strength (minimum value)
• column shear strength (minimum value)
• allowable movement at expansion joints
• isolator type if known, otherwise to be determined
		A1. Bridge Properties, Example 1.4
		• Number of supports, $m = 4$
		  o North Abutment ($m = 1$)
		  o Pier 1 ($m = 2$)
		  o Pier 2 ($m = 3$)
		  o South Abutment ($m = 4$)
		• Number of girders per support, $n = 6$
		• Number of columns per support = 3
		• Angle of skew = 0^0
		• Weight of superstructure including permanent loads, $W_{SS} = 650.52$ k
		• Weight of superstructure at each support:
		  o $W_1 = 44.95$ k
		  o $W_2 = 280.31$ k
		  o $W_3 = 280.31$ k
		  o $W_4 = 44.95$ k
		• Participating weight of piers, $W_{PP} = 107.16$ k
		• Effective weight (for calculation of period), $W_{eff} = Wss + W_{PP} = 757.68$ k
		• Stiffness of each pier in the longitudinal direction:
		  o $K_{sub,pier1,long} = 172.0$ k/in
		  o $K_{sub,pier2,long} = 172.0$ k/in
		• Stiffness of each pier in the transverse direction:
		  o $K_{sub,pier1,trans} = 687.0$ k/in
		  o $K_{sub,pier2,trans} = 687.0$ k/in
		• Minimum flexural yield strength of one column = 425 kft (plastic moment capacity)
		• Displacement capacity of expansion joints (longitudinal) = 2.0 in for thermal and other movements.
		• Eradiquake isolators

A2. Seismic Hazard Determine seismic hazard at site: • acceleration coefficients • site class and site factors • seismic zone Plot response spectrum. Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of PGA, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD). Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for		**A2. Seismic Hazard, Example 1.4** Acceleration coefficients for bridge site are given in design statement as follows: • $PGA = 0.40$ • $S_1 = 0.20$ • $S_S = 0.75$ Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec. Table 3.10.3.1-1 LRFD gives Site Class as B. Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors: • $F_{pga} = 1.0$ • $F_a = 1.0$ • $F_v = 1.0$

<table>
<tr>
<td>conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD.

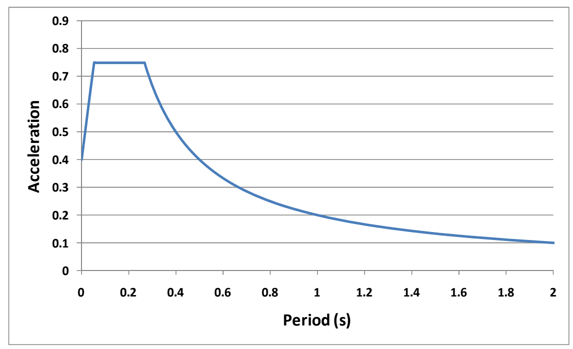
Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows:
• $A_s = F_{pga}\, PGA$
• $S_{DS} = F_a\, S_S$
• $S_{D1} = F_v\, S_1$

Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table5-1 GSID.

These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.</td>
<td></td>
<td>• $A_s = F_{pga}\, PGA = 1.0(0.40) = 0.40$
• $S_{DS} = F_a\, S_S = 1.0(0.75) = 0.75$
• $S_{D1} = F_v\, S_1 = 1.0(0.20) = 0.20$

Since $0.15 < S_{D1} \leq 0.30$, bridge is located in Seismic Zone 2.

Design Response Spectrum is as below: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<td>A3. Performance Requirements
Determine required performance of isolated bridge during Design Earthquake (1000-yr return period).

Examples of performance that might be specified by the Owner include:
• Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake.
• Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake.
• For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements)
• Reduced substructure forces for bridges on weak soils to reduce foundation costs.</td>
<td></td>
<td>A3. Performance Requirements, Example 1.4
In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield).

To remain elastic the maximum lateral load on the pier must be less than the load to yield the column. This load is taken as the plastic moment capacity (strength) of the column (425 kft, see above) divided by the overall column height (17 ft). This calculation assumes the column is acting as a simple cantilever in single curvature in the longitudinal direction.

Hence load to yield column = 425 /17 = 25.0 k

The maximum shear in the column must therefore be less than 25 k in order to keep the column elastic and meet the required performance criterion.</td>
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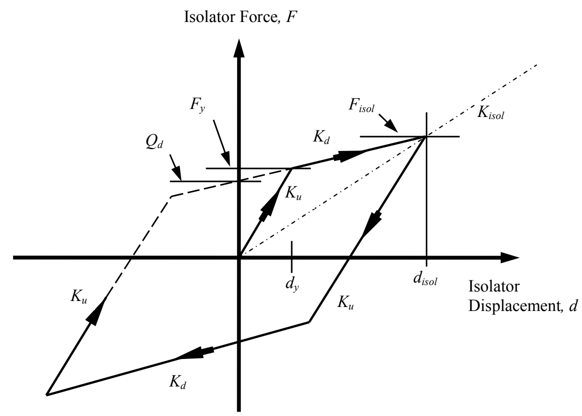
</table>

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties

To begin the iterative solution, an estimate is required of :

(1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)

Art C7.1 GSID

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \qquad \text{(B-1)}$$

(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not occur under non-seismic loads (e.g. wind) but

B1.1 Initial System Displacement and Properties, Example 1.4

$$d \cong 10 S_{D1} = 10(0.20) \cong 2.0\ in$$

low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e.

$$Q_d = 0.05W \qquad \text{(B-2)}$$

(3) Post-yield stiffness, K_d
Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3.

Art. 12.2 GSID

$$K_{d,min} \geq \frac{0.025W}{d} \qquad \text{(B-3)}$$

Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$

$$Q_d = 0.05W = 0.05(650.52) = 32.53\ k$$

$$K_d = 0.05\frac{W}{d} = 0.05\frac{650.52}{2.0} = 16.26\ k/in$$

B1.2 Initial Isolator Properties at Supports

Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support:

$$Q_{d,j} = Q_d\frac{W_j}{W} \qquad \text{(B-4)}$$

and

$$K_{d,j} = K_d\frac{W_j}{W} \qquad \text{(B-5)}$$

B1.2 Initial Isolator Properties at Supports, Example 1.4

$$Q_{d,j} = Q_d\frac{W_j}{W}$$

- $Q_{d,1}$ = 2.25 k
- $Q_{d,2}$ = 14.02 k
- $Q_{d,3}$ = 14.02 k
- $Q_{d,4}$ = 2.25 k

and

$$K_{d,j} = K_d\frac{W_j}{W}$$

- $K_{d,1}$ = 1.12 k/in
- $K_{d,2}$ = 7.01 k/in
- $K_{d,3}$ = 7.01 k/in
- $K_{d,4}$ = 1.12 k/in

B1.3 Effective Stiffness of Combined Pier and Isolator System

Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID).

An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula as follows (MCEER 2006):

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j} \qquad \text{(B-6)}$$

where

$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}} \qquad \text{(B-7)}$$

and $K_{sub,j}$ for the piers are given in Step A1. For the abutments, take $K_{sub,j}$ to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high

B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 1.4

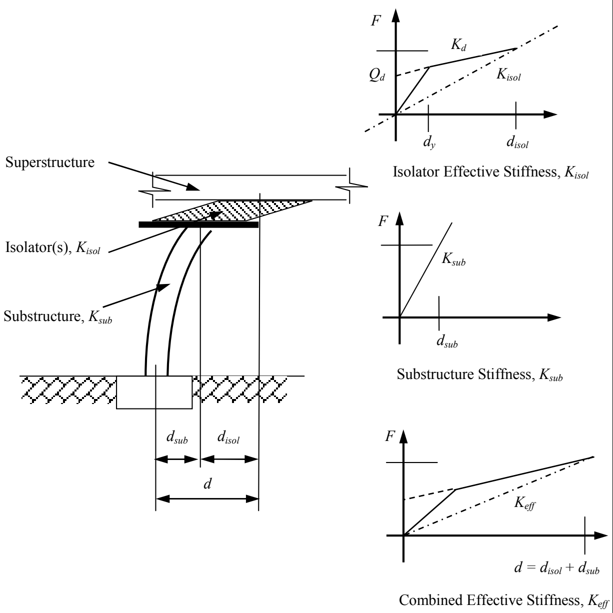
$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$

- $\alpha_1 = 2.25\text{x}10^{-4}$
- $\alpha_2 = 8.49\text{x}10^{-2}$
- $\alpha_3 = 8.49\text{x}10^{-2}$
- $\alpha_4 = 2.25\text{x}10^{-4}$

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$$

- $K_{eff,1}$ = 2.25 k/in
- $K_{eff,2}$ = 13.47 k/in
- $K_{eff,3}$ = 13.47 k/in
- $K_{eff,4}$ = 2.25 k/in

values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.



B1.4 Total Effective Stiffness

Calculate the total effective stiffness, K_{eff}, of the bridge:

Eq. 7.1-6 GSID

$$K_{eff} = \sum_{j=1}^{m} K_{eff,j} \qquad \text{(B-8)}$$

B1.4 Total Effective Stiffness, Example 1.4

$$K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 31.43\ k/in$$

B1.5 Isolation System Displacement at Each Support

Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports:

$$d_{isol,j} = \frac{d}{1+\alpha_j} \qquad \text{(B-9)}$$

B1.5 Isolation System Displacement at Each Support, Example 1.4

$$d_{isol,j} = \frac{d}{1+\alpha_j}$$

- $d_{isol,1}$ = 2.00 in
- $d_{isol,2}$ = 1.84 in
- $d_{isol,3}$ = 1.84 in
- $d_{isol,4}$ = 2.00 in

B1.6 Isolation System Stiffness at Each Support

Calculate the effective stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports:

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \qquad \text{(B-10)}$$

B1.6 Isolation System Stiffness at Each Support, Example 1.4

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$

- $K_{isol,1}$ = 2.25 k/in
- $K_{isol,2}$ = 14.61 k/in
- $K_{isol,3}$ = 14.61 k/in
- $K_{isol,4}$ = 2.25 k/in

B1.7 Substructure Displacement at Each Support Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports: $d_{sub,j} = d - d_{isol,j}$ (B-11)		**B1.7 Substructure Displacement at Each Support, Example 1.4** $d_{sub,j} = d - d_{isol,j}$ o $d_{sub,1} = 4.49\text{x}10^{-4}$ in o $d_{sub,2} = 1.57\text{x}10^{-1}$ in o $d_{sub,3} = 1.57\text{x}10^{-1}$ in o $d_{sub,4} = 4.49\text{x}10^{-4}$ in
B1.8 Lateral Load in Each Substructure Support Calculate the shear at support 'j', $F_{sub,j}$, for all supports: $F_{sub,j} = K_{sub,j} d_{sub,j}$ (B-12) where values for $K_{sub,j}$ are given in Step A1.		**B1.8 Lateral Load in Each Substructure, Example 1.4** $F_{sub,j} = K_{sub,j} d_{sub,j}$ o $F_{sub,1} =$ 4.49 k o $F_{sub,2} =$ 26.93 k o $F_{sub,3} =$ 26.93 k o $F_{sub,4} =$ 4.49 k
B1.9 Column Shear Force at Each Support Calculate the shear in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j': $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ (B-13) Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.		**B1.9 Column Shear Force at Each Support, Example 1.4** $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ o $F_{col,2,1\text{-}3} \approx$ 8.98 k o $F_{col,3,1\text{-}3} \approx$ 8.98 k These column shears are less than the yield capacity of each column (25 k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.
B1.10 Effective Period and Damping Ratio Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge: Eq. 7.1-5 GSID $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}}$ (B-14) and Eq. 7.1-10 GSID $\xi = \frac{2\sum_j(Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi\sum_j(K_{eff,j}(d_{isol,j} + d_{sub,j})^2}$ (B-15) where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j} = 0$. For other types of isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for the Simplified Method, set $d_{y,j} = 0$ for all isolator		**B1.10 Effective Period and Damping Ratio, Example 1.4** $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{757.68}{386.4(31.43)}}$ = 1.57 sec and taking $d_{y,j} = 0$: $\xi = \frac{2\sum_j(Q_{d,j}(d_{isol,j} - 0)}{\pi\sum_j(K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.31$

types. See Step B2.2 where the value of $d_{y,j}$ is revisited for the Multimode Spectral Analysis Method.		
B1.11 Damping Factor Calculate the damping factor, B_L, and the displacement, d, of the bridge: Eq. 7.1-3 GSID $$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases} \quad \text{(B-16)}$$ Eq. 7.1-4 GSID $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \quad \text{(B-17)}$$		**B1.11 Damping Factor, Example 1.4** Since $\xi = 0.31 \geq 0.3$ $$B_L = 1.70$$ and $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.2)1.57}{1.70} = 1.81\, in$$
B1.12 Convergence Check Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement. This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5). After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both. Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)		**B1.12 Convergence Check, Example 1.4** Since the calculated value for displacement, d (=1.81 in) is not close to that assumed at the beginning of the cycle (Step B1.1, d = 2.0), use the value of 1.81 in as the new assumed displacement and repeat from Step B1.3. After one iteration, convergence is reached at a superstructure displacement of 1.76 in, with an effective period of 1.52 seconds, and a damping factor of 1.70 (33% damping ratio). The displacement in the isolators at Pier 1 is 1.61 in and the effective stiffness of the same isolators is 15.69 k/in. See spreadsheet in Table B1.12-1 for results of final iteration. Ignoring the weight of the cap beams, the sum of the column shears at a pier must equal the total isolator shear force. Hence, approximate column shear (per column) = 15.69(1.61)/3 = 8.42 k which is less than the maximum allowable (25k) if elastic behavior is to be achieved (as required in Step A3). Also the superstructure displacement = 1.76 in, which is less than the available clearance of 2.0 in. Therefore the above solution is acceptable and go to Step B2. Note that available clearance (2.0 in) is greater than minimum required which is given by: $$= \frac{8\, S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.52}{1.7} = 1.43 in$$

Table B1.12-1 Simplified Method Solution for Design Example 1.4 – Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	650.52	107.16	757.68	0.2	6							
Step B1.1	d	1.72	Assumed displacement									
	Q_d	32.53	Characteristic strength									
	K_d	16.26	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j}+d_{sub,j})^2$
Abut 1	44.95	2.25	1.12	10000	2.40E-04	2.40	1.76	2.40	4.23E-04	4.23	3.958	7.444
Pier 1	280.31	14.02	7.01	172.0	9.12E-02	14.38	1.61	15.69	1.47E-01	25.33	22.623	44.611
Pier 2	280.31	14.02	7.01	172.0	9.12E-02	14.38	1.61	15.69	1.47E-01	25.33	22.623	44.611
Abut 2	44.95	2.25	1.12	10000	2.40E-04	2.40	1.76	2.40	4.23E-04	4.23	3.958	7.444
Total	650.52	32.526	16.263		$\Sigma K_{eff,j}$	33.557				59.107	53.161	104.109
					Step	**B1.4**						
Step B1.10	T_{eff}	1.52	Effective period									
	ξ	0.33	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.75										
	B_L	1.70	Damping Factor									
	d	1.75	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	0.375	0.187	0.400		1.66	0.413						
Pier 1	2.336	1.168	2.615		1.47	2.757						
Pier 2	2.336	1.168	2.615		1.47	2.757						
Abut 2	0.375	0.187	0.400		1.66	0.413						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength

Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \qquad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \qquad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 1.4

Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-1) by the number of isolators at each support (n = 6), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,\,1}$ = 2.25/6 = 0.37 k
- $Q_{d,\,2}$ = 14.02/6= 2.34 k
- $Q_{d,\,3}$ = 14.02/6= 2.34 k
- $Q_{d,\,4}$ = 2.25/6 = 0.37 k

and

- $K_{d,1}$ = 1.12/6 = 0.19 k/in
- $K_{d,2}$ = 7.01/6 = 1.17 k/in
- $K_{d,3}$ = 7.01/6 = 1.17 k/in
- $K_{d,4}$ = 1.12/6 = 0.19 k/in

B2.2 Initial Stiffness and Yield Displacement

Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

(1) For friction-based isolators: $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \qquad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \qquad \text{(B-22)}$$

B2.2 Initial Stiffness and Yield Displacement, Example 1.4

Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(1.17) = 11.7\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{2.34}{(11.7 - 1.17)} = 0.22\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2 in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i} = 0$.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$

Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

$$k_{isol,i} = \frac{K_{isol,j}}{n} \qquad \text{(B -23)}$$

B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 1.4

Dividing the results for K_{isol} (Step B1.12) among the 6 isolators at each support, the following values for K_{isol} /isolator are obtained:

- $K_{isol,1}$ = 2.40/6 = 0.40 k/in
- $K_{isol,2}$ = 15.69/6 = 2.62 k/in
- $K_{isol,3}$ = 15.69/6 = 2.62 k/in
- $K_{isol,4}$ = 2.40/6 = 0.40 k/in

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B2.4 Three-Dimensional Bridge Model
Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single ‘stick’ provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator.   If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam,   a more rigorous model is justified.

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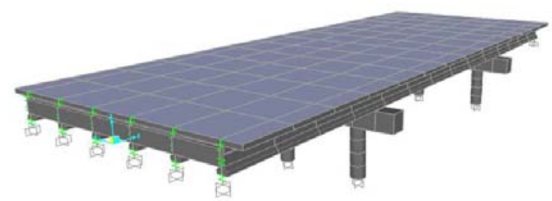
B2.4 Three-Dimensional Bridge Model, Example 1.4
Although the bridge in this Design Example is regular and is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below.



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B2.5 Composite Design Response Spectrum
Modify the response spectrum obtained in Step A2 to obtain a ‘composite’ response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, $B_{L.}$

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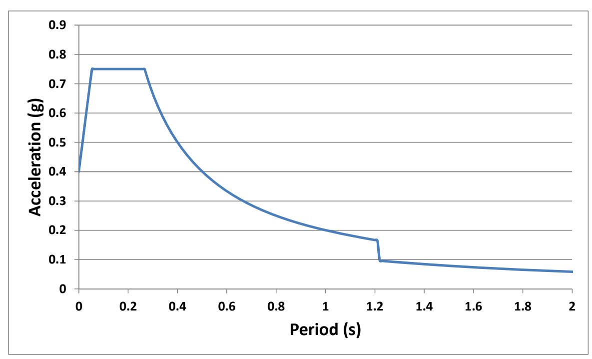
B.2.5 Composite Design Response Spectrum, Example 1.4
From the final results of Simplified Method (Step B1.12), B_L = 1.70 and T_{eff} = 1.52 sec. Hence the transition in the composite spectrum from 5% to 33% damping occurs at 0.8 T_{eff} = 0.8 (1.46) = 1.22 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods $\geq$ 1.22 sec by 1.70.



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B2.6 Multimodal Analysis of Finite Element Model
Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

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B2.6 Multimodal Analysis of Finite Element Model, Example 1.4
Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first two modes are the principal longitudinal and transverse modes with periods of 1.41 and 1.35 sec respectively. The period of the longitudinal mode (1.41 sec) is close to that calculated in the Simplified Method.

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Table B2.6-1 Modal Properties of Bridge Example 1.4 – First Iteration

Mode	Period	Modal Participating Mass Ratios					
No.	Sec	UX	UY	UZ	RX	RY	RZ
1	1.410	0.761	0.000	0.000	0.000	0.003	0.000
2	1.346	0.000	0.738	0.031	0.059	0.000	0.534
3	1.325	0.000	0.000	0.000	0.000	0.000	0.217
4	0.187	0.000	0.000	0.000	0.000	0.000	0.000
5	0.186	0.125	0.000	0.000	0.000	0.000	0.000
6	0.121	0.000	0.000	0.000	0.000	0.000	0.000
7	0.121	0.000	0.000	0.000	0.000	0.000	0.006
8	0.104	0.000	0.034	0.183	0.107	0.000	0.064
9	0.101	0.000	0.000	0.000	0.000	0.226	0.000
10	0.095	0.000	0.121	0.081	0.184	0.000	0.041
11	0.094	0.000	0.000	0.000	0.000	0.000	0.000
12	0.074	0.000	0.002	0.000	0.003	0.000	0.000

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 1.65 (1.76) in
- $d_{isol,2}$ = 1.47 (1.61) in
- $d_{isol,3}$ = 1.47 (1.61) in
- $d_{isol,4}$ = 1.65 (1.76) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 1.4

The new superstructure displacement is 1.65 in, more than a 5% difference from the displacement assumed at the start of the Multimode Spectral Analysis.

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

$$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \qquad \text{(B-24)}$$

Recalculate $K_{eff,j}$:

Eq. 7.1-6 GSID

$$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \qquad \text{(B-25)}$$

Recalculate system damping ratio, ξ :

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \qquad \text{(B-26)}$$

Recalculate system damping factor, B_L:

Eq. 7.1-3

$$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \qquad \text{(B-27)}$$

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 1.4

Updated values for $K_{isol,i}$ are given below (previous values are in parentheses):

- $K_{isol,1}$ = 0.41 (0.40) k/in
- $K_{isol,2}$ = 2.76 (2.62) k/in
- $K_{isol,3}$ = 2.76 (2.62) k/in
- $K_{isol,4}$ = 0.41 (0.40) k/in

Updated values for $K_{eff,j}$, ξ, B_L and T_{eff} are given below (previous values are in parentheses):

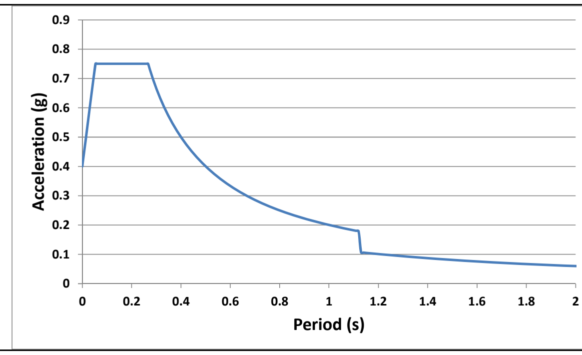
- $K_{eff,1}$ = 2.48 (2.40) k/in
- $K_{eff,2}$ = 15.48 (14.38) k/in
- $K_{eff,3}$ = 15.48 (14.38) k/in
- $K_{eff,4}$ = 2.48 (2.40) k/in

- ξ = 27% (33%)
- B_L = 1.66 (1.70)
- T_{eff} = 1.41 (1.52) sec

The updated composite response spectrum is shown below:

GSID

Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.



B2.6 Multimodal Analysis Second Iteration, Example 1.4

Reanalysis gives the following values for the isolator displacements (numbers in parentheses are from the previous cycle):

- $d_{isol,1}$ = 1.66 (1.65) in
- $d_{isol,2}$ = 1.47 (1.47) in
- $d_{isol,3}$ = 1.47 (1.47) in
- $d_{isol,4}$ = 1.66 (1.65) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 1.4

The new superstructure displacement is 1.66 in, less than a 1% difference from the displacement assumed at the start of the second cycle of Multimode Spectral Analysis.

B2.9 Superstructure and Isolator Displacements

Once convergence has been reached, obtain

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and
- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.

B2.9 Superstructure and Isolator Displacements, Example 1.4

From the above analysis:

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are:

  x_L= 1.66 in

  y_L= 0.0 in

- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are:
  - Abutments: u_L = 1.66 in, v_L = 0.00 in
  - Piers: u_L = 1.47 in, v_L = 0.00 in

  All isolators at same support have the same displacements.

B2.10 Pier Bending Moments and Shear Forces

Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.

B2.10 Pier Bending Moments and Shear Forces, Example 1.4

Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are:

Exterior Columns:

- M_{PLL}= 0 kft

		o M_{PTL}= 240 kft o V_{PLL}= 15.81 k o V_{PTL}= 0 k Interior Columns: o M_{PLL}= 0 kft o M_{PTL}= 235 kft o V_{PLL}= 15.24 k o V_{PTL}= 0 k Both piers have the same distribution of bending moments and shear forces among the columns.
B2.11 Isolator Shear and Axial Forces Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.		**B2.11 Isolator Shear and Axial Forces, Example 1.4** Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abutment	1	0.69	0.00	0.36
	2	0.69	0.00	0.39
	3	0.69	0.00	0.39
	4	0.69	0.00	0.39
	5	0.69	0.00	0.39
	6	0.69	0.00	0.36
Pier	1	4.04	0.00	0.13
	2	4.05	0.00	0.19
	3	4.05	0.00	0.22
	4	4.05	0.00	0.22
	5	4.05	0.00	0.19
	6	4.04	0.00	0.13

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 1.4

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.43 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows: x_T = 0 and y_T = 1.53 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions are as follows:
  Abutments u_T = 0.00 in, v_T = 1.53 in
  Piers u_T = 0.00 in, v_T = 1.49 in
- Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are:
  Exterior Columns:
  - M_{PLT}= 108 kft
  - M_{PTT}= 1 kft
  - V_{PLT}= 0.06 k
  - V_{PTT}= 14.87 k

  Interior Columns:
  - M_{PLT}= 120 kft
  - M_{PTT}= 0 kft
  - V_{PLT}= 0 k
  - V_{PTT}= 17.29 k

  Both piers have the same distribution of bending moments and shear forces among the columns.

- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abutment	1	0.00	0.65	3.50
	2	0.00	0.65	1.93
	3	0.00	0.65	0.68
	4	0.00	0.65	0.68
	5	0.00	0.65	1.93
	6	0.00	0.65	3.50
Pier	1	0.02	3.98	12.56
	2	0.01	4.00	1.14
	3	0.00	4.01	2.29
	4	0.00	4.01	2.29
	5	0.01	4.00	1.14
	6	0.02	3.98	12.56

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

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D1. Design Isolator Displacements
Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
$v_1 = v_L + 0.3v_T$ (D-2)
$R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)
- $u_2 = 0.3u_L + u_T$ (D-4)
$v_2 = 0.3v_L + v_T$ (D-5)
$R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)
- $d_t = \max(R_1, R_2)$ (D-7)

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D1. Design Isolator Displacements at Pier 1, Example 1.4
To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

<u>Load Case 1</u>:
$u_1 = u_L + 0.3u_T = 1.0(1.47) + 0.3(0) = 1.47$ in
$v_1 = v_L + 0.3v_T = 1.0(0) + 0.3(1.49) = 0.45$ in
$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.47^2 + 0.45^2} = 1.54$ in

<u>Load Case 2</u>:
$u_2 = 0.3u_L + u_T = 0.3(1.47) + 1.0(0) = 0.44$ in
$v_2 = 0.3v_L + v_T = 0.3(0) + 1.0(1.49) = 1.49$in
$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{0.44^2 + 1.49^2} = 1.55$in

<u>Governing Case</u>:
Total design displacement, $d_t = \max(R_1, R_2)$
$= 1.55$ in

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D2. Design Moments and Shears
Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

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D2. Design Moments and Shears in Pier 1, Example 1.4
Design moments and shear forces are calculated for Pier 1, Column 1, below to illustrate the process.

<u>Load Case 1</u>:
$V_{PL1} = V_{PLL} + 0.3V_{PLT} = 1.0(15.81) + 0.3(0.06)$
$= 15.83$ k
$V_{PT1} = V_{PTL} + 0.3V_{PTT} = 1.0(0.02) + 0.3(14.87)$
$= 4.48$ k
$R_1 = \sqrt{V_{PL1}^2 + V_{PT1}^2} = \sqrt{15.83^2 + 4.48^2} = 16.45$ k

<u>Load Case 2</u>:
$V_{PL2} = 0.3V_{PLL} + V_{PLT} = 0.3(15.81) + 1.0(0.06)$
$= 4.80$ k
$V_{PT2} = 0.3V_{PTL} + V_{PTT} = 0.3(0.02) + 1.0(14.87)$
$= 14.88$ k
$R_2 = \sqrt{V_{PL2}^2 + V_{PT2}^2} = \sqrt{4.80^2 + 14.88^2} = 15.63$ k

<u>Governing Case</u>:
Design column shear $= \max(R1, R2)$
$= 16.45$ k

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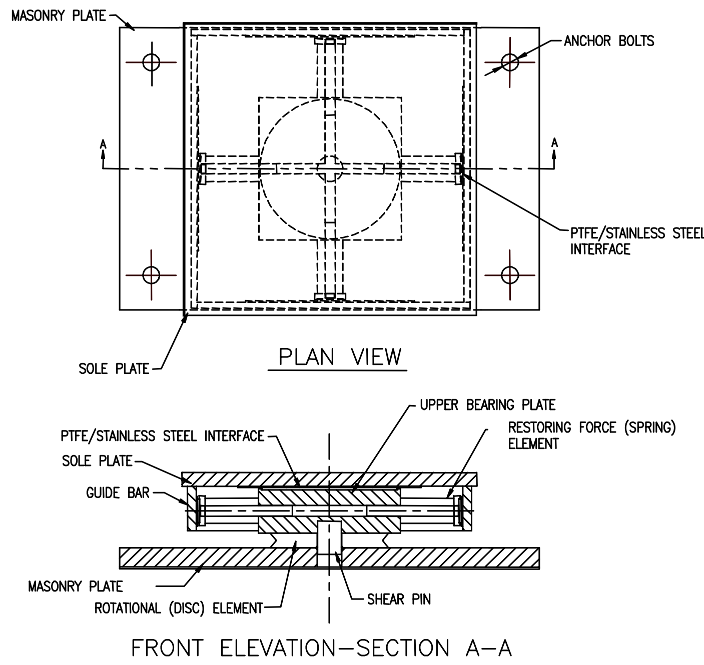
STEP E. DESIGN OF ERADIQUAKE ISOLATORS



An EradiQuake Isolator (EQS) is a sliding isolation bearing composed of a multi-directional sliding disc bearing and lateral springs. Each spring assembly consists of a cylindrical polyurethane spring and a spring piston. The piston keeps the spring straight as the isolator moves in different directions. The disc bearing and springs are housed in a mirror-finished stainless steel lined box.

The required values for Q_d and K_d determine the coefficient of friction at the sliding interface and the properties of the springs.

The sliding interface is typically comprised of stainless steel and PTFE. Energy is dissipated during sliding while the springs provide a restoring force. PTFE is an attractive material in that at sliding slow speeds it has a low coefficient of friction which is ideal for accommodating thermal effects, and at higher speeds the friction becomes greater and acts as an effective energy dissipator during seismic events. The polyurethane springs are designed such that they are never in tension. Their basic design and composition is derived from the die-spring industry.

Design and materials conform to the LRFD Specifications. Steel components are designed in accordance with Section 6, while the disc bearing is designed and constructed per Section 14.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

Notation

A	Area	K_d	Stiffness when sliding (Total spring rate)
A_1	Area based on dead load	k_1	Stiffness (spring rate) for one spring
A_2	Area based on total load	L	Length
A_S	Spring area	L_{GB}	Guide bar length
B_{BB}	Bearing block plan dimension	L_S	Spring length
B_{Box}	Guide box plan dimension (out to out dimension of guide bars)	L_{SI}	Installed spring length
		LL	Live Load
B_{BP}	Base plate length	L_1	Spring length based on max long term displacement
B_{PTFE}	PTFE dimension		
B_{SP}	Slide plate (guide box top) length	L_2	Spring length based on max short term displacement
D_D	Disc outer diameter		
D_{PTFE}	PTFE diameter	M	Moment
D_S	Spring outer diameter	M_N	Factored moment
d_L	Service (long term) displacement	P_{DL}	Dead load
d_T	Total seismic displacement	P_{LL}	Live load
E	Elastic modulus	P_{SL}	Seismic live load
F	Spring force	P_{WL}	Wind load
F_Y	Yield stress	Q_d	Characteristic strength
H	Isolator height	S_G	Gross shape factor of disc
ID_S	Spring inner diameter	T_{BB}	Bearing block thickness

T_{BP} Base plate thickness T_D Disc thickness T_{GB} Guide bar thickness T_{SP} Slide plate thickness W Isolator weight W Plan width of isolator W_{BP} Base plate length WL Wind load Z Plastic modulus	α Bearing rotation β Inner to outer diameter ratio δ_W Wind displacement ε_C Maximum average compression strain μ Coefficient of friction

E1. Required Properties

Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1).

- the required characteristic strength, Q_d, per isolator
- the required post-elastic stiffness, K_d, per isolator
- the total design displacement, d_t, for each isolator
- the maximum applied dead and live load, P_{DL}, P_{LL}, and seismic load, P_{SL},which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator
- the wind load per isolator, P_{WL}, and
- the thermal displacement of the superstructure at each isolator, d_L.

E1. Required Properties, Example 1.4

The design of one of the exterior isolators on a pier is given below to illustrate the design process for an EQS isolator.

From previous work:

- Q_d/isolator = 2.34 k
- K_d/isolator = 1.17 k/in
- Total design displacement, d_t = 1.55 in
- P_{DL} = 45.52 k
- P_{LL} = 15.50 k
- P_{SL} = 12.56 k

Calculated for this design:

- P_{WL} = 1.76 k
- d_L = +/- 0.25 in

E2. Isolator Sizing

E2.1 Size the Disc

Estimate the disc outer diameter based on an average compressive stress of 4.5 ksi using the gross plan area.

$$D_D = \sqrt{\frac{4(P_{DL} + P_{LL})}{\pi\,(4.5)}} = 0.53\sqrt{(P_{DL} + P_{LL})} \qquad \text{(E-1)}$$

Estimate disc thickness based on a gross shape factor of 2.4:

$$S_G = \frac{D_D}{4T_D} \qquad \text{(E-2)}$$

Check rotational capacity and adjust disc thickness if required. Use standard disc design rotation, α, of 0.02 radians, and a maximum compression strain, ε_c, of 0.10 for this calculation.

$$\frac{D_D}{2}\alpha \le \varepsilon_c T_D \qquad \text{(E-3)}$$

E2.1 Size Disc, Example 1.4

$$D_D = 0.53\sqrt{62} = 4.17 \approx 4.50\ in$$

$$T_D = \frac{4.5}{4(2.4)} = 0.47 \approx 0.50\ in$$

$$T_D \ge \frac{\alpha D_D}{2\varepsilon_c} = \frac{0.02(4.50)}{2(0.10)} = 0.45\ in\ OK$$

E2.2 Size the Springs	**E2.2 Size the Springs, Example 1.4**
E2.2.1 Calculate Installed Spring Length Assume 60% max compressive strain on the MER spring for short term loading, 40% max compressive strain for long term loading. Add 20% of long term loading strain for elastomer compression set. Then $L_1 = 2.5\ d_L$ (E-4) And using a load combination of two times seismic total design displacement (for Seismic Zone 2) plus 50% service (thermal): $L_2 = \dfrac{2\ d_t + 0.5 d_L}{0.60}$ (E-5) Then required spring length is given by: $L_S = \max\ (L_1, L_2)$ (E-6)	**E2.2 Calculate Installed Spring Length, Example 1.4** $L_1 = 2.5(0.25) = 0.63\ in$ $L_2 = \dfrac{2(1.55) + 0.5(0.25)}{0.60} = 5.38\ in$ $L_S = 5.38\ in$
E2.2.2 Check Wind Displacements Calculate displacement due to wind as follows: $\delta_W = \dfrac{P_{WL} - 0.25 Q_d}{K_d}$ (E-7) If the displacement due to wind is too large, add spring precompression equal to the wind displacement to the spring length in the transverse direction. Precompression doubles the stiffness over the precompressed displacement. If the wind displacements are still too large, consider increasing the spring stiffness in the transverse direction, or using sacrificial shear keys.	**E2.2.2 Check Wind Displacements, Example 1.4** $\delta_W = \dfrac{1.8 - 0.25(2.3)}{1.2} = 1.02\ in$ In this example, the wind displacement is acceptable and no adjustment of spring length is required.
E2.2.3 Calculate Spring Diameter Assume only one spring per side is used to meet spring rate requirements, i.e. let $k_1 = K_d$, and take the elastic modulus for polyurethane spring to be 6.0 ksi. Since $k_1 = \dfrac{EA_S}{L_S} = \dfrac{6.0 A_S}{L_S}$ (E-8) it follows that $A_S = \dfrac{k_1 L_S}{6.0}$ (E-9) and $D_S = \sqrt{\dfrac{4A_S}{\pi(1-\beta^2)}}$ (E-10)	**E2.2.3 Calculate Spring Diameter, Example 1.4** Since K_d = 1.17 k/in $A_S = \dfrac{1.2(5.38)}{6.0} = 1.08\ in^2$ Take initial value for β = 0.20 and then $D_S = \sqrt{\dfrac{4(1.08)}{\pi(1-0.2^2)}} = 1.19 \approx 1.25\ in$

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E2.2.4 Adjust Spring Length Using Nominal Diameters

For manufacturing purposes it is advantageous to use standard diameters and adjust the spring length according to the actual value of β to fine tune the stiffness (spring rate).

$$\beta = \frac{ID_s}{D_S} \quad \text{(E-11)}$$

$$A_S = \frac{\pi}{4} D_S^2 (1 - \beta^2) \quad \text{(E-12)}$$

$$L_S = \frac{EA_S}{k_1} \quad \text{(E-13)}$$

Check that L_s is greater than either L_1 and L_2

Note that L_S is the installed spring length. The actual size of the springs may be slightly different than the installed size. Springs are pre-compressed to provide additional wind resistance if needed and account for compression set in the elastomer.

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E2.2.4 Adjust Spring Length Using Nominal Diameters, Example 1.4

Use 1-1/4 in for the spring OD, and 7/16 in for the spring ID then

$$\beta = \frac{0.44}{1.25} = 0.35$$

$$A_S = \frac{\pi}{4} 1.25^2 (1 - 0.35^2) = 1.08\ in^2$$

$$L_S = \frac{6.0(1.08)}{1.20} = 5.38\ in$$

$$L_S \geq \max(L_1, L_2)\ OK$$

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E2.3 Size the PTFE Pad

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E2.3 Size the PTFE Pad, Example 1.4

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E2.3.1 Calculate Coefficient of Friction

Calculate the required coefficient of friction from:

$$\mu = \frac{Q_d}{W} \quad \text{(E-14)}$$

Select PTFE and polished stainless steel as the sliding surfaces. Low coefficients of friction are possible with these materials at high contact stresses. In general the friction coefficient decreases with increasing pressure.

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E2.3.1 Calculate Required Coefficient of Friction, Example 1.4

$$\mu = \frac{2.3}{46} = 0.05$$

A value of 0.05 is lower than the dry PTFE sliding material can achieve at design pressures. Two alternatives are available: (1) design with a higher Q_d and then reanalyzing bridge response, and (2) use EQS bearings with lubricated surfaces at some isolator locations to reduce the global coefficient of friction. However, because displacements are small, two pieces of PTFE can be used, one dimpled and lubricated ($\mu = 0.02$), the other dry ($\mu = 0.07$).

The dry PTFE area will need to comprise 60% of the total area to achieve an overall coefficient of 0.05, assuming the same contact stress across both pieces of PTFE.

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E2.3.2 Calculate Required Area of PTFE

Calculate required area of PTFE using allowable contact stresses in GSID Table 16.4.1-1. For service loads (i.e. dead load) allowable average stress is 3.5 ksi, and then:

$$A_1 = \frac{P_{DL}}{3.5} \quad \text{(E-15)}$$

Check area required under dead plus live load using

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E2.3.2 Calculate Required Area of PTFE, Example 1.4

$$A_1 = \frac{46}{3.5} = 13.1\ in^2$$

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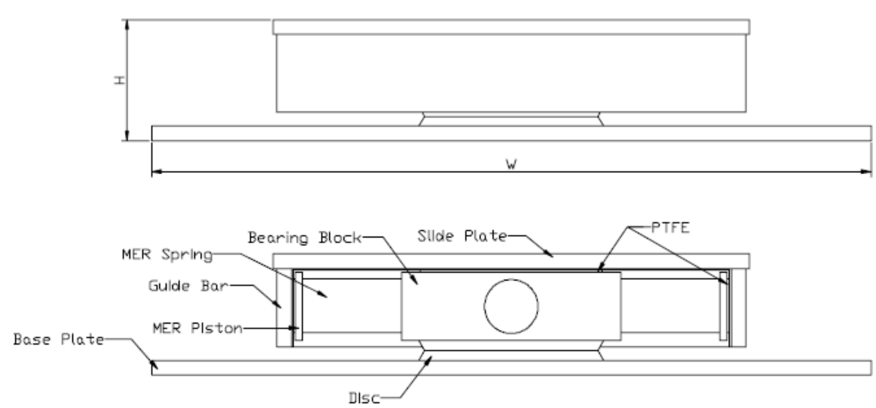
an allowable average stress of 4.5 ksi (as permitted in LRFD Sec 14.) $$A_2 = \frac{(P_{DL} + P_{LL})}{4.5} \quad \text{(E-16)}$$ Then required area is $$A = \max(A_1, A_2) \quad \text{(E-17)}$$ Since the structure design rotation of 0.01 radians is only one-half of the disc design rotation, the limits on the PTFE edge contact stresses (GSID Table 16.4.1-1) do not govern.		$$A_2 = \frac{(46+16)}{4.5} = 13.8\ in^2$$ $$A = \max(A_1, A_2) = 13.8\ in^2$$
E2.3.3 Calculate Size of PTFE Pad For a circular PTFE pad, the diameter is given by: $$D_{PTFE} = \sqrt{\frac{4}{\pi}A} \quad \text{(E-18a)}$$ For a square PTFE pad, the side dimension is given by: $$B_{PTFE} = \sqrt{A} \quad \text{(E-18b)}$$		**E2.3.3 Calculate Size of PTFE Pad, Example 1.4** In this example, two rectangular pieces of PTFE with different friction coefficients, are being used to achieve the particularly low coefficient of friction that is required overall. These pieces are separated by a distance of $2d_t$ to prevent the 'dry' side becoming lubricated during seismic excitation. The dimensions are such that the two pieces form a square of side B_{PTFE} which is given by: $$B_{PTFE} = d_t + \sqrt{d_t^2 + A}$$ $$B_{PTFE} = 1.55 + \sqrt{1.55^2 + 13.8} = 5.57 \approx 5.63in$$
E2.4 Size the Bearing Block		**E2.4 Size the Bearing Block, Example 1.4**
E2.4.1 Calculate Bearing Plan Dimension Two criteria must be checked to determine the bearing block plan dimension. The disc must fit under the block with some clearance, and the PTFE must fit on top of the block with at least 1/8 in edge clearance. $$B_{BB1} = 1.15D_D \quad \text{(E-19)}$$ $$B_{BB2} = B_{PTFE} + 2(0.125) \quad \text{(E-20)}$$ $$B_{BB} = \max(B_{BB1}, B_{BB2}) \quad \text{(E-21)}$$		**E2.4.1 Calculate Bearing Plan Dimension, Example 1.4** $$B_{BB1} = 1.15(4.50) = 5.18\ in$$ $$B_{BB2} = 5.63 + 2(0.125) = 5.88\ in$$ $$B_{BB} = \max(B_{BB1}, B_{BB2}) = 5.88 \approx 6.00\ in$$
E2.4.2 Calculate Bearing Block Thickness The thickness of bearing block must be sufficient to ensure that the springs can be attached on each side of the block, allowing for a 30% increase in diameter upon spring compression. $$T_{BB} = 1.3D_S \quad \text{(E-22)}$$ Note that if T_{BB} is too large, reduce the diameter of the springs and increase their number.		**E2.4.2 Calculate Bearing Block Thickness, Example 1.4** $$T_{BB} = 1.3(1.25) = 1.63 \approx 1.75\ in$$ Size is ok. No need to resize spring diameters.

E2.5 Size the Box	E2.5 Size the Box, Example 1.4
E2.5 .1 Calculate Guide Bar Thickness (a) <u>Guide Bar Force</u> Guide bars resist the spring forces. They are modeled as cantilever beams, with the fixed end of the cantilever located where the guide bar meets the slide plate. Assume the resisting length of guide bar to be three times the diameter of the spring. The moment arm is one-half of the bearing block thickness, plus 0.20 in. Forces corresponding to two times the seismic displacement, imposed during prototype testing, are used to design the guide bar. $F = k_1(2d_t + 0.5d_L)$ (E-23) (b) <u>Guide Bar Moment</u> $M = (0.5T_{BB} + 0.20)F$ (E-24) Since the effective length of guide bar resisting this moment is assumed to be $3D_s$, the bending moment per inch of guide bar is: $M_1 = \frac{M}{3D_S}$ (E-25) (c) <u>Guide Bar Thickness</u> Using a load factor of 1.75, a resistance factor of 1.00, and assuming 50 ksi steel: $M_N = 1.75M_1 = 1.00F_YZ$ (E-26) then $Z = \frac{1.75M_1}{F_Y}$ (E-27) But since $Z = \frac{T_{GB}^2}{4}$ (E-28) then $T_{GB} = \sqrt{4Z}$ (E-29)	**E2.5.1 Calculate Guide Bar Thickness, Example 1.4** $F = 1.2\big(2(1.55) + 0.5(0.25)\big) = 3.87\ k$ $M = (0.5(1.75) + 0.20)3.87 = 4.16\ kin$ $M_1 = \frac{4.16}{3(1.25)} = 1.11\ kin/in$ $Z = \frac{1.75(1.11)}{50} = 0.039\ in^3/in$ $T_{GB} = \sqrt{4(0.039)} = 0.39 \approx 0.50\ in$
E2.5.2 Calculate Guide Bar Length $L_{GB} = T_{GB} + B_{BB} + 2L_s$ (E-30)	**E2.5.2 Calculate Guide Bar Length, Example 1.4** $L_{GB} = 0.50 + 6.00 + 2(5.38) = 17.26 \approx 17.25in$
E2.5.3 Calculate Guide Bar Width $W_{GB} = T_{BB} + 0.5T_D$ (E-31)	**E2.5.3 Calculate Guide Bar Width, Example 1.4** $W_{GB} = 1.75 + 0.5(0.50) = 2.0\ in$
E2.5.4 Calculate Size of Box and Slide Plate Calculate side dimension of box $B_{Box} = L_{GB} + T_{GB}$ (E-32) Choose plan dimension of slide plate equal to, or slightly larger than, the box, i.e.	**E2.5.4 Calculate Size of Box and Slide Plate, Example 1.4** $B_{Box} = 17.25 + 0.50 = 17.75\ in$

$B_{SP} \geq B_{Box}$ (E-33)	Take $B_{SP} = 18.00\ in$
E2.5.5 Calculate Box Top (Slide Plate) Thickness Make the slide plate (guide box top) the same thickness as the guide bars, with a minimum value of ¾ in.	**E2.5.5 Calculate Box Top (Slide Plate) Thickness, Example 1.4** $T_{SP} = 0.75\ in$
E2.6 Size the Lower Plate (a) <u>Thickness</u> Use ¾ inch minimum thickness unless otherwise required by State DOT specifications. (b) <u>Width</u> Since GSID provisions for prototype testing require the isolator to be displaced to twice the design displacement (for Seismic Zone 2), the base plate must be wide enough to allow such movement without interference from the anchor bolts. $W_{BP} = B_{Box} + 4d_t + 8$ (E-34) (c) <u>Length</u> Take $B_{BP} = B_{SP}$ (E-35) (d) <u>Anchor Bolts</u> Design anchor bolts per LRFD specifications, increase W_{BP} if necessary.	**E2.6 Size the Lower Plate, Example 1.4** $T_{LP} = 0.75\ in$ $W_{BP} = 17.75 + 4(1.55) + 8 = 31.95 \approx 32.0\ in$ $B_{BP} = 18.00\ in$
E3. Design Summary Overall dimensions of isolator are: Width = W_{BP} Length = B_{SP} Height is given by: $H = T_{LP} + T_D + T_{BB} + T_{SP} + 0.20$ (E-36)	**E3 Design Summary, Example 1.4** Width = W_{BP} = 32.0 in Length = B_{SP} = 18.00 in Height is given by: $H = 0.75 + 0.50 + 1.75 + 0.75 + 0.20 = 3.95\ in$



E4. Minimum and Maximum Performance Check
Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E4-1.

Table E4-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-37)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-38)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-39)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-40)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E4-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E4-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})(\lambda_{min,a,Kd})(\lambda_{min,v,Kd})(\lambda_{min,tr,Kd})(\lambda_{min,c,Kd})(\lambda_{min,scrag,Kd})$	(E-41)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})(\lambda_{max,a,Kd})(\lambda_{max,v,Kd})(\lambda_{max,tr,Kd})(\lambda_{max,c,Kd})(\lambda_{max,scrag,Kd})$	(E-42)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})(\lambda_{min,a,Qd})(\lambda_{min,v,Qd})(\lambda_{min,tr,Qd})(\lambda_{min,c,Qd})(\lambda_{min,scrag,Qd})$	(E-43)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})(\lambda_{max,a,Qd})(\lambda_{max,v,Qd})(\lambda_{max,tr,Qd})(\lambda_{max,c,Qd})(\lambda_{max,scrag,Qd})$	(E-44)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

E4. Minimum and Maximum Performance Check, Example 1.4
For Eradiquake isolators, Modification Factors are applied to both Q_d and K_d, because both frictional and elastomeric (urethane) elements are used in these isolators.

Minimum Property Modification factors are:
$\lambda_{min,Kd} = 1.0$
$\lambda_{min,Qd} = 1.0$

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are (GSID Appendix A):
$\lambda_{max,a,Kd} = 1.0$
$\lambda_{max,a,Qd} = 1.2$

$\lambda_{max,t,Kd} = 1.3$
$\lambda_{max,t,Qd} = 1.5$

$\lambda_{max,tr,Kd} = 1.0$
$\lambda_{max,tr,Qd} = 1.0$

$\lambda_{max,c,Kd} = 1.0$
$\lambda_{max,c,Qd} = 1.1$

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd} = 1.0 + 0.0(0.66) = 1.00$
$\lambda_{max,a,Qd} = 1.0 + 0.2(0.66) = 1.13$

1.00 for critical bridges
0.75 for essential bridges
0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

$\lambda_{max,t,Kd}$ = 1.0 + 0.3(0.66) = 1.20
$\lambda_{max,t,Qd}$ = 1.0 + 0.5(0.66) = 1.33

$\lambda_{max,tr,Kd}$ = 1.0 + 0.0(0.66) = 1.00
$\lambda_{max,tr,Qd}$ = 1.0 + 0.0(0.66) =1.00

$\lambda_{max,c,Kd}$ = 1.0 + 0.0(0.66) = 1.00
$\lambda_{max,c,Qd}$ = 1.0 + 0.0(0.66) =1.00

Therefore the maximum overall modification factors
$\lambda_{max,Kd}$ = (1.00)(1.20)(1.00)(1.00) = 1.20
$\lambda_{max,Qd}$ = (1.13)(1.33)(1.00)(1.00) = 1.50

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:
$Q_{d,max}$ = 1.50(2.34) = 3.51 k
and
$K_{d,max}$=1.20(1.17) = 1.40 k/in

E5. Design and Performance Summary

E5. Design and Performance Summary, Example 1.4

E5.1 Isolator dimensions

Summarize final dimensions of isolators:

- Overall size of lower plate
- Overall size of box (top plate)
- Overall height
- Size of disc
- Size of PTFE pad
- Number of polyurethane springs
- Diameter of polyurethane springs

Check all dimensions with manufacturer.

E5.1 Isolator dimensions, Example 1.4

Isolator dimensions are summarized in Table E5.1-1.

Table E5.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plate (in)	Overall size without mounting plate (in)	Diam. Disc (in)
Under edge girder on Pier 1	32.0 x 18.0 x 4.0(H)	17.75x17.75 x 4.0(H)	4.50

Isolator Location	Size PTFE pad (in)	No. poly-urethane springs	Diam. poly-urethane springs (in)
Under edge girder on Pier 1	5.63 x 5.63	4	1.25

E5.2 Bridge Performance

Summarize bridge performance

- Maximum superstructure displacement (longitudinal)
- Maximum superstructure displacement (transverse)
- Maximum superstructure displacement (resultant)
- Maximum column shear (resultant)

E5.2 Bridge Performance, Example 1.4

Bridge performance is summarized in Table E5.2-1 where it is seen that the maximum column shear is 18.03 k. This less than the column plastic shear (25 k) and therefore the required performance criterion is satisfied (fully elastic behavior). Furthermore the maximum longitudinal displacement is 1.66 in which is less than the 2.0 in available at the abutment expansion joints and is therefore acceptable.

- Maximum column moment (about transverse axis)
- Maximum column moment (about longitudinal axis)
- Maximum column torque

Check required performance as determined in Step A3, is satisfied.

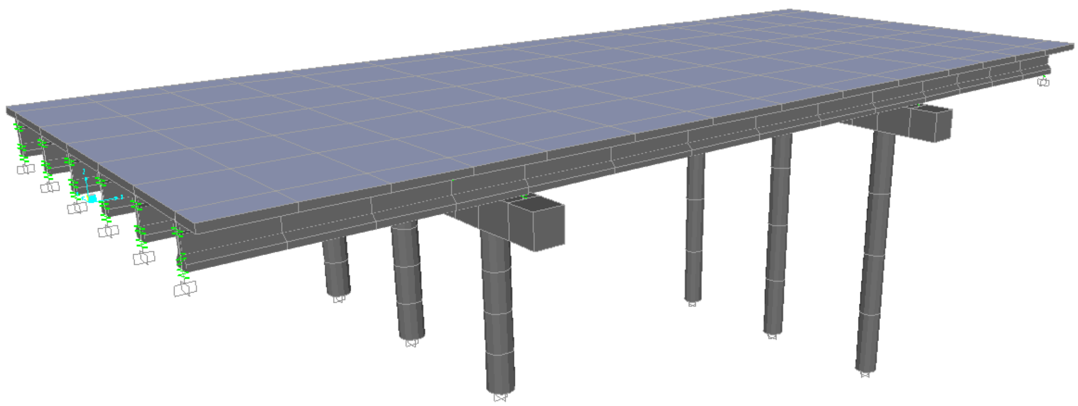
Table E5.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	1.66 in
Maximum superstructure displacement (transverse)	1.54 in
Maximum superstructure displacement (resultant)	1.72 in
Maximum column shear (resultant)	18.03 k
Maximum column moment about transverse axis	242 kft
Maximum column moment about longitudinal axis	121 kft
Maximum column torque	1.82 kft

SECTION 1

PC Girder Bridge, Short Spans, Multi-Column Piers

DESIGN EXAMPLE 1.5: H_1=0.5 H_2



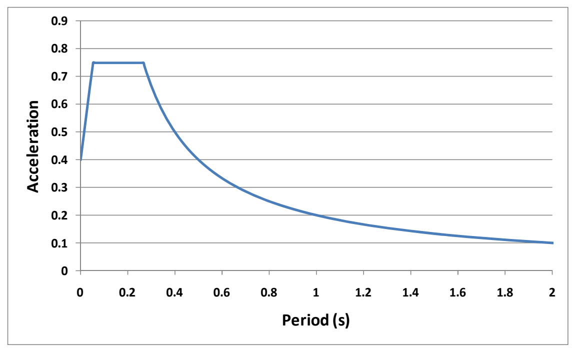
Design Examples in Section 1

ID	Description	S_1	Site Class	Pier height	Skew	Isolator type
1.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
1.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
1.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
1.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
1.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
1.5	Change column height	0.2g	B	H_1=0.5 H_2	0	Lead rubber bearing
1.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

DESIGN PROCEDURE	DESIGN EXAMPLE 1.5 (Unequal Pier Heights: H_1=0.5H_2)
STEP A: BRIDGE AND SITE DATA	

A1. Bridge Properties
Determine properties of the bridge:
• number of supports, *m*
• number of girders per support, *n*
• angle of skew
• weight of superstructure including railings, curbs, barriers and to the permanent loads, W_{SS}
• weight of piers participating with superstructure in dynamic response, W_{PP}
• weight of superstructure, W_j, at each support
• stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column heights.
• column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
• allowable movement at expansion joints
• isolator type if known, otherwise to be determined
	A1. Bridge Properties, Example 1.5
	• Number of supports, m = 4
	  o North Abutment (m = 1)
	  o Pier 1 (m = 2)
	  o Pier 2 (m = 3)
	  o South Abutment (m =4)
	• Number of girders per support, n = 6
	• Number of columns per support = 3
	• Angle of skew = 0^0
	• Weight of superstructure including permanent loads, W_{SS} = 650.52 k
	• Weight of superstructure at each support:
	  o W_1 = 44.95 k
	  o W_2 = 280.31 k
	  o W_3 = 280.31 k
	  o W_4 = 44.95 k
	• Participating weight of piers, W_{PP} = 107.16 k
	• Effective weight (for calculation of period), $W_{eff} = W_{SS} + W_{PP}$ = 757.68 k
	• Stiffness of each pier in the longitudinal direction:
	  o $K_{sub,pier1,long}$ = 172.0 k/in
	  o $K_{sub,pier2,long}$ = 21.5 k/in
	• Stiffness of each pier in the transverse direction:
	  o $K_{sub,pier1,trans}$ = 687.0 k/in
	  o $K_{sub,pier2,trans}$ = 86.0 k/in
	• Minimum column shear strength of shorter column based on flexural yield capacity of column = 25 k
	• Displacement capacity of expansion joints (longitudinal) = 2.0 in for thermal and other movements.
	• Lead rubber isolators

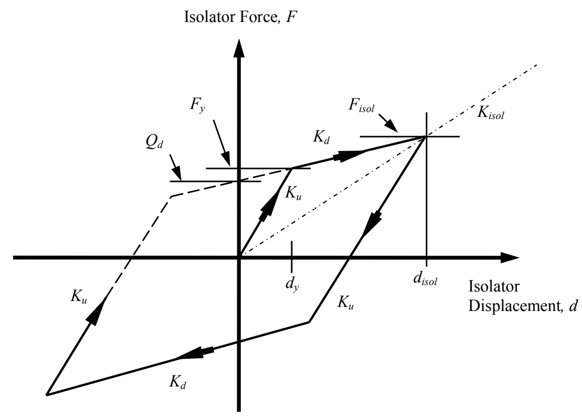
A2. Seismic Hazard Determine seismic hazard at site: • acceleration coefficients • site class and site factors • seismic zone Plot response spectrum. Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of *PGA*, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD). Use Art. 3.2 to obtain Site Class and corresponding Site	**A2. Seismic Hazard, Example 1.5** Acceleration coefficients for bridge site are given in design statement as follows: • PGA = 0.40 • S_1 = 0.20 • S_S = 0.75 Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec. Table 3.10.3.1-1 LRFD gives Site Class as B. Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors: • F_{pga} = 1.0 • F_a = 1.0

Factors (F_{pga}, F_a and F_v). These data are the same as for conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD. Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows: • $A_s = F_{pga}\ PGA$ • $S_{DS} = F_a\ S_S$ • $S_{D1} = F_v\ S_1$ Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table5-1 GSID. These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.		• $F_v = 1.0$ • $A_s = F_{pga}\ PGA = 1.0(0.40) = 0.40$ • $S_{DS} = F_a\ S_S = 1.0(0.75) = 0.75$ • $S_{D1} = F_v\ S_1 = 1.0(0.20) = 0.20$ Since $0.15 < S_{D1} \leq 0.30$, bridge is located in Seismic Zone 2. Design Response Spectrum is as below: 
A3. Performance Requirements Determine required performance of isolated bridge during Design Earthquake (1000-yr return period). Examples of performance that might be specified by the Owner include: • Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake. • Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake. • For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements) • Reduced substructure forces for bridges on weak soils to reduce foundation costs.		**A3. Performance Requirements, Example 1.5** In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield). To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (25 k). The maximum shear in any of the shorter columns must therefore be less than 25 k in order to keep these columns elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

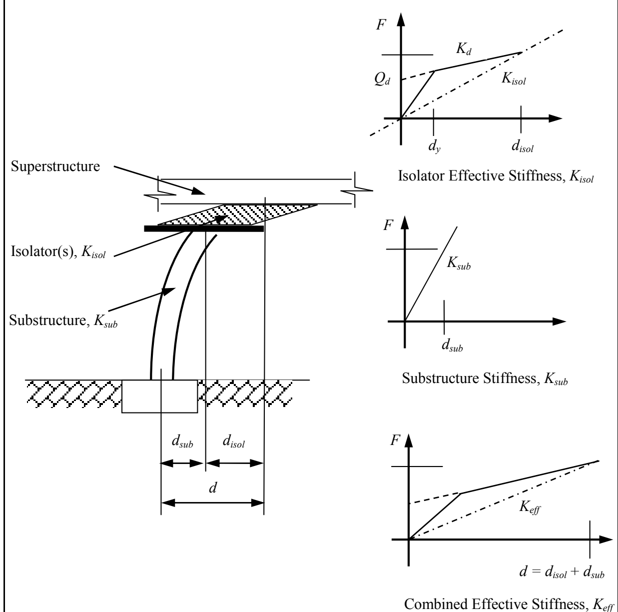
Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties		B1.1 Initial System Displacement and Properties, Example 1.5
To begin the iterative solution, an estimate is required of : (1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)		
Art C7.1 GSID $d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1}$ (B-1)		$d \cong 10 S_{D1} = 10(0.20) \cong 2.0\ in$
(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not occur under non-seismic loads (e.g. wind)		

but low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e. $Q_d = 0.05W$ (B-2) (3) Post-yield stiffness, K_d Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3. Art. 12.2 GSID $K_{d,min} \geq \frac{0.025W}{d}$ (B-3) Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$		$Q_d = 0.05W = 0.05(650.52) = 32.53\ k$ $K_d = 0.05\frac{W}{d} = 0.05\frac{650.52}{2.0} = 16.26\ k/in$
B1.2 Initial Isolator Properties at Supports Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support: $Q_{d,j} = Q_d \frac{W_j}{W}$ (B-4) and $K_{d,j} = K_d \frac{W_j}{W}$ (B-5)		**B1.2 Initial Isolator Properties at Supports, Example 1.5** $Q_{d,j} = Q_d \frac{W_j}{W}$ o $Q_{d,1}$ = 2.25 k o $Q_{d,2}$ = 14.02 k o $Q_{d,3}$ = 14.02 k o $Q_{d,4}$ = 2.25 k and $K_{d,j} = K_d \frac{W_j}{W}$ o $K_{d,1}$ = 1.12 k/in o $K_{d,2}$ = 7.01 k/in o $K_{d,3}$ = 7.01 k/in o $K_{d,4}$ = 1.12 k/in
B1.3 Effective Stiffness of Combined Pier and Isolator System Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID). An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula as follows (MCEER 2006): $K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$ (B-6) where $\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$ (B-7) and $K_{sub,j}$ for the piers are given in Step A1. For the abutments, take $K_{sub,j}$ to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high		**B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 1.5** $\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$ o $\alpha_1 = 2.25\text{x}10^{-4}$ o $\alpha_2 = 8.49\text{x}10^{-2}$ o $\alpha_3 = 9.67\text{x}10^{-1}$ o $\alpha_4 = 2.25\text{x}10^{-4}$ $K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$ o $K_{eff,1}$ = 2.25 k/in o $K_{eff,2}$ = 13.47 k/in o $K_{eff,3}$ = 10.57 k/in o $K_{eff,4}$ = 2.25 k/in

values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.  		
B1.4 Total Effective Stiffness Calculate the total effective stiffness, K_{eff}, of the bridge: Eq. 7.1-6 GSID $\quad K_{eff} = \sum_{j=1}^{m} K_{eff,j}$ (B-8)		**B1.4 Total Effective Stiffness, Example 1.5** $K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 28.53\ k/in$
B1.5 Isolation System Displacement at Each Support Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports: $d_{isol,j} = \frac{d}{1+\alpha_j}$ (B-9)		**B1.5 Isolation System Displacement at Each Support, Example 1.5** $d_{isol,j} = \frac{d}{1+\alpha_j}$ o $d_{isol,1}$ = 2.00 in o $d_{isol,2}$ = 1.84 in o $d_{isol,3}$ = 1.02 in o $d_{isol,4}$ = 2.00 in
B1.6 Isolation System Stiffness at Each Support Calculate the effective stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports: $K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$ (B-10)		**B1.6 Isolation System Stiffness at Each Support, Example 1.5** $K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$ o $K_{isol,1}$ = 2.25 k/in o $K_{isol,2}$ = 14.61 k/in o $K_{isol,3}$ = 20.79 k/in o $K_{isol,4}$ = 2.25 k/in

B1.7 Substructure Displacement at Each Support Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports: $$d_{sub,j} = d - d_{isol,j} \qquad \text{(B-11)}$$		**B1.7 Substructure Displacement at Each Support, Example 1.5** $$d_{sub,j} = d - d_{isol,j}$$ o $d_{sub,1}$ = 4.49x10^{-4} in o $d_{sub,2}$ = 1.57x10^{-1} in o $d_{sub,3}$ = 9.83x10^{-1} in o $d_{sub,4}$ = 4.49x10^{-4} in
B1.8 Lateral Load in Each Substructure Support Calculate the shear at support 'j', $F_{sub,j}$, for all supports: $$F_{sub,j} = K_{sub,j} d_{sub,j} \qquad \text{(B-12)}$$ where values for $K_{sub,j}$ are given in Step A1.		**B1.8 Lateral Load in Each Substructure, Example 1.5** $$F_{sub,j} = K_{sub,j} d_{sub,j}$$ o $F_{sub,1}$ = 4.49 k o $F_{sub,2}$ = 26.93 k o $F_{sub,3}$ = 21.14 k o $F_{sub,4}$ = 4.49 k
B1.9 Column Shear Force at Each Support Calculate the shear in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j': $$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j} \qquad \text{(B-13)}$$ Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.		**B1.9 Column Shear Force at Each Support, Example 1.5** $$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$$ o $F_{col,2,1\text{-}3}$ ≈ 8.98 k o $F_{col,3,1\text{-}3}$ ≈ 7.05 k These column shears are less than the yield capacity of each column (25 k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.
B1.10 Effective Period and Damping Ratio Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge: Eq. 7.1-5 GSID $$T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} \qquad \text{(B-14)}$$ and Eq. 7.1-10 GSID $$\xi = \frac{2\sum_j(Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi\sum_j(K_{eff,j}(d_{isol,j} + d_{sub,j})^2} \qquad \text{(B-15)}$$ where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j}$ = 0. For other types of isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for the Simplified Method, set $d_{y,j}$ = 0 for all isolator		**B1.10 Effective Period and Damping Ratio, Example 1.5** $$T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{757.68}{386.4(28.53)}}$$ = 1.65 sec and taking $d_{y,j}$ = 0: $$\xi = \frac{2\sum_j(Q_{d,j}(d_{isol,j} - 0)}{\pi\sum_j(K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.27$$

types. See Step B2.2 where the value of $d_{y,j}$ is revisited for the Multimode Spectral Analysis Method.		
B1.11 Damping Factor Calculate the damping factor, B_L, and the displacement, d, of the bridge: Eq. 7.1-3 GSID $B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases}$ (B-16) Eq. 7.1-4 GSID $d = \frac{9.79 S_{D1} T_{eff}}{B_L}$ (B-17)		**B1.11 Damping Factor, Example 1.5** Since $\xi = 0.27 < 0.3$ $B_L = 1.67$ and $d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.2)1.65}{1.67} = 1.94\ in$
B1.12 Convergence Check Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement. This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5). After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both. Note that Art 9 GSID requires that a minimum clearance be provided equal to $8\ S_{D1}\ T_{eff}/\ B_L$. (B-18)		**B1.12 Convergence Check, Example 1.5** Since the calculated value for displacement, d (=1.94 in) is close to that assumed at the beginning of the cycle (Step B1.1, d = 2.0), use the value of 1.94 in as the assumed displacement. See spreadsheet in Table B1.12-1 for results of simplified method. Ignoring the weight of the cap beams, the sum of the column shears at a pier must equal the total isolator shear force. Hence, approximate column shear (per column) = 14.87(1.78)/3 = 8.82 k which is less than the maximum allowable (25k) if elastic behavior is to be achieved (as required in Step A3). Also the superstructure displacement = 1.94 in, which is less than the available clearance of 2.0 in. Therefore the above solution is acceptable and go to Step B2. Note that available clearance (2.0 in) is greater than minimum required which is given by: $= \frac{8\ S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.63}{1.67} = 1.95 in$

Table B1.12-1 Simplified Method Solution for Design Example 1.5 – Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	650.52	107.16	757.68	0.2	6							
Step B1.1	d	1.94	Assumed displacement									
	Q_d	32.53	Characteristic strength									
	K_d	16.26	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	44.95	2.25	1.12	10000	2.28E-04	2.28	1.94	2.28	4.42E-04	4.42	4.351	8.565
Pier 1	280.31	14.02	7.01	172.0	8.64E-02	13.69	1.78	14.87	1.54E-01	26.51	24.983	51.330
Pier 2	280.31	14.02	7.01	21.5	9.99E-01	10.74	0.97	21.47	9.68E-01	20.80	13.581	40.291
Abut 2	44.95	2.25	1.12	10000	2.28E-04	2.28	1.94	2.28	4.42E-04	4.42	4.351	8.565
Total	650.52	32.526	16.260		$\Sigma K_{eff,j}$	28.996				56.155	47.267	108.752
					Step	**B1.4**						
Step B1.10	T_{eff}	1.63	Effective period									
	ξ	0.28	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.67										
	B_L	1.67	Damping Factor									
	d	1.92	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	0.375	0.187	0.381		1.88	0.386						
Pier 1	2.336	1.168	2.478		1.68	2.558						
Pier 2	2.336	1.168	3.579		0.94	3.653						
Abut 2	0.375	0.187	0.381		1.88	0.386						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength

Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \tag{B-19}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \tag{B-20}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 1.5

Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-1) by the number of isolators at each support (n = 6), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,1}$ = 2.25/6 = 0.37 k
- $Q_{d,2}$ = 14.02/6= 2.34 k
- $Q_{d,3}$ = 14.02/6= 2.34 k
- $Q_{d,4}$ = 2.25/6 = 0.37 k

and

- $K_{d,1}$ = 1.12/6 = 0.19 k/in
- $K_{d,2}$ = 7.01/6 = 1.17 k/in
- $K_{d,3}$ = 7.01/6 = 1.17 k/in
- $K_{d,4}$ = 1.12/6 = 0.19 k/in

B2.2 Initial Stiffness and Yield Displacement

Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

(1) For friction-based isolators: $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \tag{B-21}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \tag{B-22}$$

B2.2 Initial Stiffness and Yield Displacement, Example 1.5

Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(1.17) = 11.7\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{2.34}{(11.7 - 1.17)} = 0.22\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2 in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i} = 0$.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$

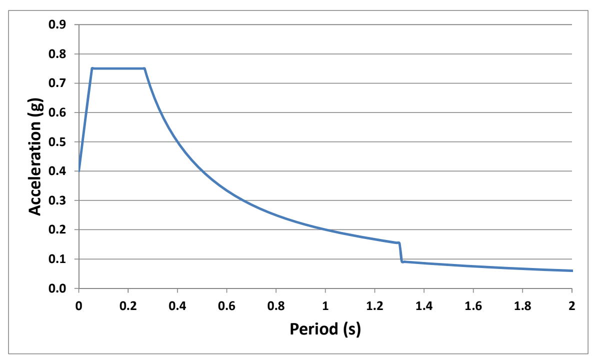
Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

$$k_{isol,i} = \frac{K_{isol,j}}{n} \tag{B -23}$$

B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 1.5

Dividing the results for K_{isol} (Step B1.12) among the 6 isolators at each support, the following values for K_{isol} /isolator are obtained:

- $K_{isol,1}$ = 2.28/6 = 0.38 k/in
- $K_{isol,2}$ = 14.87/6 = 2.48 k/in
- $K_{isol,3}$ = 21.47/6 = 3.58 k/in

		o $K_{isol,4}$ = 2.28/6 = 0.38 k/in
B2.4 Three-Dimensional Bridge Model Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is justified.		**B2.4 Three-Dimensional Bridge Model, Example 1.5** Although the bridge in this Design Example is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below.  .
B2.5 Composite Design Response Spectrum Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.		**B.2.5 Composite Design Response Spectrum, Example 1.5** From the final results of Simplified Method (Step B1.12), B_L = 1.67 and T_{eff} = 1.63 sec. Hence the transition in the composite spectrum from 5% to 28% damping occurs at 0.8 T_{eff} = 0.8 (1.63) = 1.30 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods ≥ 1.30 sec by 1.67. 
B2.6 Multimodal Analysis of Finite Element Model Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.		**B2.6 Multimodal Analysis of Finite Element Model, Example 1.5** Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first two modes are the principal longitudinal and transverse modes with periods of 1.54 and 1.37 sec respectively. While no significant coupling is observed in the first two modes, the third mode is a strongly coupled transverse and rotational mode, as might be expected due to the lack of

symmetry in the transverse direction.

Table B2.6-1 Modal Properties of Bridge Example 1.5 – First Iteration

Mode	Period	Modal Participating Mass Ratios					
No.	Sec	UX	UY	UZ	RX	RY	RZ
1	1.543	0.792	0.000	0.000	0.000	0.001	0.000
2	1.365	0.000	0.408	0.000	0.015	0.000	0.057
3	1.286	0.000	0.340	0.000	0.010	0.000	0.698
4	0.370	0.025	0.000	0.000	0.000	0.001	0.000
5	0.252	0.000	0.068	0.000	0.015	0.000	0.107
6	0.243	0.000	0.000	0.000	0.000	0.000	0.003
7	0.187	0.062	0.000	0.000	0.000	0.000	0.000
8	0.120	0.000	0.000	0.000	0.000	0.000	0.003
9	0.104	0.000	0.007	0.000	0.226	0.000	0.001
10	0.102	0.000	0.000	0.246	0.000	0.197	0.000
11	0.096	0.000	0.069	0.000	0.041	0.000	0.013
12	0.095	0.000	0.000	0.021	0.000	0.017	0.000

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the SimplifiedMethod):

- $d_{isol,1}$ = 1.88 (1.94) in
- $d_{isol,2}$ = 1.68 (1.78) in
- $d_{isol,3}$ = 0.94 (0.97) in
- $d_{isol,4}$ = 1.88 (1.94) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (*d*) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 1.5

The new superstructure displacement is 1.88 in, a 3% difference from the displacement assumed at the start of the Multimode Spectral Analysis.

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

$$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \qquad \text{(B-24)}$$

Recalculate $K_{eff,j}$:

Eq. 7.1-6 GSID

$$K_{eff,j} = \frac{K_{sub,j}\,\sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \qquad \text{(B-25)}$$

Recalculate system damping ratio, ξ :

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \qquad \text{(B-26)}$$

Recalculate system damping factor, B_L:

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 1.5

As convergence was reached at the first iteration, it is unnecessary to calculate updated properties.

Eq. 7.1-3 GSID $$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \quad \text{(B-27)}$$ Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.		
B2.9 Superstructure and Isolator Displacements Once convergence has been reached, obtain o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.		**B2.9 Superstructure and Isolator Displacements, Example 1.5** From the above analysis: o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are: x_L= 1.88 in y_L= 0.0 in o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are: o Abutments: u_L = 1.88 in, v_L = 0.00 in o Pier 1: u_L = 1.68 in, v_L = 0.00 in o Pier 2: u_L = 0.94 in, v_L = 0.00 in All isolators at same support have the same displacements.
B2.10 Pier Bending Moments and Shear Forces Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.		**B2.10 Pier Bending Moments and Shear Forces, Example 1.5** Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are: Pier 1, Exterior Columns: o M_{PLL}= 1 kft o M_{PTL}= 246 kft o V_{PLL}= 16.21 k o V_{PTL}= 0.28 k Pier 1, Interior Column: o M_{PLL}= 1 kft o M_{PTL}= 241 kft o V_{PLL}= 15.63 k o V_{PTL}= 0.27 k Pier 2, Exterior Columns: o M_{PLL}= 1 kft o M_{PTL}= 233 kft o V_{PLL}= 8.77 k o V_{PTL}= 0.13 k Pier 2, Interior Column: o M_{PLL}= 0 kft o M_{PTL}= 232 kft o V_{PLL}= 8.57 k o V_{PTL}= 0.12 k

B2.11 Isolator Shear and Axial Forces

Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 1.5

Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
North Abutment	1	0.72	0.00	0.39
	2	0.72	0.00	0.45
	3	0.72	0.00	0.43
	4	0.72	0.00	0.43
	5	0.72	0.00	0.45
	6	0.72	0.00	0.39
Pier 1	1	4.15	0.00	0.20
	2	4.16	0.00	0.17
	3	4.16	0.00	0.15
	4	4.16	0.00	0.15
	5	4.16	0.00	0.17
	6	4.15	0.00	0.20
Pier 2	1	3.34	0.00	0.18
	2	3.35	0.00	0.22
	3	3.35	0.00	0.25
	4	3.35	0.00	0.25
	5	3.35	0.00	0.22
	6	3.34	0.00	0.18
South Abutment	1	0.72	0.00	0.33
	2	0.72	0.00	0.40
	3	0.72	0.00	0.40
	4	0.72	0.00	0.40
	5	0.72	0.00	0.40
	6	0.72	0.00	0.33

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 1.5

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.43 sec
- Maximum superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows:

  North Abutment x_T = 0.63 and y_T = 1.93 in
  Pier 1 x_T = 0.63 and y_T = 1.31 in
  Pier 2 x_T = 0.63 and y_T = 1.54 in
  South Abutment x_T = 0.63 and y_T = 2.24 in

- Maximum isolator displacements in the longitudinal (u_T) and transverse (v_T) directions are as follows:

  North Abutment u_T = 0.63 and v_T = 1.93 in
  Pier 1 u_T = 0.60 and v_T = 1.27 in
  Pier 2 u_T = 0.53 and v_T = 1.39 in
  South Abutment u_T = 0.63 and v_T = 2.24 in

- Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are:

  Pier 1, Exterior Columns:
  - M_{PLT}= 116 kft
  - M_{PTT}= 25 kft
  - V_{PLT}= 2.69 k
  - V_{PTT}= 16.13 k

  Pier 1, Interior Column:
  - M_{PLT}= 129 kft
  - M_{PTT}= 3 kft
  - V_{PLT}= 0.74 k
  - V_{PTT}= 18.71 k

  Pier 2, Exterior Columns:
  - M_{PLT}= 241 kft
  - M_{PTT}= 27 kft
  - V_{PLT}= 1.65 k
  - V_{PTT}= 17.58 k

  Pier 2, Interior Column:
  - M_{PLT}= 252 kft
  - M_{PTT}= 2 kft
  - V_{PLT}= 0.38 k
  - V_{PTT}= 18.75 k

- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
North Abutment	1	0.24	0.74	2.50
	2	0.15	0.75	1.07
	3	0.05	0.75	0.37
	4	0.05	0.75	0.37
	5	0.15	0.75	1.07
	6	0.24	0.74	2.50
Pier 1	1	1.80	3.79	6.35
	2	1.08	3.81	0.17
	3	0.36	3.81	2.15
	4	0.36	3.81	2.15
	5	1.08	3.81	0.17
	6	1.80	3.79	6.35
Pier 2	1	1.43	3.69	7.29
	2	0.85	3.71	4.95
	3	0.28	3.71	2.37
	4	0.28	3.71	2.37
	5	0.85	3.71	4.95
	6	1.43	3.69	7.29
South Abutment	1	0.22	0.76	1.84
	2	0.13	0.76	1.18
	3	0.04	0.76	0.55
	4	0.04	0.76	0.55
	5	0.13	0.76	1.18
	6	0.22	0.76	1.84

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements
Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)

- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)

- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements at Pier 1, Example 1.5
To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

Load Case 1:
$u_1 = u_L + 0.3u_T$ = 1.0(1.68) + 0.3(0.60) = 1.86 in
$v_1 = v_L + 0.3v_T$ = 1.0(0) + 0.3(1.26) = 0.38 in
$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.86^2 + 0.38^2}$ = 1.89 in

Load Case 2:
$u_2 = 0.3u_L + u_T$ = 0.3(1.68) + 1.0(0.60) = 1.10 in
$v_2 = 0.3v_L + v_T$ = 0.3(0) + 1.0(1.26) = 1.26 in
$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{1.10^2 + 1.26^2}$ = 1.68 in

Governing Case:
Total design displacement, $d_t = \max(R_1, R_2)$
= 1.89 in

D2. Design Moments and Shears
Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

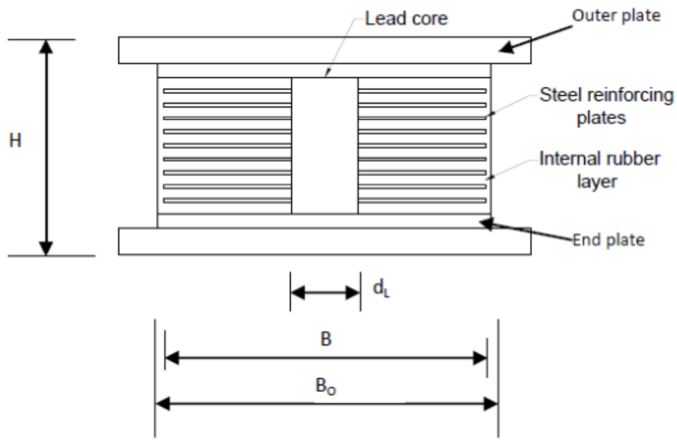
D2. Design Moments and Shears in Pier 1, Example 1.5
Design moments and shear forces are calculated for Pier 1, Column 1, below to illustrate the process.

Load Case 1:
$V_{PL1} = V_{PLL} + 0.3V_{PLT}$ = 1.0(16.21) + 0.3(2.69)
= 17.01 k
$V_{PT1} = V_{PTL} + 0.3V_{PTT}$ = 1.0(0.28) + 0.3(16.13)
= 5.12 k
$R_1 = \sqrt{V_{PL1}^2 + V_{PT1}^2} = \sqrt{17.01^2 + 5.12^2}$ = 17.77 k

Load Case 2:
$V_{PL2} = 0.3V_{PLL} + V_{PLT}$ = 0.3(16.21) + 1.0(2.69)
= 7.55 k
$V_{PT2} = 0.3V_{PTL} + V_{PTT}$ = 0.3(0.28) + 1.0(16.13)
= 16.22 k
$R_2 = \sqrt{V_{PL2}^2 + V_{PT2}^2} = \sqrt{7.55^2 + 16.22^2}$ = 17.89 k

Governing Case:
Design column shear = max (*R1*, *R2*)
= 17.89 k

STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS



A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness (K_u).With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness K_d is given by the rubber alone. More details are given in MCEER 2006.

While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary. When sizing the physical dimensions of the bearing, plan dimensions (B, d_L) should be rounded up to the next $^1/_4$" increment, while the total thickness of elastomer, T_r, is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are $^1/_4$" and $^3/_8$". High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in MCEER 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See MCEER 2006.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties

Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1).

- required characteristic strength, Q_d / isolator
- required post-elastic stiffness, K_d / isolator
- total design displacement, d_t, for each isolator
- maximum applied dead and live load (P_{DL}, P_{LL}) and seismic load (P_{SL}) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator, and
- maximum wind load, P_{WL}

E1. Required Properties, Example 1.5

The design of one of the exterior isolators on a pier is given below to illustrate the design process for lead-rubber isolators.

From previous work

- Q_d/isolator = 2.34 k
- K_d/isolator = 1.17 k/in
- Total design displacement, d_t = 1.89 in
- P_{DL} = 45.52 k
- P_{LL} = 15.50 k
- P_{SL} = 6.35 k
- P_{WL} =1.76 k < Q_d OK

E2. Isolator Sizing

E2.1 Lead Core Diameter

Determine the required diameter of the lead plug, d_L, using:

$$d_L = \sqrt{\frac{Q_d}{0.9}} \qquad \text{(E-1)}$$

See Step E2.5 for limitations on d_L

E2.1 Lead Core Diameter, Example 1.5

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{2.34}{0.9}} = 1.61\ in$$

E2.2 Plan Area and Isolator Diameter Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.0 ksi. Then the bonded area of the isolator is given by: $A_b = \frac{P_{DL} + P_{LL}}{1.0}\ in^2$ (E-2) and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by: $B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2}$ (E-3) Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using $A_b = \frac{\pi}{4}(B^2 - d_L^2)$ (E-4) Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, B_o is given by: $B_o = B + 1.0$ (E-5)		**E2.2 Plan Area and Isolator Diameter, Example 1.5** $A_b = \frac{P_{DL} + P_{LL}}{1.0}\ in^2 = \frac{45.52 + 15.50}{1.0} = 61.02\ in^2$ $B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} = \sqrt{\frac{4\ (61.02)}{\pi} + 1.61^2}$ = 8.96 in Round B up to 9.0 in and the actual bonded area is: $A_b = \frac{\pi}{4}(9.0^2 - 1.61^2) = 61.57\ in^2$ B_o = 9.0 + 2(0.5) = 10.0 in
E2.3 Elastomer Thickness and Number of Layers Since the shear stiffness of the elastomeric bearing is given by: $K_d = \frac{GA_b}{T_r}$ (E-6) where G = shear modulus of the rubber, and T_r = the total thickness of elastomer, it follows Eq. E-5 may be used to obtain T_r given a required value for K_d $T_r = \frac{GA_b}{K_d}$ (E-7) A typical range for shear modulus, G, is 60-120 psi. Higher and lower values are available and are used in special applications. If the layer thickness is t_r, the number of layers, n, is given by: $n = \frac{T_r}{t_r}$ (E-8) rounded up to the nearest integer.		**E2.3 Elastomer Thickness and Number of Layers, Example 1.5** Select G, shear modulus of rubber, = 100 psi (0.1 ksi) Then $T_r = \frac{GA_b}{K_d} = \frac{0.1(61.57)}{1.17} = 5.27\ in$ $n = \frac{5.27}{0.25} = 21.09$ Round to nearest integer, i.e. n = 22

<table>
<tr><td>Note that because of rounding the plan dimensions and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be necessary if the differences are large.</td><td></td></tr>
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E2.4 Overall Height

The overall height of the isolator, H, is given by:

$$H = n\,t_r + (n-1)t_s + 2t_c \qquad \text{(E-9)}$$

where t_s = thickness of an internal shim (usually about 1/8 in), and
t_c = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in)

</td><td>

E2.4 Overall Height, Example 1.5

$$H = 22(0.25) + 21(0.125) + 2*1.5 = 11.125\ in$$

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E2.5 Size Checks

Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows:

$$\frac{B}{3} \geq d_L \geq \frac{B}{6} \qquad \text{(E-10)}$$

Art. 12.2 GSID requires that the isolation system provides a lateral restoring force at d_t greater than the restoring force at $0.5d_t$ by not less than $W/80$. This equates to a minimum K_d of $0.025W/d$.

$$K_{d,min} = \frac{0.025W}{d}$$

</td><td>

E2.5 Size Checks, Example 1.5

Since B=9.0 check

$$\frac{9.0}{3} \geq d_L \geq \frac{9.0}{6}$$

i.e., $3.0 \geq d_L \geq 1.5$

Since d_L = 1.61, lead core size is acceptable.

$$K_{d,min} = \frac{0.025W}{d} = \frac{0.025(45.52)}{2.27} = 0.40\ k/in$$

As

$$K_d = \frac{GA_b}{T_r} = \frac{0.1(61.56)}{5.5} = 1.12k/in > K_{d,min}$$

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E3. Strain Limit Check

Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e.,

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5 \qquad \text{(E-11)}$$

where γ_c, $\gamma_{s,eq}$, and γ_r are defined below.

(a) γ_c is the maximum shear strain in the layer due to compression and is given by:

$$\gamma_c = \frac{D_c \sigma_S}{GS} \qquad \text{(E-12)}$$

where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = {P_{DL}}/{A_b}$, G is shear modulus, and S is the layer shape factor given by:

$$S = \frac{A_b}{\pi B t_r} \qquad \text{(E-13)}$$

(b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by:

</td><td>

E3. Strain Limit Check, Example 1.5

Since

$$\sigma_S = \frac{45.52}{61.57} = 0.739\ ksi$$

G = 0.1 ksi

and

$$S = \frac{61.57}{\pi 9.0(0.25)} = 8.71$$

then

$$\gamma_c = \frac{1.0(0.739)}{0.1(8.71)} = 0.849$$

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</table>

$\gamma_{s,eq} = \frac{d_t}{T_r}$ (E-14) (c) γ_r is the shear strain due to rotation and is given by: $\gamma_r = \frac{D_r B^2 \theta}{t_r T_r}$ (E-15) where *Dr* is shape coefficient for rotation in circular bearings = 0.375, and θ is design rotation due to DL, LL and construction effects Actual value for θ may not be known at this time and value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1)		$\gamma_{s,eq} = \frac{1.89}{5.5} = 0.344$ $\gamma_r = \frac{0.375(9.0^2)(0.01)}{0.25(5.5)} = 0.221$ Substitution in Eq E-11 gives $\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.849 + 0.344 + 0.5(0.221) = 1.30 \leq 5.5\ OK$
E4. Vertical Load Stability Check Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state. Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either 2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or 1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4.		**E4. Vertical Load Stability Check, Example 1.5**
E4.1 Vertical Load Stability in Undeformed State The critical load capacity of an elastomeric isolator at zero shear displacement is given by $P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2}\left[\sqrt{(1 + \frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2})} - 1\right]$ (E-16) where $H_{eff} = T_r + T_s$ T_s = total shim thickness $K_\theta = {E_b I}/{T_r}$ $E_b = E(1 + 0.67S^2)$ E = elastic modulus of elastomer = $3G$ $I = {\pi B^4}/{64}$ It is noted that typical elastomeric isolators have high shape factors, *S*, in which case: $\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1$ (E-17) and Eq. E-16 reduces to: $P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta}$ (E-18) Check that:		**E4.1 Vertical Load Stability in Undeformed State, Example 1.5** $E = 3G = 3(0.1) = 0.3\ ksi$ $E_b = 0.3(1 + 0.67(8.71^2)) = 15.55\ ksi$ $I = \pi\frac{9.0^4}{64} = 322.1\ in^4$ $K_\theta = \frac{15.55(322.1)}{5.5} = 910.15\ kin/rad$ $K_d = \frac{GA_b}{T_r} = \frac{0.1(61.57)}{5.5} = 1.12\ k/in$ $P_{cr(\Delta=0)} = \pi\sqrt{1.12(910.15)} = 100.27\ k$

$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} \geq 3 \quad \text{(E-19)}$$		$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \frac{100.27}{(45.52+15.5)} = 1.64 \ngeq 3 \quad NOK$$
E4.2 Vertical Load Stability in Deformed State The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by: $$P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)} \quad \text{(E-20)}$$ where A_r = overlap area between top and bottom plates of isolator at displacement Δ (Fig. 2.2-1 GSID) = $B^2(\delta - sin\delta)/_4$ δ = $2cos^{-1}(^{\Delta}/_{B})$ A_{gross} = $\pi\, ^{B^2}/_4$ It follows that: $$\frac{A_r}{A_{gross}} = \frac{(\delta - sin\delta)}{\pi} \quad \text{(E-21)}$$ Check that: $$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} \geq 1 \quad \text{(E-22)}$$		**E4.2 Vertical Load Stability in Deformed State, Example 1.5** Since bridge is in Zone 2, $\Delta = 2d_t = 2(1.89) = 3.79\ in$ $$\delta = 2cos^{-1}\left(\frac{3.79}{9.0}\right) = 2.27$$ $$\frac{A_r}{A_{gross}} = \frac{(2.27 - sin2.27)}{\pi} = 0.480$$ $$P_{cr(\Delta)} = 0.480(100.27) = 48.15\ k$$ $$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} = \frac{48.15}{1.2(45.52)+6.35} = 0.79 \ngeq 1 \quad NOK$$
E5. Design Review		**E5. Design Review, Example 1.5** The basic dimensions of the isolator designed above are as follows: 10.00 in (od) x 11.125 in (high) x 1.61 in dia. lead core and the volume, excluding steel end and cover plates, is 638 in^3. This design satisfies the shear strain limit criteria, but not the vertical load stability ratio in the undeformed and deformed states. A redesign is therefore required and the easiest way to increase the P_{cr} is to increase the shape factor, *S*, since the bending stiffness of an isolator is a function of the shape factor squared. See equations in Step E4.1. To increase *S*, increase the bonded area A_b while keeping t_r constant (Eq. E-13). But to keep K_d constant while increasing A_b and T_r is constant, decrease the shear modulus, *G* (Eq. E-6). This redesign is outlined below. After repeating the calculation for diameter of lead core, the process begins by reducing the shear modulus to 60 psi (0.06

ksi) and increasing the bonded diameter to 12 in.

E2.1

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{2.34}{0.9}} = 1.61\ in$$

E2.2

$$A_b = \frac{T_r K_d}{G}\ in^2 = \frac{5.5(1.17)}{0.06} = 107.25\ in^2$$

$$B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} = \sqrt{\frac{4\ (107.25)}{\pi} + 1.61^2}$$
$$= 11.80\ in$$

Round B to 12 in and the actual bonded area becomes:

$$A_b = \frac{\pi}{4}(12^2 - 1.61^2) = 111.06\ in^2$$

$$B_o = 12 + 2(0.5) = 13 \text{ in}$$

E2.3

$$T_r = \frac{GA_b}{K_d} = \frac{0.06(111.06)}{1.17} = 5.71\ in$$

$$n = \frac{5.71}{0.25} = 22.82$$

Round to nearest integer, i.e. $n = 23$.

E2.4

$$H = 23(0.25) + 22(0.125) + \ 2 * 1.5 = 11.5\ in$$

E2.5

Since B=12 check

$$\frac{12}{3} \geq d_L \geq \frac{12}{6}$$

$$\text{i.e., } 4 \geq d_L \geq 2$$

Since d_L = 1.61, the size of lead core is too small, and there are 2 options: (1) Accept the undersize and check for adequate performance during the Quality Control Tests required by GSID Art. 15.2.2; or (2) Only have lead cores in every second isolator, in which case the core diameter, in those isolators with cores, will be √2 x 1.61 = 2.27 in (which satisfies above criterion).

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(111.06)}{5.75} = 1.16k/in\ > K_{d,min}$$

E3.

$$\sigma_S = \frac{45.52}{111.06} = 0.41\ ksi$$

$$S = \frac{111.06}{\pi 12(0.25)} = 11.78$$

$$\gamma_c = \frac{1.0(0.41)}{0.06(11.78)} = 0.580$$

$$\gamma_{s,eq} = \frac{1.89}{5.75} = 0.329$$

$$\gamma_r = \frac{0.375(12^2)(0.01)}{0.25(5.75)} = 0.376$$

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.580 + 0.329 + 0.5(0.376) = 1.10 \leq 5.5\ OK$$

E4.1

$$E = 3G = 3(0.06) = 0.18\ ksi$$

$$E_b = 0.18(1 + 0.67(11.78^2)) = 16.93\ ksi$$

$$I = \pi \frac{12^4}{64} = 1017.88\ in^4$$

$$K_\theta = \frac{16.93(1017.88)}{5.75} = 2996.42\ kin/rad$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(111.06)}{5.75} = 1.159\ k/in$$

$$P_{cr(\Delta=0)} = \pi\sqrt{1.159(2996.42)} = 185.13\ k$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} = \frac{185.13}{(45.52 + 15.50)} = 3.03 \geq 3\quad OK$$

E4.2

$$\delta = 2cos^{-1}\left(\frac{3.79}{12}\right) = 2.50$$

$$\frac{A_r}{A_{gross}} = \frac{(2.50 - sin2.50)}{\pi} = 0.605$$

$$P_{cr(\Delta)} = 0.605(185.13) = 111.96\ k$$

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{111.96}{1.2(45.52) + 6.35} = 1.84 \geq 1\ OK$$

E5.

The basic dimensions of the redesigned isolator are as follows:

13.0 in (od) x 11.5 in (high) x 1.61 in dia. lead core and its volume (excluding steel end and cover plates) is 1128 in^3.

This design meets all the design criteria but is about 75% larger by volume than the previous design. This increase in size is driven by the need to satisfy the vertical load stability ratio of 3.0 in the undeformed state.

E6. Minimum and Maximum Performance Check

Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E6-1.

Table E6-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-23)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-24)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-25)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-26)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E6-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})(\lambda_{min,a,Kd})(\lambda_{min,v,Kd})(\lambda_{min,tr,Kd})(\lambda_{min,c,Kd})(\lambda_{min,scrag,Kd})$	(E-27)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})(\lambda_{max,a,Kd})(\lambda_{max,v,Kd})(\lambda_{max,tr,Kd})(\lambda_{max,c,Kd})(\lambda_{max,scrag,Kd})$	(E-28)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})(\lambda_{min,a,Qd})(\lambda_{min,v,Qd})(\lambda_{min,tr,Qd})(\lambda_{min,c,Qd})(\lambda_{min,scrag,Qd})$	(E-29)

E6. Minimum and Maximum Performance Check, Example 1.5

Minimum Property Modification factors are:

$\lambda_{min,Kd} = 1.0$
$\lambda_{min,Qd} = 1.0$

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are:

$\lambda_{max,a,Kd} = 1.1$
$\lambda_{max,a,Qd} = 1.1$

$\lambda_{max,t,Kd} = 1.1$
$\lambda_{max,t,Qd} = 1.4$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})(\lambda_{max,a,Qd})(\lambda_{max,v,Qd})(\lambda_{max,tr,Qd})(\lambda_{max,c,Qd})(\lambda_{max,scrag,Qd})$	(E-30)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

- 1.00 for critical bridges
- 0.75 for essential bridges
- 0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd}$ = 1.0 + 0.1(0.66) = 1.066
$\lambda_{max,a,Qd}$ = 1.0 + 0.1(0.66) = 1.066

$\lambda_{max,t,Kd}$ = 1.0 + 0.1(0.66) = 1.066
$\lambda_{max,t,Qd}$ = 1.0 + 0.4(0.66) = 1.264

$\lambda_{max,scrag,Kd}$ = 1.0
$\lambda_{max,scrag,Qd}$ = 1.0

Therefore the maximum overall modification factors

$\lambda_{max,Kd}$ = 1.066(1.066)1.0 = 1.14
$\lambda_{max,Qd}$ = 1.066(1.264)1.0 = 1.35

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:
$Q_{d,max}$ = 1.35 (2.34) = 3.16 k
and
$K_{d,max}$=1.14(1.16) = 1.32 k/in

E7. Design and Performance Summary

E7.1 Isolator dimensions

Summarize final dimensions of isolators:

- Overall diameter (includes cover layer)
- Overall height
- Diameter lead core
- Bonded diameter
- Number of rubber layers
- Thickness of rubber layers
- Total rubber thickness
- Thickness of steel shims
- Shear modulus of elastomer

Check all dimensions with manufacturer.

E7. Design and Performance Summary, Example 1.5

E7.1 Isolator dimensions, Example 1.5

Isolator dimensions are summarized in Table E7.1-1.

Table E7.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Diam. lead core (in)
Under edge girder on Pier 1	17.0 x 17.0 x 11.50(H)	13.0 dia. x 10.0(H)	1.61

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in)	Total rubber thick-ness (in)	Steel shim thick-ness (in)
Under edge girder on Pier 1	23	0.25	5.75	0.125

	Shear modulus of elastomer = 60 psi
E7.2 Bridge Performance Summarize bridge performance • Maximum superstructure displacement (longitudinal) • Maximum superstructure displacement (transverse) • Maximum superstructure displacement (resultant) • Maximum column shear (resultant) • Maximum column moment (about transverse axis) • Maximum column moment (about longitudinal axis) • Maximum column torque Check required performance as determined in Step A3, is satisfied.	**E7.2 Bridge Performance, Example 1.5** Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 19.56 k. This less than the column plastic shear strength (25 k) and therefore the required performance criterion is satisfied (fully elastic behavior). The maximum longitudinal displacement is 2.07 in, which is slightly more than the 2.0 in available at the abutment expansion joints, and is barely acceptable (light pounding may occur but not likely to cause damage to the back wall).

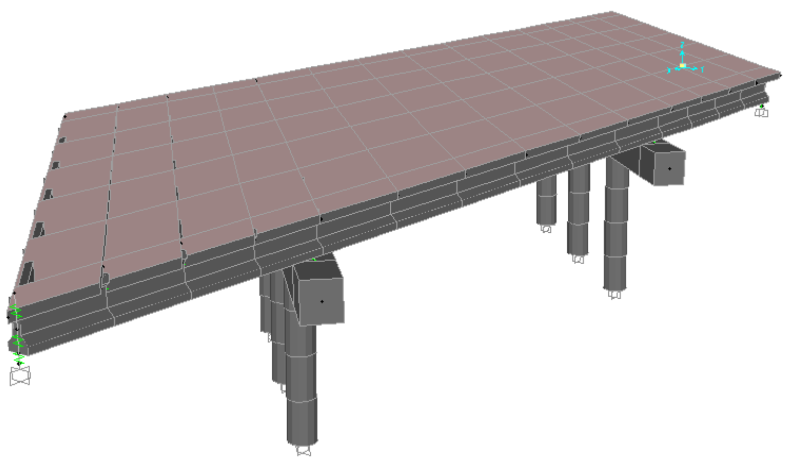
Table E7.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	2.07 in
Maximum superstructure displacement (transverse)	2.24 in
Maximum superstructure displacement (resultant)	2.32 in
Maximum column shear (resultant)	19.56 k
Maximum column moment about transverse axis	254 kft
Maximum column moment about longitudinal axis	256 kft
Maximum column torque	6.84 kft

SECTION 1

PC Girder Bridge, Short Spans, Multi-Column Piers

DESIGN EXAMPLE 1.6: Skew = 45^0



Design Examples in Section 1

ID	Description	S_1	Site Class	Pier height	Skew	Isolator type
1.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
1.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
1.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
1.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
1.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
1.5	Change column height	0.2g	B	H_1=0.5 H_2	0	Lead rubber bearing
1.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

DESIGN PROCEDURE	DESIGN EXAMPLE 1.6 (Skew = 45^0)
STEP A: BRIDGE AND SITE DATA	

A1. Bridge Properties
Determine properties of the bridge:
• number of supports, m
• number of girders per support, n
• angle of skew
• weight of superstructure including railings, curbs, barriers and to the permanent loads, W_{SS}
• weight of piers participating with superstructure in dynamic response, W_{PP}
• weight of superstructure, W_j, at each support
• stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
• column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
• allowable movement at expansion joints
• isolator type if known, otherwise to be determined
	A1. Bridge Properties, Example 1.6
	• Number of supports, $m = 4$
	  o North Abutment ($m = 1$)
	  o Pier 1 ($m = 2$)
	  o Pier 2 ($m = 3$)
	  o South Abutment ($m = 4$)
	• Number of girders per support, $n = 6$
	• Number of columns per support = 3
	• Angle of skew = 45^0
	• Weight of superstructure including permanent loads, $W_{SS} = 678.62$ k
	• Weight of superstructure at each support:
	  o $W_1 = 51.98$ k
	  o $W_2 = 287.33$ k
	  o $W_3 = 287.33$ k
	  o $W_4 = 51.98$ k
	• Participating weight of piers, $W_{PP} = 151.52$ k
	• Effective weight (for calculation of period), $W_{eff} = W_{SS} + W_{PP} = 830.14$ k
	• Stiffness of each pier in the longitudinal direction:
	  o $K_{sub,pier1,long} = 307$ k/in
	  o $K_{sub,pier2,long} = 307$ k/in
	• Stiffness of each pier in the transverse direction:
	  o $K_{sub,pier1,trans} = 307$ k/in
	  o $K_{sub,pier2,trans} = 307$ k/in
	• Minimum column shear strength based on flexural yield capacity of column = 25 k
	• Displacement capacity of expansion joints (longitudinal) = 2.0 in for thermal and other movements.
	• Lead rubber isolators

A2. Seismic Hazard Determine seismic hazard at site: • acceleration coefficients • site class and site factors • seismic zone Plot response spectrum. Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of PGA, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD). Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for	**A2. Seismic Hazard, Example 1.6** Acceleration coefficients for bridge site are given in design statement as follows: • $PGA = 0.40$ • $S_1 = 0.20$ • $S_S = 0.75$ Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec. Table 3.10.3.1-1 LRFD gives Site Class as B. Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors: • $F_{pga} = 1.0$ • $F_a = 1.0$ • $F_v = 1.0$

conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD.

Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows:

- $A_s = F_{pga}\ PGA$
- $S_{DS} = F_a\ S_S$
- $S_{D1} = F_v\ S_1$

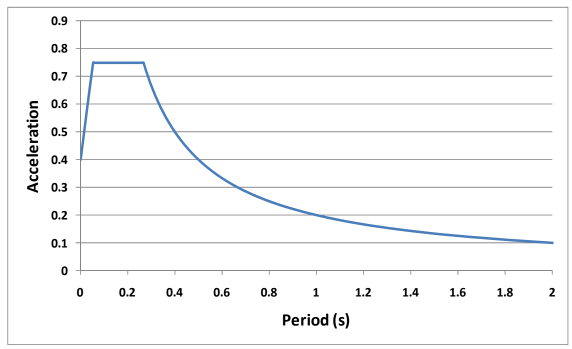
Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table5-1 GSID.

These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.

- $A_s = F_{pga}\ PGA = 1.0(0.40) = 0.40$
- $S_{DS} = F_a\ S_S = 1.0(0.75) = 0.75$
- $S_{D1} = F_v\ S_1 = 1.0(0.20) = 0.20$

Since $0.15 < S_{D1} \leq 0.30$, bridge is located in Seismic Zone 2.

Design Response Spectrum is as below:



A3. Performance Requirements

Determine required performance of isolated bridge during Design Earthquake (1000-yr return period).

Examples of performance that might be specified by the Owner include:

- Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake.
- Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake.
- For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements)
- Reduced substructure forces for bridges on weak soils to reduce foundation costs.

A3. Performance Requirements, Example 1.6

In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield).

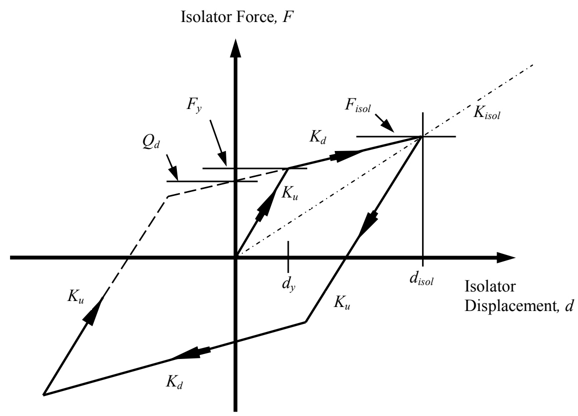
To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (25 k).

The maximum shear in the column must therefore be less than 25 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties

To begin the iterative solution, an estimate is required of :

(1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)

Art C7.1 GSID

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \qquad \text{(B-1)}$$

(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not occur under non-seismic loads (e.g. wind)

B1.1 Initial System Displacement and Properties, Example 1.6

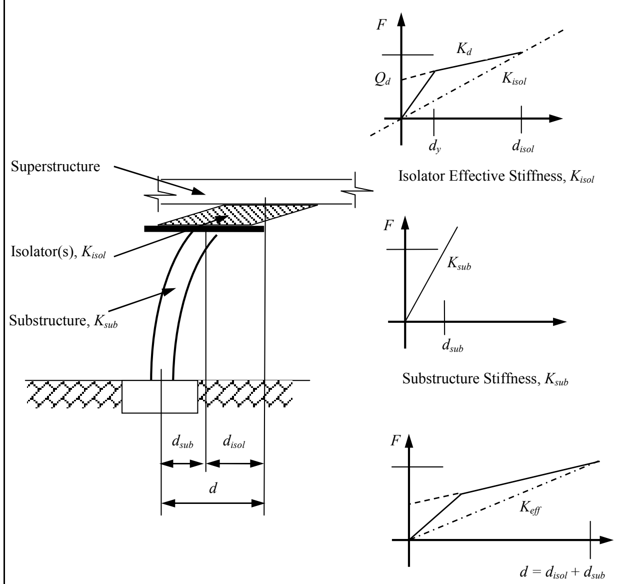
$$d \cong 10 S_{D1} = 10(0.20) \cong 2.0\ in$$

<table>
<tr><td>but low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e.
$$Q_d = 0.05W \quad \text{(B-2)}$$
(3) Post-yield stiffness, K_d
Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3.

Art. 12.2 GSID
$$K_{d,min} \geq \frac{0.025W}{d} \quad \text{(B-3)}$$
Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$</td><td></td><td>$$Q_d = 0.05W = 0.05(678.62) = 33.93\ k$$
$$K_d = 0.05\frac{W}{d} = 0.05\frac{678.62}{2.0} = 16.97\ k/in$$</td></tr>
<tr><td>B1.2 Initial Isolator Properties at Supports
Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support:
$$Q_{d,j} = Q_d\frac{W_j}{W} \quad \text{(B-4)}$$
and
$$K_{d,j} = K_d\frac{W_j}{W} \quad \text{(B-5)}$$</td><td></td><td>B1.2 Initial Isolator Properties at Supports, Example 1.6
$$Q_{d,j} = Q_d\frac{W_j}{W}$$
o $Q_{d,\,1}$ = 2.60 k
o $Q_{d,\,2}$ = 14.37 k
o $Q_{d,\,3}$ = 14.37 k
o $Q_{d,\,4}$ = 2.60 k

and
$$K_{d,j} = K_d\frac{W_j}{W}$$
o $K_{d,1}$ = 1.30 k/in
o $K_{d,2}$ = 7.18 k/in
o $K_{d,3}$ = 7.18 k/in
o $K_{d,4}$ = 1.30 k/in</td></tr>
<tr><td>B1.3 Effective Stiffness of Combined Pier and Isolator System
Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID).

An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula as follows (MCEER 2006):
$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j} \quad \text{(B-6)}$$
where
$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}} \quad \text{(B-7)}$$
and $K_{sub,j}$ for the piers are given in Step A1. For the abutments, take $K_{sub,j}$ to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high</td><td></td><td>B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 1.6
$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$
o $\alpha_1 = 2.60\text{x}10^{-4}$
o $\alpha_2 = 4.79\text{x}10^{-2}$
o $\alpha_3 = 4.79\text{x}10^{-2}$
o $\alpha_4 = 2.60\text{x}10^{-4}$
$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$$
o $K_{eff,1}$ = 2.60 k/in
o $K_{eff,2}$ = 14.04 k/in
o $K_{eff,3}$ = 14.04 k/in
o $K_{eff,4}$ = 2.60 k/in</td></tr>
</table>

values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.  Isolator Effective Stiffness, K_{isol} Substructure Stiffness, K_{sub} Combined Effective Stiffness, K_{eff}		
B1.4 Total Effective Stiffness Calculate the total effective stiffness, K_{eff}, of the bridge: Eq. 7.1-6 GSID $\qquad K_{eff} = \sum_{j=1}^{m} K_{eff,j} \qquad$ (B-8)		**B1.4 Total Effective Stiffness, Example 1.6** $K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 33.27\ k/in$
B1.5 Isolation System Displacement at Each Support Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports: $d_{isol,j} = \dfrac{d}{1+\alpha_j} \qquad$ (B-9)		**B1.5 Isolation System Displacement at Each Support, Example 1.6** $d_{isol,j} = \dfrac{d}{1+\alpha_j}$ o $d_{isol,1}$ = 2.00 in o $d_{isol,2}$ = 1.91 in o $d_{isol,3}$ = 1.91 in o $d_{isol,4}$ = 2.00 in
B1.6 Isolation System Stiffness at Each Support Calculate the effective stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports: $K_{isol,j} = \dfrac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \qquad$ (B-10)		**B1.6 Isolation System Stiffness at Each Support, Example 1.6** $K_{isol,j} = \dfrac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$ o $K_{isol,1}$ = 2.60 k/in o $K_{isol,2}$ = 14.71 k/in o $K_{isol,3}$ = 14.71 k/in o $K_{isol,4}$ = 2.60 k/in

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B1.7 Substructure Displacement at Each Support
Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports:

$$d_{sub,j} = d - d_{isol,j} \quad \text{(B-11)}$$

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B1.7 Substructure Displacement at Each Support, Example 1.6

$$d_{sub,j} = d - d_{isol,j}$$

- $d_{sub,1}$ = 5.20x10^{-4} in
- $d_{sub,2}$ = 9.15x10^{-2} in
- $d_{sub,3}$ = 9.15x10^{-2} in
- $d_{sub,4}$ = 5.20x10^{-4} in

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B1.8 Lateral Load in Each Substructure Support
Calculate the shear at support 'j', $F_{sub,j}$, for all supports:

$$F_{sub,j} = K_{sub,j} d_{sub,j} \quad \text{(B-12)}$$

where values for $K_{sub,j}$ are given in Step A1.

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B1.8 Lateral Load in Each Substructure, Example 1.6

$$F_{sub,j} = K_{sub,j} d_{sub,j}$$

- $F_{sub,1}$ = 5.20 k
- $F_{sub,2}$ = 28.08 k
- $F_{sub,3}$ = 28.08 k
- $F_{sub,4}$ = 5.20 k

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B1.9 Column Shear Force at Each Support
Calculate the shear in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j':

$$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j} \quad \text{(B-13)}$$

Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.

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B1.9 Column Shear Force at Each Support, Example 1.6

$$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$$

- $F_{col,2,1\text{-}3} \approx$ 9.36 k
- $F_{col,3,1\text{-}3} \approx$ 9.36 k

These column shears are less than the yield capacity of each column (25 k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.

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B1.10 Effective Period and Damping Ratio
Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge:

Eq. 7.1-5 GSID

$$T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} \quad \text{(B-14)}$$

and

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} \quad \text{(B-15)}$$

where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j}$ = 0. For other types of isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for the Simplified Method, set $d_{y,j}$ = 0 for all isolator

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B1.10 Effective Period and Damping Ratio, Example 1.6

$$T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{830.14}{386.4(33.27)}}$$

= 1.60 sec

and taking $d_{y,j}$ = 0:

$$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - 0)}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.31$$

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types. See Step B2.2 where the value of $d_{y,j}$ is revisited for the Multimode Spectral Analysis Method.

B1.11 Damping Factor

Calculate the damping factor, B_L, and the displacement, d, of the bridge:

Eq. 7.1-3 GSID
$$B_L = \begin{cases} \left(\frac{\xi}{0.05}\right)^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases} \quad \text{(B-16)}$$

Eq. 7.1-4 GSID
$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \quad \text{(B-17)}$$

B1.11 Damping Factor, Example 1.6

Since $\xi = 0.31 > 0.3$

$$B_L = 1.70$$

and

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.2)1.60}{1.70} = 1.84\ in$$

B1.12 Convergence Check

Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement.

This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5).

After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both.

Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)

B1.12 Convergence Check, Example 1.6

Since the calculated value for displacement, d (=1.84 in) is not close to that assumed at the beginning of the cycle (Step B1.1, d = 2.0), use the value of 1.84 in as the new assumed displacement and repeat from Step B1.3.

After one iteration, convergence is reached at a superstructure displacement of 1.80 in, with an effective period of 1.55 seconds, and a damping factor of 1.70 (33% damping ratio). The displacement in the isolators at Pier 1 is 1.71 in and the effective stiffness of the same isolators is 15.57 k/in.

See spreadsheet in Table B1.12-1 for results of final iteration.

Ignoring the weight of the cap beams, the sum of the column shears at a pier must equal the total isolator shear force. Hence, approximate column shear (per column) = 15.57(1.71)/3 = 8.87 k which is less than the maximum allowable (25k) if elastic behavior is to be achieved (as required in Step A3).

Also the superstructure displacement = 1.80 in, which is less than the available clearance of 2.0 in.

Therefore the above solution is acceptable, and go to Step B2.

Note that available clearance (2.0 in) is greater than minimum required which is given by:

$$= \frac{8\, S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.55}{1.7} = 1.46 in$$

Table B1.12-1 Simplified Method Solution for Design Example 1.6 – Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	678.62	151.52	830.137	0.2	6							
Step B1.1	d	1.80	Assumed displacement									
	Q_d	32.53	Characteristic strength									
	K_d	16.26	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	51.98	2.60	1.30	10000	2.74E-04	2.74	1.80	2.74	4.94E-04	4.94	4.677	8.887
Pier 1	287.33	14.37	7.18	307.0	5.07E-02	14.82	1.71	15.57	8.69E-02	26.67	24.609	48.004
Pier 2	287.33	14.37	7.18	307.0	5.07E-02	14.82	1.71	15.57	8.69E-02	26.67	24.609	48.004
Abut 2	51.98	2.60	1.30	10000	2.74E-04	2.74	1.80	2.74	4.94E-04	4.94	4.677	8.887
Total	678.62	33.931	16.965		$\Sigma K_{eff,j}$	35.123				63.217	58.572	113.783
					Step	**B1.4**						
Step B1.10	T_{eff}	1.55	Effective period									
	ξ	0.33	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.76										
	B_L	1.70	Damping Factor									
	d	1.79	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	0.433	0.217	0.457		1.65	0.479						
Pier 1	2.394	1.197	2.595		1.54	2.752						
Pier 2	2.394	1.197	2.595		1.54	2.752						
Abut 2	0.433	0.217	0.457		1.65	0.479						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength
Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \qquad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \qquad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 1.6
Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-1) by the number of isolators at each support (n = 6), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,\,1}$ = 2.60/6 = 0.43 k
- $Q_{d,\,2}$ = 14.37/6= 2.39 k
- $Q_{d,\,3}$ = 14.37/6= 2.39 k
- $Q_{d,\,4}$ = 2.60/6 = 0.43 k

and

- $K_{d,1}$ = 1.30/6 = 0.22 k/in
- $K_{d,2}$ = 7.18/6 = 1.20 k/in
- $K_{d,3}$ = 7.18/6 = 1.20 k/in
- $K_{d,4}$ = 1.30/6 = 0.22 k/in

B2.2 Initial Stiffness and Yield Displacement
Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

(1) For friction-based isolators: $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \qquad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \qquad \text{(B-22)}$$

B2.2 Initial Stiffness and Yield Displacement, Example 1.6
Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(1.20) = 12.0\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{2.39}{(12.0 - 1.20)} = 0.22\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2 in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i} = 0$.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$
Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

$$k_{isol,i} = \frac{K_{isol,j}}{n} \qquad \text{(B -23)}$$

B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 1.6
Dividing the results for K_{isol} (Step B1.12) among the 6 isolators at each support, the following values for K_{isol} /isolator are obtained:

- $K_{isol,1}$ = 2.74/6 = 0.46 k/in
- $K_{isol,2}$ = 15.57/6 = 2.60 k/in

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- $K_{isol,3}$ = 15.57/6 = 2.60 k/in
- $K_{isol,4}$ = 2.74/6 = 0.46 k/in

</td></tr>
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B2.4 Three-Dimensional Bridge Model
Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is justified.

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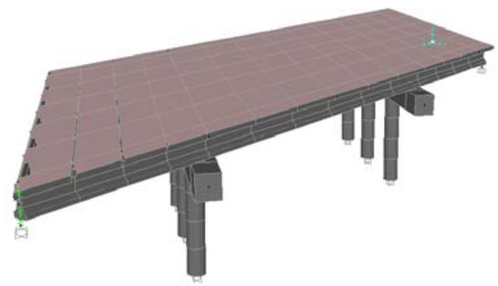
B2.4 Three-Dimensional Bridge Model, Example 1.6
A 3-dimensional finite element model was developed for this Step, as shown below.



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B2.5 Composite Design Response Spectrum
Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.

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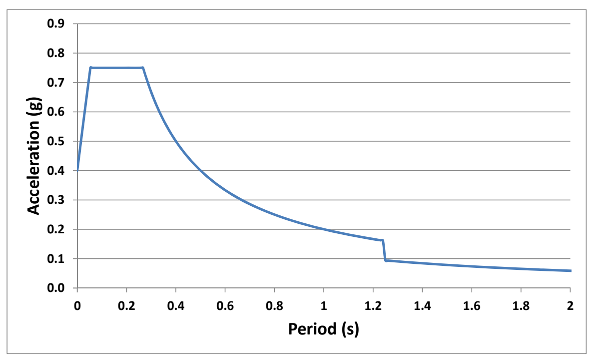
B.2.5 Composite Design Response Spectrum, Example 1.6
From the final results of Simplified Method (Step B1.12), B_L = 1.70 and T_{eff} = 1.55 sec. Hence the transition in the composite spectrum from 5% to 33% damping occurs at 0.8 T_{eff} = 0.8 (1.55) = 1.24 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods $\geq$ 1.24 sec by 1.70.



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B2.6 Multimodal Analysis of Finite Element Model
Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

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B2.6 Multimodal Analysis of Finite Element Model, Example 1.6
Results of modal analysis of the example bridge are summarized in Table B2.6-1, in which the X-direction is along the bridge (longitudinal), and the Y-direction is across the bridge (transverse). Here the modal periods and mass participation factors of the first 12

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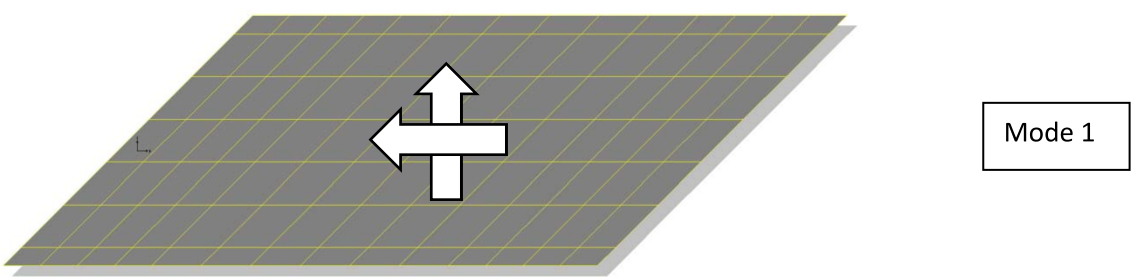
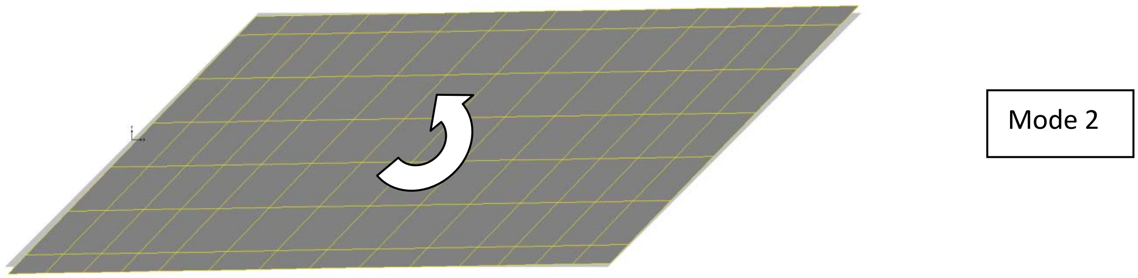
modes are given. The first three modes are the principal isolation modes with periods of 1.47, 1.40 and 1.39 sec respectively. Mode shapes corresponding to these three modes are plotted in Figure B2.6-1. As can be seen, the first and third modes are coupled translational modes whereas the second mode is a pure torsional mode (rotation about the Z-axis). Figure B2.6-1 also shows that the first and second modes have approximately equal displacement in the longitudinal and transverse directions, and this is confirmed by the relative sizes of the mass participation factors in Table B2.6-1. The strong nature of coupling in these modes would explain the presence of significant discrepancies between the Simplified Method and the Multimodal Analysis.

Table B2.6-1 Modal Properties of Bridge Example 1.6 – First Iteration

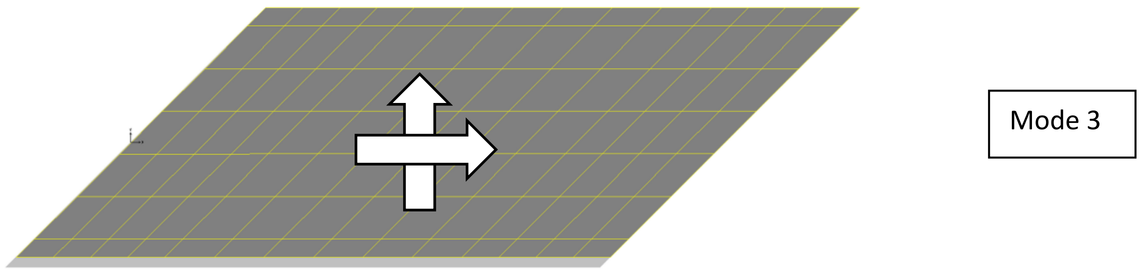
Mode	Period	Modal Participating Mass Ratios					
No.	Sec	**UX**	**UY**	**UZ**	**RX**	**RY**	**RZ**
1	1.466	0.352	0.399	0.000	0.016	0.001	0.274
2	1.401	0.000	0.000	0.000	0.000	0.000	0.236
3	1.392	0.384	0.337	0.000	0.016	0.001	0.232
4	0.227	0.000	0.000	0.000	0.000	0.000	0.012
5	0.227	0.071	0.071	0.000	0.006	0.000	0.048
6	0.201	0.000	0.000	0.000	0.000	0.000	0.000
7	0.201	0.000	0.000	0.000	0.000	0.000	0.012
8	0.149	0.006	0.008	0.000	0.537	0.024	0.006
9	0.140	0.000	0.000	0.313	0.000	0.232	0.000
10	0.114	0.000	0.000	0.000	0.000	0.000	0.014
11	0.108	0.086	0.085	0.000	0.063	0.002	0.058
12	0.103	0.003	0.003	0.000	0.000	0.000	0.002



Combined longitudinal & transverse translational mode



In-plane rotational mode



Combined longitudinal & transverse translational mode

Figure B2.6-1 First Three Mode Shapes for Isolated Bridge with 45° Skew (Example 1.6)

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 1.67 (1.80) in
- $d_{isol,2}$ = 1.56 (1.71) in
- $d_{isol,3}$ = 1.56 (1.71) in
- $d_{isol,4}$ = 1.67 (1.80) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 1.6

The new superstructure displacement is 1.67 in, a 7% difference from the displacement assumed at the start of the Multimode Spectral Analysis.

Go to Step B2.8 and update properties for a second cycle of iteration.

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

$$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \qquad \text{(B-24)}$$

Recalculate $K_{eff,j}$:

Eq. 7.1-6 GSID

$$K_{eff,j} = \frac{K_{sub,j}\sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \qquad \text{(B-25)}$$

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 1.6

Updated values for $K_{isol,I}$ (per isolator) are given below (previous values are in parentheses):

- $K_{isol,1}$ = 0.48 (0.46) k/in
- $K_{isol,1}$ = 2.73 (2.60) k/in
- $K_{isol,1}$ = 2.73 (2.60) k/in
- $K_{isol,1}$ = 0.48 (0.46) k/in

Updated values for $K_{eff,j}$(per support), ξ, B_L and T_{eff} are given below (previous values are in parentheses):

Recalculate system damping ratio, ξ :

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \quad \text{(B-26)}$$

Recalculate system damping factor, B_L:

Eq. 7.1-3 GSID

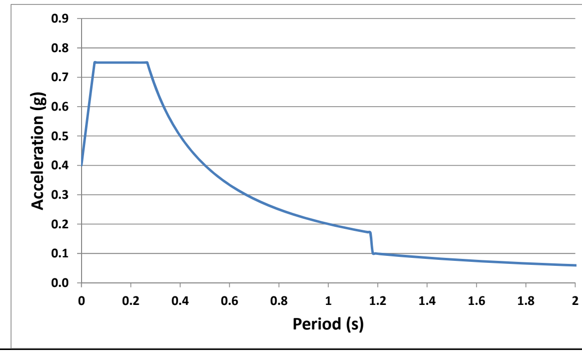
$$B_L = \begin{cases} \left(\frac{\xi}{0.05}\right)^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \quad \text{(B-27)}$$

Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.

- $K_{eff,1}$ = 2.86 (2.74) k/in
- $K_{eff,2}$ = 15.80 (14.82) k/in
- $K_{eff,3}$ = 15.80 (14.82) k/in
- $K_{eff,4}$ = 2.86 (2.74) k/in

- ξ = 28% (33%)
- B_L = 1.68 (1.70)
- T_{eff} = 1.47 (1.55) sec

The updated composite response spectrum is shown below:



B2.9 Superstructure and Isolator Displacements
Once convergence has been reached, obtain

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and
- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.

B2.9 Superstructure and Isolator Displacements, Example 1.6
From the above analysis:

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are:

  x_L= 1.17 in
  y_L= 1.17 in

- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are:
  - Abutments: u_L = 1.17 in, v_L = 1.17 in
  - Pier2: u_L = 1.09 in, v_L = 1.09 in

  All isolators at same support have the same displacements.

B2.10 Pier Bending Moments and Shear Forces
Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.

B2.10 Pier Bending Moments and Shear Forces, Example 1.6
Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are:

Exterior Columns:
- M_{PLL}= 170 kft
- M_{PTL}= 170 kft
- V_{PLL}= 15.34 k
- V_{PTL}= 15.28 k

Interior Columns:
- M_{PLL}= 147 kft
- M_{PTL}= 147 kft
- V_{PLL}= 13.40 k
- V_{PTL}= 13.34 k

Both piers have the same distribution of bending

moments and shear forces among the columns.

B2.11 Isolator Shear and Axial Forces
Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 1.6
Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
North Abutment	1	0.56	0.55	1.10
	2	0.56	0.56	1.07
	3	0.56	0.56	0.79
	4	0.56	0.56	0.42
	5	0.56	0.56	0.79
	6	0.56	0.56	1.18
Pier 1	1	2.98	2.95	5.69
	2	2.98	2.96	5.84
	3	2.99	2.97	0.16
	4	2.99	2.97	0.42
	5	2.98	2.96	4.91
	6	2.97	2.95	6.43
Pier 2	1	2.97	2.95	6.43
	2	2.98	2.96	4.91
	3	2.99	2.98	0.42
	4	2.99	2.98	0.16
	5	2.98	2.96	5.84
	6	2.97	2.95	5.69
South Abutment	1	0.56	0.56	1.18
	2	0.56	0.56	0.79
	3	0.56	0.56	0.41
	4	0.56	0.56	0.79
	5	0.56	0.56	1.06
	6	0.56	0.56	1.10

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 1.6

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.40 sec
- Maximum superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows:

  North Abutment x_T = 1.17 and y_T = 1.18 in
  Pier 1 x_T = 1.18 and y_T = 1.19 in
  Pier 1 x_T = 1.18 and y_T = 1.19 in
  North Abutment x_T = 1.18 and y_T = 1.19 in

- Maximum isolator displacements in the longitudinal (u_T) and transverse (v_T) directions are as follows:

  North Abutment u_T = 1.18 and v_T = 1.20 in
  Pier 1 u_T = 1.10 and v_T = 1.11 in
  Pier 1 u_T = 1.10 and v_T = 1.10 in
  North Abutment u_T = 1.19 and v_T = 1.19 in

- Maximum bending moments and shear forces in the columns in the longitudinal (M_{PLL},V_{PLL}) and transverse (M_{PTL},V_{PTL}) directions are:

  Exterior Columns:
  - M_{PLT}= 171 kft
  - M_{PTT}= 171 kft
  - V_{PLT}= 15.30 k
  - V_{PTT}= 15.31 k

  Interior Columns:
  - M_{PLT}= 147 kft
  - M_{PTT}= 147 kft
  - V_{PLT}= 13.23 k
  - V_{PTT}= 13.23 k

- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
North Abutment	1	0.56	0.57	1.19
	2	0.56	0.57	1.26
	3	0.56	0.57	0.98
	4	0.56	0.57	0.49
	5	0.56	0.57	0.89
	6	0.56	0.57	1.13
Pier 1	1	2.98	3.00	5.88
	2	3.00	3.02	6.66
	3	3.01	3.03	0.30
	4	3.01	3.03	0.55
	5	3.01	3.01	5.49
	6	3.00	3.00	6.69
Pier 2	1	2.97	2.98	6.69
	2	2.99	3.00	5.49
	3	3.00	3.01	0.55
	4	3.01	3.01	0.30
	5	3.00	2.99	6.66
	6	2.99	2.97	5.88
South Abutment	1	0.56	0.56	1.13
	2	0.56	0.56	0.88
	3	0.56	0.56	0.48
	4	0.56	0.56	0.97
	5	0.56	0.56	1.26
	6	0.56	0.56	1.19

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements

Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)
- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)
- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements at Pier 1, Example 1.6

To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

Load Case 1:

$u_1 = u_L + 0.3u_T$ = 1.0(1.09) + 0.3(1.09) = 1.42 in

$v_1 = v_L + 0.3v_T$ = 1.0(1.08) + 0.3(1.10) = 1.41 in

$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.42^2 + 1.41^2}$ = 2.00 in

Load Case 2:

$u_2 = 0.3u_L + u_T$ = 0.3(1.09) + 1.0(1.09) = 1.42 in

$v_2 = 0.3v_L + v_T$ = 0.3(1.08) + 1.0(1.10) = 1.42 in

$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{1.42^2 + 1.42^2}$ = 2.01 in

Governing Case:

Total design displacement, $d_t = \max(R_1, R_2)$
= 2.01 in

D2. Design Moments and Shears

Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

D2. Design Moments and Shears in Pier 1, Example 1.6

Design moments and shear forces are calculated for Pier 1, Column 1, below to illustrate the process.

Load Case 1:

$V_{PL1} = V_{PLL} + 0.3V_{PLT}$ = 1.0(15.29) + 0.3(15.18)
= 19.85 k

$V_{PT1} = V_{PTL} + 0.3V_{PTT}$ = 1.0(15.22) + 0.3(15.16)
= 19.77 k

$R_1 = \sqrt{V_{PL1}^2 + V_{PT1}^2} = \sqrt{19.85^2 + 19.77^2}$ = 28.01 k

Load Case 2:

$V_{PL2} = 0.3V_{PLL} + V_{PLT}$ = 0.3(15.29) + 1.0(15.18)
= 19.77 k

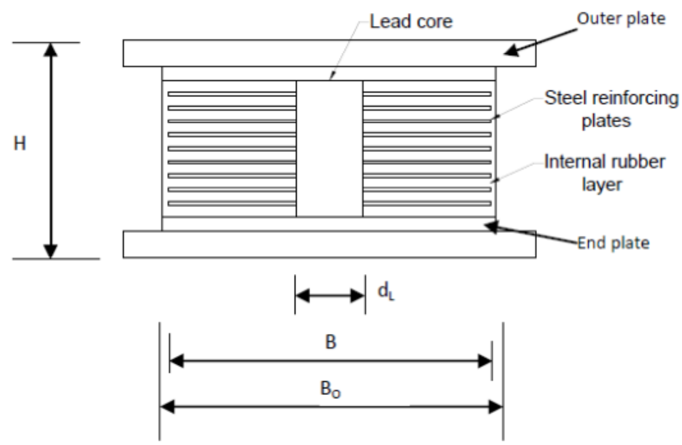
$V_{PT2} = 0.3V_{PTL} + V_{PTT}$ = 0.3(15.22) + 1.0(15.16)
= 19.73 k

$R_2 = \sqrt{V_{PL2}^2 + V_{PT2}^2} = \sqrt{19.77^2 + 19.73^2}$ = 27.93 k

Governing Case:

Design column shear = max (*R1*, *R2*)
= 28.01 k

STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS



A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness (K_u).With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness K_d is given by the rubber alone. More details are given in MCEER 2006.

While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary. When sizing the physical dimensions of the bearing, plan dimensions (B, d_L) should be rounded up to the next $^1/_4$" increment, while the total thickness of elastomer, T_r, is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are $^1/_4$" and $^3/_8$". High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in MCEER 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See MCEER 2006.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1). • required characteristic strength, Q_d / isolator • required post-elastic stiffness, K_d / isolator • total design displacement, d_t, for each isolator • maximum applied dead and live load (P_{DL}, P_{LL}) and seismic load (P_{SL}) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator, and • maximum wind load, P_{WL}		**E1. Required Properties, Example 1.6** The design of one of the exterior isolators on a pier is given below to illustrate the design process for lead-rubber isolators. From previous work • Q_d/isolator = 2.39 k • K_d/isolator = 1.20 k/in • Total design displacement, d_t = 2.01 in • P_{DL} = 38.42k • P_{LL} = 12.37 k • P_{SL} = 6.69 k • P_{WL} = 1.76 k < Q_d OK
E2. Isolator Sizing		
E2.1 Lead Core Diameter Determine the required diameter of the lead plug, d_L, using: $$d_L = \sqrt{\frac{Q_d}{0.9}} \quad \text{(E-1)}$$ See Step E2.5 for limitations on d_L		**E2.1 Lead Core Diameter, Example 1.6** $$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{2.39}{0.9}} = 1.63\ in$$

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E2.2 Plan Area and Isolator Diameter
Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.0 ksi.

Then the bonded area of the isolator is given by:

$$A_b = \frac{P_{DL} + P_{LL}}{1.0}\ in^2 \qquad \text{(E-2)}$$

and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by:

$$B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} \qquad \text{(E-3)}$$

Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using

$$A_b = \frac{\pi}{4}(B^2 - d_L^2) \qquad \text{(E-4)}$$

Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, B_o is given by:

$$B_o = B + 1.0 \qquad \text{(E-5)}$$

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E2.2 Plan Area and Isolator Diameter, Example 1.6

$$A_b = \frac{P_{DL} + P_{LL}}{1.0}\ in^2 = \frac{45.52 + 15.50}{1.0} = 50.79\ in^2$$

$$B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} = \sqrt{\frac{4\ (50.79)}{\pi} + 1.63^2}$$
= 8.21 in

Round B up to 8.25 in and the actual bonded area is:

$$A_b = \frac{\pi}{4}(8.25^2 - 1.63^2) = 51.37\ in^2$$

B_o = 8.25 + 2(0.5) = 9.25 in

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E2.3 Elastomer Thickness and Number of Layers
Since the shear stiffness of the elastomeric bearing is given by:

$$K_d = \frac{GA_b}{T_r} \qquad \text{(E-6)}$$

where G = shear modulus of the rubber, and
T_r = the total thickness of elastomer,
it follows Eq. E-5 may be used to obtain T_r given a required value for K_d

$$T_r = \frac{GA_b}{K_d} \qquad \text{(E-7)}$$

A typical range for shear modulus, G, is 60-120 psi. Higher and lower values are available and are used in special applications.

If the layer thickness is t_r, the number of layers, n, is given by:

$$n = \frac{T_r}{t_r} \qquad \text{(E-8)}$$

rounded up to the nearest integer.

Note that because of rounding the plan dimensions

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E2.3 Elastomer Thickness and Number of Layers, Example 1.6

Select G, shear modulus of rubber, = 100 psi (0.1 ksi)

Then

$$T_r = \frac{GA_b}{K_d} = \frac{0.1(51.37)}{1.20} = 4.29\ in$$

$$n = \frac{4.29}{0.25} = 17.16$$

Round to nearest integer, i.e. n = 18

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and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be necessary if the differences are large.	
E2.4 Overall Height The overall height of the isolator, H, is given by: $$H = n\,t_r + (n-1)t_s + 2t_c \quad \text{(E-9)}$$ where t_s = thickness of an internal shim (usually about 1/8 in), and t_c = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in)	**E2.4 Overall Height, Example 1.6** $$H = 18(0.25) + 17(0.125) + 2 * 1.5 = 9.625\ in$$
E2.5 Size Checks Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows: $$\frac{B}{3} \geq d_L \geq \frac{B}{6} \quad \text{(E-10)}$$ Art. 12.2 GSID requires that the isolation system provides a lateral restoring force at d_t greater than the restoring force at $0.5d_t$ by not less than $W/80$. This equates to a minimum K_d of $0.025W/d$. $$K_{d,min} = \frac{0.025W}{d}$$	**E2.5 Size Checks, Example 1.6** Since B=8.25 check $$\frac{8.25}{3} \geq d_L \geq \frac{8.25}{6}$$ i.e., $2.75 \geq d_L \geq 1.38$ Since d_L = 1.63, lead core size is acceptable. $$K_{d,min} = \frac{0.025W}{d} = \frac{0.025(38.42)}{2.17} = 0.44\ k/in$$ As $$K_d = \frac{GA_b}{T_r} = \frac{0.1(51.37)}{4.5} = 1.14k/in > K_{d,min}$$
E3. Strain Limit Check Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e., $$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5 \quad \text{(E-11)}$$ where $\gamma_c,\ \gamma_{s,eq}, and\ \gamma_r$ are defined below. (a) γ_c is the maximum shear strain in the layer due to compression and is given by: $$\gamma_c = \frac{D_c\sigma_S}{GS} \quad \text{(E-12)}$$ where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = {}^{P_{DL}}/_{A_b}$, G is shear modulus, and S is the layer shape factor given by: $$S = \frac{A_b}{\pi B t_r} \quad \text{(E-13)}$$ (b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by:	**E3. Strain Limit Check, Example 1.6** Since $$\sigma_S = \frac{38.42}{51.37} = 0.75\ ksi$$ G = 0.1 ksi and $$S = \frac{51.37}{\pi 8.25(0.25)} = 7.93$$ then $$\gamma_c = \frac{1.0(0.75)}{0.1(7.93)} = 0.943$$

$$\gamma_{s,eq} = \frac{d_t}{T_r} \qquad \text{(E-14)}$$

(c) γ_r is the shear strain due to rotation and is given by:

$$\gamma_r = \frac{D_r B^2 \theta}{t_r T_r} \qquad \text{(E-15)}$$

where *Dr* is shape coefficient for rotation in circular bearings = 0.375, and θ is design rotation due to DL, LL and construction effects Actual value for θ may not be known at this time and value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1)

$$\gamma_{s,eq} = \frac{2.01}{4.5} = 0.446$$

$$\gamma_r = \frac{0.375(8.25^2)(0.01)}{0.25(4.5)} = 0.227$$

Substitution in Eq E-11 gives

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.943 + 0.446 + 0.5(0.227)$$
$$= 1.50 \leq 5.5\ OK$$

E4. Vertical Load Stability Check

Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state.

Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either

2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or

1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4.

E4. Vertical Load Stability Check, Example 1.6

E4.1 Vertical Load Stability in Undeformed State

The critical load capacity of an elastomeric isolator at zero shear displacement is given by

$$P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2}\left[\sqrt{(1+\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2})} - 1\right] \qquad \text{(E-16)}$$

where

$H_{eff} = T_r + T_s$

T_s = total shim thickness

$K_\theta = {E_b I}/{T_r}$

$E_b = E(1 + 0.67S^2)$

E = elastic modulus of elastomer = $3G$

$I = {\pi B^4}/{64}$

It is noted that typical elastomeric isolators have high shape factors, *S*, in which case:

$$\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1 \qquad \text{(E-17)}$$

and Eq. E-16 reduces to:

$$P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta} \qquad \text{(E-18)}$$

Check that:

E4.1 Vertical Load Stability in Undeformed State, Example 1.6

$$E = 3G = 3(0.1) = 0.3\ ksi$$

$$E_b = 0.3(1 + 0.67(7.93^2)) = 12.93\ ksi$$

$$I = \pi\frac{8.25^4}{64} = 227.40\ in^4$$

$$K_\theta = \frac{12.93(227.40)}{4.5} = 653.56\ kin/rad$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.1(51.37)}{4.5} = 1.14\ k/in$$

$$P_{cr(\Delta=0)} = \pi\sqrt{1.14(653.56)} = 85.81\ k$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} \geq 3 \qquad \text{(E-19)}$$	$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \frac{85.81}{(38.42+12.37)} = 1.69 \ngeq 3 \quad NOK$$
E4.2 Vertical Load Stability in Deformed State The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by: $$P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)} \qquad \text{(E-20)}$$ where A_r = overlap area between top and bottom plates of isolator at displacement Δ (Fig. 2.2-1 GSID) = $B^2(\delta - sin\delta)/4$ δ = $2cos^{-1}(\Delta/B)$ A_{gross} = $\pi B^2/4$ It follows that: $$\frac{A_r}{A_{gross}} = \frac{(\delta - sin\delta)}{\pi} \qquad \text{(E-21)}$$ Check that: $$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} \geq 1 \qquad \text{(E-22)}$$	**E4.2 Vertical Load Stability in Deformed State, Example 1.6** Since bridge is in Zone 2, $\Delta = 2d_t = 2(2.01) = 4.02\ in$ $$\delta = 2cos^{-1}\left(\frac{4.02}{8.25}\right) = 2.13$$ $$\frac{A_r}{A_{gross}} = \frac{(2.13 - sin2.13)}{\pi} = 0.406$$ $$P_{cr(\Delta)} = 0.406(85.81) = 34.83\ k$$ $$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} = \frac{34.83}{1.2(38.42)+6.69} = 0.66 \ngeq 1 \quad NOK$$
E5. Design Review	**E5. Design Review, Example 1.6** The basic dimensions of the isolator designed above are as follows: 9.25 in (od) x 9.625 in (high) x 1.63 in dia. lead core and the volume, excluding steel end and cover plates, is 445 in^3. This design satisfies the shear strain limit criteria, but not the vertical load stability ratio in the undeformed and deformed states. A redesign is therefore required and the easiest way to increase the P_{cr} is to increase the shape factor, S, since the bending stiffness of an isolator is a function of the shape factor squared. See equations in Step E4.1. To increase S, increase the bonded area A_b while keeping t_r constant (Eq. E-13). But to keep K_d constant while increasing A_b and T_r is constant, decrease the shear modulus, G (Eq. E-6). This redesign is outlined below. After repeating the calculation for diameter of lead core, the process begins by reducing the shear modulus to 60 psi (0.06 ksi) and increasing the bonded diameter to 11 in.

E2.1

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{2.39}{0.9}} = 1.63\ in$$

E2.2

$$A_b = \frac{T_r K_d}{G}\ in^2 = \frac{4.5(1.20)}{0.06} = 90\ in^2$$

$$B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} = \sqrt{\frac{4\ (90)}{\pi} + 1.63^2} = 10.83\ in$$

Round B to 11 in and the actual bonded area becomes:

$$A_b = \frac{\pi}{4}(11^2 - 1.61^2) = 92.95\ in^2$$

$$B_o = 11 + 2(0.5) = 12 \text{ in}$$

E2.3

$$T_r = \frac{GA_b}{K_d} = \frac{0.06(92.95)}{1.20} = 4.66\ in$$

$$n = \frac{4.66}{0.25} = 18.63$$

Round to nearest integer, i.e. n = 19.

E2.4

$$H = 19(0.25) + 18(0.125) + 2 * 1.5 = 10\ in$$

E2.5

Since B=11 check

$$\frac{11}{3} \geq d_L \geq \frac{11}{6}$$

$$\text{i.e., } 3.67 \geq d_L \geq 1.83$$

Since d_L = 1.63, the size of lead core is too small, and there are 2 options: (1) Accept the undersize and check for adequate performance during the Quality Control Tests required by GSID Art. 15.2.2; or (2) Only have lead cores in every second isolator, in which case the core diameter, in those isolators with cores, will be √2 x 1.63 = 2.31 in (which satisfies above criterion).

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(92.95)}{4.75} = 1.17k/in > K_{d,min}$$

E3.

$$\sigma_S = \frac{38.42}{92.95} = 0.41\ ksi$$

$$S = \frac{92.95}{\pi 11(0.25)} = 10.76$$

$$\gamma_c = \frac{1.0(0.41)}{0.06(10.76)} = 0.640$$

$$\gamma_{s,eq} = \frac{2.01}{4.75} = 0.422$$

$$\gamma_r = \frac{0.375(11^2)(0.01)}{0.25(4.75)} = 0.382$$

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.640 + 0.422 + 0.5(0.382)$$
$$= 1.25 \le 5.5\ OK$$

E4.1

$$E = 3G = 3(0.06) = 0.18\ ksi$$

$$E_b = 0.18(1 + 0.67(10.76^2)) = 14.14\ ksi$$

$$I = \pi\frac{11^4}{64} = 718.69\ in^4$$

$$K_\theta = \frac{14.14(718.69)}{4.75} = 2139.24\ kin/rad$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(92.95)}{4.75} = 1.174\ k/in$$

$$P_{cr(\Delta=0)} = \pi\sqrt{1.174(2139.24)} = 157.44\ k$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} = \frac{157.44}{(38.42 + 12.37)} = 3.10 \ge 3\quad OK$$

E4.2

$$\delta = 2cos^{-1}\left(\frac{4.02}{11}\right) = 2.39$$

$$\frac{A_r}{A_{gross}} = \frac{(2.39 - sin2.39)}{\pi} = 0.546$$

$$P_{cr(\Delta)} = 0.546(157.44) = 85.96\ k$$

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{85.96}{1.2(38.42) + 6.69} = 1.63 \ge 1\ \ OK$$

E5.

The basic dimensions of the redesigned isolator are as follows:

12 in (od) x 10 in (high) x 1.63 in dia. lead core and its volume (excluding steel end and cover plates) is 792 in^3.

This design meets all the design criteria but is about 75% larger by volume than the previous design. This increase in size is dictated by the need to satisfy the vertical load stability ratio of 3.0 in the undeformed state.

E6. Minimum and Maximum Performance Check

Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E6-1.

Table E6-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-23)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-24)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-25)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-26)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E6-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})(\lambda_{min,a,Kd})(\lambda_{min,v,Kd})(\lambda_{min,tr,Kd})(\lambda_{min,c,Kd})(\lambda_{min,scrag,Kd})$	(E-27)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})(\lambda_{max,a,Kd})(\lambda_{max,v,Kd})(\lambda_{max,tr,Kd})(\lambda_{max,c,Kd})(\lambda_{max,scrag,Kd})$	(E-28)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})(\lambda_{min,a,Qd})(\lambda_{min,v,Qd})(\lambda_{min,tr,Qd})(\lambda_{min,c,Qd})(\lambda_{min,scrag,Qd})$	(E-29)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})(\lambda_{max,a,Qd})(\lambda_{max,v,Qd})(\lambda_{max,tr,Qd})(\lambda_{max,c,Qd})(\lambda_{max,scrag,Qd})$	(E-30)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the

E6. Minimum and Maximum Performance Check, Example 1.6

Minimum Property Modification factors are:

$\lambda_{min,Kd} = 1.0$
$\lambda_{min,Qd} = 1.0$

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are:

$\lambda_{max,a,Kd} = 1.1$
$\lambda_{max,a,Qd} = 1.1$

$\lambda_{max,t,Kd} = 1.1$
$\lambda_{max,t,Qd} = 1.4$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:

minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

1.00 for critical bridges
0.75 for essential bridges
0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

$\lambda_{max,a,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,a,Qd} = 1.0 + 0.1(0.66) = 1.066$

$\lambda_{max,t,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,t,Qd} = 1.0 + 0.4(0.66) = 1.264$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Therefore the maximum overall modification factors

$\lambda_{max,Kd} = 1.066(1.066)1.0 = 1.14$
$\lambda_{max,Qd} = 1.066(1.264)1.0 = 1.35$

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:
$Q_{d,max} = 1.35\ (2.39) = 3.23$ k
and
$K_{d,max} = 1.14(1.17) = 1.34$ k/in

E7. Design and Performance Summary

E7.1 Isolator dimensions
Summarize final dimensions of isolators:

- Overall diameter (includes cover layer)
- Overall height
- Diameter lead core
- Bonded diameter
- Number of rubber layers
- Thickness of rubber layers
- Total rubber thickness
- Thickness of steel shims
- Shear modulus of elastomer

E7. Design and Performance Summary, Example 1.6

E7.1 Isolator dimensions, Example 1.6
Isolator dimensions are summarized in Table E7.1-1.

Table E7.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Diam. lead core (in)
Under edge girder on Pier 1	16.0 x 16.0 x 10.0 (H)	12.0 dia. x 8.5(H)	1.63

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in)	Total rubber thick-ness (in)	Steel shim thick-ness (in)
Under edge girder on Pier 1	19	0.25	4.75	0.125

Shear modulus of elastomer = 60 psi

E7.2 Bridge Performance
Summarize bridge performance

- Maximum superstructure displacement (longitudinal)

E7.2 Bridge Performance, Example 1.6
Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 28.32 k. This is more than the column plastic shear

- Maximum superstructure displacement (transverse)
- Maximum superstructure displacement (resultant)
- Maximum column shear (resultant)
- Maximum column moment (about transverse axis)
- Maximum column moment (about longitudinal axis)
- Maximum column torque

Check required performance as determined in Step A3, is satisfied.

strength (25 k) and therefore the required performance criterion is not satisfied (fully elastic behavior). Clearly the additional column shear forces due to the skew are too high to be reduced to below yield and give a fully elastic response. However the displacement is only 1.61 in and the displacement ductility demand on the columns is likely to be less than 2, thus indicating 'essentially' elastic behavior. If this is not acceptable, options include: (1) jacketing the column if an existing bridge, or (2) increasing the size of the column if a new bridge.

It is noted that the maximum longitudinal displacement is 1.54 in which is less than the 2.0 in available at the abutment expansion joints and therefore acceptable.

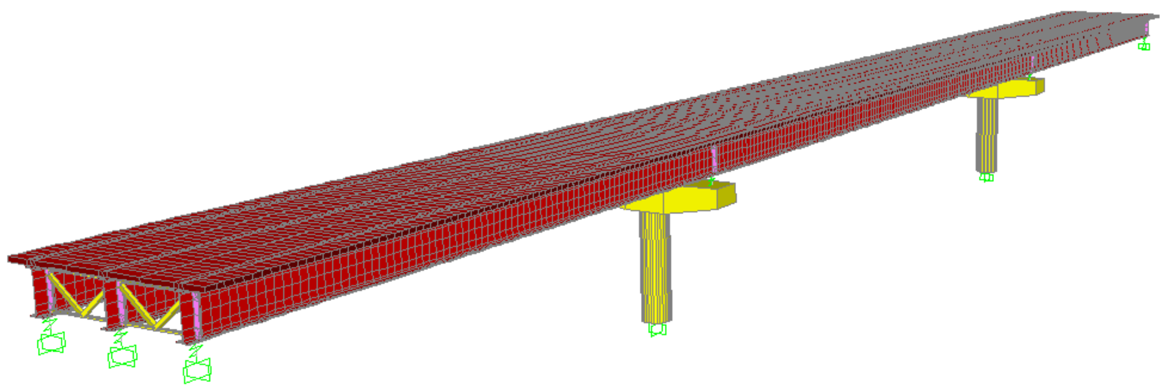
Table E7.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	1.54 in
Maximum superstructure displacement (transverse)	1.54 in
Maximum superstructure displacement (resultant)	1.61 in
Maximum column shear (resultant)	28.32 k
Maximum column moment about transverse axis	227 kft
Maximum column moment about longitudinal axis	227 kft
Maximum column torque	23.25 kft

SECTION 2

Steel Plate Girder Bridge, Long Spans, Single-Column Pier

DESIGN EXAMPLE 2.0: Benchmark Bridge #2



Design Examples in Section 2

ID	Description	S_1	Site Class	Column height	Skew	Isolator type
2.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
2.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
2.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
2.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
2.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
2.5	Change column height	0.2g	B	H_1=0.5H_2	0	Lead rubber bearing
2.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

<table>
<tr><th>DESIGN PROCEDURE</th><th>DESIGN EXAMPLE 2.0 (Benchmark #2)</th></tr>
<tr><td colspan="2">STEP A: BRIDGE AND SITE DATA</td></tr>
<tr><td>A1. Bridge Properties
Determine properties of the bridge:
• number of supports, m
• number of girders per support, n
• angle of skew
• weight of superstructure including railings, curbs, barriers and other permanent loads, W_{SS}
• weight of piers participating with superstructure in dynamic response, W_{PP}
• weight of superstructure, W_j, at each support
• stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
• column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
• allowable movement at expansion joints
• isolator type if known, otherwise ‘to be selected’</td><td>A1. Bridge Properties, Example 2.0
• Number of supports, m = 4
o North Abutment (m = 1)
o Pier 1 (m = 2)
o Pier 2 (m = 3)
o South Abutment (m =4)
• Number of girders per support, n = 3
• Angle of skew = 0^0
• Number of columns per support = 1
• Weight of superstructure including permanent loads, W_{SS} = 1651.32 k
• Weight of superstructure at each support:
o W_1 = 168.48 k
o W_2 = 657.18 k
o W_3 = 657.18 k
o W_4 = 168.48 k
• Participating weight of piers, W_{PP} = 256.26 k
• Effective weight (for calculation of period), $W_{eff} = Wss + W_{PP}$ = 1907.58 k
• Pier heights are both 19ft (clear)
• Stiffness of each pier in the both directions (assume fixed at footing and single curvature behavior) :
o $K_{sub,pier1}$ = 288.87 k/in
o $K_{sub,pier2}$ = 288.87 k/in
• Minimum column shear strength based on flexural yield capacity of column = 128 k
• Displacement capacity of expansion joints (longitudinal) = 2.5 in for thermal and other movements
• Lead-rubber isolators</td></tr>
<tr><td>A2. Seismic Hazard
Determine seismic hazard at site:
• acceleration coefficients
• site class and site factors
• seismic zone
Plot response spectrum.

Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of PGA, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD).

Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for conventional bridges and Art 3.2 refers the designer to</td><td>A2. Seismic Hazard, Example 2.0
Acceleration coefficients for bridge site are given in design statement as follows:
• PGA = 0.40
• S_1 = 0.20
• S_S = 0.75

Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec.

Table 3.10.3.1-1 LRFD gives Site Class as B.

Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors:
• F_{pga} = 1.0
• F_a = 1.0
• F_v = 1.0</td></tr>
</table>

the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD.

Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows:

- $A_s = F_{pga}\,PGA$
- $S_{DS} = F_a\,S_S$
- $S_{D1} = F_v\,S_1$

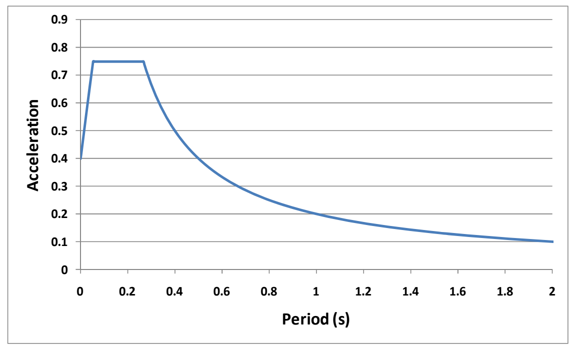
Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table 5-1 GSID.

These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.

- $A_s = F_{pga}\,PGA = 1.0(0.40) = 0.40$
- $S_{DS} = F_a\,S_S = 1.0(0.75) = 0.75$
- $S_{D1} = F_v\,S_1 = 1.0(0.20) = 0.20$

Since $0.15 < S_{D1} \leq 0.30$, bridge is located in Seismic Zone 2.

Design Response Spectrum is as below:



A3. Performance Requirements

Determine required performance of isolated bridge during Design Earthquake (1000-yr return period).

Examples of performance that might be specified by the Owner include:

- Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake.
- Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake.
- For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements)
- Reduced substructure forces for bridges on weak soils to reduce foundation costs.

A3. Performance Requirements, Example 2.0

In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield).

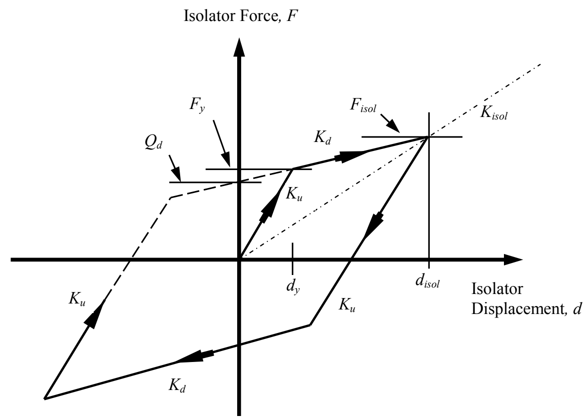
To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (128 k).

The maximum shear in the column must therefore be less than 128 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of $k_{sub.}$



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties

To begin the iterative solution, an estimate is required of :

(1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)

Art C7.1 GSID

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \qquad \text{(B-1)}$$

(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not

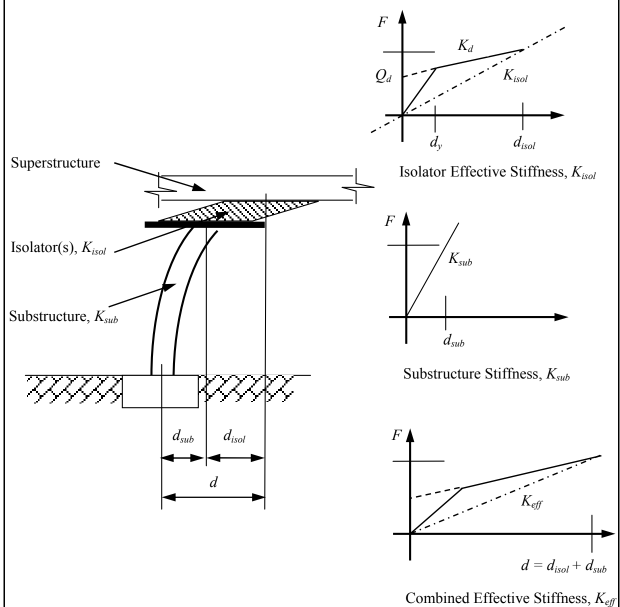
B1.1 Initial System Displacement and Properties, Example 2.0

$$d \cong 10 S_{D1} = 10(0.20) \cong 2.0\ in$$

<table>
<tr><td>occur under non-seismic loads (e.g. wind) but low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e.
$$Q_d = 0.05W \quad \text{(B-2)}$$
(3) Post-yield stiffness, K_d
Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3.
Art. 12.2 GSID
$$K_{d,min} \geq \frac{0.025W}{d} \quad \text{(B-3)}$$
Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$</td><td></td><td>$$Q_d = 0.05W = 0.05(1651.32) = 82.56\ k$$
$$K_d = 0.05\frac{W}{d} = 0.05\frac{1651.32}{2.0} = 41.28\ k/in$$</td></tr>
<tr><td>B1.2 Initial Isolator Properties at Supports
Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support ‘j’ by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support:
$$Q_{d,j} = Q_d\frac{W_j}{W} \quad \text{(B-4)}$$
and
$$K_{d,j} = K_d\frac{W_j}{W} \quad \text{(B-5)}$$</td><td></td><td>B1.2 Initial Isolator Properties at Supports, Example 2.0
$$Q_{d,j} = Q_d\frac{W_j}{W}$$
o $Q_{d,1}$ = 8.42 k
o $Q_{d,2}$ = 32.86 k
o $Q_{d,3}$ = 32.86 k
o $Q_{d,4}$ = 8.42 k
and
$$K_{d,j} = K_d\frac{W_j}{W}$$
o $K_{d,1}$ = 4.21 k/in
o $K_{d,2}$ = 16.43 k/in
o $K_{d,3}$ = 16.43 k/in
o $K_{d,4}$ = 4.21 k/in</td></tr>
<tr><td>B1.3 Effective Stiffness of Combined Pier and Isolator System
Calculate the effective stiffness, $K_{eff,j}$, of each support ‘j’ for all supports, taking into account the stiffness of the isolators at support ‘j’ ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID).

An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula is as follows (MCEER 2006):
$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j} \quad \text{(B-6)}$$
where
$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}} \quad \text{(B-7)}$$
and $K_{sub,j}$ for the piers are given in Step A1. For the</td><td></td><td>B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 2.0
$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$
o $\alpha_1 = 8.43\text{x}10^{-4}$
o $\alpha_2 = 1.21\text{x}10^{-1}$
o $\alpha_3 = 1.21\text{x}10^{-1}$
o $\alpha_4 = 8.43\text{x}10^{-4}$
$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$$
o $K_{eff,1}$ = 8.42 k/in
o $K_{eff,2}$ = 31.09 k/in
o $K_{eff,3}$ = 31.09 k/in
o $K_{eff,4}$ = 8.42 k/in</td></tr>
</table>

abutments, take $K_{sub,j}$ to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.



B1.4 Total Effective Stiffness
Calculate the total effective stiffness, K_{eff}, of the bridge:

Eq. 7.1-6 GSID

$$K_{eff} = \sum_{j=1}^{m} K_{eff,j} \qquad \text{(B-8)}$$

B1.4 Total Effective Stiffness, Example 2.0

$$K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 79.02\ k/in$$

B1.5 Isolation System Displacement at Each Support
Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports:

$$d_{isol,j} = \frac{d}{1+\alpha_j} \qquad \text{(B-9)}$$

B1.5 Isolation System Displacement at Each Support, Example 2.0

$$d_{isol,j} = \frac{d}{1+\alpha_j}$$

- $d_{isol,1}$ = 2.00 in
- $d_{isol,2}$ = 1.79 in
- $d_{isol,3}$ = 1.79 in
- $d_{isol,4}$ = 2.00 in

B1.6 Isolation System Stiffness at Each Support
Calculate the stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports:

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \qquad \text{(B-10)}$$

B1.6 Isolation System Stiffness at Each Support, Example 2.0

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$

- $K_{isol,1}$ = 8.43 k/in
- $K_{isol,2}$ = 34.84 k/in
- $K_{isol,3}$ = 34.84 k/in
- $K_{isol,4}$ = 8.43 k/in

B1.7 Substructure Displacement at Each Support Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports: $d_{sub,j} = d - d_{isol,j}$ (B-11)	**B1.7 Substructure Displacement at Each Support, Example 2.0** $d_{sub,j} = d - d_{isol,j}$ o $d_{sub,1}$ = 0.002 in o $d_{sub,2}$ = 0.215 in o $d_{sub,3}$ = 0.215 in o $d_{sub,4}$ = 0.002 in
B1.8 Lateral Load in Each Substructure Calculate the lateral load in substructure 'j', $F_{sub,j}$, for all supports: $F_{sub,j} = K_{sub,j} d_{sub,j}$ (B-12) where values for $K_{sub,j}$ are given in Step A1.	**B1.8 Lateral Load in Each Substructure, Example 2.0** $F_{sub,j} = K_{sub,j} d_{sub,j}$ o $F_{sub,1}$ = 16.84 k o $F_{sub,2}$ = 62.18 k o $F_{sub,3}$ = 62.18 k o $F_{sub,4}$ = 16.84 k
B1.9 Column Shear Force at Each Support Calculate the shear force in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j': $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ (B-13) Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.	**B1.9 Column Shear Force at Each Support, Example 2.0** $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ o $F_{col,2,1}$ = 62.18 k o $F_{col,3,1}$ = 62.18 k These column shears are less than the plastic shear capacity of each column (128k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.
B1.10 Effective Period and Damping Ratio Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge: Eq. 7.1-5 GSID $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}}$ (B-14) and Eq. 7.1-10 GSID $\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2}$ (B-15) where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j}$ = 0. For other types of isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for the Simplified Method, set $d_{y,j}$ = 0 for all isolator types. See Step B2.2 where the value of $d_{y,j}$ is revisited	**B1.10 Effective Period and Damping Ratio, Example 2.0** $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{1907.58}{386.4(79.02)}}$ = 1.57 sec and taking $d_{y,j}$ = 0: $\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - 0)}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.30$

for the Multimode Spectral Analysis Method.		
B1.11 Damping Factor Calculate the damping factor, B_L, and the displacement, d, of the bridge: Eq. 7.1-3 GSID $$B_L = \begin{cases} \left(\frac{\xi}{0.05}\right)^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases} \quad \text{(B-16)}$$ Eq. 7.1-4 GSID $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \quad \text{(B-17)}$$		**B1.11 Damping Factor, Example 2.0** Since $\xi = 0.30 \geq 0.3$ $$B_L = 1.70$$ and $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.2)1.57}{1.70} = 1.81\ in$$
B1.12 Convergence Check Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement. This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5). After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both. Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)		**B1.12 Convergence Check, Example 2.0** Since the calculated value for displacement, d (=1.81) is not close to that assumed at the beginning of the cycle (Step B1.1, d = 2.0), use the value of 1.81 as the new assumed displacement and repeat from Step B1.3. After three iterations, convergence is reached at a superstructure displacement of 1.65 in, with an effective period of 1.43 seconds, and a damping factor of 1.7 (30% damping ratio). The displacement in the isolators at Pier 1 is 1.44 in and the effective stiffness of the same isolators is 42.78 k/in. See spreadsheet in Table B1.12-1 for results of final iteration. Ignoring the weight of the hammerhead, the column shear force must equal the isolator shear force for equilibrium. Hence column shear = 42.78 (1.44) = 61.60 k which is less than the maximum allowable (128 k) if elastic behavior is to be achieved (as required in Step A3). Also the superstructure displacement = 1.65 in, which is less than the available clearance of 2.5 in. Therefore the above solution is acceptable and go to Step B2. Note that available clearance (2.5 in) is greater than minimum required which is given by: $$= \frac{8\ S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.43}{1.7} = 1.35\ in$$

Table B1.12-1 Simplified Method Solution for Design Example 2.0 – Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	1651.32	256.26	1907.58	0.2	3							
Step B1.1	d	1.65	Assumed displacement									
	Q_d	82.57	Characteristic strength									
	K_d	50.04	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\, d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	168.48	8.424	5.105	10,000.00	0.001022	10.206	1.648	10.216	0.002	16.839	13.885	27.785
Pier 1	657.18	32.859	19.915	288.87	0.148088	37.260	1.437	42.778	0.213	61.480	47.224	101.441
Pier 2	657.18	32.859	19.915	288.87	0.148088	37.260	1.437	42.778	0.213	61.480	47.224	101.441
Abut 2	168.48	8.424	5.105	10,000.00	0.001022	10.206	1.648	10.216	0.002	16.839	13.885	27.785
Total	1651.32	82.566	50.040		$\Sigma K_{eff,j}$	94.932				156.638	122.219	258.453
					Step	**B1.4**						
Step B1.10	T_{eff}	1.43	Effective period									
	ξ	0.30	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.71										
	B_L	1.70	Damping Factor									
	d	1.65	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	2.808	1.702	3.405		1.69	3.363						
Pier 1	10.953	6.638	14.259		1.20	15.766						
Pier 2	10.953	6.638	14.259		1.20	15.766						
Abut 2	2.808	1.702	3.405		1.69	3.363						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength

Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \qquad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \qquad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 2.0

Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-1) by the number of isolators at each support (n = 3), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,\,1}$ = 8.42/3 = 2.81 k
- $Q_{d,\,2}$ = 32.86/3=10.95 k
- $Q_{d,\,3}$ = 32.86/3 = 10.95 k
- $Q_{d,\,4}$ = 8.42/3 = 2.81 k

and

- $K_{d,1}$ = 5.10/3 = 1.70 k/in
- $K_{d,2}$ = 19.92/3 = 6.64 k/in
- $K_{d,3}$ = 19.92/3 = 6.64 k/in
- $K_{d,4}$ = 5.10/3 = 1.70 k/in

Note that the K_d values per support used above are from the final iteration given in Table B1.12-1. These are not the same as the initial values in Step B1.2, because they have been adjusted from cycle to cycle, such that the total K_d summed over all the isolators satisfies the minimum lateral restoring force requirement for the bridge, i.e. K_{dtotal} = 0.05 W/d. See Step B1.1. Since d varies from cycle to cycle, $K_{d,j}$ varies from cycle to cycle.

B2.2 Initial Stiffness and Yield Displacement

Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

(1) For friction-based isolators $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \qquad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \qquad \text{(B-22)}$$

B2.2 Initial Stiffness and Yield Displacement, Example 2.0

Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(6.64) = 66.4\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{10.95}{(66.4 - 6.64)} = 0.18\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2

in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i}$ = 0.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$

Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

$$K_{isol,i} = \frac{K_{isol,j}}{n} \qquad \text{(B -23)}$$

B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 2.0

Dividing the results for K_{isol} (Step B1.12) among the 3 isolators at each support, the following values for K_{isol} /isolator are obtained:

- $K_{isol,1}$ = 10.22/3 = 3.41 k/in
- $K_{isol,2}$ = 42.78/3 = 14.26 k/in
- $K_{isol,3}$ = 42.78/3 = 14.26 k/in
- $K_{isol,4}$ = 10.22/3 = 3.41 k/in

B2.4 Three-Dimensional Bridge Model

Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is recommended.

B2.4 Three-Dimensional Bridge Model, Example 2.0

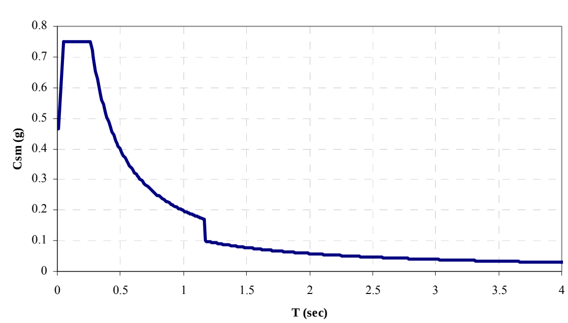
Although the bridge in this Design Example is regular and is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below.

B2.5 Composite Design Response Spectrum

Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.

B.2.5 Composite Design Response Spectrum, Example 2.0

From the final results of Simplified Method (Step B1.12), B_L = 1.70 and T_{eff} = 1.43 sec. Hence the transition in the composite spectrum from 5% to 30% damping occurs at 0.8 T_{eff} = 0.8 (1.43) = 1.14 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods ≥ 1.14 sec by 1.70.



B2.6 Multimodal Analysis of Finite Element Model

Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

B2.6 Multimodal Analysis of Finite Element Model, Example 2.0

Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first three modes are the principal transverse, longitudinal, and torsion modes with periods of 1.60, 1.46 and 1.39 sec respectively. The period of the longitudinal mode (1.46 sec) is very close to that calculated in the Simplified Method. The mass participation factors indicate there is no coupling between these three modes (probably due to the symmetric nature of the bridge) and the high values for the first and second modes (92% and 94% respectively) indicate the bridge is responding essentially in a single mode of vibration in each

Mode No	Period Sec	Mass Participation Ratios UX	UY	UZ	RX	RY	RZ
1	1.604	0.000	0.919	0.000	0.952	0.000	0.697
2	1.463	0.941	0.000	0.000	0.000	0.020	0.000
3	1.394	0.000	0.000	0.000	0.000	0.000	0.231
4	0.479	0.000	0.003	0.000	0.013	0.000	0.002
5	0.372	0.000	0.000	0.076	0.000	0.057	0.000
6	0.346	0.000	0.000	0.000	0.000	0.000	0.000
7	0.345	0.000	0.001	0.000	0.010	0.000	0.000
8	0.279	0.000	0.003	0.000	0.013	0.000	0.002
9	0.268	0.000	0.000	0.000	0.000	0.000	0.000
10	0.267	0.058	0.000	0.000	0.000	0.000	0.000
11	0.208	0.000	0.000	0.000	0.000	0.129	0.000
12	0.188	0.000	0.000	0.000	0.000	0.000	0.001

direction. Similar results to that obtained by the Simplified Method are therefore expected.

Table B2.6-1 Modal Properties of Bridge Example 2.0 – First Iteration

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 1.69 (1.65) in
- $d_{isol,2}$ = 1.20 (1.44) in
- $d_{isol,3}$ = 1.20 (1.44) in
- $d_{isol,4}$ = 1.69 (1.65) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 2.0

The results for isolator displacements are close but not close enough (15% difference at the piers)

Go to Step B2.8 and update properties for a second cycle of iteration.

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows: $$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \quad \text{(B-24)}$$ Recalculate $K_{eff,j}$: Eq. 7.1-6 GSID $$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \quad \text{(B-25)}$$ Recalculate system damping ratio, ξ : Eq. 7.1-10 GSID $$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \quad \text{(B-26)}$$ Recalculate system damping factor, B_L: Eq. 7.1-3 GSID $$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \quad \text{(B-27)}$$ Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.	**B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 2.0** Updated values for $K_{isol,i}$ are given below (previous values are in parentheses): o $K_{isol,1}$ = 3.36 (3.41) k/in o $K_{isol,2}$ = 15.77 (14.26) k/in o $K_{isol,3}$ = 15.77 (14.26) k/in o $K_{isol,4}$ = 3.36 (3.41) k/in Since the isolator displacements are relatively close to previous results no significant change in the damping ratio is expected. Hence $K_{eff,j}$ and ξ are not recalculated and B_L is taken at 1.70. Since the change in effective period is very small (1.43 to 1.46 sec) and no change has been made to B_L, there is no need to construct a new composite response spectrum in this case. Go back to Step B2.6 (see immediately below).
	B2.6 Multimodal Analysis Second Iteration, Example 2.0 Reanalysis gives the following values for the isolator displacements (numbers in parentheses are those from the previous cycle): o $d_{isol,1}$ = 1.66 (1.69) in o $d_{isol,2}$ = 1.15 (1.20) in o $d_{isol,3}$ = 1.15 (1.20) in o $d_{isol,4}$ = 1.66 (1.69) in Go to Step B2.7
B2.7 Convergence Check Compare results and determine if convergence has been reached. If so go to Step B2.9. Otherwise Go to Step B2.8.	**B2.7 Convergence Check, Example 2.0** Satisfactory agreement has been reached on this second cycle. Go to Step B2.9
B2.9 Superstructure and Isolator Displacements Once convergence has been reached, obtain o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for	**B2.9 Superstructure and Isolator Displacements, Example 2.0** From the above analysis: o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are: x_L= 1.69 in

each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.

y_L= 0.0 in

- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are:
  - Abutments: u_L = 1.66 in, v_L = 0.00 in
  - Piers: u_L = 1.15 in, v_L = 0.00 in

  All isolators at same support have the same displacements.

B2.10 Pier Bending Moments and Shear Forces

Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.

B2.10 Pier Bending Moments and Shear Forces, Example 2.0

Bending moments in single column pier in the longitudinal (M_{PLL}) and transverse (M_{PTL}) directions are:

M_{PLL}= 0
M_{PTL}= 1602 kft

Shear forces in single column pier the longitudinal (V_{PLL}) and transverse (V_{PTL}) directions are

V_{PLL}=67.16 k
V_{PTL}=0

B2.11 Isolator Shear and Axial Forces

Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 2.0

Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abutment	1	5.63	0	1.29
	2	5.63	0	1.30
	3	5.63	0	1.29
Pier	1	18.19	0	0.77
	2	18.25	0	1.11
	3	18.19	0	0.77

The difference between the longitudinal shear force in the column (V_{PLL} = 67.16k) and the sum of the isolator shear forces at the same Pier (54.63 k) is about 12.5 k. This is due to the inertia force developed in the hammerhead cap beam which weighs about 128 k and can generate significant additional demand on the column (about a 23% increase in this case).

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 2.0

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.52 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows:
  x_T = 0 and y_T = 1.75 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions as follows:
  Abutments u_T = 0.00 in, v_T = 1.75 in
  Piers u_T = 0.00 in, v_T = 0.71 in
- Pier bending moments in the longitudinal (M_{PLT}) and transverse (M_{PTT}) directions are as follows:
  M_{PLT}= 1548.33 kft and M_{PTT}= 0
- Pier shear forces in the longitudinal (V_{PLT}) and transverse (V_{PTT}) directions are as follows:
  V_{PLT}= 0 and V_{PTT}= 60.75 k
- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abutment	1	0.0	5.82	13.51
	2	0.0	5.83	0
	3	0.0	5.82	13.51
Pier	1	0.0	15.40	26.40
	2	0.0	15.57	0
	3	0.0	15.40	26.40

The difference between the transverse shear force in the column (V_{PLL} = 60.75k) and the sum of the isolator shear forces at the same Pier (46.37 k) is about 14.4 k. This is due to the inertia force developed in the hammerhead cap beam which weighs about 128 k and can generate significant additional demand on the column (about 31%).

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements
Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)

- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)

- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements at Pier 1, Example 2.0
To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

Load Case 1:
$u_1 = u_L + 0.3u_T = 1.0(1.15) + 0.3(0) = 1.15$ in
$v_1 = v_L + 0.3v_T = 1.0(0) + 0.3(0.71) = 0.21$ in
$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.15^2 + 0.21^2} = 1.17$ in

Load Case 2:
$u_2 = 0.3u_L + u_T = 0.3(1.15) + 1.0(0) = 0.35$ in
$v_2 = 0.3v_L + v_T = 0.3(0) + 1.0(0.71) = 0.71$in
$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{0.35^2 + 0.71^2} = 0.79$ in

Governing Case:
Total design displacement, $d_t = \max(R_1, R_2)$
$= 1.17$ in

D2. Design Moments and Shears
Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

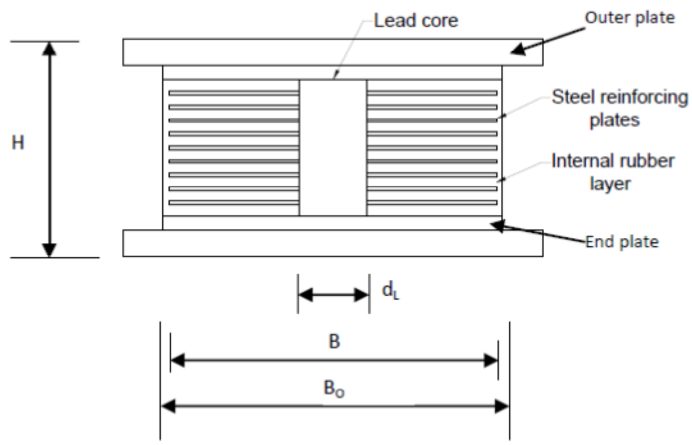
D2. Design Moments and Shears in Pier 1, Example 2.0
Design moments and shear forces are calculated for Pier 1 below, to illustrate the process.

Load Case 1:
$V_{PL1} = V_{PLL} + 0.3V_{PLT} = 1.0(67.16) + 0.3(0) = 67.16$ k
$V_{PT1} = V_{PTL} + 0.3V_{PTT} = 1.0(0) + 0.3(60.75) = 18.23$ k
$R_1 = \sqrt{V_{L1}^2 + V_{T1}^2} = \sqrt{67.16^2 + 18.23^2} = 69.59$ k

Load Case 2:
$V_{PL2} = 0.3V_{PLL} + V_{PLT} = 0.3(67.16) + 1.0(0) = 20.15$ k
$V_{PT2} = 0.3V_{PTL} + V_{PTT} = 0.3(0) + 1.0(60.75) = 60.75$ k
$R_2 = \sqrt{V_{L2}^2 + V_{T2}^2} = \sqrt{20.15^2 + 60.75^2} = 64.00$ k

Governing Case:
Design column shear $= \max(R1, R2)$
$= 69.59$ k

STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS



A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness (K_u).With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness $\underline{K_d}$ is given by the rubber alone. More details are given in MCEER 2006.

While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary. When sizing the physical dimensions of the bearing, plan dimensions (B, d_L) should be rounded up to the next $^1/_4$" increment, while the total thickness of elastomer, T_r, is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are $^1/_4$" and $^3/_8$". High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in MCEER 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See MCEER 2006.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties

Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1).

- required characteristic strength, Q_d / isolator
- required post-elastic stiffness, K_d / isolator
- total design displacement, d_t, for each isolator
- maximum applied dead and live load (P_{DL}, P_{LL}) and seismic load (P_{SL}) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator, and
- maximum wind load, P_{WL}.

E1. Required Properties, Example 2.0

The design of one of the exterior isolators on a pier is given below to illustrate the design process for lead-rubber isolators.

From previous work:

- Q_d / isolator = 10.95 k
- K_d / isolator = 6.76 k/in
- Total design displacement, d_t = 1.17 in
- P_{DL} = 187 k
- P_{LL} = 123 k and P_{SL} = 26.4 k (Table C1-1)
- P_{WL} = 8.21 k < Q_d OK

Note that the K_d value per isolator used above is from the final iteration of the analysis. It is not the same as the initial value in Step B2.1 (6.64 k/in) , because it has been adjusted from cycle to cycle, such that the total K_d summed over all the isolators satisfies the minimum lateral restoring force requirement for the bridge, i.e. K_{dtotal} = 0.05 W/d. See Step B1.1. Since d varies from cycle to cycle, $K_{d,j}$ varies from cycle to cycle.

<table>
<tr><td>E2. Isolator Sizing</td><td></td><td></td></tr>
<tr><td>E2.1 Lead Core Diameter
Determine the required diameter of the lead plug, d_L, using:
$$d_L = \sqrt{\frac{Q_d}{0.9}} \qquad \text{(E-1)}$$
See Step E2.5 for limitations on d_L</td><td></td><td>E2.1 Lead Core Diameter, Example 2.0
$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{10.95}{0.9}} = 3.49\ in$$</td></tr>
<tr><td>E2.2 Plan Area and Isolator Diameter
Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.6 ksi.

Then the bonded area of the isolator is given by:
$$A_b = \frac{P_{DL} + P_{LL}}{1.6}\ in^2 \qquad \text{(E-2)}$$
and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by:
$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} \qquad \text{(E-3)}$$
Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using
$$A_b = \frac{\pi}{4}(B^2 - d_L^2) \qquad \text{(E-4)}$$
Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, B_o is given by:
$$B_o = B + 1.0 \qquad \text{(E-5)}$$</td><td></td><td>E2.2 Plan Area and Isolator Diameter, Example 2.0
$$A_b = \frac{P_{DL} + P_{LL}}{1.6}\ in^2 = \frac{187 + 123}{1.6} = 193.75\ in^2$$
$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} = \sqrt{\frac{4\,(193.75)}{\pi} + 3.49^2}$$
= 16.09 in

Round B up to 16.25 in and the actual bonded area is:
$$A_b = \frac{\pi}{4}(16.25^2 - 3.49^2) = 197.84\ in^2$$
B_o = 16.25 + 2(0.5) = 17.25 in</td></tr>
<tr><td>E2.3 Elastomer Thickness and Number of Layers
Since the shear stiffness of the elastomeric bearing is given by:
$$K_d = \frac{GA_b}{T_r} \qquad \text{(E-6)}$$
where G = shear modulus of the rubber, and
T_r = the total thickness of elastomer,
it follows Eq. E-6 may be used to obtain T_r given a required value for K_d
$$T_r = \frac{GA_b}{K_d} \qquad \text{(E-7)}$$
A typical range for shear modulus, G, is 60-120 psi. Higher and lower values are available and are used in special applications.</td><td></td><td>E2.3 Elastomer Thickness and Number of Layers, Example 2.0

Select G, shear modulus of rubber, = 100 psi (0.1 ksi)

Then
$$T_r = \frac{GA_b}{K_d} = \frac{0.1(197.84)}{6.76} = 2.93\ in$$</td></tr>
</table>

If the layer thickness is t_r, the number of layers, n, is given by: $$n = \frac{T_r}{t_r} \quad \text{(E-8)}$$ rounded up to the nearest integer. Note that because of rounding the plan dimensions and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be necessary if the differences are large.		$$n = \frac{2.93}{0.25} = 11.72$$ Round up to nearest integer, i.e. $n = 12$
E2.4 Overall Height The overall height of the isolator, H, is given by: $$H = n\,t_r + (n-1)t_s + 2t_c \quad \text{(E-9)}$$ where t_s = thickness of an internal shim (usually about 1/8 in), and t_c = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in)		**E2.4 Overall Height, Example 2.0** $$H = 12(0.25) + 11(0.125) + 2 * 1.5 = 7.375\ in$$
E2.5 Lead Core Size Check Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows: $$\frac{B}{3} \geq d_L \geq \frac{B}{6} \quad \text{(E-10)}$$		**E2.5 Lead Core Size Check, Example 2.0** Since B=16.25 check $$\frac{16.25}{3} \geq d_L \geq \frac{16.25}{6}$$ i.e., $5.41 \geq d_L \geq 2.71$ Since d_L = 3.49, lead core size is acceptable.
E3. Strain Limit Check Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e., $$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5 \quad \text{(E-11)}$$ where γ_c, $\gamma_{s,eq}$, and γ_r are defined below. (a) γ_c is the maximum shear strain in the layer due to compression and is given by: $$\gamma_c = \frac{D_c \sigma_S}{GS} \quad \text{(E-12)}$$ where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = {P_{DL}}/{A_b}$, G is shear modulus, and S is the layer shape factor given by: $$S = \frac{A_b}{\pi B t_r} \quad \text{(E-13)}$$ (b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by: $$\gamma_{s,eq} = \frac{d_t}{T_r} \quad \text{(E-14)}$$		**E3. Strain Limit Check, Example 2.0** Since $$\sigma_S = \frac{187.0}{197.84} = 0.945\ ksi$$ $$G = 0.1 \text{ ksi}$$ and $$S = \frac{197.84}{\pi 16.25(0.25)} = 15.50$$ then $$\gamma_c = \frac{1.0(0.945)}{0.1(15.50)} = 0.61$$ $$\gamma_{s,eq} = \frac{1.17}{3.0} = 0.39$$

(c) γ_r is the shear strain due to rotation and is given by: $$\gamma_r = \frac{D_r B^2 \theta}{t_r T_r} \qquad \text{(E-15)}$$ where D_r is shape coefficient for rotation in circular bearings = 0.375, and θ is design rotation due to DL, LL and construction effects. Actual value for θ may not be known at this time and a value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1).		$$\gamma_r = \frac{0.375(16.25^2)(0.01)}{0.25(3.0)} = 1.32$$ Substitution in Eq E-11 gives $$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.61 + 0.39 + 0.5(1.32) = 1.66 \leq 5.5\ OK$$
E4. Vertical Load Stability Check Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state. Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either 2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or 1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4.		**E4. Vertical Load Stability Check, Example 2.0**
E4.1 Vertical Load Stability in Undeformed State The critical load capacity of an elastomeric isolator at zero shear displacement is given by $$P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2}\left[\sqrt{\left(1 + \frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2}\right)} - 1\right] \qquad \text{(E-16)}$$ where $H_{eff} = T_r + T_s$ T_s = total shim thickness $K_\theta = {E_b I}/{T_r}$ $E_b = E(1 + 0.67S^2)$ E = elastic modulus of elastomer = $3G$ $I = {\pi B^4}/{64}$ It is noted that typical elastomeric isolators have high shape factors, S, in which case: $$\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1 \qquad \text{(E-17)}$$ and Eq. E-16 reduces to: $$P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta} \qquad \text{(E-18)}$$ Check that:		**E4.1 Vertical Load Stability in Undeformed State, Example 2.0** $$E = 3G = 3(0.1) = 0.3\ ksi$$ $$E_b = 0.3(1 + 0.67(15.50^2)) = 48.38\ ksi$$ $$I = \pi\frac{16.25^4}{64} = 3{,}422.8\ in^4$$ $$K_\theta = \frac{48.38(3{,}422.8)}{3.0} = 55{,}201\ kin/rad$$ $$K_d = \frac{GA_b}{T_r} = \frac{0.1(197.84)}{3.0} = 6.59\ k/in$$ $$P_{cr(\Delta=0)} = \pi\sqrt{6.59(55{,}201)} = 1895.5\ k$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} \geq 3 \qquad \text{(E-19)}$$	$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \frac{1895.5}{(187+123)} = 6.11 \geq 3 \quad OK$$
E4.2 Vertical Load Stability in Deformed State The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by: $$P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)} \qquad \text{(E-20)}$$ where A_r = overlap area between top and bottom plates of isolator at displacement Δ (Fig. 2.2-1 GSID) = $B^2(\delta - sin\delta)/4$ δ = $2cos^{-1}(\Delta/B)$ $A_{gross} = \pi B^2/4$ It follows that: $$\frac{A_r}{A_{gross}} = \frac{(\delta - sin\delta)}{\pi} \qquad \text{(E-21)}$$ Check that: $$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} \geq 1 \qquad \text{(E-22)}$$	**E4.2 Vertical Load Stability in Deformed State, Example 2.0** Since bridge is in Zone 2, $\Delta = 2d_t = 2(1.17) = 2.34$ $$\delta = 2cos^{-1}\left(\frac{2.34}{16.25}\right) = 2.85$$ $$\frac{A_r}{A_{gross}} = \frac{(2.85 - sin2.85)}{\pi} = 0.817$$ $$P_{cr(\Delta)} = 0.817(1895.5) = 1548.6\ k$$ $$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} = \frac{1548.6}{1.2(187)+26.4} = 6.17 \geq 1 \quad OK$$
E5. Design Review	**E5. Design Review, Example 2.0** The basic dimensions of the isolator designed above are as follows: 17.25 in (od) x 7.375in (high) x 3.49 in dia. lead core and the volume, excluding steel end and cover plates, = 1,022 in^3 Although this design satisfies all the required criteria, the vertical load stability ratios (Eq. E-19 and E-22) are much higher than required (6.11 vs 3.0) and total rubber shear strain (1.66) is much less than the maximum allowable (5.5), as shown in Step E3. In other words, the isolator is not working very hard and a redesign appears to be indicated to obtain a smaller isolator with more optimal properties (as well as less cost). This redesign is outlined below. It begins by increasing the allowable compressive stress from 1.6 to 3.2 ksi to obtain initial sizes. Remember that no

limits are placed on compressive stress in GSID, only a limit on strain.

E2.1

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{10.95}{0.9}} = 3.49\ in$$

E2.2

$$A_b = \frac{P_{DL} + P_{LL}}{3.2}\ in^2 = \frac{187 + 123}{3.2} = 96.87\ in^2$$

$$B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} = \sqrt{\frac{4\ (96.87)}{\pi} + 3.49^2} = 11.64$$

Round *B* up to 12.5 in and the actual bonded area becomes:

$$A_b = \frac{\pi}{4}(12.5^2 - 3.49^2) = 113.16\ in^2$$

B_o = 12.5 + 2(0.5) = 13.5 in

E2.3

$$T_r = \frac{GA_b}{K_d} = \frac{0.1(113.16)}{6.76} = 1.67\ in$$

$$n = \frac{1.67}{0.25} = 6.7$$

Round up to nearest integer, i.e. *n* = 7.

E2.4

$$H = 7(0.25) + 6(0.125) + 2 * 1.5 = 5.5\ in$$

E2.5

Since *B*=12.5 check

$$\frac{12.5}{3} \geq d_L \geq \frac{12.5}{6}$$

i.e., $4.17 \geq d_L \geq 2.08$

Since d_L = 3.49, size of lead core is acceptable.

E3.

$$\sigma_S = \frac{187.0}{113.16} = 1.652\ ksi$$

$$S = \frac{113.16}{\pi 12.5(0.25)} = 11.53$$

$$\gamma_c = \frac{1.0(1.652)}{0.1(11.53)} = 1.43$$

$$\gamma_{s,eq} = \frac{1.17}{1.75} = 0.67$$

		$$\gamma_r = \frac{0.375(12.5^2)(0.01)}{0.25(1.75)} = 1.34$$ $$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 1.43 + 0.67 + 0.5(1.34) = 2.77 \leq 5.5\ OK$$ **E4.1** $$E = 3G = 3(0.1) = 0.3\ ksi$$ $$E_b = 0.3(1 + 0.67(11.53^2)) = 26.89\ ksi$$ $$I = \frac{12.5^4}{64} = 1{,}198.4\ in^4$$ $$K_\theta = \frac{26.89(1198.4)}{1.75} = 18{,}411.9\ kin/rad$$ $$K_d = \frac{GA_b}{T_r} = \frac{0.1(113.16)}{1.75} = 6.47\ k/in$$ $$P_{cr(\Delta=0)} = \pi\sqrt{6.47(18411.9)} = 1084.0\ k$$ $$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} = \frac{1084.0}{(187 + 123)} = 3.50 \geq 3\quad OK$$ **E4.2** $$\delta = 2cos^{-1}\left(\frac{2.34}{12.5}\right) = 2.765$$ $$\frac{A_r}{A_{gross}} = \frac{(2.76 - sin2.76)}{\pi} = 0.763$$ $$P_{cr(\Delta)} = 0.763(1084.0) = 827.15k$$ $$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{827.15}{1.2(187) + 26.4} = 3.30 \geq 1\ OK$$ **E5.** The basic dimensions of the redesigned isolator are as follows: 13.5 in (od) x 5.5 in (high) x 3.49 in dia. lead core and the volume, excluding steel end and cover plates, = 358 in^3 This design reduces the excessive vertical stability ratio of the previous design (it is now 3.50 vs 3.0

required) and the total layer shear strain is increased (2.77 vs 5.5 max allowable). Furthermore, the isolator volume is decreased from 1,022 in^3 to 358 in^3. This design is clearly more efficient than the previous one.

E6. Minimum and Maximum Performance Check

Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E6-1.

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E6-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-23)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-24)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-25)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-26)

Table E6-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})(\lambda_{min,a,Kd})(\lambda_{min,v,Kd})(\lambda_{min,tr,Kd})(\lambda_{min,c,Kd})(\lambda_{min,scrag,Kd})$	(E-27)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})(\lambda_{max,a,Kd})(\lambda_{max,v,Kd})(\lambda_{max,tr,Kd})(\lambda_{max,c,Kd})(\lambda_{max,scrag,Kd})$	(E-28)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})(\lambda_{min,a,Qd})(\lambda_{min,v,Qd})(\lambda_{min,tr,Qd})(\lambda_{min,c,Qd})(\lambda_{min,scrag,Qd})$	(E-29)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})(\lambda_{max,a,Qd})(\lambda_{max,v,Qd})(\lambda_{max,tr,Qd})(\lambda_{max,c,Qd})(\lambda_{max,scrag,Qd})$	(E-30)

E6. Minimum and Maximum Performance Check, Example 2.0

Minimum Property Modification factors are:

$\lambda_{min,Kd} = 1.0$
$\lambda_{min,Qd} = 1.0$

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are:

$\lambda_{max,a,Kd} = 1.1$
$\lambda_{max,a,Qd} = 1.1$

$\lambda_{max,t,Kd} = 1.1$
$\lambda_{max,t,Qd} = 1.4$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,a,Qd} = 1.0 + 0.1(0.66) = 1.066$

$\lambda_{max,t,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,t,Qd} = 1.0 + 0.4(0.66) = 1.264$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Therefore the maximum overall modification factors

$\lambda_{max,Kd} = 1.066(1.066)1.0 = 1.14$
$\lambda_{max,Qd} = 1.066(1.264)1.0 = 1.35$

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:

$Q_{d,max} = 1.35\ (10.95) = 14.78$ k
and
$K_{d,max} = 1.14(6.76) = 7.71$ k/in

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

1.00 for critical bridges
0.75 for essential bridges
0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

E7. Design and Performance Summary

E7.1 Isolator dimensions

Summarize final dimensions of isolators:

- Overall diameter (includes cover layer)
- Overall height
- Diameter lead core
- Bonded diameter
- Number of rubber layers
- Thickness of rubber layers
- Total rubber thickness
- Thickness of steel shims
- Shear modulus of elastomer

Check all dimensions with manufacturer.

E7. Design and Performance Summary, Example 2.0

E7.1 Isolator dimensions, Example 2.0

Isolator dimensions are summarized in Table E7.1-1.

Table E7.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Diam. lead core (in)
Under edge girder on Pier 1	17.5 x 17.5 x 5.5(H)	13.5 dia. x 4.0(H)	3.49

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in)	Total rubber thick-ness (in)	Steel shim thick-ness (in)
Under edge girder on Pier 1	7	0.25	1.75	0.125

Shear modulus of elastomer = 100 psi

E7.2 Bridge Performance

Summarize bridge performance

- Maximum superstructure displacement (longitudinal)
- Maximum superstructure displacement (transverse)
- Maximum superstructure displacement

E7.2 Bridge Performance, Example 2.0

Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 71.74k. This less than the column plastic shear (128k) and therefore the required performance criterion is satisfied (fully elastic behavior). Furthermore the maximum longitudinal displacement is 1.69 in which

(resultant)

- Maximum column shear (resultant)
- Maximum column moment (about transverse axis)
- Maximum column moment (about longitudinal axis)
- Maximum column torque

Check required performance as determined in Step A3, is satisfied.

is less than the 2.5in available at the abutment expansion joints and is therefore acceptable.

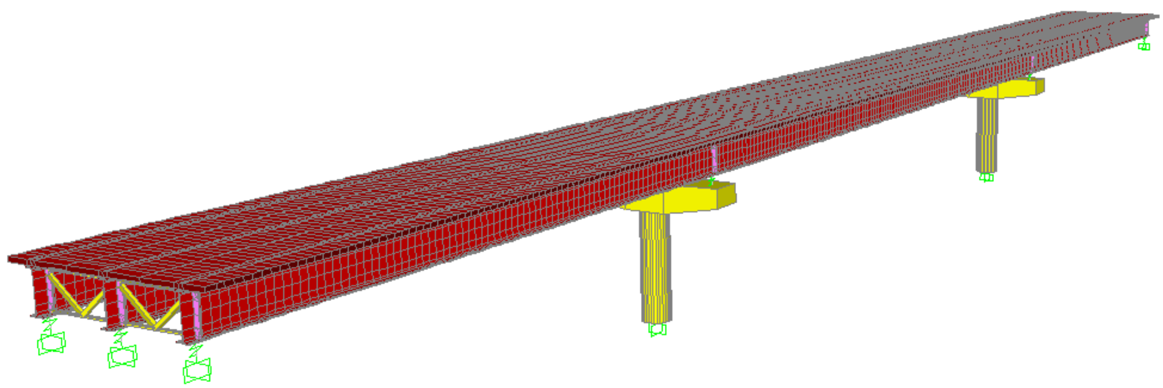
Table E7.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	1.69 in
Maximum superstructure displacement (transverse)	1.75 in
Maximum superstructure displacement (resultant)	1.82 in
Maximum column shear (resultant)	71.74 k
Maximum column moment about transverse axis	1,657 kft
Maximum column moment about longitudinal axis	1,676 kft
Maximum column torque	21.44 kft

SECTION 2

Steel Plate Girder Bridge, Long Spans, Single-Column Pier

DESIGN EXAMPLE 2.1: Site Class D



Design Examples in Section 2

ID	Description	S_1	Site Class	Column height	Skew	Isolator type
2.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
2.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
2.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
2.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
2.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
2.5	Change column height	0.2g	B	H_1=0.5H_2	0	Lead rubber bearing
2.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

<table>
<tr><th>DESIGN PROCEDURE</th><th></th><th>DESIGN EXAMPLE 2.1 (Site Class D)</th></tr>
<tr><td colspan="3">STEP A: BRIDGE AND SITE DATA</td></tr>
<tr><td>A1. Bridge Properties
Determine properties of the bridge:
• number of supports, m
• number of girders per support, n
• angle of skew
• weight of superstructure including railings, curbs, barriers and other permanent loads, W_{SS}
• weight of piers participating with superstructure in dynamic response, W_{PP}
• weight of superstructure, W_j, at each support
• stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
• column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
• allowable movement at expansion joints
• isolator type if known, otherwise ‘to be selected’</td><td></td><td>A1. Bridge Properties, Example 2.1
• Number of supports, $m = 4$
o North Abutment ($m = 1$)
o Pier 1 ($m = 2$)
o Pier 2 ($m = 3$)
o South Abutment ($m = 4$)
• Number of girders per support, $n = 3$
• Angle of skew = 0^0
• Number of columns per support = 1
• Weight of superstructure including permanent loads,, $W_{SS} = 1651.32$ k
• Weight of superstructure at each support:
o $W_1 = 168.48$ k
o $W_2 = 657.18$ k
o $W_3 = 657.18$ k
o $W_4 = 168.48$ k
• Participating weight of piers, $W_{PP} = 256.26$ k
• Effective weight (for calculation of period), $W_{eff} = W_{SS} + W_{PP} = 1907.58$ k
• Stiffness of each pier in the both directions:
o $K_{sub,pier1} = 288.87$ k/in
o $K_{sub,pier2} = 288.87$ k/in
• Minimum column shear strength based on flexural yield capacity of column = 128 k
• Displacement capacity of expansion joints (longitudinal) = 2.5 in (required to accommodate thermal expansion and other movements)
• Lead-rubber isolators</td></tr>
<tr><td>A2. Seismic Hazard
Determine seismic hazard at site:
• acceleration coefficients
• site class and site factors
• seismic zone
Plot response spectrum.

Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of PGA, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD).

Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications,</td><td></td><td>A2. Seismic Hazard, Example 2.1
Acceleration coefficients for bridge site are given in design statement as follows:
• $PGA = 0.40$
• $S_1 = 0.20$
• $S_S = 0.75$

Bridge is on a stiff soil site with a shear wave velocity in upper 100 ft less than 1,200 ft/sec.

Table 3.10.3.1-1 LRFD gives Site Class as D.

Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors:
• $F_{pga} = 1.1$
• $F_a = 1.2$
• $F_v = 2.0$</td></tr>
</table>

i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD.

Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows:

- $A_s = F_{pga}\,PGA$
- $S_{DS} = F_a\,S_S$
- $S_{D1} = F_v\,S_1$

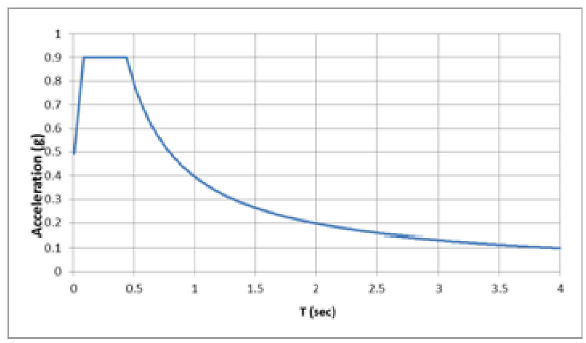
Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table 5-1 GSID.

These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.

- $A_s = F_{pga}\,PGA = 1.1(0.40) = 0.44$
- $S_{DS} = F_a\,S_S = 1.2(0.75) = 0.90$
- $S_{D1} = {}_{Fv}\,S_1 = 2.0(0.20) = 0.40$

Since $0.30 < S_{D1} \leq 0.50$, bridge is located in Seismic Zone 3.

Design Response Spectrum is as below :



A3. Performance Requirements

Determine required performance of isolated bridge during Design Earthquake (1000-yr return period).

Examples of performance that might be specified by the Owner include:

- Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake.
- Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake.
- For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements)
- Reduced substructure forces for bridges on weak soils to reduce foundation costs.

A3. Performance Requirements, Example 2.1

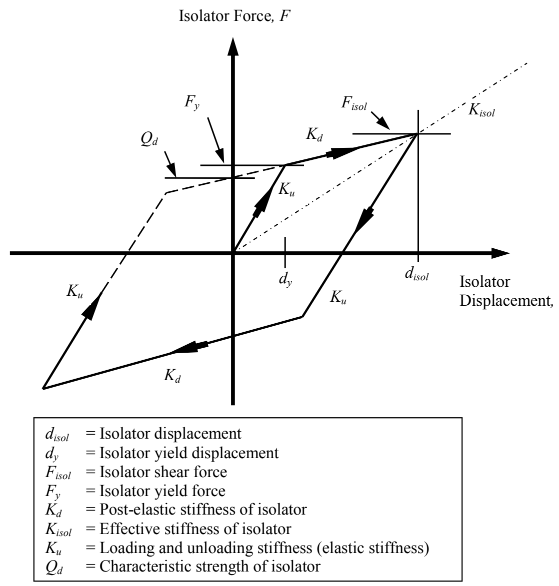
As in previous examples, the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield).

To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (128 k).

The maximum shear in the column must therefore be less than 128 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.



Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of $k_{sub.}$

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties	B1.1 Initial System Displacement and Properties, Example 2.1
To begin the iterative solution, an estimate is required of : (1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)	
Art C7.1 GSID $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \quad \text{(B-1)}$$	$$d \cong 10S_{D1} = 10(0.40) \cong 4.0\ in$$
(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not	

occur under non-seismic loads (e.g. wind) but low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e. $Q_d = 0.05W$ (B-2) (3) Post-yield stiffness, K_d Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3. Art. 12.2 GSID $K_{d,min} \geq \frac{0.025W}{d}$ (B-3) Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$		$Q_d = 0.05W = 0.05(1651.32) = 82.56\ k$ $K_d = 0.05\frac{W}{d} = 0.05\frac{1651.32}{4.0} = 20.64\ k/in$
B1.2 Initial Isolator Properties at Supports Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support: $Q_{d,j} = Q_d\frac{W_j}{W}$ (B-4) and $K_{d,j} = K_d\frac{W_j}{W}$ (B-5)		**B1.2 Initial Isolator Properties at Supports, Example 2.1** $Q_{d,j} = Q_d\frac{W_j}{W}$ o $Q_{d,1}$ = 8.42 k o $Q_{d,2}$ = 32.86 k o $Q_{d,3}$ = 32.86 k o $Q_{d,4}$ = 8.42 k and $K_{d,j} = K_d\frac{W_j}{W}$ o $K_{d,1}$ = 2.11 k/in o $K_{d,2}$ = 8.25 k/in o $K_{d,3}$ = 8.25 k/in o $K_{d,4}$ = 2.11 k/in
B1.3 Effective Stiffness of Combined Pier and Isolator System Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below (after Fig. 7.1-1 GSID). An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula as follows (MCEER 2006): $K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$ (B-6)		**B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 2.1**

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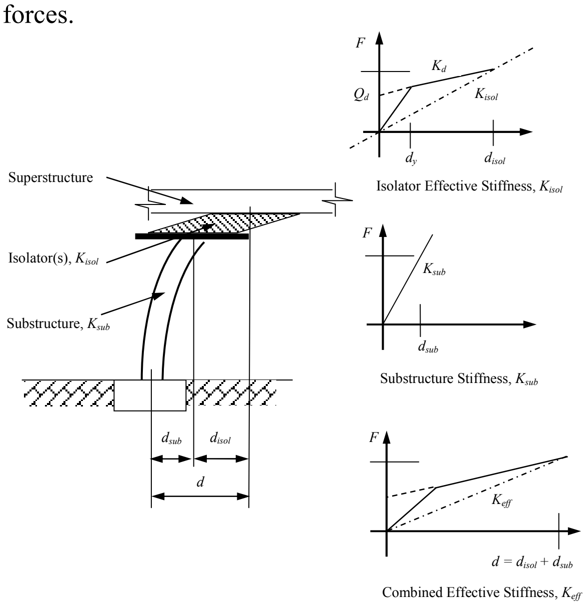
where

$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}} \qquad \text{(B-7)}$$

and $K_{sub,j}$ for the piers are given in Step A1. For the abutments, take $Ksub,j$ to be a large number, say 10,000 k/in (unless actual stiffness values are available). Note that if the default option is chosen, unrealistically high values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.



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$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$

- $\alpha_1 = 4.21\text{x}10^{-4}$
- $\alpha_2 = 5.85\text{x}10^{-2}$
- $\alpha_3 = 5.85\text{x}10^{-2}$
- $\alpha_4 = 4.21\text{x}10^{-4}$

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1 + \alpha_j}$$

- $K_{eff,1}$ = 4.21 k/in
- $K_{eff,2}$ = 15.98 k/in
- $K_{eff,3}$ = 15.98 k/in
- $K_{eff,4}$ = 4.21 k/in
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B1.4 Total Effective Stiffness

Calculate the total effective stiffness, K_{eff}, of the bridge:

Eq. 7.1-6 GSID

$$K_{eff} = \sum_{j=1}^{m} K_{eff,j} \qquad \text{(B-8)}$$
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B1.4 Total Effective Stiffness, Example 2.1

$$K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 40.37\ k/in$$
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B1.5 Isolation System Displacement at Each Support

Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports:

$$d_{isol,j} = \frac{d}{1 + \alpha_j} \qquad \text{(B-9)}$$
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B1.5 Isolation System Displacement at Each Support, Example 2.1

$$d_{isol,j} = \frac{d}{1 + \alpha_j}$$

- $d_{isol,1}$ = 4.00 in
- $d_{isol,2}$ = 3.78 in
- $d_{isol,3}$ = 3.78 in
- $d_{isol,4}$ = 4.00 in
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B1.6 Isolation System Stiffness at Each Support

Calculate the stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports:

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \qquad \text{(B-10)}$$
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B1.6 Isolation System Stiffness at Each Support, Example 2.1

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$

- $K_{isol,1}$ = 4.21 k/in
- $K_{isol,2}$ = 16.91 k/in
- $K_{isol,3}$ = 16.91 k/in
- $K_{isol,4}$ = 4.21 k/in
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B1.7 Substructure Displacement at Each Support Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports: $d_{sub,j} = d - d_{isol,j}$ (B-11)	**B1.7 Substructure Displacement at Each Support, Example 2.1** $d_{sub,j} = d - d_{isol,j}$ o $d_{sub,1}$ = 0.002 in o $d_{sub,2}$ = 0.221 in o $d_{sub,3}$ = 0.221 in o $d_{sub,4}$ = 0.002 in
B1.8 Lateral Load in Each Substructure Calculate the shear at support 'j', $F_{sub,j}$, for all supports: $F_{sub,j} = K_{sub,j} d_{sub,j}$ (B-12) where values for $K_{sub,j}$ are given in Step A1.	**B1.8 Lateral Load in Each Substructure, Example 2.1** $F_{sub,j} = K_{sub,j} d_{sub,j}$ o $F_{sub,1}$ = 16.84 k o $F_{sub,2}$ = 63.91 k o $F_{sub,3}$ = 63.91 k o $F_{sub,4}$ = 16.84 k
B1.9 Column Shear Force at Each Support Calculate the shear in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j': $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ (B-13) Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.	**B1.9 Column Shear Force at Each Support, Example 2.1** $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ o $F_{col,2,1}$ = 63.91 k o $F_{col,3,1}$ = 63.91 k These column shears are less than the plastic shear capacity of each column (128k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.
B1.10 Effective Period and Damping Ratio Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge: Eq. 7.1-5 GSID $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}}$ (B-14) and Eq. 7.1-10 GSID $\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2}$ (B-15) where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j}$ = 0. For other types of	**B1.10 Effective Period and Damping Ratio, Example 2.1** $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{1907.58}{386.4(40.37)}}$ = 2.20 sec and taking $d_{y,j}$ = 0: $\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - 0)}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.31$

isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for the Simplified Method, set $d_{y,j} = 0$ for all isolator types. See Step B2.2 where the value of $d_{y,j}$ is revisited for the Multimode Spectral Analysis Method.	
B1.11 Damping Factor Calculate the damping factor, B_L, and the displacement, *d*, of the bridge: Eq. 7.1-3 GSID $B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases}$ (B-16) Eq. 7.1-4 GSID $d = \frac{9.79 S_{D1} T_{eff}}{B_L}$ (B-17)	**B1.11 Damping Factor, Example 2.1** Since $\xi = 0.31 \geq 0.3$ $B_L = 1.70$ and $d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.4)2.20}{1.70} = 5.06\ in$
B1.12 Convergence Check Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement. This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5). After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both. Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)	**B1.12 Convergence Check, Example 2.1** Since the calculated value for displacement, *d* (= 5.06) is not close to that assumed at the beginning of the cycle (Step B1.1, *d* = 4.0), use a value of say 5.0 in as the new assumed displacement and repeat from Step B1.3. After three iterations, convergence is reached at a superstructure displacement of 6.38 in, with an effective period of 2.76 seconds, and a damping factor of 1.7 (31% damping ratio). The displacement in the isolators at Pier 1 is 6.16 in and the effective stiffness of the same isolators is 10.49 k/in. See spreadsheet in Table B1.12-1 for results of final iteration. Since the column shear force must equal the isolator shear force for equilibrium, the column shear = 10.49 (6.16) = 64.62 k which is less than the maximum allowable (128 k) if elastic behavior is to be achieved (as required in Step A3). But the superstructure displacement = 6.38 in, which exceeds the available clearance of 2.5 in. There are three choices here: 1. Increase the clearance at the abutment to say 7 in to avoid impact. (Note that the minimum required is $\frac{8\,S_{D1} T_{eff}}{B_L} = \frac{8(0.40)2.76}{1.7} = 5.20\ in.$) 2. Allow impact to happen which will damage abutment back wall and require repair. This option would violate the elastic performance

requirement in Step A3.

3. Redesign the isolators. Since the column shear is only one-half of the yield capacity (64.62 vs 128 k) there is room to increase the characteristic strength Q_d and post yield stiffness K_d to increase this shear and reduce the displacements.

One of the many possible solutions here is to increase Q_d to $0.07W$ and K_d to 0.07 W/d. In this case, it will be found that the superstructure displacement reduces to 4.50 in, the effective period is 1.97 seconds, and the damping factor remains at 1.7 (31% damping ratio).

See spreadsheet in Table B1.12-2 for results of the final iteration for this solution.

The displacement in the isolators at Pier 1 is 4.19 in and the effective stiffness of the same isolators is 21.2 k/in. The column shear force is therefore = 21.2 (4.19) = 88.83 k which is much closer to the capacity of 128 k, but remains elastic as required. Although this is a much more efficient design, the superstructure displacement (4.5 in) still exceeds the capacity (2.5 in) and the recommended option is to increase the clearance at the abutments to, say, 5.0 in (the minimum required using 8 $S_{D1}T_{eff}/B_L$ is 3.71 in). This option is revisited in Step E7.2.

Option 3 is recommended and the following properties are assumed for the isolation system going forward to the next step (Step B2):

$$Q_d = 0.07W = 0.07(1651.32) = 115.59\ k$$

and

$$K_d = \frac{0.07W}{d} = \frac{0.07(1651.32)}{4.5} = 25.69\ k/in$$

Table B1.12-1 Simplified Method Solution for Design Example 2.1 - First Solution, Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	1651.32	256.26	1907.58	0.4	3							
Step B1.1	d	6.38	Assumed displacement									
	Q_d	82.57	Characteristic strength									
	K_d	12.94	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	168.48	8.424	1.320	10,000.00	0.000264	2.640	6.378	2.641	0.002	16.846	53.731	107.476
Pier 1	657.18	32.859	5.150	288.87	0.036306	10.120	6.156	10.488	0.224	64.567	202.296	411.936
Pier 2	657.18	32.859	5.150	288.87	0.036306	10.120	6.156	10.488	0.224	64.567	202.296	411.936
Abut 2	168.48	8.424	1.320	10,000.00	0.000264	2.640	6.378	2.641	0.002	16.846	53.731	107.476
Total	1651.32	82.566	12.941		$\Sigma K_{eff,j}$	25.521				162.825	512.054	1,038.825
					Step	**B1.4**						
Step B1.10	T_{eff}	2.76	Effective period									
	ξ	0.31	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.74										
	B_L	1.70	Damping Factor									
	d	6.37	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1												
Pier 1												
Pier 2												
Abut 2												

Table B1.12-2 Simplified Method Solution for Design Example 2.1 - Second Solution, Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	1651.32	256.26	1907.58	0.4	3							
Step B1.1	d	4.50	Assumed displacement									
	Q_d	115.59	Characteristic strength									
	K_d	25.69	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\, d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	168.48	11.794	2.621	10,000.00	0.000524	5.240	4.498	5.243	0.002	23.581	53.043	106.115
Pier 1	657.18	46.003	10.223	288.87	0.073375	19.747	4.192	21.196	0.308	88.861	192.861	399.872
Pier 2	657.18	46.003	10.223	288.87	0.073375	19.747	4.192	21.196	0.308	88.861	192.861	399.872
Abut 2	168.48	11.794	2.621	10,000.00	0.000524	5.240	4.498	5.243	0.002	23.581	53.043	106.115
Total	1651.32	115.592	25.687		$\Sigma K_{eff,j}$	49.974				224.883	491.808	1,011.974
					Step	**B1.4**						
Step B1.10	T_{eff}	1.97	Effective period									
	ξ	0.31	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.73										
	B_L	1.70	Damping Factor									
	d	4.55	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	3.931	0.874	1.748		4.37	1.773						
Pier 1	15.334	3.408	7.065		3.78	7.464						
Pier 2	15.334	3.408	7.065		3.78	7.464						
Abut 2	3.931	0.874	1.748		4.37	1.773						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength
Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \qquad \text{(B-19)}$$

And

$$K_{d,i} = \frac{K_{d,j}}{n} \qquad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 2.1
Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-2) by the number of isolators at each support (n = 3), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,1}$ = 11.79/3 = 3.93 k
- $Q_{d,2}$ = 46.00/3= 15.33 k
- $Q_{d,3}$ = 46.00/3 = 15.33 k
- $Q_{d,4}$ = 11.79/3 = 3.93 k

and

- $K_{d,1}$ = 2.62/3 = 0.87 k/in
- $K_{d,2}$ = 10.22/3 = 3.41 k/in
- $K_{d,3}$ = 10.22/3 = 3.41 k/in
- $K_{d,4}$ = 2.62/3 = 0.87 k/in

Note that the K_d values per support used above are from the final iteration in Table B1.12-2 and are not the same as the initial values in Step B1.2.This is principally because Q_d and K_d were changed in Step B1.12 to reduce the superstructure displacements from 6.38 in to 4.50in. Even if these parameters had not been changed, the above K_d values would not be the same as the values in Step B1.2, because they are adjusted from cycle to cycle in the iteration process, such that the total K_d summed over all the isolators satisfies the minimum lateral restoring force requirement for the bridge, i.e. K_{dtotal} = 0.05 W/d. See Step B1.1. Since d varies from cycle to cycle, $K_{d,j}$ varies from cycle to cycle.

B2.2 Initial Stiffness and Yield Displacement
Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

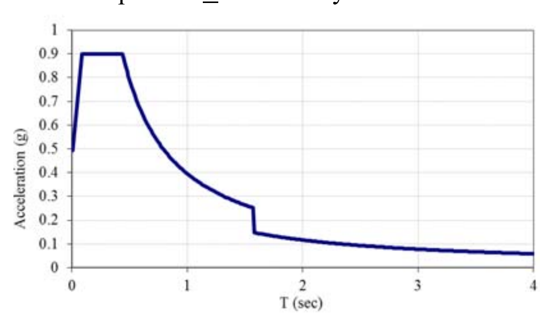
(1) For friction-based isolators $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \qquad \text{(B-21)}$$

and then

B2.2 Initial Stiffness and Yield Displacement, Example 2.1
Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1as follows:

$$K_{u,i} = 10K_{d,i} = 10(3.41) = 34.1\ k/in$$

(B-22)		and As expected, the yield displacement is small compared to the expected isolator displacement (~ 5 in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i} = 0$.
B2.3 Isolator Effective Stiffness, $K_{isol,i}$ Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i': (B -23)		**B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 2.1** Dividing the results for K_{isol} (Step B1.12) among the 3 isolators at each support, the following values for K_{isol} /isolator are obtained: o $K_{isol,1}$ = 5.24/3 = 1.75 k/in o $K_{isol,2}$ = 21.20/3 = 7.07 k/in o $K_{isol,3}$ = 21.20/3 = 7.07 k/in o $K_{isol,4}$ = 5.24/3 = 1.75 k/in
B2.4 Three Dimensional Bridge Model Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator.		**B2.4 Three Dimensional Bridge Model, Example 2.1** Although the bridge in this Design Example is regular and is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below. 
B2.5 Composite Design Response Spectrum Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.		**B.2.5 Composite Design Response Spectrum, Example 2.1** From the final results of Simplified Method (Step B1.12), B_L = 1.70 and T_{eff} = 1.97 sec. Hence the transition in the composite spectrum from 5% to 30% damping occurs at 0.8 T_{eff} = 0.8 (1.97) = 1.58 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods ≥ 1.58 sec by 1.70. 

B2.6 Multimodal Analysis of Finite Element Model Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.		**B2.6 Multimodal Analysis of Finite Element Model, Example 2.1** Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first three modes are the principal transverse, longitudinal, and torsion modes with periods of 2.0, 1.91 and 1.87 sec respectively. The period of the longitudinal mode (1.91 sec) is the almost the same as calculated in the Simplified Method (1.97 sec). The mass participation factors indicate there is no coupling between these three modes (probably due to the symmetric nature of the bridge) and the high values for the first and second modes (91% for each mode) indicate the bridge is responding essentially in a single mode of vibration in each direction. Similar results to those obtained by the Simplified Method are therefore to be expected.

Table B2.6-1 Modal Properties of Bridge Example 2.1 – First Iteration

Mode No	Period Sec	UX Unitless	UY Unitless	UZ Unitless	RX Unitless	RY Unitless	RZ Unitless
1	1.998	0.000	0.909	0.000	0.911	0.000	0.689
2	1.905	0.910	0.000	0.000	0.000	0.020	0.000
3	1.874	0.000	0.000	0.000	0.000	0.000	0.229
4	0.488	0.000	0.001	0.000	0.041	0.000	0.001
5	0.372	0.000	0.000	0.074	0.000	0.055	0.000
6	0.354	0.000	0.000	0.000	0.000	0.000	0.002
7	0.346	0.000	0.003	0.000	0.017	0.000	0.002
8	0.283	0.000	0.006	0.000	0.018	0.000	0.004
9	0.251	0.000	0.000	0.000	0.000	0.000	0.000
10	0.251	0.089	0.000	0.000	0.000	0.001	0.000
11	0.207	0.000	0.000	0.000	0.000	0.128	0.000
12	0.188	0.000	0.000	0.000	0.000	0.000	0.001

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 4.37 (4.50) in
- $d_{isol,2}$ = 3.78 (4.19) in
- $d_{isol,3}$ = 3.78 (4.19) in
- $d_{isol,4}$ = 4.37 (4.50) in

B2.7 Convergence Check Compare the resulting displacements at the superstructure level (*d*) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.		**B2.7 Convergence Check, Example 2.1** The results for the isolator displacements are close but not close enough (10% difference at piers). Go to Step B2.8 and update properties for a second cycle of iteration.

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows: $$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \quad \text{(B-24)}$$ Recalculate $K_{eff,j}$: Eq. 7.1-6 GSID $$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \quad \text{(B-25)}$$ Recalculate system damping ratio, ξ : Eq. 7.1-10 GSID $$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \quad \text{(B-26)}$$ Recalculate system damping factor, B_L: Eq. 7.1-3 GSID $$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \quad \text{(B-27)}$$ Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.	**B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 2.1** Updated values for $K_{isol,i}$ are given below (previous values are in parentheses): - $K_{isol,1}$ = 1.77 (1.75) k/in - $K_{isol,2}$ = 7.46 (7.07) k/in - $K_{isol,3}$ = 7.46 (7.07) k/in - $K_{isol,4}$ = 1.77 (1.75) k/in Since the isolator displacements are relatively close to previous results no significant change in the damping ratio is expected. Hence $K_{eff,j}$ and ξ are not recalculated and B_L is taken at 1.70. Since the change in effective period is very small (1.91 to 1.97 sec) and no change has been made to B_L, there is no need to construct a new composite response spectrum in this case. Go back to Step B2.6 (see immediately below).
	B2.6 Multimodal Analysis Second Iteration, Example 2.1 Reanalysis gives the following values for the isolator displacements (numbers in parentheses are those from the previous cycle): - $d_{isol,1}$ = 4.29 (4.37) in - $d_{isol,2}$ = 3.68 (3.78) in - $d_{isol,3}$ = 3.68 (3.78) in - $d_{isol,4}$ = 4.29 (4.29) in Go to Step B2.7
B2.7 Convergence Check Compare results and determine if convergence has been reached. If so go to Step B2.9. Otherwise Go to Step B2.8.	**B2.7 Convergence Check, Example 2.1** Satisfactory agreement has been reached on this second cycle (better than 2% at the abutments and 3% at the piers). Go to Step B2.9
B2.9 Superstructure and Isolator Displacements Once convergence has been reached, obtain - superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and - isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e.	**B2.9 Superstructure and Isolator Displacements, Example 2.1** From the above analysis: - superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are: x_L= 4.32 in y_L= 0.0 in

longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.

- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are:
  - Abutments: u_L = 4.29 in, v_L = 0.00 in
  - Piers: u_L = 3.68 in, v_L = 0.00 in

  All isolators at same support have the same displacements.

B2.10 Pier Bending Moments and Shear Forces
Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.

B2.10 Pier Bending Moments and Shear Forces, Example 2.1
Bending moments in single column pier in the longitudinal (M_{PLL}) and transverse (M_{PTL}) directions are:

M_{PLL}= 0
M_{PTL}= 2,661 kft

Shear forces in single column pier the longitudinal (V_{PLL}) and transverse (V_{PTL}) directions are:

V_{PLL}= 111.54 k
V_{PTL}= 0

B2.11 Isolator Shear and Axial Forces
Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 2.1
Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abutment	1	7.60	0	1.56
	2	13.98	0	1.54
	3	13.98	0	1.56
Pier	1	27.47	0	0.64
	2	27.51	0	0.63
	3	27.47	0	0.64

The difference between the longitudinal shear force in the column (V_{PLL} = 111.54 k) and the sum of the isolator shear forces at the same Pier (82.45 k) is about 29.1 k. This is due to the inertia force developed in the hammerhead cap beam which weighs about 128 k and can generate significant additional demand on the column (about a 34% increase in this case).

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 2.1

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.92 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows:
  x_T = 0 and y_T = 4.45 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions are as follows:
  Abutments u_T = 0.00 in, v_T = 4.48in
  Piers u_T = 0.00 in, v_T = 3.20 in
- Pier bending moments in the longitudinal (M_{PLT}) and transverse (M_{PTT}) directions are as follows:
  M_{PLT} = 2,506 kft and M_{PTT} = 0
- Pier shear forces in the longitudinal (V_{PLT}) and transverse (V_{PTT}) directions as follows:
  V_{PLT} = 0 and V_{PTT} = 92.06 k
- Isolator shear and axial forces are summarized in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abutment	1	0.0	7.79	18.04
	2	0.0	7.80	0
	3	0.0	7.79	18.04
Pier	1	0.0	25.87	33.64
	2	0.0	25.98	0
	3	0.0	25.87	33.64

The difference between the longitudinal shear force in the column (V_{PTT} = 92.06 k) and the sum of the isolator shear forces at the same Pier (77.72 k) is about 14.3 k. This is due to the inertia force developed in the hammerhead cap beam which weighs about 128 k and can generate significant additional demand on the column (about 18%).

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements

Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator, by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)
- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)
- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements at Pier 1, Example 2.1

To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

<u>Load Case 1</u>:

$u_1 = u_L + 0.3u_T$ = 1.0(3.67) + 0.3(0) = 3.67 in
$v_1 = v_L + 0.3v_T$ = 1.0(0) + 0.3(3.20) = 0.96 in
$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{3.67^2 + 0.96^2}$ = 3.79 in

<u>Load Case 2</u>:

$u_2 = 0.3u_L + u_T$ = 0.3(3.67) + 1.0(0) = 1.10 in
$v_2 = 0.3v_L + v_T$ = 0.3(0) + 1.0(3.20) = 3.20 in
$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{1.10^2 + 3.20^2}$ = 3.38 in

<u>Governing Case</u>:

Total design displacement, d_t = max(R_1, R_2)
= 3.79 in

D2. Design Moments and Shears

Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

D2. Design Moments and Shears at Pier 1, Example 2.1

Design moments and shear forces are calculated for Pier 1 below, to illustrate the process.

<u>Load Case 1</u>:

$V_{PL1} = V_{PLL} + 0.3V_{PLT}$ = 1.0(111.54) + 0.3(0) = 111.5 k
$V_{PT1} = V_{PTL} + 0.3V_{PTT}$ = 1.0(0) + 0.3(92.06) = 27.62 k
$R_1 = \sqrt{V_{L1}^2 + V_{T1}^2} = \sqrt{111.5^2 + 27.62^2}$ = 114.9 k

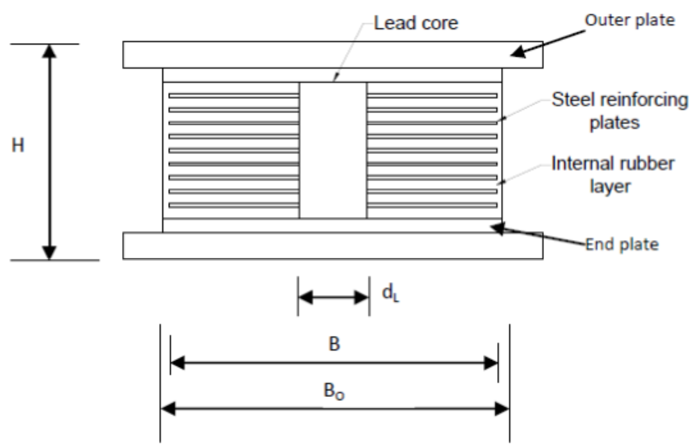
<u>Load Case 2</u>:

$V_{PL2} = 0.3V_{PLL} + V_{PLT}$ = 0.3(111.54) + 1.0(0) = 33.46 k
$V_{PT2} = 0.3V_{PTL} + V_{PTT}$ = 0.3(0) + 1.0(92.06) = 92.06k
$R_2 = \sqrt{V_{L2}^2 + V_{T2}^2} = \sqrt{33.46^2 + 92.06^2}$ = 97.95k

<u>Governing Case</u>:

Design column shear = max (*R1*, *R2*)
= 114.9 k

STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS



A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness (K_u).With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness K_d is given by the rubber alone. More details are given in MCEER 2006.

While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary. When sizing the physical dimensions of the bearing, plan dimensions (B, d_L) should be rounded up to the next ¼ -inch increment, while the total thickness of elastomer, T_r, is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are ¼ inch and $^3/_8$ in. High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in MCEER 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See MCEER 2006.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1). • required characteristic strength, Q_d / isolator • required post-elastic stiffness, K_d / isolator • total design displacement, d_t, for each isolator • maximum applied dead and live load (P_{DL}, P_{LL}) and seismic load (P_{SL}) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator, and • maximum wind load, P_{WL}		**E1. Required Properties, Example 2.1** The design of one of the exterior isolators on a pier is given below to illustrate the design process for a lead-rubber isolator. From previous work • Q_d / isolator = 15.33 k • K_d / isolator = 3.41 k/in • Total design displacement, d_t = 3.79 in • P_{DL} = 187 k • P_{LL} = 123 k and P_{SL} = 33.64 k (Table C1-1) • P_{WL} = 8.21k < Q_d OK
E2. Isolator Sizing		
E2.1 Lead Core Diameter Determine the required diameter of the lead plug, d_L, using: $$d_L = \sqrt{\frac{Q_d}{0.9}} \qquad \text{(E-1)}$$ See Step E2.5 for limitations on d_L		**E2.1 Lead Core Diameter, Example 2.1** $$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{15.33}{0.9}} = 4.13\ in$$

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E2.2 Plan Area and Isolator Diameter
Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.6 ksi.

Then the bonded area of the isolator is given by:

$$A_b = \frac{P_{DL} + P_{LL}}{1.6}\ in^2 \qquad \text{(E-2)}$$

and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by:

$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} \qquad \text{(E-3)}$$

Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using

$$A_b = \frac{\pi}{4}(B^2 - d_L^2) \qquad \text{(E-4)}$$

Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, B_o is given by:

$$B_o = B + 1.0 \qquad \text{(E-5)}$$

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E2.2 Plan Area and Isolator Diameter, Example 2.1

$$A_b = \frac{P_{DL} + P_{LL}}{1.6}\ in^2 = \frac{187 + 123}{1.6} = 193.75\ in^2$$

$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} = \sqrt{\frac{4\,(193.75)}{\pi} + 4.13^2}$$
= 16.24 in

Round B up to 16.25 in and the actual bonded area is:

$$A_b = \frac{\pi}{4}(16.25^2 - 4.13^2) = 194.02\ in^2$$

B_o = 16.25 + 2(0.5) = 17.25 in

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E2.3 Elastomer Thickness and Number of Layers
Since the shear stiffness of an elastomeric bearing is given by:

$$K_d = \frac{GA_b}{T_r} \qquad \text{(E-6)}$$

where G = shear modulus of the rubber, and
T_r = the total thickness of elastomer,
it follows Eq. E-6 may be used to obtain T_r given a required value for K_d

$$T_r = \frac{GA_b}{K_d} \qquad \text{(E-7)}$$

A typical range for shear modulus, G, is 60-120 psi. Higher and lower values are available and are used in special applications.

If the layer thickness is t_r, the number of layers, n, is given by:

$$n = \frac{T_r}{t_r} \qquad \text{(E-8)}$$

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E2.3 Elastomer Thickness and Number of Layers, Example 2.1

Select G, shear modulus of rubber = 100 psi (0.1ksi), then

$$T_r = \frac{GA_b}{K_d} = \frac{0.1(194.02)}{3.41} = 5.69\ in$$

$$n = \frac{5.69}{0.25} = 22.8$$

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rounded up to the nearest integer. Note that because of rounding the plan dimensions and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be necessary if the differences are large.		Round up to nearest integer, i.e. n = 23
E2.4 Overall Height The overall height of the isolator, H, is given by: $H = n\,t_r + (n-1)t_s + 2t_c$ (E-9) where t_s = thickness of an internal shim (usually about 1/8 in), and t_c = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in)		**E2.4 Overall Height, Example 2.1** $H = 23(0.25) + 22(0.125) + 2 * 1.5 = 11.50\ in$
E2.5 Lead Core Size Check Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows: $\frac{B}{3} \geq d_L \geq \frac{B}{6}$ (E-10)		**E2.5 Lead Core Size Check, Example 2.1** Since B=16.25 check $\frac{16.25}{3} \geq d_L \geq \frac{16.25}{6}$ i.e., $5.42 \geq d_L \geq 2.71$ Since d_L = 4.13, lead core size is acceptable.
E3. Strain Limit Check Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e., $\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5$ (E-11) where γ_c, $\gamma_{s,eq}$, and γ_r are defined below. (a) γ_c is the maximum shear strain in the layer due to compression and is given by: $\gamma_c = \frac{D_c \sigma_S}{GS}$ (E-12) where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = {P_{DL}}/{A_b}$, G is shear modulus, and S is the layer shape factor given by: $S = \frac{A_b}{\pi B t_r}$ (E-13) (b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by: $\gamma_{s,eq} = \frac{d_t}{T_r}$ (E-14) (c) γ_r is the shear strain due to rotation and is given		**E3. Strain Limit Check, Example 2.1** Since $\sigma_S = \frac{187.0}{194.02} = 0.964\ ksi$ G = 0.1 ksi and $S = \frac{194.02}{\pi 16.25(0.25)} = 15.20$ then $\gamma_c = \frac{1.0(0.964)}{0.1(15.2)} = 0.63$

by:

$$\gamma_r = \frac{D_r B^2 \theta}{t_r T_r} \qquad \text{(E-15)}$$

where D_r is shape coefficient for rotation in circular bearings = 0.375, and θ is design rotation due to DL, LL and construction effects . Actual value for θ may not be known at this time and a value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1).

$$\gamma_{s,eq} = \frac{3.79}{5.75} = 0.66$$

$$\gamma_r = \frac{0.375(16.25^2)(0.01)}{0.25(5.75)} = 0.69$$

Substitution in Eq E-11 gives

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.63 + 0.66 + 0.5(0.69) = 1.64 \leq 5.5\ OK$$

E4. Vertical Load Stability Check

Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state.

Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either
2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or
1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4.

E4. Vertical Load Stability Check, Example 2.1

E4.1 Vertical Load Stability in Undeformed State

The critical load capacity of an elastomeric isolator at zero shear displacement is given by

$$P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2}\left[\sqrt{(1 + \frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2})} - 1\right] \qquad \text{(E-16)}$$

where
$H_{eff} = T_r + T_s$
T_s = total shim thickness
$K_\theta = {E_b I}/{T_r}$
$E_b = E(1 + 0.67S^2)$
E = elastic modulus of elastomer = $3G$
$I = {\pi B^4}/{64}$

It is noted that typical elastomeric isolators have high shape factors, S, in which case:

$$\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1 \qquad \text{(E-17)}$$

E4.1 Vertical Load Stability in Undeformed State, Example 2.1

$$E = 3G = 3(0.1) = 0.3\ ksi$$

$$E_b = 0.3(1 + 0.67(15.20^2)) = 46.54\ ksi$$

$$I = \pi\frac{16.25^4}{64} = 3{,}422.8\ in^4$$

$$K_\theta = \frac{46.54(3{,}422.8)}{5.75} = 27{,}705\ kin/rad$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.1(194.02)}{5.75} = 3.37\ k/in$$

and Eq. E-16 reduces to:

$$P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta} \qquad \text{(E-18)}$$

Check that:

$$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} \geq 3 \qquad \text{(E-19)}$$

$$P_{cr(\Delta=0)} = \pi\sqrt{3.37(27{,}705)} = 960.54\ k$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} = \frac{960.54}{(187 + 123)} = 3.10 \geq 3 \quad OK$$

E4.2 Vertical Load Stability in Deformed State

The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by:

$$P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)} \qquad \text{(E-20)}$$

where

A_r = overlap area between top and bottom plates of isolator at displacement Δ (Fig. 2.2-1 GSID)

= $B^2(\delta - sin\delta)/4$

δ = $2cos^{-1}(\Delta/B)$

A_{gross} = $\pi B^2/4$

It follows that:

$$\frac{A_r}{A_{gross}} = \frac{(\delta - sin\delta)}{\pi} \qquad \text{(E-21)}$$

Check that:

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} \geq 1 \qquad \text{(E-22)}$$

E4.2 Vertical Load Stability in Deformed State, Example 2.1

Since bridge is in Zone 3,

$$\Delta = 1.5d_t = 1.5(3.79) = 5.69\ in$$

$$\delta = 2cos^{-1}\left(\frac{5.69}{16.25}\right) = 2.43$$

$$\frac{A_r}{A_{gross}} = \frac{(2.43 - sin2.43)}{\pi} = 0.564$$

$$P_{cr(\Delta)} = 0.564(960.54) = 541.57\ k$$

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{541.57}{1.2(187) + 33.64} = 2.10 \geq 1 \quad OK$$

E5. Design Review

E5. Design Review, Example 2.1

The basic dimensions of the isolator designed above are as follows:

17.25 in (od) x 11.50 in (high) x 4.13 in dia. lead core

and the volume, excluding the steel end and cover plates = 2,337 in^3

Although this design satisfies all the required criteria, the total rubber shear strain (1.64) is much less than the maximum allowable (5.5), as shown in Step E3. In other words, the isolator is not working very hard and a redesign appears to be worth exploring to see if a more optimal design can be found. Since the plan dimension appears to be about right to satisfy both vertical stability requirements (Step E4.1 and E4.2) the best way to optimize the design is to reduce its

height. But reducing the height will increase the stiffness, K_d, unless the shear modulus of the elastomer is likewise reduced. In the redesign below, the plan dimensions remain the same but the shear modulus is reduced from 100 to 60 psi.

E2.1

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{15.33}{0.9}} = 4.13\ in$$

E2.2

$$A_b = \frac{P_{DL} + P_{LL}}{1.6}\ in^2 = \frac{187 + 123}{1.6} = 193.75\ in^2$$

$$B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} = \sqrt{\frac{4\ (193.75)}{\pi} + 4.13^2}$$

= 16.24 in

Round B up to 16.25 in and the actual bonded area is:

$$A_b = \frac{\pi}{4}(16.25^2 - 4.13^2) = 194.02\ in^2$$

B_o = 16.25 + 2(0.5) = 17.25 in

E2.3

$$T_r = \frac{GA_b}{K_d} = \frac{0.06(194.02)}{3.41} = 3.41\ in$$

$$n = \frac{3.41}{0.25} = 13.7$$

Round n up to 14

E2.4

$$H = 14(0.25) + 13(0.125) + 2 * 1.5 = 8.125\ in$$

E2.5

Since B = 16.25 check

$$\frac{16.25}{3} \geq d_L \geq \frac{16.25}{6}$$

$$5.42 \geq d_L \geq 2.71$$

Since d_L = 4.13 in, size of lead core is acceptable.

E3.

$$\sigma_S = \frac{187.0}{194.02} = 0.964\ ksi$$

$$S = \frac{194.02}{\pi 16.25(0.25)} = 15.20$$

$$\gamma_c = \frac{1.0(0.964)}{0.06(15.20)} = 1.06$$

$$\gamma_{s,eq} = \frac{3.79}{3.5} = 1.08$$

$$\gamma_r = \frac{0.375(16.25^2)(0.01)}{0.25(3.5)} = 1.13$$

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 1.06 + 1.08 + 0.5(1.13)$$
$$= 2.71 \le 5.5\ OK$$

E4.1

$$E = 3G = 3(0.06) = 0.18\ ksi$$

$$E_b = 0.18(1 + 0.67(15.20^2)) = 27.93\ ksi$$

$$I = \frac{16.25^4}{64} = 3{,}422.8in^4$$

$$K_\theta = \frac{27.93(3{,}422.8)}{3.5} = 27{,}309\ kin/rad$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(194.02)}{3.5} = 3.33\ k/in$$

$$P_{cr(\Delta=0)} = \pi\sqrt{3.33(27{,}309)} = 946.82\ k$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} = \frac{946.82}{(187 + 123)} = 3.05 \ge 3\quad OK$$

E4.2

$$\delta = 2cos^{-1}\left(\frac{5.68}{16.25}\right) = 2.43$$

$$\frac{A_r}{A_{gross}} = \frac{(2.43 - sin2.43)}{\pi} = 0.564$$

$$P_{cr(\Delta)} = 0.564(946.82) = 533.84\ k$$

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{533.84}{1.2(187) + 33.64} = 2.07 \ge 1\ \ OK$$

E5.
The basic dimensions of the redesigned isolator are as

follows:

17.25 in (od) x 8.125 in (high) x 4.13 in dia. lead core

and the volume, excluding steel end and cover plates, = 1,548 in^3

This is a more optimal design. It is smaller than the previous design (1,548 in^3 vs 2,337 in^3) while still satisfying all the design criteria. Being smaller it works harder to satisfy these requirements, as can be seen by the increase in the total shear strain from 1.64 to 2.71 (but still less than the maximum allowable of 5.5).

E6. Minimum and Maximum Performance Check
Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E6-1.

Table E6-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-23)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-24)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-25)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-26)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity.

E6. Minimum and Maximum Performance Check, Example 2.1
Minimum Property Modification factors are
$\lambda_{min,Kd} = 1.0$
$\lambda_{min,Qd} = 1.0$

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are

$\lambda_{max,a,Kd} = 1.1$
$\lambda_{max,a,Qd} = 1.1$

$\lambda_{max,t,Kd} = 1.1$
$\lambda_{max,t,Qd} = 1.4$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,a,Qd} = 1.0 + 0.1(0.66) = 1.066$

$\lambda_{max,t,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,t,Qd} = 1.0 + 0.4(0.66) = 1.264$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Therefore the maximum overall modification factors

$\lambda_{max,Kd} = 1.066(1.066)1.0 = 1.14$
$\lambda_{max,Qd} = 1.066(1.264)1.0 = 1.35$

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

Art. 8.2.2 GSID gives these factors as follows:

- 1.00 for critical bridges
- 0.75 for essential bridges
- 0.66 for all other bridges

Table E6-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})\ (\lambda_{min,a,Kd})\ (\lambda_{min,v,Kd})\ (\lambda_{min,tr,Kd})\ (\lambda_{min,c,Kd})\ (\lambda_{min,scrag,Kd})$	(E-27)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})\ (\lambda_{max,a,Kd})\ (\lambda_{max,v,Kd})\ (\lambda_{max,tr,Kd})\ (\lambda_{max,c,Kd})\ (\lambda_{max,scrag,Kd})$	(E-28)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})\ (\lambda_{min,a,Qd})\ (\lambda_{min,v,Qd})\ (\lambda_{min,tr,Qd})\ (\lambda_{min,c,Qd})\ (\lambda_{min,scrag,Qd})$	(E-29)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})\ (\lambda_{max,a,Qd})\ (\lambda_{max,v,Qd})\ (\lambda_{max,tr,Qd})\ (\lambda_{max,c,Qd})\ (\lambda_{max,scrag,Qd})$	(E-30)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

- 1.00 for critical bridges
- 0.75 for essential bridges
- 0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

The upper-bound properties are:
$Q_{d,max}$ = 1.35 (15.33) = 20.70 k
and
$K_{d,max}$=1.14(3.41) = 3.89 k/in

E7. Design and Performance Summary

E7.1 Isolator dimensions

Summarize final dimensions of isolators:

- Overall diameter (includes cover layer)
- Overall height
- Diameter lead core
- Bonded diameter
- Number of rubber layers
- Thickness of rubber layers
- Total rubber thickness
- Thickness of steel shims
- Shear modulus of elastomer

Check all dimensions with manufacturer.

E7.2 Bridge Performance

Summarize bridge performance

- Maximum superstructure displacement (longitudinal)
- Maximum superstructure displacement (transverse)
- Maximum superstructure displacement (resultant)
- Maximum column shear (resultant)
- Maximum column moment (about transverse axis)
- Maximum column moment (about longitudinal axis)
- Maximum column torque

Check required performance as determined in Step A3, is satisfied.

E7. Design and Performance Summary, Example 2.1

E7.1 Isolator dimensions, Example 2.1

Isolator dimensions are summarized in Table E7.1-1.

Table E7.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Diam. lead core (in)
Under edge girder on Pier 1	21.25 x 21.25 x 8.125(H)	17.25 dia. x 6.625(H)	4.13

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in)	Total rubber thick-ness (in)	Steel shim thick-ness (in)
Under edge girder on Pier 1	14	0.25	3.50	0.125

Shear modulus of elastomer = 60 psi

E7.2 Bridge Performance, Example 2.1

Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 121 k. This less than the column plastic shear (128k) and therefore the required performance criterion is satisfied (fully elastic behavior).

However the maximum longitudinal displacement is 3.79 in which is more than the 2.5 in available at the abutment expansion joints. As discussed in Step B1.12, the best option here is to increase the clearance at the abutment to allow for this movement. Anything less will lead to impact at the abutments and whereas such damage will not be life threatening, it will not satisfy the performance requirement in Step A3 (fully elastic behavior).

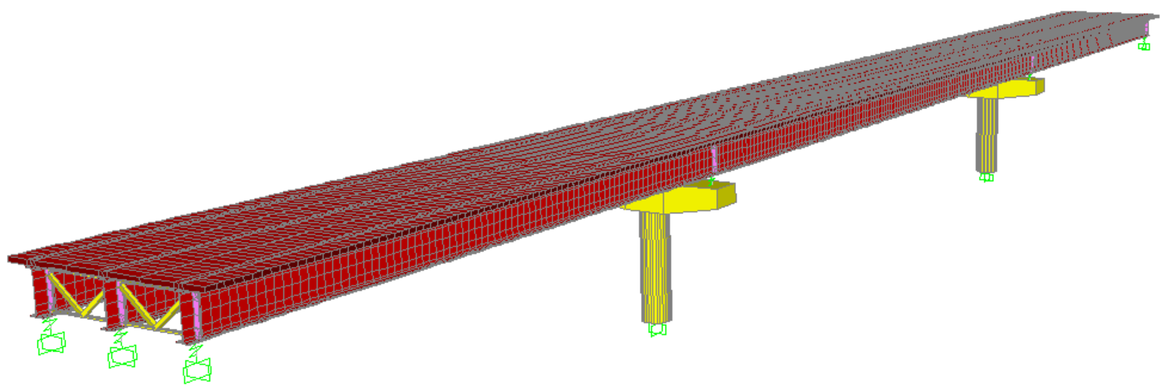
. **Table E7.2-1 Summary of Bridge Performance**

Maximum superstructure displacement (longitudinal)	3.67 in
Maximum superstructure displacement (transverse)	3.20 in
Maximum superstructure displacement (resultant)	3.79 in
Maximum column shear (resultant)	121 k
Maximum column moment about transverse axis	2809 kft
Maximum column moment about longitudinal axis	2873 kft
Maximum column torque	29 kft

SECTION 2

Steel Plate Girder Bridge, Long Spans, Single-Column Pier

DESIGN EXAMPLE 2.2: S_1 = 0.6g



Design Examples in Section 2

ID	Description	S_1	Site Class	Column height	Skew	Isolator type
2.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
2.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
2.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
2.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
2.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
2.5	Change column height	0.2g	B	H_1=0.5H_2	0	Lead rubber bearing
2.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

DESIGN PROCEDURE	DESIGN EXAMPLE 2.2 (S_1 = 0.6g)

STEP A: BRIDGE AND SITE DATA

A1. Bridge Properties

Determine properties of the bridge:

- number of supports, *m*
- number of girders per support, *n*
- angle of skew
- weight of superstructure including railings, curbs, barriers and other permanent loads, W_{SS}
- weight of piers participating with superstructure in dynamic response, W_{PP}
- weight of superstructure, W_j, at each support
- stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
- column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
- allowable movement at expansion joints
- isolator type if known, otherwise 'to be selected'

A1. Bridge Properties, Example 2.2

- Number of supports, $m = 4$
  - North Abutment ($m = 1$)
  - Pier 1 ($m = 2$)
  - Pier 2 ($m = 3$)
  - South Abutment ($m = 4$)
- Number of girders per support, $n = 3$
- Angle of skew = 0^0
- Number of columns per support = 1
- Weight of superstructure including permanent loads, W_{SS} = 1651.32 k
- Weight of superstructure at each support:
  - W_1 = 168.48 k
  - W_2 = 657.18 k
  - W_3 = 657.18 k
  - W_4 = 168.48 k
- Participating weight of piers, W_{PS} = 256.26 k
- Effective weight (for calculation of period), $W_{eff} = Wss + Wps$ = 1907.58 k
- Stiffness of each pier in the both directions:
  - $K_{sub,pier1}$ = 288.87 k/in
  - $K_{sub,pier2}$ = 288.87 k/in
- Minimum column shear strength based on flexural yield capacity of column = 128 k
- Displacement capacity of expansion joints (longitudinal) = 2.5 in for thermal and other movements
- Lead-rubber isolators

A2. Seismic Hazard

Determine seismic hazard at site:

- acceleration coefficients
- site class and site factors
- seismic zone

Plot response spectrum.

Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of *PGA*, S_S and S_1 are given in both printed and CD formats (Figures 3.10.2.1-1 – 3.10.2.1-21LRFD).

Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications,

A2. Seismic Hazard, Example 2.2

Acceleration coefficients for bridge site are given in design statement as follows:

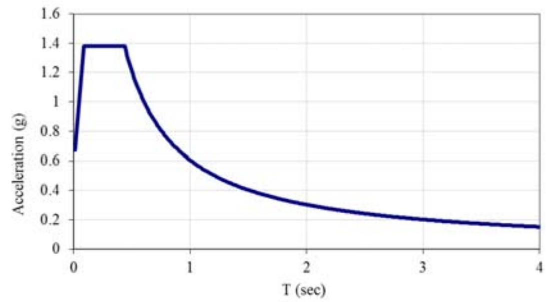
- PGA = 0.58
- S_1 = 0.60
- S_S = 1.38

Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec.

Table 3.10.3.1-1 LRFD gives Site Class as B.

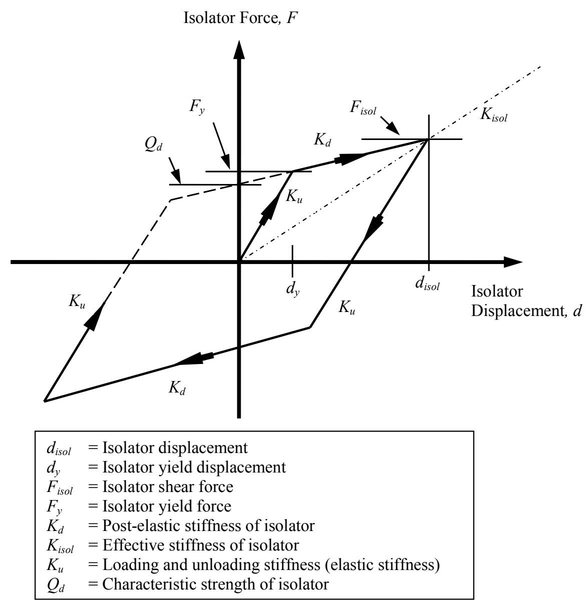
Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors:

- F_{pga} = 1.0
- F_a = 1.0
- F_v = 1.0

i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD. Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows: • $A_s = F_{pga}\, PGA$ • $S_{DS} = F_a\, S_S$ • $S_{D1} = F_v\, S_1$ Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table 5-1 GSID. These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.		• $A_s = F_{pga}\, PGA = 1.0(0.58) = 0.58$ • $S_{DS} = F_a\, S_S = 1.0(1.38) = 1.38$ • $S_{D1} = F_v\, S_1 = 1.0(0.60) = 0.60$ Since $0.60 \leq S_{D1}$, bridge is located in Seismic Zone 4. Design Response Spectrum is as below: 
A3. Performance Requirements Determine required performance of isolated bridge during Design Earthquake (1000-yr return period). Examples of performance that might be specified by the Owner include: • Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake. • Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake. • For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements) • Reduced substructure forces for bridges on weak soils to reduce foundation costs.		**A3. Performance Requirements, Example 2.2** As in previous examples, the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield). To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (128 k). The maximum shear in the column must therefore be less than 128 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.



Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties

To begin the iterative solution, an estimate is required of :

(1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)

Art C7.1 GSID

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \qquad \text{(B-1)}$$

(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not

B1.1 Initial System Displacement and Properties, Example 2.2

$$d \cong 10 S_{D1} = 10(0.60) \cong 6.0\ in$$

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occur under non-seismic loads (e.g. wind) but low enough that yield does occur under earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e.

$$Q_d = 0.05W \qquad \text{(B-2)}$$

(3) Post-yield stiffness, K_d
Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3.

Art. 12.2 GSID

$$K_{d,min} \geq \frac{0.025W}{d} \qquad \text{(B-3)}$$

Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$

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$$Q_d = 0.05W = 0.05(1651.32) = 82.56\ k$$

$$K_d = 0.05\frac{W}{d} = 0.05\frac{1651.32}{6.0} = 13.76\ k/in$$

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B1.2 Initial Isolator Properties at Supports
Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support:

$$Q_{d,j} = Q_d\frac{W_j}{W} \qquad \text{(B-4)}$$

and

$$K_{d,j} = K_d\frac{W_j}{W} \qquad \text{(B-5)}$$

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B1.2 Initial Isolator Properties at Supports, Example 2.2

$$Q_{d,j} = Q_d\frac{W_j}{W}$$

- $Q_{d,1} = 8.42$ k
- $Q_{d,2} = 32.86$ k
- $Q_{d,3} = 32.86$ k
- $Q_{d,4} = 8.42$ k

and

$$K_{d,j} = K_d\frac{W_j}{W}$$

- $K_{d,1} = 1.40$ k/in
- $K_{d,2} = 5.48$ k/in
- $K_{d,3} = 5.48$ k/in
- $K_{d,4} = 1.40$ k/in

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B1.3 Effective Stiffness of Combined Pier and Isolator System
Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below (after Fig. 7.1-1 GSID).

An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula as follows (MCEER 2006):

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j} \qquad \text{(B-6)}$$

where

$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}} \qquad \text{(B-7)}$$

and $K_{sub,j}$ for the piers are given in Step A1. For abutments, take $Ksub,j$ to be a large number, say

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B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 2.2

$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$

- $\alpha_1 = 2.81\text{x}10^{-6}$
- $\alpha_2 = 3.86\text{x}10^{-2}$
- $\alpha_3 = 3.86\text{x}10^{-2}$
- $\alpha_4 = 2.81\text{x}10^{-6}$

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$$

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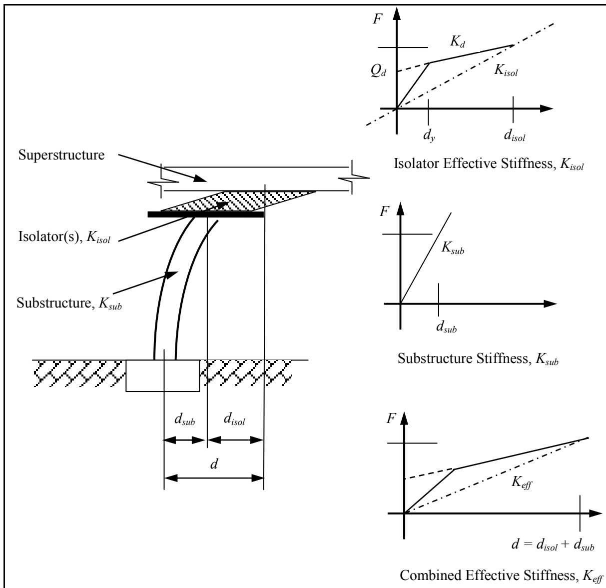

10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.

- $K_{eff,1}$ = 2.81k/in
- $K_{eff,2}$ = 10.75 k/in
- $K_{eff,3}$ = 10.75 k/in
- $K_{eff,4}$ = 2.81 k/in

B1.4 Total Effective Stiffness

Calculate the total effective stiffness, K_{eff}, of the bridge:

Eq. 7.1-6 GSID

$$K_{eff} = \sum_{j=1}^{m} K_{eff,j} \qquad \text{(B-8)}$$

B1.4 Total Effective Stiffness, Example 2.2

$$K_{eff} = \sum_{j=1}^{4} K_{eff,j}$$

= 27.11 k/in

B1.5 Isolation System Displacement at Each Support

Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports:

$$d_{isol,j} = \frac{d}{1+\alpha_j} \qquad \text{(B-9)}$$

B1.5 Isolation System Displacement at Each Support, Example 2.2

$$d_{isol,j} = \frac{d}{1+\alpha_j}$$

- $d_{isol,1}$ = 6.00 in
- $d_{isol,2}$ = 5.78 in
- $d_{isol,3}$ = 5.78 in
- $d_{isol,4}$ = 6.00 in

B1.6 Isolation System Stiffness at Each Support

Calculate the stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports:

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \qquad \text{(B-10)}$$

B1.6 Isolation System Stiffness at Each Support, Example 2.2

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$

- $K_{isol,1}$ = 2.81 k/in
- $K_{isol,2}$ = 10.75 k/in
- $K_{isol,3}$ = 10.75 k/in
- $K_{isol,4}$ = 2.81 k/in

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B1.7 Substructure Displacement at Each Support
Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports:

$$d_{sub,j} = d - d_{isol,j} \quad \text{(B-11)}$$

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B1.7 Substructure Displacement at Each Support, Example 2.2

$$d_{sub,j} = d - d_{isol,j}$$

- $d_{sub,1}$ = 0 in
- $d_{sub,2}$ = 0.22 in
- $d_{sub,3}$ = 0.22 in
- $d_{sub,4}$ = 0 in

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B1.8 Substructure Shear at Each Support
Calculate the shear at support 'j', $F_{sub,j}$, for all supports:

$$F_{sub,j} = K_{sub,j} d_{sub,j} \quad \text{(B-12)}$$

where values for $K_{sub,j}$ are given in Step A1.

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B1.8 Lateral Load in Each Substructure, Example 2.2

$$F_{sub,j} = K_{sub,j} d_{sub,j}$$

- $F_{sub,1}$ = 16.85 k
- $F_{sub,2}$ = 64.49 k
- $F_{sub,3}$ = 64.49 k
- $F_{sub,4}$ = 16.85 k

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B1.9 Column Shear at Each Support
Calculate the shear in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j':

$$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j} \quad \text{(B-13)}$$

Use these approximate column shears as a check on the validity of the chosen strength and stiffness characteristics.

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B1.9 Column Shear Force at Each Support, Example 2.2

$$F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$$

- $F_{col,2,1}$ = 64.49 k
- $F_{col,3,1}$ = 64.49 k

These column shears are less than the plastic shear capacity of each column (128 k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.

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B1.10 Effective Period and Damping Ratio
Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge:

Eq. 7.1-5 GSID

$$T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} \quad \text{(B-14)}$$

and

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} \quad \text{(B-15)}$$

where d_y is the yield displacement of the isolator and assumed to be negligible compared to d, i.e., take d_y =

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B1.10 Effective Period and Damping Ratio, Example 2.2

$$T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{1907.58}{386.4(27.11)}}$$

= 2.68 sec

and assuming $d_{y,j} = 0$:

$$\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.31$$

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<tr><td>0 for the Simplified Method.</td><td></td><td></td></tr>
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B1.11 Damping Factor
Calculate the damping factor, B_L, and the displacement, d, of the bridge:

Eq. 7.1-3 GSID $$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases} \quad \text{(B-16)}$$
Eq. 7.1-4 GSID $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \quad \text{(B-17)}$$
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B1.11 Damping Factor, Example 2.2
Since $\xi = 0.31 \geq 0.3$
$$B_L = 1.70$$
and
$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.6)2.68}{1.70} = 9.34\ in$$
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B1.12 Convergence Check
Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement.

This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5).

After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both.

Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)
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B1.12 Convergence Check, Example 2.2
Since the calculated value for displacement, d (= 9.34) is not close to that assumed at the beginning of the cycle (Step B1.1, d = 6.0), use the value of 9.34 as the new assumed displacement and repeat from Step B1.3.

After several iterations, convergence is reached at a superstructure displacement of 14.19 in, with an effective period of 4.11 seconds, and a damping factor of 1.7 (32% damping ratio). The displacement in the isolators at Pier 1 is 13.97 in and the effective stiffness of the same isolators is 4.59 k/in.

See spreadsheet in Figure B1.12-1 for results of final iteration.

Since the column shear must equal the isolator shear for equilibrium, the column shear = 4.59 (13.97) = 64.12 k which is less than the maximum allowable (128 k) if elastic behavior is to be achieved (as required in Step A3).

But the superstructure displacement = 14.19 in, which far exceeds the available clearance of 2.5 in.

There are three choices here:

1. Increase the clearance at the abutment to say 15 in to avoid impact. (Note that the minimum required is $$\frac{8\, S_{D1} T_{eff}}{B_L} = \frac{8(0.60)4.11}{1.7} = 11.60\ in.)$$ This could be expensive.

2. Allow impact to happen which will damage abutment back wall and require repair. This option would violate the elastic performance requirement in Step A3.

3. Redesign the isolators. Since the column
</td></tr>
</table>

shear force is only one-half of the capacity (64.12 vs 128 k) there is room to increase the characteristic strength Q_d and post yield stiffness K_d to increase this shear and reduce the displacements.

One of the many possible solutions here is to increase Q_d to $0.09W$ and K_d to $0.09W/d$. In this case, it will be found that the superstructure displacement reduces to 8.00 in, the effective period is 2.31 seconds, and the damping factor remains at 1.7 (31% damping ratio).

See spreadsheet in Figure B1.12-2 for results of final iteration.

The displacement in the isolators at Pier 1 is 7.60 in and the effective stiffness of the same isolators is 14.42 k/in. The column shear is therefore = 14.42 (7.60) = 109.6 k which is still below the capacity of 128 k, so that the column remains elastic as required. Although this is a much more efficient design, the superstructure displacement (8.0 in) still exceeds the capacity (2.5 in) and the recommended option is to increase the clearance at the abutments to, say, 9.0 in (the minimum required using 8 $S_{D1}T_{eff}/B_L$ is 6.6 in). This option is revisited in Step E7.2.

Option 3 is recommended and the following properties are assumed for the isolation system going forward to the next step (Step B2):

$$Q_d = 0.09W = 0.09(1651.32) = 148.62\ k$$

and

$$K_d = \frac{0.09W}{d} = \frac{0.09(1651.32)}{8.00} = 18.57\ k/in$$

Table B1.12-1 Simplified Method Solution for Design Example 2.2 - Final Iteration, First Solution

$Q_d = 0.05W$

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	1651.32	256.26	1907.58	0.6	3							
Step B1.1	d	14.20	Assumed displacement									
	Q_d	82.57	Characteristic strength									
	K_d	5.81	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	168.48	8.424	0.593	10,000.00	0.000119	1.186	14.198	1.187	0.002	16.847	119.607	239.227
Pier 1	657.18	32.859	2.314	288.87	0.016151	4.591	13.974	4.665	0.226	65.196	459.182	925.780
Pier 2	657.18	32.859	2.314	288.87	0.016151	4.591	13.974	4.665	0.226	65.196	459.182	925.780
Abut 2	168.48	8.424	0.593	10,000.00	0.000119	1.186	14.198	1.187	0.002	16.847	119.607	239.227
Total	1651.32	82.566	5.815		$\Sigma K_{eff,j}$	11.555				164.085	1,157.577	2,330.014
					Step	**B1.4**						
Step B1.10	T_{eff}	4.11	Effective period									
	ξ	0.32	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.74										
	B_L	1.70	Damping Factor									
	d	14.19	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1												
Pier 1												
Pier 2												
Abut 2												

Table B1.12-2 Simplified Method Solution for Design Example 2.2 - Final Iteration, Second Solution

$$Q_d = 0.09W$$

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PS}	W_{eff}	S_{D1}	n							
	1651.32	256.26	1907.58	0.6	3							
Step B1.1	d	8.00	Assumed displacement									
	Q_d	148.62	Characteristic strength									
	K_d	18.58	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j}+d_{sub,j})^2$
Abut 1	168.48	15.163	1.895	10,000.00	0.000379	3.790	7.997	3.792	0.003	30.321	121.260	242.565
Pier 1	657.18	59.146	7.393	288.87	0.052532	14.418	7.601	15.175	0.399	115.340	449.554	922.723
Pier 2	657.18	59.146	7.393	288.87	0.052532	14.418	7.601	15.175	0.399	115.340	449.554	922.723
Abut 2	168.48	15.163	1.895	10,000.00	0.000379	3.790	7.997	3.792	0.003	30.321	121.260	242.565
Total	1651.32	148.619	18.577		$\Sigma K_{eff,j}$	36.415				291.322	1,141.626	2,330.577
					Step	**B1.4**						
Step B1.10	T_{eff}	2.31	Effective period									
	ξ	0.31	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.73										
	B_L	1.70	Damping Factor									
	d	7.99	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	5.054	0.632	1.264		7.70	1.288						
Pier 1	19.715	2.464	5.058		7.04	5.265						
Pier 2	19.715	2.464	5.058		7.04	5.265						
Abut 2	5.054	0.632	1.264		7.70	1.288						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength
Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \quad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \quad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 2.2
Dividing the results for Q_d and K_d in Step B1.12 by the number of isolators at each support (n = 3), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,1}$ = 15.16/3 = 5.05 k
- $Q_{d,2}$ = 59.15/3= 19.72 k
- $Q_{d,3}$ = 59.15/3 = 19.72 k
- $Q_{d,4}$ = 15.16/3 = 5.05 k

and

- $K_{d,1}$ =1.89/3 = 0.63 k/in
- $K_{d,2}$ = 7.39/3 = 2.43 k/in
- $K_{d,3}$ = 7.39/3 = 2.43 k/in
- $K_{d,4}$ = 1.89/3 = 0.63 k/in

Note that the K_d values per support used above are from the final iteration given in Table B1.12-1. These are not the same as the initial values in Step B1.2, because they have been adjusted from cycle to cycle, such that the total K_d summed over all the isolators satisfies the minimum lateral restoring force requirement for the bridge, i.e. K_{dtotal} = 0.05 W/d. See Step B1.1. Since d varies from cycle to cycle, $K_{d,j}$ varies from cycle to cycle.

B2.2 Initial Stiffness and Yield Displacement
Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

In the absence of isolator-specific information take

$$K_{u,i} = 10K_{d,i} \quad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \quad \text{(B-22)}$$

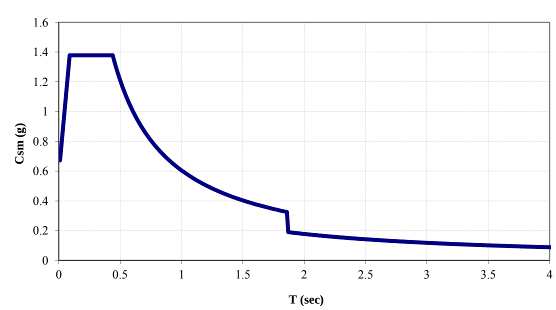
B2.2 Initial Stiffness and Yield Displacement, Example 2.2
Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(2.43) = 24.3\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{19.72}{(24.3 - 2.43)} = 0.90\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~8

		in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i} = 0$.
B2.3 Isolator Effective Stiffness, $k_{isol,i}$ Calculate the isolator stiffness, $k_{isol,i}$, of each isolator 'i': $K_{isol,i} = \frac{K_{isol,j}}{n}$ (B -23)		**B2.3 Isolator Effective Stiffness, $k_{isol,i}$, Example 2.2** Dividing the results for K_{isol} (Step B1.12) among the 3 isolators at each support, the following values for K_{isol} /isolator are obtained: o $K_{isol,1}$ = 3.79/3 = 1.26 k/in o $K_{isol,2}$ = 15.18/3 = 5.06 k/in o $K_{isol,3}$ = 15.18/3 = 5.06 k/in o $K_{isol,4}$ = 3.79/3 = 1.26 k/in
B2.4 Finite Element Model Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is recommended.		**B2.4 Finite Element Model, Example 2.2** 
B2.5 Composite Design Response Spectrum Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.		**B.2.5 Composite Design Response Spectrum, Ex 2.2** From the final results of Simplified Method (Step B1.12), B_L = 1.70 and T_{eff} = 2.31 sec. Hence the transition in the composite spectrum from 5% to 30% damping occurs at 0.8 T_{eff} = 0.8 (2.31) = 1.85 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods ≥ 1.85 sec by 1.70. 

B2.6 Multimodal Analysis of Finite Element Model

Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

B2.6 Multimodal Analysis of Finite Element Model, Example 2.2

Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first three modes are the principal transverse, longitudinal, and torsion modes with periods of 2.304, 2.224 and 2.192 sec respectively. The period of the longitudinal mode (2.30 sec) is close to the period calculated in the Simplified Method (2.31 sec). The mass participation factors indicate there is no coupling between these three modes (probably due to the symmetric nature of the bridge) and the high values for the first and second modes (90% for each mode) indicate the bridge is behaving essentially in a single mode of vibration in each direction. Similar results to those obtained by the Simplified Method are therefore to be expected.

Table B2.6-1 Modal Properties of Bridge Example 2.2 – First Iteration

Mode	Period	Mass Participating Ratios					
No	Sec	UX	UY	UZ	RX	RY	RZ
1	2.304	0.000	0.904	0.000	0.898	0.000	0.686
2	2.224	0.903	0.000	0.000	0.000	0.020	0.000
3	2.192	0.000	0.000	0.000	0.000	0.000	0.228
4	0.499	0.000	0.004	0.000	0.052	0.000	0.003
5	0.372	0.000	0.000	0.074	0.000	0.055	0.000
6	0.364	0.000	0.000	0.000	0.000	0.000	0.003
7	0.347	0.000	0.004	0.000	0.019	0.000	0.003
8	0.285	0.000	0.007	0.000	0.018	0.000	0.005
9	0.255	0.000	0.000	0.000	0.000	0.000	0.000
10	0.255	0.096	0.000	0.000	0.000	0.001	0.000
11	0.207	0.000	0.000	0.000	0.000	0.128	0.000
12	0.188	0.000	0.000	0.000	0.000	0.000	0.001

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 7.80 (8.00) in
- $d_{isol,2}$ = 7.02 (7.60) in
- $d_{isol,3}$ = 7.02 (7.60) in
- $d_{isol,4}$ = 7.80 (8.00) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 2.2

The results for isolator displacements could be considered close enough (better than 4% at abutments and 9% at piers). But for illustrative purposes a second cycle of iteration is performed.

Go to Step B2.8 and update properties.

B2.8 Update $K_{isol,i}$ $K_{eff,j}$, ξ and B_L

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

$$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \qquad \text{(B-24)}$$

Recalculate $K_{eff,j}$:

Eq. 7.1-6 GSID
$$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \qquad \text{(B-25)}$$

Recalculate system damping ratio, ξ :

Eq. 7.1-10 GSID
$$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \qquad \text{(B-26)}$$

Recalculate system damping factor, B_L:

Eq. 7.1-3 GSID
$$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \qquad \text{(B-27)}$$

Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.

B2.8 Update $K_{isol,i}$ $K_{eff,j}$, ξ and B_L, Example 2.2

Updated values for $K_{isol,i}$ are given below (previous values are in parentheses):

- $K_{isol,1}$ = 1.27 (1.26) k/in
- $K_{isol,2}$ = 5.24 (5.06) k/in
- $K_{isol,3}$ = 5.24 (5.06) k/in
- $K_{isol,4}$ = 1.27 (1.26) k/in

Since the isolator displacements are relatively close to previous results no significant change in the damping ratio is expected. Hence $K_{eff,j}$ and ξ are not recalculated and B_L is taken at 1.70.

Since the change in effective period is very small (2.224 to 2.183 sec) and no change has been made to B_L, there is no need to construct a new composite response spectrum in this case. Go back to Step B2.6 (see immediately below).

B2.6 Multimodal Analysis Second Iteration, Example 2.2

Reanalysis gives the following values for the isolator displacements (numbers in parentheses are those from the previous cycle):

- $d_{isol,1}$ = 7.65 (7.80) in
- $d_{isol,2}$ = 6.86 (7.02) in
- $d_{isol,3}$ = 6.86 (7.02) in
- $d_{isol,4}$ = 7.65 (7.80) in

Go to Step B2.7

B2.7 Convergence Check

Compare results and determine if convergence has been reached. If so go to Step B2.9. Otherwise Go to Step B2.8.

B2.7 Convergence Check, Example 2.2

Satisfactory agreement has been reached on this second cycle (better than 2% at the abutments and 3% at the piers). Go to Step B2.9

B2.9 Superstructure and Isolator Displacements

Once convergence has been reached, obtain

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and
- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e. longitudinal loading). These displacements

B2.9 Superstructure and Isolator Displacements, Example 2.2

From the above analysis:

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are:

  x_L= 7.69 in
  y_L= 0.0 in

may be found by subtracting the nodal displacements at each end of each isolator spring element.

- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are:
  - Abutments: u_L = 7.65 in, v_L = 0.00 in
  - Piers: u_L = 6.86 in, v_L = 0.00 in

  All isolators at same support have the same displacements.

B2.10 Pier Bending Moments and Shear Forces

Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.

B2.10 Pier Bending Moments and Shear Forces, Example 2.2

Bending moments in single column pier in the longitudinal (M_{PLL}) and transverse (M_{PTL}) directions are:

M_{PLL}= 0
M_{PTL}= 3,897 kft

Shear forces in single column pier the longitudinal (V_{PLL}) and transverse (V_{PTL}) directions are

V_{PLL}= 163.79 k
V_{PTL}= 0

The above column shear (163.8 k) is larger than the calculated shear in the Simplified Method (115.3 k) because inertia load from the pier cap is not included in the Simplified Method. The pier cap weighs 92 k.

B2.11 Isolator Shear and Axial Forces

Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 2.2

Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

		V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abutment	Isol. 1	9.72	0	1.93
	Isol. 2	9.72	0	1.91
	Isol. 3	9.72	0	1.93
Pier	Isol. 1	35.93	0	0.68
	Isol. 2	35.97	0	0.98
	Isol. 3	35.93	0	0.68

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 2.2

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) for transverse loading , are as follows:

- T_{eff} = 2.23 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions due to transverse load are as follows:
  x_T = 0 in
  y_T = 7.88 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions due to transverse loading are as follows:
  Abutments u_T = 0.00 in, v_T = 7.93 in
  Piers u_T = 0.00 in, v_T = 6.25 in
- Pier bending moments in the longitudinal (M_{PLT}) and transverse (M_{PTT}) directions due to transverse load are as follows:
  M_{PLT} = 3,345 kft
  M_{PTT} = 0 kft
- Pier shear forces in the longitudinal (V_{PLT}) and transverse (V_{PTT}) directions due to transverse load are as follows:
  V_{PLT} = 0 k
  V_{PTT} = 127.1 k
- Isolator shear and axial forces are summarized in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

		V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abut ment	Isol. 1	0.0	9.91	26.67
	Isol. 2	0.0	9.92	0
	Isol. 3	0.0	9.91	26.67
Pier	Isol. 1	0.0	34.41	50.0
	Isol. 2	0.0	34.51	0
	Isol. 3	0.0	34.41	50.0

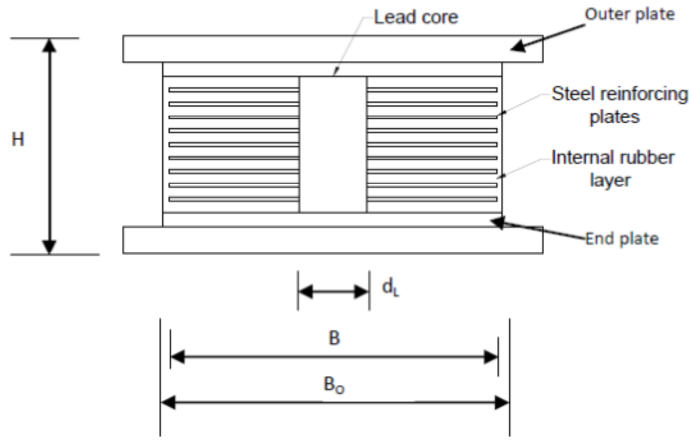
STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows: • $u_1 = u_L + 0.3u_T$ (D-1) $v_1 = v_L + 0.3v_T$ (D-2) $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3) • $u_2 = 0.3u_L + u_T$ (D-4) $v_2 = 0.3v_L + v_T$ (D-5) $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6) • $d_t = \max(R_1, R_2)$ (D-7)		**D1. Design Isolator Displacements at Pier, Example 2.2** Load Case 1: $u_1 = u_L + 0.3u_T = 1.0(6.86) + 0.3(0) = 6.86$ in $v_1 = v_L + 0.3v_T = 1.0(0) + 0.3(6.25) = 1.88$ in $R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{6.86^2 + 1.88^2} = 7.11$ in Load Case 2: $u_2 = 0.3u_L + u_T = 0.3(6.86) + 1.0(0) = 2.06$ in $v_2 = 0.3v_L + v_T = 0.3(0) + 1.0(6.25) = 6.25$ in $R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{2.06^2 + 6.25^2} = 6.58$ in Governing Case: Total design displacement, $d_t = \max(R_1, R_2)$ $= 7.11$ in
D2. Design Moments and Shears Calculate design values for column bending moments and shear forces using the same combination rules as for displacements. Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.		**D2. Design Moments and Shears at Pier, Example 2.2** Load Case 1: $V_{PL1} = V_{PLL} + 0.3V_{PLT} = 1.0(163.79) + 0.3(0) = 163.79$ k $V_{PT1} = V_{PTL} + 0.3V_{PTT} = 1.0(0) + 0.3(127.10) = 38.13$ k $R_1 = \sqrt{V_{L1}^2 + V_{T1}^2} = \sqrt{163.79^2 + 38.13^2} = 168.17$ k Load Case 2: $V_{PL2} = 0.3V_{PLL} + V_{PLT} = 0.3(163.79) + 1.0(0) = 49.14$ k $V_{PT2} = 0.3V_{PTL} + V_{PTT} = 0.3(0) + 1.0(127.10) = 127.10$ k $R_2 = \sqrt{V_{L2}^2 + V_{T2}^2} = \sqrt{49.14^2 + 127.10^2} = 136.27$ k Governing Case: Design column shear $= \max(R1, R2)$ $= 168.17$ K Note that this column shear force (168.17 k) is larger than the plastic shear (128 k) and fully elastic behavior might not be achievable with this seismic demand (S_1=0.6g) and column size.

STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS



A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness (K_u).With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness $\underline{K_d}$ is given by the rubber alone. More details are given in Buckle et.al, 2006.

While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary. When sizing the physical dimensions of the bearing, plan dimensions (B, d_L) should be rounded up to the next $^1/_4$" increment, while the total thickness of elastomer, T_r, is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are $^1/_4$" and $^3/_8$". High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in Buckle et.al., 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See Buckle et al, 2006.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1). • required characteristic strength, Q_d / isolator • required post-elastic stiffness, K_d / isolator • total design displacement, d_t, for each isolator • maximum applied dead and live load (P_{DL}, P_{LL}) and seismic load (P_{SL}) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator, and • maximum wind load, P_{WL}	**E1. Required Properties, Example 2.2** The design of one of the exterior isolators on a pier is given below to illustrate the design process for a lead-rubber isolator. From previous work • Q_d / isolator = 19.72 k • K_d / isolator = 2.43 k/in • Total design displacement, dt = 7.11 in • P_{DL} = 187 k • P_{LL} = 123 k and P_{SL} = 50 k (Table C1-1) • P_{WL} = 8.21 k < Q_d OK
E2. Isolator Sizing	
E2.1 Lead Core Diameter Determine the required diameter of the lead plug, d_L, using: $$d_L = \sqrt{\frac{Q_d}{0.9}} \qquad \text{(E-1)}$$ See Step E2.5 for limitations on d_L.	**E2.1 Lead Core Diameter, Example 2.2** $$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{19.72}{0.9}} = 4.68\ in$$

E2.2 Plan Area and Isolator Diameter

Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.6 ksi.

Then the bonded area of the isolator is given by:

$$A_b = \frac{P_{DL} + P_{LL}}{1.6}\ in^2 \qquad \text{(E-2)}$$

and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by:

$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} \qquad \text{(E-3)}$$

Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using

$$A_b = \frac{\pi}{4}(B^2 - d_L^2) \qquad \text{(E-4)}$$

Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, B_o is given by:

$$B_o = B + 1.0 \qquad \text{(E-5)}$$

E2.2 Plan Area and Isolator Diameter, Example 2.2

Looking ahead (and based on experience from previous examples) the isolator must be stable at $\Delta = 1.5\ d_t = 1.5(7.11) = 10.67$ in (Step E4.2).

This is a much larger displacement than in the benchmark example (where it was only 2.34 in). It is therefore likely that in this example, this value for Δ will dictate the size of the isolator and a rule of thumb is to choose a diameter, B, between 1.5 and 2 times Δ, in order to provide sufficient vertical load capacity when the isolator is deformed to 1.5 d_t.

For this reason choose $B = 1.75\ \Delta = 18.6 \sim 19.0$ in.

Then

$$A_b = \frac{\pi}{4}(B^2 - d_L^2)$$
$$= \frac{\pi}{4}(19.00^2 - 4.68^2) = 266.32 in^2$$

$B_o = 19.0 + 2(0.5) = 20.0$ in

E2.3 Elastomer Thickness and Number of Layers

Since the shear stiffness of an elastomeric bearing is given by:

$$K_d = \frac{GA_b}{T_r} \qquad \text{(E-6)}$$

where G = shear modulus of the rubber, and
T_r = the total thickness of elastomer,
it follows Eq. E-5 may be used to obtain T_r given a required value for K_d

$$T_r = \frac{GA_b}{K_d} \qquad \text{(E-7)}$$

A typical range for shear modulus, G, is 60-120 psi. Higher and lower value are available and used in special applications.

If the layer thickness is t_r, the number of layers, n, is given by:

$$n = \frac{T_r}{t_r} \qquad \text{(E-8)}$$

rounded to the nearest integer.

Note that because of rounding the plan dimensions and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be

E2.3 Elastomer Thickness and Number of Layers, Example 2.2

$$T_r = \frac{GA_b}{K_d} = \frac{0.1(266.32)}{2.43} = 10.96\ in$$

$$n = \frac{10.96}{0.25} = 43.8$$

Round up to nearest integer, i.e. n = 44

necessary if the differences are large.		
E2.4 Overall Height The overall height of the isolator, H, is given by: $$H = n\,t_r + (n-1)t_s + 2t_c \qquad \text{(E-9)}$$ where t_s = thickness of an internal shim (usually about 1/8 in), and t_c = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in)		**E2.4 Overall Height, Example 2.2** $$H = 44(0.25) + 43(0.125) + 2(1.5) = 19.375\ in$$
E2.5 Lead Core Size Check Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows: $$\frac{B}{3} \geq d_L \geq \frac{B}{6} \qquad \text{(E-10)}$$		**E2.5 Lead Core Size Check, Example 2.2** Since B = 22.00 check $$\frac{20.0}{3} \geq d_L \geq \frac{20.0}{6}$$ i.e., $6.67 \geq d_L \geq 3.33$ Since d_L = 4.68, lead core size is acceptable.
E3. Strain Limit Check Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e., $$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5 \qquad \text{(E-11)}$$ where $\gamma_c,\ \gamma_{s,eq}, and\ \gamma_r$ are defined below. (a) γ_c is the maximum shear strain in the layer due to compression and is given by: $$\gamma_c = \frac{D_c\sigma_S}{GS} \qquad \text{(E-12)}$$ where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = {P_{DL}}/{A_b}$, G is shear modulus, and S is the layer shape factor given by: $$S = \frac{A_b}{\pi B t_r} \qquad \text{(E-13)}$$ (b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by: $$\gamma_{s,eq} = \frac{d_t}{T_r} \qquad \text{(E-14)}$$ (c) γ_r is the shear strain due to rotation and is given by: $$\gamma_r = \frac{D_r B^2\theta}{t_r T_r} \qquad \text{(E-15)}$$ where D_r is shape coefficient for rotation in circular		**E3. Strain Limit Check, Example 2.2** Since $$\sigma_S = \frac{187.0}{266.32} = 0.702\ ksi$$ and $$S = \frac{266.32}{\pi 19.0(0.25)} = 17.85$$ then $$\gamma_c = \frac{1.0(0.702)}{0.1(17.85)} = 0.393$$ $$\gamma_{s,eq} = \frac{7.11}{11.0} = 0.646$$ $$\gamma_r = \frac{0.375(19.00^2)(0.01)}{0.25(11.0)} = 0.492$$

bearings = 0.375, and θ is design rotation due to DL, LL and construction effects. Actual value for θ may not be known at this time and a value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1).

Substitution in Eq E-11 gives

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.39 + 0.65 + 0.5(0.49) = 1.28 \leq 5.5\ OK$$

E4. Vertical Load Stability Check

Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state.

Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either

2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or

1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4.

E4. Vertical Load Stability Check, Example 2.2

E4.1 Vertical Load Stability in Undeformed State

The critical load capacity of an elastomeric isolator at zero shear displacement is given by

$$P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2}\left[\sqrt{(1+\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2})} - 1\right] \qquad \text{(E-16)}$$

where

$H_{eff} = T_r + T_s$

T_s = total shim thickness

$K_\theta = {E_b I}/{T_r}$

$E_b = E(1+0.67S^2)$

E = elastic modulus of elastomer = $3G$

$I = {\pi B^4}/{64}$

It is noted that typical elastomeric isolators have high shape factors, S, in which case:

$$\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1 \qquad \text{(E-17)}$$

and Eq. E-16 reduces to:

$$P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta} \qquad \text{(E-18)}$$

Check that:

$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} \geq 3 \qquad \text{(E-19)}$$

E4.1 Vertical Load Stability in Undeformed State, Example 2.2

$$E = 3G = 3(0.1) = 0.3ksi$$

$$E_b = 0.3(1+0.67(17.85^2)) = 64.03ksi$$

$$I = \pi\frac{19.0^4}{64} = 6397.1in^4$$

$$K_\theta = \frac{64.03(6397)}{11.0} = 37{,}239kin/rad$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.1(266.32)}{11.0} = 2.42k/in$$

$$P_{cr(\Delta=0)} = \pi\sqrt{2.42(37{,}239)} = 943.1k$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \frac{943}{(187+123)} = 3.04 \geq 3\quad OK$$

E4.2 Vertical Load Stability in Deformed State

The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by:

E4.2 Vertical Load Stability in Deformed State, Example 2.2

Since bridge is in Zone 4,

$$P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)} \qquad \text{(E-20)}$$

where

A_r = overlap area between top and bottom plates of isolator at displacement Δ (Fig. 2.2-1 GSID)

= $B^2(\delta - sin\delta)/_4$

δ = $2cos^{-1}(\Delta/_B)$

A_{gross} = $\pi B^2/_4$

It follows that:

$$\frac{A_r}{A_{gross}} = \frac{(\delta - sin\delta)}{\pi} \qquad \text{(E-21)}$$

Check that:

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} \geq 1 \qquad \text{(E-22)}$$

$$\Delta = 1.5d_t = 1.5(7.11) = 10.65\ in$$

$$\delta = 2cos^{-1}\left(\frac{10.65}{19.0}\right) = 1.95$$

$$\frac{A_r}{A_{gross}} = \frac{(1.95 - sin1.95)}{\pi} = 0.325$$

$$P_{cr(\Delta)} = 0.325(943.1) = 306.5k$$

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{306.5}{1.2(187) + 50.0} = 1.12 \geq 1\ \ OK$$

E5. Design Review

E5. Design Review, Example 2.2

The basic dimensions of the isolator designed above are as follows:

20.0 in (od) x 19.375 in (high) x 4.68 in dia. lead core

and the volume, excluding the steel end and cover plates = 5,144 in^3

Although this design satisfies all the required criteria, the total rubber shear strain (1.28) is much less than the maximum allowable (5.5), as shown in Step E3. In other words, the isolator is not working very hard and a redesign appears to be worth exploring to see if a more optimal design can be found. Since the plan dimension was set at the beginning (Step E2.2) to satisfy vertical load stability requirements (successfully as it turned out) the only way to optimize the design is to reduce its height. But reducing the height will increase the stiffness, K_d, unless the shear modulus of the elastomer is likewise reduced. In the redesign below, the plan dimensions remain the same but the shear modulus is reduced from 100 to 60 psi.

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{19.72}{0.9}} = 4.68\ in$$

Choose B = 19.00 in as before, then

$$A_b = \frac{\pi}{4}(19.0^2 - 4.68^2) = 266.32\ in^2$$

$B_o = 19.0 + 2(0.5) = 20.0$ in

$$T_r = \frac{GA_b}{K_d} = \frac{0.06(266.32)}{2.43} = 6.58\ in$$

$$n = \frac{6.58}{0.25} = 26.3$$

Round to nearest integer: $n = 26$

$$H = 26(0.25) + 25(0.125) + 2(1.5) = 12.625\ in$$

Since $B = 19.0$ check

$$\frac{19}{3} \geq d_L \geq \frac{19}{6}$$

$$6.33 \geq d_L \geq 3.17$$

Since $d_L = 4.68$ in, size of lead core is acceptable.

$$\sigma_S = \frac{187.0}{266.32} = 0.702\ ksi$$

$$S = \frac{266.32}{\pi 19.0(0.25)} = 17.85$$

$$\gamma_c = \frac{1.0(0.702)}{0.06(17.85)} = 0.66$$

$$\gamma_{s,eq} = \frac{7.11}{6.5} = 1.09$$

$$\gamma_r = \frac{0.375(19.0^2)(0.01)}{0.25(6.5)} = 0.833$$

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.66 + 1.09 + 0.5(0.83) = 2.17 \leq 5.5\ OK$$

$$E = 3G = 3(0.06) = 0.18\ ksi$$

$$E_b = 0.18(1 + 0.67(17.85^2)) = 38.42 ksi$$

$$I = \pi\frac{19.0^4}{64} = 6{,}397.1 in^4$$

		$$K_\theta = \frac{38.42(6{,}397.1)}{6.5} = 37{,}812 kin/rad$$ $$K_d = \frac{GA_b}{T_r} = \frac{0.06(266.32)}{6.5} = 2.46k/in$$ $$P_{cr(\Delta=0)} = \pi\sqrt{2.46(37{,}812)} = 957.8k$$ $$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \frac{957.8}{(187+123)} = 3.09 \geq 3 \quad OK$$ $$\delta = 2cos^{-1}\left(\frac{10.67}{19.0}\right) = 1.95$$ $$\frac{A_r}{A_{gross}} = \frac{(1.95 - sin1.95)}{\pi} = 0.325$$ $$P_{cr(\Delta)} = 0.325(957.8) = 311.1k$$ $$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}} = \frac{311.1}{1.2(187)+50.0} = 1.13 \geq 1 \quad OK$$ The basic dimensions of the redesigned isolator are as follows: 20.0 in (od) x 12.625in (high) x 4.68 in dia. lead core and the volume, excluding steel end and cover plates, = 3,024 in^3 This is a more optimal design. It is smaller than the previous design (3,024 in^3 vs 5,144 in^3) while still satisfying all the design criteria. Being smaller it works harder to satisfy these requirements, as can be seen by an increase in the total shear strain from 1.28 to 2.17 (but still less than the maximum allowable of 5.5).
E6. Minimum and Maximum Performance Check Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E6-1. **Table E6-1. Minimum and maximum values for K_d and Q_d.**		**E6. Minimum and Maximum Performance Check, Example 2.2** Minimum Property Modification factors are $\lambda_{min,Kd} = 1.0$ $\lambda_{min,Qd} = 1.0$ which means there is no need to reanalyze the bridge with a set of minimum values. Maximum Property Modification factors are $\lambda_{max,a,Kd} = 1.1$

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-23)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-24)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-25)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-26)

Determination of the system property modification factors shall include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E6-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})\ (\lambda_{min,a,Kd})\ (\lambda_{min,v,Kd})\ (\lambda_{min,tr,Kd})\ (\lambda_{min,c,Kd})\ (\lambda_{min,scrag,Kd})$	(E-27)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})\ (\lambda_{max,a,Kd})\ (\lambda_{max,v,Kd})\ (\lambda_{max,tr,Kd})\ (\lambda_{max,c,Kd})\ (\lambda_{max,scrag,Kd})$	(E-28)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})\ (\lambda_{min,a,Qd})\ (\lambda_{min,v,Qd})\ (\lambda_{min,tr,Qd})\ (\lambda_{min,c,Qd})\ (\lambda_{min,scrag,Qd})$	(E-29)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})\ (\lambda_{max,a,Qd})\ (\lambda_{max,v,Qd})\ (\lambda_{max,tr,Qd})\ (\lambda_{max,c,Qd})\ (\lambda_{max,scrag,Qd})$	(E-30)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

1.00 for critical bridges
0.75 for essential bridges
0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second

$\lambda_{max,a,Qd} = 1.1$

$\lambda_{max,t,Kd} = 1.1$
$\lambda_{max,t,Qd} = 1.4$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,a,Qd} = 1.0 + 0.1(0.66) = 1.066$

$\lambda_{max,t,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,t,Qd} = 1.0 + 0.4(0.66) = 1.264$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Therefore the maximum overall modification factors

$\lambda_{max,Kd} = 1.066(1.066)1.0 = 1.14$
$\lambda_{max,Qd} = 1.066(1.264)1.0 = 1.35$

case ($K_{d,max}$ and $Q_{d,max}$).

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties of pier isolators are:
$Q_{d,max}$ = 1.35 (19.72) = 26.62 k
and
$K_{d,max}$=1.14(2.43) = 2.77 k/in

E7. Design and Performance Summary

E7. Design and Performance Summary

E7.1 Isolator dimensions
Summarize final dimensions of isolators:

- Overall diameter (includes cover layer)
- Overall height
- Diameter lead core
- Bonded diameter
- Number of rubber layers
- Thickness of rubber layers
- Total rubber thickness
- Thickness of steel shims
- Shear modulus of elastomer

Check all dimensions with manufacturer.

E7.1 Isolator dimensions, Example 2.2
Isolator dimensions are summarized in Table E7.1-1.

Table E7.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Diam. lead core (in)
Under edge girder on Pier	24.0 x 24.0 x 12.625(H)	20.0 dia. x 11.125(H)	4.68

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in)	Total rubber thick-ness (in)	Steel shim thick-ness (in)
Under edge girder on Pier	26	0.25	6.50	0.125

Shear modulus of elastomer = 60 psi

E7.2 Bridge Performance
Summarize bridge performance

- Maximum superstructure displacement (longitudinal)
- Maximum superstructure displacement (transverse)
- Maximum superstructure displacement (resultant)
- Maximum column shear (resultant)
- Maximum column moment (about transverse axis)
- Maximum column moment (about longitudinal axis)
- Maximum column torque

Check required performance as determined in Step A3, is satisfied.

E7.2 Bridge Performance, Example 2.2
Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 175 k. This is more than the column plastic shear (128 k) and therefore the required performance criterion is not satisfied (fully elastic behavior). Clearly the seismic demand (S_1 = 0.6g) is too high and the column too small for isolation to give fully elastic response. The next step might be to a conduct pushover analysis of the column to determine if the ductility demand at 8.21 in is acceptable. If not, and this is an existing bridge, jacket the column (as well as isolate the bridge). If a new design, increase the size of the column and thereby increase its strength. It is noted that the maximum longitudinal displacement (7.69 in) exceeds the available clearance, and as noted in Section B1.12, this gap needs to be increased to say

9.0 in to meet the requirements for isolation. An alternative is to accept pounding at the abutments. The consequential damage is not likely to be life-threatening and easily repaired.

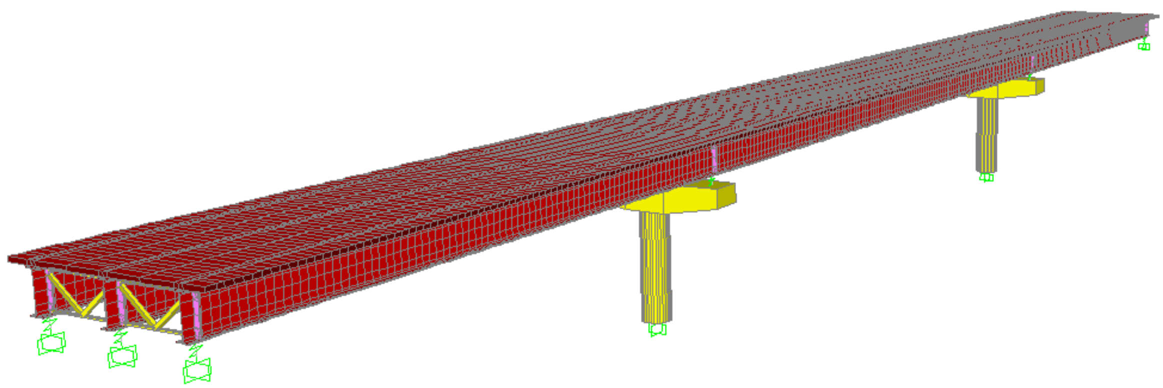
Table E7.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	7.69 in
Maximum superstructure displacement (transverse)	7.88 in
Maximum superstructure displacement (resultant)	8.21 in
Maximum column shear (resultant)	175 k
Maximum column moment about transverse axis	4049 k-ft
Maximum column moment about longitudinal axis	3814 k-ft
Maximum column torque	14 k-ft

SECTION 2

Steel Plate Girder Bridge, Long Spans, Single-Column Pier

DESIGN EXAMPLE 2.3: Spherical Friction Isolators



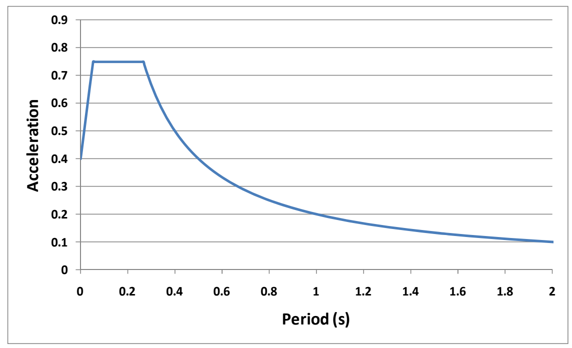
Design Examples in Section 2

ID	Description	S_1	Site Class	Column height	Skew	Isolator type
2.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
2.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
2.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
2.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
2.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
2.5	Change column height	0.2g	B	H_1=0.5H_2	0	Lead rubber bearing
2.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

DESIGN PROCEDURE	**DESIGN EXAMPLE 2.3 (Spherical Friction Isolators)**
STEP A: BRIDGE AND SITE DATA	

A1. Bridge Properties
Determine properties of the bridge:
• number of supports, *m*
• number of girders per support, *n*
• angle of skew
• weight of superstructure including railings, curbs, barriers and other permanent loads, W_{SS}
• weight of piers participating with superstructure in dynamic response, W_{PP}
• weight of superstructure, W_j, at each support
• stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
• column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
• allowable movement at expansion joints
• isolator type if known, otherwise 'to be selected'
	A1. Bridge Properties, Example 2.3
	• Number of supports, $m = 4$
	  o North Abutment ($m = 1$)
	  o Pier 1 ($m = 2$)
	  o Pier 2 ($m = 3$)
	  o South Abutment ($m = 4$)
	• Number of girders per support, $n = 3$
	• Angle of skew = 0^0
	• Number of columns per support = 1
	• Weight of superstructure including permanent loads, W_{SS} = 1651.32 k
	• Weight of superstructure at each support:
	  o W_1 = 168.48 k
	  o W_2 = 657.18 k
	  o W_3 = 657.18 k
	  o W_4 = 168.48 k
	• Participating weight of piers, W_{PP} = 256.26 k
	• Effective weight (for calculation of period), $W_{eff} = Wss + W_{PP}$ = 1907.58 k
	• Stiffness of each pier in the both directions (assume fixed at footing and single curvature behavior) :
	  o $K_{sub,pier1}$ = 288.87 k/in
	  o $K_{sub,pier2}$ = 288.87 k/in
	• Minimum column shear strength based on flexural yield capacity of column = 128 k
	• Displacement capacity of expansion joints (longitudinal) = 2.5 in for thermal and other movements
	• Spherical Friction Isolators

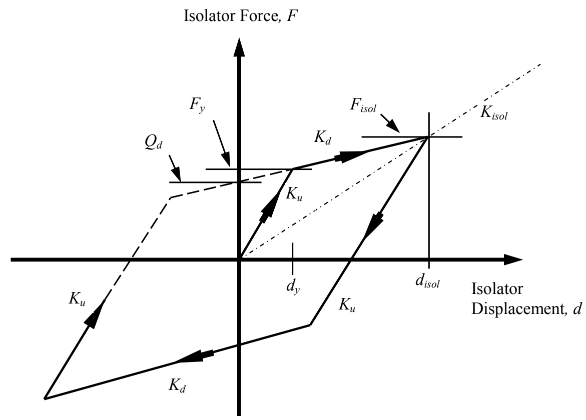
A2. Seismic Hazard Determine seismic hazard at site: • acceleration coefficients • site class and site factors • seismic zone Plot response spectrum. Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of *PGA*, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD). Use Art. 3.2 to obtain Site Class and corresponding Site	**A2. Seismic Hazard, Example 2.3** Acceleration coefficients for bridge site are given in design statement as follows: • PGA = 0.40 • $S_1 = 0.20$ • $S_S = 0.75$ Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec. Table 3.10.3.1-1 LRFD gives Site Class as B. Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors: • $F_{pga} = 1.0$ • $F_a = 1.0$

Factors (F_{pga}, F_a and F_v). These data are the same as for conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD. Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows: • $A_s = F_{pga}\ PGA$ • $S_{DS} = F_a\ S_S$ • $S_{D1} = F_v\ S_1$ Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table5-1 GSID. These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.		• $F_v = 1.0$ • $A_s = F_{pga}\ PGA = 1.0(0.40) = 0.40$ • $S_{DS} = F_a\ S_S = 1.0(0.75) = 0.75$ • $S_{D1} = F_v\ S_1 = 1.0(0.20) = 0.20$ Since $0.15 < S_{D1} \leq 0.30$, bridge is located in Seismic Zone 2. Design Response Spectrum is as below: 
A3. Performance Requirements Determine required performance of isolated bridge during Design Earthquake (1000-yr return period). Examples of performance that might be specified by the Owner include: • Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake. • Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake. • For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements) • Reduced substructure forces for bridges on weak soils to reduce foundation costs.		**A3. Performance Requirements, Example 2.3** In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield). To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (128 k). The maximum shear in the column must therefore be less than 128 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

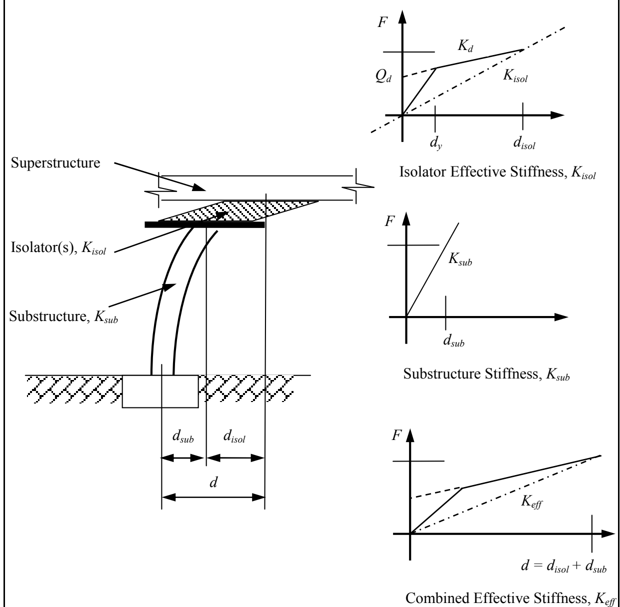
Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties To begin the iterative solution, an estimate is required of : (1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.) Art C7.1 GSID $d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1}$ (B-1) (2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not	**B1.1 Initial System Displacement and Properties, Example 2.3** $d \cong 10 S_{D1} = 10(0.20) \cong 2.0\ in$

occur under non-seismic loads (e.g. wind) but low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e. $$Q_d = 0.05W \qquad \text{(B-2)}$$ (3) Post-yield stiffness, K_d Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3. Art. 12.2 GSID $$K_{d,min} \geq \frac{0.025W}{d} \qquad \text{(B-3)}$$ Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$		$$Q_d = 0.05W = 0.05(1651.32) = 82.56\ k$$ $$K_d = 0.05\frac{W}{d} = 0.05\frac{1651.32}{2.0} = 41.28\ k/in$$
B1.2 Initial Isolator Properties at Supports Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support: $$Q_{d,j} = Q_d\frac{W_j}{W} \qquad \text{(B-4)}$$ and $$K_{d,j} = K_d\frac{W_j}{W} \qquad \text{(B-5)}$$		**B1.2 Initial Isolator Properties at Supports, Example 2.3** $$Q_{d,j} = Q_d\frac{W_j}{W}$$ o $Q_{d,1}$ = 8.42 k o $Q_{d,2}$ = 32.86 k o $Q_{d,3}$ = 32.86 k o $Q_{d,4}$ = 8.42 k and $$K_{d,j} = K_d\frac{W_j}{W}$$ o $K_{d,1}$ = 4.21 k/in o $K_{d,2}$ = 16.43 k/in o $K_{d,3}$ = 16.43 k/in o $K_{d,4}$ = 4.21 k/in
B1.3 Effective Stiffness of Combined Pier and Isolator System Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID). An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula is as follows (MCEER 2006): $$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j} \qquad \text{(B-6)}$$ where $$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}} \qquad \text{(B-7)}$$ and $K_{sub,j}$ for the piers are given in Step A1. For the		**B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 2.3** $$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$ o $\alpha_1 = 8.43\text{x}10^{-4}$ o $\alpha_2 = 1.21\text{x}10^{-1}$ o $\alpha_3 = 1.21\text{x}10^{-1}$ o $\alpha_4 = 8.43\text{x}10^{-4}$ $$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$$ o $K_{eff,1}$ = 8.42 k/in o $K_{eff,2}$ = 31.09 k/in o $K_{eff,3}$ = 31.09 k/in o $K_{eff,4}$ = 8.42 k/in

abutments, take $K_{sub,j}$ to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.  		
B1.4 Total Effective Stiffness Calculate the total effective stiffness, K_{eff}, of the bridge: Eq. 7.1-6 GSID $$K_{eff} = \sum_{j=1}^{m} K_{eff,j}$$ (B-8)		**B1.4 Total Effective Stiffness, Example 2.3** $$K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 79.02\ k/in$$
B1.5 Isolation System Displacement at Each Support Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports: $$d_{isol,j} = \frac{d}{1+\alpha_j}$$ (B-9)		**B1.5 Isolation System Displacement at Each Support, Example 2.3** $$d_{isol,j} = \frac{d}{1+\alpha_j}$$ o $d_{isol,1}$ = 2.00 in o $d_{isol,2}$ = 1.79 in o $d_{isol,3}$ = 1.79 in o $d_{isol,4}$ = 2.00 in
B1.6 Isolation System Stiffness at Each Support Calculate the stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports: $$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$ (B-10)		**B1.6 Isolation System Stiffness at Each Support, Example 2.3** $$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$ o $K_{isol,1}$ = 8.43 k/in o $K_{isol,2}$ = 34.84 k/in o $K_{isol,3}$ = 34.84 k/in o $K_{isol,4}$ = 8.43 k/in

B1.7 Substructure Displacement at Each Support Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports: $d_{sub,j} = d - d_{isol,j}$ (B-11)	**B1.7 Substructure Displacement at Each Support, Example 2.3** $d_{sub,j} = d - d_{isol,j}$ o $d_{sub,1}$ = 0.002 in o $d_{sub,2}$ = 0.215 in o $d_{sub,3}$ = 0.215 in o $d_{sub,4}$ = 0.002 in
B1.8 Lateral Load in Each Substructure Calculate the lateral load in substructure 'j', $F_{sub,j}$, for all supports: $F_{sub,j} = K_{sub,j} d_{sub,j}$ (B-12) where values for $K_{sub,j}$ are given in Step A1.	**B1.8 Lateral Load in Each Substructure, Example 2.3** $F_{sub,j} = K_{sub,j} d_{sub,j}$ o $F_{sub,1}$ = 16.84 k o $F_{sub,2}$ = 62.18 k o $F_{sub,3}$ = 62.18 k o $F_{sub,4}$ = 16.84 k
B1.9 Column Shear Force at Each Support Calculate the shear force in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j': $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ (B-13) Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.	**B1.9 Column Shear Force at Each Support, Example 2.3** $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ o $F_{col,2,1}$ = 62.18 k o $F_{col,3,1}$ = 62.18 k These column shears are less than the plastic shear capacity of each column (128k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.
B1.10 Effective Period and Damping Ratio Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge: Eq. 7.1-5 GSID $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}}$ (B-14) and Eq. 7.1-10 GSID $\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2}$ (B-15) where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j}$ = 0. For other types of isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for the Simplified Method, set $d_{y,j}$ = 0 for all isolator types. See Step B2.2 where the value of $d_{y,j}$ is revisited	**B1.10 Effective Period and Damping Ratio, Example 2.3** $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{1907.58}{386.4(79.02)}}$ = 1.57 sec and taking $d_{y,j}$ = 0: $\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - 0)}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.30$

<table>
<tr><td>for the Multimode Spectral Analysis Method.</td><td></td><td></td></tr>
<tr><td>B1.11 Damping Factor
Calculate the damping factor, B_L, and the displacement, d, of the bridge:

Eq. 7.1-3 GSID $$B_L = \begin{cases} \left(\frac{\xi}{0.05}\right)^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases} \quad \text{(B-16)}$$

Eq. 7.1-4 GSID $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \quad \text{(B-17)}$$</td><td></td><td>B1.11 Damping Factor, Example 2.3
Since $\xi = 0.30 \geq 0.3$

$$B_L = 1.70$$
and
$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.2)1.57}{1.70} = 1.81\ in$$</td></tr>
<tr><td>B1.12 Convergence Check
Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement.

This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5).

After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both.

Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)</td><td></td><td>B1.12 Convergence Check, Example 2.3
Since the calculated value for displacement, d (=1.81) is not close to that assumed at the beginning of the cycle (Step B1.1, d = 2.0), use the value of 1.81 as the new assumed displacement and repeat from Step B1.3.

After three iterations, convergence is reached at a superstructure displacement of 1.65 in, with an effective period of 1.43 seconds, and a damping factor of 1.7 (30% damping ratio). The displacement in the isolators at Pier 1 is 1.44 in and the effective stiffness of the same isolators is 42.78 k/in.

See spreadsheet in Table B1.12-1 for results of final iteration.

Ignoring the weight of the hammerhead, the column shear force must equal the isolator shear force for equilibrium. Hence column shear = 42.78 (1.44) = 61.60 k which is less than the maximum allowable (128 k) if elastic behavior is to be achieved (as required in Step A3).

Also the superstructure displacement = 1.65 in, which is less than the available clearance of 2.5 in.

Therefore the above solution is acceptable and go to Step B2.

Note that available clearance (2.5 in) is greater than minimum required which is given by:
$$= \frac{8\ S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.43}{1.7} = 1.35\ in$$</td></tr>
</table>

Table B1.12-1 Simplified Method Solution for Design Example 2.3 – Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	1651.32	256.26	1907.58	0.2	3							
Step B1.1	d	1.65	Assumed displacement									
	Q_d	82.57	Characteristic strength									
	K_d	50.04	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	168.48	8.424	5.105	10,000.00	0.001022	10.206	1.648	10.216	0.002	16.839	13.885	27.785
Pier 1	657.18	32.859	19.915	288.87	0.148088	37.260	1.437	42.778	0.213	61.480	47.224	101.441
Pier 2	657.18	32.859	19.915	288.87	0.148088	37.260	1.437	42.778	0.213	61.480	47.224	101.441
Abut 2	168.48	8.424	5.105	10,000.00	0.001022	10.206	1.648	10.216	0.002	16.839	13.885	27.785
Total	1651.32	82.566	50.040		$\Sigma K_{eff,j}$	94.932				156.638	122.219	258.453
					Step	**B1.4**						
Step B1.10	T_{eff}	1.43	Effective period									
	ξ	0.30	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.71										
	B_L	1.70	Damping Factor									
	d	1.65	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	2.808	1.702	3.405		1.69	3.363						
Pier 1	10.953	6.638	14.259		1.20	15.766						
Pier 2	10.953	6.638	14.259		1.20	15.766						
Abut 2	2.808	1.702	3.405		1.69	3.363						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength

Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \qquad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \qquad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 2.3

Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-1) by the number of isolators at each support (n = 3), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,1}$ = 8.42/3 = 2.81 k
- $Q_{d,2}$ = 32.86/3=10.95 k
- $Q_{d,3}$ = 32.86/3 = 10.95 k
- $Q_{d,4}$ = 8.42/3 = 2.81 k

and

- $K_{d,1}$ = 5.10/3 = 1.70 k/in
- $K_{d,2}$ = 19.92/3 = 6.64 k/in
- $K_{d,3}$ = 19.92/3 = 6.64 k/in
- $K_{d,4}$ = 5.10/3 = 1.70 k/in

Note that the K_d values per support used above are from the final iteration given in Table B1.12-1. These are not the same as the initial values in Step B1.2, because they have been adjusted from cycle to cycle, such that the total K_d summed over all the isolators satisfies the minimum lateral restoring force requirement for the bridge, i.e. K_{dtotal} = 0.05 W/d. See Step B1.1. Since d varies from cycle to cycle, $K_{d,j}$ varies from cycle to cycle.

B2.2 Initial Stiffness and Yield Displacement

Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

(1) For friction-based isolators $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \qquad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \qquad \text{(B-22)}$$

B2.2 Initial Stiffness and Yield Displacement, Example 2.3

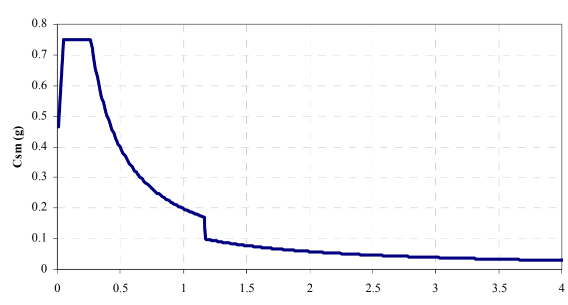
Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(6.64) = 66.4\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{10.95}{(66.4 - 6.64)} = 0.18\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2

		in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i} = 0$.
B2.3 Isolator Effective Stiffness, $K_{isol,i}$ Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i': $$k_{isol,i} = \frac{K_{isol,j}}{n} \quad \text{(B -23)}$$		**B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 2.3** Dividing the results for K_{isol} (Step B1.12) among the 3 isolators at each support, the following values for K_{isol} /isolator are obtained: o $K_{isol,1}$ = 10.22/3 = 3.41 k/in o $K_{isol,2}$ = 42.78/3 = 14.26 k/in o $K_{isol,3}$ = 42.78/3 = 14.26 k/in o $K_{isol,4}$ = 10.22/3 = 3.41 k/in
B2.4 Three-Dimensional Bridge Model Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is recommended.		**B2.4 Three-Dimensional Bridge Model, Example 2.3** Although the bridge in this Design Example is regular and is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below. 
B2.5 Composite Design Response Spectrum Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.		**B.2.5 Composite Design Response Spectrum, Example 2.3** From the final results of Simplified Method (Step B1.12), B_L = 1.70 and T_{eff} = 1.43 sec. Hence the transition in the composite spectrum from 5% to 30% damping occurs at 0.8 T_{eff} = 0.8 (1.43) = 1.14 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods ≥ 1.14 sec by 1.70. 

B2.6 Multimodal Analysis of Finite Element Model

Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

B2.6 Multimodal Analysis of Finite Element Model, Example 2.3

Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first three modes are the principal transverse, longitudinal, and torsion modes with periods of 1.60, 1.46 and 1.39 sec respectively. The period of the longitudinal mode (1.46 sec) is very close to that calculated in the Simplified Method. The mass participation factors indicate there is no coupling between these three modes (probably due to the symmetric nature of the bridge) and the high values for the first and second modes (92% and 94% respectively) indicate the bridge is responding essentially in a single mode of vibration in each direction. Similar results to that obtained by the Simplified Method are therefore expected.

Mode	Period	Mass Participation Ratios					
No	Sec	UX	UY	UZ	RX	RY	RZ
1	1.604	0.000	0.919	0.000	0.952	0.000	0.697
2	1.463	0.941	0.000	0.000	0.000	0.020	0.000
3	1.394	0.000	0.000	0.000	0.000	0.000	0.231
4	0.479	0.000	0.003	0.000	0.013	0.000	0.002
5	0.372	0.000	0.000	0.076	0.000	0.057	0.000
6	0.346	0.000	0.000	0.000	0.000	0.000	0.000
7	0.345	0.000	0.001	0.000	0.010	0.000	0.000
8	0.279	0.000	0.003	0.000	0.013	0.000	0.002
9	0.268	0.000	0.000	0.000	0.000	0.000	0.000
10	0.267	0.058	0.000	0.000	0.000	0.000	0.000
11	0.208	0.000	0.000	0.000	0.000	0.129	0.000
12	0.188	0.000	0.000	0.000	0.000	0.000	0.001

Table B2.6-1 Modal Properties of Bridge Example 2.3- First Iteration

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 1.69 (1.65) in
- $d_{isol,2}$ = 1.20 (1.44) in
- $d_{isol,3}$ = 1.20 (1.44) in
- $d_{isol,4}$ = 1.69 (1.65) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 2.3

The results for isolator displacements are close but not close enough (15% difference at the piers)

Go to Step B2.8 and update properties for a second cycle of iteration.

<table>
<tr><td>

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

$$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \quad \text{(B-24)}$$

Recalculate $K_{eff,j}$:

Eq. 7.1-6 GSID
$$K_{eff,j} = \frac{K_{sub,j}\sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \quad \text{(B-25)}$$

Recalculate system damping ratio, ξ :

Eq. 7.1-10 GSID
$$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \quad \text{(B-26)}$$

Recalculate system damping factor, B_L:

Eq. 7.1-3 GSID
$$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \le 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \quad \text{(B-27)}$$

Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.

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B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 2.3

Updated values for $K_{isol,i}$ are given below (previous values are in parentheses):

- $K_{isol,1}$ = 3.36 (3.41) k/in
- $K_{isol,2}$ = 15.77 (14.26) k/in
- $K_{isol,3}$ = 15.77 (14.26) k/in
- $K_{isol,4}$ = 3.36 (3.41) k/in

Since the isolator displacements are relatively close to previous results no significant change in the damping ratio is expected. Hence $K_{eff,j}$ and ξ are not recalculated and B_L is taken at 1.70.

Since the change in effective period is very small (1.43 to 1.46 sec) and no change has been made to B_L, there is no need to construct a new composite response spectrum in this case. Go back to Step B2.6 (see immediately below).

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B2.6 Multimodal Analysis Second Iteration, Example 2.3

Reanalysis gives the following values for the isolator displacements (numbers in parentheses are those from the previous cycle):

- $d_{isol,1}$ = 1.66 (1.69) in
- $d_{isol,2}$ = 1.15 (1.20) in
- $d_{isol,3}$ = 1.15 (1.20) in
- $d_{isol,4}$ = 1.66 (1.69) in

Go to Step B2.7

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B2.7 Convergence Check

Compare results and determine if convergence has been reached. If so go to Step B2.9. Otherwise Go to Step B2.8.

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B2.7 Convergence Check, Example 2.3

Satisfactory agreement has been reached on this second cycle. Go to Step B2.9

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B2.9 Superstructure and Isolator Displacements

Once convergence has been reached, obtain

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and
- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for

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B2.9 Superstructure and Isolator Displacements, Example 2.3

From the above analysis:

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are:

  x_L= 1.69 in

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each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.		y_L= 0.0 in o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are: o Abutments: u_L = 1.66 in, v_L = 0.00 in o Piers: u_L = 1.15 in, v_L = 0.00 in All isolators at same support have the same displacements.
B2.10 Pier Bending Moments and Shear Forces Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.		**B2.10 Pier Bending Moments and Shear Forces, Example 2.3** Bending moments in single column pier in the longitudinal (M_{PLL}) and transverse (M_{PTL}) directions are: M_{PLL}= 0 M_{PTL}= 1602 kft Shear forces in single column pier the longitudinal (V_{PLL}) and transverse (V_{PTL}) directions are V_{PLL}=67.16 k V_{PTL}=0
B2.11 Isolator Shear and Axial Forces Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.		**B2.11 Isolator Shear and Axial Forces, Example 2.3** Isolator shear and axial forces are summarized in Table B2.11-1 **Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.** (table below) The difference between the longitudinal shear force in the column (V_{PLL} = 67.16k) and the sum of the isolator shear forces at the same Pier (54.63 k) is about 12.5 k. This is due to the inertia force developed in the hammerhead cap beam which weighs about 128 k and can generate significant additional demand on the column (about a 23% increase in this case).

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abutment	1	5.63	0	1.29
	2	5.63	0	1.30
	3	5.63	0	1.29
Pier	1	18.19	0	0.77
	2	18.25	0	1.11
	3	18.19	0	0.77

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 2.3

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.52 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows: x_T = 0 and y_T = 1.75 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions as follows:
  Abutments u_T = 0.00 in, v_T = 1.75 in
  Piers u_T = 0.00 in, v_T = 0.71 in
- Pier bending moments in the longitudinal (M_{PLT}) and transverse (M_{PTT}) directions are as follows: M_{PLT} = 1548.33 kft and M_{PTT} = 0
- Pier shear forces in the longitudinal (V_{PLT}) and transverse (V_{PTT}) directions are as follows: V_{PLT} = 0 and V_{PTT} = 60.75 k
- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abutment	1	0.0	5.82	13.51
	2	0.0	5.83	0
	3	0.0	5.82	13.51
Pier	1	0.0	15.40	26.40
	2	0.0	15.57	0
	3	0.0	15.40	26.40

The difference between the transverse shear force in the column (V_{PLL} = 60.75k) and the sum of the isolator shear forces at the same Pier (46.37 k) is about 14.4 k. This is due to the inertia force developed in the hammerhead cap beam which weighs about 128 k and can generate significant additional demand on the column (about 31%).

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements
Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)

- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)

- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements at Pier 1, Example 2.3
To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

Load Case 1:
$u_1 = u_L + 0.3u_T = 1.0(1.15) + 0.3(0) = 1.15$ in
$v_1 = v_L + 0.3v_T = 1.0(0) + 0.3(0.71) = 0.21$ in
$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.15^2 + 0.21^2} = 1.17$ in

Load Case 2:
$u_2 = 0.3u_L + u_T = 0.3(1.15) + 1.0(0) = 0.35$ in
$v_2 = 0.3v_L + v_T = 0.3(0) + 1.0(0.71) = 0.71$in
$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{0.35^2 + 0.71^2} = 0.79$ in

Governing Case:
Total design displacement, $d_t = \max(R_1, R_2)$
$= 1.17$ in

D2. Design Moments and Shears
Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

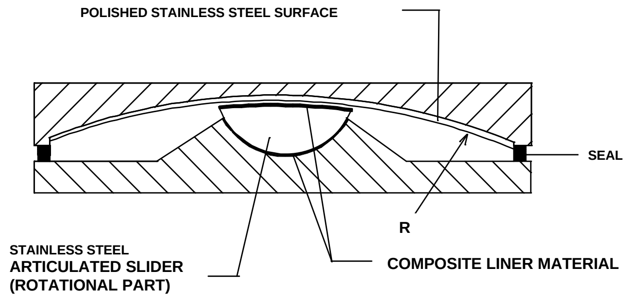
D2. Design Moments and Shears in Pier 1, Example 2.3
Design moments and shear forces are calculated for Pier 1 below, to illustrate the process.

Load Case 1:
$V_{PL1} = V_{PLL} + 0.3V_{PLT} = 1.0(67.16) + 0.3(0) = 67.16$ k
$V_{PT1} = V_{PTL} + 0.3V_{PTT} = 1.0(0) + 0.3(60.75) = 18.23$ k
$R_1 = \sqrt{V_{L1}^2 + V_{T1}^2} = \sqrt{67.16^2 + 18.23^2} = 69.59$ k

Load Case 2:
$V_{PL2} = 0.3V_{PLL} + V_{PLT} = 0.3(67.16) + 1.0(0) = 20.15$ k
$V_{PT2} = 0.3V_{PTL} + V_{PTT} = 0.3(0) + 1.0(60.75) = 60.75$ k
$R_2 = \sqrt{V_{L2}^2 + V_{T2}^2} = \sqrt{20.15^2 + 60.75^2} = 64.00$ k

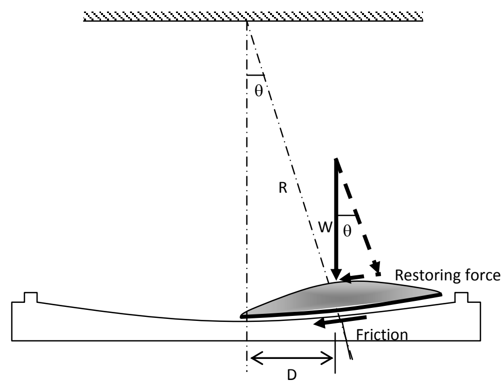
Governing Case:
Design column shear $= \max(R1, R2)$
$= 69.59$ k

STEP E. DESIGN OF SPHERICAL FRICTION ISOLATORS



The Spherical Friction Bearing (SFB) isolator has an articulated slider to permit rotation, and a spherical sliding interface. It has lateral stiffness due to the curvature of this interface. These isolators are capable of carrying very large axial loads and can be designed to have long periods of vibration (5 seconds or longer).

The main components of an SFB isolator are a stainless steel concave spherical plate, an articulated slider and a housing plate as illustrated in figure above. In this figure, the concave spherical plate is facing down. The bearings may also be installed with this surface facing up as in the figure below. The side of the articulated slider in contact with the concave spherical surface is coated with a low-friction composite material, usually PTFE. The other side of the slider is also spherical but lined with stainless steel and sits in a spherical cavity coated with PTFE.



Spherical friction bearings are described by the same equation of motion as conventional pendulums. As a consequence their period of vibration is directly proportional to the radius of curvature of the concave surface. See figure at right. Long period shifts are therefore possible with surfaces that have large radii of curvature. Friction between the articulated slider and the concave surface dissipates energy and the weight of the bridge acts as a restoring force, due to the curvature of the sliding surface.

The required values for Q_d and K_d determine the coefficient of friction at the sliding interface and the radius of curvature.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties

Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1).

- required characteristic strength, Q_d / isolator
- required post-elastic stiffness, K_d / isolator
- the total design displacement, d_t, for each isolator
- maximum applied dead load, P_{DL}
- maximum live load, P_{LL} and
- maximum wind load, P_{WL}

E1. Required Properties, Example 2.3

The design of one of the pier isolators is given below to illustrate the design process for spherical friction isolators.

From previous work

- Q_d / isolator = 2.34 k
- K_d / isolator = 1.17 k/in
- Total design displacement, d_t = 1.55 in
- P_{DL} = 45.52 k
- P_{LL} = 15.50 k
- P_{WL} = 1.76 k < Q_d OK

E2. Isolator Dimensions

E2.1 Radius of Curvature

Determine the required radius of curvature, R, using:

$$R = \frac{P_{DL}}{K_d} \qquad \text{(E-1)}$$

E2.1 Radius of Curvature, Example 2.3

$$R = \frac{187}{6.76} = 27.66 \approx 27.75\ in$$

E2.2 Coefficient of Friction
Determine the required coefficient of friction, μ, using:

$$\mu = \frac{Q_d}{P_{DL}} \qquad \text{(E-2)}$$

E2.2 Coefficient of Friction, Example 2.3

$$\mu = \frac{10.95}{187} = 0.0585 = 5.85\%$$

E2.3 Material Selection
Based on the required coefficient of friction select an appropriate PTFE compound and contact pressure, σ_c, from Table E2.3-1.

Table E2.3-1 Material Properties

PTFE Compound (Filled and Unfilled Teflon)	Contact Pressure, σ_c (psi)	μ (%)
Unfilled (UF)	1,000	11.93
	2,000	8.70
	3,000	7.03
	6,500	5.72
Glass-filled 15% by weight (15GF)	1,000	14.61
	2,000	10.08
	3,000	8.49
	6,500	5.27
Glass-filled 25% by weight (25GF)	1,000	13.20
	2,000	11.20
	3,000	9.60
	6,500	5.89

E2.3 Material Selection, Example 2.3
Select 25GF Teflon and size disc to achieve required contact pressure of 6,500 psi (Step E2.4).

E2.4 Disk Diameter
Determine the required contact area, A_c, and disk diameter, d, using:

$$A_c = \frac{P_{DL}}{\sigma_c} \qquad \text{(E-3)}$$

and

$$d_d = \sqrt{\frac{4A_c}{\pi}} \qquad \text{(E-4)}$$

E2.4 Disk Diameter, Example 2.3

$$A_c = \frac{187}{6.5} = 28.77\ in^2$$

$$d_d = \sqrt{\frac{4(7.00)}{\pi}} = 6.05 \approx 6.00\ in$$

E2.5 Isolator Diameter
Determine the required diameter of the concave surface, L_{chord}, and overall isolator width, B, using:

$$L_{chord} = 2(\Delta + {}^{d_d}/_2) \qquad \text{(E-5)}$$

E2.5 Isolator Diameter, Example 2.3
As the bridge is in Seismic Zone 2,

$$\Delta = 2(d_t) = 2(1.17) = 2.34 \text{ in}$$

$$L_{chord} = 2(2.34 + 3.0) = 10.68\ in$$

and $B = L_{chord} + 2s$ (E-6) where: Δ = 2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or 1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4. s = width of shoulder of concave plate.		Select s = 1.5 in: $B = 10.68 + 2(1.5) = 13.68 \approx 13.75\ in$
E2.6 Isolator Height		**E2.6 Isolator Height, Example 2.3**
E2.6.1 Rise Determine the rise of the concave surface, h, using: $h = \frac{(L_{chord})^2}{8R}$ (E-7)		**E2.6.1 Rise, Example 2.3** $h = \frac{(10.68)^2}{8(27.66)} = 0.52\ in$
E2.6.2 Throat Thickness Determine the required throat thickness, t, based on the minimum required bearing area, A_b, such that the maximum allowable bearing stress, $\sigma_{bearing}$, is not exceeded on either the sole plate above or the masonry plate below, depending on whether the isolator is installed with concave surface facing up or down. $A_b = \frac{P_{DL} + P_{LL}}{\sigma_{bearing}}$ (E-8) $d_b = \sqrt{\frac{4A_b}{\pi}}$ (E-9) $t = 0.5(d_b - d_d)$ (E-10) This assumes a 45° distribution of compressive stress through the throat to the support plates.		**E2.6.2 Throat Thickness, Example 2.3** Assume safe bearing stress below isolator: $\sigma_{bearing}$ = 2.0 ksi. $A_b = \frac{187 + 123}{2.0} = 155.0\ in^2$ $d_b = \sqrt{\frac{4(155)}{\pi}} = 14.05\ in$ $t = 0.5(14.05 - 6.0) = 4.02 \approx 4.0\ in$
E2.6.3 Total Height Determine the thickness of concave plate, T_1, using: $T_1 = h + t$ (E-11) Thickness of slider plate (T_2) will vary with detail for socket that holds articulated slider and rotation requirement. Check with manufacturer for value. For estimating purposes take $T_2 = T_1$. Then total height of isolator: $H = T_1 + T_2$ (E-12)		**E2.6.3 Total Height, Example 2.3** $T_1 = 0.52 + 4.0 = 4.52 \approx 4.50\ in$ $T_2 = 4.50\ in(est)$ $H = 4.50 + 4.50 = 9.00\ in(est)$
E3. Design Summary		**E3. Design Summary, Example 2.3** Overall diameter = 13.75 in Overall height = 9.0 in (est.)

Radius concave surface = 27.75 in
PTFE is 25% GF; contact pressure = 6,500 psi
Diameter PTFE sliding disc = 6.00 in

E4. Minimum and Maximum Performance Check
Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E4-1.

Table E4-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-13)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-14)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-15)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-16)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E4-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E4-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})(\lambda_{min,a,Kd})(\lambda_{min,v,Kd})(\lambda_{min,tr,Kd})(\lambda_{min,c,Kd})(\lambda_{min,scrag,Kd})$	(E-21)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})(\lambda_{max,a,Kd})(\lambda_{max,v,Kd})(\lambda_{max,tr,Kd})(\lambda_{max,c,Kd})(\lambda_{max,scrag,Kd})$	(E-18)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})(\lambda_{min,a,Qd})(\lambda_{min,v,Qd})(\lambda_{min,tr,Qd})(\lambda_{min,c,Qd})(\lambda_{min,scrag,Qd})$	(E-19)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})(\lambda_{max,a,Qd})(\lambda_{max,v,Qd})(\lambda_{max,tr,Qd})(\lambda_{max,c,Qd})(\lambda_{max,scrag,Qd})$	(E-20)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied

E4. Minimum and Maximum Performance Check, Example 2.3
For spherical friction isolators, property modification factors are applied to Q_d only.

Minimum Property Modification factors are:
$\lambda_{min} = 1.0$

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are (GSID Appendix A.1):

$\lambda_{max,a} = 1.1$
$\lambda_{max,c} = 1.0$
$\lambda_{max,tr} = 1.2$
$\lambda_{max,t} = 1.2$

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:
$\lambda_{max,a} = 1.0 + 0.1(0.66) = 1.066$

to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows: 1.00 for critical bridges 0.75 for essential bridges 0.66 for all other bridges As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).		$\lambda_{max,c}$ = 1.0 $\lambda_{max,t}$ = 1.0 + 0.2(0.66) = 1.132 $\lambda_{max,a}$ = 1.0 + 0.2(0.66) = 1.132 Therefore the maximum overall modification factors λ_{max} = 1.066(1.0)(1.132)(1.132) = 1.37 Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties. The upper-bound properties are: $Q_{d,max}$ = 1.37 (10.95) = 15.0 k K_{dmax} = K_d = 6.76 k/in
E5. Design and Performance Summary		**E5. Design and Performance Summary, Example 2.3**
E5.1 Isolator dimensions Summarize final dimensions of isolators: • Overall diameter of isolator • Overall height • Radius of curvature of concave plate • Diameter of PTFE disc • PTFE Compound • PTFE contact pressure Check all dimensions with manufacturer.		**E5.1 Isolator dimensions, Example 2.3** Isolator dimensions are summarized in Table E5.1-1. **Table E5.1-1 Isolator Dimensions** (see table below) PTFE is 25% Glass-filled; 6,500 psi contact pressure; disc diameter is 6.00 in.
E5.2 Bridge Performance Summarize bridge performance • Maximum superstructure displacement (longitudinal) • Maximum superstructure displacement (transverse) • Maximum superstructure displacement (resultant) • Maximum column shear (resultant) • Maximum column moment (about transverse axis) • Maximum column moment (about longitudinal axis) • Maximum column torque Check required performance as determined in Step A3, is satisfied.		**E5.2 Bridge Performance, Example 2.3** Bridge performance is summarized in Table E5.2-1 where it is seen that the maximum column shear is 71.74k. This less than the column plastic shear (128k) and therefore the required performance criterion is satisfied (fully elastic behavior). Furthermore the maximum longitudinal displacement is 1.69 in which is less than the 2.5in available at the abutment expansion joints and is therefore acceptable.

Table E5.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Radius (in)
Under edge girder on Pier 1	17.75x17.75 x 9.0(H)	13.75 dia. x 7.0(H)	27.75

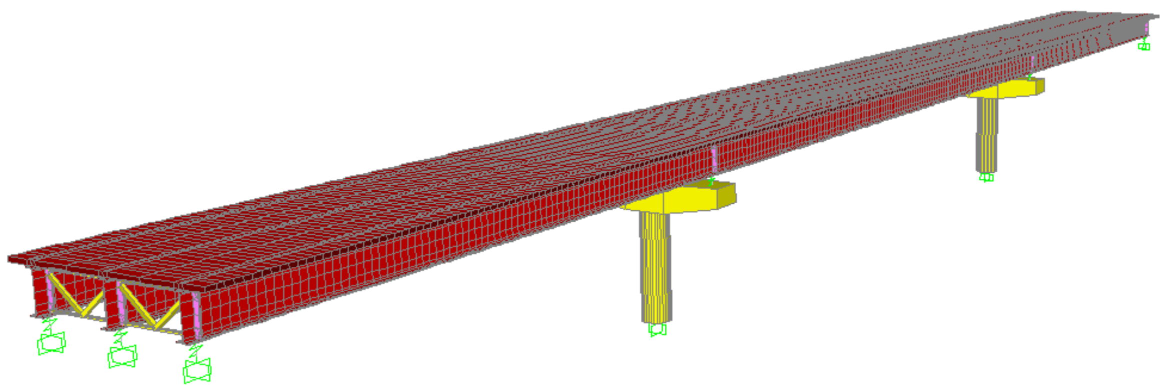
Table E5.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	1.69 in
Maximum superstructure displacement (transverse)	1.75 in
Maximum superstructure displacement (resultant)	1.82 in
Maximum column shear (resultant)	71.74 k
Maximum column moment about transverse axis	1,657 kft
Maximum column moment about longitudinal axis	1,676 kft
Maximum column torque	21.44 kft

SECTION 2

Steel Plate Girder Bridge, Long Spans, Single-Column Pier

DESIGN EXAMPLE 2.4: Eradiquake Isolators



Design Examples in Section 2

ID	**Description**	**S_1**	**Site Class**	**Column height**	**Skew**	**Isolator type**
2.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
2.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
2.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
2.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
2.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
2.5	Change column height	0.2g	B	H_1=0.5H_2	0	Lead rubber bearing
2.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

DESIGN PROCEDURE	DESIGN EXAMPLE 2.4 (EQS Isolators)

STEP A: BRIDGE AND SITE DATA

A1. Bridge Properties

Determine properties of the bridge:

- number of supports, *m*
- number of girders per support, *n*
- angle of skew
- weight of superstructure including railings, curbs, barriers and other permanent loads, W_{SS}
- weight of piers participating with superstructure in dynamic response, W_{PP}
- weight of superstructure, W_j, at each support
- stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
- column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
- allowable movement at expansion joints
- isolator type if known, otherwise ‘to be selected’

A1. Bridge Properties, Example 2.4

- Number of supports, $m = 4$
  - North Abutment ($m = 1$)
  - Pier 1 ($m = 2$)
  - Pier 2 ($m = 3$)
  - South Abutment ($m = 4$)
- Number of girders per support, $n = 3$
- Angle of skew = 0^0
- Number of columns per support = 1
- Weight of superstructure including permanent loads, W_{SS} = 1651.32 k
- Weight of superstructure at each support:
  - W_1 = 168.48 k
  - W_2 = 657.18 k
  - W_3 = 657.18 k
  - W_4 = 168.48 k
- Participating weight of piers, W_{PP} = 256.26 k
- Effective weight (for calculation of period), $W_{eff} = W_{SS} + W_{PP}$ = 1907.58 k
  - Stiffness of each pier in the both directions $K_{sub,pier1}$ = 288.87 k/in
  - $K_{sub,pier2}$ = 288.87 k/in
- Minimum column shear strength based on flexural yield capacity of column = 128 k
- Displacement capacity of expansion joints (longitudinal) = 2.5 in for thermal and other movements
- Eradiquake isolators

A2. Seismic Hazard

Determine seismic hazard at site:

- acceleration coefficients
- site class and site factors
- seismic zone

Plot response spectrum.

Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of *PGA*, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD).

Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for conventional bridges and Art 3.2 refers the designer to

A2. Seismic Hazard, Example 2.4

Acceleration coefficients for bridge site are given in design statement as follows:

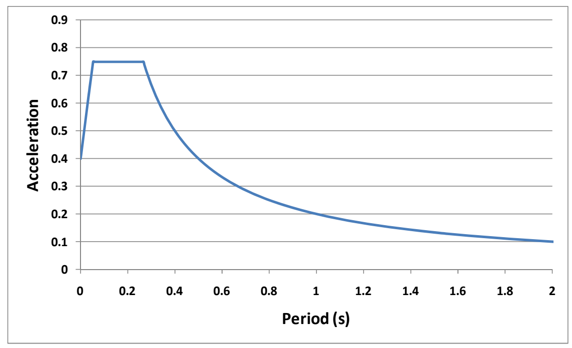
- PGA = 0.40
- $S_1 = 0.20$
- $S_S = 0.75$

Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec.

Table 3.10.3.1-1 LRFD gives Site Class as B.

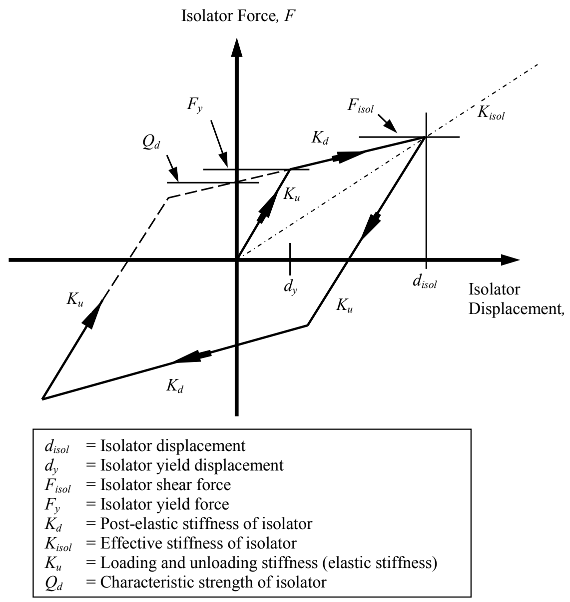
Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors:

- $F_{pga} = 1.0$
- $F_a = 1.0$
- $F_v = 1.0$

the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD. Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows: • $A_s = F_{pga}\, PGA$ • $S_{DS} = F_a\, S_S$ • $S_{D1} = F_v\, S_1$ Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table5-1 GSID. These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.		• $A_s = F_{pga}\, PGA = 1.0(0.40) = 0.40$ • $S_{DS} = F_a\, S_S = 1.0(0.75) = 0.75$ • $S_{D1} = F_v\, S_1 = 1.0(0.20) = 0.20$ Since $0.15 < S_{D1} \leq 0.30$, bridge is located in Seismic Zone 2. Design Response Spectrum is as below: 
A3. Performance Requirements Determine required performance of isolated bridge during Design Earthquake (1000-yr return period). Examples of performance that might be specified by the Owner include: • Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake. • Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake. • For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements) • Reduced substructure forces for bridges on weak soils to reduce foundation costs.		**A3. Performance Requirements, Example 2.4** In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield). To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (128 k). The maximum shear in the column must therefore be less than 128 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.



Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.

The methodology below uses the <u>Simplified Method</u> (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the <u>Multimode Spectral Analysis Method</u> (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties	B1.1 Initial System Displacement and Properties, Example 2.4
To begin the iterative solution, an estimate is required of : (1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.) Art C7.1 GSID $\quad d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1}$ (B-1)	$d \cong 10 S_{D1} = 10(0.20) \cong 2.0\ in$

(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not occur under non-seismic loads (e.g. wind) but low enough that yield will occur during an earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e.

$$Q_d = 0.05W \tag{B-2}$$

(3) Post-yield stiffness, K_d
Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3.

Art. 12.2 GSID

$$K_{d,min} \geq \frac{0.025W}{d} \tag{B-3}$$

Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$

$$Q_d = 0.05W = 0.05(1651.32) = 82.56\ k$$

$$K_d = 0.05\frac{W}{d} = 0.05\frac{1651.32}{2.0} = 41.28\ k/in$$

B1.2 Initial Isolator Properties at Supports
Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support:

$$Q_{d,j} = Q_d\frac{W_j}{W} \tag{B-4}$$

and

$$K_{d,j} = K_d\frac{W_j}{W} \tag{B-5}$$

B1.2 Initial Isolator Properties at Supports, Example 2.4

$$Q_{d,j} = Q_d\frac{W_j}{W}$$

- $Q_{d,1}$ = 8.42 k
- $Q_{d,2}$ = 32.86 k
- $Q_{d,3}$ = 32.86 k
- $Q_{d,4}$ = 8.42 k

and

$$K_{d,j} = K_d\frac{W_j}{W}$$

- $K_{d,1}$ = 4.21 k/in
- $K_{d,2}$ = 16.43 k/in
- $K_{d,3}$ = 16.43 k/in
- $K_{d,4}$ = 4.21 k/in

B1.3 Effective Stiffness of Combined Pier and Isolator System
Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below for definitions (after Fig. 7.1-1 GSID).

An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula is as follows (MCEER 2006):

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j} \tag{B-6}$$

where

$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}} \tag{B-7}$$

B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 2.4

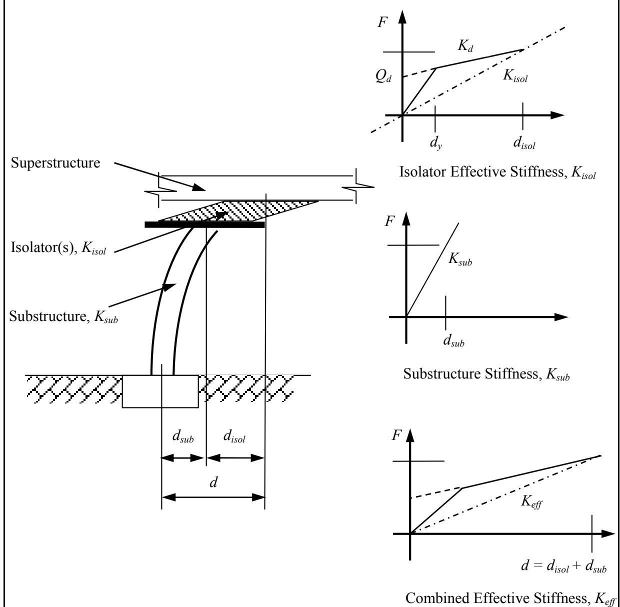
$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$

- $\alpha_1 = 8.43\text{x}10^{-4}$
- $\alpha_2 = 1.21\text{x}10^{-1}$
- $\alpha_3 = 1.21\text{x}10^{-1}$
- $\alpha_4 = 8.43\text{x}10^{-4}$

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$$

- $K_{eff,1}$ = 8.42 k/in
- $K_{eff,2}$ = 31.09 k/in

and $K_{sub,j}$ for the piers are given in Step A1. For the abutments, take $K_{sub,j}$ to be a large number, say 10,000 k/in, unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high



values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.

- o $K_{eff,3}$ = 31.09 k/in
- o $K_{eff,4}$ = 8.42 k/in

B1.4 Total Effective Stiffness

Calculate the total effective stiffness, K_{eff}, of the bridge:

Eq. 7.1-6 GSID

$$K_{eff} = \sum_{j=1}^{m} K_{eff,j} \quad \text{(B-8)}$$

B1.4 Total Effective Stiffness, Example 2.4

$$K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 79.02\ k/in$$

B1.5 Isolation System Displacement at Each Support

Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports:

$$d_{isol,j} = \frac{d}{1+\alpha_j} \quad \text{(B-9)}$$

B1.5 Isolation System Displacement at Each Support, Example 2.4

$$d_{isol,j} = \frac{d}{1+\alpha_j}$$

- o $d_{isol,1}$ = 2.00 in
- o $d_{isol,2}$ = 1.79 in
- o $d_{isol,3}$ = 1.79 in
- o $d_{isol,4}$ = 2.00 in

B1.6 Isolation System Stiffness at Each Support

Calculate the stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports:

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \quad \text{(B-10)}$$

B1.6 Isolation System Stiffness at Each Support, Example 2.4

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$

- o $K_{isol,1}$ = 8.43 k/in
- o $K_{isol,2}$ = 34.84 k/in

		o $K_{isol,3}$ = 34.84 k/in o $K_{isol,4}$ = 8.43 k/in
B1.7 Substructure Displacement at Each Support Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports: $d_{sub,j} = d - d_{isol,j}$ (B-11)		**B1.7 Substructure Displacement at Each Support, Example 2.4** $d_{sub,j} = d - d_{isol,j}$ o $d_{sub,1}$ = 0.002 in o $d_{sub,2}$ = 0.215 in o $d_{sub,3}$ = 0.215 in o $d_{sub,4}$ = 0.002 in
B1.8 Lateral Load in Each Substructure Calculate the lateral load in substructure 'j', $F_{sub,j}$, for all supports: $F_{sub,j} = K_{sub,j} d_{sub,j}$ (B-12) where values for $K_{sub,j}$ are given in Step A1.		**B1.8 Lateral Load in Each Substructure, Example 2.4** $F_{sub,j} = K_{sub,j} d_{sub,j}$ o $F_{sub,1}$ = 16.84 k o $F_{sub,2}$ = 62.18 k o $F_{sub,3}$ = 62.18 k o $F_{sub,4}$ = 16.84 k
B1.9 Column Shear Force at Each Support Calculate the shear force in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j': $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ (B-13) Use these approximate column shear forces as a check on the validity of the chosen strength and stiffness characteristics.		**B1.9 Column Shear Force at Each Support, Example 2.4** $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ o $F_{col,2,1}$ = 62.18 k o $F_{col,3,1}$ = 62.18 k These column shears are less than the plastic shear capacity of each column (128k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.
B1.10 Effective Period and Damping Ratio Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge: Eq. 7.1-5 GSID $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}}$ (B-14) and Eq. 7.1-10 GSID $\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2}$ (B-15) where $d_{y,j}$ is the yield displacement of the isolator. For friction-based isolators, $d_{y,j}$ = 0. For other types of isolators $d_{y,j}$ is usually small compared to $d_{isol,j}$ and has negligible effect on ξ, Hence it is suggested that for		**B1.10 Effective Period and Damping Ratio, Example 2.4** $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{1907.58}{386.4(79.02)}}$ = 1.57 sec and taking $d_{y,j}$ = 0: $\xi = \frac{2\sum_j (Q_{d,j}(d_{isol,j} - 0)}{\pi \sum_j (K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.30$

the Simplified Method, set $d_{y,j}$ = 0 for all isolator types. See Step B2.2 where the value of $d_{y,j}$ is revisited for the Multimode Spectral Analysis Method.		
B1.11 Damping Factor Calculate the damping factor, B_L, and the displacement, d, of the bridge: Eq. 7.1-3 GSID $$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases} \quad \text{(B-16)}$$ Eq. 7.1-4 GSID $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \quad \text{(B-17)}$$		**B1.11 Damping Factor, Example 2.4** Since $\xi = 0.30 \geq 0.3$ $$B_L = 1.70$$ and $$d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.2)1.57}{1.70} = 1.81\ in$$
B1.12 Convergence Check Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement. This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5). After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both. Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)		**B1.12 Convergence Check, Example 2.4** Since the calculated value for displacement, d (=1.81) is not close to that assumed at the beginning of the cycle (Step B1.1, d = 2.0), use the value of 1.81 as the new assumed displacement and repeat from Step B1.3. After three iterations, convergence is reached at a superstructure displacement of 1.65 in, with an effective period of 1.43 seconds, and a damping factor of 1.7 (30% damping ratio). The displacement in the isolators at Pier 1 is 1.44 in and the effective stiffness of the same isolators is 42.78 k/in. See spreadsheet in Table B1.12-1 for results of final iteration. Ignoring the weight of the hammerhead, the column shear force must equal the isolator shear force for equilibrium. Hence column shear = 42.78 (1.44) = 61.60 k which is less than the maximum allowable (128 k) if elastic behavior is to be achieved (as required in Step A3). Also the superstructure displacement = 1.65 in, which is less than the available clearance of 2.5 in. Therefore the above solution is acceptable and go to Step B2. Note that available clearance (2.5 in) is greater than minimum required which is given by: $$= \frac{8\ S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.43}{1.7} = 1.35\ in$$

Table B1.12-1 Simplified Method Solution for Design Example 2.4 – Final Iteration

SIMPLIFIED METHOD SOLUTION

Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n
	1651.32	256.26	1907.58	0.2	3

Step B1.1	d	1.65	Assumed displacement
	Q_d	82.57	Characteristic strength
	K_d	50.04	Post-yield stiffness

Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	168.48	8.424	5.105	10,000.00	0.001022	10.206	1.648	10.216	0.002	16.839	13.885	27.785
Pier 1	657.18	32.859	19.915	288.87	0.148088	37.260	1.437	42.778	0.213	61.480	47.224	101.441
Pier 2	657.18	32.859	19.915	288.87	0.148088	37.260	1.437	42.778	0.213	61.480	47.224	101.441
Abut 2	168.48	8.424	5.105	10,000.00	0.001022	10.206	1.648	10.216	0.002	16.839	13.885	27.785
Total	1651.32	82.566	50.040		$\Sigma K_{eff,j}$	94.932				156.638	122.219	258.453
					Step	**B1.4**						

Step B1.10	T_{eff}	1.43	Effective period
	ξ	0.30	Equivalent viscous damping ratio
Step B1.11	B_L (B-15)	1.71	
	B_L	1.70	Damping Factor
	d	1.65	Compare with Step B1.1

Step	**B2.1**	**B2.1**	**B2.3**	**B2.6**	**B2.8**
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$	$d_{isol,i}$	$K_{isol,i}$
Abut 1	2.808	1.702	3.405	1.69	3.363
Pier 1	10.953	6.638	14.259	1.20	15.766
Pier 2	10.953	6.638	14.259	1.20	15.766
Abut 2	2.808	1.702	3.405	1.69	3.363

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength

Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \qquad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \qquad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 2.4

Dividing the results for Q_d and K_d in Step B1.12 (see Table B1.12-1) by the number of isolators at each support (n = 3), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,1}$ = 8.42/3 = 2.81 k
- $Q_{d,2}$ = 32.86/3=10.95 k
- $Q_{d,3}$ = 32.86/3 = 10.95 k
- $Q_{d,4}$ = 8.42/3 = 2.81 k

and

- $K_{d,1}$ = 5.10/3 = 1.70 k/in
- $K_{d,2}$ = 19.92/3 = 6.64 k/in
- $K_{d,3}$ = 19.92/3 = 6.64 k/in
- $K_{d,4}$ = 5.10/3 = 1.70 k/in

Note that the K_d values per support used above are from the final iteration given in Table B1.12-1. These are not the same as the initial values in Step B1.2, because they have been adjusted from cycle to cycle, such that the total K_d summed over all the isolators satisfies the minimum lateral restoring force requirement for the bridge, i.e. K_{dtotal} = 0.05 W/d. See Step B1.1. Since d varies from cycle to cycle, $K_{d,j}$ varies from cycle to cycle.

B2.2 Initial Stiffness and Yield Displacement

Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

(1) For friction-based isolators $K_{u,i} = \infty$ and $d_{y,i} = 0$.
(2) For other types of isolators, and in the absence of isolator-specific information, take

$$K_{u,i} = 10K_{d,i} \qquad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \qquad \text{(B-22)}$$

B2.2 Initial Stiffness and Yield Displacement, Example 2.4

Since the isolator type has been specified in Step A1 to be an elastomeric bearing (i.e. not a friction-based bearing), calculate $K_{u,i}$ and $d_{y,i}$ for an isolator on Pier 1 as follows:

$$K_{u,i} = 10K_{d,i} = 10(6.64) = 66.4\ k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{10.95}{(66.4 - 6.64)} = 0.18\ in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2

in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i}$ = 0.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$

Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

$$k_{isol,i} = \frac{K_{isol,j}}{n} \qquad \text{(B -23)}$$

B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 2.4

Dividing the results for K_{isol} (Step B1.12) among the 3 isolators at each support, the following values for K_{isol} /isolator are obtained:

- $K_{isol,1}$ = 10.22/3 = 3.41 k/in
- $K_{isol,2}$ = 42.78/3 = 14.26 k/in
- $K_{isol,3}$ = 42.78/3 = 14.26 k/in
- $K_{isol,4}$ = 10.22/3 = 3.41 k/in

B2.4 Three-Dimensional Bridge Model

Using computer-based structural analysis software, create a 3-dimensional model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step. For bridges with regular geometry and minimal skew or curvature, the superstructure may be represented by a single 'stick' provided the load path to each individual isolator at each support is explicitly modeled, usually by a rigid cap beam and a set of rigid links. If the geometry is irregular, or if the bridge is skewed or curved, a finite element model is recommended to accurately capture the load carried by each individual isolator. If the piers have an unusual weight distribution, such as a pier with a hammerhead cap beam, a more rigorous model is recommended.

B2.4 Three-Dimensional Bridge Model, Example 2.4

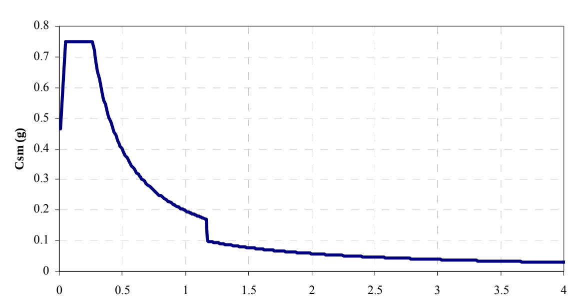
Although the bridge in this Design Example is regular and is without skew or curvature, a 3-dimensional finite element model was developed for this Step, as shown below.

B2.5 Composite Design Response Spectrum

Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.

B.2.5 Composite Design Response Spectrum, Example 2.4

From the final results of Simplified Method (Step B1.12), B_L = 1.70 and T_{eff} = 1.43 sec. Hence the transition in the composite spectrum from 5% to 30% damping occurs at 0.8 T_{eff} = 0.8 (1.43) = 1.14 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods ≥ 1.14 sec by 1.70.



B2.6 Multimodal Analysis of Finite Element Model

Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

B2.6 Multimodal Analysis of Finite Element Model, Example 2.4

Results of modal analysis of the example bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first three modes are the principal transverse, longitudinal, and torsion modes with periods of 1.60, 1.46 and 1.39 sec respectively. The period of the longitudinal mode (1.46 sec) is very close to that calculated in the Simplified Method. The mass participation factors indicate there is no coupling between these three modes (probably due to the symmetric nature of the bridge) and the high values for the first and second modes (92% and 94% respectively) indicate the bridge is responding essentially in a single mode of vibration in each

Mode No	Period Sec	Mass Participation Ratios UX	UY	UZ	RX	RY	RZ
1	1.604	0.000	0.919	0.000	0.952	0.000	0.697
2	1.463	0.941	0.000	0.000	0.000	0.020	0.000
3	1.394	0.000	0.000	0.000	0.000	0.000	0.231
4	0.479	0.000	0.003	0.000	0.013	0.000	0.002
5	0.372	0.000	0.000	0.076	0.000	0.057	0.000
6	0.346	0.000	0.000	0.000	0.000	0.000	0.000
7	0.345	0.000	0.001	0.000	0.010	0.000	0.000
8	0.279	0.000	0.003	0.000	0.013	0.000	0.002
9	0.268	0.000	0.000	0.000	0.000	0.000	0.000
10	0.267	0.058	0.000	0.000	0.000	0.000	0.000
11	0.208	0.000	0.000	0.000	0.000	0.129	0.000
12	0.188	0.000	0.000	0.000	0.000	0.000	0.001

direction. Similar results to that obtained by the Simplified Method are therefore expected.

Table B2.6-1 Modal Properties of Bridge Example 2.4 – First Iteration

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 1.69 (1.65) in
- $d_{isol,2}$ = 1.20 (1.44) in
- $d_{isol,3}$ = 1.20 (1.44) in
- $d_{isol,4}$ = 1.69 (1.65) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (*d*) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 2.4

The results for isolator displacements are close but not close enough (15% difference at the piers)

Go to Step B2.8 and update properties for a second cycle of iteration.

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows: $$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \qquad \text{(B-24)}$$ Recalculate $K_{eff,j}$: Eq. 7.1-6 GSID $$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \qquad \text{(B-25)}$$ Recalculate system damping ratio, ξ : Eq. 7.1-10 GSID $$\xi = \frac{2 \sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \qquad \text{(B-26)}$$ Recalculate system damping factor, B_L: Eq. 7.1-3 GSID $$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \qquad \text{(B-27)}$$ Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.	**B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 2.4** Updated values for $K_{isol,i}$ are given below (previous values are in parentheses): o $K_{isol,1}$ = 3.36 (3.41) k/in o $K_{isol,2}$ = 15.77 (14.26) k/in o $K_{isol,3}$ = 15.77 (14.26) k/in o $K_{isol,4}$ = 3.36 (3.41) k/in Since the isolator displacements are relatively close to previous results no significant change in the damping ratio is expected. Hence $K_{eff,j}$ and ξ are not recalculated and B_L is taken at 1.70. Since the change in effective period is very small (1.43 to 1.46 sec) and no change has been made to B_L, there is no need to construct a new composite response spectrum in this case. Go back to Step B2.6 (see immediately below).
	B2.6 Multimodal Analysis Second Iteration, Example 2.4 Reanalysis gives the following values for the isolator displacements (numbers in parentheses are those from the previous cycle): o $d_{isol,1}$ = 1.66 (1.69) in o $d_{isol,2}$ = 1.15 (1.20) in o $d_{isol,3}$ = 1.15 (1.20) in o $d_{isol,4}$ = 1.66 (1.69) in Go to Step B2.7
B2.7 Convergence Check Compare results and determine if convergence has been reached. If so go to Step B2.9. Otherwise Go to Step B2.8.	**B2.7 Convergence Check, Example 2.4** Satisfactory agreement has been reached on this second cycle. Go to Step B2.9
B2.9 Superstructure and Isolator Displacements Once convergence has been reached, obtain o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for	**B2.9 Superstructure and Isolator Displacements, Example 2.4** From the above analysis: o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are: x_L= 1.69 in

each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.

y_L= 0.0 in

- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are:
  - Abutments: u_L = 1.66 in, v_L = 0.00 in
  - Piers: u_L = 1.15 in, v_L = 0.00 in

  All isolators at same support have the same displacements.

B2.10 Pier Bending Moments and Shear Forces
Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.

B2.10 Pier Bending Moments and Shear Forces, Example 2.4
Bending moments in single column pier in the longitudinal (M_{PLL}) and transverse (M_{PTL}) directions are:
M_{PLL}= 0
M_{PTL}= 1602 kft

Shear forces in single column pier the longitudinal (V_{PLL}) and transverse (V_{PTL}) directions are
V_{PLL}=67.16 k
V_{PTL}=0

B2.11 Isolator Shear and Axial Forces
Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 2.4
Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

Sub-structure	Isol-ator	V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abutment	1	5.63	0	1.29
	2	5.63	0	1.30
	3	5.63	0	1.29
Pier	1	18.19	0	0.77
	2	18.25	0	1.11
	3	18.19	0	0.77

The difference between the longitudinal shear force in the column (V_{PLL} = 67.16k) and the sum of the isolator shear forces at the same Pier (54.63 k) is about 12.5 k. This is due to the inertia force developed in the hammerhead cap beam which weighs about 128 k and can generate significant additional demand on the column (about a 23% increase in this case).

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 2.4

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) are:

- T_{eff} = 1.52 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions are as follows: x_T = 0 and y_T = 1.75 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions as follows:
  Abutments u_T = 0.00 in, v_T = 1.75 in
  Piers u_T = 0.00 in, v_T = 0.71 in
- Pier bending moments in the longitudinal (M_{PLT}) and transverse (M_{PTT}) directions are as follows: M_{PLT} = 1548.33 kft and M_{PTT} = 0
- Pier shear forces in the longitudinal (V_{PLT}) and transverse (V_{PTT}) directions are as follows: V_{PLT} = 0 and V_{PTT} = 60.75 k
- Isolator shear and axial forces are in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

Sub-structure	Isol-ator	V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abutment	1	0.0	5.82	13.51
	2	0.0	5.83	0
	3	0.0	5.82	13.51
Pier	1	0.0	15.40	26.40
	2	0.0	15.57	0
	3	0.0	15.40	26.40

The difference between the transverse shear force in the column (V_{PLL} = 60.75k) and the sum of the isolator shear forces at the same Pier (46.37 k) is about 14.4 k. This is due to the inertia force developed in the hammerhead cap beam which weighs about 128 k and can generate significant additional demand on the column (about 31%).

STEP D. CALCULATE DESIGN VALUES

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements
Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, for each isolator by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)
- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)
- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements at Pier 1, Example 2.4
To illustrate the process, design displacements for the outside isolator on Pier 1 are calculated below.

Load Case 1:
$u_1 = u_L + 0.3u_T$ = 1.0(1.15) + 0.3(0) = 1.15 in
$v_1 = v_L + 0.3v_T$ = 1.0(0) + 0.3(0.71) = 0.21 in
$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.15^2 + 0.21^2}$ = 1.17 in

Load Case 2:
$u_2 = 0.3u_L + u_T$ = 0.3(1.15) + 1.0(0) = 0.35 in
$v_2 = 0.3v_L + v_T$ = 0.3(0) + 1.0(0.71) = 0.71in
$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{0.35^2 + 0.71^2}$ = 0.79 in

Governing Case:
Total design displacement, $d_t = \max(R_1, R_2)$
= 1.17 in

D2. Design Moments and Shears
Calculate design values for column bending moments and shear forces for all piers using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

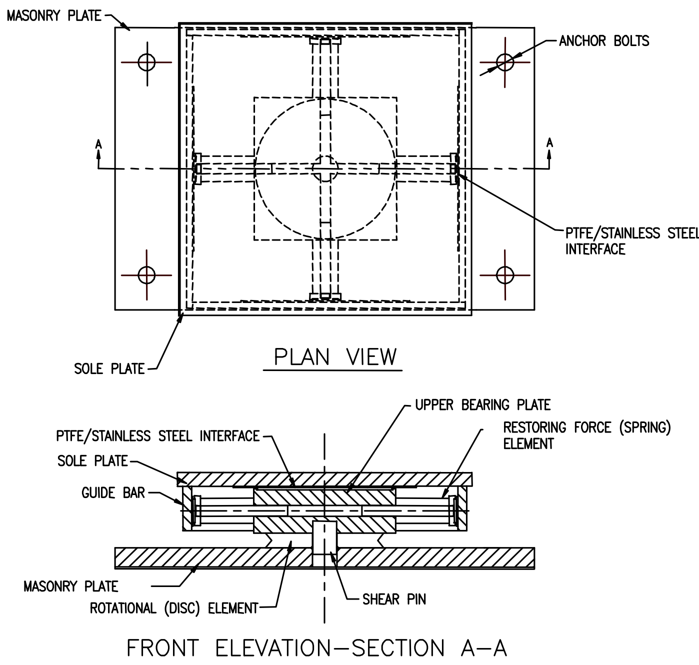
D2. Design Moments and Shears in Pier 1, Example 2.4
Design moments and shear forces are calculated for Pier 1 below, to illustrate the process.

Load Case 1:
$V_{PL1} = V_{PLL} + 0.3V_{PLT}$ = 1.0(67.16) + 0.3(0) = 67.16 k
$V_{PT1} = V_{PTL} + 0.3V_{PTT}$ = 1.0(0) + 0.3(60.75) = 18.23 k
$R_1 = \sqrt{V_{L1}^2 + V_{T1}^2} = \sqrt{67.16^2 + 18.23^2}$ = 69.59 k

Load Case 2:
$V_{PL2} = 0.3V_{PLL} + V_{PLT}$ = 0.3(67.16) + 1.0(0) = 20.15 k
$V_{PT2} = 0.3V_{PTL} + V_{PTT}$ = 0.3(0) + 1.0(60.75) = 60.75 k
$R_2 = \sqrt{V_{L2}^2 + V_{T2}^2} = \sqrt{20.15^2 + 60.75^2}$ = 64.00 k

Governing Case:
Design column shear = max ($R1$, $R2$)
= 69.59 k

STEP E. DESIGN OF ERADIQUAKE ISOLATORS



An EradiQuake Isolator (EQS) is a sliding isolation bearing composed of a multi-directional sliding disc bearing and lateral springs. Each spring assembly consists of a cylindrical polyurethane spring and a spring piston. The piston keeps the spring straight as the isolator moves in different directions. The disc bearing and springs are housed in a mirror-finished stainless steel lined box.

The required values for Q_d and K_d determine the coefficient of friction at the sliding interface and the properties of the springs.

The sliding interface is typically comprised of stainless steel and PTFE. Energy is dissipated during sliding while the springs provide a restoring force. PTFE is an attractive material in that at slow sliding speeds it has a low coefficient of friction, which is ideal for accommodating thermal effects, and at higher speeds the friction becomes greater and acts as an effective energy dissipator during seismic events. The polyurethane springs are designed such that they are never in tension. Their basic design and composition is derived from the die-spring industry.

Design and materials conform to the LRFD Specifications. Steel components are designed in accordance with Section 6, while the disc bearing is designed and constructed per Section 14.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

Notation

A	Area	K_d	Stiffness when sliding (Total spring rate)
A_1	Area based on dead load	k_1	Stiffness (spring rate) for one spring
A_2	Area based on total load	L	Length
A_S	Spring area	L_{GB}	Guide bar length
B_{BB}	Bearing block plan dimension	L_S	Spring length
B_{Box}	Guide box plan dimension (out to out dimension of guide bars)	L_{SI}	Installed spring length
		LL	Live Load
B_{BP}	Base plate length	L_1	Spring length based on max long term displacement
B_{PTFE}	PTFE dimension		
B_{SP}	Slide plate (guide box top) length	L_2	Spring length based on max short term displacement
D_D	Disc outer diameter		
D_{PTFE}	PTFE diameter	M	Moment
D_S	Spring outer diameter	M_N	Factored moment
d_L	Service (long term) displacement	P_{DL}	Dead load
d_T	Total seismic displacement	P_{LL}	Live load
E	Elastic modulus	P_{SL}	Seismic live load
F	Spring force	P_{WL}	Wind load
F_Y	Yield stress	Q_d	Characteristic strength
H	Isolator height	S_G	Gross shape factor of disc
ID_S	Spring inner diameter	T_{BB}	Bearing block thickness

T_{BP} Base plate thickness T_D Disc thickness T_{GB} Guide bar thickness T_{SP} Slide plate thickness W Isolator weight W Plan width of isolator W_{BP} Base plate length WL Wind load Z Plastic modulus	α Bearing rotation β Inner to outer diameter ratio δ_W Wind displacement ε_C Maximum average compression strain μ Coefficient of friction

E1. Required Properties

Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1).

- required characteristic strength, Q_d, per isolator
- required post-elastic stiffness, K_d, per isolator
- total design displacement, d_t, for each isolator
- maximum applied dead and live load, P_{DL}, P_{LL}, and seismic load, P_{SL}, which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator
- wind load per isolator, P_{WL}, and
- thermal displacement of the superstructure at each isolator, d_L.

E1. Required Properties, Example 2.4

The design of one of the exterior isolators on a pier is given below to illustrate the design process for an EQS isolator.

From previous work:

- Q_d/isolator = 10.95 k
- K_d/isolator = 6.76 k/in
- Total design displacement, d_t = 1.17 in
- P_{DL} = 187.0 k
- P_{LL} = 123.0k
- P_{SL} = 26.4 k

Calculated for this design:

- P_{WL} = 8.21 k
- d_L = +/- 0.53 in

E2. Isolator Sizing

E2.1 Size the Disc

Estimate the disc outer diameter based on an average compressive stress of 4.5 ksi using the gross plan area.

$$D_D = \sqrt{\frac{4(P_{DL} + P_{LL})}{\pi\,(4.5)}} = 0.53\sqrt{(P_{DL} + P_{LL})} \qquad \text{(E-1)}$$

Estimate disc thickness based on a gross shape factor of 2.4:

$$S_G = \frac{D_D}{4T_D} \qquad \text{(E-2)}$$

Check rotational capacity and adjust disc thickness if required. Use standard disc design rotation, α, of 0.02 radians, and a maximum compression strain, ε_c, of 0.10 for this calculation.

$$\frac{D_D}{2}\alpha \le \varepsilon_c T_D \qquad \text{(E-3)}$$

E2.1 Size Disc, Example 2.4

$$D_D = 0.53\sqrt{310} = 9.33 \approx 9.50\ in$$

$$T_D = \frac{9.5}{4(2.4)} = 0.99 \approx 1.00\ in$$

$$T_D \ge \frac{\alpha D_D}{2\varepsilon_c} = \frac{0.02(9.50)}{2(0.10)} = 0.95\ in\ OK$$

E2.2 Size the Springs		**E2.2 Size the Springs, Example 2.4**
E2.2.1 Calculate Installed Spring Length Assume 60% max compressive strain on the MER spring for short term loading, 40% max compressive strain for long term loading. Add 20% of long term loading strain for elastomer compression set. Then $$L_1 = 2.5\, d_L \qquad \text{(E-4)}$$ And using a load combination of two times the total seismic design displacement (for Seismic Zone 2) plus 50% service (thermal): $$L_2 = \frac{2\, d_t + 0.5 d_L}{0.60} \qquad \text{(E-5)}$$ Then required spring length is given by: $$L_S = \max(L_1, L_2) + 0.2 d_L \qquad \text{(E-6)}$$		**E2.2 Calculate Installed Spring Length, Example 2.4** $$L_1 = 2.5(0.53) = 1.33\ in$$ $$L_2 = \frac{2(1.17) + 0.5(0.53)}{0.60} = 4.34\ in$$ $$L_S = 4.34 + 0.2(0.53) = 4.45 \approx 5.0\ in$$
E2.2.2 Check Wind Displacements Calculate displacement due to wind as follows: $$\delta_W = \frac{P_{WL} - 0.25 Q_d}{K_d} \qquad \text{(E-7)}$$ If the displacement due to wind is too large, add spring precompression equal to the wind displacement to the spring length in the transverse direction. Precompression doubles the stiffness over the precompressed displacement. If the wind displacements are still too large, consider increasing the spring stiffness in the transverse direction, or using sacrificial shear keys.		**E2.2.2 Check Wind Displacements, Example 2.4** $$\delta_W = \frac{8.2 - 0.25(11.0)}{6.76} = 0.81\ in$$ In this example, the wind displacement is acceptable and no adjustment of spring length is required.
E2.2.3 Calculate Spring Diameter Assume only one spring per side is used to meet spring rate requirements, i.e. let $k_1 = K_d$, and take the elastic modulus for polyurethane spring to be 6.0 ksi. Since $$k_1 = \frac{E A_S}{L_S} = \frac{6.0 A_S}{L_S} \qquad \text{(E-8)}$$ it follows that $$A_S = \frac{k_1 L_S}{6.0} \qquad \text{(E-9)}$$ and $$D_S = \sqrt{\frac{4 A_S}{\pi(1-\beta^2)}} \qquad \text{(E-10)}$$		**E2.2.3 Calculate Spring Diameter, Example 2.4** Since K_d = 6.76 k/in $$A_S = \frac{6.76(5.00)}{6.0} = 5.63\ in^2$$ Take initial value for β = 0.20 and then $$D_S = \sqrt{\frac{4(5.63)}{\pi(1-0.2^2)}} = 2.73 \approx 2.75\ in$$

E2.2.4 Adjust Spring Length Using Nominal Diameters For manufacturing purposes it is advantageous to use standard diameters and adjust the spring length according to the actual value of β to fine tune the stiffness (spring rate). $$\beta = \frac{ID_s}{D_S} \quad \text{(E-11)}$$ $$A_S = \frac{\pi}{4} D_S^2 (1-\beta^2) \quad \text{(E-12)}$$ $$L_S = \frac{EA_S}{k_1} \quad \text{(E-13)}$$ Check that L_s is greater than either L_1 and L_2 Note that L_S is the installed spring length. The actual size of the springs may be slightly different than the installed size. Springs are pre-compressed to provide additional wind resistance if needed and account for compression set in the elastomer.	**E2.2.4 Adjust Spring Length Using Nominal Diameters, Example 2.4** Use 2-3/4 in for the spring OD, and 0.50 in for the spring ID then $$\beta = \frac{0.50}{2.75} = 0.18$$ $$A_S = \frac{\pi}{4} 2.75^2 (1-0.18^2) = 5.75\ in^2$$ $$L_S = \frac{6.0(5.75)}{6.76} = 5.10\ in$$ $$L_S \geq \max(L_1, L_2)\ OK$$
E2.3 Size the PTFE Pad	**E2.3 Size the PTFE Pad, Example 2.4**
E2.3.1 Calculate Coefficient of Friction Calculate the required coefficient of friction from: $$\mu = \frac{Q_d}{W} \quad \text{(E-14)}$$ Select PTFE and polished stainless steel as the sliding surfaces. Low coefficients of friction are possible with these materials at high contact stresses. In general the friction coefficient decreases with increasing pressure.	**E2.3.1 Calculate Required Coefficient of Friction, Example 2.4** $$\mu = \frac{11.0}{187} = 0.059$$ A value of 0.059 is at the low end of the spectrum for virgin PTFE/stainless steel materials and will require use of the highest contact stresses allowed in the GSID and LRFD Specifications to achieve this value; i.e. 3.5 ksi under dead load and 4.5 ksi under (dead + live) load.
E2.3.2 Calculate Required Area of PTFE Calculate required area of PTFE using allowable contact stresses in GSID Table 16.4.1-1. For service loads (i.e. dead load) allowable average stress is 3.5 ksi and then: $$A_1 = \frac{P_{DL}}{3.5} \quad \text{(E-15)}$$ Check area required under dead plus live load using an allowable average stress of 4.5 ksi (as permitted in LRFD Sec 14.) $$A_2 = \frac{(P_{DL} + P_{LL})}{4.5} \quad \text{(E-16)}$$ Then required area is $$A = \max(A_1, A_2) \quad \text{(E-17)}$$ Since the structure design rotation of 0.01 radians is	**E2.3.2 Calculate Required Area of PTFE, Example 2.4** $$A_1 = \frac{187}{3.5} = 53.4\ in^2$$ $$A_2 = \frac{(187+123)}{4.5} = 68.9\ in^2$$ $$A = \max(A_1, A_2) = 68.9\ in^2$$

only one-half of the disc design rotation, the limits on the PTFE edge contact stresses (GSID Table 16.4.1-1) do not govern.		
E2.3.3 Calculate Size of PTFE Pad For a circular PTFE pad, the diameter is given by: $$D_{PTFE} = \sqrt{\frac{4}{\pi}A} \quad \text{(E-18)}$$		**E2.3.3 Calculate Size of PTFE Pad, Example 2.4** $$D_{PTFE} = \sqrt{\frac{4}{\pi}68.9} = 9.36 \approx 9.375\ in$$
E2.4 Size the Bearing Block		**E2.4 Size the Bearing Block, Example 2.4**
E2.4.1 Calculate Bearing Plan Dimension Two criteria must be checked to determine the bearing block plan dimension. The disc must fit under the block with some clearance, and the PTFE must fit on top of the block with at least 1/8 in edge clearance. $$B_{BB1} = 1.15D_D \quad \text{(E-19)}$$ $$B_{BB2} = D_{PTFE} + 2(0.125) \quad \text{(E-20)}$$ $$B_{BB} = \max\,(B_{BB1}, B_{BB2}) \quad \text{(E-21)}$$		**E2.4.1 Calculate Bearing Plan Dimension, Example 2.4** $$B_{BB1} = 1.15(9.50) = 10.9\ in$$ $$B_{BB2} = 9.375 + 2(0.125) = 9.625\ in$$ $$B_{BB} = \max(B_{BB1}, B_{BB2}) = 10.9 \approx 11.00\ in$$
E2.4.2 Calculate Bearing Block Thickness The thickness of bearing block must be sufficient to ensure that the springs can be attached on each side of the block, allowing for a 30% increase in diameter upon spring compression. $$T_{BB} = 1.3D_S \quad \text{(E-22)}$$ Note that if T_{BB} is too large, reduce the diameter of the springs and increase their number.		**E2.4.2 Calculate Bearing Block Thickness, Example 2.4** $$T_{BB} = 1.3(2.75) = 3.58 \approx 3.50\ in$$ Size is ok. No need to resize spring diameters.

<table>
<tr><td>E2.5 Size the Box</td><td></td><td>E2.5 Size the Box, Example 2.4</td></tr>
<tr><td>E2.5 .1 Calculate Guide Bar Thickness
(a) <u>Guide Bar Force</u>
Guide bars resist the spring forces. They are modeled as cantilever beams, with the fixed end of the cantilever located where the guide bar meets the slide plate. Assume the resisting length of guide bar to be three times the diameter of the spring. The moment arm is one-half of the bearing block thickness, plus 0.20 in. Forces corresponding to two times the seismic displacement, imposed during prototype testing, are used to design the guide bar.

$$F = k_1(2d_t + 0.5d_L) \quad \text{(E-23)}$$
(b) <u>Guide Bar Moment</u>
$$M = (0.5T_{BB} + 0.20)F \quad \text{(E-24)}$$
Since the effective length of guide bar resisting this moment is assumed to be $3D_s$, the bending moment per inch of guide bar is:
$$M_1 = \frac{M}{3D_S} \quad \text{(E-25)}$$
(c) <u>Guide Bar Thickness</u>
Using a load factor of 1.75, a resistance factor of 1.00, and assuming 50 ksi steel:

$$M_N = 1.75M_1 = 1.00F_YZ \quad \text{(E-26)}$$
then
$$Z = \frac{1.75M_1}{F_Y} \quad \text{(E-27)}$$
But since
$$Z = \frac{T_{GB}^2}{4} \quad \text{(E-28)}$$
then
$$T_{GB} = \sqrt{4Z} \quad \text{(E-29)}$$</td><td></td><td>E2.5.1 Calculate Guide Bar Thickness, Example 2.4

$$F = 6.76\big(2(1.17) + 0.5(0.53)\big) = 17.6\ k$$
$$M = (0.5(3.50) + 0.20)17.6 = 34.33\ kin$$
$$M_1 = \frac{34.33}{3(2.75)} = 4.16\ kin/in$$
$$Z = \frac{1.75(4.16)}{50} = 0.146\ in^3/in$$
$$T_{GB} = \sqrt{4(0.146)} = 0.76 \approx 0.75\ in$$</td></tr>
<tr><td>E2.5.2 Calculate Guide Bar Length

$$L_{GB} = T_{GB} + B_{BB} + 2L_s \quad \text{(E-30)}$$</td><td></td><td>E2.5.2 Calculate Guide Bar Length, Example 2.4

$$L_{GB} = 0.75 + 11.00 + 2(5.10) = 21.95 \approx 22.0in$$</td></tr>
<tr><td>E2.5.3 Calculate Guide Bar Width

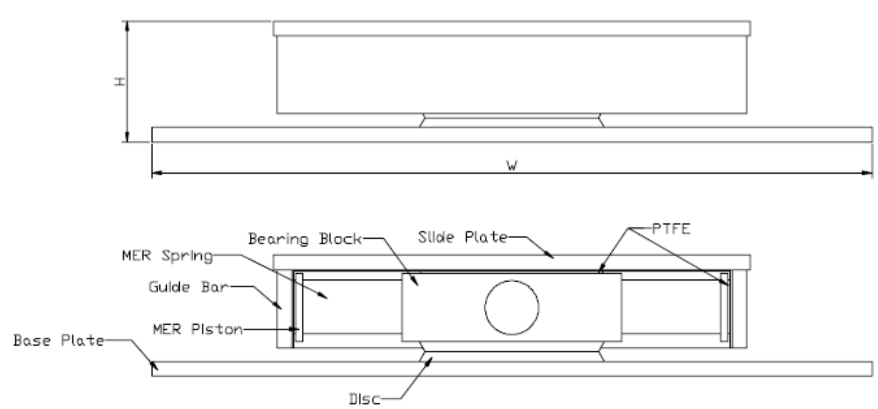
$$W_{GB} = T_{BB} + 0.5T_D \quad \text{(E-31)}$$</td><td></td><td>E2.5.3 Calculate Guide Bar Width, Example 2.4

$$W_{GB} = 3.50 + 0.5(1.00) = 4.0\ in$$</td></tr>
<tr><td>E2.5.4 Calculate Size of Box and Slide Plate
Calculate side dimension of box
$$B_{Box} = L_{GB} + T_{GB} \quad \text{(E-32)}$$
Choose plan dimension of slide plate equal to, or slightly larger than, the box, i.e.</td><td></td><td>E2.5.4 Calculate Size of Box and Slide Plate, Example 2.4

$$B_{Box} = 22.0 + 0.75 = 22.75\ in$$

Take</td></tr>
</table>

$$B_{SP} \geq B_{Box} \qquad \text{(E-33)}$$	$$B_{SP} = 23.0\ in$$
E2.5.5 Calculate Box Top (Slide Plate) Thickness Make the slide plate (guide box top) the same thickness as the guide bars, with a minimum value of ¾ in.	**E2.5.5 Calculate Box Top (Slide Plate) Thickness, Example 2.4** $$T_{SP} = 0.75\ in$$
E2.6 Size the Lower Plate (a) <u>Thickness</u> Use ¾ inch minimum thickness unless otherwise required by State DOT specifications. (b) <u>Width</u> Since GSID provisions for prototype testing require the isolator to be displaced to twice the design displacement (for Seismic Zone 2), the base plate must be wide enough to allow such movement without interference from the anchor bolts. $$W_{BP} = B_{Box} + 4d_t + 8 \qquad \text{(E-34)}$$ (c) <u>Length</u> Take $$B_{BP} = B_{SP} \qquad \text{(E-35)}$$ (d) <u>Anchor Bolts</u> Design anchor bolts per LRFD specifications, increase W_{BP} if necessary.	**E2.6 Size the Lower Plate, Example 2.4** $$T_{LP} = 0.75\ in$$ $$W_{BP} = 23.0 + 4(1.17) + 8 = 35.68 \approx 36.00\ in$$ $$B_{BP} = 23.00\ in$$
E3. Design Summary Overall dimensions of isolator are: Width = W_{BP} Length = B_{SP} Height is given by: $$H = T_{LP} + T_D + T_{BB} + T_{SP} + 0.20 \qquad \text{(E-36)}$$	**E3 Design Summary, Example 1.4** Width = W_{BP} = 36.00 in Length = B_{SP} = 23.00 in Height is given by: $$H = 0.75 + 1.00 + 3.50 + 0.75 + 0.20 = 6.20\ in$$



E4. Minimum and Maximum Performance Check
Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E4-1.

Table E4-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-37)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-38)
Eq. 8.1.2-3 GSID	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-39)
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-40)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E4-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E4-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})(\lambda_{min,a,Kd})(\lambda_{min,v,Kd})(\lambda_{min,tr,Kd})(\lambda_{min,c,Kd})(\lambda_{min,scrag,Kd})$	(E-41)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})(\lambda_{max,a,Kd})(\lambda_{max,v,Kd})(\lambda_{max,tr,Kd})(\lambda_{max,c,Kd})(\lambda_{max,scrag,Kd})$	(E-42)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})(\lambda_{min,a,Qd})(\lambda_{min,v,Qd})(\lambda_{min,tr,Qd})(\lambda_{min,c,Qd})(\lambda_{min,scrag,Qd})$	(E-43)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})(\lambda_{max,a,Qd})(\lambda_{max,v,Qd})(\lambda_{max,tr,Qd})(\lambda_{max,c,Qd})(\lambda_{max,scrag,Qd})$	(E-44)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as

E4. Minimum and Maximum Performance Check, Example 2.4
For Eradiquake isolators, Modification Factors are applied to both Q_d and K_d , because both frictional and elastomeric (urethane) elements are used in these isolators.

Minimum Property Modification factors are:
$\lambda_{min,Kd} = 1.0$
$\lambda_{min,Qd} = 1.0$

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are (GSID Appendix A):
$\lambda_{max,a,Kd} = 1.0$
$\lambda_{max,a,Qd} = 1.2$

$\lambda_{max,t,Kd} = 1.3$
$\lambda_{max,t,Qd} = 1.5$

$\lambda_{max,tr,Kd} = 1.0$
$\lambda_{max,tr,Qd} = 1.0$

$\lambda_{max,c,Kd} = 1.0$
$\lambda_{max,c,Qd} = 1.1$

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd} = 1.0 + 0.0(0.66) = 1.00$
$\lambda_{max,a,Qd} = 1.0 + 0.2(0.66) = 1.13$

follows:
1.00 for critical bridges
0.75 for essential bridges
0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

$\lambda_{max,t,Kd}$ = 1.0 + 0.3(0.66) = 1.20
$\lambda_{max,t,Qd}$ = 1.0 + 0.5(0.66) = 1.33

$\lambda_{max,tr,Kd}$ = 1.0 + 0.0(0.66) = 1.00
$\lambda_{max,tr,Qd}$ = 1.0 + 0.0(0.66) =1.00

$\lambda_{max,c,Kd}$ = 1.0 + 0.0(0.66) = 1.00
$\lambda_{max,c,Qd}$ = 1.0 + 0.0(0.66) =1.00

Therefore the maximum overall modification factors

$\lambda_{max,Kd}$ = (1.00)(1.20)(1.00)(1.00) = 1.20
$\lambda_{max,Qd}$ = (1.13)(1.33)(1.00)(1.00) = 1.50

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:
$Q_{d,max}$ = 1.50(11.0) = 16.5 k
and
$K_{d,max}$=1.20(6.76) = 8.11 k/in

E5. Design and Performance Summary

E5. Design and Performance Summary, Example 2.4

E5.1 Isolator dimensions
Summarize final dimensions of isolators:
- Overall size of lower plate
- Overall size of box (top plate)
- Overall height
- Size of disc
- Size of PTFE pad
- Number of polyurethane springs
- Diameter of polyurethane springs

E5.1 Isolator dimensions, Example 2.4
Isolator dimensions are summarized in Table E5.1-1.

Table E5.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Diam. Disc (in)
Under edge girder on Pier 1	36.0 x 23.0 x 6.20(H)	23.0 x 23.0 x 6.20 (H)	9.50

Isolator Location	Diam. PTFE pad (in)	No. poly-urethane springs	Diam. poly-urethane springs (in)
Under edge girder on Pier 1	9.375	4	2.75

E5.2 Bridge Performance
Summarize bridge performance
- Maximum superstructure displacement (longitudinal)
- Maximum superstructure displacement (transverse)
- Maximum superstructure displacement

E5.2 Bridge Performance, Example 2.4
Bridge performance is summarized in Table E5.2-1 where it is seen that the maximum column shear is 71.74k. This less than the column plastic shear (128k) and therefore the required performance criterion is satisfied (fully elastic behavior). Furthermore the maximum longitudinal displacement is 1.69 in which

(resultant)

- Maximum column shear (resultant)
- Maximum column moment (about transverse axis)
- Maximum column moment (about longitudinal axis)
- Maximum column torque

Check required performance as determined in Step A3, is satisfied.

is less than the 2.5in available at the abutment expansion joints and is therefore acceptable.

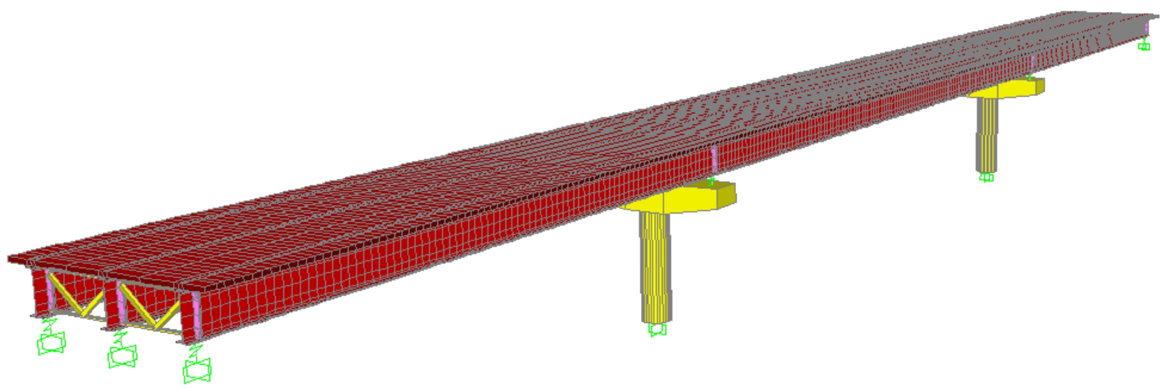
Table E5.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	1.69 in
Maximum superstructure displacement (transverse)	1.75 in
Maximum superstructure displacement (resultant)	1.82 in
Maximum column shear (resultant)	71.74 k
Maximum column moment about transverse axis	1,657 kft
Maximum column moment about longitudinal axis	1,676 kft
Maximum column torque	21.44 kft

SECTION 2

Steel Plate Girder Bridge, Long Spans, Single-Column Pier

DESIGN EXAMPLE 2.5: H_1=0.5H_2



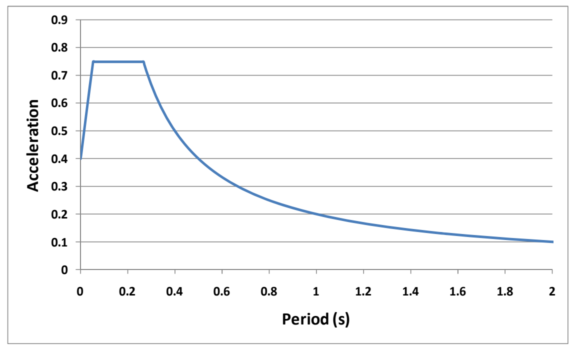
Design Examples in Section 2

ID	Description	S_1	Site Class	Column height	Skew	Isolator type
2.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
2.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
2.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
2.3	Change isolator to FPS	0.2g	B	Same	0	Spherical friction bearing
2.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
2.5	Change column height	0.2g	B	H_1=0.5H_2	0	Lead rubber bearing
2.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

DESIGN PROCEDURE	DESIGN EXAMPLE 2.5 (H_1= 0.5H_2)
STEP A: BRIDGE AND SITE DATA	

A1. Bridge Properties
Determine properties of the bridge:
• number of supports, m
• number of girders per support, n
• weight of superstructure including railings, curbs, barriers and other permanent loads, W_{SS}
• weight of piers participating with superstructure in dynamic response, W_{PP}
• weight of superstructure, W_j, at each support
• stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
• column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
• allowable movement at expansion joints
• isolator type if known, otherwise 'to be selected'
	A1. Bridge Properties, Example 2.5
	• Number of supports, m = 4
	  o North Abutment (m = 1)
	  o Pier 1 (m = 2)
	  o Pier 2 (m = 3)
	  o South Abutment (m =4)
	• Number of girders per support, n = 3
	• Number of columns per support = 1
	• Weight of superstructure including permanent loads, W_{SS} = 1651.32 k
	• Weight of superstructure at each support:
	  o W_1 = 168.48 k
	  o W_2 = 657.18 k
	  o W_3 = 657.18 k
	  o W_4 = 168.48 k
	• Participating weight of piers, W_{PP} = 256.26 k
	• Effective weight (for calculation of period), $W_{eff} = Wss + W_{PP}$ = 1907.58 k
	• Stiffness of each pier in the both directions:
	  o $K_{sub,pier1}$ = 288.87 k/in
	  o $K_{sub,pier2}$ = 36.58 k/in
	• Minimum column shear strength based on flexural yield capacity of column = 128 k
	• Displacement capacity of expansion joints (longitudinal) = 2.5 in (required to accommodate thermal expansion and other movements)
	• Lead rubber isolators

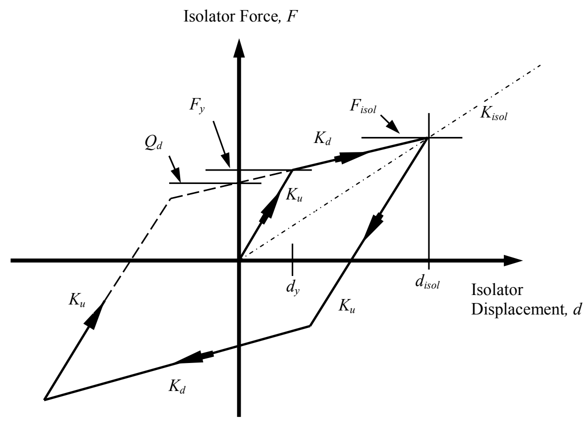
A2. Seismic Hazard Determine seismic hazard at site: • acceleration coefficients • site class and site factors • seismic zone Plot response spectrum. Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of PGA, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD). Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications, i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3,	**A2. Seismic Hazard, Example 2.5** Acceleration coefficients for bridge site are given in design statement as follows: • PGA = 0.40 • S_1 = 0.20 • S_S = 0.75 Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec. Table 3.10.3.1-1 LRFD gives Site Class as B. Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors: • F_{pga} = 1.0 • F_a = 1.0 • F_v = 1.0

LRFD. Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows: • $A_s = F_{pga}\ PGA$ • $S_{DS} = F_a\ S_S$ • $S_{D1} = F_v\ S_1$ Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table 5-1 GSID. These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.		• $A_s = F_{pga}\ PGA = 1.0(0.40) = 0.40$ • $S_{DS} = F_a\ S_S = 1.0(0.75) = 0.75$ • $S_{D1} = F_v\ S_1 = 1.0(0.20) = 0.20$ Since $0.15 < S_{D1} \leq 0.30$, bridge is located in Seismic Zone 2. Design Response Spectrum is as below: 
A3. Performance Requirements Determine required performance of isolated bridge during Design Earthquake (1000-yr return period). Examples of performance that might be specified by the Owner include: • Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake. • Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake. • For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements) • Reduced substructure forces for bridges on weak soils to reduce foundation costs.		**A3. Performance Requirements, Example 2.5** In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield). To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (128 k). The maximum shear in the column must therefore be less than 128 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties

To begin the iterative solution, an estimate is required of :

(1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)

Art C7.1 GSID

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \qquad \text{(B-1)}$$

(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not

B1.1 Initial System Displacement and Properties, Example 2.5

$$d \cong 10 S_{D1} = 10(0.20) \cong 2.0\ in$$

occur under non-seismic loads (e.g. wind) but low enough that yield does occur under earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e. $$Q_d = 0.05W \quad \text{(B-2)}$$ (3) Post-yield stiffness, K_d Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3. Art. 12.2 GSID $$K_{d,min} \geq \frac{0.025W}{d} \quad \text{(B-3)}$$ Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$		$$Q_d = 0.05W = 0.05(1651.32) = 82.56\ k$$ $$K_d = 0.05\frac{W}{d} = 0.05\frac{1651.32}{2.0} = 41.28\ k/in$$
B1.2 Initial Isolator Properties at Supports Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support: $$Q_{d,j} = Q_d\frac{W_j}{W} \quad \text{(B-4)}$$ and $$K_{d,j} = K_d\frac{W_j}{W} \quad \text{(B-5)}$$		**B1.2 Initial Isolator Properties at Supports, Example 2.5** $$Q_{d,j} = Q_d\frac{W_j}{W}$$ o $Q_{d,1}$ = 8.42 k o $Q_{d,2}$ = 32.86 k o $Q_{d,3}$ = 32.86 k o $Q_{d,4}$ = 8.42 k and $$K_{d,j} = K_d\frac{W_j}{W}$$ o $K_{d,1}$ = 4.21 k/in o $K_{d,2}$ = 16.43 k/in o $K_{d,3}$ = 16.43 k/in o $K_{d,4}$ = 4.21 k/in
B1.3 Effective Stiffness of Combined Pier and Isolator System Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below (after Fig. 7.1-1 GSID). An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula is as follows (MCEER 2006): $$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j} \quad \text{(B-6)}$$ where $$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}} \quad \text{(B-7)}$$ and $K_{sub,j}$ for the piers are given in Step A1. For abutments, take $Ksub,j$ to be a large number, say		**B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 2.5** $$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$ o $\alpha_1 = 8.43\text{x}10^{-4}$ o $\alpha_2 = 1.21\text{x}10^{-1}$ o $\alpha_3 = 1.63$ o $\alpha_4 = 8.43\text{x}10^{-4}$ $$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$$

10,000 k/in , unless actual stiffness values are available. Note that if the default option is chosen, unrealistically high values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.		o $K_{eff,1}$ = 8.42 k/in o $K_{eff,2}$ = 31.09 k/in o $K_{eff,3}$ = 22.67 k/in o $K_{eff,4}$ = 8.42 k/in
B1.4 Total Effective Stiffness Calculate the total effective stiffness, K_{eff}, of the bridge: Eq. 7.1-6 GSID $$K_{eff} = \sum_{j=1}^{m} K_{eff,j} \quad \text{(B-8)}$$		**B1.4 Total Effective Stiffness, Example 2.5** $$K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 70.61\ k/in$$
B1.5 Isolation System Displacement at Each Support Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports: $$d_{isol,j} = \frac{d}{1+\alpha_j} \quad \text{(B-9)}$$		**B1.5 Isolation System Displacement at Each Support, Example 2.5** $$d_{isol,j} = \frac{d}{1+\alpha_j}$$ o $d_{isol,1}$ = 2.00 in o $d_{isol,2}$ = 1.79 in o $d_{isol,3}$ = 0.76 in o $d_{isol,4}$ = 2.00 in
B1.6 Isolation System Stiffness at Each Support Calculate the stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports: $$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \quad \text{(B-10)}$$		**B1.6 Isolation System Stiffness at Each Support, Example 2.5** $$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$ o $K_{isol,1}$ = 8.43 k/in o $K_{isol,2}$ = 34.84 k/in o $K_{isol,3}$ = 59.65 k/in o $K_{isol,4}$ = 8.43 k/in
B1.7 Substructure Displacement at Each Support Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports: $$d_{sub,j} = d - d_{isol,j} \quad \text{(B-11)}$$		**B1.7 Substructure Displacement at Each Support, Example 2.5** $$d_{sub,j} = d - d_{isol,j}$$ o $d_{sub,1}$ = 0.002 in o $d_{sub,2}$ = 0.215 in o $d_{sub,3}$ = 1.240 in o $d_{sub,4}$ = 0.002 in
B1.8 Substructure Shear at Each Support Calculate the shear at support 'j', $F_{sub,j}$, for all supports: $$F_{sub,j} = K_{sub,j} d_{sub,j} \quad \text{(B-12)}$$ where values for $K_{sub,j}$ are given in Step A1.		**B1.8 Lateral Load in Each Substructure, Example 2.5** $$F_{sub,j} = K_{sub,j} d_{sub,j}$$ o $F_{sub,1}$ = 16.84 k o $F_{sub,2}$ = 62.18k o $F_{sub,3}$ = 45.35 k o $F_{sub,4}$ = 16.84 k

B1.9 Column Shear at Each Support Calculate the shear in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j': $F_{col,j,k} = \dfrac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ (B-13) Use these approximate column shears as a check on the validity of the chosen strength and stiffness characteristics.		**B1.9 Column Shear Forces at Each Support, Example 2.5** $F_{col,j,k} = \dfrac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ o $F_{col,2,1}$ = 62.18 k o $F_{col,3,1}$ = 45.35 k These column shears are less than the plastic shear capacity of each column (128k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.
B1.10 Effective Period and Damping Ratio Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge: Eq. 7.1-5 GSID $T_{eff} = 2\pi\sqrt{\dfrac{W_{eff}}{gK_{eff}}}$ (B-14) and Eq. 7.1-10 GSID $\xi = \dfrac{2\sum_j(Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi\sum_j(K_{eff,j}(d_{isol,j} + d_{sub,j})^2}$ (B-15) where $d_{y,j}$ is the yield displacement of the isolator and assumed to be small compared to $d_{isol,j}$ with negligible effect on ξ, i.e., take $d_{y,j} = 0$.		**B1.10 Effective Period and Damping Ratio, Example 2.5** $T_{eff} = 2\pi\sqrt{\dfrac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\dfrac{1907.58}{386.4(70.61)}}$ = 1.66 sec and taking $d_{y,j} = 0$: $\xi = \dfrac{2\sum_j(Q_{d,j}(d_{isol,j} - 0)}{\pi\sum_j(K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.26$
B1.11 Damping Factor Calculate the damping factor, B_L, and the displacement, d, of the bridge: Eq. 7.1-3 GSID $B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases}$ (B-16) Eq. 7.1-4 GSID $d = \dfrac{9.79 S_{D1} T_{eff}}{B_L}$ (B-17)		**B1.11 Damping Factor, Example 2.5** Since $\xi = 0.26 < 0.3$ $B_L = \dfrac{0.26}{0.05}^{0.3} = 1.65$ and $d = \dfrac{9.79 S_{D1} T_{eff}}{B_L} = \dfrac{9.79(0.2)1.66}{1.65} = 1.97\ in$
B1.12 Convergence Check Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement. This iterative process is amenable to solution using a		**B1.12 Convergence Check, Example 2.5** Since the calculated value for displacement, d (=1.97) is close to that assumed at the beginning of the cycle (Step B1.1, d = 2.0), this solution could be taken as final. However to see the effect of iteration, Step B1.3 was repeated using 1.97 as the new assumed displacement.

spreadsheet and usually converges in a few cycles (less than 5). After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both. Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)		After two iterations, convergence is reached at a superstructure displacement of 1.95 in, with an effective period of 1.64 seconds, and a damping factor of 1.65 (26% damping ratio). The displacement in the isolators on Pier 1 (19 ft column) is 1.73 in and the effective stiffness of the same isolators is 35.79 k/in. For Pier 2 (38 ft column), these values are 0.72 in and 62.49 k respectively. See spreadsheet in Table B1.12-1 for results of final iteration. Since the column shear must equal the isolator shear for equilibrium, the column shear in Pier 1 = 35.79 (1.73) = 61.92 k. Likewise the column shear in Pier 2 is 62.49(0.72) = 44.99 k. Both values are less than the maximum allowable (128k) for elastic behavior in the columns as required in Step A3. It is seen that the taller pier attracts less shear because of its greater flexibility. Also the superstructure displacement = 1.95 in, which is less than the available clearance of 2.5 in. Therefore the above solution is acceptable and go to Step B2. Note that available clearance (2.5 in) is greater than minimum required $= \frac{8\, S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.64}{1.65} = 1.59\ in$

Table B1.12-1 Simplified Method Solution for Design Example 2.5– Final Iteration

SIMPLIFIED METHOD SOLUTION												
Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n							
	1651.32	256.26	1907.58	0.2	3							
Step B1.1	d	1.95	Assumed displacement									
	Q_d	82.57	Characteristic strength									
	K_d	42.34	Post-yield stiffness									
Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\, d_{isol,j}$	$K_{eff,j}(d_{isol,j} + d_{sub,j})^2$
Abut 1	168.48	8.424	4.320	10,000.00	0.000864	8.636	1.948	8.644	0.002	16.841	16.413	32.839
Pier 1	657.18	32.859	16.851	288.87	0.123894	31.844	1.735	35.789	0.215	62.096	57.012	121.087
Pier 2	657.18	32.859	16.851	36.58	1.708203	23.073	0.720	62.486	1.230	44.992	23.660	87.735
Abut 2	168.48	8.424	4.320	10,000.00	0.000864	8.636	1.948	8.644	0.002	16.841	16.413	32.839
Total	1651.32	82.566	42.342		$\Sigma K_{eff,j}$	72.189				140.769	113.496	274.500
					Step	**B1.4**						
Step B1.10	T_{eff}	1.64	Effective period									
	ξ	0.26	Equivalent viscous damping ratio									
Step B1.11	B_L (B-15)	1.65										
	B_L	1.65	Damping Factor									
	d	1.95	Compare with Step B1.1									
Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**						
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$						
Abut 1	2.808	1.440	2.881		1.69	3.102						
Pier 1	10.953	5.617	11.930		1.20	14.744						
Pier 2	10.953	5.617	20.829		1.20	14.744						
Abut 2	2.808	1.440	2.881		1.69	3.102						

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength

Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \quad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \quad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 2.5

Dividing the results for Q_d and K_d in Step B1.12 by the number of isolators at each support (n = 3), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,1}$ = 8.42/3 = 2.81 k
- $Q_{d,2}$ = 32.86/3=10.95 k
- $Q_{d,3}$ = 32.86/3 = 10.95 k
- $Q_{d,4}$ = 8.42/3 = 2.81 k

and

- $K_{d,1}$ = 4.32/3 = 1.44 k/in
- $K_{d,2}$ = 16.85/3 = 5.62 k/in
- $K_{d,3}$ = 16.85/3 = 5.62 k/in
- $K_{d,4}$ = 4.32/3 = 1.44 k/in

Note that the K_d values per support used above are from the final iteration given in Table B1.12-1. These are not the same as the initial values in Step B1.2, because they have been adjusted from cycle to cycle, such that the total K_d summed over all the isolators satisfies the minimum lateral restoring force requirement for the bridge, i.e. K_{dtotal} = 0.05 W/d. See Step B1.1. Since d varies from cycle to cycle, $K_{d,j}$ varies from cycle to cycle.

B2.2 Initial Stiffness and Yield Displacement

Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

In the absence of isolator-specific information take

$$K_{u,i} = 10K_{d,i} \quad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \quad \text{(B-22)}$$

B2.2 Initial Stiffness and Yield Displacement, Example 2.5

For an isolator on Pier 1:

$$K_{u,i} = 10K_{d,i} = 10(5.62) = 56.2k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{10.95}{(56.2 - 5.62)} = 0.22in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2 in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i}$ = 0.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$
Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

(B -23)

B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 2.5
Dividing the results for K_{isol} (Step B1.12) among the 3 isolators at each support, the following values for K_{isol} /isolator are obtained:

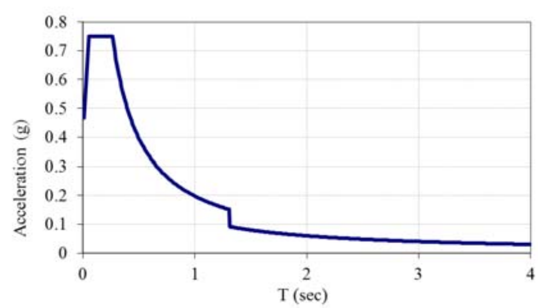
- $K_{isol,1}$ = 8.64/3 = 2.88 k/in
- $K_{isol,2}$ = 35.79/3 = 11.93 k/in
- $K_{isol,3}$ = 62.498/3 = 20.83 k/in
- $K_{isol,4}$ = 8.64/3 = 2.88 k/in

B2.4 Finite Element Model
Using computer-based structural analysis software, create a finite element model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step.

B2.4 Finite Element Model, Example 2.5

B2.5 Composite Design Response Spectrum
Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.

B.2.5 Composite Design Response Spectrum, Example 2.5
From the final results of Simplified Method (Step B1.12), B_L = 1.65 and T_{eff} = 1.64 sec. Hence the transition in the composite spectrum from 5% to 26% damping occurs at 0.8 T_{eff} = 0.8 (1.64) = 1.31 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods ≥ 1.31 sec by 1.65.



B2.6 Multimodal Analysis of Finite Element Model
Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

B2.6 Multimodal Analysis of Finite Element Model, Example 2.5
Results of modal analysis of this bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first three modes are the principal 'isolation' modes with periods of 1.85, 1.68 and 1.52 sec respectively. The mass participation factors in Table B2.6-1 gives the following information about these three modes:

The first mode (1.85 sec) is a strongly coupled mode in the transverse and torsional directions (rotation

		about z-axis) due to a lack of symmetry in the column stiffness in this direction. The second mode is the longitudinal mode and its period (1.68 sec) is very close to that calculated in the Simplified Method (1.64 sec). It is not coupled with a torsional mode because the column stiffness is symmetric in this direction, and a single degree-of-freedom model (as assumed in the Simplified Method) should give good answers. The third mode is also a coupled mode (transverse and torsional) but not as strongly coupled as the first mode. It follows that in the longitudinal direction spectral modal analysis should give similar answers to the Simplified Method and converge quickly to a final solution. But the same might not be true for loading in the transverse direction (See Step C).

Table B2.6-1 Modal Properties of Bridge Example 2.5 – First Iteration

Mode	Period	Mass Participating Ratios					
No	Sec	UX	UY	UZ	RX	RY	RZ
1	1.848	0.000	0.838	0.000	0.863	0.000	0.911
2	1.681	0.950	0.000	0.000	0.000	0.020	0.000
3	1.516	0.000	0.088	0.000	0.091	0.000	0.037
4	0.467	0.000	0.001	0.000	0.013	0.000	0.002
5	0.400	0.005	0.000	0.003	0.000	0.002	0.000
6	0.372	0.000	0.001	0.000	0.000	0.000	0.003
7	0.371	0.000	0.000	0.071	0.000	0.054	0.000
8	0.343	0.000	0.002	0.000	0.012	0.000	0.000
9	0.286	0.000	0.008	0.000	0.009	0.000	0.006
10	0.261	0.043	0.000	0.000	0.000	0.001	0.000
11	0.223	0.000	0.015	0.000	0.003	0.000	0.023
12	0.208	0.000	0.000	0.001	0.000	0.143	0.000

Computed values for the isolator displacements due to a longitudinal earthquake are as follows (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 2.10 (1.5) in
- $d_{isol,2}$ = 1.62 (1.73) in
- $d_{isol,3}$ = 0.53 (0.72) in
- $d_{isol,4}$ = 2.10 (1.95) in

B2.7 Convergence Check Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.		**B2.7 Convergence Check, Example 2.5** The results for isolator displacements are not close. Go to Step B2.8 and update properties for a second cycle of iteration.

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows: $$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \quad \text{(B-24)}$$ Recalculate $K_{eff,j}$: Eq. 7.1-6 GSID $$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \quad \text{(B-25)}$$ Recalculate system damping ratio, ξ : Eq. 7.1-10 GSID $$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \quad \text{(B-26)}$$ Recalculate system damping factor, B_L: Eq. 7.1-3 GSID $$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \quad \text{(B-27)}$$ Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.		**B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 2.5** Updated values for $K_{isol,i}$ are given below (previous values are in parentheses): o $K_{isol,1}$ = 2.73(2.88) k/in o $K_{isol,2}$ = 12.38 (11.93) k/in o $K_{isol,3}$ = 26.28 (20.83) k/in o $K_{isol,4}$ = 2.78 (2.88) k/in Since the isolator displacements are relatively close to previous results no significant change in the damping ratio is expected. Hence $K_{eff,j}$ and ξ are not recalculated and B_L is taken at 1.65. Since the change in effective period is very small (1.64 to 1.68 sec) and no change has been made to B_L, there is no need to construct a new composite response spectrum in this case. Go back to Step B2.6 (see immediately below).
		B2.6 Multimodal Analysis Second and Third Iteration, Example 2.5 Reasonable convergence was not obtained after the second iteration and a third cycle was performed. Results for the isolator displacements at the end of the third cycle (numbers in parentheses are those from the first cycle): o $d_{isol,1}$ = 1.96 (2.10) in o $d_{isol,2}$ = 1.56 (1.62) in o $d_{isol,3}$ = 0.33 (0.53) in o $d_{isol,4}$ = 1.96 (2.10) in Go to Step B2.7
B2.7 Convergence Check Compare results and determine if convergence has been reached. If so go to Step B2.9. Otherwise Go to Step B2.8.		**B2.7 Convergence Check, Example 2.5** Satisfactory agreement has been reached on the third cycle. Go to Step B2.9
B2.9 Superstructure and Isolator Displacements Once convergence has been reached, obtain o superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge,		**B2.9 Superstructure and Isolator Displacements, Example 2.5** From the above analysis: o superstructure displacements in the

<table>
<tr><td>and
o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.</td><td></td><td>longitudinal (x_L) and transverse (y_L) directions are:
x_L= 1.98 in
y_L= 0.0 in

o isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are:
o Abutment 1: u_L = 1.96 in, v_L = 0.00 in
o Pier 1: u_L = 1.56 in, v_L = 0.00 in
o Pier 2: u_L = 0.33 in, v_L = 0.00 in
o Abutment 2: u_L = 1.96 in, v_L = 0.00 in

All isolators at same support have the same displacements.</td></tr>
<tr><td>B2.10 Pier Bending Moments and Shear Forces
Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.</td><td></td><td>B2.10 Pier Bending Moments and Shear Forces, Example 2.5
Bending moments in single column pier in the longitudinal (M_{PLL}) and transverse (M_{PTL}) directions are:
Pier 1: M_{P1LL}= 0
M_{P1TL}= 2020 kft
Pier 2: M_{P2LL}= 0
M_{P2TL}= 1789 kft

Shear forces in single column pier the longitudinal (V_{PLL}) and transverse (V_{PTL}) directions are:
Pier 1: V_{P1LL} = 84.79 k
V_{P1TL} = 0
Pier 2: V_{P2LL} = 41.69 k
V_{P2TL} = 0</td></tr>
<tr><td>B2.11 Isolator Shear and Axial Forces
Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.</td><td></td><td>B2.11 Isolator Shear and Axial Forces, Example 2.5
Isolator shear and axial forces are summarized in Table B2.11-1</td></tr>
</table>

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

		V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abut 1	Isol.1	5.63	0	1.31
	Isol.2	5.63	0	1.33
	Isol.3	5.63	0	1.33
Pier 1	Isol.1	19.62	0	0.99
	Isol.2	19.65	0	1.36
	Isol.3	19.62	0	0.99
Pier 2	Isol.1	11.61	0	0.84
	Isol.2	11.67	0	1.14
	Isol.3	11.61	0	0.84
Abut 2	Isol.1	5.63	0	1.04
	Isol.2	5.63	0	1.05
	Isol.3	5.63	0	1.02

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 2.5

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) for transverse loading , are as follows:

- T_{eff} = 1.86 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions due to transverse load are as follows:
  x_T = 0
  y_T = 1.81 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions due to transverse loading are as follows:
-
  Abutment1 u_T = 0.15 in, v_T = 2.91 in
  Pier1 u_T = 0.14 in, v_T = 1.04 in
  Pier2 u_T = 0.08 in, v_T = 0.22 in
  Abutment2 u_T = 0.14 in, v_T = 3.40 in

- Pier bending moments in the longitudinal (M_{PLT}) and transverse (M_{PTT}) directions due to transverse load are as follows:
  Pier 1: M_{PLT} = 1906 kft
  M_{PTT} = 2 kft
  Pier 2: M_{PLT} = 1722 kft
  M_{PTT} = 1 kft

- Pier shear forces in the longitudinal (V_{PLT}) and transverse (V_{PTT}) directions due to transverse load are as follows:
  Pier 1: V_{PLT} = 0.33k
  V_{PTT} = 70.13 k
  Pier 2: V_{PLT} = 0.09k
  V_{PTT} = 45.85 k

- Isolator shear and axial forces are summarized in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

		V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abut 1	Isol. 1	0.45	8.84	15.36
	Isol. 2	0	8.85	1.31
	Isol. 3	0.45	8.84	15.27
Pier 1	Isol. 1	2.61	19.45	26.77
	Isol. 2	0	19.63	1.58
	Isol. 3	2.61	19.45	26.87
Pier 2	Isol.1	4.18	11.55	26.32
	Isol.2	0	11.82	1.71
	Isol.3	4.18	11.55	26.58
Abut 2	Isol.1	0.32	7.70	16.26
	Isol.2	0	7.71	1.78
	Isol.3	0.32	7.70	16.44

<table>
<tr><td colspan="3">STEP D. CALCULATE DESIGN VALUES (Exterior Isolator at Pier 1)</td></tr>
<tr><td colspan="3">Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.</td></tr>
<tr><td>D1. Design Isolator Displacements
Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:
• $u_1 = u_L + 0.3u_T$ (D-1)
$v_1 = v_L + 0.3v_T$ (D-2)
$R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)

• $u_2 = 0.3u_L + u_T$ (D-4)
$v_2 = 0.3v_L + v_T$ (D-5)
$R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)

• $d_t = \max(R_1, R_2)$ (D-7)</td><td></td><td>D1. Design Isolator Displacements, Example 2.5
<u>Load Case 1</u>:
$u_1 = u_L + 0.3u_T$ = 1.0(1.57) + 0.3(0.14) = 1.61 in
$v_1 = v_L + 0.3v_T$ = 1.0(0) + 0.3(1.04) = 0.31 in
$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.61^2 + 0.31^2}$ = 1.64 in

<u>Load Case 2</u>:
$u_2 = 0.3u_L + u_T$ = 0.3(1.57) + 1.0(0.14) = 0.61 in
$v_2 = 0.3v_L + v_T$ = 0.3(0) + 1.0(1.04) = 1.04 in
$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{0.61^2 + 1.04^2}$ = 1.21 in

<u>Governing Case</u>:
Total design displacement, d_t = max(R_1, R_2)
= 1.64 in
This is the design displacement for an exterior isolator at Pier 1.</td></tr>
<tr><td>D2. Design Moments and Shears
Calculate design values for column bending moments and shear forces using the same combination rules as for displacements.

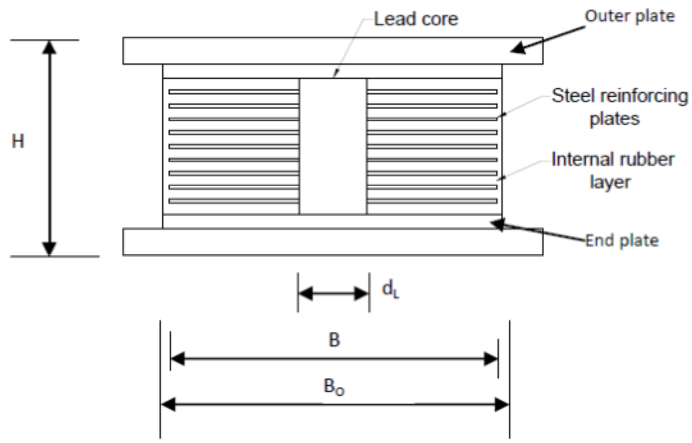
Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.</td><td></td><td>D2. Design Moments and Shears (Pier 1), Example 2.5
<u>Load Case 1</u>:
$V_{PL1} = V_{PLL} + 0.3V_{PLT}$ = 1.0(19.62) + 0.3(2.61) = 20.40 k
$V_{PT1} = V_{PTL} + 0.3V_{PTT}$ = 1.0(0) + 0.3(19.45) = 5.84 k
$R_1 = \sqrt{V_{L1}^2 + V_{T1}^2} = \sqrt{20.40^2 + 5.84^2}$ = 21.22 k

<u>Load Case 2</u>:
$V_{PL2} = 0.3V_{PLL} + V_{PLT}$ = 0.3(19.62) + 1.0(2.61) = 8.5 k
$V_{PT2} = 0.3V_{PTL} + V_{PTT}$ = 0.3(0) + 1.0(19.45) = 19.45 k
$R_2 = \sqrt{V_{L2}^2 + V_{T2}^2} = \sqrt{8.5^2 + 19.45^2}$ = 21.22 k

<u>Governing Case</u>:
Design column shear = max ($R1$, $R2$)
= 21.22 k</td></tr>
</table>

STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS

A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness (K_u).With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness $\underline{K_d}$ is given by the rubber alone. More details are given in MCEER 2006.



While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary. When sizing the physical dimensions of the bearing, plan dimensions (B, d_L) should be rounded up to the next $^{1}/_{4}$" increment, while the total thickness of elastomer, T_r, is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are $^{1}/_{4}$" and $^{3}/_{8}$". High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in MCEER 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See MCEER 2006.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties

Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1).

- required characteristic strength, Q_d / isolator
- required post-elastic stiffness, K_d / isolator
- total design displacement, d_t, for each isolator
- maximum applied dead and live load (P_{DL}, P_{LL}) and seismic load (P_{SL}) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator, and
- maximum wind load, P_{WL}

E1. Required Properties, Example 2.5

The design of one of the exterior isolators on a pier is given below to illustrate the design process for lead-rubber isolators.

From previous work

- Q_d / isolator = 10.95 k
- K_d / isolator = 5.62 k/in
- Total design displacement dt = 1.64 in
- P_{DL} = 187 k
- P_{LL} = 123 k
- P_{SL} = 27 k (Table C1-1)
- P_{WL} = 8.21 k < Q_d OK

E2. Isolator Sizing

E2.1 Lead Core Diameter

Determine the required diameter of the lead plug, d_L, using:

$$d_L = \sqrt{\frac{Q_d}{0.9}} \qquad \text{(E-1)}$$

See Step E2.5 for limitations on d_L

E2.1 Lead Core Diameter, Example 2.5

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{10.95}{0.9}} = 3.49\ in$$

E2.2 Plan Area and Isolator Diameter Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.6 ksi. Then the bonded area of the isolator is given by: $A_b = \frac{P_{DL} + P_{LL}}{1.6}\ in^2$ (E-2) and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by: $B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2}$ (E-3) Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using $A_b = \frac{\pi}{4}(B^2 - d_L^2)$ (E-4) Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, B_o is given by: $B_o = B + 1.0$ (E-5)	**E2.2 Plan Area and Isolator Diameter, Example 2.5** Based on the final design of the isolators for Example 2.0, increase the allowable stress to 3.2 ksi. $A_b = \frac{P_{DL} + P_{LL}}{3.2}\ in^2 = \frac{187 + 123}{3.2} = 96.88\ in^2$ $B = \sqrt{\frac{4\ A_b}{\pi} + d_L^2} = \sqrt{\frac{4\ (96.88)}{\pi} + 3.49^2}$ = 11.64 in Round B up to 12.5 in (based on experience with Example 2.0) and the actual bonded area is: $A_b = \frac{\pi}{4}(12.50^2 - 3.49^2) = 113.16\ in^2$ B_o = 12.50 + 2(0.5) = 13.50 in
E2.3 Elastomer Thickness and Number of Layers Since the shear stiffness of the elastomeric bearing is given by: $K_d = \frac{GA_b}{T_r}$ (E-6) where G = shear modulus of the rubber, and T_r = the total thickness of elastomer, it follows Eq. E-5 may be used to obtain T_r given a required value for K_d $T_r = \frac{GA_b}{K_d}$ (E-7) If the layer thickness is t_r, the number of layers, n, is given by: $n = \frac{T_r}{t_r}$ (E-8) rounded up to the nearest integer. Note that because of rounding the plan dimensions and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be necessary if the differences are large.	**E2.3 Elastomer Thickness and Number of Layers, Example 2.5** $T_r = \frac{GA_b}{K_d} = \frac{0.1(113.16)}{5.62} = 2.01\ in$ $n = \frac{2.01}{0.25} = 8.05$ Round to nearest integer, i.e. n = 8

E2.4 Overall Height The overall height of the isolator, H, is given by: $$H = n\,t_r + (n-1)t_s + 2t_c \qquad \text{(E-9)}$$ where t_s = thickness of an internal shim (usually about 1/8 in), and t_c = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in)		**E2.4 Overall Height, Example 2.5** $$H = 8(0.25) + 7(0.125) + 2*1.5 = 5.875\ in$$
E2.5 Lead Core Size Check Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows: $$\frac{B}{3} \geq d_L \geq \frac{B}{6} \qquad \text{(E-10)}$$		**E2.5 Lead Core Size Check, Example 2.5** Since *B*=12.5 check $$\frac{12.5}{3} \geq d_L \geq \frac{12.5}{6}$$ i.e., $4.16 \geq d_L \geq 2.08$ Since d_L = 3.49, lead core size is acceptable.
E3. Strain Limit Check Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e., $$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5 \qquad \text{(E-11)}$$ where $\gamma_c,\ \gamma_{s,eq}, and\ \gamma_r$ are defined below. (a) γ_c is the maximum shear strain in the layer due to compression and is given by: $$\gamma_c = \frac{D_c \sigma_S}{GS} \qquad \text{(E-12)}$$ where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = {P_{DL}}/{A_b}$, *G* is shear modulus, and *S* is the layer shape factor given by: $$S = \frac{A_b}{\pi B t_r} \qquad \text{(E-13)}$$ (b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by: $$\gamma_{s,eq} = \frac{d_t}{T_r} \qquad \text{(E-14)}$$ (c) γ_r is the shear strain due to rotation and is given by: $$\gamma_r = \frac{D_r B^2 \theta}{t_r T_r} \qquad \text{(E-15)}$$ where D_r is shape coefficient for rotation in circular bearings = 0.375, and θ is design rotation due to DL, LL and construction effects . Actual value for θ may not be known at this time and a value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1).		**E3. Strain Limit Check, Example 2.5** Since $$\sigma_S = \frac{187.0}{113.16} = 1.65\ ksi$$ and $$S = \frac{113.16}{\pi 12.5(0.25)} = 11.53$$ then $$\gamma_c = \frac{1.0(1.65)}{0.1(11.53)} = 1.43$$ $$\gamma_{s,eq} = \frac{1.64}{2.0} = 0.82$$ $$\gamma_r = \frac{0.375(12.5^2)(0.01)}{0.25(2.0)} = 1.17$$ Substitution in Eq E-11 gives $$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 1.43 + 0.82 + 0.5(1.17) = 2.84 \leq 5.5\ OK$$

E4. Vertical Load Stability Check Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state. Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either 2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or 1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4.		**E4. Vertical Load Stability Check, Example 2.5**
E4.1 Vertical Load Stability in Undeformed State The critical load capacity of an elastomeric isolator at zero shear displacement is given by $P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2}\left[\sqrt{(1+\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2})} - 1\right]$ (E-16) where $H_{eff} = T_r + T_s$ T_s = total shim thickness $K_\theta = {E_b I}/{T_r}$ $E_b = E(1+0.67S^2)$ E = elastic modulus of elastomer = $3G$ $I = {\pi B^4}/{64}$ It is noted that typical elastomeric isolators have high shape factors, S, in which case: $\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1$ (E-17) and Eq. E-16 reduces to: $P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta}$ (E-18) Check that: $\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} \geq 3$ (E-19)		**E4.1 Vertical Load Stability in Undeformed State, Example 2.5** $E = 3G = 3(0.1) = 0.3\ ksi$ $E_b = 0.3(1+0.67(11.53^2)) = 26.89 ksi$ $I = \pi\frac{12.50^4}{64} = 1198.4 in^4$ $K_\theta = \frac{26.89(1198.4)}{2.0} = 16{,}110 kin/rad$ $K_d = \frac{GA_b}{T_r} = \frac{0.1(113.16)}{2.0} = 5.66 k/in$ $P_{cr(\Delta=0)} = \pi\sqrt{5.66(16{,}110)} = 948.5k$ $\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \frac{948.5}{(187+123)} = 3.06 \geq 3\ \ OK$
E4.2 Vertical Load Stability in Deformed State The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by: $P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)}$ (E-20) where A_r = overlap area between top and bottom plates of isolator at displacement Δ (Fig. 2.2-1 GSID)		**E4.2 Vertical Load Stability in Deformed State, Example 2.5** Since bridge is in Zone 2, $\Delta = 2d_t = 2(1.64) = 3.28$ $\delta = 2cos^{-1}\left(\frac{3.28}{12.50}\right) = 2.61$

$= B^2(\delta - sin\delta)/_4$ $\delta \quad = 2cos^{-1}(^{\Delta}/_{B})$ $A_{gross} = \pi\, {B^2}/_4$ It follows that: $$\frac{A_r}{A_{gross}} = \frac{(\delta - sin\delta)}{\pi} \qquad \text{(E-21)}$$ Check that: $$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} \geq 1 \qquad \text{(E-22)}$$	$$\frac{A_r}{A_{gross}} = \frac{(2.61 - sin2.61)}{\pi} = 0.669$$ $$P_{cr(\Delta)} = 0.67(948.5) = 635.3k$$ $$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{635.3}{1.2(187) + 27} = 2.53 \geq 1 \;\; OK$$
E5. Design Review	**E5. Design Review, Example 2.5** The basic dimensions of the isolator designed above are as follows: 13.50 in (od) x 5.875in (high) x 3.49 in dia. lead core and the volume, excluding steel end and cover plates, = 412 in^3 This design is considered satisfactory since both the total strain (Eq E-11) and the vertical load stability factors are reasonable values (not excessively low or excessively high).
E6. Minimum and Maximum Performance Check Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E6-1. **Table E6-1. Minimum and maximum values for K_d and Q_d.** <table><tr><td>Eq. 8.1.2-1 GSID</td><td>$K_{d,max} = K_d\lambda_{max,Kd}$</td><td>(E-23)</td></tr><tr><td>Eq. 8.1.2-2 GSID</td><td>$K_{d,min} = K_d\lambda_{min,Kd}$</td><td>(E-24)</td></tr><tr><td>Eq. 8.1.2-3 GSID</td><td>$Q_{d,max} = Q_d\lambda_{max,Qd}$</td><td>(E-25)</td></tr><tr><td>Eq. 8.1.2-4 GSID</td><td>$Q_{d,min} = Q_d\lambda_{min,Qd}$</td><td>(E-26)</td></tr></table> Determination of the system property modification	**E6. Minimum and Maximum Performance Check, Example 2.5** Minimum Property Modification factors are $\lambda_{min,Kd} = 1.0$ $\lambda_{min,Qd} = 1.0$ which means there is no need to reanalyze the bridge with a set of minimum values. Maximum Property Modification factors are $\lambda_{max,a,Kd} = 1.1$ $\lambda_{max,a,Qd} = 1.1$ $\lambda_{max,t,Kd} = 1.1$ $\lambda_{max,t,Qd} = 1.4$ $\lambda_{max,scrag,Kd} = 1.0$ $\lambda_{max,scrag,Qd} = 1.0$

factors shall include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E6-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})\ (\lambda_{min,a,Kd})\ (\lambda_{min,v,Kd})\ (\lambda_{min,tr,Kd})\ (\lambda_{min,c,Kd})\ (\lambda_{min,scrag,Kd})$	(E-27)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})\ (\lambda_{max,a,Kd})\ (\lambda_{max,v,Kd})\ (\lambda_{max,tr,Kd})\ (\lambda_{max,c,Kd})\ (\lambda_{max,scrag,Kd})$	(E-28)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})\ (\lambda_{min,a,Qd})\ (\lambda_{min,v,Qd})\ (\lambda_{min,tr,Qd})\ (\lambda_{min,c,Qd})\ (\lambda_{min,scrag,Qd})$	(E-29)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})\ (\lambda_{max,a,Qd})\ (\lambda_{max,v,Qd})\ (\lambda_{max,tr,Qd})\ (\lambda_{max,c,Qd})\ (\lambda_{max,scrag,Qd})$	(E-30)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

- 1.00 for critical bridges
- 0.75 for essential bridges
- 0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

Applying a system adjustment factor of 0.66 for an ‘other’ bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,a,Qd} = 1.0 + 0.1(0.66) = 1.066$

$\lambda_{max,t,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,t,Qd} = 1.0 + 0.4(0.66) = 1.264$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Therefore the maximum overall modification factors

$\lambda_{max,Kd} = 1.066(1.066)1.0 = 1.14$
$\lambda_{max,Qd} = 1.066(1.264)1.0 = 1.35$

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:
$Q_{d,max} = 1.35\ (10.95) = 14.78$ k
and
$K_{d,max} = 1.14(5.62) = 6.41$ k/in

E7. Design and Performance Summary	E7. Design and Performance Summary
E7.1 Isolator dimensions Summarize final dimensions of isolators: • Overall diameter (includes cover layer) • Overall height • Diameter lead core • Bonded diameter • Number of rubber layers • Thickness of rubber layers • Total rubber thickness • Thickness of steel shims • Shear modulus of elastomer Check all dimensions with manufacturer.	**E7.1 Isolator dimensions, Example 2.5** Isolator dimensions are summarized in Table E7.1-1. Shear modulus of elastomer = 100 psi
E7.2 Bridge Performance Summarize bridge performance • Maximum superstructure displacement (longitudinal) • Maximum superstructure displacement (transverse) • Maximum superstructure displacement (resultant) • Maximum column shear (resultant) • Maximum column moment (about transverse axis) • Maximum column moment (about longitudinal axis) • Maximum column torque Check required performance as determined in Step A3, is satisfied.	**E7.2 Bridge Performance, Example 2.5** Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 87.36k (Pier1) and 47.53k (Pier2). This less than the column plastic shear (128k at Pier 1 and 72k at Pier2) and therefore the required performance criterion is satisfied (fully elastic behavior). Furthermore the maximum longitudinal displacement is 1.98 in which is less than the 2.5in which is available at the abutment expansion joints and therefore acceptable.

Table E7.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates. (in)	Overall size without mounting plates (in)	Diam. lead core (in)
Under edge girder on Pier	17.5 x17.5 x 5.875(H)	13.5 dia x 4.375(H)	3.49

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in)	Total rubber thick-ness (in)	Steel shim thick-ness (in)
Under edge girder on Pier	8	0.25	2.0	0.125

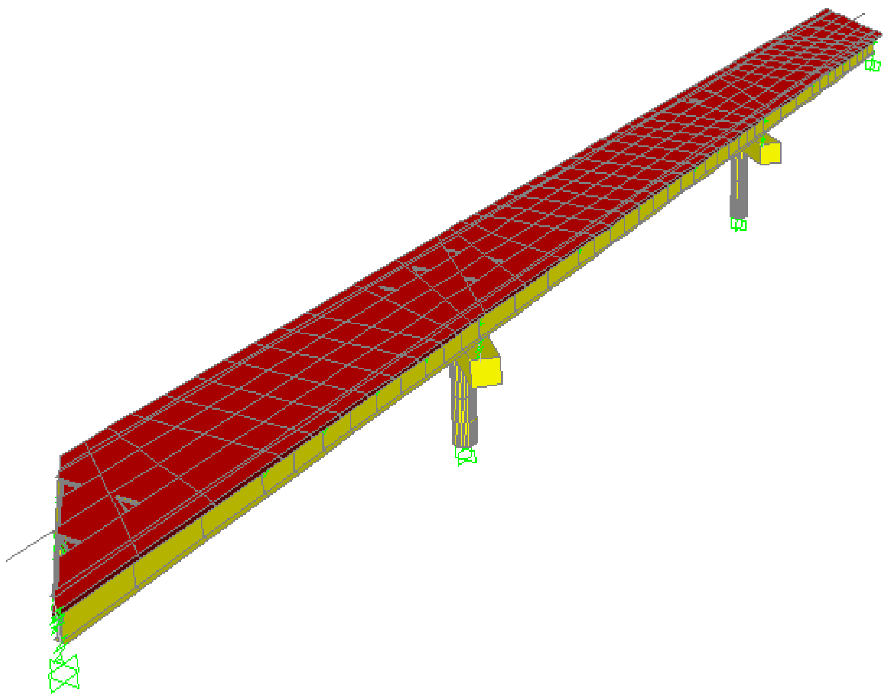
Table E7.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	1.98 in
Maximum superstructure displacement (transverse)	1.81 in
Maximum superstructure displacement (resultant)	2.05 in
Maximum column shear (resultant)	87.36k (Pier1) 47.53k (Pier2)
Maximum column moment about transverse axis	2020 kft (Pier1) 1789 kft (Pier2)
Maximum column moment about longitudinal axis	1906 kft (Pier1) 1722 kft (Pier2)
Maximum column torque	72 kft (Pier1) 99 kft (Pier2)

SECTION 2

Steel Plate Girder Bridge, Long Spans, Single-Column Pier

DESIGN EXAMPLE 2.6: Skew = 45^0



Design Examples in Section 2

ID	Description	S_1	Site Class	Column height	Skew	Isolator type
2.0	Benchmark bridge	0.2g	B	Same	0	Lead-rubber bearing
2.1	Change site class	0.2g	D	Same	0	Lead rubber bearing
2.2	Change spectral acceleration, S_1	0.6g	B	Same	0	Lead rubber bearing
2.3	Change isolator to SFB	0.2g	B	Same	0	Spherical friction bearing
2.4	Change isolator to EQS	0.2g	B	Same	0	Eradiquake bearing
2.5	Change column height	0.2g	B	H_1=0.5H_2	0	Lead rubber bearing
2.6	Change angle of skew	0.2g	B	Same	45^0	Lead rubber bearing

<table>
<tr><th>DESIGN PROCEDURE</th><th>DESIGN EXAMPLE 2.6 (45^0 skew)</th></tr>
<tr><td colspan="2">STEP A: BRIDGE AND SITE DATA</td></tr>
<tr><td>A1. Bridge Properties
Determine properties of the bridge:
• Number of supports, m
• Number of girders per support, n
• Angle of skew
• Weight of superstructure including railings, curbs, barriers and other permanent loads, W_{SS}
• Weight of piers participating with superstructure in dynamic response, W_{PP}
• Weight of superstructure, W_j, at each support
• Stiffness, $K_{sub,j}$, of each support in both longitudinal and transverse directions of the bridge. The calculation of these quantities requires careful consideration of several factors such as the use of cracked sections when estimating column or wall flexural stiffness, foundation flexibility, and effective column height.
• Column shear strength (minimum value). This will usually be derived from the minimum value of the column flexural yield strength, the column height, and whether the column is acting in single or double curvature in the direction under consideration.
• Allowable movement at expansion joints
• Isolator type if known, otherwise 'to be selected'</td>
<td>A1. Bridge Properties, Example 2.6
• Number of supports, $m = 4$
  o North Abutment ($m = 1$)
  o Pier 1 ($m = 2$)
  o Pier 2 ($m = 3$)
  o South Abutment ($m = 4$)
• Number of girders per support, $n = 3$
• Number of columns per support = 1
• Angle of skew = 45^0
• Weight of superstructure including permanent loads, $W_{SS} = 1651.3$ k
• Weight of superstructure at each support:
  o $W_1 = 168.48$ k
  o $W_2 = 657.18$ k
  o $W_3 = 657.18$ k
  o $W_4 = 168.48$ k
• Participating weight of piers, $W_{PP} = 256.3$ k
• Effective weight (for calculation of period), $W_{eff} = W_{SS} + W_{PP} = 1907.6$ k
• Stiffness of each pier in the both directions:
  o $K_{sub,pier1} = 288.87$ k/in
  o $K_{sub,pier2} = 288.87$ k/in
• Minimum column shear strength based on flexural yield capacity of column = 128 k
• Displacement capacity of expansion joints (longitudinal) = 2.5 in (required to accommodate thermal expansion and other movements)
• Lead rubber isolators</td></tr>
<tr><td>A2. Seismic Hazard
Determine seismic hazard at site:
• acceleration coefficients
• site class and site factors
• seismic zone
Plot response spectrum.

Use Art. 3.1 GSID to obtain peak ground and spectral acceleration coefficients. These coefficients are the same as for conventional bridges and Art 3.1 refers the designer to the corresponding articles in the LRFD Specifications. Mapped values of PGA, S_S and S_1 are given in both printed and CD formats (e.g. Figures 3.10.2.1-1 to 3.10.2.1-21 LRFD).

Use Art. 3.2 to obtain Site Class and corresponding Site Factors (F_{pga}, F_a and F_v). These data are the same as for conventional bridges and Art 3.2 refers the designer to the corresponding articles in the LRFD Specifications,</td>
<td>A2. Seismic Hazard, Example 2.6
Acceleration coefficients for bridge site are given in design statement as follows:
• PGA = 0.40
• $S_1 = 0.20$
• $S_S = 0.75$

Bridge is on a rock site with shear wave velocity in upper 100 ft of soil = 3,000 ft/sec.

Table 3.10.3.1-1 LRFD gives Site Class as B.

Tables 3.10.3.2-1, -2 and -3 LRFD give following Site Factors:
• $F_{pga} = 1.0$
• $F_a = 1.0$</td></tr>
</table>

i.e. to Tables 3.10.3.1-1 and 3.10.3.2-1, -2, and -3, LRFD.
Art. 4 GSID and Eq. 4-2, -3, and -8 GSID give modified spectral acceleration coefficients that include site effects as follows:

- $A_s = F_{pga}\, PGA$
- $S_{DS} = F_a\, S_S$
- $S_{D1} = F_v\, S_1$

Seismic Zone is determined by value of S_{D1} in accordance with provisions in Table 5-1 GSID.

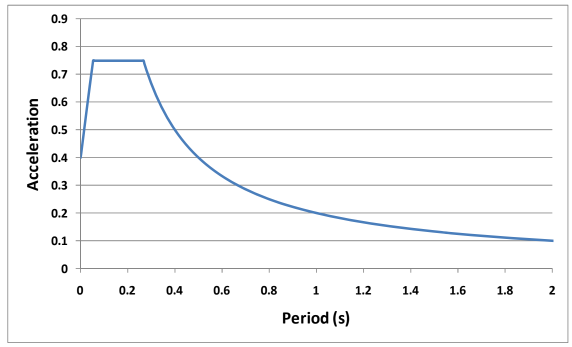
These coefficients are used to plot design response spectrum as shown in Fig. 4-1 GSID.

- $F_v = 1.0$

- $A_s = F_{pga}\, PGA = 1.0(0.40) = 0.40$
- $S_{DS} = F_a\, S_S = 1.0(0.75) = 0.75$
- $S_{D1} = F_v\, S_1 = 1.0(0.20) = 0.20$

Since $0.15 < S_{D1} \leq 0.30$, bridge is located in Seismic Zone 2.

Design Response Spectrum is as below:



A3. Performance Requirements
Determine required performance of isolated bridge during Design Earthquake (1000-yr return period).

Examples of performance that might be specified by the Owner include:

- Reduced displacement ductility demand in columns, so that bridge is open for emergency vehicles immediately following earthquake.
- Fully elastic response (i.e., no ductility demand in columns or yield elsewhere in bridge), so that bridge is fully functional and open to all vehicles immediately following earthquake.
- For an existing bridge, minimal or zero ductility demand in the columns and no impact at abutments (i.e., longitudinal displacement less than existing capacity of expansion joint for thermal and other movements)
- Reduced substructure forces for bridges on weak soils to reduce foundation costs.

A3. Performance Requirements, Example 2.6
In this example, assume the owner has specified full functionality following the earthquake and therefore the columns must remain elastic (no yield).

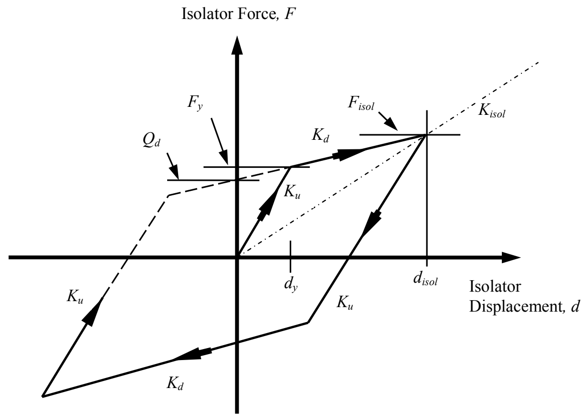
To remain elastic the maximum lateral load on the pier must be less than the load to yield any one column (128 k).

The maximum shear in the column must therefore be less than 128 k in order to keep the column elastic and meet the required performance criterion.

STEP B: ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN LONGITUDINAL DIRECTION

In most applications, isolation systems must be stiff for non-seismic loads but flexible for earthquake loads (to enable required period shift). As a consequence most have bilinear properties as shown in figure at right.

Strictly speaking nonlinear methods should be used for their analysis. But a common approach is to use equivalent linear springs and viscous damping to represent the isolators, so that linear methods of analysis may be used to determine response. Since equivalent properties such as K_{isol} are dependent on displacement (d), and the displacements are not known at the beginning of the analysis, an iterative approach is required. Note that in Art 7.1, GSID, k_{eff} is used for the effective stiffness of an isolator unit and K_{eff} is used for the effective stiffness of a combined isolator and substructure unit. To minimize confusion, K_{isol} is used in this document in place of k_{eff}. There is no change in the use of K_{eff} and $K_{eff,j}$, but K_{sub} is used in place of k_{sub}.



d_{isol} = Isolator displacement
d_y = Isolator yield displacement
F_{isol} = Isolator shear force
F_y = Isolator yield force
K_d = Post-elastic stiffness of isolator
K_{isol} = Effective stiffness of isolator
K_u = Loading and unloading stiffness (elastic stiffness)
Q_d = Characteristic strength of isolator

The methodology below uses the Simplified Method (Art 7.1 GSID) to obtain initial estimates of displacement for use in an iterative solution involving the Multimode Spectral Analysis Method (Art 7.3 GSID).

Alternatively nonlinear time history analyses may be used which explicitly include the nonlinear properties of the isolator without iteration, but these methods are outside the scope of the present work.

B1. SIMPLIFIED METHOD

In the Simplified Method (Art. 7.1, GSID) a single degree-of-freedom model of the bridge with equivalent linear properties and viscous dampers to represent the isolators, is analyzed iteratively to obtain estimates of superstructure displacement (d_{isol} in above figure, replaced by d below to include substructure displacements) and the required properties of each isolator necessary to give the specified performance (i.e. find d, characteristic strength, $Q_{d,j}$, and post elastic stiffness, $K_{d,j}$ for each isolator 'j' such that the performance is satisfied). For this analysis the design response spectrum (Step A2 above) is applied in longitudinal direction of bridge.

B1.1 Initial System Displacement and Properties

To begin the iterative solution, an estimate is required of :

(1) Structure displacement, d. One way to make this estimate is to assume the effective isolation period, T_{eff}, is 1.0 second, take the viscous damping ratio, ξ, to be 5% and calculate the displacement using Eq. B-1. (The damping factor, B_L, is given by Eq.7.1-3 GSID, and equals 1.0 in this case.)

Art C7.1 GSID

$$d = \frac{9.79 S_{D1} T_{eff}}{B_L} \cong 10 S_{D1} \qquad \text{(B-1)}$$

B1.1 Initial System Displacement and Properties, Example 2.6

$$d \cong 10 S_{D1} = 10(0.20) \cong 2.0\ in$$

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(2) Characteristic strength, Q_d. This strength needs to be high enough that yield does not occur under service loads (e.g. wind) but low enough that yield does occur under earthquake. Experience has shown that taking Q_d to be 5% of the bridge weight is a good starting point, i.e.

$$Q_d = 0.05W \quad \text{(B-2)}$$

(3) Post-yield stiffness, K_d
Art 12.2 GSID requires that all isolators exhibit a minimum lateral restoring force at the design displacement, which translates to a minimum post yield stiffness $K_{d,min}$ given by Eq. B-3.

Art. 12.2 GSID

$$K_{d,min} \geq \frac{0.025W}{d} \quad \text{(B-3)}$$

Experience has shown that a good starting point is to take K_d equal to twice this minimum value, i.e. $K_d = 0.05W/d$

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$$Q_d = 0.05W = 0.05(1651.32) = 82.56\ k$$

$$K_d = 0.05\frac{W}{d} = 0.05\frac{1651.32}{2.0} = 41.28\ k/in$$

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B1.2 Initial Isolator Properties at Supports
Calculate the characteristic strength, $Q_{d,j}$, and post-elastic stiffness, $K_{d,j}$, of the isolation system at each support 'j' by distributing the total calculated strength, Q_d, and stiffness, K_d, values in proportion to the dead load applied at that support:

$$Q_{d,j} = Q_d\frac{W_j}{W} \quad \text{(B-4)}$$

and

$$K_{d,j} = K_d\frac{W_j}{W} \quad \text{(B-5)}$$

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B1.2 Initial Isolator Properties at Supports, Example 2.6

$$Q_{d,j} = Q_d\frac{W_j}{W}$$

- $Q_{d,1}$ = 8.42 k
- $Q_{d,2}$ = 32.86 k
- $Q_{d,3}$ = 32.86 k
- $Q_{d,4}$ = 8.42 k

and

$$K_{d,j} = K_d\frac{W_j}{W}$$

- $K_{d,1}$ = 4.21 k/in
- $K_{d,2}$ = 16.43 k/in
- $K_{d,3}$ = 16.43 k/in
- $K_{d,4}$ = 4.21 k/in

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B1.3 Effective Stiffness of Combined Pier and Isolator System
Calculate the effective stiffness, $K_{eff,j}$, of each support 'j' for all supports, taking into account the stiffness of the isolators at support 'j' ($K_{isol,j}$) and the stiffness of the substructure $K_{sub,j}$. See figure below (after Fig. 7.1-1 GSID).

An expression for $K_{eff,j}$, is given in Eq.7.1-6 GSID, but a more useful formula as follows (MCEER 2006):

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j} \quad \text{(B-6)}$$

where

$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}} \quad \text{(B-7)}$$

and $K_{sub,j}$ for the piers are given in Step A1.

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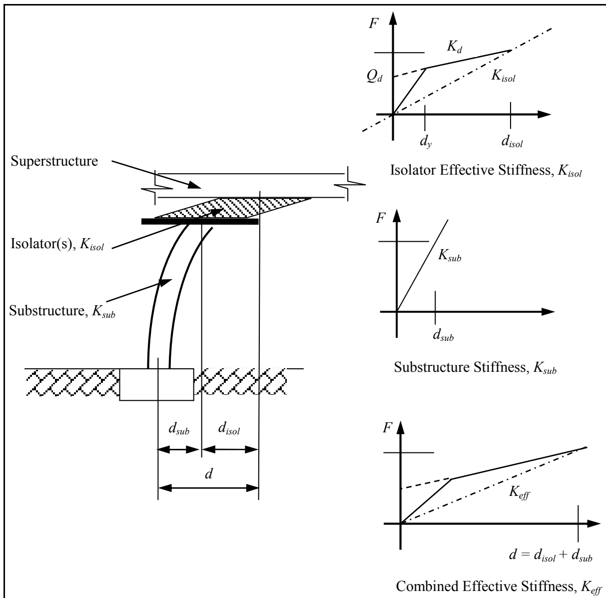
B1.3 Effective Stiffness of Combined Pier and Isolator System, Example 2.6

$$\alpha_j = \frac{K_{d,j}d + Q_{d,j}}{K_{sub,j}d - Q_{d,j}}$$

- $\alpha_1 = 8.43\text{x}10^{-4}$
- $\alpha_2 = 1.21\text{x}10^{-1}$
- $\alpha_3 = 1.21\text{x}10^{-1}$
- $\alpha_4 = 8.43\text{x}10^{-4}$

</td>
</tr>
</table>



For abutments, take *Ksub,j* to be a large number, say 10,000 k/in, unless an actual stiffness values are available. Note that if the default option is chosen, unrealistically high values for $K_{sub,j}$ will give unconservative results for column moments and shear forces.

$$K_{eff,j} = \frac{\alpha_j K_{sub,j}}{1+\alpha_j}$$

- $K_{eff,1}$ = 8.42 k/in
- $K_{eff,2}$ = 31.09 k/in
- $K_{eff,3}$ = 31.09 k/in
- $K_{eff,4}$ = 8.42 k/in

B1.4 Total Effective Stiffness

Calculate the total effective stiffness, K_{eff}, of the bridge:

Eq. 7.1-6 GSID

$$K_{eff} = \sum_{j=1}^{m} K_{eff,j} \qquad \text{(B-8)}$$

B1.4 Total Effective Stiffness, Example 2.6

$$K_{eff} = \sum_{j=1}^{4} K_{eff,j} = 79.02\ k/in$$

B1.5 Isolation System Displacement at Each Support

Calculate the displacement of the isolation system at support 'j', $d_{isol,j}$, for all supports:

$$d_{isol,j} = \frac{d}{1+\alpha_j} \qquad \text{(B-9)}$$

B1.5 Isolation System Displacement at Each Support, Example 2.6

$$d_{isol,j} = \frac{d}{1+\alpha_j}$$

- $d_{isol,1}$ = 2.00 in
- $d_{isol,2}$ = 1.79 in
- $d_{isol,3}$ = 1.79 in
- $d_{isol,4}$ = 2.00 in

B1.6 Isolation System Stiffness at Each Support

Calculate the stiffness of the isolation system at support 'j', $K_{isol,j}$, for all supports:

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j} \qquad \text{(B-10)}$$

B1.6 Isolation System Stiffness at Each Support, Example 2.6

$$K_{isol,j} = \frac{Q_{d,j}}{d_{isol,j}} + K_{d,j}$$

- $K_{isol,1}$ = 8.43 k/in
- $K_{isol,2}$ = 34.84 k/in
- $K_{isol,3}$ = 34.84 k/in
- $K_{isol,4}$ = 8.43 k/in

B1.7 Substructure Displacement at Each Support Calculate the displacement of substructure 'j', $d_{sub,j}$, for all supports: $d_{sub,j} = d - d_{isol,j}$ (B-11)	**B1.7 Substructure Displacement at Each Support, Example 2.6** $d_{sub,j} = d - d_{isol,j}$ o $d_{sub,1}$ = 0.002 in o $d_{sub,2}$ = 0.215 in o $d_{sub,3}$ = 0.215 in o $d_{sub,4}$ = 0.002 in
B1.8 Substructure Shear at Each Support Calculate the shear at support 'j', $F_{sub,j}$, for all supports: $F_{sub,j} = K_{sub,j} d_{sub,j}$ (B-12) where values for $K_{sub,j}$ are given in Step A1.	**B1.8 Lateral Load at Each Support, Example 2.6** $F_{sub,j} = K_{sub,j} d_{sub,j}$ o $F_{sub,1}$ = 16.84 k o $F_{sub,2}$ = 62.18k o $F_{sub,3}$ = 62.18 k o $F_{sub,4}$ = 16.84 k
B1.9 Column Shear at Each Support Calculate the shear in column 'k' at support 'j', $F_{col,j,k}$, assuming equal distribution of shear for all columns at support 'j': $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ (B-13) Use these approximate column shears as a check on the validity of the chosen strength and stiffness characteristics.	**B1.9 Column Shear Forces at Each Support, Example 2.6** $F_{col,j,k} = \frac{F_{sub,j}}{\#\ of\ columns\ at\ support\ j}$ o $F_{col,2,1}$ = 62.18 k o $F_{col,3,1}$ = 62.18 k These column shears are less than the plastic shear capacity of each column (128k) as required in Step A3 and the chosen strength and stiffness values in Step B1.1 are therefore satisfactory.
B1.10 Effective Period and Damping Ratio Calculate the effective period, T_{eff}, and the viscous damping ratio, ξ, of the bridge: Eq. 7.1-5 GSID $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}}$ (B-14) and Eq. 7.1-10 GSID $\xi = \frac{2\sum_j(Q_{d,j}(d_{isol,j} - d_{y,j})}{\pi\sum_j(K_{eff,j}(d_{isol,j} + d_{sub,j})^2}$ (B-15) where $d_{y,j}$ is the yield displacement of the isolator and assumed to be small compared to $d_{isol,j}$ with negligible effect on ξ, i.e., take $d_{y,j} = 0$.	**B1.10 Effective Period and Damping Ratio, Example 2.6** $T_{eff} = 2\pi\sqrt{\frac{W_{eff}}{gK_{eff}}} = 2\pi\sqrt{\frac{1907.58}{386.4(79.02)}}$ = 1.57 sec and taking $d_{y,j} = 0$: $\xi = \frac{2\sum_j(Q_{d,j}(d_{isol,j} - 0)}{\pi\sum_j(K_{eff,j}(d_{isol,j} + d_{sub,j})^2} = 0.30$

B1.11 Damping Factor Calculate the damping factor, B_L, and the displacement, d, of the bridge: Eq. 7.1-3 GSID $B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3}, & \xi < 0.3 \\ 1.7, & \xi \geq 0.3 \end{cases}$ (B-16) Eq. 7.1-4 GSID $d = \frac{9.79 S_{D1} T_{eff}}{B_L}$ (B-17)	**B1.11 Damping Factor, Example 2.6** Since $\xi = 0.30 \geq 0.3$ $B_L = 1.70$ and $d = \frac{9.79 S_{D1} T_{eff}}{B_L} = \frac{9.79(0.2)1.57}{1.70} = 1.81\ in$
B1.12 Convergence Check Compare the new displacement with the initial value assumed in Step B1.1. If there is close agreement, go to the next step; otherwise repeat the process from Step B1.3 with the new value for displacement as the assumed displacement. This iterative process is amenable to solution using a spreadsheet and usually converges in a few cycles (less than 5). After convergence the performance objective and the displacement demands at the expansion joints (abutments) should be checked. If these are not satisfied adjust Q_d and K_d (Step B1.1) and repeat. It may take several attempts to find the right combination of Q_d and K_d. It is also possible that the performance criteria and the displacement limits are mutually exclusive and a solution cannot be found. In this case a compromise will be necessary, such as increasing the clearance at the expansion joints or allowing limited yield in the columns, or both. Note that Art 9 GSID requires that a minimum clearance be provided equal to 8 S_{D1} T_{eff}/ B_L. (B-18)	**B1.12 Convergence Check, Example 2.6** Since the calculated value for displacement, d (=1.81) is not close to that assumed at the beginning of the cycle (Step B1.1, d = 2.0), use the value of 1.81 as the new assumed displacement and repeat from Step B1.3. After three iterations, convergence is reached at a superstructure displacement of 1.65 in, with an effective period of 1.43 seconds, and a damping factor of 1.7 (30% damping ratio). The displacement in the isolators at Pier 1 is 1.44 in and the effective stiffness of the same isolators is 42.78 k/in. See spreadsheet in Table B1.12-1 for results of final iteration. Since the column shear must equal the isolator shear for equilibrium, the column shear = 42.78 (1.44) = 61.60 k which is less than the maximum allowable (128k) if elastic behavior is to be achieved (as required in Step A3). Also the superstructure displacement = 1.65 in, which is less than the available clearance of 2.5 in. Therefore the above solution is acceptable and go to Step B2. Note that available clearance (2.5 in) is greater than minimum required $= \frac{8\ S_{D1} T_{eff}}{B_L} = \frac{8(0.20)1.43}{1.7} = 1.35\ in$

Table B1.12-1 Simplified Method Solution for Design Example 2.6 – Final Iteration

SIMPLIFIED METHOD SOLUTION

Step A1,A2	W_{SS}	W_{PP}	W_{eff}	S_{D1}	n
	1651.32	256.26	1907.58	0.2	3

Step B1.1	d	1.65	Assumed displacement
	Q_d	82.57	Characteristic strength
	K_d	50.04	Post-yield stiffness

Step	**A1**	**B1.2**	**B1.2**	**A1**	**B1.3**	**B1.3**	**B1.5**	**B1.6**	**B1.7**	**B1.8**	**B1.10**	**B1.10**
	W_j	$Q_{d,j}$	$K_{d,j}$	$K_{sub,j}$	α_j	$K_{eff,j}$	$d_{isol,j}$	$K_{isol,j}$	$d_{sub,j}$	$F_{sub,j}$	$Q_{d,j}\ d_{isol,j}$	$K_{eff,j}(d_{isol,j}+d_{sub,j})^2$
Abut 1	168.48	8.424	5.105	10,000.00	0.001022	10.206	1.648	10.216	0.002	16.839	13.885	27.785
Pier 1	657.18	32.859	19.915	288.87	0.148088	37.260	1.437	42.778	0.213	61.480	47.224	101.441
Pier 2	657.18	32.859	19.915	288.87	0.148088	37.260	1.437	42.778	0.213	61.480	47.224	101.441
Abut 2	168.48	8.424	5.105	10,000.00	0.001022	10.206	1.648	10.216	0.002	16.839	13.885	27.785
Total	1651.32	82.566	50.040		$\Sigma K_{eff,j}$	94.932				156.638	122.219	258.453
					Step	**B1.4**						

Step B1.10	T_{eff}	1.43	Effective period
	ξ	0.30	Equivalent viscous damping ratio

Step B1.11	B_L (B-15)	1.71	
	B_L	1.70	Damping Factor
	d	1.65	Compare with Step B1.1

Step	**B2.1**	**B2.1**	**B2.3**		**B2.6**	**B2.8**
	$Q_{d,i}$	$K_{d,i}$	$K_{isol,i}$		$d_{isol,i}$	$K_{isol,i}$
Abut 1	2.808	1.702	3.405		1.69	3.363
Pier 1	10.953	6.638	14.259		1.20	15.766
Pier 2	10.953	6.638	14.259		1.20	15.766
Abut 2	2.808	1.702	3.405		1.69	3.363

B2. MULTIMODE SPECTRAL ANALYSIS METHOD

In the Multimodal Spectral Analysis Method (Art.7.3), a 3-dimensional, multi-degree-of-freedom model of the bridge with equivalent linear springs and viscous dampers to represent the isolators, is analyzed iteratively to obtain final estimates of superstructure displacement and required properties of each isolator to satisfy performance requirements (Step A3). The results from the Simplified Method (Step B1) are used to determine initial values for the equivalent spring elements for the isolators as a starting point in the iterative process. The design response spectrum is modified for the additional damping provided by the isolators (see Step B2.5) and then applied in longitudinal direction of bridge.
Once convergence has been achieved, obtain the following:

- longitudinal and transverse displacements (u_L, v_L) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

B2.1 Characteristic Strength
Calculate the characteristic strength, $Q_{d,i}$, and post-elastic stiffness, $K_{d,i}$, of each isolator 'i' as follows:

$$Q_{d,i} = \frac{Q_{d,j}}{n} \qquad \text{(B-19)}$$

and

$$K_{d,i} = \frac{K_{d,j}}{n} \qquad \text{(B-20)}$$

where values for $Q_{d,j}$ and $K_{d,j}$ are obtained from the final cycle of iteration in the Simplified Method (Step B1. 12

B2.1Characteristic Strength, Example 2.6
Dividing the results for Q_d and K_d in Step B1.12 by the number of isolators at each support (n = 3), the following values for Q_d /isolator and K_d /isolator are obtained:

- $Q_{d,\,1}$ = 8.42/3 = 2.81 k
- $Q_{d,\,2}$ = 32.86/3=10.95 k
- $Q_{d,\,3}$ = 32.86/3 = 10.95 k
- $Q_{d,\,4}$ = 8.42/3 = 2.81 k

and

- $K_{d,1}$ = 5.10/3 = 1.70 k/in
- $K_{d,2}$ = 19.92/3 = 6.64 k/in
- $K_{d,3}$ = 19.92/3 = 6.64 k/in
- $K_{d,4}$ = 5.10/3 = 1.70 k/in

Note that the K_d values per support used above are from the final iteration given in Table B1.12-1. These are not the same as the initial values in Step B1.2, because they have been adjusted from cycle to cycle, such that the total K_d summed over all the isolators satisfies the minimum lateral restoring force requirement for the bridge, i.e. K_{dtotal} = 0.05 W/d. See Step B1.1. Since d varies from cycle to cycle, $K_{d,j}$ varies from cycle to cycle.

B2.2 Initial Stiffness and Yield Displacement
Calculate the initial stiffness, $K_{u,i}$, and the yield displacement, $d_{y,i}$, for each isolator 'i' as follows:

In the absence of isolator-specific information take

$$K_{u,i} = 10K_{d,i} \qquad \text{(B-21)}$$

and then

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} \qquad \text{(B-22)}$$

B2.2 Initial Stiffness and Yield Displacement, Example 2.6
For an isolator on Pier 1:

$$K_{u,i} = 10K_{d,i} = 10(6.64) = 66.4k/in$$

and

$$d_{y,i} = \frac{Q_{d,i}}{K_{u,i} - K_{d,i}} = \frac{10.95}{(66.4 - 6.64)} = 0.18in$$

As expected, the yield displacement is small compared to the expected isolator displacement (~2 in) and will have little effect on the damping ratio (Eq B-15). Therefore take $d_{y,i}$ = 0.

B2.3 Isolator Effective Stiffness, $K_{isol,i}$

Calculate the isolator stiffness, $K_{isol,i}$, of each isolator 'i':

$$k_{isol,i} = \frac{K_{isol,j}}{n} \qquad \text{(B -23)}$$

B2.3 Isolator Effective Stiffness, $K_{isol,i}$, Example 2.6

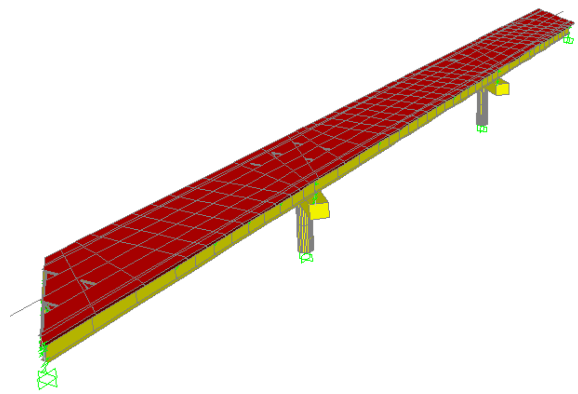
Dividing the results for K_{isol} (Step B1.12) among the 3 isolators at each support, the following values for K_{isol} /isolator are obtained:

- $K_{isol,1}$ = 10.22/3 = 3.41 k/in
- $K_{isol,2}$ = 42.78/3 = 14.26 k/in
- $K_{isol,3}$ = 42.78/3 = 14.26 k/in
- $K_{isol,4}$ = 10.22/3 = 3.41 k/in

B2.4 Finite Element Model

Using computer-based structural analysis software, create a finite element model of the bridge with the isolators represented by spring elements. The stiffness of each isolator element in the horizontal axes (K_x and K_y in global coordinates, K_2 and K_3 in typical local coordinates) is the K_{isol} value calculated in the previous step.

B2.4 Finite Element Model, Example 2.6

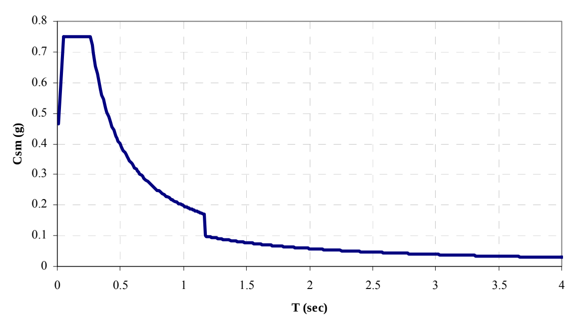


B2.5 Composite Design Response Spectrum

Modify the response spectrum obtained in Step A2 to obtain a 'composite' response spectrum, as illustrated in Figure C1-5 GSID. The spectrum developed in Step A2 is for a 5% damped system. It is modified in this step to allow for the higher damping (ξ) in the fundamental modes of vibration introduced by the isolators. This is done by dividing all spectral acceleration values at periods above 0.8 x the effective period of the bridge, T_{eff}, by the damping factor, B_L.

B.2.5 Composite Design Response Spectrum, Ex 2.6

From the final results of Simplified Method (Step B1.12), B_L = 1.70 and T_{eff} = 1.43 sec. Hence the transition in the composite spectrum from 5% to 30% damping occurs at 0.8 T_{eff} = 0.8 (1.43) = 1.14 sec. The spectrum below is obtained from the 5% spectrum in Step A2, by dividing all acceleration values with periods ≥ 1.14 sec by 1.70.



B2.6 Multimodal Analysis of Finite Element Model

Input the composite response spectrum as a user-specified spectrum in the software, and define a load case in which the spectrum is applied in the longitudinal direction. Analyze the bridge for this load case.

B2.6 Multimodal Analysis of Finite Element Model, Example 2.6

Results of the modal analysis of this bridge are summarized in Table B2.6-1 Here the modal periods and mass participation factors of the first 12 modes are given. The first three modes are the principal isolation modes with periods of 1.57, 1.39 and 1.38 sec respectively. Mode shapes corresponding to these three modes are plotted in Figure B2.6-1. The first

and second modes are seen to be coupled translational modes whereas the third mode is a pure torsional mode (rotation about the Z-axis). It is clear in Figure B2.6-1 that the first mode is predominantly transverse with some longitudinal displacement, and the second mode is predominantly longitudinal with some transverse displacement. This observation is confirmed by the relative sizes of the mass participation factors in Table B2.6-1. Because the coupling is not strong, the results from the Simplified Method are considered to be a good starting point for the iterative Multimodal Analysis.

Table B2.6-1 Modal Properties of Bridge Example 2.6 – First Iteration

Mode	Period	Mass Participating Ratio					
No	Sec	UX	UY	UZ	RX	RY	RZ
1	1.573	0.057	0.772	0.000	0.878	0.002	0.591
2	1.385	0.791	0.062	0.000	0.056	0.018	0.048
3	1.375	0.000	0.000	0.000	0.000	0.000	0.212
4	0.522	0.001	0.007	0.000	0.012	0.001	0.006
5	0.403	0.000	0.000	0.001	0.000	0.001	0.000
6	0.372	0.000	0.000	0.064	0.000	0.048	0.000
7	0.340	0.000	0.001	0.000	0.002	0.000	0.001
8	0.303	0.000	0.000	0.000	0.000	0.000	0.000
9	0.296	0.000	0.000	0.000	0.000	0.000	0.001
10	0.285	0.064	0.061	0.000	0.024	0.000	0.047
11	0.283	0.000	0.000	0.001	0.000	0.001	0.009
12	0.255	0.001	0.009	0.000	0.013	0.007	0.007

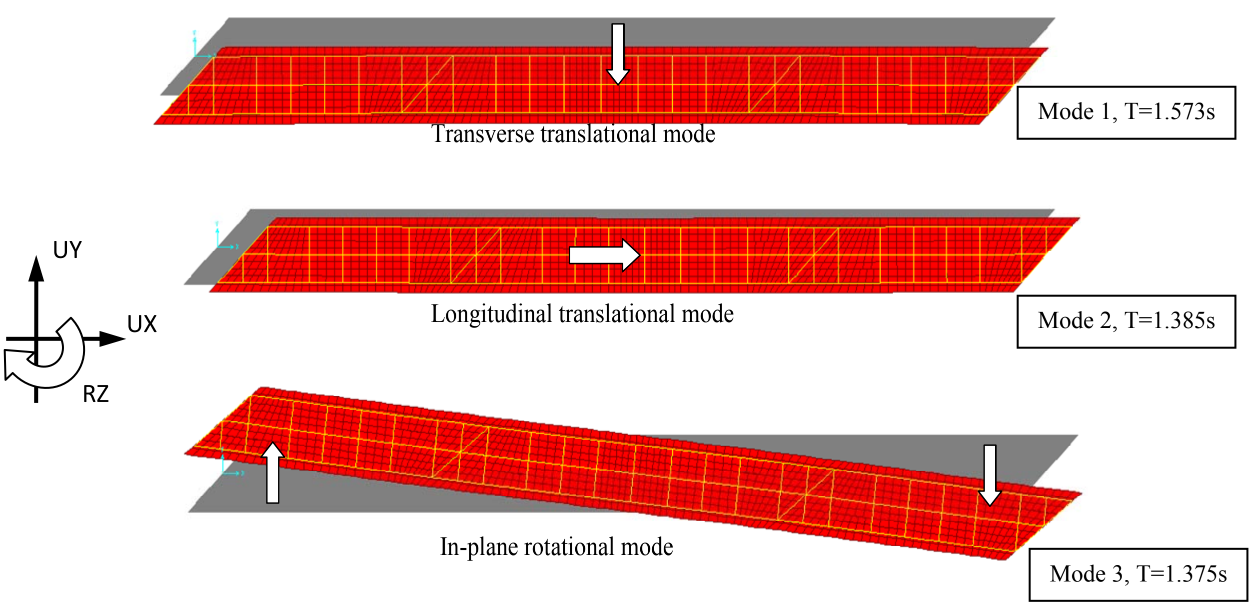


Figure B2.6-1 First Three Mode Shapes for Isolated Bridge with 45^0 Skew (Example 2.6)

Computed values for the isolator displacements due to a longitudinal earthquake are as follows:

Loc.	Isol. #	u_L	v_L	R
Abut1	Isol. 1	1.57	0.45	1.63
	Isol. 2	1.56	0.46	1.63
	Isol. 3	1.55	0.46	1.62
Pier 1	Isol. 1	1.23	0.37	1.29
	Isol. 2	1.23	0.38	1.29
	Isol. 3	1.22	0.38	1.28
Pier 2	Isol. 1	1.22	0.38	1.28
	Isol. 2	1.23	0.38	1.29
	Isol. 3	1.23	0.37	1.29
Abut 2	Isol. 1	1.55	0.46	1.62
	Isol. 2	1.56	0.46	1.63
	Isol. 3	1.57	0.45	1.63

Because of coupling between modes, there is displacement in both the longitudinal and transverse directions even when the earthquake is applied in the longitudinal direction only.

The resultant isolator displacements which will be used to calculate the effective isolator stiffness are (numbers in parentheses are those used to calculate the initial properties to start iteration from the Simplified Method):

- $d_{isol,1}$ = 1.63 (1.65) in
- $d_{isol,2}$ = 1.29 (1.44) in
- $d_{isol,3}$ = 1.29 (1.44) in
- $d_{isol,4}$ = 1.63 (1.65) in

B2.7 Convergence Check

Compare the resulting displacements at the superstructure level (d) to the assumed displacements. These displacements can be obtained by examining the joints at the top of the isolator spring elements. If in close agreement, go to Step B2.9. Otherwise go to Step B2.8.

B2.7 Convergence Check, Example 2.6

The results for isolator displacements are close but not close enough (10% difference at the piers)

Go to Step B2.8 and update properties for a second cycle of iteration.

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L

Use the calculated displacements in each isolator element to obtain new values of $K_{isol,i}$ for each isolator as follows:

$$K_{isol,i} = \frac{Q_{d,i}}{d_{isol,i}} + K_{d,i} \qquad \text{(B-24)}$$

Recalculate $K_{eff,j}$:

Eq. 7.1-6 GSID

$$K_{eff,j} = \frac{K_{sub,j} \sum K_{isol,i}}{(K_{sub,j} + \sum K_{isol,i})} \qquad \text{(B-25)}$$

B2.8 Update $K_{isol,i}$, $K_{eff,j}$, ξ and B_L, Example 2.6

Updated values for $K_{isol,i}$ are given below (previous values are in parentheses):

- $K_{isol,1}$ = 3.43 (3.41) k/in
- $K_{isol,2}$ = 15.17 (14.26) k/in
- $K_{isol,3}$ = 15.17 (14.26) k/in
- $K_{isol,4}$ = 3.43 (3.41) k/in

Since the isolator displacements are relatively close to previous results no significant change in the damping ratio is expected. Hence $K_{eff,j}$ and ξ are not

Recalculate system damping ratio, ξ :

Eq. 7.1-10 GSID

$$\xi = \frac{2\sum_j \sum_i (Q_{d,i}(d_{isol,i} - d_{y,i})}{\pi \sum_j \sum_i (K_{eff,j}(d_{isol,i} + d_{sub,j})^2} \quad \text{(B-26)}$$

Recalculate system damping factor, B_L:

Eq. 7.1-3 GSID

$$B_L = \begin{cases} (\frac{\xi}{0.05})^{0.3} & \xi \leq 0.3 \\ 1.7 & \xi > 0.3 \end{cases} \quad \text{(B-27)}$$

Obtain the effective period of the bridge from the multi-modal analysis and with the revised damping factor (Eq. B-27), construct a new composite response spectrum. Go to Step B2.6.

recalculated and B_L is taken at 1.70.

Since the change in effective period is very small (1.38 to 1.36 sec) and no change has been made to B_L, there is no need to construct a new composite response spectrum in this case. Go back to Step B2.6 (see immediately below).

B2.6 Multimodal Analysis Second Iteration, Example 2.6

Reanalysis gives the following values for the isolator displacements (numbers in parentheses are those from the previous cycle):

Loc.	Isol. #	u_L	v_L	R
Abut1	Isol. 1	1.54	0.46	1.60
	Isol. 2	1.53	0.47	1.60
	Isol. 3	1.52	0.47	1.59
Pier 1	Isol. 1	1.19	0.37	1.25
	Isol. 2	1.19	0.37	1.25
	Isol. 3	1.18	0.38	1.24
Pier 2	Isol. 1	1.18	0.38	1.24
	Isol. 2	1.19	0.37	1.25
	Isol. 3	1.19	0.37	1.25
Abut 2	Isol. 1	1.52	0.47	1.59
	Isol. 2	1.53	0.47	1.60
	Isol. 3	1.54	0.46	1.60

The resultant isolator displacements which will be used to calculate the effective isolator stiffness are:

- $d_{isol,1}$ = 1.60 (1.63) in
- $d_{isol,2}$ = 1.25 (1.29) in
- $d_{isol,3}$ = 1.25 (1.29) in
- $d_{isol,4}$ = 1.60 (1.63) in

Go to Step B2.7

B2.7 Convergence Check

Compare results and determine if convergence has been reached. If so go to Step B2.9. Otherwise Go to Step B2.8.

B2.7 Convergence Check, Example 2.6

Satisfactory agreement has been reached on this second cycle. Go to Step B2.9

B2.9 Superstructure and Isolator Displacements
Once convergence has been reached, obtain

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions of the bridge, and
- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions of the bridge, for each isolator, for this load case (i.e. longitudinal loading). These displacements may be found by subtracting the nodal displacements at each end of each isolator spring element.

B2.9 Superstructure and Isolator Displacements, Example 2.6
From the above analysis:

- superstructure displacements in the longitudinal (x_L) and transverse (y_L) directions are:

  x_L= 1.53 in
  y_L= 0.48 in

- isolator displacements in the longitudinal (u_L) and transverse (v_L) directions are:
  - Abutments: u_L = 1.54 in, v_L = 0.47 in
  - Piers: u_L = 1.19 in, v_L = 0.37 in

  All isolators at same support have the same displacements.

B2.10 Pier Bending Moments and Shear Forces
Obtain the pier bending moments and shear forces in the longitudinal (M_{PLL}, V_{PLL}) and transverse (M_{PTL}, V_{PTL}) directions at the critical locations for the longitudinally-applied seismic loading.

B2.10 Pier Bending Moments and Shear Forces, Example 2.6
Bending moments in single column pier in the longitudinal (M_{PLL}) and transverse (M_{PTL}) directions are:

M_{PLL}= 884
M_{PTL}= 1508 kft

Shear forces in single column pier the longitudinal (V_{PLL}) and transverse (V_{PTL}) directions are

V_{PLL} = 70.63 k
V_{PTL} = 43.85 k

B2.11 Isolator Shear and Axial Forces
Obtain the isolator shear (V_{LL}, V_{TL}) and axial forces (P_L) for the longitudinally-applied seismic loading.

B2.11 Isolator Shear and Axial Forces, Example 2.6
Isolator shear and axial forces are summarized in Table B2.11-1

Table B2.11-1. Maximum Isolator Shear and Axial Forces due to Longitudinal Earthquake.

		V_{LL} (k) Long. shear due to long. EQ	V_{TL} (k) Transv. shear due to long. EQ	P_L (k) Axial forces due to long. EQ
Abut ment	Isol. 1	5.27	1.58	7.42
	Isol. 2	5.26	1.60	9.99
	Isol. 3	5.21	1.61	14.52
Pier	Isol. 1	18.05	5.57	20.77
	Isol. 2	18.03	5.68	15.28
	Isol. 3	17.86	5.69	16.21

STEP C. ANALYZE BRIDGE FOR EARTHQUAKE LOADING IN TRANSVERSE DIRECTION

Repeat Steps B1 and B2 above to determine bridge response for transverse earthquake loading. Apply the composite response spectrum in the transverse direction and obtain the following response parameters:

- longitudinal and transverse displacements (u_T, v_T) for each isolator
- longitudinal and transverse displacements for superstructure
- biaxial column moments and shears at critical locations

C1. Analysis for Transverse Earthquake

Repeat the above process, starting at Step B1, for earthquake loading in the transverse direction of the bridge. Support flexibility in the transverse direction is to be included, and a composite response spectrum is to be applied in the transverse direction. Obtain isolator displacements in the longitudinal (u_T) and transverse (v_T) directions of the bridge, and the biaxial bending moments and shear forces at critical locations in the columns due to the transversely-applied seismic loading.

C1. Analysis for Transverse Earthquake, Example 2.6

Key results from repeating Steps B1 and B2 (Simplified and Mulitmode Spectral Methods) for transverse loading , are as follows:

- T_{eff} = 1.51 sec
- Superstructure displacements in the longitudinal (x_T) and transverse (y_T) directions due to transverse load are as follows:
  x_T = 0.49 in
  y_T = 1.63 in
- Isolator displacements in the longitudinal (u_T) and transverse (v_T) directions due to transverse loading are as follows:

Loc.	Isol. #	u_T	v_T	R
Abut1	Isol. 1	0.51	1.59	1.67
	Isol. 2	0.49	1.62	1.69
	Isol. 3	0.46	1.64	1.70
Pier 1	Isol. 1	0.38	0.87	0.95
	Isol. 2	0.38	0.87	0.95
	Isol. 3	0.38	0.85	0.93
Pier 2	Isol. 1	0.38	0.85	0.93
	Isol. 2	0.38	0.87	0.95
	Isol. 3	0.38	0.87	0.95
Abut 2	Isol. 1	0.46	1.64	1.70
	Isol. 2	0.49	1.62	1.69
	Isol. 3	0.51	1.59	1.67

Because of coupling between modes, there is displacement in both the longitudinal and transverse directions even when the earthquake is applied in the transverse direction only.

- Pier bending moments in the longitudinal (M_{PLT}) and transverse (M_{PTT}) directions due to transverse load are as follows:
  M_{PLT} = 1645 kft
  M_{PTT} = 953 kft
- Pier shear forces in the longitudinal (V_{PLT}) and transverse (V_{PTT}) directions due to transverse load are as follows:

V_{PLT} = 43.01 k
V_{PTT} = 63.67 k

- Isolator shear and axial forces are summarized in Table C1-1.

Table C1-1. Maximum Isolator Shear and Axial Forces due to Transverse Earthquake.

		V_{LT} (k) Long. shear due to transv. EQ	V_{TT} (k) Transv. shear due to transv. EQ	P_T(k) Axial forces due to transv. EQ
Abutment	Isol. 1	1.70	5.34	8.77
	Isol. 2	1.65	5.43	14.68
	Isol. 3	1.55	5.49	23.32
Pier	Isol. 1	6.87	15.48	19.78
	Isol. 2	6.74	15.50	29.00
	Isol. 3	6.71	15.25	31.33

STEP D. CALCULATE DESIGN VALUES (For Isolator 1 at Pier 1)

Combine results from longitudinal and transverse analyses using the (1.0L+0.3T) and (0.3L+1.0T) rules given in Art 3.10.8 LRFD, to obtain design values for isolator and superstructure displacements, column moments and shears.

Check that required performance is satisfied.

D1. Design Isolator Displacements

Following the provisions in Art. 2.1 GSID, and illustrated in Fig. 2.1-1 GSID, calculate the total design displacement, d_t, by combining the displacements from the longitudinal (u_L and v_L) and transverse (u_T and v_T) cases as follows:

- $u_1 = u_L + 0.3u_T$ (D-1)
  $v_1 = v_L + 0.3v_T$ (D-2)
  $R_1 = \sqrt{u_1^2 + v_1^2}$ (D-3)

- $u_2 = 0.3u_L + u_T$ (D-4)
  $v_2 = 0.3v_L + v_T$ (D-5)
  $R_2 = \sqrt{u_2^2 + v_2^2}$ (D-6)

- $d_t = \max(R_1, R_2)$ (D-7)

D1. Design Isolator Displacements, Example 2.6

Load Case 1:

$u_1 = u_L + 0.3u_T$ = 1.0(1.19) + 0.3(0.38) = 1.30 in

$v_1 = v_L + 0.3v_T$ = 1.0(0.37) + 0.3(0.87) = 0.63 in

$R_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{1.30^2 + 0.63^2}$ = 1.44 in

Load Case 2:

$u_2 = 0.3u_L + u_T$ = 0.3(1.19) + 1.0(0.38) = 0.74 in

$v_2 = 0.3v_L + v_T$ = 0.3(0.37) + 1.0(0.88) =0.98 1in

$R_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{0.74^2 + 0.98^2}$ = 1.23 in

Governing Case:

Total design displacement, $d_t = \max(R_1, R_2)$
= 1.44 in

D2. Design Moments and Shears

Calculate design values for column bending moments and shear forces using the same combination rules as for displacements.

Alternatively this step may be deferred because the above results may not be final. Upper and lower bound analyses are required after the isolators have been designed as described in Art 7. GSID. These analyses are required to determine the effect of possible variations in isolator properties due age, temperature and scragging in elastomeric systems. Accordingly the results for column shear in Steps B2.10 and C are likely to increase once these analyses are complete.

D2. Design Moments and Shears, Example 2.6

Load Case 1:

$V_{PL1} = V_{PLL} + 0.3V_{PLT}$ = 1.0(70.63) + 0.3(43.01) = 83.53 k

$V_{PT1} = V_{PTL} + 0.3V_{PTT}$ = 1.0(43.85) + 0.3(63.67) = 62.95 k

$R_1 = \sqrt{V_{L1}^2 + V_{T1}^2} = \sqrt{83.53^2 + 62.95^2}$ = 104.59 k

Load Case 2:

$V_{PL2} = 0.3V_{PLL} + V_{PLT}$ = 0.3(70.63) + 1.0(43.01) = 64.20 k

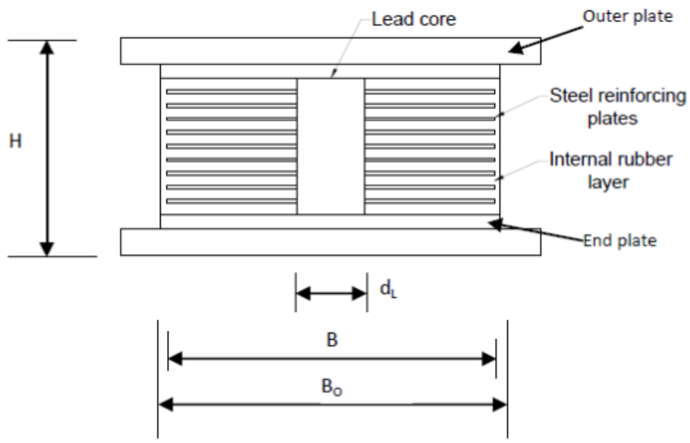
$V_{PT2} = 0.3V_{PTL} + V_{PTT}$ = 0.3(43.85) + 1.0(63.67) = 76.83 k

$R_2 = \sqrt{V_{L2}^2 + V_{T2}^2} = \sqrt{64.20^2 + 76.83^2}$ = 100.12 k

Governing Case:

Design column shear = max ($R1$, $R2$)
= 104.59 K

STEP E. DESIGN OF LEAD-RUBBER (ELASTOMERIC) ISOLATORS



A lead-rubber isolator is an elastomeric bearing with a lead core inserted on its vertical centreline. When the bearing and lead core are deformed in shear, the elastic stiffness of the lead provides the initial stiffness (K_u).With increasing lateral load the lead yields almost perfectly plastically, and the post-yield stiffness $\underline{K_d}$ is given by the rubber alone. More details are given in MCEER 2006.

While both circular and rectangular bearings are commercially available, circular bearings are more commonly used. Consequently the procedure given below focuses on circular bearings. The same steps can be followed for rectangular bearings, but some modifications will be necessary. When sizing the physical dimensions of the bearing, plan dimensions (B, d_L) should be rounded up to the next $^1/_4$" increment, while the total thickness of elastomer, T_r, is specified in multiples of the layer thickness. Typical layer thicknesses for bearings with lead cores are $^1/_4$" and $^3/_8$". High quality natural rubber should be specified for the elastomer. It should have a shear modulus in the range 60-120 psi and an ultimate elongation-at-break in excess of 5.5. Details can be found in rubber handbooks or in MCEER 2006.

The following design procedure assumes the isolators are bolted to the masonry and sole plates. Isolators that use shear-only connections (and not bolts) require additional design checks for stability which are not included below. See MCEER 2006.

Note that the procedure given in this step is intended for preliminary design only. Final design details and material selection should be checked with the manufacturer.

E1. Required Properties Obtain from previous work the properties required of the isolation system to achieve the specified performance criteria (Step A1). • required characteristic strength, Q_d / isolator • required post-elastic stiffness, K_d / isolator • total design displacement, d_t, for each isolator • maximum applied dead and live load (P_{DL}, P_{LL}) and seismic load (P_{SL}) which includes seismic live load (if any) and overturning forces due to seismic loads, at each isolator, and • maximum wind load, P_{WL}		**E1. Required Properties, Example 2.6** The design of one of the exterior isolators on a pier is given below to illustrate the design process for lead-rubber isolators. From previous work • Q_d / isolator = 10.95 k • K_d / isolator = 6.64 k/in • Total design displacement dt = 1.44 in • P_{DL} = 187 k • P_{LL} = 123 k • P_{SL} = 29 k (Table C1-1) • P_{WL} = 8.21 k < Q_d OK
E2. Isolator Sizing		
E2.1 Lead Core Diameter Determine the required diameter of the lead plug, d_L, using:		**E2.1 Lead Core Diameter, Example 2.6**

$$d_L = \sqrt{\frac{Q_d}{0.9}} \quad \text{(E-1)}$$ See Step E2.5 for limitations on d_L		$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{10.95}{0.9}} = 3.49\ in$$
E2.2 Plan Area and Isolator Diameter Although no limits are placed on compressive stress in the GSID, (maximum strain criteria are used instead, see Step E3) it is useful to begin the sizing process by assuming an allowable stress of, say, 1.6 ksi. Then the bonded area of the isolator is given by: $$A_b = \frac{P_{DL} + P_{LL}}{1.6}\ in^2 \quad \text{(E-2)}$$ and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by: $$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} \quad \text{(E-3)}$$ Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using $$A_b = \frac{\pi}{4}(B^2 - d_L^2) \quad \text{(E-4)}$$ Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually 1/2 inch each side). In this case the overall diameter, B_o is given by: $$B_o = B + 1.0 \quad \text{(E-5)}$$		**E2.2 Plan Area and Isolator Diameter, Example 2.6** Based on the final design of the isolators for Example 2.0, increase the allowable stress to 3.2 ksi. $$A_b = \frac{P_{DL} + P_{LL}}{3.2}\ in^2 = \frac{187 + 123}{3.2} = 96.88\ in^2$$ $$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} = \sqrt{\frac{4\,(96.88)}{\pi} + 3.49^2}$$ = 11.64 in Round B up to 12.5 in (based on experience with Example 2.0) and the actual bonded area is: $$A_b = \frac{\pi}{4}(12.50^2 - 3.49^2) = 113.16\ in^2$$ B_o = 12.50 + 2(0.5) = 13.50in
E2.3 Elastomer Thickness and Number of Layers Since the shear stiffness of the elastomeric bearing is given by: $$K_d = \frac{GA_b}{T_r} \quad \text{(E-6)}$$ where G = shear modulus of the rubber, and T_r = the total thickness of elastomer, it follows Eq. E-5 may be used to obtain T_r given a required value for K_d $$T_r = \frac{GA_b}{K_d} \quad \text{(E-7)}$$ If the layer thickness is t_r, the number of layers, n, is given by: $$n = \frac{T_r}{t_r} \quad \text{(E-8)}$$ rounded up to the nearest integer.		**E2.3 Elastomer Thickness and Number of Layers, Example 2.6** $$T_r = \frac{GA_b}{K_d} = \frac{0.1(113.16)}{6.64} = 1.70\ in$$ $$n = \frac{1.70}{0.25} = 6.8$$ Round up to nearest integer, i.e. $n = 7$

<table>
<tr>
<td>Note that because of rounding the plan dimensions and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be necessary if the differences are large.</td>
<td></td>
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</tr>
<tr>
<td>E2.4 Overall Height
The overall height of the isolator, H, is given by:

$$H = n\,t_r + (n-1)t_s + 2t_c \qquad \text{(E-9)}$$
where t_s = thickness of an internal shim (usually about 1/8 in), and
t_c = combined thickness of end cover plate (0.5 in) and outer plate (1.0 in)</td>
<td></td>
<td>E2.4 Overall Height, Example 2.6

$$H = 7(0.25) + 6(0.125) + 2 * 1.5 = 5.50\ in$$</td>
</tr>
<tr>
<td>E2.5 Lead Core Size Check
Experience has shown that for optimum performance of the lead core it must not be too small or too large. The recommended range for the diameter is as follows:
$$\frac{B}{3} \geq d_L \geq \frac{B}{6} \qquad \text{(E-10)}$$</td>
<td></td>
<td>E2.5 Lead Core Size Check, Example 2.6
Since B=16.25 check
$$\frac{12.50}{3} \geq d_L \geq \frac{12.50}{6}$$
i.e., $4.16 \geq d_L \geq 2.08$

Since d_L = 3.49, lead core size is acceptable.</td>
</tr>
<tr>
<td>E3. Strain Limit Check
Art. 14.2 and 14.3 GSID requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e.,

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5 \qquad \text{(E-11)}$$
where γ_c, $\gamma_{s,eq}$, and γ_r are defined below.
(a) γ_c is the maximum shear strain in the layer due to compression and is given by:
$$\gamma_c = \frac{D_c \sigma_S}{GS} \qquad \text{(E-12)}$$
where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = {P_{DL}}/{A_b}$, G is shear modulus, and S is the layer shape factor given by:
$$S = \frac{A_b}{\pi B t_r} \qquad \text{(E-13)}$$
(b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by:
$$\gamma_{s,eq} = \frac{d_t}{T_r} \qquad \text{(E-14)}$$
(c) γ_r is the shear strain due to rotation and is given by:
$$\gamma_r = \frac{D_r B^2 \theta}{t_r T_r} \qquad \text{(E-15)}$$</td>
<td></td>
<td>E3. Strain Limit Check, Example 2.6
Since
$$\sigma_S = \frac{187.0}{113.16} = 1.65\ ksi$$
and
$$S = \frac{113.16}{\pi 12.50(0.25)} = 11.53$$
then
$$\gamma_c = \frac{1.0(1.65)}{0.1(11.53)} = 1.43$$

$$\gamma_{s,eq} = \frac{1.44}{1.75} = 0.82$$

$$\gamma_r = \frac{0.375(12.50^2)(0.01)}{0.25(1.75)} = 1.34$$
Substitution in Eq E-11 gives</td>
</tr>
</table>

where D_r is shape coefficient for rotation in circular bearings = 0.375, and θ is design rotation due to DL, LL and construction effects. Actual value for θ may not be known at this time and a value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Art. 14.4.2.1).		$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 1.43 + 0.82 + 0.5(1.34)$ $= 2.92$ $\leq 5.5\ OK$
E4. Vertical Load Stability Check Art 12.3 GSID requires the vertical load capacity of all isolators be at least 3 times the applied vertical loads (DL and LL) in the laterally undeformed state. Further, the isolation system shall be stable under 1.2(DL+SL) at a horizontal displacement equal to either 2 x total design displacement, d_t, if in Seismic Zone 1 or 2, or 1.5 x total design displacement, d_t, if in Seismic Zone 3 or 4.		**E4. Vertical Load Stability Check, Example 2.6**
E4.1 Vertical Load Stability in Undeformed State The critical load capacity of an elastomeric isolator at zero shear displacement is given by $$P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2}\left[\sqrt{(1+\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2})} - 1\right] \quad \text{(E-16)}$$ where $H_{eff} = T_r + T_s$ T_s = total shim thickness $K_\theta = {E_b I}/{T_r}$ $E_b = E(1 + 0.67S^2)$ E = elastic modulus of elastomer = $3G$ $I = {\pi B^4}/{64}$ It is noted that typical elastomeric isolators have high shape factors, S, in which case: $$\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1 \quad \text{(E-17)}$$ and Eq. E-16 reduces to: $$P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta} \quad \text{(E-18)}$$ Check that: $$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} \geq 3 \quad \text{(E-19)}$$		**E4.1 Vertical Load Stability in Undeformed State, Example 2.6** $E = 3G = 3(0.1) = 0.3\ ksi$ $E_b = 0.3(1 + 0.67(11.53^2)) = 26.89\ ksi$ $I = \pi\frac{12.50^4}{64} = 1198.4 in^4$ $K_\theta = \frac{26.89(1198.4)}{1.75} = 18{,}412 kin/rad$ $K_d = \frac{GA_b}{T_r} = \frac{0.1(113.16)}{1.75} = 6.47 k/in$ $P_{cr(\Delta=0)} = \pi\sqrt{6.47(18{,}412)} = 1084k$ $\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} = \frac{1084}{(187 + 123)} = 3.49 \geq 3\ \ OK$
E4.2 Vertical Load Stability in Deformed State The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by:		**E4.2 Vertical Load Stability in Deformed State, Example 2.6** Since bridge is in Zone 2, $\Delta = 2d_t = 2(1.44) = 2.88$

$$P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)} \qquad \text{(E-20)}$$

where

A_r = overlap area between top and bottom plates of isolator at displacement Δ (Fig. 2.2-1 GSID)

= $B^2(\delta - sin\delta)/_4$

δ = $2cos^{-1}(\Delta/_B)$

A_{gross} = $\pi B^2/_4$

It follows that:

$$\frac{A_r}{A_{gross}} = \frac{(\delta - sin\delta)}{\pi} \qquad \text{(E-21)}$$

Check that:

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} \geq 1 \qquad \text{(E-22)}$$

$$\delta = 2cos^{-1}\left(\frac{2.88}{12.50}\right) = 2.68$$

$$\frac{A_r}{A_{gross}} = \frac{(2.68 - sin2.68)}{\pi} = 0.711$$

$$P_{cr(\Delta)} = 0.711(1084) = 769\ k$$

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{769}{1.2(187) + 29} = 3.03 \geq 1 \ \ OK$$

E5. Design Review

E5. Design Review, Example 2.6

The basic dimensions of the isolator designed above are as follows:

13.50 in (od) x 5.50 in (high) x 3.49 in dia. lead core

and the volume, excluding steel end and cover plates, = 358 in^3

This design is considered satisfactory since both the total strain (Eq E-11) and the vertical load stability factors are reasonable values (not excessively low or excessively high).

E6. Minimum and Maximum Performance Check

Art. 8 GSID requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table E6-1.

Table E6-1. Minimum and maximum values for K_d and Q_d.

Eq. 8.1.2-1 GSID	$K_{d,max} = K_d \lambda_{max,Kd}$	(E-23)
Eq. 8.1.2-2 GSID	$K_{d,min} = K_d \lambda_{min,Kd}$	(E-24)
Eq. 8.1.2-3	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(E-25)

E6. Minimum and Maximum Performance Check, Example 2.6

Minimum Property Modification factors are

$\lambda_{min,Kd}$ = 1.0
$\lambda_{min,Qd}$ = 1.0

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are

$\lambda_{max,a,Kd}$ = 1.1
$\lambda_{max,a,Qd}$ = 1.1

$\lambda_{max,t,Kd}$ = 1.1
$\lambda_{max,t,Qd}$ = 1.4

$\lambda_{max,scrag,Kd}$ = 1.0
$\lambda_{max,scrag,Qd}$ = 1.0

GSID		
Eq. 8.1.2-4 GSID	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(E-26)

Determination of the system property modification factors shall include consideration of the effects of temperature, aging, scragging, velocity, travel (wear) and contamination as shown in Table E6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A, GSID.

Table E6-2. Minimum and maximum values for system property modification factors.

Eq. 8.2.1-1 GSID	$\lambda_{min,Kd} = (\lambda_{min,t,Kd}) (\lambda_{min,a,Kd}) (\lambda_{min,v,Kd}) (\lambda_{min,tr,Kd}) (\lambda_{min,c,Kd}) (\lambda_{min,scrag,Kd})$	(E-27)
Eq. 8.2.1-2 GSID	$\lambda_{max,Kd} = (\lambda_{max,t,Kd}) (\lambda_{max,a,Kd}) (\lambda_{max,v,Kd}) (\lambda_{max,tr,Kd}) (\lambda_{max,c,Kd}) (\lambda_{max,scrag,Kd})$	(E-28)
Eq. 8.2.1-3 GSID	$\lambda_{min,Qd} = (\lambda_{min,t,Qd}) (\lambda_{min,a,Qd}) (\lambda_{min,v,Qd}) (\lambda_{min,tr,Qd}) (\lambda_{min,c,Qd}) (\lambda_{min,scrag,Qd})$	(E-29)
Eq. 8.2.1-4 GSID	$\lambda_{max,Qd} = (\lambda_{max,t,Qd}) (\lambda_{max,a,Qd}) (\lambda_{max,v,Qd}) (\lambda_{max,tr,Qd}) (\lambda_{max,c,Qd}) (\lambda_{max,scrag,Qd})$	(E-30)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity but only to the portion of the λ-factor that is greater than, or less than, unity. Art. 8.2.2 GSID gives these factors as follows:

- 1.00 for critical bridges
- 0.75 for essential bridges
- 0.66 for all other bridges

As required in Art. 7 GSID and shown in Fig. C7-1 GSID, the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Fig C7-1 GSID, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

Applying a system adjustment factor of 0.66 for an 'other' bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd}$ = 1.0 + 0.1(0.66) = 1.066
$\lambda_{max,a,Qd}$ = 1.0 + 0.1(0.66) = 1.066

$\lambda_{max,t,Kd}$ = 1.0 + 0.1(0.66) = 1.066
$\lambda_{max,t,Qd}$ = 1.0 + 0.4(0.66) = 1.264

$\lambda_{max,scrag,Kd}$ = 1.0
$\lambda_{max,scrag,Qd}$ = 1.0

Therefore the maximum overall modification factors

$\lambda_{max,Kd}$ = 1.066(1.066)1.0 = 1.14
$\lambda_{max,Qd}$ = 1.066(1.264)1.0 = 1.35

Since the possible variation in upper bound properties exceeds 15% a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:
$Q_{d,max}$ = 1.35 (10.95) = 14.78 k
and
$K_{d,max}$=1.14(6.64) = 7.57 k/in

E7. Design and Performance Summary

E7. Design and Performance Summary

E7.1 Isolator dimensions
Summarize final dimensions of isolators:

- Overall diameter (includes cover layer)
- Overall height
- Diameter lead core
- Bonded diameter
- Number of rubber layers
- Thickness of rubber layers
- Total rubber thickness
- Thickness of steel shims
- Shear modulus of elastomer

Check all dimensions with manufacturer.

E7.1 Isolator dimensions, Example 2.6
Isolator dimensions are summarized in Table E7.1-1.

Table E7.1-1 Isolator Dimensions

Isolator Location	Overall size including mounting plates (in)	Overall size without mounting plates (in)	Diam. lead core (in)
Under edge girder on Pier	17.5 x 17.5 x 5.5(H)	13.5 dia. x 4.0(H)	3.49

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in)	Total rubber thick-ness (in)	Steel shim thick-ness (in)
Under edge girder on Pier	7	0.25	1.75	0.125

Shear modulus of elastomer = 100 psi

E7.2 Bridge Performance
Summarize bridge performance

- Maximum superstructure displacement (longitudinal)
- Maximum superstructure displacement (transverse)
- Maximum superstructure displacement (resultant)
- Maximum column shear (resultant)
- Maximum column moment (about transverse axis)
- Maximum column moment (about longitudinal axis)
- Maximum column torque

Check required performance as determined in Step A3, is satisfied.

E7.2 Bridge Performance, Example 2.6
Bridge performance is summarized in Table E7.2-1 where it is seen that the maximum column shear is 106.8 k. This less than the column plastic shear (128k) and therefore the required performance criterion is satisfied (fully elastic behavior). Furthermore the maximum longitudinal displacement is 1.53 in which is less than the 2.5 in which is available at the abutment expansion joints and therefore acceptable.

Table E7.2-1 Summary of Bridge Performance

Maximum superstructure displacement (longitudinal)	1.53 in
Maximum superstructure displacement (transverse)	1.63 in
Maximum superstructure displacement (resultant)	1.69 in
Maximum column shear (resultant)	106.8 k
Maximum column moment about transverse axis	1,621 kft
Maximum column moment about longitudinal axis	1,692 kft
Maximum column torque	14.2 kft