Appendices B through E are supplemental to *NCHRP Research Report 1078: MASH Railing Load Requirements for Bridge Deck Overhang* (NCHRP Project 12-119). The full report can be found by searching on *NCHRP Research Report 1078* on the National Academies Press website (nap.nationalacademies.org)

The National Cooperative Highway Research Program (NCHRP) is sponsored by the individual state departments of transportation of the American Association of State Highway and Transportation Officials. NCHRP is administered by the Transportation Research Board (TRB), part of the National Academies of Sciences, Engineering, and Medicine, under a cooperative agreement with the Federal Highway Administration (FHWA). Any opinions and conclusions expressed or implied in resulting research products are those of the individuals and organizations who performed the research and are not necessarily those of TRB; the National Academies of Sciences, Engineering, and Medicine; the FHWA; or NCHRP sponsors.

APPENDIX B

Concrete Barrier Railing on Deck Example

The following design example includes the full analysis of an overhang supporting a barrier railing configured for TL-4 loading consistent with MASH criteria. The evaluation includes both interior and end-region calculations. Extreme event design loading is taken from proposed revisions to Section 13 of the AASHTO *LRFD Bridge Design Specifications* available at the time this example was prepared. The proposed draft language has been provided to AASHTO for consideration.

Note that this example was prepared in Mathcad Prime 8. Due to limitations in variable formatting, commas in subscripts were omitted (e.g., "*cctop*" was used in examples in place of "*cc,top*" in the body of this report and the proposed revisions).

In this example, the adequacy of a deck overhang design to support the attached barrier bridge railing and corresponding impact loads is evaluated. The overhang and barrier design, which was configured for MASH TL-4 loading, is shown in Figure B1.

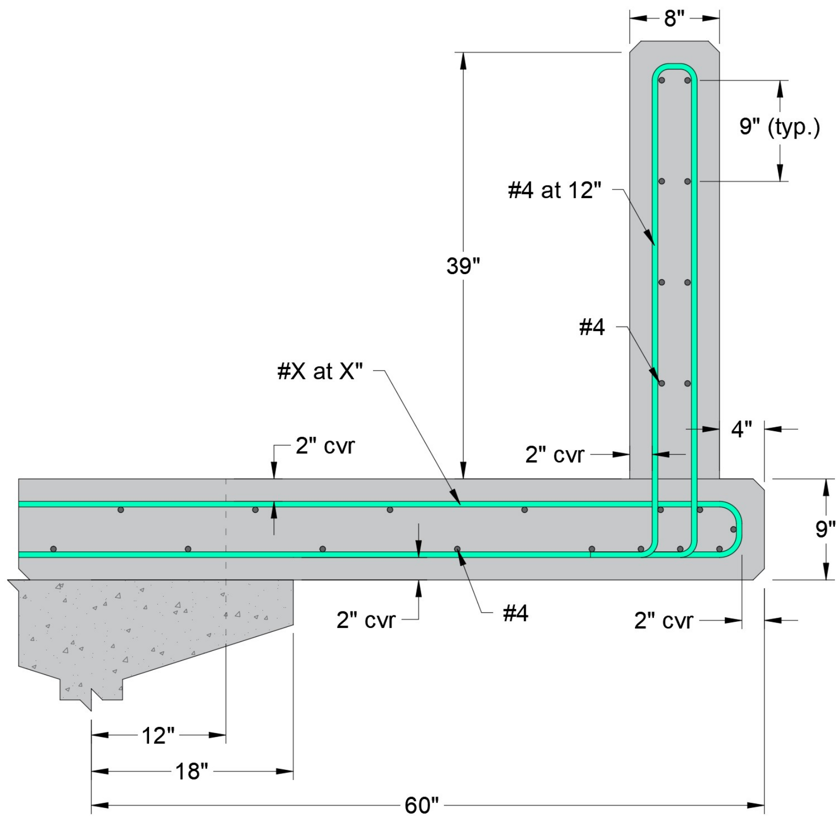


Figure B1. Example overhang with barrier railing

The general design/analysis procedure is as follows:

1. Identify critical overhang regions
2. Configure transverse deck steel for strength limit state
3. Establish design loads and effective tensile demands
4. Estimate bending strength of deck slab at each critical region
5. Calculate critical length and capacity of barrier yield-line mechanism
6. Estimate distributed demands at each Design Region and compare to slab strength
7. Repeat process for end region

Known system parameters

System dimensions and characteristics known prior to the overhang design process are shown below. For this example, it is assumed that the bridge railing design is known, and certain aspects of the slab design are continued from the interior deck region into the overhang. Therefore, deck thickness and deck steel covers are assumed to be known design parameters.

Materials

Concrete compressive strength	$f'_c := 5$ ksi
Effective concrete strength	$f'_{ce} := 1.3\ f'_c = 6.5$ ksi
Concrete design crush strain	$\varepsilon_{cu} := 0.003$
Steel yield stress	$f_y := 60$ ksi
Effective steel yield stress	$f_{ye} := 1.1\ f_y = 66$ ksi

Optional effective material strength factors taken from Table 13.7.3-1

Overhang parameters

Distance from girder CL to edge	$X_G := 60$ in
Girder flange width	$b_{fg} := 36$ in
Slab thickness	$t_s := 9$ in > *8-in. recommended minimum (13.10.2.1)*
Top cover	$c_{ct} := 2$ in
Bottom cover	$c_{cb} := 2$ in

Barrier parameters (interior)

Barrier edge distance	$e_p := 4$ in
Barrier height	$H := 39$ in
Barrier thickness	$t_p := 8$ in
Cover (all faces)	$c_{cp} := 2$ in
Vertical steel	
Bar diameter	$d_{bv} := 0.50$ in
Bar spacing	$s_{bv} := 12$ in
Unit-length layer area	$A_{sv} := 0.25\ \pi \cdot d_{bv}^{\ 2} \cdot 12 \cdot \text{in} \cdot \text{ft}^{-1} \cdot s_{bv}^{\ -1} = 0.2\ \text{in}^2 \cdot \text{ft}^{-1}$
Traffic-side depth	$d_{sv} := t_p - c_{cp} - 0.5\ d_{bv} = 5.75$ in
Field-side depth	$d'_{sv} := c_{cp} + 0.5\ d_{bv} = 2.25$ in
Longitudinal steel	
Bar diameter	$d_{bl} := 0.50$ in
Bar quantity per face	$n_{bl} := 4$
Total layer steel area	$A_{sl} := 0.25\ \pi \cdot d_{bl}^{\ 2} \cdot n_{bl} = 0.79\ \text{in}^2$
Traffic-side depth	$d_{sl} := d_{sv} - 0.5\ d_{bv} - 0.5\ d_{bl} = 5.25$ in
Field-side depth	$d'_{sl} := d'_{sv} + 0.5\ d_{bv} + 0.5\ d_{bl} = 2.75$ in

1. Identify Design Cases and critical overhang regions

The deck overhang is evaluated for two load cases under the extreme event limit state, with vehicular collision forces, CT, taken as:

Design Case 1: transverse and longitudinal forces specified in Article 13.7.2. Longitudinal forces are not discussed in this example. Design Case 1 is evaluated at Design Region A-A, which is a plane coincident with the traffic-side vertical steel, and Design Region B-B, which is a plane coincident with the critical section of the exterior girder. Design Regions for overhangs with barrier railings are shown in Figure B2. For Design Case 1, the system is evaluated in its pre-overlay state to maximize the lateral design load - thus, the effective barrier height about the riding surface is 39 in.

Design Case 2: vertical forces specified in Article 13.7.2. Design Case 2 is evaluated only at Design Region B-B, as the distance to Design Region A-A is insufficient to develop a significant moment demand at that location. For Design Case 2, the system is evaluated assuming a 3-in. overlay has been applied to maximize vertical design load and dead load.

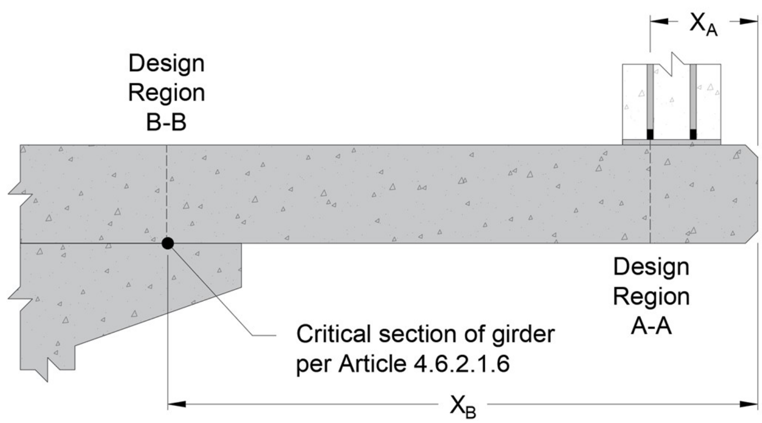


Figure B2. Design Regions for overhang with barrier railing

For this system, Design Region A-A is a section through the center of the vertical bar, which is:

$X_A := e_p + d_{sv} = 9.8 \text{ in}$ from the field edge of the slab.

Design Region B-B is over the critical section of the supporting element. The overhang is supported by a concrete girder with a total top flange width of 36 in. The critical section of the flange is offset from the exterior girder centerline by the lesser of one-third the flange width and 15 in. In this case, the flange width is 36 in.; therefore, the offset distance is 12 in. The distance from the field edge of the slab to Design Region B-B is therefore:

$$X_B := X_G - \frac{b_{fg}}{3} = 48.0 \text{ in}$$

The distance between the two Design Regions is:

$$X_{AB} := X_B - X_A = 38.3 \text{ in}$$

2. Configure transverse steel for strength limit state

The factored moment demand at Design Region B-B under the strength limit state will be used to set an initial transverse steel area requirement. After configuring the steel for adequacy in this limit state, the slab's adequacy in the extreme event limit state will be evaluated.

If the attached railing is structurally continuous, the continuous length of the slab and barrier is at least 25 ft, and the distance from the exterior girder center to the field edge of the slab is less than 6 ft, the design load for the strength limit state may be taken as 1 kip/ft line load placed 1 ft from the traffic face of the railing. This allowance, which is stipulated in Article 3.6.1.3.4, results in a significantly reduced moment demand at Design Region B-B relative to using a distributed wheel load of 16 kips. Strength limit state loading is shown in Figure B3.

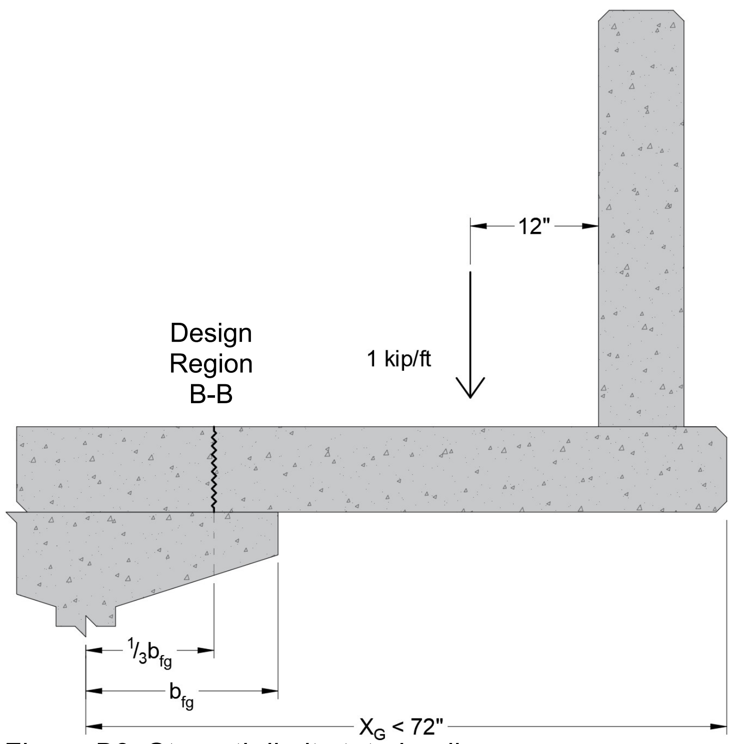


Figure B3. Strength limit state loading

The unfactored moment at Design Region B-B due to the applied wheel load is:

$$M_{LL} := 1\ \frac{kip}{ft} \cdot \left(X_B - e_p - t_p - 12\ in\right) = 2.0\ \frac{kip \cdot ft}{ft}$$

The basic wheel load moment must be increased by the following factors:

Multiple presence factor	Live load factor	Dynamic load allowance
$m := 1.20$	$\gamma_{LL} := 1.75$	$IM := 0.33$

The self-weight moment of the slab and barrier at Design Region B-B is:

$$M_{DC} := 150\ \text{pcf} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot \left(t_s \cdot 0.5\ X_B^{\ 2} + H \cdot t_p \cdot \left(X_B - e_p - 0.5\ t_p\right)\right) = 2\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The dead load factor for structural elements is:

$$\gamma_{DC} := 1.25$$

If an eventual 3-in. wearing surface were added to the slab, the induced moment at region B-B would be:

$$H_{wear} := 3\ \text{in}$$

$$M_{DW} := 140\ \text{pcf} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot \left(H_{wear} \cdot 0.5\ \left(X_B - e_p - t_p\right)^2\right) = 0.2\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The dead load factor for wearing surfaces is:

$$\gamma_{DW} := 1.50$$

Therefore, the design moment at Design Region B-B for the strength limit state is:

$$M_u := M_{LL} \cdot m \cdot (1 + IM) \cdot \gamma_{LL} + M_{DC} \cdot \gamma_{DC} + M_{DW} \cdot \gamma_{DW} = 8.3\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

If #4 transverse bars were placed at 6 in., the transverse bending strength of the slab would be 17.7 kip-ft/ft. Therefore, this trial steel configuration will be evaluated against demands imposed by lateral and vertical MASH loading (extreme event limit state). It is recommended to use a transverse bar spacing of 6 in. or less in the slab to limit debris size.

The transverse bar diameter, spacing, and unit-length area are:

$$d_{bt} := 0.5\ \text{in} \qquad s_{bt} := 6\ \text{in} \qquad A_{st} := 0.25\ \pi \cdot d_{bt}^{\ 2} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot s_{bt}^{\ -1} = 0.39\ \frac{\text{in}^2}{\text{ft}}$$

It should be noted that calculating the live load moment using the 16-kip wheel load distributed over the equivalent strip width results in a significantly greater moment demand than using the 1 k/ft assumption. For this system, spreading the 16-kip wheel load over the 75-in. equivalent strip results in an unfactored region B-B moment of 7.7 k-ft/ft. After applying load factors and considering self-weight, the Strength I demand is 24.2 kip-ft/ft.

3. Establish MASH design loads and overhang tensile demands

The trial design, which is shown below, was deemed in the previous step to be adequate for the strength limit state assuming a 1 k/ft wheel load placed 1 ft from the traffic face of the barrier. In this section, extreme event design loads are established. Further, distributed tensile demands in the slab are estimated, as they are required for the following slab bending strength calculations. The overhang and barrier system with transverse slab steel configured for the Strength I is shown in Figure B4.

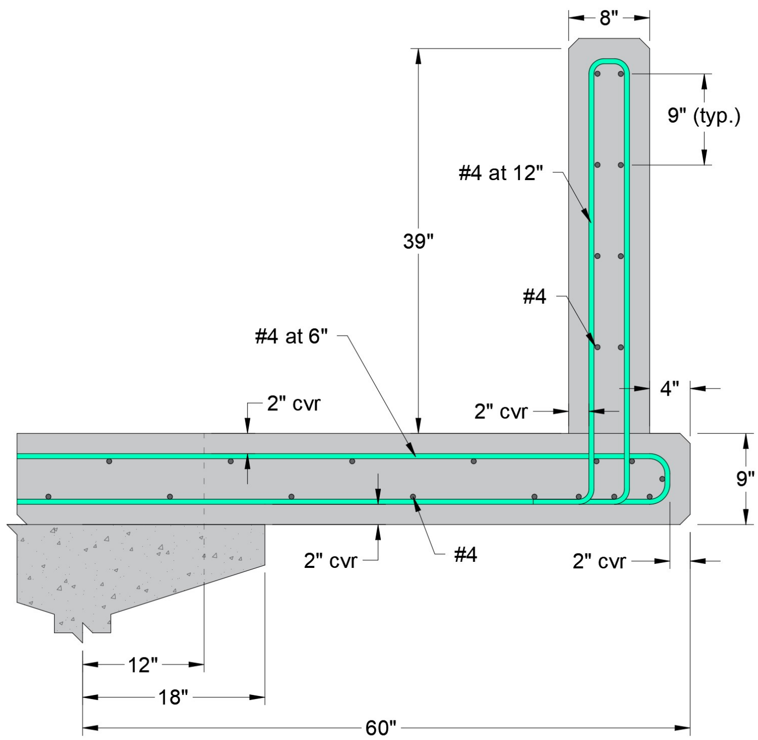


Figure B4. Example system with overhang designed to strength limit state

Design loads are taken from Table 13.7.2-1, which is provided below. In this case, the attached barrier is a MASH TL-4 system - therefore, Table 13.7.2-2 is used, in which design loads and application dimensions are calculated as a function of barrier height.

Design Case 1 - Lateral Loads

For Design Case 1, the system is evaluated in its pre-overlay state - thus, the barrier height used to calculate F_t is 39 in. This state was chosen for evaluation in this design case because TL-4 lateral design loads increase with increasing railing height, due to increased box contact.

Lateral design load	$F_t := 2\ H \cdot \frac{kip}{in} - 4\ kip = 74\ kip$	(Table 13.7.2-2)
Load application height	$H_e := 1.33\ H - 23\ in = 28.9\ in$	(Table 13.7.2-2)
Load application length	$L_t := 5\ ft$	(Table 13.7.2-2)

The distributed tension demand in the slab at Design Region A-A can be taken as:

$$N_{1A} := \frac{F_t}{L_t} = 14.8\ \frac{\text{kip}}{\text{ft}} \tag{13.10.2.3-1}$$

For overhangs supporting barrier railings, distribution of tensile loads with inward transmission through the slab is not permitted. Thus, the distributed tension demand at Design Region B-B is equal to that at A-A:

$$N_{1B} := N_{1A} = 14.8\ \frac{\text{kip}}{\text{ft}} \tag{13.10.2.3-2}$$

Distributed moment demands are not calculated in this step, as the critical length of the barrier yield-line mechanism used to determine distribution lengths is a function of the slab strength. Therefore, the bending strength of the slab must first be calculated.

Design Case 2 - Vertical Loads

For Design Case 2, the system is analyzed after the addition of a 3-in. wearing surface, which decreases the effective barrier height from 39 in. to 36 in. This state was chosen for this design case because TL-4 vertical design loads increase with decreasing railing height due to increased roll of the vehicle onto the railing.

Vertical design load $F_v := 101\ \text{kip} - 1.75\ \left(H - H_{wear}\right) \cdot \frac{\text{kip}}{\text{in}} = 38\ \text{kip}$ (Table 13.7.2-2)

Load application length $L_v := 18\ \text{ft}$ (Table 13.7.2-2)

Note: vertical design loads are applied at the top-field edge of the barrier railing.

Design Case 1 and Design Case 2 loads are shown in Figure B5.

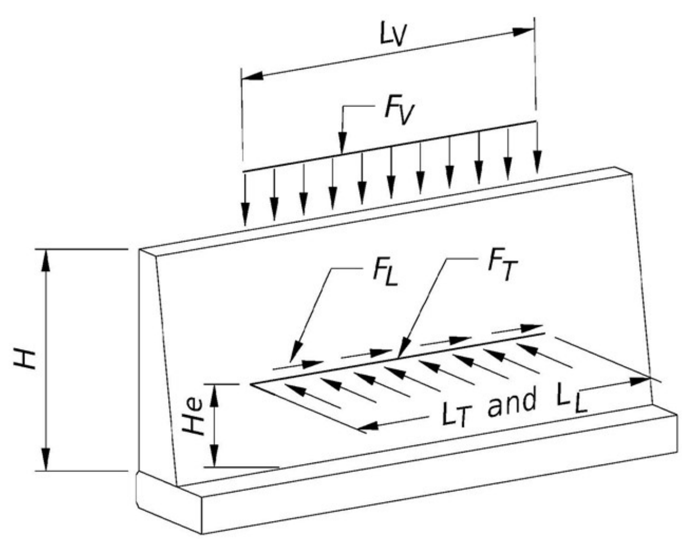


Figure B5. Extreme event limit state Design Case 1 and Design Case 2 design loads and application

Design loads are taken from Table 13.7.2-1 and 13.7.2-2.

Table 13.7.2-1

Design Force or Application Dimension	Railing Test Level				
	TL-1 $H \geq 20$ in.	TL-2 $H \geq 24$ in.	TL-3 $H \geq 30$ in.	TL-4 and TL-5	TL-6 $H \geq 90$ in.
F_t (kip)	17	35	70	See Table 13.7.2-2	350
F_L (kip)	4.5	9	18	See Table 13.7.2-2	75
F_v (kip)	4.5	4.5	4.5	See Table 13.7.2-2	NA
L_t and L_L (ft)	4	4	4	See Table 13.7.2-2	10
L_v (ft)	18	18	18	See Table 13.7.2-2	40
H_e (in.)	18	20	19	See Table 13.7.2-2	64

Table 13.7.2-2

Design Force or Application Dimension	Railing Test Level	
	TL-4 $H \geq 36$ in.	TL-5 $H \geq 42$ in.
F_t (kip)	$2H - 4$ for $36 \leq H \leq 42$ $0.15H + 74$ for $H > 42$	$17.2H - 560$ for $42 \leq H \leq 48$ $5.7H - 8$ for $48 < H \leq 54$ $0.2H + 289$ for $H > 54$
F_L (kip)	$0.87H - 9.6$ for $36 \leq H \leq 42$ $0.01H + 26.5$ for $H > 42$	$0.31H + 60.6$ for $42 \leq H \leq 54$ $79.6 - 0.04H$ for $H > 54$
F_v (kip)	$101 - 1.75H$ for $36 \leq H \leq 45$ $32.7 - 0.23H$ for $H > 45$	$496 - 8H$ for $42 \leq H \leq 54$ $97.4 - 0.62H$ for $H > 54$
L_t and L_L (ft)	4 for $36 \leq H < 39$ 5 for $39 \leq H \leq 42$ $0.09H + 1.2$ for $H > 42$	10
L_v (ft)	18	40
H_e (in.)	$1.33H - 23$ for $36 \leq H \leq 40$ $0.15H + 24.3$ for $H > 40$	$1.43H - 25.9$ for $42 \leq H \leq 51$ $0.04H + 45$ for $H > 51$

4. Estimate bending strength of slab

Prior to calculating the transverse bending strength of the slab, the ability of the slab edge to transfer the barrier loads to Design Region A-A must first be checked. A common damage mechanism observed in deck overhangs supporting bridge railings is a diagonal tension failure of the slab-barrier joint, as shown below.

Frosch & Morel (2016)

NCHRP 12-119 barrier test

Depending on the slab configuration, this damage mechanism can be caused by either compression strut splitting or vertical shear. If adequate edge distance is provided, and hooked transverse bars are used in the slab, a strut-and-tie model may be able to develop in the joint. In this case, diagonal tension failure would be due to transverse splitting of the effectively unreinforced compression strut due to the Poisson effect. If edge distance is minimal and/or transverse slab bars are not hooked, a strut-and-tie behavior cannot be expected, as the top-mat tension tie cannot develop. In this case, diagonal tension failure would be due to vertical shear, as the compression at back edge of the barrier is resisted purely by the vertical shear resistance of the slab. These two cases are discussed below.

In the first case, it is assumed that the transverse bar is able to develop a sufficient tensile stress at the compression-compression-tension (CCT) node to act as a tension tie. This assumption relies on the transverse bar being hooked around a longitudinal bar. Additionally, it is assumed that the top-mat longitudinal bar coincident with the node bears a significant compressive force, bearing on the longitudinal bar and further aiding in development of the tension tie. Diagonal tension damage in this case would be the result of transverse strut splitting. In addition to special steel detailing, a sufficient edge distance must be provided for this mechanism to occur - if the barrier compressive force is centered over the hooked portion of the bar, spalling of the field-edge slab cover would likely occur. An example of a strut-and-tie mechanism which may form in an overhang supporting a barrier railing is shown in Figure B6.

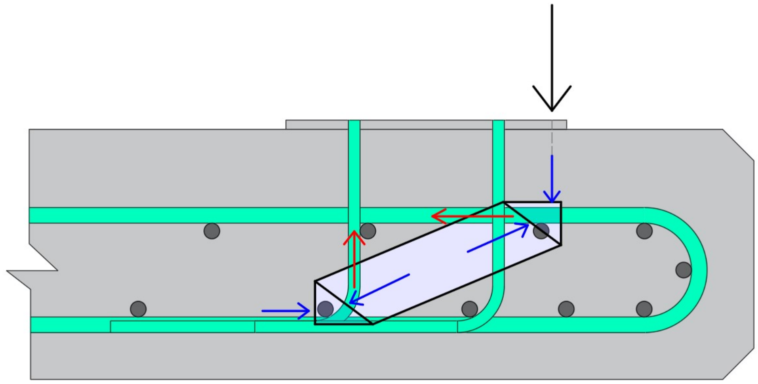


Figure B6. Slab strut-and-tie mechanism (requires special detailing)

To demonstrate the second case, a design variant is shown in Figure B7 in which the edge distance was reduced, and the transverse bars were unhooked. In this case, a top-mat tension tie cannot develop. Diagonal tension damage in this case would be the result of punching shear and a lateral sliding of the field edge concrete off of the straight transverse bars. In cases where the strut-and-tie behavior cannot be justified, this behavior should be assumed.

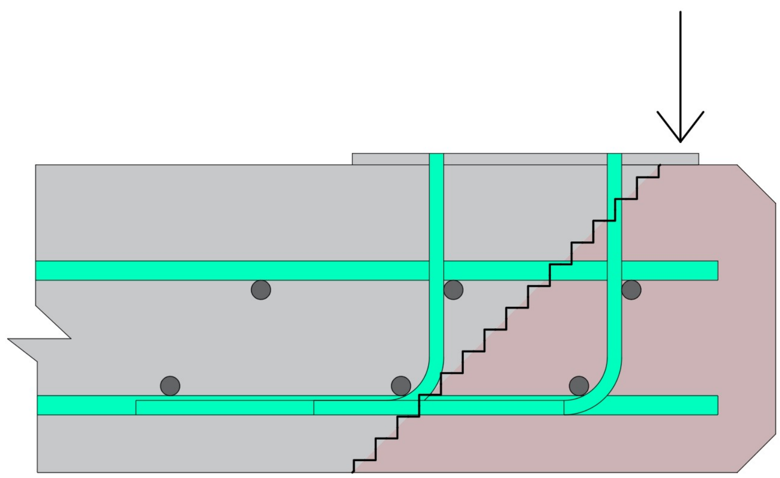


Figure B7. Slab vertical shear mechanism (general case)

For this design, it is assumed that a strut-and-tie behavior can be developed. The generalized orientation of the compression strut is shown in Figure B8. It should be noted that the compression strut orientation shown above is more realistic; however, that shown below is conservative and presented in terms of basic design parameters.

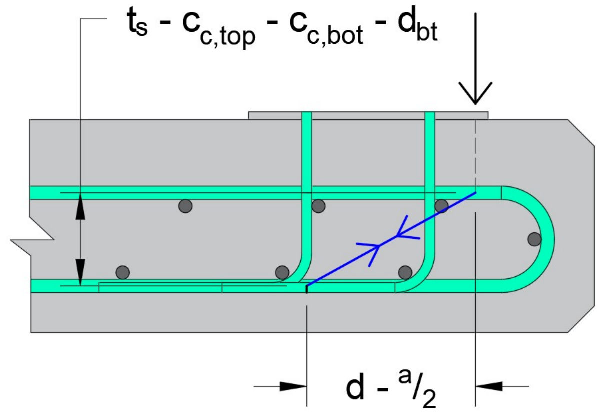


Figure B8. Generalized compression strut orientation

The vertical resistance of the compression strut must be greater than or equal to the required compressive force at the back of the barrier when it reaches its plastic bending strength. Further, in order to orient the compression strut, the compression block depth associated with the barrier bending strength must be estimated. Therefore, the bending strength of the barrier at its base is next calculated in order to define the compression demand acting on the slab joint.

When calculating section strengths for extreme event limit state evaluation, effective material properties (Table 13.7.3-1) are used in lieu of nominal properties. The effective concrete compressive strength and steel yield stress are:

$$f'_{ce} = 6.5 \text{ ksi} \qquad f_{ye} = 66 \text{ ksi} \qquad (13.7.3\text{-}1)$$

Considering effective material properties, the development length of a hooked, 66-ksi #4 bar in 6.5-ksi concrete is:

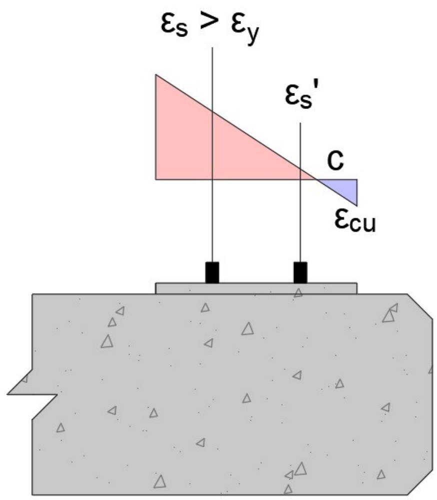
$$l_{dhv} := \frac{38\ d_{bv}}{60} \cdot \frac{f_{ye} \cdot \text{ksi}^{-1}}{\sqrt{f'_{ce} \cdot \text{ksi}^{-1}}} \cdot 0.8 \cdot 1.2 = 7.87 \text{ in} \qquad (5.10.8.2.4\text{a-1 and a-2})$$

where the 0.8 and 1.2 factors are confinement and coating factors, respectively. The distance from the deck surface to the bottom face of the hook is 7 in.; therefore, the maximum achievable stress of the vertical bar, based on AASHTO LRFD BDS development length, is:

$$\frac{7 \text{ in}}{l_{dhv}} f_{ye} = 58.7 \text{ ksi}$$

Full development of vertical barrier bars with embedment less than the code minimum was observed in both the interior and end-region bogie impact tests performed in NCHRP Project 12-119. In those tests, #4 vertical barrier bars reached peak axial strains of 0.8% and 1.2% at the deck surface, respectively, although their embedment depth in the slab was only 7 in. This finding is consistent with those of Alberson (2005), in which hooked #4 vertical bars reached a peak axial strain of 0.23% with 6.75-in. slab embedment. Further, in pullout testing of hooked #5 vertical bars with 6-in. embedment performed by FDOT (2002), seven of seven test specimens were able to reach their yield stress.

The results discussed above suggest that hooked vertical barrier bars can develop their full yield stress without being embedded in the slab to the code minimum if hooked around longitudinal slab steel. For overhang design, the barrier bending strength is a demand, rather than a capacity - as such, it is conservative to assume that the vertical barrier bars can achieve their full yield stress. This assumption results in greater expected overhang demands. For this example, the full bending strength of the barrier is assumed. Assuming the vertical barrier bars can develop their full yield strength at the deck surface, the cantilever bending strength of the barrier at its base is calculated as shown below. In all concrete bending strength calculations performed in this example, both tension-side and compression-side steel layers are considered, as shown in Figure B9.



Consistent with Article 5.6.2

Figure B9.1. Assumed strain distribution

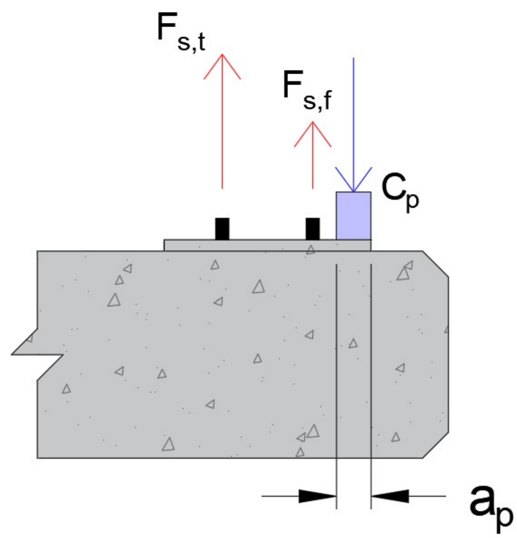


Figure B9.2. Base couple forces at full cantilever bending strength of barrier

The location of the neutral axis was determined iteratively. In this calculation, the location of the neutral axis was varied until steel forces were in equilibrium with concrete compression. This process is a modified form of Eqn. 5.7.3.1.1-4, in which prestressing area is equal to zero, and the analysis is performed on a unit-length basis. The neutral axis depth was found to be:

$$c_p := 0.54 \text{ in}$$

Therefore, the compression block depth is:

$$\beta_1 := 0.85 - 0.05 \cdot \left(f'_{ce} - 4 \text{ ksi}\right) \text{ ksi}^{-1} = 0.725$$

$$a_p := \beta_1 \cdot c_p = 0.39 \text{ in}$$

The effective depth to each layer of vertical barrier steel is:

$$d_{straf} := 8\ \text{in} - 2\ \text{in} - 0.5 \cdot 0.5\ \text{in} = 5.75\ \text{in}$$

8-in. thick rail, 2-in. cover, #4 vertical bars

$$d_{sfield} := 2\ \text{in} + 0.5 \cdot 0.5\ \text{in} = 2.25\ \text{in}$$

The traffic-side steel is assumed to be yielded. At this neutral axis depth, the strain in the field-side steel is:

$$\varepsilon_s' := \frac{\varepsilon_{cu}}{c_p}\left(d_{sfield} - c_p\right) = 0.01$$

The effective yield strain is:

$$\varepsilon_{ye} := \frac{f_{ye}}{29000\ \text{ksi}} = 0.0023$$

Which is less than the field-side steel strain. Therefore, the field-side steel is yielded at the full cantilever bending strength of the railing, and the force in each layer of vertical railing steel is:

$$F_s := f_{ye} \cdot A_{sv} = 13\ \frac{\text{kip}}{\text{ft}}$$

The concrete compressive force is:

$$C_p := 0.85 \cdot f'_{ce} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot a_p = 26\ \frac{\text{kip}}{\text{ft}}$$

Which is in vertical equilibrium with steel forces:

$$F_s' := F_s \qquad F_s + F_s' - C_p = 0.0\ \frac{\text{kip}}{\text{ft}}$$

The cantilever bending strength of the railing at its base is calculated using Eqn. 5.6.3.2.2-1:

$$M_n := \boxed{A_{ps} f_{ps}} \cdot \left(d_p - \frac{a}{2}\right) + A_s f_s \cdot \left(d_s - \frac{a}{2}\right) - A_s' f_s' \cdot \left(d_s' - \frac{a}{2}\right) + \alpha_1 f'_c \cdot \left(b - b_w\right) \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

As the section is rectangular, b_w is equal to b. Additionally, the section does not include prestressing strands. Therefore, the equation is reduced to:

$$M_n := \boxed{A_s f_s} \cdot \left(d_s - \frac{a}{2}\right) - A_s' f_s' \cdot \left(d_s' - \frac{a}{2}\right)$$

In this case, $A_s f_s$ is the force in the traffic-side vertical steel, and $A_s' f_s'$ is the force in the field-side steel. The preceding strain calculations predicted that both layers of steel are yielded in tension. Therefore the stress in the traffic-side steel is:

$$f_s := f_{ye} = 66\ \text{ksi}$$

and the stress in the field-side steel is:

$$f_s' := -f_{ye} = -66 \text{ ksi}$$

It should be noted that the stress in the field-side steel was taken as negative, as Eqn. 5.6.3.2.2-1 assumes the compression-side steel is in compression. Substituting variable names used in this example results in a barrier bending strength of:

$$M_{cbase} := A_{sv} \cdot f_s \cdot \left(d_{straf} - \frac{a_p}{2}\right) - A_{sv} \cdot f_s' \cdot \left(d_{sfield} - \frac{a_p}{2}\right) = 8.2 \ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The procedure used above to calculate $M_{c,base}$ is consistent with Article 5.6.3.2.5, which states "The stress and corresponding strain in any given layer of reinforcement may be taken from any representative stress-strain formula or graph for nonprestressed reinforcement..." In this case, a rectangular compression block and linear strain diagram were assumed.

The compressive force at this moment and the associated compression block depth are:

$$C_p := 25.9 \ \frac{\text{kip}}{\text{ft}} \qquad a_p := 0.4 \text{ in}$$

With the bending strength at the barrier base defined, the diagonal slab compression strut can be evaluated. The angle of the assumed compression strut from horizontal is:

$$\theta_s := \text{atan}\left(\frac{t_s - c_{ct} - c_{cb} - d_{bt}}{d_{sv} - 0.5 \ a_p}\right) = 39 \text{ deg} \qquad \text{(C13.10.2.3-1)}$$

The length of the node perpendicular to the bridge span is assumed equal to twice the barrier compression block depth. The height of the node is assumed equal to the top slab cover. The bearing length of the node is therefore:

$$l_b := \sqrt{\left(2 \ a_p\right)^2 + \left(c_{ct}\right)^2} = 2.15 \text{ in} \qquad \text{(C13.10.2.3-2)}$$

Article 5.8.2.5.3 of the AASHTO LRFD BDS specifies the limiting compressive stress at the node face. The notional area was accounted for in the above calculation of the bearing length, therefore *m* is taken as 1. Specific crack control reinforcement is not provided across the strut, therefore, the limiting compressive stress in the strut is:

$$f_{cu} := (1.0) \ (0.45) \ \left(f'_{ce}\right) = 2.925 \text{ ksi} \qquad \text{(5.8.2.5.3)}$$

It is assumed that, at this stress, the strut will sustain transverse splitting damage. The effective cross-section area of the node face is:

$$A_{cn} := l_b \cdot 12 \ \frac{\text{in}}{\text{ft}} = 25.8 \ \frac{\text{in}^2}{\text{ft}}$$

Therefore, the maximum unit-length load which can be applied along the axis of the strut is:

$$P_{ns} := A_{cn} \cdot f_{cu} = 75.6 \ \frac{\text{kip}}{\text{ft}} \qquad (5.8.2.5.1\text{-}1)$$

The vertical component of this load must be greater than or equal to the compressive force that develops at the back of the barrier at its full capacity. If the vertical component of the strut splitting load is less than the required barrier compressive force, diagonal tension damage of the slab joint is expected. In this case:

$$\frac{C_p}{P_{ns} \cdot \sin(\theta_s)} = 0.54$$

Therefore, the compression strut is assumed to be adequate, and diagonal tension failure will not occur.

If the assumed strut-and-tie model could not develop, the vertical shear capacity of the slab would limit the load transfer from the barrier into Design Region A-A. For comparison purposes, this calculation is performed herein. As the load patch is effectively a strip load, the concrete shear strength is assumed to be equal to the one-way beam shear value, which is:

$$v_c := 0.0633 \cdot \sqrt{f'_{ce} \cdot \text{ksi}^{-1}} \ \text{ksi} = 161 \ \text{psi} \qquad (13.10.2.3\text{-}3)$$

The critical perimeter (unit-length) and capacity of the punching shear mechanism are:

$$b_o := 12 \ \frac{\text{in}}{\text{ft}} \qquad V_n := v_c \cdot b_o \cdot t_s = 17.4 \ \frac{\text{kip}}{\text{ft}} \qquad (13.10.2.3\text{-}4)$$

Which must be greater than or equal to the required compressive force at the back of the barrier to prevent diagonal tension damage in the slab. In this case:

$$\frac{C_p}{V_n} = 1.49$$

Therefore, if the slab design was not configured such that a strut-and-tie behavior could be developed, the slab would sustain diagonal tension damage prior to developing the full capacity of the barrier. In this case, slab bending strengths would be calculated using a reduced slab depth accounting for delamination of the bottom slab cover.

For this design, it is assumed that the strut-and-tie model shown can develop, and the slab will not sustain diagonal tension damage. The bending strength of the slab is therefore calculated as follows. The assumed equilibrium state is shown in Figure B10.

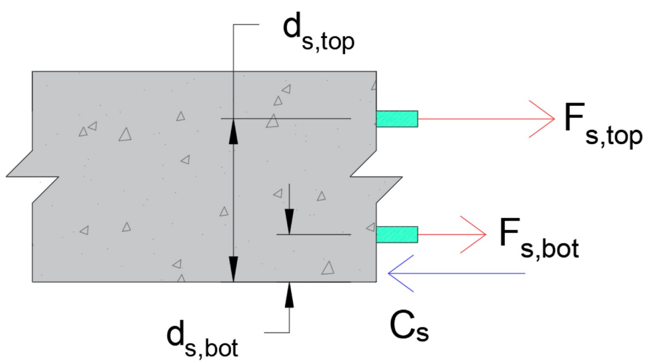


Figure B10. Slab couple forces at full bending strength (passed diagonal tension damage check)

If the preceding diagonal tension damage check were failed, the assumed equilibrium state is as shown in Figure B11.

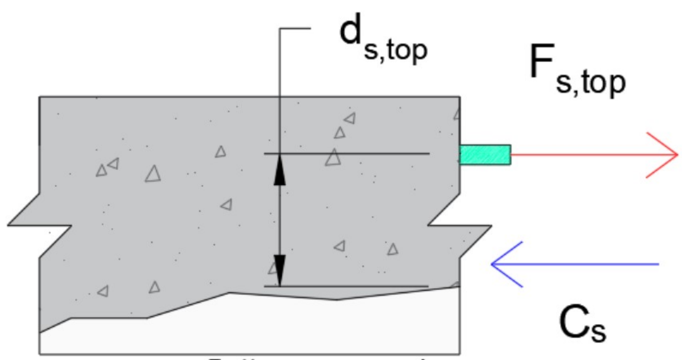


Figure B11. Slab couple forces at full bending strength (failed diagonal tension damage check)

The neutral axis location was iteratively determined to be:

$$c_s := 1.08 \text{ in}$$

The compression block depth is therefore:

$$a_s := \beta_1 \cdot c_s = 0.78 \text{ in}$$

The effective depth to each layer of transverse slab steel is:

$$d_{stop} := 9 \text{ in} - 2 \text{ in} - 0.5 \cdot 0.5 \text{ in} = 6.75 \text{ in}$$ *9-in. thick slab, 2-in. cover, #4 bars*

$$d_{sbot} := 2 \text{ in} + 0.5 \cdot 0.5 \text{ in} = 2.25 \text{ in}$$

The top-mat steel is assumed to be yielded. At this neutral axis depth, the strain in the bottom-mat steel is

$$\varepsilon_{sbot} := \frac{\varepsilon_{cu}}{c_s} \left(d_{sbot} - c_s\right) = 0.003$$

The effective yield strain is:

$$\varepsilon_{ye} := \frac{f_{ye}}{29000\ \text{ksi}} = 0.0023$$

Which is less than the bottom-mat steel strain. Therefore, the field-side steel is yielded at the full cantilever bending strength of the slab, and the force in each layer of transverse steel is:

$$F_{stop} := f_{ye} \cdot A_{st} = 25.9\ \frac{\text{kip}}{\text{ft}}$$

The concrete compressive force is:

$$C_s := 0.85 \cdot f'_{ce} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot a_s = 51.9\ \frac{\text{kip}}{\text{ft}}$$

Which is in vertical equilibrium with steel forces:

$$2\ F_{stop} - C_s = 0\ \frac{\text{kip}}{\text{ft}}$$

The transverse bending strength of the slab is therefore:

$$M_{st} := F_{stop} \cdot \left(d_{stop} - d_{sbot}\right) + C_s \cdot \left(d_{sbot} - 0.5\ a_s\right) = 17.8\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (5.6.3.2.2\text{-}1)$$

For this design, the steel configuration between Design Region A-A and B-B is unchanged, and the #4 bars are fully developed at Design Region A-A, as the distance from A-A to the hook apex is 10 in. Therefore, the bending strength is equal at both regions. The distributed tension acting on the section due to the lateral load is:

$$N_{1A} = 14.8\ \frac{\text{kip}}{\text{ft}}$$

The effect of extreme event, Design Case 1 tension on the bending strength of the slab is accounted for by determining the capacity of the section in pure tension, then linearly interpolating between the pure tension and pure flexure strengths.

The tensile capacity of the slab in pure tension is:

$$P_n := 2\ F_{stop} = 51.8\ \frac{\text{kip}}{\text{ft}} \qquad (5.6.4.4)$$

The flexural capacity of the slab in pure flexure is:

$$M_{st} = 17.8\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The tension acting on the slab is:

$$N_{1A} = 14.8\ \frac{\text{kip}}{\text{ft}}$$

Therefore, the effective bending strength of the slab under the imposed tension is:

$$M_{str} := \left(1 - \frac{N_{1A}}{P_n}\right) M_{st} = 12.7\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The process used to determine the effective bending strength of the slab is shown in Figure B12.

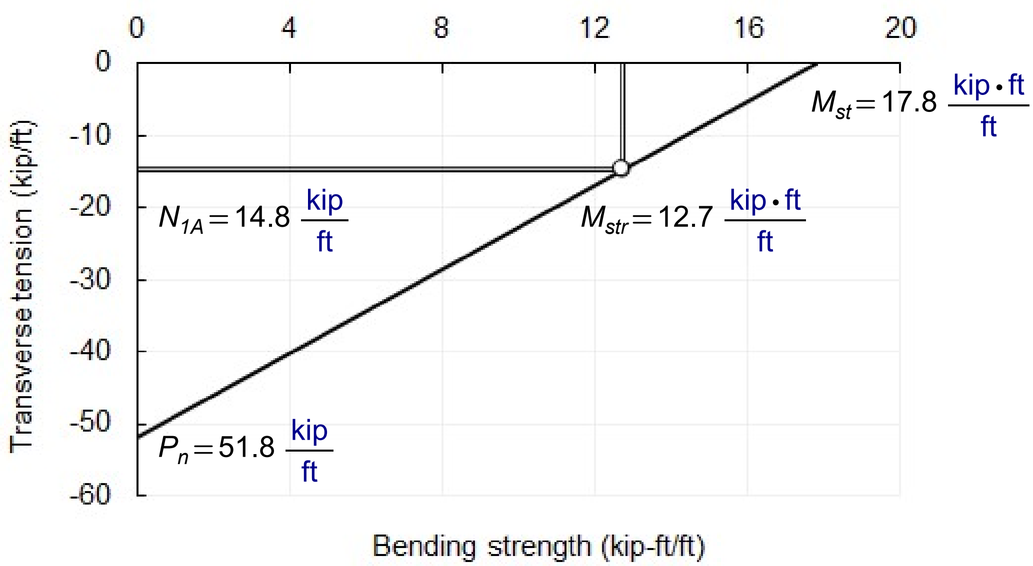


Figure B12. Calculation of effective bending strength under applied axial tension

This method of considering axial tension is consistent with Article 5.6.2.

As the distributed tension demand is conservatively assumed to be equal at both regions, this value is unchanged between Design Regions A-A and B-B.

5. Calculate critical length and capacity of barrier yield-line mechanism

In this section, the barrier yield-line mechanism is defined, and the mechanism critical length and redirective capacity are estimated. The redirective capacity is then compared to the lateral design load, and the critical length of the mechanism is used in the following section to establish effective distribution lengths for flexural demands in the overhang.

The assumed interior yield-line mechanism is shown in Figure B13. For this barrier, the cantilever bending strengths of the horizontal and diagonal yield-lines are equal, as the steel configuration is unchanged over the height of the wall, and the bars are fully developed at the deck surface. The cantilever (span-axis) and wall (vertical-axis) bending strengths are:

$$M_c := M_{cbase} = 8.2\ \frac{kip \cdot ft}{ft} \qquad M_w := 32.5\ kip \cdot ft$$

(39-in. tall, 8-in. thick railing with four #4 bars per face and 2-in. cover)

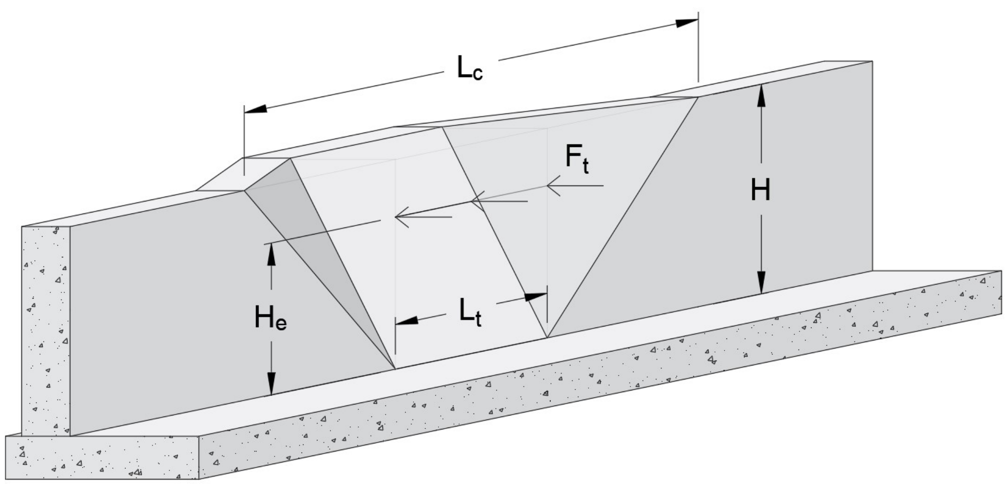


Figure B13. Interior yield-line mechanism

As the horizontal yield-line forms at the deck surface, its flexural strength is limited to the lesser of the barrier bending strength and the slab bending strength. If the slab bending strength is less than that of the barrier, it is assumed that the horizontal yield-line will extend into the transverse slab steel. Therefore, in the following calculations, $M_{c,base}$ is limited to a maximum value of M_{str}. For this design, no adjustment is needed.

The length of the critical interior yield-line mechanism is:

$$L_c := L_t + \sqrt{\frac{8\ M_w \cdot H}{M_c}} = 15.1\ ft \qquad (13.7.3.1.1\text{-}2)$$

The redirective capacity of the barrier is:

$$R_w := \frac{H}{H_e}\left(min\left(M_{cbase}, M_{str}\right) \cdot \frac{L_t}{H} + M_c \cdot \frac{L_c - L_t}{H} + M_w \cdot \frac{8}{L_c - L_t}\right) = 86\ kip \qquad (13.7.3.1.1\text{-}1)$$

The redirective capacity of the barrier, as-calculated via the method shown above, must be greater than the lateral design load. In this case:

$$\frac{F_t}{R_w} = 0.86$$

Therefore, the redirective capacity of the barrier is adequate for TL-4 loading.

---

6. Compare distributed overhang demands to slab strength

In this section, distributed flexural demands are compared to the transverse bending strength of the slab at each critical region. Required checks are summarized below.

Design Case 1 - lateral loads

- Distributed moment demand compared to tension-penalized slab strength at A-A and B-B
- Excludes eventual 3-in. wearing surface to maximize lateral load

Design Case 2 - vertical loads

Distributed moment demand compared to unpenalized slab strength at B-B only
- Includes eventual 3-in. wearing surface to maximize vertical load

For each design case at each critical region, moment demands are estimated by dividing the total applied moment over an effective distribution length.

The effective distribution lengths for Design Case 1 moment at each Design Region are shown in Figure B14.

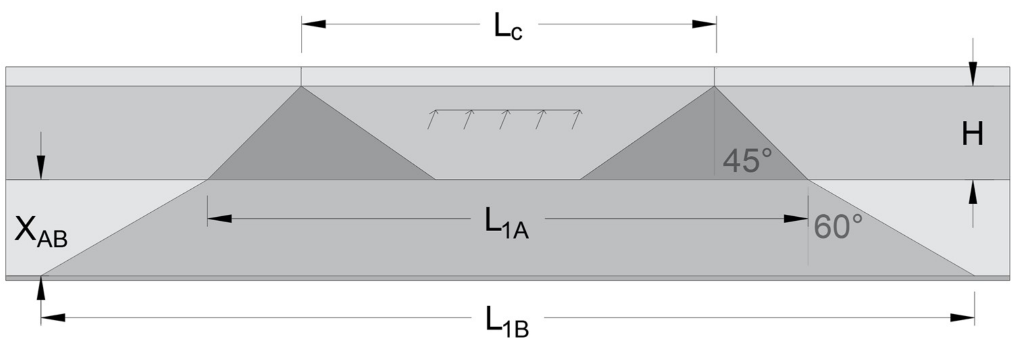


Figure B14. Effective distribution lengths for Design Case 1 moment demands

Distribution length for Design Case 1 at A-A:

$$L_{A1} := L_c + 2\ H = 21.6\ \text{ft} \qquad (13.10.2.3\text{-}5)$$

Self-weight moment at A-A:

$$M_{swA} := 0.1\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

Distributed moment demand for Design Case 1 at A-A:

$$M_{A1} := min\left(\frac{F_t \cdot (H_e + 0.5\ t_s)}{L_{A1}}, M_{cbase}\right) + M_{swA} = 8.3\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.3\text{-}11)$$

Distribution length for Design Case 1 at B-B:

$$L_{B1} := L_{A1} + 2\ X_{AB} \cdot \tan(60\ \text{deg}) = 32.7\ \text{ft} \qquad (13.10.2.3\text{-}6)$$

Distributed moment demand for Design Case 1 at B-B:

$$M_{B1} := \frac{F_t \cdot (H_e + 0.5\ t_s)}{L_{B1}} + M_{DC} = 8.3\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.3\text{-}12)$$

The effective distribution length for Design Case 2 moment at Design Region B-B is shown in Figure B15. The distribution angle for transmission from Design Region A-A to B-B is reduced from 60 degrees to 45 degrees.

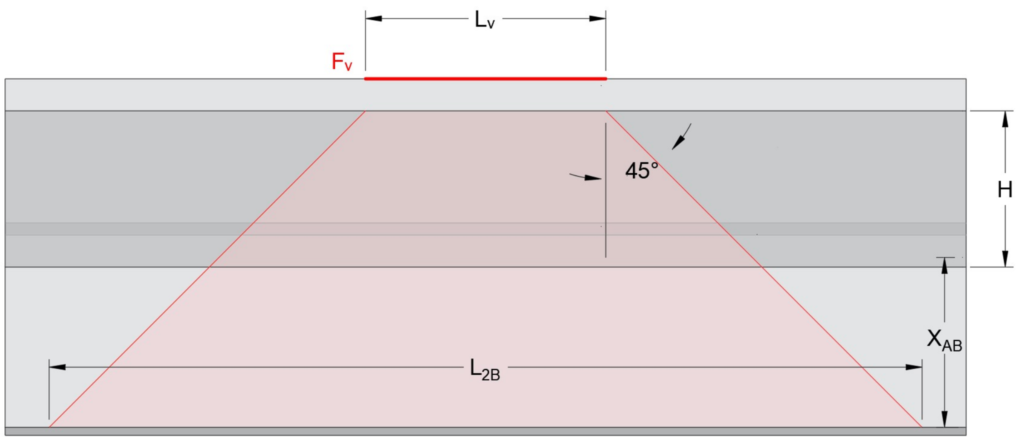


Figure B15. Effective distribution lengths for Design Case 2 moment demands

Distribution length for Design Case 2 at B-B:

$$L_{2B} := L_v + 2\ H + 2\ X_{AB} = 30.9\ \text{ft} \qquad (13.10.2.3\text{-}13)$$

Distributed moment demand for Design Case 2 at B-B:

$$M_{B2} := \frac{F_v \cdot (X_B - e_p)}{L_{2B}} + M_{DC} + M_{DW} = 6.7\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.3\text{-}16)$$

Demands are summarized and compared to corresponding slab strengths below:

Design Case 1, Design Region A-A (13.10.2.3-9)

Demand $M_{A1} = 8.317 \frac{kip \cdot ft}{ft}$

Capacity $M_{str} = 12.7 \frac{kip \cdot ft}{ft}$

Utility ratio $\frac{M_{A1}}{M_{str}} = 0.66$

Design Case 1, Design Region B-B (13.10.2.3-10)

Demand $M_{B1} = 8.3 \frac{kip \cdot ft}{ft}$

Capacity $M_{str} = 12.7 \frac{kip \cdot ft}{ft}$

Utility ratio $\frac{M_{B1}}{M_{str}} = 0.65$

Design Case 2, Design Region B-B (13.10.2.3-15)

Demand $M_{B2} = 6.7 \frac{kip \cdot ft}{ft}$

Capacity $M_{st} = 17.8 \frac{kip \cdot ft}{ft}$

Utility ratio $\frac{M_{B2}}{M_{st}} = 0.37$ *Tension not considered for Design Case 2*

As extreme event limit states are satisfied for Design Cases 1 and 2, the overhang design is **adequate** to support the attached railing in the interior region.

7. End region evaluation

In this step, the barrier design is evaluated for the extreme event limit state (Design Cases 1 and 2) for an end-region impact. In an end-region event, load distributions are restricted to one direction, resulting in higher-magnitude moment and tension demands in the slab. Further, the cantilever bending strength of the barrier is increased at the free end, resulting in a greater maximum moment transfer from the barrier into the slab. For this example, the barrier vertical steel spacing was reduced from 12 in. to 8 in., and the number of longitudinal bars was doubled. Increasing the number of barrier longitudinal bars at the end region allows for increased barrier capacity without increasing the demand on the slab. Relying only on vertical steel increases to reach the end-region capacity may result in demands which cannot be supported by the slab. As shown in Figure B16, transverse slab steel spacing was reduced from 6 in. to 4 in.

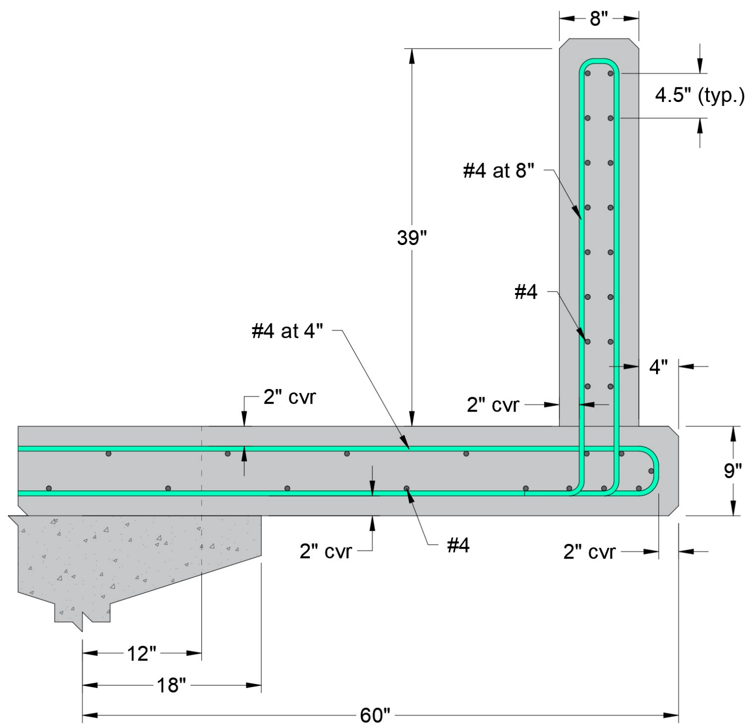


Figure B16. End-region overhang and railing design

Critical regions and design loads are unchanged for the end region. Additional calculations required for the end region include the barrier directional bending strengths and yield-line definition, the transverse bending strength of the slab, and distributed moment demands in the slab at each critical region.

Bending strength of barrier at base

Spacing of vertical #4 bars was reduced from 12 in. to 8 in. The bending strength associated with this configuration and the associated compressive force and block depth are:

$$M_{cbase} := 12.0 \ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad C_p := 38.9 \ \frac{\text{kip}}{\text{ft}} \qquad a_p := 0.6 \ \text{in}$$

Slab joint evaluation

As the cantilever bending strength of the barrier increased, the compression strut must be re-evaluated.

$$\theta_s := \operatorname{atan}\left(\frac{t_s - c_{ct} - c_{cb} - d_{bt}}{d_{sv} - 0.5\ a_p}\right) = 39.5\ \text{deg} \qquad \text{(C13.10.2.3-1)}$$

$$l_b := \sqrt{(2\ a_p)^2 + (c_{ct})^2} = 2.33\ \text{in} \qquad \text{(C13.10.2.3-2)}$$

$$A_{cn} := l_b \cdot 12\ \frac{\text{in}}{\text{ft}} = 28\ \frac{\text{in}^2}{\text{ft}}$$

$$P_{nsy} := A_{cn} \cdot f_{cu} \cdot \sin(\theta_s) = 52.1\ \frac{\text{kip}}{\text{ft}} \qquad \text{(C13.10.2.3-3)}$$

$$\frac{C_p}{P_{nsy}} = 0.746$$

Therefore, diagonal tension failure is not expected for the end-region impact event.

Distributed tension

The effective tensile demand in the slab for end-region loading can be estimated as:

$$N_A := \frac{F_t}{L_t} = 14.8\ \frac{\text{kip}}{\text{ft}} \qquad \text{(13.10.2.3-3)}$$

Slab transverse bending strength

$M_{st} := 24.5\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$ Basic bending strength of full slab depth

$M_{str} := 16.6\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$ Bending strength of reduced slab depth accounting for axial tension by linearly interpolating between pure tensile strength and pure bending strength

Barrier yield-line mechanism

Directional bending strengths of the barrier at the end region are:

$\boxed{M_w} := 52.6\ \text{kip} \cdot \text{ft}$ *Four #4 longitudinal bars added per face*

$\boxed{M_c} := M_{cbase} = 12.0\ \dfrac{\text{kip} \cdot \text{ft}}{\text{ft}}$ *Spacing reduced from 12 in. to 8 in.*

The end-region barrier yield-line mechanism is shown in Figure B17.

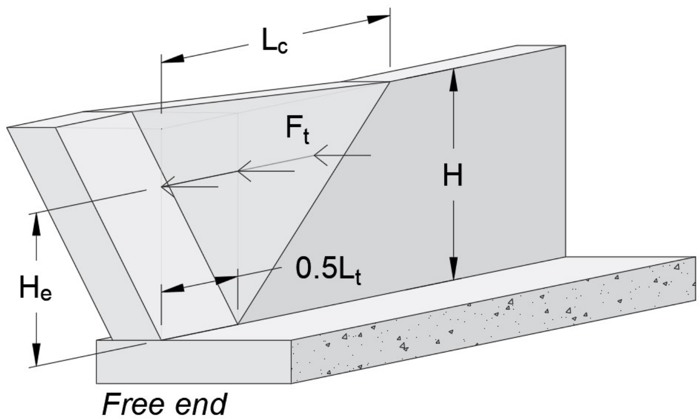


Figure B17. End-region barrier yield-line mechanism

The length of the critical end-region yield-line mechanism is:

$$\boxed{L_c} := \frac{1}{8 \cdot M_c}\left(5 \cdot M_c \cdot L_t + \sqrt{M_c \cdot \left(M_c \cdot L_t^{\,2} + 4 \cdot M_{cbase} \cdot L_t^{\,2} + 128 \cdot H \cdot M_w\right)}\right) = 8.6\ \text{ft}$$

(13.7.3.1.1-4)

The redirective capacity of the barrier is:

$$\boxed{R_w} := \frac{H}{H_e} \cdot \left(3 + \frac{L_c - L_t}{L_c - 0.5\ L_t}\right)^{-1} \cdot \left(\frac{8\ M_w}{L_c - 0.5\ L_t} + \frac{4\ M_c \cdot \left(L_c - 0.5\ L_t\right)}{H} + \frac{2\ M_{cbase} \cdot L_t}{H}\right) = 74\ \text{kip}$$

(13.7.3.1.1-3)

Therefore, the demand-to-capacity ratio for the MASH impact load is:

$$\frac{F_t}{R_w} = 1.00$$

Distributed moment demands for Design Case 1 at A-A

Distribution length and demand at A-A (13.10.2.3-7)

$$L_{1A} := L_c + H = 11.9 \text{ ft}$$

$$M_{1A} := min\left(\frac{F_t \cdot (H_e - 0.5\ t_s)}{L_{1A}}, M_{cbase}\right) + M_{swA} = 12.1\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.3\text{-}11)$$

Utility ratio

$$\frac{M_{1A}}{M_{str}} = 0.73 \qquad (13.10.2.3\text{-}9)$$

Distributed moment demands for Design Case 1 at B-B

Distribution length and demand at B-B (13.10.2.3-8)

$$L_{1B} := L_c + H + X_{AB} \cdot \tan(60 \text{ deg}) = 17.4 \text{ ft}$$

$$M_{1B} := \frac{F_t \cdot (H_e - 0.5\ t_s)}{L_{1B}} + M_{DC} = 10.6\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.3\text{-}12)$$

Utility ratio

$$\frac{M_{1B}}{M_{str}} = 0.64 \qquad (13.10.2.3\text{-}10)$$

Distributed moment demands for Design Case 2 at B-B

Distribution length and moment demand at A-A (13.10.2.3-14)

$$\boxed{L_{2B}} := L_v + H = 21.3 \text{ ft}$$

$$M_{2B} := \frac{F_v \cdot (X_B - e_p)}{L_{2B}} + M_{DC} + M_{DW} = 8.7\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.3\text{-}16)$$

Utility ratio

$$\frac{M_{2B}}{M_{st}} = 0.36$$ *Tension not considered in Design Case 2* (13.10.2.3-15)

APPENDIX C

Open Concrete Railing Post on Deck Example

The following design example includes the full analysis of an overhang supporting an open concrete railing configured for MASH TL-4 loading. Extreme event design loading is taken from proposed revisions to Section 13 of the AASHTO LRFD Bridge Design Specifications available at the time this example was prepared. The proposed draft language has been provided to AASHTO for consideration.

Note that this example was prepared in Mathcad Prime 8. Due to limitations in variable formatting, commas in subscripts were omitted (e.g., "*cctop*" was used in examples in place of "*cc,top*" in the body of this report and the proposed revisions). Similarly, Y was used in examples in place of $\overline{Y}$.

In this example, the adequacy of a deck overhang design to support the attached concrete post-and-beam railing is evaluated. The overhang and railing design, which was configured for TL-4 loading, is shown in Figure C1.

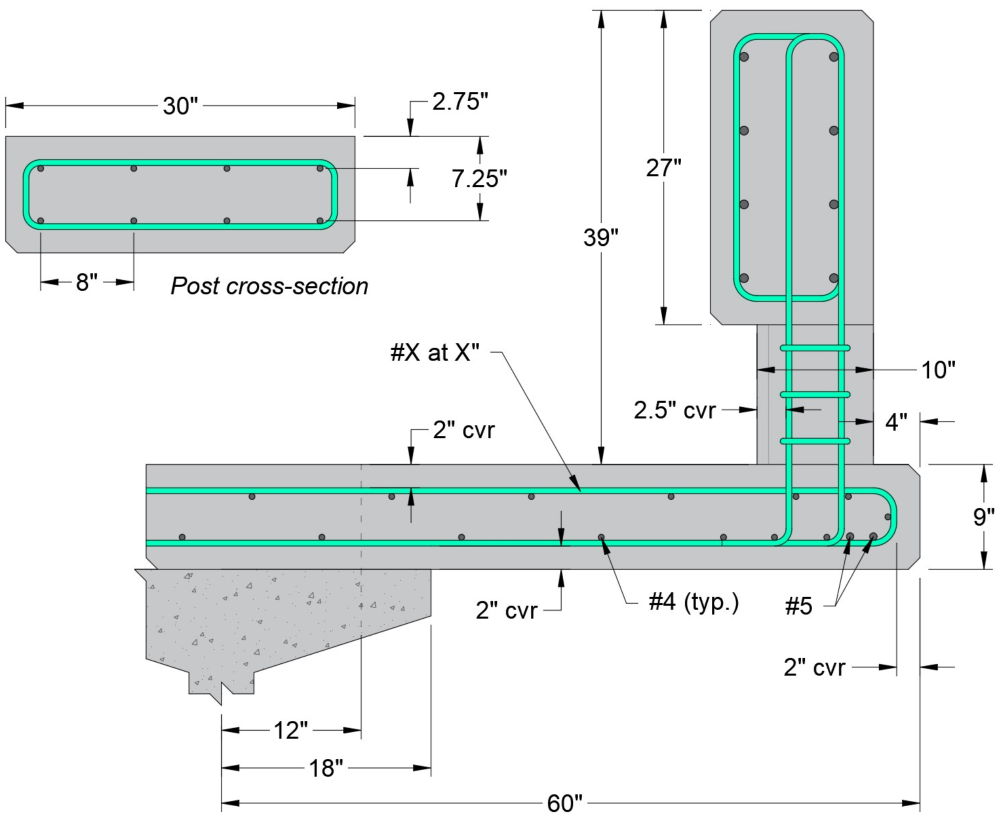


Figure C1. Example concrete post-and-beam railing system

The general design/analysis procedure is as follows:

1. Identify critical overhang regions
2. Configure transverse deck steel for strength limit state
3. Establish ultimate capacity of post and associated deck overhang demands
4. Check the deck-post joint for diagonal tension damage
5. Calculate the yield-line capacity of the slab and compare to required compressive force
6. Estimate distributed overhang demands at Region B-B and compare to slab strength

Known system parameters

System dimensions and characteristics known prior to the overhang design process are shown below. For this example, it is assumed that the bridge railing design is known, and certain aspects of the slab design are continued from the interior deck region into the overhang. Therefore, deck thickness and deck steel covers are assumed to be known design parameters.

Materials

Concrete compressive strength	$f'_c := 5\ \text{ksi}$
Effective concrete strength	$f'_{ce} := 1.3\ f'_c = 6.5\ \text{ksi}$
Concrete design crush strain	$\varepsilon_{cu} := 0.003$
Compression block depth factor	$\beta_1 := 0.85 - 0.05\ \left(f'_{ce} \cdot \text{ksi}^{-1} - 4\right) = 0.73$
Steel yield stress	$f_y := 60\ \text{ksi}$
Effective steel yield stress	$f_{ye} := 1.1\ f_y = 66\ \text{ksi}$

<u>Optional</u> effective material strength factors taken from Table 13.7.3-1

Overhang parameters

Distance from girder CL to edge	$X_G := 60\ \text{in}$
Girder flange width	$b_{fg} := 36\ \text{in}$
Slab thickness	$t_s := 9\ \text{in}$ *> 8-in. recommended minimum (13.10.2.1)*
Top cover	$c_{ctop} := 2\ \text{in}$
Bottom cover	$c_{cbot} := 2\ \text{in}$

Concrete post-and-beam railing parameters

Total railing height	$H := 39\ \text{in}$
Beam centroid height	$Y := H - 0.5 \cdot 27\ \text{in} = 25.5\ \text{in}$
Post edge distance	$e_p := 4\ \text{in}$
Post thickness	$t_p := 10\ \text{in}$
Post width	$W_p := 30\ \text{in}$
Vertical bar diameter	$d_{bp} := 0.5\ \text{in}$
Vertical bar qty. per face	$n_{bp} := 4$
Vertical bar area per face	$A_{sp} := 0.25\ \pi \cdot d_{bp}^{\ 2} \cdot n_{bp} = 0.8\ \text{in}^2$
Vertical bar cover	$c_{cp} := 2.5\ \text{in}$
Depth to traffic-side bars	$d_s := t_p - c_{cp} - 0.5\ d_{bp} = 7.3\ \text{in}$
Depth to field-side bars	$d_s' := c_{cp} + 0.5\ d_{bp} = 2.8\ \text{in}$
Post spacing	$L := 9\ \text{ft}$

1. Identify Design Cases and critical overhang regions

The deck overhang is evaluated for two load cases under the extreme event limit state, with vehicular collision forces, CT, taken as:

Design Case 1: transverse and longitudinal forces developed at the plastic moment capacity of the post. Longitudinal forces are not discussed in this example. Design Case 1 is evaluated at Design Region A-A, which, for overhangs supporting posts, is a trapezoidal yield-line mechanism, and Design Region B-B, which is a plane coincident with the critical section of the exterior girder. Design Regions for overhangs with concrete post-and-beam railings are shown in Figure C2.

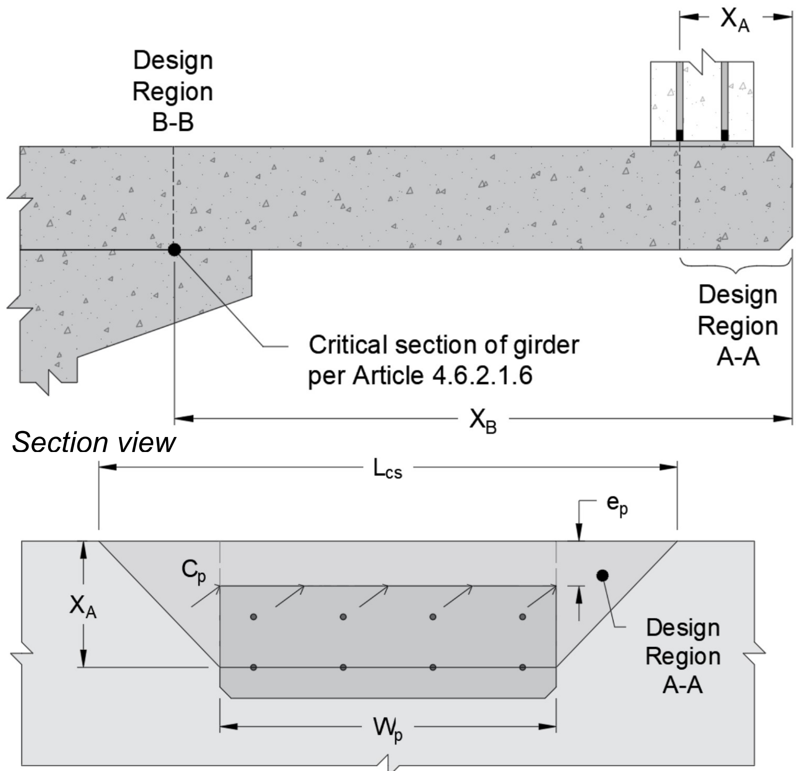


Figure C2. Design Regions for overhangs with concrete posts

Design Case 2: vertical forces specified in Article 13.7.2. Design Case 2 is evaluated only at Design Region B-B, as the distance to Design Region A-A is insufficient to develop a significant moment demand at that location.

For both design cases, the system is evaluated assuming a maximum overlay thickness of 3-in. is present. Although TL-4 lateral design loads decrease with decreasing railing height, overhangs supporting posts are designed to withstand the plastic moment capacity of the post. Therefore, the deck design procedure is independent of F_t and H_e, and the additional weight of the wearing surface makes the post-overlay state the critical design case. Additionally, TL-4 vertical design loads increase with decreasing railing height - thus, the vertical design load is greatest when the overlay is present and the railing height above the vehicle riding surface is at its minimum of 36 in.

For this system, Design Region A-A is a trapezoidal yield-line region in the slab. The longitudinal yield-line is located at the traffic-side vertical post steel, which is a distance:

$X_A := e_p + d_s = 11.3 \text{ in}$ from the field edge of the slab.

Design Region B-B is over the critical section of the supporting element. The overhang is supported by a concrete girder with a total top flange width of 36 in. The critical section of the flange is offset from the exterior girder centerline by the lesser of one-third the flange width and 15 in. In this case, the flange width is 36 in.; therefore, the offset distance is 12 in. The distance from the field edge of the slab to Design Region B-B is therefore:

$$X_B := X_G - \frac{b_{fg}}{3} = 48.0 \text{ in}$$

The distance between the two Design Regions is:

$$X_{AB} := X_B - X_A = 36.8 \text{ in}$$

2. Configure transverse steel for strength limit state

The factored moment demand at Design Region B-B under the strength limit state will be used to set an initial transverse steel spacing requirement. After configuring the steel for adequacy in this limit state, the slab's adequacy in the extreme event limit state (Design Cases 1 and 2) will be evaluated.

If the attached railing is structurally continuous, the continuous length of the slab and railing is at least 25 ft, and the distance from the exterior girder center to the field edge of the slab is less than 6 ft, the design load for the strength limit state may be taken as a uniform 1 kip/ft line load placed 1 ft from the traffic face of the railing. This allowance, which is stipulated in Article 3.6.1.3.4, results in a significantly reduced moment demand at Design Region B-B relative to using a distributed wheel load of 16 kips.

Although analytical modeling has suggested that concrete post-and-beam railings may effectively stiffen the deck edge and reduce the effective wheel load moment at Design Region B-B, this effect is conservatively neglected in this example. Therefore, the worst case between the single 16-kip wheel load and 25-kip is used in the strength limit state evaluation. Per Articles 3.6.1.3.1 and 3.6.1.2.5, the 16-kip wheel load is positioned 1 ft from the face of the railing over a longitudinal distance of 10 in. Per Articles 3.6.1.2.3 and 3.6.1.2.5, the 25-kip tandem load is positioned 1 ft from the face of the railing as two 12.5-kip wheel loads separated by 4 ft, each applied over 10-in. lengths.

The distance from the load application point to region B-B is:

$$X := X_B - e_p - 14\ \text{in} - 12\ \text{in} = 18\ \text{in}$$

The equivalent strip width of overhang resisting the applied load is:

$$W_{patch} := 45\ \text{in} + 10\ X \cdot \frac{\text{in}}{\text{ft}} = 60\ \text{in}$$

Therefore, the distributed moment demand at Design Region B-B due to the 16-kip wheel load is:

$$M_{LL} := \frac{16\ \text{kip} \cdot X}{W_{patch}} = 4.8\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The equivalent strip width for the 10-in. wide single wheel patch was 70 in. Therefore, it is assumed that the equivalent strip width for the 58-in. wide tandem wheel patch set is:

$$W_{tandem} := 48\ \text{in} + 60\ \text{in} = 108\ \text{in}$$

The distributed moment demand at Design Region B-B due to the 25-kip tandem load is therefore:

$$M_{LL'} := \frac{25\ \text{kip} \cdot X}{W_{tandem}} = 4.2\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

As the demand calculated for the single 16-kip wheel load was greater than that of the 25-kip tandem load, the design live load for the strength limit state is taken as:

$$M_{LL} = 4.8\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The basic wheel load moment must be increased by the following factors:

Multiple presence factor
$m := 1.20$

Dynamic load allowance
$IM := 0.33$

Live load factor
$\gamma_{LL} := 1.75$

The self-weight moment at Design Region B-B due to the slab is:

$$M_{DCs} := 150\ \text{pcf} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot t_s \cdot 0.5\ X_B^{\ 2} = 0.9\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The post and beam weight supported by one post is:

$$W_{post} := 150\ \text{pcf} \cdot \left(\underbrace{\left(12\ \text{in} \cdot t_p \cdot 30\ \text{in}\right)}_{\textit{Post weight}} + \underbrace{\left(27\ \text{in} \cdot 14\ \text{in} \cdot 9\ \text{ft}\right)}_{\textit{Beam weight}} \right) = 3.9\ \text{kip}$$

Which generates a total moment at Design Region B-B of:

$$M_{DCp'} := W_{post} \cdot \left(X_B - e_p - 0.5\ t_p\right) = 12.5\ \text{kip} \cdot \text{ft}$$

If this demand is assumed to distribute at a 45-degree angle through the slab from the post location to Design Region B-B without overlapping with an adjacent post, the distributed moment at Design Region B-B is:

$$M_{DCp} := \frac{M_{DCp'}}{W_p + 2\ \left(X_B - e_p - 0.5\ t_p\right)\ \tan\left(45\ \text{deg}\right)} = 1.4\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

Therefore, the total dead-load moment at Design Region B-B is:

$$M_{DC} := M_{DCs} + M_{DCp} = 2.3\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The dead load factor for structural elements is:

$\gamma_{DC} := 1.25$

If a 3-in. wearing surface were added to the slab, the moment at Design Region B-B would be:

$$M_{DW} := 140\ \text{pcf} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot \left(3\ \text{in} \cdot 0.5\ \left(X_B - e_p - t_p\right)^2\right) = 0.1\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The dead load factor for wearing surfaces is:

$$\gamma_{DW} := 1.50$$

Therefore, the design moment at Design Region B-B for the strength limit state is:

$$M_u := M_{LL} \cdot m \cdot (1 + IM) \cdot \gamma_{LL} + M_{DC} \cdot \gamma_{DC} + M_{DW} \cdot \gamma_{DW} = 16.5\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

If #5 transverse bars were placed at 8 in., the factored transverse bending strength of the slab would be 16.2 k-ft/ft. Although this value is roughly 2% lower than the strength limit state demand, it is deemed acceptable due to the conservative assumption that the concrete railing does not act as an edge-stiffening element for resisting wheel loads (accounting for this effect would result in a strength limit state demand of just 7.2 k-ft/ft). Therefore, this trial steel configuration will be evaluated against demands imposed by lateral and vertical loading for the extreme event limit state. It should be noted that steel spacing cannot be widened away from post locations in this configuration, as it was assumed that railing dead loads distributed through the slab at a 45-degree angle. As shown in Figure C3, at a 9-ft post spacing, these distribution patches are nearly coincident; therefore, the 16.5 kip-ft/ft strength limit state demand must be satisfied along the entire bridge span.

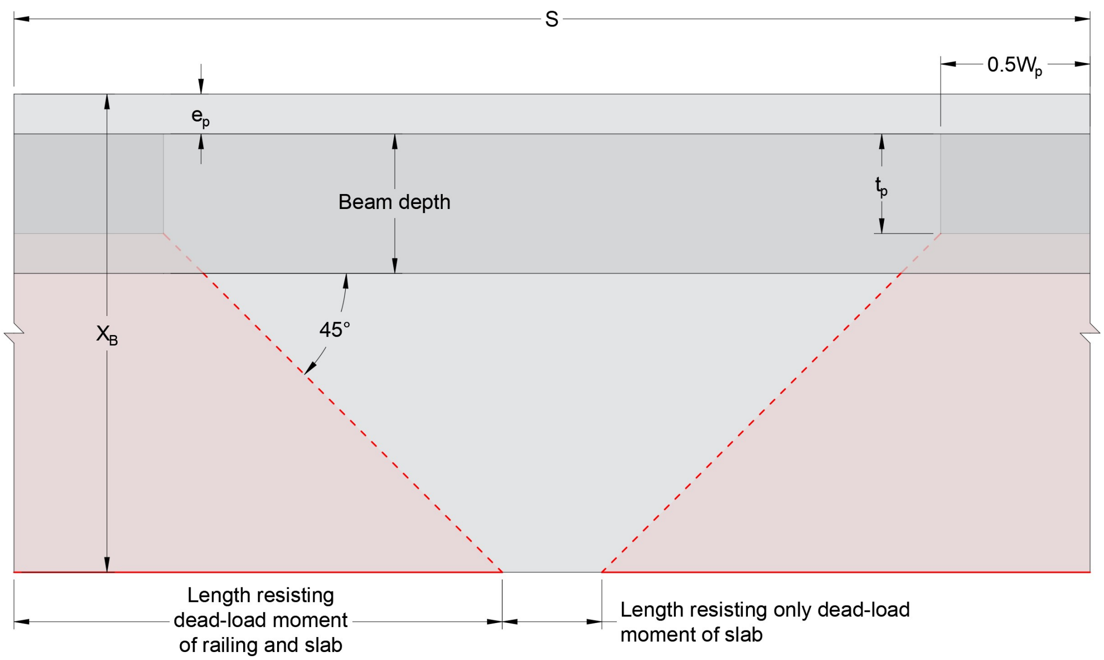


Figure C3. Assumed distribution of railing dead load moment through slab

The trial transverse slab bar diameter, spacing, and unit-length area are:

$$d_{bt} := 0.625\ \text{in} \qquad s_{bt} := 8\ \text{in} \qquad A_{st} := 0.25\ \pi \cdot d_{bt}^{\ 2} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot s_{bt}^{\ -1} = 0.46\ \frac{\text{in}^2}{\text{ft}}$$

3. Establish ultimate post capacity and associated overhang demands

The railing and overhang trial design, which uses #5 transverse bars at 8 in. in the slab, is shown in Figure C4.

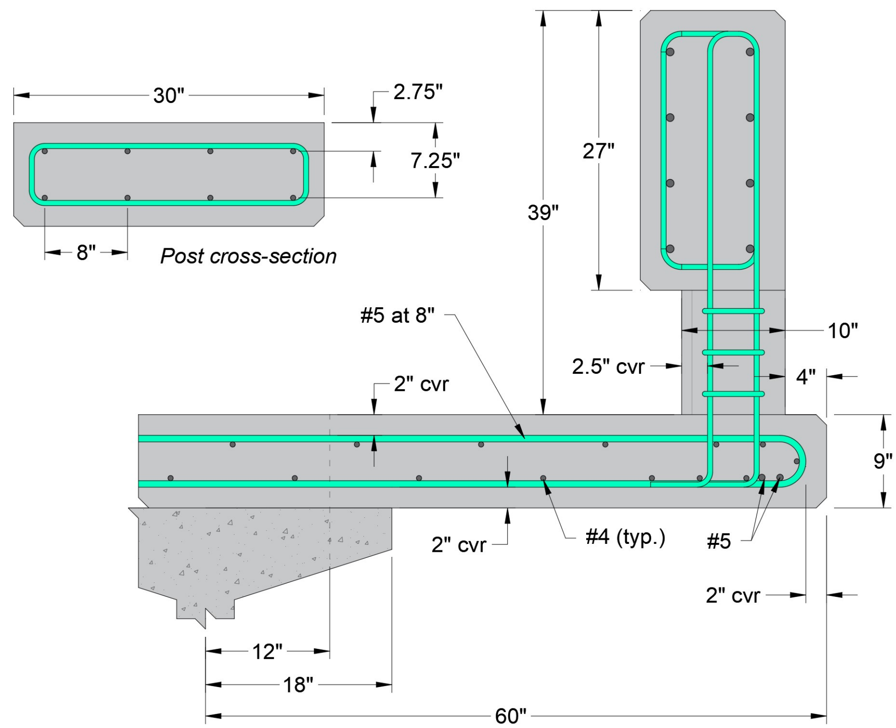


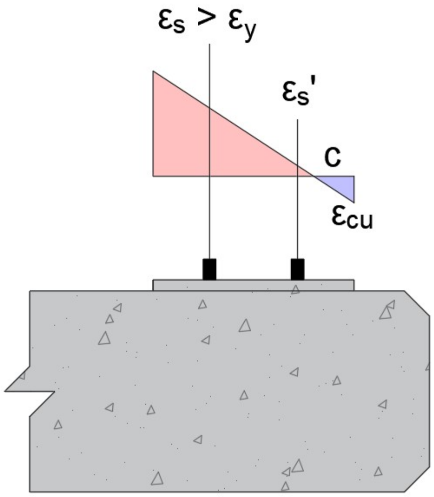
Figure C4. Overhang and railing trial design

In this section, overhang design demands for extreme event Design Case 1 are calculated. Unlike for overhangs with concrete barriers, which are designed for the lateral impact load spread over an effective distribution length, it is recommended that overhangs with concrete posts are designed to support the ultimate capacity of the post. It should be noted that, when calculating section strengths in the extreme event limit state, effective material properties (Table 13.3.7.3-1) may <u>optionally</u> be used.

$f'_{ce}=6.5$ ksi $\qquad f_{ye}=66$ ksi (13.3.7.3-1)

The AASHTO LRFD BDS development length of a hooked, #4 bar with a yield stress of 66 ksi in 6,500-psi concrete is 7.8 in. In this design, the vertical post bars are embedded in 7 in. of slab concrete. Based on this calculation, the code-permitted maximum stress which can be developed in the vertical bars is 59 ksi. However, as testing performed in NCHRP Project 12-119 indicated that #4 bars embedded in 7 in. of concrete were able to reach their rupture stress, it is assumed in this example that these bars can reach their full effective yield stress of 66 ksi. This finding was consistent with testing performed by Ansley (2002) and Alberson (2005). Additionally, the assumption of full post bar development is conservative, as the post strength in this case is a demand acting on the slab, rather than a capacity.

The plastic moment capacity of the attached post is determined assuming the strain distribution and vertical equilibrium state shown in Figure C5:



Consistent with Article 5.6.2 and 5.6.3

Figure C5.1. Assumed strain distribution at full strength of post

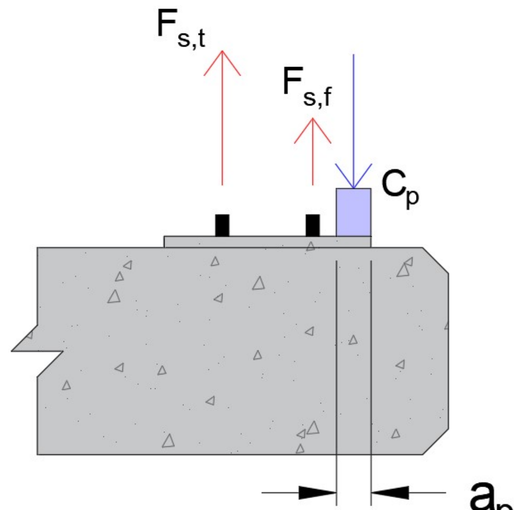


Figure C5.2. Base couple forces at full cantilever bending strength of post

The location of the neutral axis was determined iteratively to account for potential elastic behavior at fs'. In this calculation, the location of the neutral axis was varied until steel forces were in equilibrium with concrete compression. This process uses Eqn. 5.6.3.1.2-4, in which prestressing area is equal to zero, and the analysis is performed on a unit-length basis. The neutral axis depth was found to be:

$$c_p := 0.863 \text{ in}$$

for which the compression block depth, ap, is:

$$\beta_1 := 0.85 - 0.05 \cdot \left(f'_{ce} - 4 \text{ ksi}\right) \text{ ksi}^{-1} = 0.7$$

$$a_p := \beta_1 \cdot c_p = 0.63 \text{ in}$$

The effective depth to each layer of vertical post steel is:

$$d_{straf} := 10 \text{ in} - 2 \text{ in} - 1.5 \cdot 0.5 \text{ in} = 7.25 \text{ in}$$

10-in. thick post, 2-in. cover, #4 hoops, #4 vertical bars

$$d_{sfield} := 2 \text{ in} + 1.5 \cdot 0.5 \text{ in} = 2.75 \text{ in}$$

The traffic-side steel is assumed to be yielded. At this neutral axis depth, the strain in the field-side steel is:

$$\varepsilon_s' := \frac{\varepsilon_{cu}}{c_p} \left(d_{sfield} - c_p\right) = 0.0066$$

The effective yield strain is:

$$\varepsilon_{ye} := \frac{f_{ye}}{29000\ \text{ksi}} = 0.0023$$

Which is less than the field-side steel strain. Therefore, the field-side steel is yielded at the full cantilever bending strength of the post, and the force in each layer of vertical post steel is:

$$F_s := f_{ye} \cdot A_{sp} = 51.8\ \text{kip}$$

The concrete compressive force is:

$$C_p := 0.85 \cdot f'_{ce} \cdot W_p \cdot a_p = 103.7\ \text{kip} \qquad \text{(C13.10.2.4.1-1)}$$

Which is in vertical equilibrium with steel forces:

$$2\ F_s - C_p = 0\ \text{kip}$$

The cantilever bending strength of the post at its base is calculated using Eqn. 5.6.3.2.2-1:

$$M_n := \boxed{A_{ps} f_{ps}} \cdot \left(d_p - \frac{a}{2}\right) + A_s f_s \cdot \left(d_s - \frac{a}{2}\right) - A_s' f_s' \cdot \left(d_s' - \frac{a}{2}\right) + \alpha_1 f'_c \cdot \left(b - b_w\right) \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

As the section is rectangular, b_w is taken equal to b. Additionally, the section does not include prestressing strands. Therefore, the equation is reduced to:

$$M_n := \boxed{A_s f_s} \cdot \left(d_s - \frac{a}{2}\right) - A_s' f_s' \cdot \left(d_s' - \frac{a}{2}\right)$$

In this case, $A_s f_s$ is the force in the traffic-side vertical steel, and $A_s' f_s'$ is the force in the field-side steel. The preceding strain calculations predicted that both layers of steel are yielded in tension. Therefore the stress in the traffic-side steel is:

$$f_s := f_{ye} = 66\ \text{ksi}$$

and the stress in the field-side steel is:

$$f_s' := -f_{ye} = -66\ \text{ksi}$$

It should be noted that the stress in the field-side steel was taken as negative, as Eqn. 5.6.3.2.2-1 assumes the compression-side steel is in compression. Substituting variable names used in this example results in a post bending strength of:

$$M_{post} := A_{sp} \cdot f_s \cdot \left(d_{straf} - \frac{a_p}{2}\right) - A_{sp} \cdot f_s' \cdot \left(d_{sfield} - \frac{a_p}{2}\right) = 40.5\ \text{kip} \cdot \text{ft}$$

The procedure used above to calculate M_{post} is consistent with Article 5.6.3.2.5, which states "The stress and corresponding strain in any given layer of reinforcement may be taken from any representative stress-strain formula or graph for nonprestressed reinforcement..." In this case, a rectangular compression block and linear strain diagram were assumed. Bending strengths calculated throughout the rest of this example follow the same procedure shown above.

The compressive force at this moment and the associated compression block depth are:

$C_p = 103.7$ kip $\qquad a_p = 0.63$ in

The centroid height of the longitudinal railing element is:

$Y = 25.5$ in

Therefore, the lateral load on the railing which induces the plastic moment capacity of the post is:

$$P_{post} := \frac{M_{post}}{Y} = 19.1 \text{ kip}$$

The total moment demand acting at the centroid of the slab is therefore:

$P_{post} \cdot (Y + 0.5\ t_s) = 47.6$ kip·ft

The compressive force at the back of the post at the plastic moment capacity is:

$C_p = 103.7$ kip

which acts at one-half the compression block depth of:

$a_p = 0.63$ in

These demands are summarized in Figure C6. Note that the figure shown does not depict an equilibrium state and is intended only to depict key overhang demands.

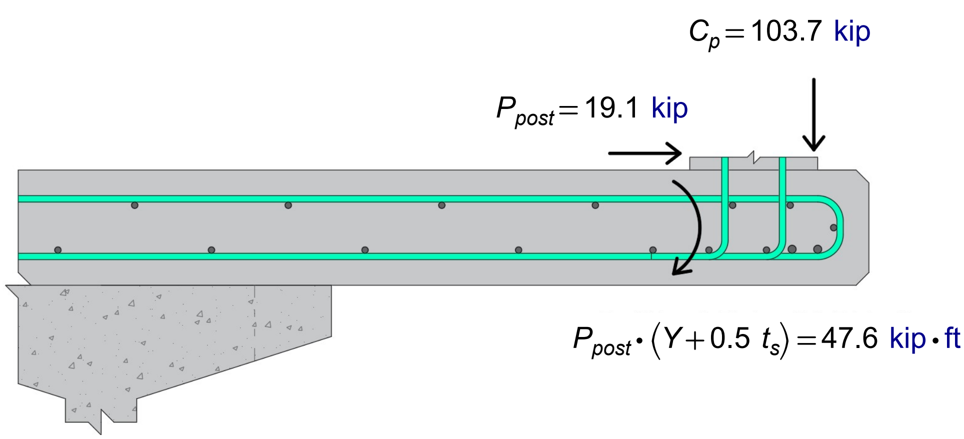


Figure C6. Design Case 1 demands at plastic strength of post

The slab must be able to transfer the compressive force from the post base into the slab through vertical shear or through a diagonal compression strut, if the slab steel is detailed such that a strut and tie mechanism can develop. The slab must also be able to support the downward compressive force in a yield-line mechanism. In this mechanism, the post base shear effectively reduces the effectiveness of transverse steel. Lastly, the tensile and flexural demand shown must be resisted as distributed loads at Design Region B-B.

---

4. Slab-post joint damage check

Prior to calculating the transverse bending strength of the slab, the ability of the slab edge to transfer the barrier loads to Design Region A-A must first be checked. A common damage mechanism observed in deck overhangs supporting bridge railings is a diagonal tension failure of the slab-barrier joint, as shown in Figure C7.

Frosch & Morel (2016)

NCHRP 12-119 barrier test

Figure C7. Examples of diagonal tension damage in slab

The damage mechanism is not fully understood, but is believed to be caused by either compression strut splitting or vertical shear. If adequate edge distance is provided, and hooked transverse bars are used in the slab, a strut-and-tie model may be able to develop in the joint. In this case, diagonal tension failure would be due to transverse splitting of a compression strut due to the Poisson effect. If edge distance is minimal and/or transverse slab bars are not hooked, a strut-and-tie behavior cannot be expected, as the top-mat tension tie cannot develop. In this case, diagonal tension failure would likely be due to vertical shear, as the compression at the back edge of the post is resisted purely by the vertical shear resistance of the slab. These two cases are discussed below.

In the first case, it is assumed that the transverse bar is able to develop a sufficient tensile stress at the compression-compression-tension (CCT) node to act as a tension tie by hooked anchorage of the transverse bar around a longitudinal bar. Additionally, it is assumed that the top-mat longitudinal bar coincident with the node bears a significant compressive force, bearing on the transverse bar and further aiding in development of the tension tie. Diagonal tension damage in this case would be the result of strut transverse splitting. In addition to special steel detailing, a sufficient edge distance must be provided for this mechanism to occur - if the post compressive force is centered over the hooked portion of the bar, spalling of the field-edge slab cover would likely occur. An example of a strut-and-tie mechanism which may form in an overhang supporting a concrete post is shown in Figure C8.

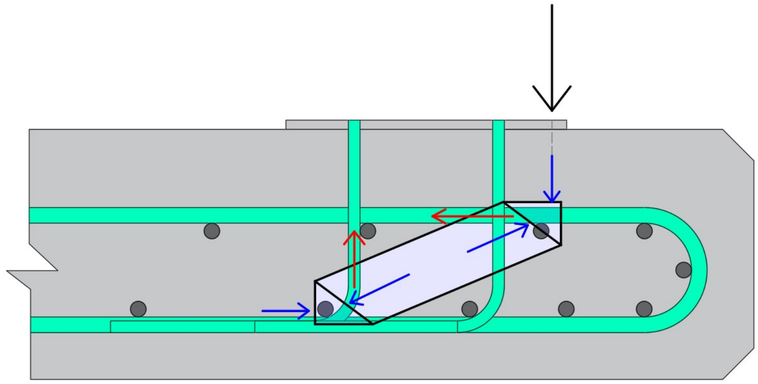


Figure C8. Slab strut-and-tie mechanism (requires appropriate detailing)

To demonstrate the second case, a design variant is shown in Figure C9 in which the edge distance was reduced, and the transverse bars were unhooked. In this case, a top-mat tension tie cannot develop. Diagonal tension damage in this case would be the result of vertical shear through the deck thickness. This is the default, conservative assumption presented in proposed Article 13.10.2.4.1.

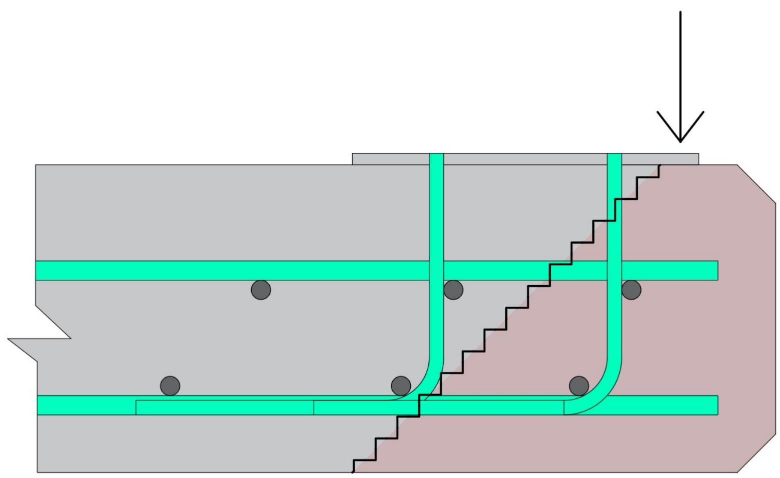


Figure C9. Slab vertical shear mechanism (general case)

For this design, it is assumed that a strut-and-tie behavior can be developed. The generalized orientation of the compression strut is shown in Figure C10. It should be noted that the compression strut orientation shown above is more realistic; however, that shown below is conservative and presented in terms of basic design parameters.

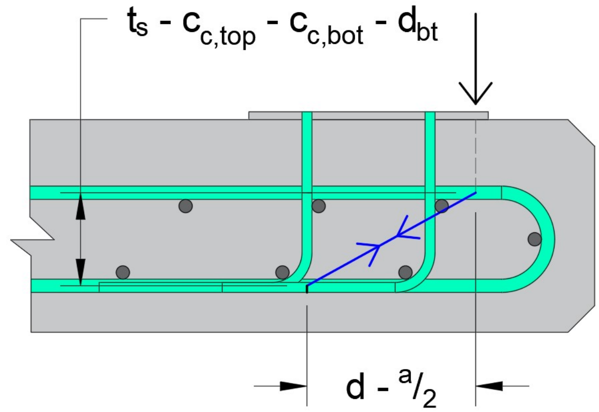


Figure C10. Generalized compression strut orientation

The vertical resistance of the compression strut must be greater than or equal to the required compressive force at the back of the post when it reaches its plastic bending strength. Further, in order to orient the compression strut, the compression block depth associated with the post bending strength must be estimated. Therefore, post bending parameters calculated in the preceding step are used to orient and evaluate the compression strut.

Aside: diagonal tension damage discussion

Several aspects of the steel configuration may justify the assumption of strut-and-tie development. First, and most importantly, the transverse slab bar is hooked around a longitudinal bar at the field edge of the slab. This detail aids in the development of the tension tie and provides improved concrete confinement under the post compression block. Additionally, the relatively long (4 in.) edge distance used in the design further aids in tension tie development and reduces the likelihood of field-edge concrete spalling, as the post compression block is directly over the horizontal portion of the bar, rather than the hook. Lastly, the longitudinal bar behind the field-side vertical post steel provides a bearing point for the diagonal compression strut.

It should be noted that an inability of the strut-and-tie model to fully equilibrate the compressive force at the back of the post does not constitute a failure of the system. Instead, this understrength indicates a likely development of diagonal cracking in the slab and delamination of the bottom cover. If vertical steel crosses the assumed compression strut, it can be assumed that the slab possesses reserve strength beyond the compression strut splitting or tie failure load. As such, if the strut-and-tie model capacity is less than the post demand, the bending strength of the slab in the trapezoidal yield-line mechanism described in the following section is penalized to account for bottom cover loss.

Many common overhang details found in DOT standards do not justify the assumption that a strut-and-tie behavior could develop. Overhangs which use straight transverse bars, for example, would not be able to sufficiently develop a horizontal tension tie. Further, shorter edge distances increase the likelihood of field-edge cover spalling and further impede development of the tension tie. Designs without at least one top-mat longitudinal bar behind the field-side vertical steel layer may also poorly develop strut-and-tie behavior due to direct bearing of the compression strut on transverse steel. Longitudinal bars in this region may aid in transfer of compression strut forces between transverse bars into the localized tension ties at bar locations.

In cases where a strut-and-tie model cannot be justified, it should be assumed that the slab must resist the downward compressive force at the back of the post in vertical punching shear. In this case, the check performed in this section would be equivalent to the traditional AASHTO LRFD BDS punching shear check for posts on slab overhangs, in which the load patch is the post compression block.

Unlike for the strut-and-tie model, where understrength did not suggest a complete failure of the system, understrength in the punching shear mechanism should be viewed as a direct limit on the moment which can be developed by the post. If the slab steel is not configured to develop a strut-and-tie model, it should be assumed that failure to provide an adequate punching shear capacity will result in a sudden and severe failure mechanism in the slab prior to reaching the nominal post strength. Further, the appropriateness of applying the yield-line method described in the following section after a punching shear failure has occurred is unknown. If transverse bars are straight, punching shear damage may also significantly interrupt bar development at the traffic face of the post.

Joint evaluation for example system design

Strut-and-tie models are highly specific to system designs. However, if the development of strut-and-tie behavior can be justified for a given design, the procedure shown in this example provides a conservative method for orienting, sizing, and evaluating the mechanism using basic design parameters. The general procedure is as follows:

1. Estimate the compression strut angle from horizontal
2. Estimate the compression strut cross-section area
3. Estimate the maximum compressive force which can act through the strut
4. Compare vertical component of strut limit to post compression force

The simplified compression strut orientation is shown in Figure C11.

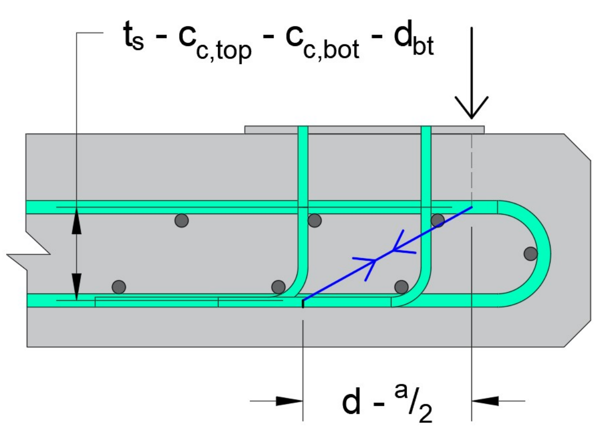


Figure C11. Simplified compression strut orientation

The angle of the strut from horizontal is:

$$\theta_s := \operatorname{atan}\left(\frac{t_s - c_{ctop} - c_{cbot} - d_{bt}}{d_s - 0.5\ a_p}\right) = 32.2\ \text{deg} \qquad \text{(C13.10.2.4.1-4)}$$

The length of the node perpendicular to the bridge span is assumed equal to twice the post compression block depth. The height of the node is assumed equal to the top slab cover. The bearing length of the node is therefore:

$$l_b := \sqrt{\left(2\ a_p\right)^2 + \left(c_{ctop}\right)^2} = 2.36\ \text{in} \qquad \text{(C13.10.2.4.1-5)}$$

Article 5.8.2.5.3 of the AASHTO LRFD BDS specifies the limiting compressive stress at the node face. The notional area was accounted for in the above calculation of the bearing length, therefore *m* is taken as 1. Specific crack control reinforcement is not provided across the strut, therefore, the limiting compressive stress in the strut is:

$$f_{cu} := (1.0)\ (0.45)\ f'_{ce} = 2.9\ \text{ksi} \qquad \text{(5.8.2.5.3)}$$

It is assumed that, at this stress, the strut will sustain transverse splitting damage. The effective cross-section area of the node face is:

$$A_{cn} := l_b \cdot W_p = 70.8\ \text{in}^2$$

Therefore, the maximum load which can be applied along the axis of the strut is:

$$P_{ns} := A_{cn} \cdot f_{cu} = 207\ \text{kip} \qquad \text{(5.8.2.5.1-1)}$$

The vertical (y) component of this load must be greater than or equal to the compressive force that develops at the back of the post to allow the post to develop its full base flexural capacity. If the vertical component of the strut splitting load does not satisfy this requirement, diagonal tension damage of the slab joint is expected. In this case:

$$P_{nsy} := f_{cu} \cdot A_{cn} \cdot \sin\left(\theta_s\right) = 110.4\ \text{kip} \qquad \text{(C13.10.2.4.1-3)}$$

$$\frac{C_p}{P_{nsy}} = 0.94 \qquad \text{(C13.10.2.4.1-2)}$$

Therefore, the compression strut is assumed to be adequate, and diagonal tension failure will not occur.

5. Slab yield-line capacity

In this section, the yield-line capacity of the slab is calculated and compared to the compression force at the back of the post associated with its ultimate strength, M_p. The assumed mechanism is shown in Figure C12.

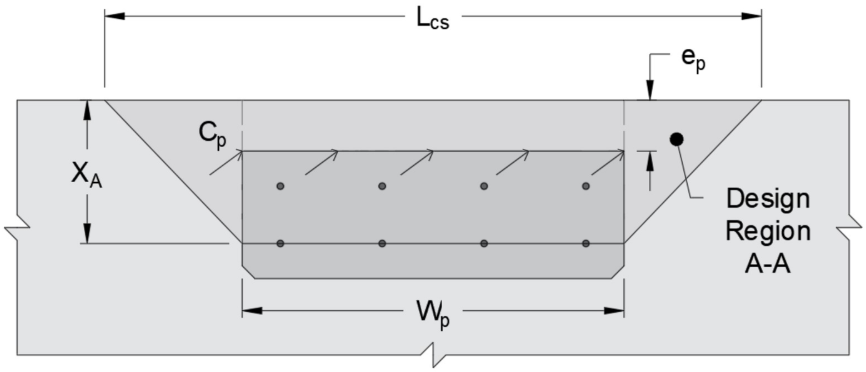


Figure C12. Yield-line mechanism for slabs with concrete posts

As the yield-line mechanism engages both transverse and longitudinal steel, two slab bending strengths must be calculated.

Transverse bending strength

As the preceding diagonal joint damage check was passed, the equilibrium state shown in Figure C13 is assumed.

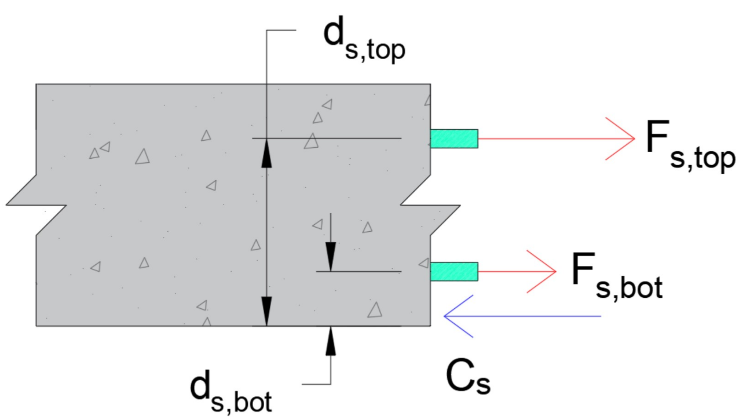


Figure C13. Slab couple forces at full bending strength
(if diagonal tension damage check passes)

If the preceding diagonal tension damage check were failed, the assumed equilibrium state is as shown in Figure C14.

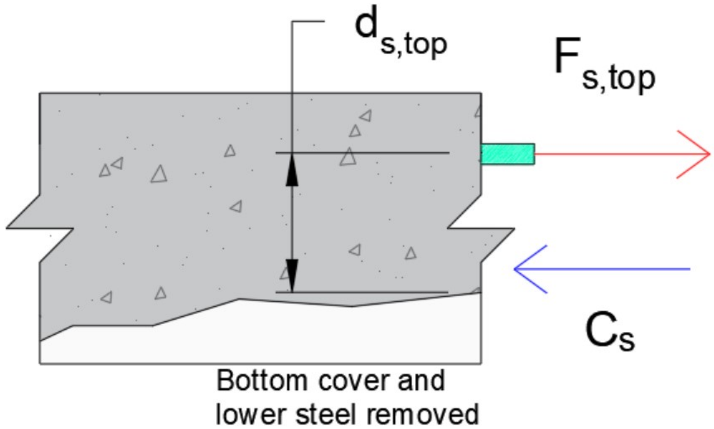


Figure C14. Slab couple forces at full bending strength (if diagonal tension damage check fails)

The neutral axis location was iteratively determined to be:

$$c_s := 1.264 \text{ in}$$

The compression block depth is therefore:

$$a_s := \beta_1 \cdot c_s = 0.92 \text{ in}$$

The effective depth to each layer of transverse slab steel is:

$$d_{stop} := 9 \text{ in} - 2 \text{ in} - 0.5 \cdot 0.625 \text{ in} = 6.7 \text{ in}$$

9-in. thick slab, 2-in. cover, #5 bars

$$d_{sbot} := 2 \text{ in} + 0.5 \cdot 0.625 \text{ in} = 2.3 \text{ in}$$

The top-mat steel is assumed to be yielded. At this neutral axis depth, the strain in the bottom-mat steel is

$$\varepsilon_{sbot} := \frac{\varepsilon_{cu}}{c_s} \left(d_{sbot} - c_s\right) = 0.0025$$

The effective yield strain is:

$$\boxed{\varepsilon_{ye}} := \frac{f_{ye}}{29000 \text{ ksi}} = 0.0023$$

Which is less than the bottom-mat steel strain. Therefore, the field-side steel is yielded at the full cantilever bending strength of the slab, and the force in each layer of transverse steel is:

$$F_{stop} := f_{ye} \cdot A_{st} = 30.4\ \frac{kip}{ft}$$

The concrete compressive force is:

$$C_s := 0.85 \cdot f'_{ce} \cdot 12\ \frac{in}{ft} \cdot a_s = 60.8\ \frac{kip}{ft}$$

Which is in vertical equilibrium with steel forces:

$$2\ F_{stop} - C_s = 0\ \frac{kip}{ft}$$

The transverse bending strength of the slab is therefore:

$$M_{stA} := F_{stop} \cdot \left(d_{stop} - d_{sbot}\right) + C_s \cdot \left(d_{sbot} - 0.5\ a_s\right) = 20.5\ \frac{kip \cdot ft}{ft}$$

For this design, the steel configuration between Design Region A-A and B-B is unchanged, and the #5 bars are assumed fully developed at Design Region A-A, as they hook around the field-edge longitudinal slab bar. Therefore, the bending strength is equal at both regions. The distributed tension acting on the section due to the lateral load is:

$$N := \frac{P_{post}}{W_p} = 7.6\ \frac{kip}{ft} \qquad (13.10.2.4.1\text{-}1)$$

The effect of extreme event, Design Case 1 tension on the bending strength of the slab is accounted for by determining the capacity of the section in pure tension, then linearly interpolating between the pure tension and pure flexure strengths.

The tensile capacity of the slab in pure tension is:

$$P_n := 2\ F_{stop} = 60.7\ \frac{kip}{ft}$$

The flexural capacity of the slab in pure flexure is:

$$M_{stA} = 20.5\ \frac{kip \cdot ft}{ft}$$

The tension acting on the slab is:

$$N = 7.6\ \frac{kip}{ft}$$

Therefore, the effective bending strength of the slab under the imposed tension is:

$$M_{strA} := \left(1 - \frac{N}{P_n}\right) M_{stA} = 17.9\ \frac{kip \cdot ft}{ft}$$

The process used to determine the effective bending strength of the slab is shown in Figure C15.

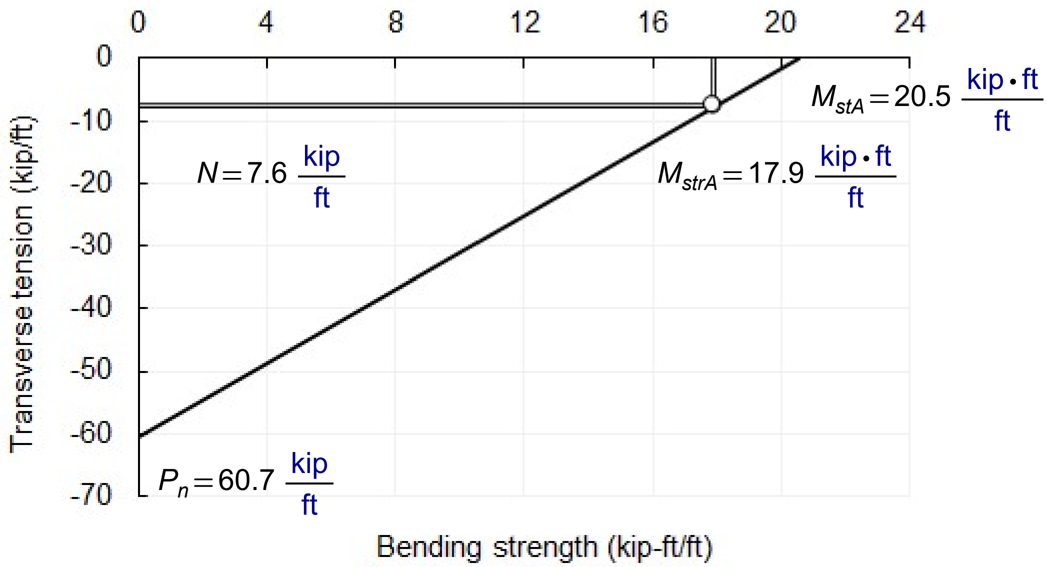


Figure C15. Calculation of effective bending strength under applied axial tension

As the distributed tension demand is conservatively assumed to be equal at both regions, this value is unchanged between Design Regions A-A and B-B.

Longitudinal bending strength

The longitudinal bending strength is asymmetric. Along the diagonal yield-lines, the top surface of the slab is in tension; along the transverse yield-lines on the bottom slab surface, the bottom surface is in tension. As the diagonal and transverse yield-lines are rotated through equal angles, the effective longitudinal bending strength of the slab can be taken as the average of the positive and negative bending strengths. Directional, longitudinal bending strengths are shown in Figure C16.

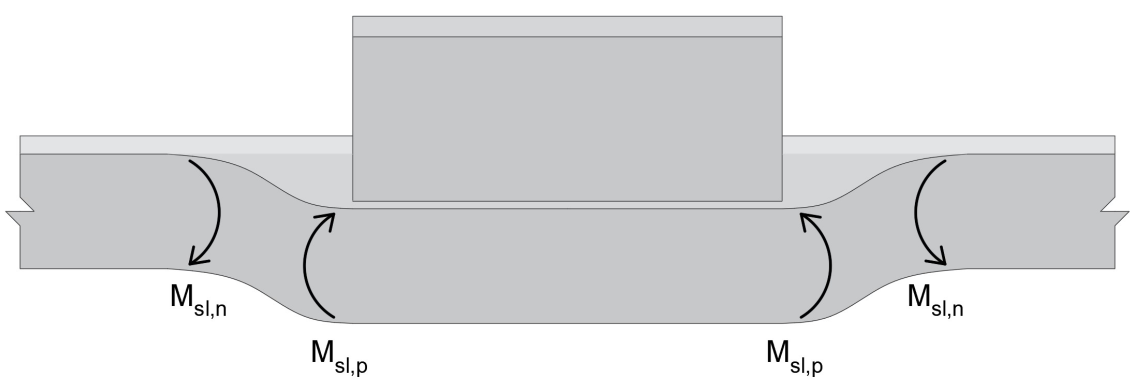
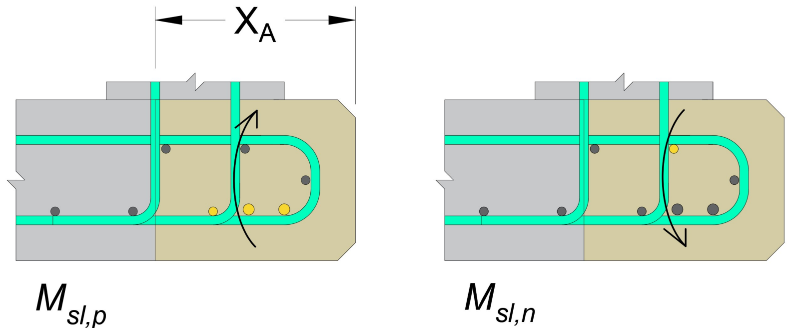


Figure C16. Directional, longitudinal bending strengths activated in slab for downward deflection of field edge

When calculating the longitudinal bending strengths of the slab, the contribution of the field-side bar is conservatively neglected. Additionally, the top-mat longitudinal bar directly behind the traffic-side vertical bar is neglected, as it is not expected to be rotated through a significant angle due to its proximity to the base of the yield-line mechanism. The effective edge beam area for longitudinal bending is shown in Figure C17. For positive and negative bending, bars assumed to contribute to the respective bending strength are highlighted.



Positive flexure (top comp.) *Negative flexure (bottom comp.)*

Figure C17. Activated area and participating bars for longitudinal slab bending

For this design:

$M_{slp} := 22.8\ \text{kip} \cdot \text{ft}$

$M_{sln} := 6.5\ \text{kip} \cdot \text{ft}$

This value is zero if preceding diagonal damage check is failed

Therefore, the effective longitudinal bending strength of the slab under the post is:

$$M_{sl} := \frac{1}{2}\left(M_{slp} + M_{sln}\right) = 14.7\ \text{kip} \cdot \text{ft}$$

The basic slab bending capacities are summarized below.

$M_{strA} = 17.9\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$ *Longitudinal yield line (assumed to take all tension effect)*

$M_{stA} = 20.5\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$ *Diagonal yield-lines (tension assumed to be taken within W_p)*

$M_{sl} = 14.7\ \text{kip} \cdot \text{ft}$ *Diagonal yield-lines*

Slab yield-line mechanism

The critical length of the yield-line mechanism is:

$$L_{cs} := W_p + \sqrt{\frac{8 \cdot M_{sl} \cdot X_A}{M_{stA}}} = 4.8\ \text{ft} \qquad (13.10.2.4.1\text{-}9)$$

The ultimate downward load, applied over W_p, which can be supported by the slab is:

$$M_{posteff} := \frac{M_{post}}{C_p} \cdot \frac{X_A}{X_A - e_p} \left(M_{strA} \cdot \frac{W_p}{X_A} + M_{stA} \cdot \frac{L_{cs} - W_p}{X_A} + M_{sl} \cdot \frac{8}{L_{cs} - W_p} \right) = 90.2\ \text{kip} \cdot \text{ft}$$

(13.10.2.4.1-8)

The utility ratio for the slab yield-line mechanism is:

$$\frac{M_{post}}{M_{posteff}} = 0.45$$

Therefore, the yield-line capacity of the slab is adequate to develop the full strength of the post and Mposteff equals Mpost.

---

6. Evaluation of slab for distributed demands at exterior girder

---

After the local capacity of the slab under the post is evaluated, the second overhang critical region must be checked against distributed demands. The two evaluations that must be performed at Design Region B-B are:

Design Case 1: distributed tension and moment demands associated with plastic moment capacity of post

Design Case 2: distributed moment demand induced by vertical loading applied to railing

In both design cases, the system is evaluated in its post-overlay state. Using this state maximizes the Design Case 1 design moment at Design Region B-B due to the additional weight and has no effect on the impact moment, as it is determined using M_{post}, not F_t. Further, for Design Case 2, using the post-overlay state maximizes the system weight and TL-4 vertical design load, as F_v increases with decreasing railing height due to increased vehicle roll.

Design Case 1 evaluation

The total moment demand acting on the slab at the ultimate strength of of the post is:

$$P_{post} \cdot (Y + 0.5\ t_s) = 47.6\ \text{kip} \cdot \text{ft}$$

Design Case 1 and 2 moments are assumed to distribute longitudinally according to the pattern shown in Figure C18.

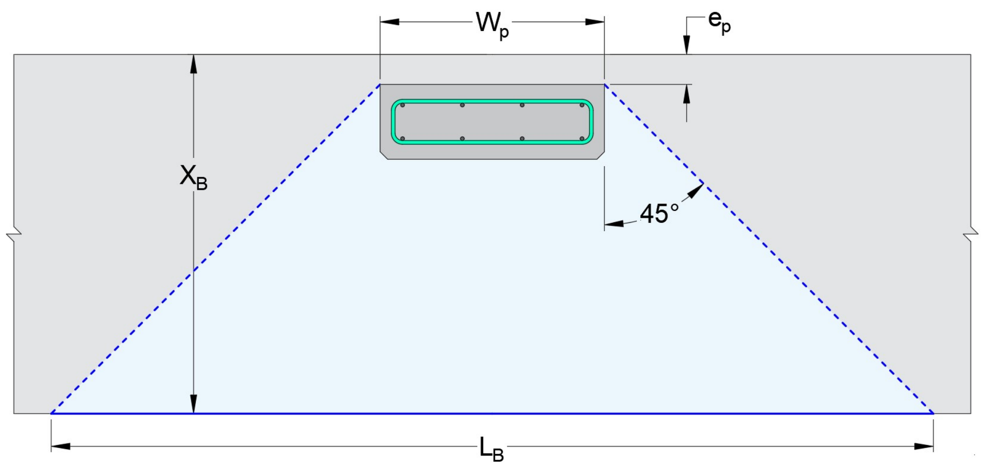


Figure C18. Effective distribution of flexural and tensile post loads through slab

The effective distribution length for the post moment demand at Design Region B-B is:

$$L_{1B} := W_p + 2\left(X_B - e_p\right) = 9.8\ \text{ft} \qquad (13.10.2.4.1\text{-}12)$$

Therefore, the Design Case 1 moment demand at Design Region B-B, which includes the distributed post moment and self-weight, is:

$$M_{swB} := M_{DC} + M_{DW} = 2.4\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

$$M_{1B} := \left(\frac{M_{posteff}}{M_{post}}\right) \cdot \frac{P_{post} \cdot \left(Y + 0.5\ t_s\right)}{L_{1B}} + M_{swB} = 13.2\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.4.1\text{-}15)$$

The distributed tension at Design Region B-B can conservatively be assumed to be equal to the tensile demand at A-A:

$$N = 7.6\ \frac{\text{kip}}{\text{ft}} \qquad (13.10.2.4.1\text{-}1)$$

Therefore, the tension-reduced bending strength of the slab at B-B is:

$$M_{strB} := M_{strA} = 17.9\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

and the utility ratio for Design Case 1 at Design Region B-B is:

$$\frac{M_{1B}}{M_{strB}} = 0.74 \qquad (13.10.2.4.1\text{-}14)$$

Design Case 2 evaluation

The assumed overlay thickness is:

$$H_{wear} := 3 \text{ in}$$

Therefore, the vertical impact load for the railing is:

$$F_v := 101 \text{ kip} - 1.75 \left(H - H_{wear}\right) \cdot \frac{\text{kip}}{\text{in}} = 38 \text{ kip} \qquad \text{(Table 13.7.2-2)}$$

applied over a length of:

$$L_v := 18 \text{ ft} \qquad \text{(Table 13.7.2-2)}$$

The portion of the total vertical load acting on a single post can be taken as:

$$\frac{L}{L_v} F_v = 19 \text{ kip}$$

which is conservatively applied at the back face of the railing. Therefore, the total moment at Design Region B-B is:

$$\frac{L}{L_v} F_v \cdot \left(X_B - e_p\right) = 69.7 \text{ kip} \cdot \text{ft}$$

which is distributed over:

$$L_{2B} := W_p + 2 \left(X_B - e_p\right) = 9.8 \text{ ft} \qquad \text{(13.10.2.4.1-16)}$$

Therefore, the Design Case 2 moment demand at Design Region B-B, which includes the distributed vertical load moment, dead load, and wearing surface moment, is:

$$M_{2B} := \frac{L \cdot F_v \cdot \left(X_B - e_p\right)}{L_v \cdot L_{2B}} + M_{swB} = 9.5 \ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad \text{(13.10.2.4.1-19)}$$

The utility ratio of this demand when compared to the bending strength at B-B (with no tension penalty) is:

$$M_{stB} := M_{stA} = 20.5 \ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

$$\frac{M_{2B}}{M_{stB}} = 0.47 \qquad \text{(13.10.2.4.1-18)}$$

As the overhang's local yield-line capacity is sufficient to develop the full capacity of the post, and Design Region B-B is sufficient to withstand distributed lateral and vertical design loads, the overhang design is **adequate** for the attached concrete post-and-beam railing.

APPENDIX D

Deck-Mounted Steel-Post Example

The following design example includes the full analysis of the TxDOT T631 railing and accompanying TxDOT overhang design. Extreme event design loading is taken from proposed revisions to Section 13 of the AASHTO *LRFD Bridge Design Specifications* available at the time this example was prepared. The proposed draft language has been provided to AASHTO for consideration.

Note that this example was prepared in Mathcad Prime 8. Due to limitations in variable formatting, commas in subscripts were omitted (e.g., "*cctop*" was used in examples in place of "*cc,top*" in the body of this report and the proposed revisions). Similarly, Y was used in examples in place of $\overline{Y}$.

In this design example, the ability of a deck overhang design to develop the full capacity of an attached steel post is evaluated. The attached steel post is a 31 in. tall S3x5.7 post, which is part of the TxDOT T631 MASH TL-2 bridge railing. The post is anchored to the deck via a through-bolt anchorage system which includes a base plate, washer plate, and no curb. The TxDOT T631 railing and overhang system is shown in Figure D1.

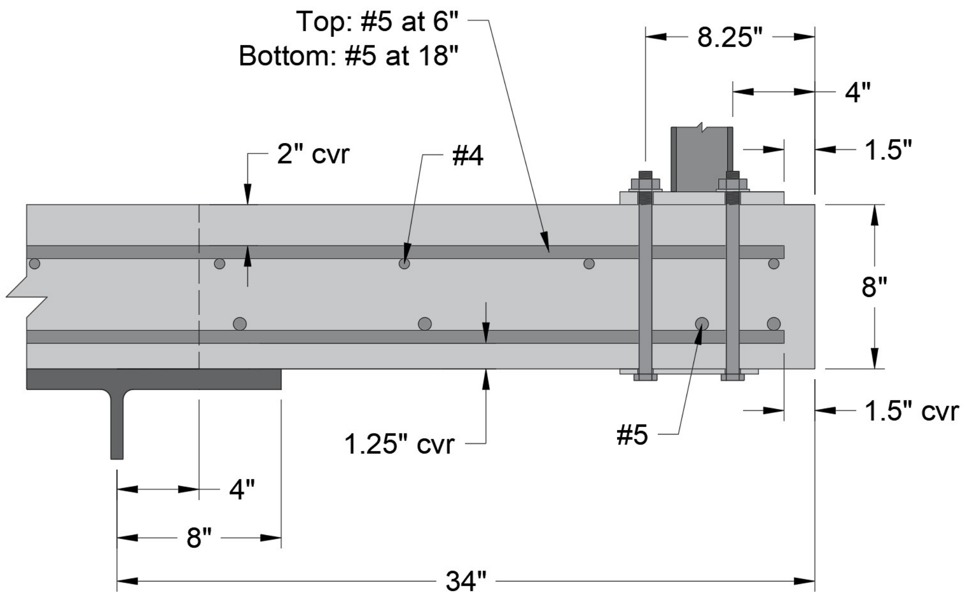


Figure D1. Example steel post and overhang system

The general design/analysis procedure is as follows:

1. Identify critical overhang regions
2. Configure transverse deck steel for strength limit state
3. Establish ultimate capacity of post and associated deck overhang demands
4. Check the deck-post joint for diagonal tension damage
5. Calculate the yield-line capacity of the slab and compare to required compressive force
6. Estimate overhang demands at Design Region B-B and compare to slab strength

Known system parameters

System dimensions and characteristics known prior to the overhang design process are shown below. For this example, it is assumed that the design of both the railing and overhang is known - the TxDOT T631 railing is attached to TxDOT's standard deck. Transverse steel has already been configured to meet the requirements of the strength limit state.

Materials

Concrete compressive strength	$f'_c := 4\ \text{ksi}$
Effective concrete strength	$f'_{ce} := 1.3\ f'_c = 5.2\ \text{ksi}$
Concrete design crush strain	$\varepsilon_{cu} := 0.003$
Compression block depth factor	$\beta_1 := 0.85 - 0.05\ \left(f'_{ce} \cdot \text{ksi}^{-1} - 4\right) = 0.79$
Steel reinforcing yield stress	$f_y := 60\ \text{ksi}$
Effective steel yield stress	$f_{ye} := 1.1\ f_y = 66\ \text{ksi}$

<u>Optional</u> effective material strength factors taken from Table 13.7.3-1

Overhang parameters

Distance from girder CL to edge	$X_G := 34\ \text{in}$
Steel girder flange width	$b_{fg} := 16\ \text{in}$
Slab thickness	$t_s := 8\ \text{in}$ *= 8-in. recommended minimum (13.10.2.1)*
Top cover	$c_{ctop} := 2\ \text{in}$
Bottom cover	$c_{cbot} := 1.25\ \text{in}$

Steel post-and-beam railing parameters

Post section	S3x5.7
Post spacing	$L := 6.25\ \text{ft}$
Centroid height of longitudinal elements	$Y := 24.375\ \text{in}$
Flange width	$b_f := 2.33\ \text{in}$
Flange thickness	$t_f := 0.26\ \text{in}$
Flange area	$A_f := b_f \cdot t_f = 0.6\ \text{in}^2$
Post yield stress	$f_{yp} := 50\ \text{ksi}$
Effective post yield stress	$f_{ype} := 1.1\ f_{yp} = 55\ \text{ksi}$
base plate width	$W_b := 8\ \text{in}$
base plate edge distance	$e_b := 1.5\ \text{in}$
Distance from field edge of plate to field edge of post	$e_p := 2.5\ \text{in}$
Distance from field edge of plate to traffic-side bolt line	$d_s := 8.25\ \text{in}$
Anchor bolt diameter	$d_a := 0.625\ \text{in}$
Anchor bolt qty. per line	$n_a := 2$
Anchor bolt yield stress	$f_{ya} := 92\ \text{ksi}$
Anchor bolt tensile stress	$f_{ua} := 120\ \text{ksi}$

<u>Optional</u> effective material strength factors taken from Table 13.7.3-1

1. Identify Design Cases and critical overhang regions

The deck overhang is evaluated for two load cases under the extreme event limit state, with vehicular collision forces, CT, taken as:

Design Case 1: transverse and longitudinal forces developed at the plastic moment capacity of the post. Longitudinal forces are not discussed in this example. Design Case 1 is evaluated at Design Region A-A, which, for overhangs supporting posts, is a trapezoidal yield-line mechanism, and Design Region B-B, which is a plane coincident with the critical section of the exterior girder. Design Regions for overhangs with deck-mounted steel post-and-beam railings are shown in Figure D2.

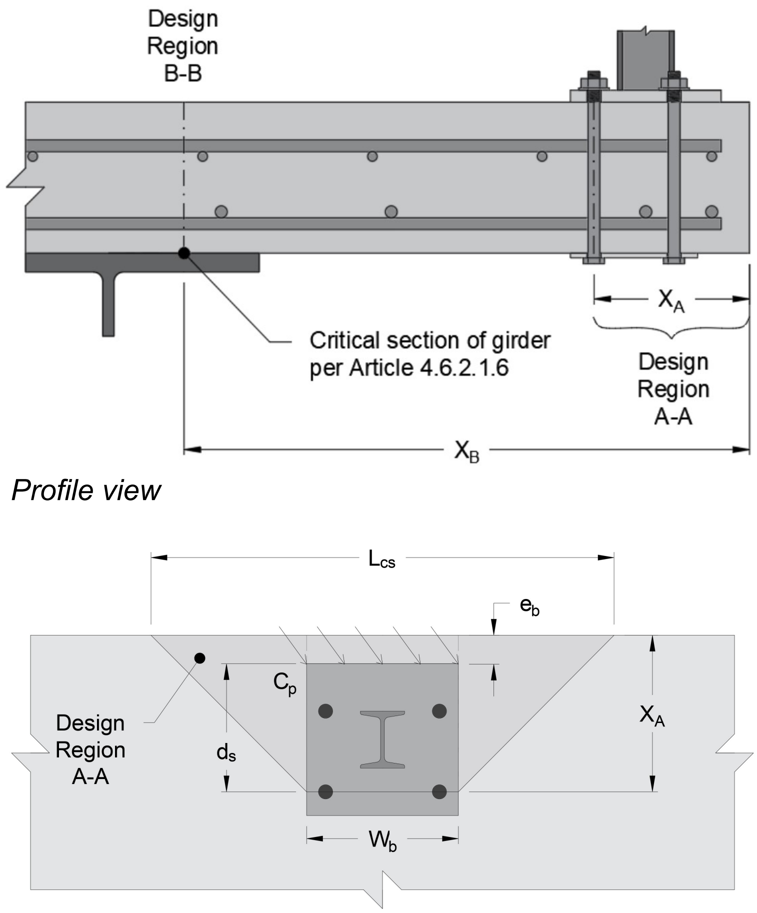


Figure D2. Design Regions for overhangs with concrete posts

Design Case 2: vertical forces specified in Article 13.7.2. Design Case 2 is evaluated only at Design Region B-B, as the distance to Design Region A-A is insufficient to develop a significant moment demand at that location.

For both design cases, the system is evaluated assuming a maximum 3-in. overlay is present to maximize system weight. Overhang design loads based on impact loading are not affected by the wearing surface. In Design Case 1, the slab is designed for M_{post}, and in Design Case 2, F_v is not affected by H, as the attached railing is TL-3.

For this system, Design Region A-A is a trapezoidal yield-line region in the slab. The longitudinal yield-line is located at the traffic-side anchor bolts, which are:

$X_A := e_b + d_s = 9.8$ in   from the field edge of the slab.

Design Region B-B is over the critical section of the supporting element. The overhang is assumed to be supported by a steel girder with a total top flange width of 16 in. The critical section of the flange is offset from the exterior steel girder centerline by the lesser of one-fourth the flange width and 15 in. In this case, the flange width is 16 in.; therefore, the offset distance is 4 in. The distance from the field edge of the slab to Design Region B-B is therefore:

$$X_B := X_G - \frac{b_{fg}}{4} = 30.0 \text{ in}$$

The distance between the two Design Regions is:

$$X_{AB} := X_B - X_A = 20.3 \text{ in}$$

2. Configure transverse steel for strength limit state

If transverse reinforcement is being configured in this process, it should first be designed for adequacy in the strength limit state. In this limit state, the worst-case moment demand induced at Design Region B-B between the 16-kip single wheel patch and 25-kip tandem wheel patch set is used as the live load moment. To determine this moment, the wheel patch is centered 1 ft inside of the innermost face of the railing. If the railing can be considered structurally continuous, i.e. it provides a significant stiffening effect at the field edge, the wheel loading can be replaced with a 1 kip/ft line load applied 1 ft from the railing face. This practice is justified for concrete barriers and may be justified for concrete post-and-beam railings. However, for deck-mounted steel post-and-beam railings, any slab stiffening effect contributed by the railing should be neglected.

As this design example uses an existing deck overhang design, this check has already been performed, and transverse steel has already been configured. Example strength limit state analyses for stiffened and unstiffened slab edges are shown in the accompanying concrete barrier and concrete post examples, respectively.

3. Establish ultimate post capacity and associated overhang demands

In this section, overhang design demands for extreme event limit state, Design Case 1 are calculated. Unlike for overhangs with concrete barriers, which are designed for the lateral impact load spread over an effective distribution length, it is recommended that overhangs with steel posts are designed to support the ultimate capacity of the post. It should be noted that, when calculating section strengths in the extreme event limit state, <u>optional</u> effective material properties (Table 13.7.3-1) are used.

$f'_{ce} = 5.2\ \text{ksi}$ $\quad$ $f_{ye} = 66\ \text{ksi}$ $\quad$ $f_{ype} = 55\ \text{ksi}$ (13.7.3-1)

The plastic section modulus of the attached post is:

$Z_x := 1.94\ \text{in}^3$

The plastic moment capacity of the attached post is:

$M_{post} := f_{ype} \cdot Z_x = 8.9\ \text{kip} \cdot \text{ft}$

The centroid height of the longitudinal railing elements is:

$Y = 24.4\ \text{in}$ *(Distance from top surface of slab to centroid of steel railing)*

Therefore, the lateral load on the railing which induces the plastic moment capacity of the post is:

$$P_{post} := \frac{M_{post}}{Y} = 4.4\ \text{kip}$$

The total moment demand acting at the centroid of the slab is therefore:

$$P_{post} \cdot (Y + 0.5\ t_s) = 10.4\ \text{kip} \cdot \text{ft}$$

The compressive force applied to the deck may be conservatively determined as the yield force of the flange:

$$C_p := A_f \cdot f_{ye} = 40\ \text{kip} \qquad \text{(C13.10.2.4.2-1)}$$

If the stress distribution on the base plate is assumed to be rectangular, and the compression flange is assumed to reach its yield force, the compression block depth at M_{post} is:

$$a_p := \frac{C_p}{0.85\ f'_c \cdot W_b} = 1.5\ \text{in} \qquad \text{(C13.10.2.4.2-2)}$$

These demands are summarized in Figure D3.

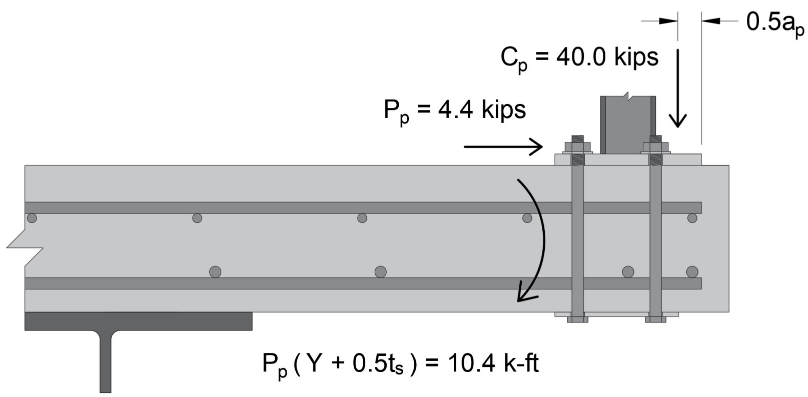


Figure D3. Loads acting on slab at plastic capacity of post

The slab must be able to transfer the compressive force from the post base into the slab through vertical shear or through a diagonal compression strut, if the slab steel is detailed such that a strut and tie mechanism can develop. The slab must also be able to support the downward compressive force in a yield-line mechanism. In this mechanism, the post base shear effectively reduces the effectiveness of transverse steel. Lastly, the tensile and flexural demand shown must be resisted as distributed loads at Design Region B-B.

Prior to performing the overhang analyses, the required force in the traffic-side bolt line at M_{post} is compared to corresponding yield force. If traffic-side bolts are expected to deform significantly, the confidence of the analysis performed herein is reduced. Significant bolt deformations may result in a decreased punching shear and/or yield-line capacities in the slab.

The traffic-side bolt line yield force is:

$$T_n := 0.76 \cdot \frac{\pi}{4}\ d_a^{\ 2} \cdot n_a \cdot f_{ya} = 42.9\ \text{kip}$$

The required force in the traffic-side bolt line at M_{post} can be approximated as:

$$T_u := \frac{M_{post}}{d_s - e_p - 0.5\ t_f} = 19\ \text{kip}$$

Therefore, the bolt yield utility ratio is:

$$\frac{T_u}{T_n} = 0.44$$

The anchor bolts are not expected to yield. If anchor bolts are expected to yield, the following procedure may still be valid; however, unexpected mechanisms and slab damage may occur, so testing to verify adequacy would be warranted.

4. Slab-post joint damage check

The local strength of the slab underneath the base plate must be sufficient to support the yield force of the compression flange. Slabs supporting posts often develop significant diagonal cracking under the post. Depending on the dimensions and steel configuration of the joint, the failure mechanism causing this damage may vary. An example of this damage is shown in Figure D4. (Frosch 2016).

Figure D4. Diagonal cracking of slab

Load transfer from the base plate into the slab is primarily a shear mechanism. However, if joint steel is specially configured, a full or partial strut-and-tie behavior may develop, reducing the reliance on the punching shear strength of the slab for load transfer. For the design used in this example, the development of a strut-and-tie mechanism cannot be justified. Therefore, it is assumed that the slab must transfer post loads via vertical shear, as shown below.

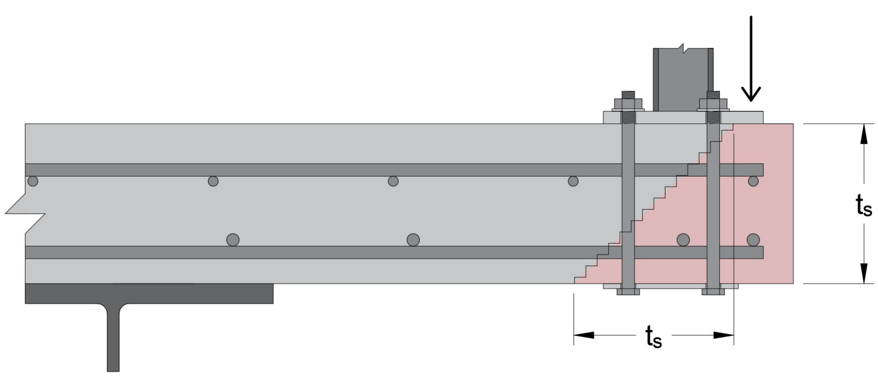


Figure D5. Shear failure of slab under base plate

A strut-and-tie behavior cannot develop for this system because the transverse slab bars are not hooked, and the edge distance is insufficient. A horizontal tension tie would not be able to develop in the top-mat transverse steel to oppose the horizontal component of the diagonal compression strut.

The effective critical perimeter of the punching shear mechanism is shown in Figure D6. The critical perimeter is offset from the assumed base plate compression zone by one-half the slab thickness.

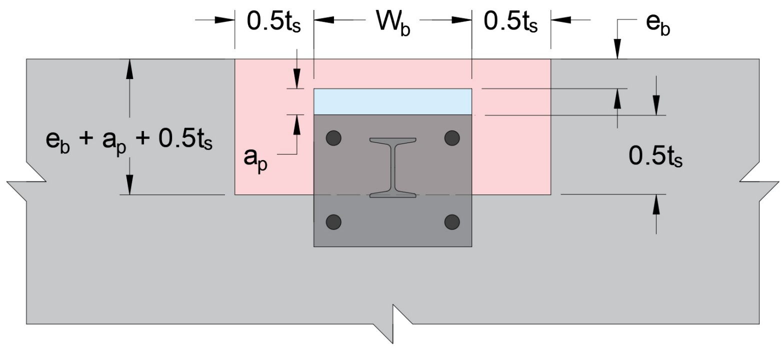


Figure D6. Punching shear mechanism in slab

It is not permitted for the longitudinal punching shear plane to extend beyond the traffic-side bolt line, as the upward tension in these bolts provides shear resistance. For this design, the half-thickness offset from the plate compression zone extends beyond the bolt line; therefore, the critical perimeter is adjusted as shown below:

The critical perimeter is calculated as:

$$b_o := W_b + t_s + 2\left(e_b + a_p + 0.5\ t_s\right) = 29.9\ \text{in} \qquad (13.10.2.4.2\text{-}5)$$

The load patch aspect ratio is taken as:

$$\beta_c := \frac{W_b}{a_p} = 5.4$$

The effective concrete shear strength is:

$$v_c := min\left(0.0633 + \frac{0.1265}{\beta_c}, 0.1265\right) \cdot \sqrt{f'_{ce} \cdot \text{ksi}^{-1}}\ \text{ksi} = 197.3\ \text{psi} \qquad (13.10.2.4.2\text{-}4)$$

Therefore, the punching shear capacity of the slab is:

$$V_n := b_o \cdot v_c \cdot t_s = 47.3\ \text{kip} \qquad (13.10.2.4.2\text{-}3)$$

The compression flange yield force is:

$$C_p = 40\ \text{kip}$$

Therefore, the punching shear utility ratio is:

$$\frac{C_p}{V_n} = 0.85 \qquad (13.10.2.4.2\text{-}2)$$

and the slab is adequate in punching shear.

If the slab steel is not configured to develop a strut-and-tie behavior, it should be assumed that failure to provide an adequate punching shear capacity will result in a sudden and severe failure prior to reaching the post strength. Further, the appropriateness of applying the yield-line method described in the following section after a punching shear failure has occurred is unknown. If transverse bars are straight, punching shear damage may also significantly interrupt bar development at the traffic face of the post.

---

5. Slab yield-line capacity

In this section, the yield-line capacity of the slab is calculated and compared to the yield force of the post compression flange. The assumed mechanism is shown in Figure D7.

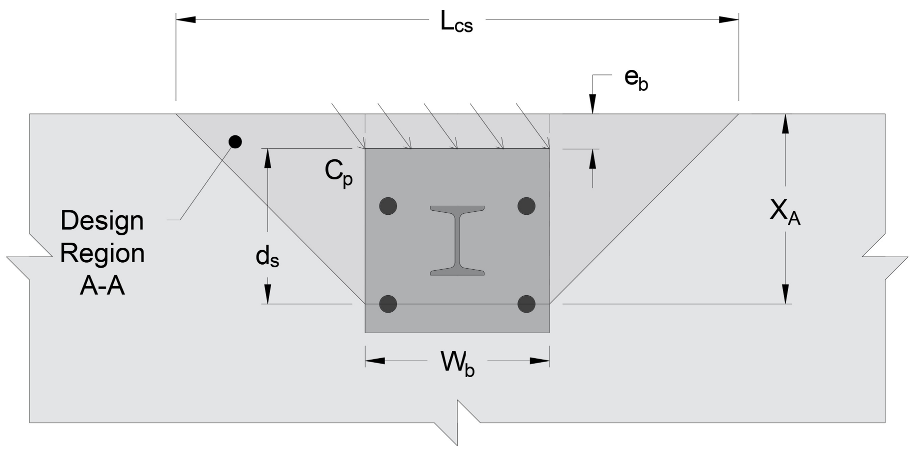


Figure D7. Slab yield-line mechanism

As the yield-line mechanism engages both transverse and longitudinal steel, two slab bending strengths must be calculated.

Transverse bending strength along longitudinal yield-line

The bending strength about the longitudinal yield-line is the transverse bending strength of the slab through the traffic-side bolt line section. The distance from the straight bar termination to this section is 6.75 in., and the development length of the #5 bar is

$$l_d := 2.4 \cdot 0.625 \text{ in} \cdot \frac{f_{ye} \cdot \text{ksi}^{-1}}{\sqrt{f'_{ce} \cdot \text{ksi}^{-1}}} \cdot 1.0 \cdot 1.0 \cdot 0.4 \cdot 1.0 = 17.4 \text{ in}$$

Therefore, the maximum stress which can be developed in the transverse bars at the longitudinal yield-line is:

$$f_{ye} \cdot \frac{6.75\ \text{in}}{l_d} = 25.7\ \text{ksi}$$

As the preceding diagonal joint damage check was passed, the equilibrium state shown in Figure D8 is assumed.

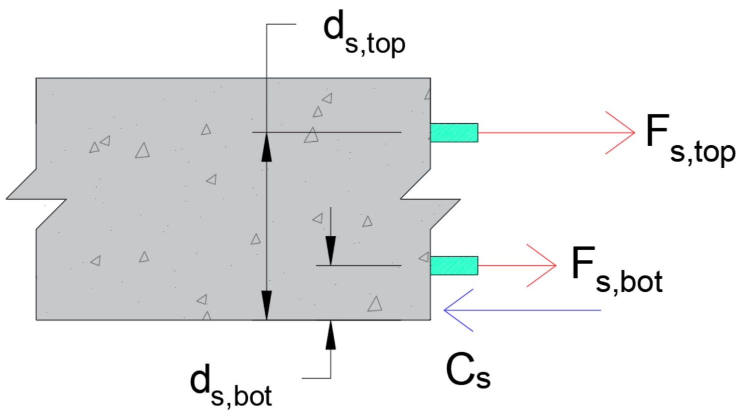


Figure D8. Slab couple forces at full bending strength (if diagonal tension damage check passes)

If the preceding diagonal tension damage check were failed, the assumed equilibrium state is as shown in Figure D9.

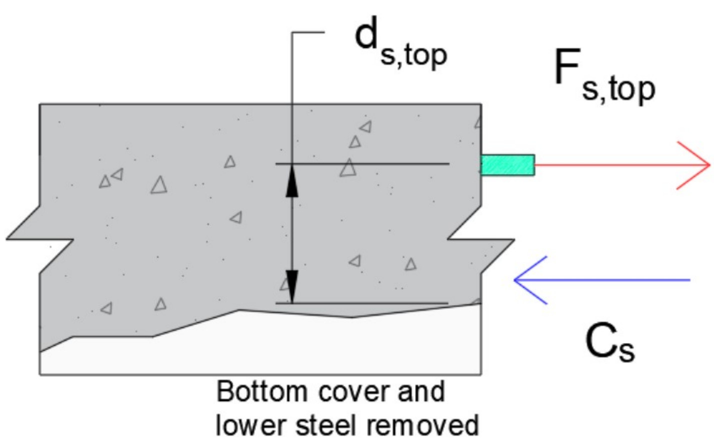


Figure D9. Slab couple forces at full bending strength (if diagonal tension damage check fails)

The neutral axis location was iteratively determined to be:

$$c_s := 0.50\ \text{in}$$

The compression block depth is therefore:

$$a_s := \beta_1 \cdot c_s = 0.40\ \text{in}$$

The effective depth to each layer of transverse slab steel is:

$d_{stop} := 8\ \text{in} - 2\ \text{in} - 0.5 \cdot 0.625\ \text{in} = 5.7\ \text{in}$ *8-in. thick slab, 2-in. cover, #5 bars*

$d_{sbot} := 1.25\ \text{in} + 0.5 \cdot 0.625\ \text{in} = 1.6\ \text{in}$ *1.25-in. cover, #5 bars*

The top-mat steel is assumed to be at yield stress. At this neutral axis depth, the strain in the bottom-mat steel is

$$\varepsilon_{sbot} := \frac{\varepsilon_{cu}}{c_s}\left(d_{sbot} - c_s\right) = 0.0064$$

The effective yield strain is:

$$\varepsilon_{ye} := \frac{f_{ye}}{29000\ \text{ksi}} = 0.0023$$

Which is less than the bottom-mat steel strain. Both layers of steel are at yield stress, and the force in each layer is:

$$F_{stop} := f_{ye} \cdot \frac{6.75\ \text{in}}{l_d} \cdot 0.614\ \frac{\text{in}^2}{\text{ft}} = 15.8\ \frac{\text{kip}}{\text{ft}} \qquad F_{sbot} := f_{ye} \cdot \frac{6.75\ \text{in}}{l_d} \cdot 0.205\ \frac{\text{in}^2}{\text{ft}} = 5.3\ \frac{\text{kip}}{\text{ft}}$$

The concrete compressive force is:

$$C_s := 0.85 \cdot f'_{ce} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot a_s = 21\ \frac{\text{kip}}{\text{ft}}$$

Which is in vertical equilibrium with steel forces:

$$F_{stop} + F_{sbot} - C_s = 0\ \frac{\text{kip}}{\text{ft}}$$

The cantilever bending strength of the slab is calculated using Eqn. 5.6.3.2.2-1:

$$M_n := \boxed{A_{ps} f_{ps}} \cdot \left(d_p - \frac{a}{2}\right) + A_s f_s \cdot \left(d_s - \frac{a}{2}\right) - A_s' f_s' \cdot \left(d_s' - \frac{a}{2}\right) + \alpha_1 f'_c \cdot \left(b - b_w\right) \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

As the section is rectangular, b_w is equal to b. Additionally, the section does not include prestressing strands. Therefore, the equation is reduced to:

$$M_n := \boxed{A_s f_s} \cdot \left(d_s - \frac{a}{2}\right) - A_s' f_s' \cdot \left(d_s' - \frac{a}{2}\right)$$

In this case, $A_s f_s$ is the force in the top-mat transverse steel, and $A_s' f_s'$ is the force in the bottom-mat transverse steel. The preceding strain calculations predicted that both layers of steel reached their maximum developable stress in tension. The stress in the top-mat steel is:

$$f_s := f_{ye} \cdot \frac{6.75\ \text{in}}{l_d} = 25.7\ \text{ksi}$$

The stress in the bottom-mat steel is:

$$f_s' := -f_{ye} \cdot \frac{6.75\ \text{in}}{l_d} = -25.7\ \text{ksi}$$

It should be noted that the stress in the bottom-mat steel was taken as negative, as Eqn. 5.6.3.2.2-1 assumes the compression-side steel is in compression. Substituting variable names used in this example results in a transverse slab bending strength of:

$$M_{stA} := 0.614\ \frac{\text{in}^2}{\text{ft}} \cdot f_s \cdot \left(d_{stop} - \frac{a_s}{2}\right) - 0.205\ \frac{\text{in}^2}{\text{ft}} \cdot f_s' \cdot \left(d_{sbot} - \frac{a_s}{2}\right) = 7.8\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The procedure used above to calculate M_{st} is consistent with Article 5.6.3.2.5, which states "The stress and corresponding strain in any given layer of reinforcement may be taken from any representative stress-strain formula or graph for nonprestressed reinforcement..." In this case, a rectangular compression block and linear strain diagram were assumed.

The distributed tension acting on the section due to the lateral load is:

$$N := \frac{P_{post}}{W_b} = 6.6\ \frac{\text{kip}}{\text{ft}} \qquad \textit{(13.10.2.4.1-1)}$$

The effect of extreme event, Design Case 1 tension on the bending strength of the slab is accounted for by determining the capacity of the section in pure tension, then linearly interpolating between the pure tension and pure flexure strengths.

The tensile capacity of the slab in pure tension is:

$$P_n := F_{stop} + F_{sbot} = 21\ \frac{\text{kip}}{\text{ft}}$$

The flexural capacity of the slab in pure flexure is:

$$M_{stA} = 7.8\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The tension acting on the slab is:

$$N = 6.6\ \frac{\text{kip}}{\text{ft}}$$

Therefore, the effective bending strength of the slab under the imposed tension is:

$$M_{strA} := \left(1 - \frac{N}{P_n}\right) M_{stA} = 5.4\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The process used to determine the effective bending strength of the slab is shown in Figure D10.

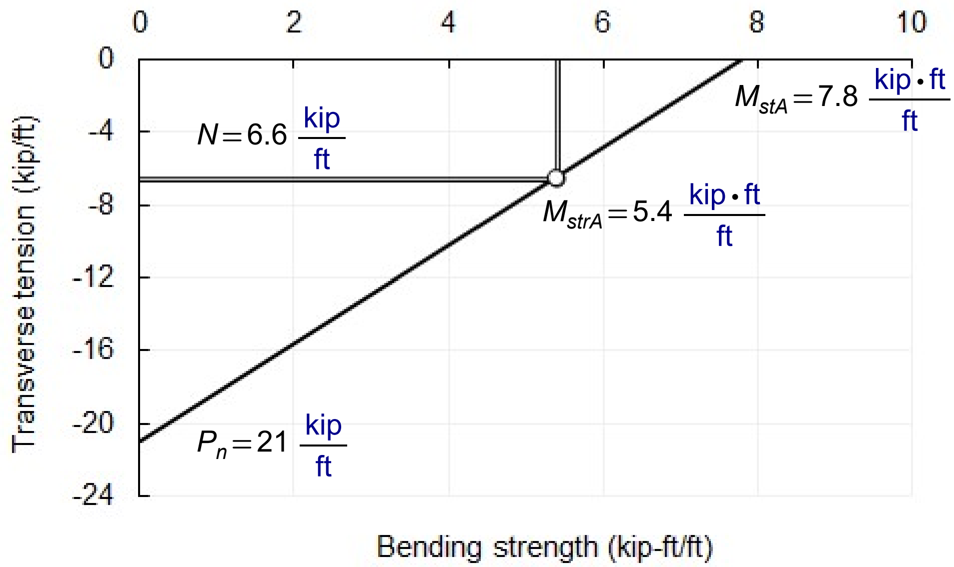


Figure D10. Calculation of effective bending strength under applied axial tension

Transverse bending strength along diagonal yield-lines

The transverse bending strength about the diagonal yield-lines engages the same steel configuration as along the longitudinal line, however, the bar embedment depth is reduced. In this case, the bar embedment depth at the mid-height of the yield-line mechanism, 2.6 in., is conservatively used to characterize an average stress available in the transverse bars along the diagonal yield-lines as:

$$f_{ye} \cdot \frac{2.62\ in}{l_d} = 10\ ksi$$

Applying the same process as for the longitudinal yield-line results in a transverse bending strength along the diagonal yield-lines of:

$$M_{stA} := 4.0\ \frac{kip \cdot ft}{ft}$$

As all tension was assumed to be taken within the width of the base plate, effects of axial slab tension are not included in the diagonal yield-line strength calculation.

Longitudinal bending strength along diagonal yield-lines

The longitudinal bending strength is asymmetric. Along the diagonal yield-lines, the top surface of the slab is in tension; along the transverse yield-lines on the bottom slab surface, the bottom surface is in tension. As the diagonal and transverse yield-lines are rotated through equal angles, the effective longitudinal bending strength of the slab can be taken as the average of the positive and negative bending strengths. Directional, longitudinal bending strengths are shown in Figure D11.

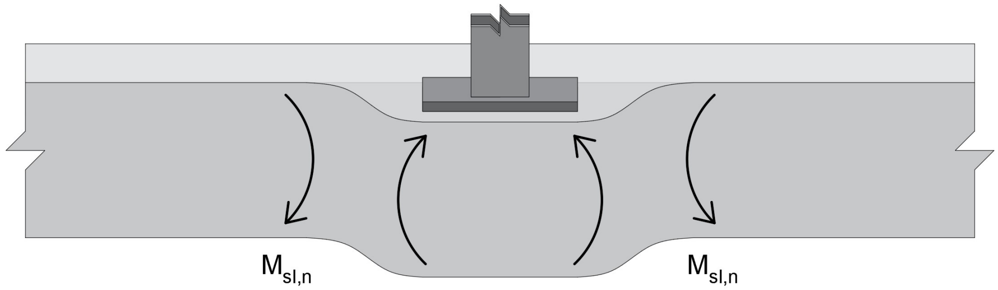
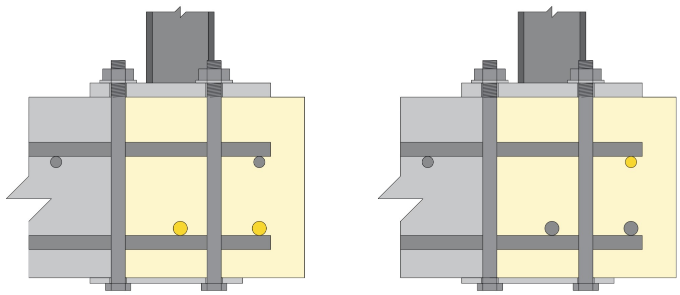


Figure D11. Directional, longitudinal bending strengths activated in slab for downward deflection of field edge

The effective edge beam area for longitudinal bending is shown in Figure D12. For positive and negative bending, bars assumed to contribute to the respective bending strength are highlighted.



Positive flexure

$M_{slp} := 17.7 \text{ kip} \cdot \text{ft}$

Negative flexure

$M_{sln} := 5.3 \text{ kip} \cdot \text{ft}$

Figure D12. Activated area and participating bars for longitudinal slab bending

This value is zero if preceding diagonal damage check is failed

The effective longitudinal bending strength of the slab under the post is taken as the average of positive and negative bending strengths:

$$M_{sl} := \frac{1}{2}\left(M_{slp} + M_{sln}\right) = 11.5 \text{ kip} \cdot \text{ft}$$

The basic slab bending capacities are summarized below.

$M_{strA} = 5.4 \ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$ *Longitudinal yield line*

$M_{stA} = 4.0 \ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$ *Diagonal yield-lines*

$M_{sl} = 11.5 \text{ kip} \cdot \text{ft}$ *Diagonal yield-lines*

Slab yield-line mechanism

The critical length of the yield-line mechanism is:

$$L_{cs} := W_b + \sqrt{\frac{8 \cdot M_{sl} \cdot X_A}{M_{stA}}} = 5 \text{ ft} \qquad (13.10.2.4.2\text{-}9)$$

The maximum post moment able to be supported by the slab in the yield-line mechanism is:

$$M_{posteff} := \frac{M_{post}}{C_p} \cdot \left(\frac{X_A}{X_A - e_b}\right) \left(M_{strA} \cdot \frac{W_b}{X_A} + M_{stA} \cdot \frac{L_{cs} - W_b}{X_A} + M_{sl} \cdot \frac{8}{L_{cs} - W_b}\right) = 12.3 \text{ kip} \cdot \text{ft}$$

(13.10.2.4.2-8)

The utility ratio for the slab yield-line mechanism is:

$$\frac{M_{post}}{M_{posteff}} = 0.72$$

Therefore, the yield-line capacity of the slab is adequate to develop the full strength of the post and Mposteff equals Mpost.

---

6. Evaluation of slab for distributed demands at exterior girder

---

After the local capacity of the slab under the post is evaluated, the second overhang critical section must be checked against distributed demands. The two evaluations that must be performed at Design Region B-B are:

Design Case 1: distributed tension and moment demands associated with plastic moment capacity of post

Design Case 2: distributed moment demand induced by vertical loading applied to railing

In both design cases, the system is evaluated in its post-overlay state. Using this state maximizes system weight and, therefore, design demands. Design loads related to vehicle impact for post-and-beam railings for TL-3 and below are not affected by railing height.

Design Case 1 evaluation

The total moment demand acting on the slab at the ultimate strength of of the post is:

$$P_{post} \cdot \left(Y + 0.5\ t_s\right) = 10.4 \text{ kip} \cdot \text{ft}$$

Design Case 1 and 2 moments are assumed to distribute longitudinally according to the pattern shown in Figure D13.

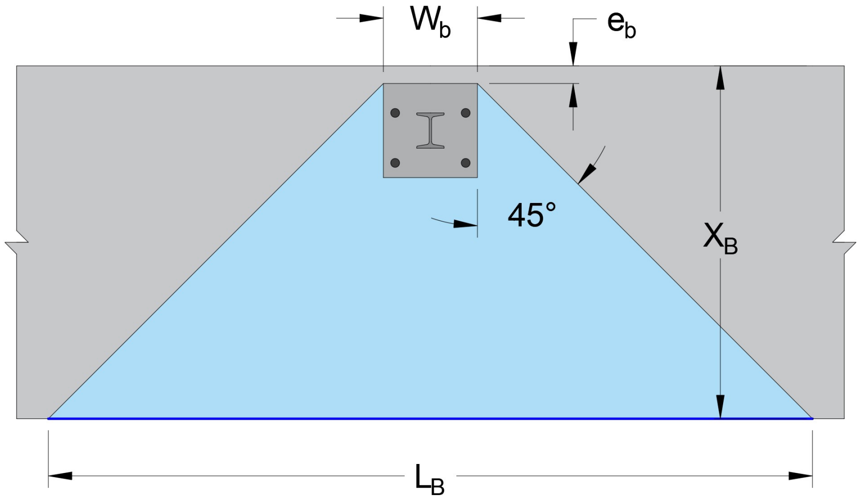


Figure D13. Distribution of moment demands through slab

The effective distribution length for the post moment demand at Design Region B-B is:

$$L_B := W_b + 2\left(X_B - e_b\right) = 5.4\ \text{ft} \qquad (13.10.2.4.2\text{-}12)$$

The slab self-weight moment at Design Region B-B is:

$$M_{DC} := 150\ \text{pcf} \cdot \left(12\ \frac{\text{in}}{\text{ft}}\right) \cdot t_s \cdot \frac{1}{2}\ X_B^{\ 2} = 0.3\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

Assuming the 3-in. wearing surface is poured flush with the traffic side of the base plate, the wearing surface moment at Design Region B-B is:

$$H_{wear} := 3\ \text{in}$$

$$M_{DW} := 140\ \text{pcf} \cdot \left(12\ \frac{\text{in}}{\text{ft}}\right) \cdot H_{wear} \cdot \frac{1}{2}\ \left(X_B - e_p - 8\ \text{in}\right)^2 = 0.05\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

Therefore, the Design Case 1 moment demand at Design Region B-B, which includes the distributed post moment and self-weight, is:

$$M_{swB} := M_{DC} + M_{DW} = 0.4\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

$$M_{1B} := \frac{P_{post} \cdot \left(Y + 0.5\ t_s\right)}{L_B} + M_{swB} = 2.3\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.4.2\text{-}15)$$

The distributed tension at Design Region B-B can conservatively be assumed to be equal to the tensile demand at A-A:

$$N = 6.6\ \frac{\text{kip}}{\text{ft}} \qquad (13.10.2.4.2\text{-}1)$$

Transverse bars are fully developed at Design Region B-B. Therefore, the pure flexural and tensile strengths at that Region are:

$$M_{stB} := 16.9\ \frac{kip \cdot ft}{ft} \qquad P_n := 44.9\ \frac{kip}{ft}$$

Using the applied tensile demand to linearly interpolate between the pure flexural and tensile strengths results in an effective bending strength at Design Region B-B of:

$$M_{strB} := \left(1 - \frac{N}{P_n}\right) M_{stB} = 14.4\ \frac{kip \cdot ft}{ft}$$

and the utility ratio for Design Case 1 at Design Region B-B is:

$$\frac{M_{1B}}{M_{strB}} = 0.16 \qquad (13.10.2.4.2\text{-}14)$$

Design Case 2 evaluation

The vertical impact load and application length for a TL-3 railing is:

$$F_v := 4.5\ kip \qquad \text{(Table 13.7.2-1)}$$

$$L_v := 18\ ft \qquad \text{(Table 13.7.2-1)}$$

The portion of the total vertical load acting on a single post can be taken as:

$$\frac{L}{L_v} F_v = 1.6\ kip$$

which is conservatively applied at the back face of the railing, which is flush with the back face of the post, as shown in Figure D14. Therefore, the total moment at Design Region B-B is:

$$\frac{L}{L_v} F_v \cdot (X_B - e_p) = 3.6\ kip \cdot ft$$

which is distributed over:

$$L_B = 5.4\ ft$$

(13.10.2.4.2-16)
note: identical to
(13.10.2.4.2-12)

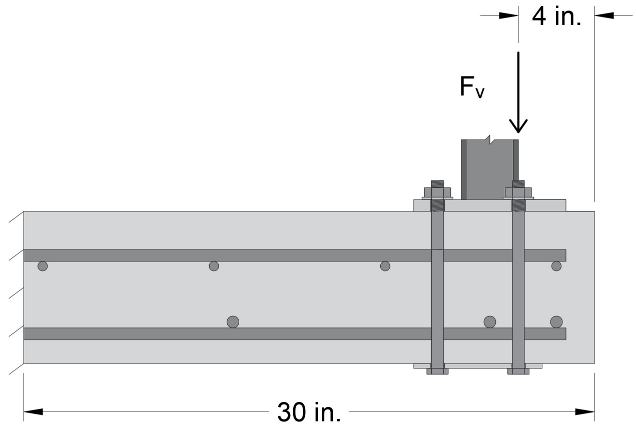


Figure D14. Application of Design Case 2 loading

Therefore, the Design Case 2 moment demand at Design Region B-B, which includes the distributed vertical load moment and self-weight moment, is:

$$M_{2B} := \frac{L \cdot F_v \cdot \left(X_B - e_p\right)}{L_v \cdot L_B} + M_{swB} = 1.0\ \frac{kip \cdot ft}{ft} \qquad (13.10.2.4.2\text{-}19)$$

The utility ratio of this demand when compared to the bending strength at B-B (with no tension penalty) is:

$$\frac{M_{2B}}{M_{stB}} = 0.06 \qquad (13.10.2.4.2\text{-}18)$$

Due to the low magnitude of the vertical post load, punching shear for Design Case 2 does not need to be evaluated.

As the overhang's local yield-line capacity is sufficient to develop the full capacity of the post, and Design Region B-B is sufficient to withstand distributed lateral and vertical design loads, the overhang design is **adequate** for the attached steel post-and-beam railing.

Note - the system used in this example was a weak post system. Stronger posts are likely to fail punching shear checks according to this methodology, which agrees with common observations in such crash-tested systems. However, numerous such systems with stronger posts have resulted in successful crash tests by containing and redirecting impacting vehicles when functioning as a system of multiple posts connected by continuous rails. Failing to satisfy the criteria of this methodology does not necessarily indicate that a particular system is inadequate, only that the simplified computational tools that can be feasibly codified are not capable of efficiently representing the complex behavior of frame-type systems when damage, such as punching shear failure at steel post-to-deck connections, is expected to occur.

APPENDIX E

Curb-Mounted Steel-Post Example

The following design example includes a full analysis of an overhang supporting a modified version of the MASSDOT S3TL4 curb-mounted steel post-and-beam railing. Extreme event design loading is taken from proposed revisions to Section 13 of the AASHTO LRFD Bridge Design Specifications available at the time this example was prepared. The proposed draft language has been provided to AASHTO for consideration.

Note that this example was prepared in Mathcad Prime 8. Due to limitations in variable formatting, commas in subscripts were omitted (e.g., "*cctop*" was used in examples in place of "*cc,top*" in the body of this report and the proposed revisions). Similarly, Y was used in examples in place of $\overline{Y}$.

In this example, the ability of a curbed deck overhang design to develop the full capacity of an attached W6x20 steel post is evaluated. The system used for this demonstration, which is loosely based on the MASSDOT S3-TL4 bridge rail, is shown in Figure E1.

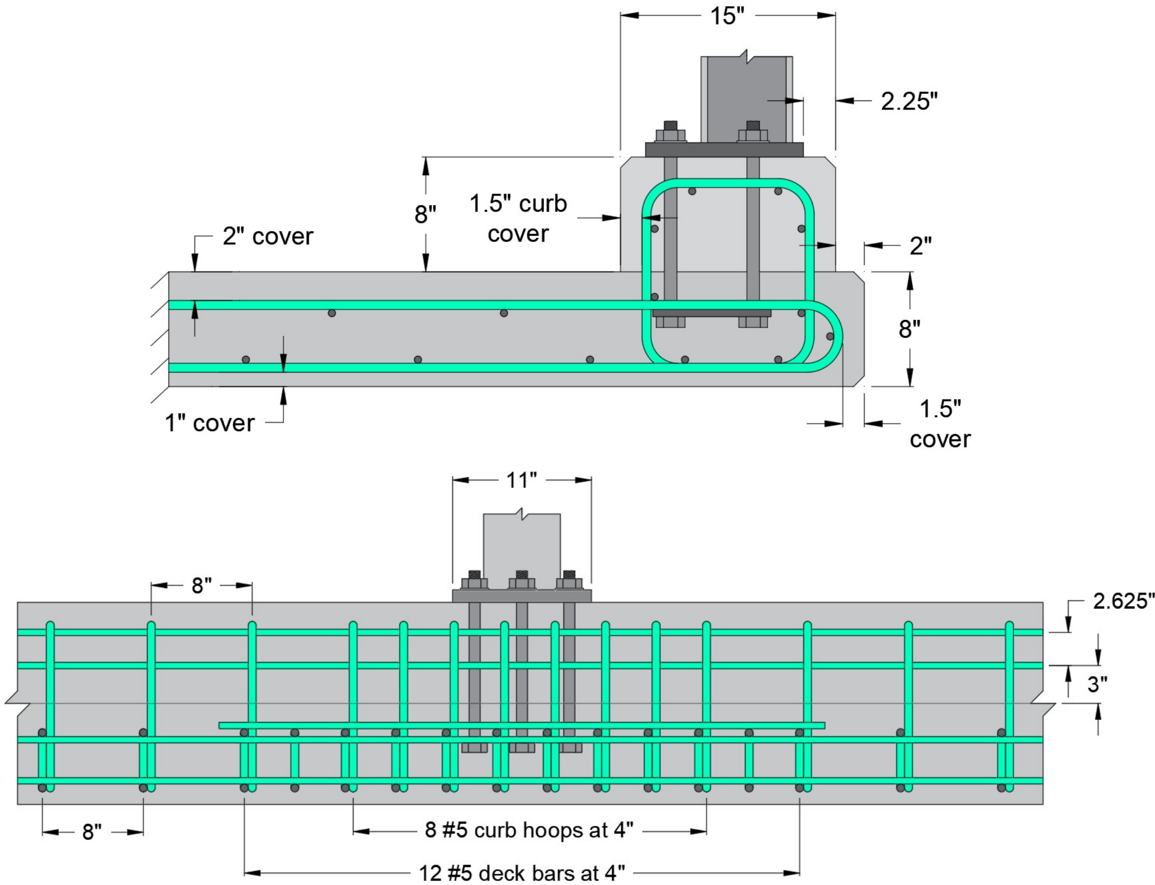


Figure E1. Example curb-mounted steel post-and-beam system

The general design/analysis procedure is as follows:

1. Identify critical overhang regions
2. Establish ultimate capacity of post and associated deck overhang demands
3. Evaluate ability of curb to transfer loads from base plate into slab
4. Check the deck joint for diagonal tension damage
5. Calculate the yield-line capacity of the slab and compare to required compressive force
6. Estimate overhang demands at Design Region B-B and compare to slab strength

Known system parameters

Design parameters required for the analysis performed herein are listed below.

Overhang dimensions

Slab thickness	$t_s := 8$ in
Top bar cover	$c_{ctop} := 2$ in
Bottom bar cover	$c_{cbot} := 1$ in
Field edge cover	$c_{cf} := 1.5$ in
Overhang distance	$X_B := 48$ in

Overhang steel

Transverse bar size	$d_{bt} := 0.625$ in
At-post spacing	$s_{s1} := 4$ in
General spacing	$s_{s2} := 8$ in
Longitudinal bar size	$d_{bl} := 0.5$ in

Post dimensions

Post section	W6x20
Post yield stress	$f_{yp} := 50$ ksi
Eff. post yield stress	$f_{ype} := 1.1\ f_{yp}$
Section depth	$d_p := 6.2$ in
Flange thickness	$t_f := 0.365$ in
Flange width	$b_f := 6.02$ in
Flange area	$A_f := t_f \cdot b_f$
Web thickness	$t_w := 0.26$ in
Post spacing	$L := 8$ ft

Materials

Concrete strength	$f'_c := 5$ ksi
Eff. concrete strength	$f'_{ce} := 1.3\ f'_c$
Concrete crush strain	$\varepsilon_{cu} := 0.003$
Concrete weight	$w_c := 150$ pcf
Rebar yield stress	$f_y := 60$ ksi
Eff. yield stress	$f_{ye} := 1.1\ f_y$
Plate yield stress	$f_{yb} := 50$ ksi
Bolt yield stress	$f_{ya} := 92$ ksi
Bolt tensile stress	$f_{ua} := 120$ ksi

Optional effective material strength factors taken from Table 13.7.3-1

Curb dimensions

Curb height	$h_{curb} := 8$ in
Curb width	$b_{curb} := 15$ in
Curb edge distance	$e_c := 2$ in
Curb bar cover	$c_{curb} := 1.5$ in

Curb and slab are separate pours

Curb steel

Vertical bar size	$d_{bct} := 0.625$ in
At post spacing	$s_{c1} := 4$ in
General spacing	$s_{c2} := 8$ in
Longitudinal bar size	$d_{bcl} := 0.625$ in

Base plate dimensions

Base plate width	$W_b := 11$ in
Base plate thickness	$t_b := 1$ in
Field edge to t-bolts	$d_t := 9.25$ in
Plate edge distance	$e_b := 2.25$ in
Post edge distance	$e_p := 0.75$ in

Washer plate dimensions

Washer plate width	$W_{wp} := 13$ in
Washer plate depth	$b_{wp} := 8.25$ in
Thickness	$t_{wp} := 0.5$ in

Anchor bolts

Anchor bolt diameter	$d_a := 0.875$ in
Traffic-side quantity	$n_a := 3$

1. Identify Design Cases and critical overhang regions

The deck overhang is evaluated for two load cases under the extreme event limit state, with vehicular collision forces, CT, taken as:

Design Case 1: transverse and longitudinal forces developed at the plastic moment capacity of the post. Longitudinal forces are not discussed in this example. Design Case 1 is evaluated at Design Region A-A, which, for overhangs supporting posts, is a trapezoidal yield-line mechanism, and Design Region B-B, which is a plane coincident with the critical section of the exterior girder. Design regions for overhangs with curb-mounted steel post-and-beam railings are shown in Figure E2.

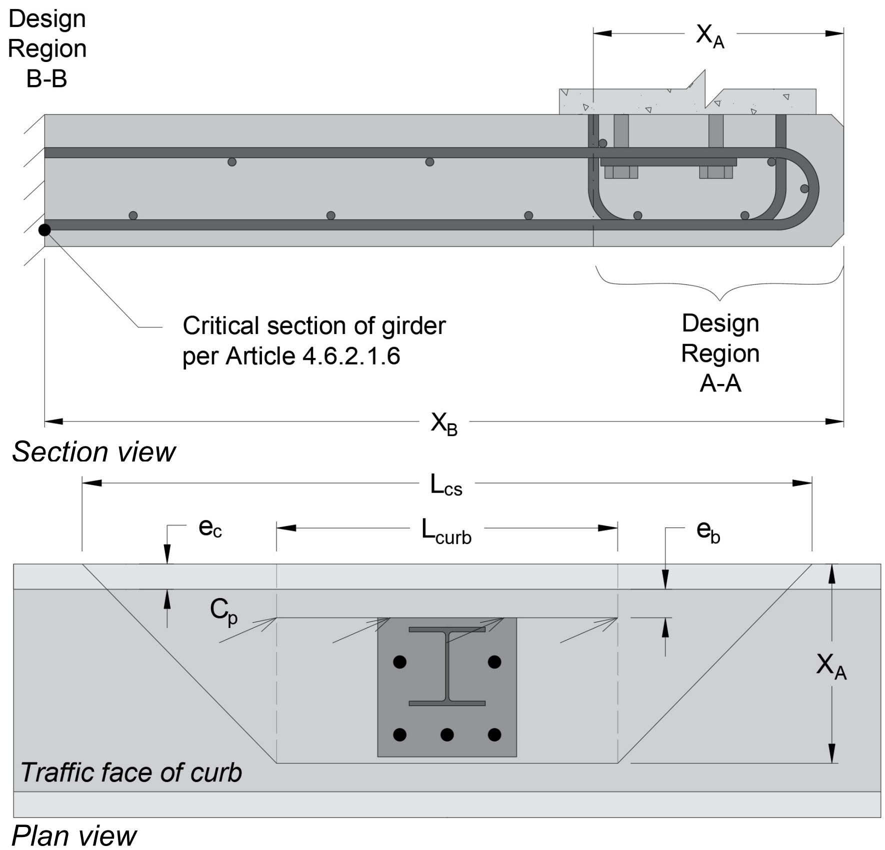


Figure E2. Design regions for overhangs with concrete posts

Design Case 2: vertical forces specified in Article 13.7.2. Design Case 2 is evaluated only at Design Region B-B, as the distance to Design Region A-A is insufficient to develop a significant moment demand at that location.

For both design cases, the system is evaluated assuming a maximum overlay thickness of 3-in. is present. Although TL-4 lateral design loads decrease with decreasing railing height, overhangs supporting posts are designed to withstand the plastic moment capacity of the post. Therefore, the deck design procedure is independent of F_t and H_e, and the additional weight of the wearing surface makes the post-overlay state the critical design case. Additionally, TL-4 vertical design loads increase with decreasing railing height - thus, the vertical design load is greatest when the overlay is present and the railing height above the vehicle riding surface is at its minimum of 36 in.

For this system, Design Region A-A is a trapezoidal yield-line region in the slab under the curb. The longitudinal yield-line is located at the traffic-side vertical curb steel, which is:

$X_A := e_c + b_{curb} - c_{curb} - 0.5\ d_{bct} = 15.2\ in$ from the field edge of the slab.

Design Region B-B is over the critical section of the supporting element which, in this case, is 48 in. from the field edge of the slab. Therefore:

$X_B = 48.0\ in$

The distance between the two design regions is:

$X_{AB} := X_B - X_A = 32.8\ in$

Aside: Configuring transverse steel for strength limit state

If transverse reinforcement is being configured in this process, it should first be designed for adequacy in the strength limit state. In this limit state, the worst-case moment demand induced at Design Region B-B between the 16-kip single wheel patch and 25-kip tandem wheel patch set is used as the live load moment. To determine this moment, the wheel patch is centered 1 ft inside of the innermost face of the railing. If the railing can be considered structurally continuous, i.e. it provides a significant stiffening effect at the field edge, the wheel loading can be replaced with a 1 kip/ft line load applied 1 ft from the railing face. This practice may be justified for curb-mounted steel post-and-beam railings, as the curb is structurally continuous along the field edge of the slab.

In this example, the deck was designed for Strength I under concentrated wheel loads. Example strength limit state analyses for stiffened and unstiffened slab edges are shown in the accompanying concrete barrier and concrete post examples, respectively.

---

2. Establish ultimate post capacity and associated overhang demands

---

In this section, overhang design demands for extreme event Design Case 1 are calculated. Unlike for overhangs with concrete barriers, which are designed for the lateral impact load spread over an effective distribution length, it is recommended that overhangs with steel posts are designed to support the ultimate capacity of the post. It should be noted that, when calculating section strengths in the extreme event limit state, optional effective material properties (Table 13.7.3-1) are used.

$f'_{ce} = 6.5\ ksi$ $\quad f_{ye} = 66\ ksi$ $\quad f_{ype} = 55\ ksi$

The plastic section modulus of the attached post is:

$Z_x := 15\ in^3$

The plastic moment capacity of the attached post is:

$$M_{post} := f_{ype} \cdot Z_x = 68.8 \text{ kip} \cdot \text{ft}$$

The centroid height of the longitudinal railing elements above the deck surface is:

$Y := 30 \text{ in}$ *(Based on railing and curb design)*

Therefore, the lateral load on the railing which induces the plastic moment capacity of the post is:

$$P_{post} := \frac{M_{post}}{Y - h_{curb}} = 37.5 \text{ kip}$$

The total moment demand acting at the centroid of the slab is therefore:

$$P_{post} \cdot \left(Y + 0.5\ t_s\right) = 106.3 \text{ kip} \cdot \text{ft}$$

The compressive force at the back of the post at the plastic moment capacity is:

$C_p := A_f \cdot f_{ype} = 120.9 \text{ kip}$ *(Flange yield force)* (C13.10.2.4.3-1)

If the compression block under the base plate is assumed rectangular, and it is assumed that the post compression flange will yield, the depth of the compression block under the plate is:

$$a_p := \frac{C_p}{0.85\ f'_c \cdot W_b} = 2.6 \text{ in} \qquad \text{(C13.10.2.4.3-2)}$$

from the field edge of the plate.

The slab must be able to transfer the compressive force from the post base into the slab through vertical shear or through a diagonal compression strut, if the slab steel is detailed such that a strut and tie mechanism can develop. The slab must also be able to support the downward compressive force in a yield-line mechanism. In this mechanism, the transverse post base shear effectively reduces the effectiveness of transverse steel. Lastly, the tensile and flexural demand shown must be resisted as distributed loads at Design Region B-B.

Prior to performing the overhang analyses, the required force in the traffic-side bolts at M_{post} is evaluated to verify elastic anchor behavior. If traffic-side bolts are expected to deform significantly, the confidence of the analysis performed herein is reduced. Significant bolt deformations may result in a decreased punching shear and/or yield-line capacities in the slab, as well as influencing the moment resistance available from the post and associated inelastic post-and-beam system capacity.

The traffic-side bolt line yield force is:

$$T_n := 0.76 \cdot \frac{\pi}{4}\ d_a^{\ 2} \cdot n_a \cdot f_{ya} = 126.1 \text{ kip}$$

The required force in the traffic-side bolt line at M_{post} can be approximated as:

$$T_u := \frac{P_{post} \cdot \left(Y - h_{curb}\right)}{d_t - 0.5\ a_p} = 103.7\ \text{kip}$$

Therefore, the bolt yield utility ratio is:

$$\frac{T_u}{T_n} = 0.82$$

Therefore, it is not expected that yielding of the traffic-side anchor bolts will occur.

---

3. Curb capacity evaluation

Prior to analyzing the deck slab, the curb's ability to transfer the post loads into the slab must first be evaluated.

Curb Limit State: flexure about longitudinal axis

In this limit state, the flexural capacity of the curb at its base is evaluated against the distributed post demands acting at that location. The total moment at the base of the curb is:

$$P_{post} \cdot Y = 94\ \text{kip} \cdot \text{ft}$$

An appropriate design moment at the base of the curb can be estimated by assuming the post demands distribute longitudinally at an angle of 45 degrees with downward transmission through the curb, as shown in Figure E3.

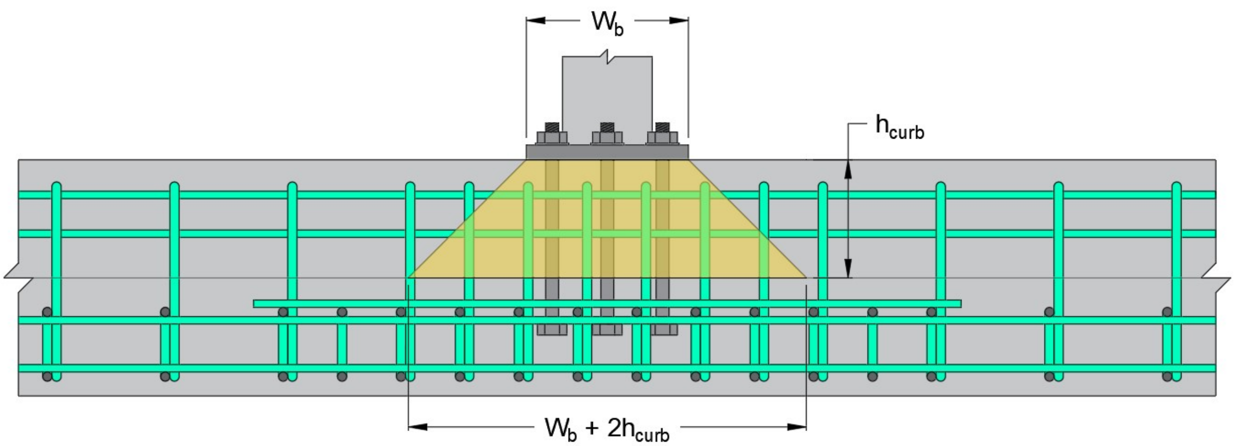


Figure E3. Effective distribution of post loads through curb

The distributed flexural demand at the base of the curb is therefore:

$$L_{curb} := W_b + 2\ h_{curb} = 27\ \text{in} \qquad (13.10.2.4.3\text{-}2)$$

$$M_{ucurb} := \frac{P_{post} \cdot Y}{L_{curb}} = 41.7\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.4.3\text{-}5)$$

Next, the unit-length flexural capacity of the curb at its base is calculated for comparison to the distributed demand. In this analysis, it is assumed that the curb vertical bars are able to develop their yield stress at the deck surface, as they are hooked around longitudinal bars within the slab. Additionally, both layers of steel were considered in the analysis. Under these assumptions, the cantilever bending strength of the curb is calculated using the process shown below.

The neutral axis location relative to the field edge of the curb was iteratively determined to be:

$$c_c := 1.55\ \text{in}$$

The compression block depth is therefore:

$$a_c := \left(0.85 - 0.05\ \left(f'_{ce} - 4\ \text{ksi}\right)\ \text{ksi}^{-1}\right) \cdot c_c = 1.12\ \text{in}$$

The effective depth to each layer of vertical curb steel is:

$$d_{straf} := b_{curb} - c_{curb} - 0.5\ d_{bct} = 13.2\ \text{in}$$

15-in. wide curb, 1.5-in. cover, #5 bar

$$d_{sfield} := c_{curb} + 0.5\ d_{bct}$$

The traffic-side steel is assumed to be at its yield stress. At this neutral axis depth, the strain in the field-side steel is

$$\varepsilon_{sfield} := \frac{\varepsilon_{cu}}{c_c}\ \left(d_{sfield} - c_c\right) = 0.0005$$

The effective yield strain is:

$$\varepsilon_{ye} := \frac{f_{ye}}{29000\ \text{ksi}} = 0.0023$$

Therefore, the field-side steel is in tension, but is not yielded. The force in the traffic-side steel is:

$$F_{straf} := \frac{\pi}{4}\ d_{bct}^{\ 2} \cdot \frac{12\ \text{in} \cdot \text{ft}^{-1}}{s_{c1}} \cdot f_{ye} = 60.7\ \frac{\text{kip}}{\text{ft}}$$

The force in the field-side steel is:

$$F_{sfield} := \frac{\pi}{4}\ d_{bct}^{\ 2} \cdot \frac{12\ \text{in} \cdot \text{ft}^{-1}}{s_{c1}} \cdot 29000\ \text{ksi}\ \varepsilon_{sfield} = 13.6\ \frac{\text{kip}}{\text{ft}}$$

The concrete compressive force is:

$$C_s := 0.85 \cdot f'_{ce} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot a_c = 74.5\ \frac{\text{kip}}{\text{ft}}$$

Which is in vertical equilibrium with steel forces:

$$F_{straf} + F_{sfield} - C_s = 0\ \frac{\text{kip}}{\text{ft}}$$

The cantilever bending strength of the slab is calculated using Eqn. 5.6.3.2.2-1:

$$M_n := \boxed{A_{ps} f_{ps}} \cdot \left(d_p - \frac{a}{2}\right) + A_s f_s \cdot \left(d_s - \frac{a}{2}\right) - A_s' f_s' \cdot \left(d_s' - \frac{a}{2}\right) + \alpha_1 f'_c \cdot \left(b - b_w\right) \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

As the section is rectangular, b_w is taken equal to b. Additionally, the section does not include prestressing strands. Therefore, the equation is reduced to:

$$M_n := \boxed{A_s f_s} \cdot \left(d_s - \frac{a}{2}\right) - A_s' f_s' \cdot \left(d_s' - \frac{a}{2}\right)$$

In this case, $A_s f_s$ is the force in the traffic-side curb steel, and $A_s' f_s'$ is the force in the field-side curb steel. The stress in the top-mat steel is:

$$f_s := f_{ye} = 66 \text{ ksi}$$

The stress in the bottom-mat steel is:

$$f_s' := -\varepsilon_{sfield} \cdot 29000 \text{ ksi} = -14.7 \text{ ksi}$$

It should be noted that the stress in the field-side curb steel was taken as negative, as Eqn. 5.6.3.2.2-1 assumes the compression-side steel is in compression. Substituting variable names used in this example results in a curb bending strength of:

$$A_{scurb} := \frac{\pi}{4} d_{bct}^{\ 2} \cdot \frac{12 \text{ in} \cdot \text{ft}^{-1}}{s_{c1}} = 0.92 \frac{\text{in}^2}{\text{ft}}$$

$$M_{ncurb} := f_s \cdot A_{scurb} \cdot \left(d_{straf} - \frac{a_c}{2}\right) - f_s' \cdot A_{scurb} \cdot \left(d_{sfield} - \frac{a_c}{2}\right) = 65.3 \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The procedure used above to calculate M_{ncurb} is consistent with Article 5.6.3.2.5, which states "The stress and corresponding strain in any given layer of reinforcement may be taken from any representative stress-strain formula or graph for nonprestressed reinforcement..." In this case, a rectangular compression block and linear strain diagram were assumed.

The bending demand about the longitudinal axis of the bridge at the deck surface is:

$$M_{ucurb} = 41.7 \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

Therefore, the demand-to-capacity ratio for span-axis bending at the curb base is:

$$\frac{M_{ucurb}}{M_{ncurb}} = 0.64 \qquad (13.10.2.4.3\text{-}4)$$

Therefore, the curb is able to transfer the full plastic moment of the W6x20 post and its associated demands into the deck overhang. Note that other limit states related to anchorage should also be evaluated as applicable to the particular design under consideration and AASHTO LRFD BDS Article 5.13.

4. Slab-post joint damage check

The local strength of the slab underneath the curb must be sufficient to support the yield force of the compression flange. Slabs supporting posts often develop significant diagonal cracking under the post. Depending on the dimensions and steel configuration of the joint, the failure mechanism causing this damage may vary. An example of this damage is shown in Figure E7 (Frosch 2016).

Figure E7. Diagonal joint damage in slab

Load transfer from the base plate into the slab is primarily a shear mechanism. However, if joint steel is configured properly, a full or partial strut-and-tie behavior may develop, reducing the reliance on the punching shear strength of the slab for load transfer. For the design used in this example, the development of a strut-and-tie mechanism cannot be justified. Therefore, it is assumed that the curb and slab must transfer post loads via punching shear, as shown below.

A strut-and-tie behavior cannot develop for this system because the edge distance is insufficient. A horizontal tension tie would not be able to develop in the top-mat transverse steel to oppose the horizontal component of the diagonal compression strut.

The punching shear mechanism is shown in Figures E8 through E10. In this example, the load is conservatively applied at the field edge of the base plate.

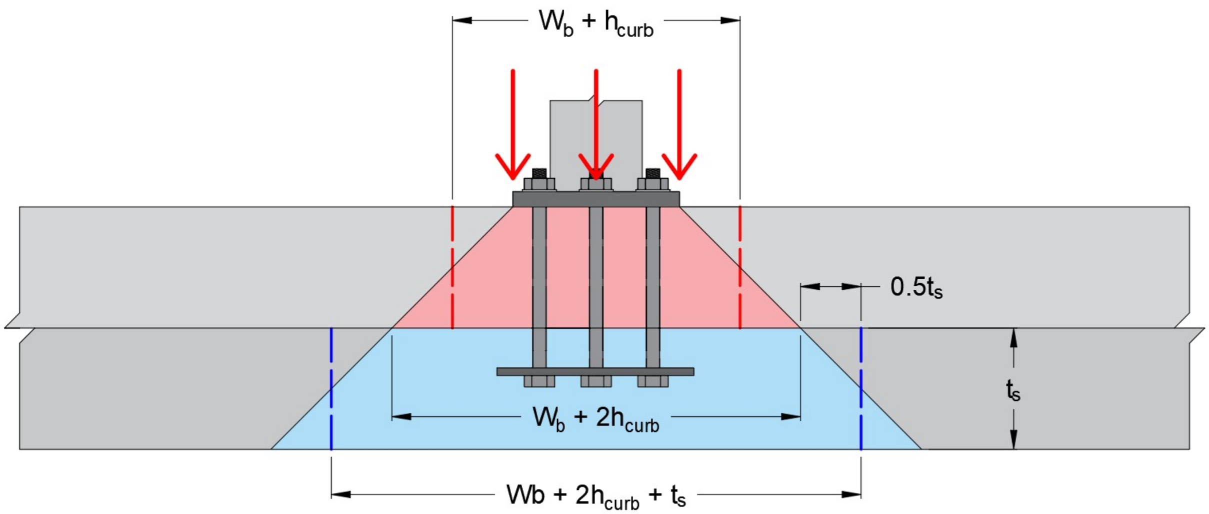


Figure E8. Elevation view of punching shear mechanism

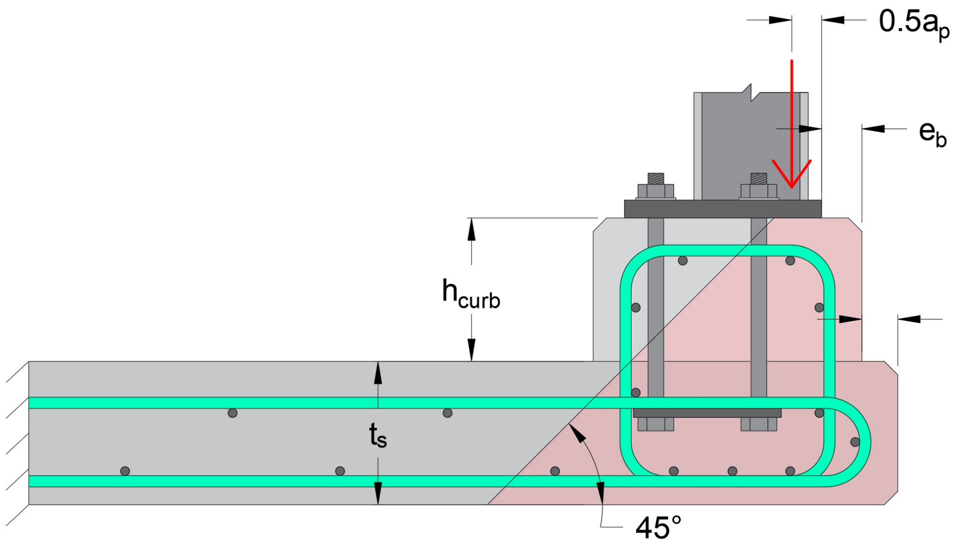


Figure E9. Section view of punching shear mechanism

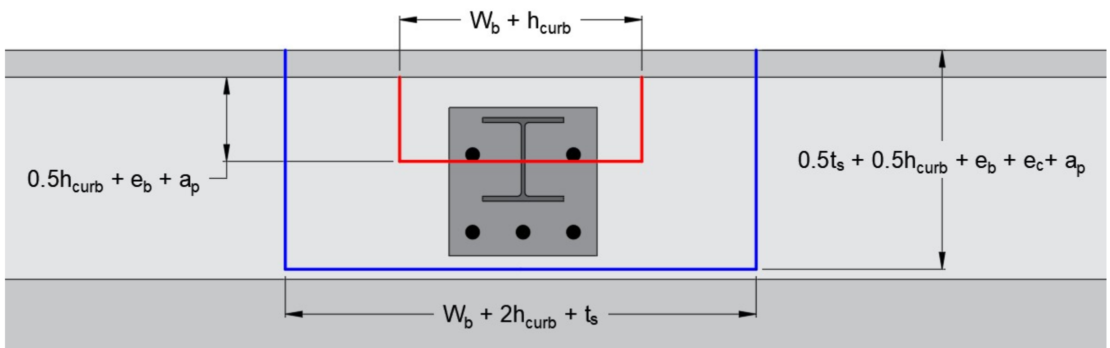


Figure E10. Plan view of punching shear mechanism

The critical perimeter of the punching shear mechanism in the curb is:

$$b_{oc} := \left(W_b + h_{curb}\right) + 2\ \left(0.5\ h_{curb} + e_b + a_p\right) = 36.7\ \text{in} \qquad (13.10.2.4.3\text{-}9)$$

The curb area resisting punching shear is:

$$A_{psc} := b_{oc} \cdot h_{curb} = 293.4\ \text{in}^2$$ included in (13.10.2.4.3-7)

The critical perimeter of the punching shear mechanism in the slab is:

$$b_{os} := \left(W_b + 2\ h_{curb} + t_s\right) + 2\ \left(0.5\ t_s + h_{curb} + e_b + e_c + a_p\right) = 72.7\ \text{in} \qquad (13.10.2.4.3\text{-}10)$$

The curb area resisting punching shear is:

$$A_{pss} := b_{os} \cdot t_s = 581.4\ \text{in}^2$$ included in (13.10.2.4.3-7)

The total area of curb and slab resting punching shear is:

$$A_{ps} := A_{psc} + A_{pss} = 874.7 \text{ in}^2$$ included in (13.10.2.4.3-7)

The load patch aspect ratio is taken as:

$$\beta_c := \frac{W_b}{a_p} = 4.3$$

The effective concrete shear strength is:

$$v_c := min\left(0.0633 + \frac{0.1265}{\beta_c}, 0.1265\right) \cdot \sqrt{f'_{ce} \cdot \text{ksi}^{-1}} \text{ ksi} = 237.2 \text{ psi} \qquad (13.10.2.4.3\text{-}8)$$

Therefore, the punching shear capacity of the curbed edge is:

$$V_n := A_{ps} \cdot v_c = 207.5 \text{ kip} \qquad (13.10.2.4.3\text{-}7)$$

The compression flange yield force is:

$$C_p = 120.9 \text{ kip} \qquad (C13.10.2.4.3\text{-}1)$$

Therefore, the punching shear utility ratio is:

$$\frac{C_p}{V_n} = 0.58 \qquad (13.10.2.4.3\text{-}6)$$

and the slab is adequate in punching shear.

If the slab steel is not configured to develop a strut-and-tie behavior, it should be assumed that failure to provide an adequate punching shear capacity will result in a sudden and severe failure prior to reaching the post strength. Further, the appropriateness of applying the yield-line method described in the following section after a punching shear failure has occurred is unknown. If transverse bars are straight, punching shear damage may also significantly interrupt bar development at the traffic face of the post.

5. Slab yield-line capacity

In this section, the yield-line capacity of the slab is calculated and compared to the yield force of the post compression flange. The assumed mechanism is shown in Figure E11.

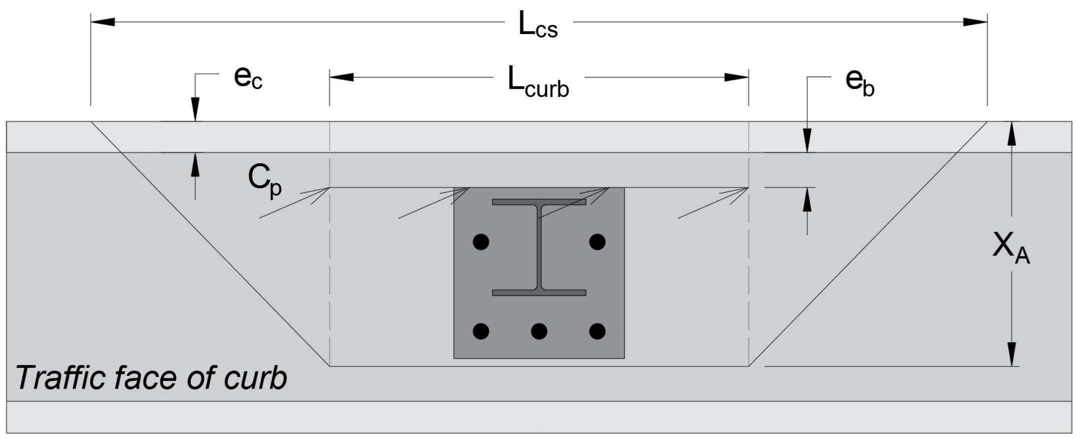


Figure E11. Yield-line mechanism for slabs with curb-mounted steel posts

Transverse bending strength along longitudinal yield-line

The hooked #5 transverse bar is fully developed at the longitudinal yield-line location.

As the preceding diagonal joint damage check was passed, the equilibrium state shown in Figure E12 is assumed.

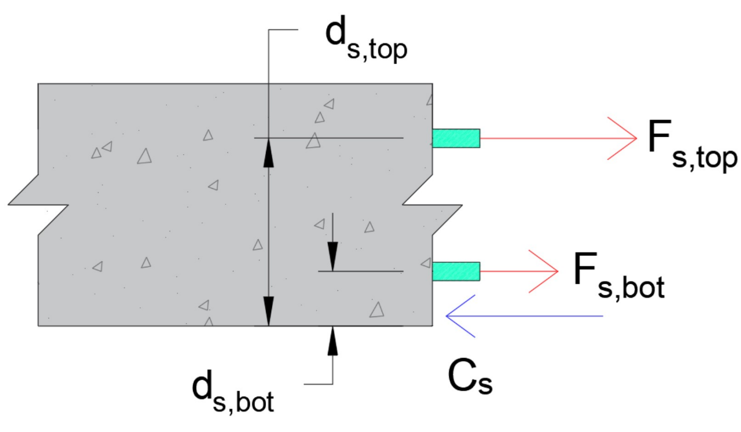


Figure E12. Slab couple forces at full bending strength
(if diagonal tension damage check passes)

If the preceding diagonal tension damage check were failed, the assumed equilibrium state is as shown in Figure E13.

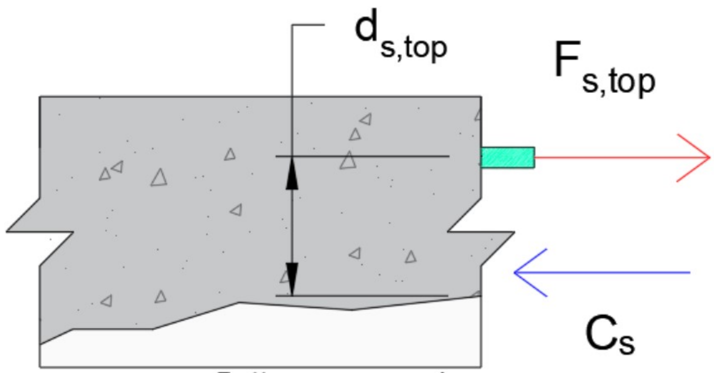


Figure E13. Slab couple forces at full bending strength (if diagonal tension damage check fails)

The neutral axis location was iteratively determined to be:

$$c_s := 1.772 \text{ in}$$

The compression block depth is therefore:

$$\beta_1 := 0.85 - 0.05 \left(f'_{ce} - 4 \text{ ksi}\right) \text{ ksi}^{-1} = 0.73$$

$$a_s := \beta_1 \cdot c_s = 1.28 \text{ in}$$

The effective depth to each layer of transverse slab steel is:

$$d_{stop} := 8 \text{ in} - 2 \text{ in} - 0.5 \cdot 0.625 \text{ in} = 5.7 \text{ in}$$ *8-in. thick slab, 2-in. cover, #5 bars*

$$d_{sbot} := 2 \text{ in} + 0.5 \cdot 0.625 \text{ in} = 2.3 \text{ in}$$ *2-in. cover, #5 bars*

The top-mat steel is assumed to be at its maximum developable stress. At this neutral axis depth, the strain in the bottom-mat steel is

$$\varepsilon_{sbot} := \frac{\varepsilon_{cu}}{c_s} \left(d_{sbot} - c_s\right) = 0.0009$$

The effective yield strain is:

$$\boxed{\varepsilon_{ye}} := \frac{f_{ye}}{29000 \text{ ksi}} = 0.0023$$

Therefore, the bottom-mat steel is not yielded at the ultimate strength of the slab. The force in each mat of steel is:

$$F_{stop} := f_{ye} \cdot 0.92\ \frac{in^2}{ft} = 60.7\ \frac{kip}{ft}$$

$$F_{sbot} := 29000\ ksi \cdot \varepsilon_{sbot} \cdot 0.92\ \frac{in^2}{ft} = 24.4\ \frac{kip}{ft}$$

The concrete compressive force is:

$$\boxed{C_s} := 0.85 \cdot f'_{ce} \cdot 12\ \frac{in}{ft} \cdot a_s = 85.2\ \frac{kip}{ft}$$

Which is in vertical equilibrium with steel forces:

$$F_{stop} + F_{sbot} - C_s = 0\ \frac{kip}{ft}$$

The cantilever bending strength of the slab is calculated using Eqn. 5.6.3.2.2-1:

$$M_n := \boxed{A_{ps}f_{ps}} \cdot \left(d_p - \frac{a}{2}\right) + A_s f_s \cdot \left(d_s - \frac{a}{2}\right) - A_s' f_s' \cdot \left(d_s' - \frac{a}{2}\right) + \alpha_1 f'_c \cdot \left(b - b_w\right) \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

As the section is rectangular, b_w is equal to b. Additionally, the section does not include prestressing strands. Therefore, the equation is reduced to:

$$M_n := \boxed{A_s f_s} \cdot \left(d_s - \frac{a}{2}\right) - A_s' f_s' \cdot \left(d_s' - \frac{a}{2}\right)$$

In this case, $A_s f_s$ is the force in the top-mat transverse steel, and $A_s' f_s'$ is the force in the bottom-mat transverse steel. The preceding strain calculations predicted that the top layer of steel was yielded in tension, therefore:

$$\boxed{f_s} := f_{ye} = 66\ ksi$$

The stress in the bottom-mat steel is:

$$\boxed{f_s'} := 29000\ ksi \cdot -\varepsilon_{sbot} = -26.5\ ksi$$

It should be noted that the stress in the bottom-mat steel was taken as negative, as Eqn. 5.6.3.2.2-1 assumes the compression-side steel is in compression. Substituting variable names used in this example results in a transverse slab bending strength of:

$$M_{stA} := 0.92\ \frac{in^2}{ft} \cdot f_s \cdot \left(d_{stop} - \frac{a_s}{2}\right) - 0.92\ \frac{in^2}{ft} \cdot f_s' \cdot \left(d_{sbot} - \frac{a_s}{2}\right) = 28.9\ \frac{kip \cdot ft}{ft}$$

The procedure used above to calculate M_{stA} is consistent with Article 5.6.3.2.5, which states "The stress and corresponding strain in any given layer of reinforcement may be taken from any representative stress-strain formula or graph for nonprestressed reinforcement..." In this case, a rectangular compression block and linear strain diagram were assumed.

The distributed tension acting on the section due to the lateral load is:

$$N := \frac{P_{post}}{L_{curb}} = 16.7\ \frac{\text{kip}}{\text{ft}} \qquad (13.10.2.4.1\text{-}1)$$

The effect of extreme event, Design Case 1 tension on the bending strength of the slab is accounted for by determining the capacity of the section in pure tension, then linearly interpolating between the pure tension and pure flexure strengths.

The tensile capacity of the slab in pure tension is:

$$P_n := 2 \cdot 0.92\ \frac{\text{in}^2}{\text{ft}} \cdot f_{ye} = 121.4\ \frac{\text{kip}}{\text{ft}}$$

The flexural capacity of the slab in pure flexure is:

$$M_{stA} = 28.9\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The tension acting on the slab is:

$$N = 16.7\ \frac{\text{kip}}{\text{ft}}$$

Therefore, the effective bending strength of the slab under the imposed tension is:

$$M_{strA} := \left(1 - \frac{N}{P_n}\right) M_{stA} = 25.0\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

The process used to determine the effective bending strength of the slab is shown in Figure E14.

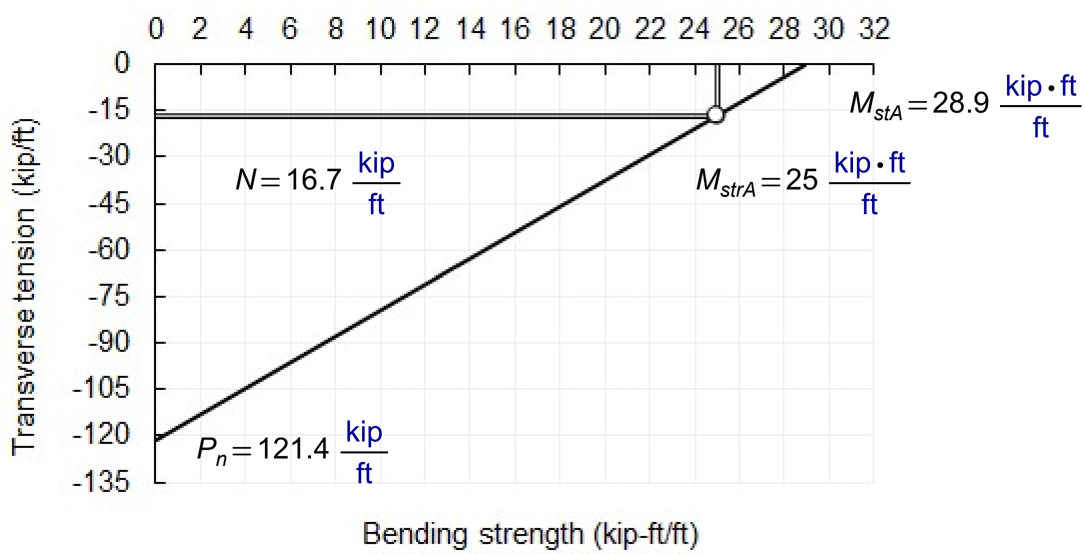


Figure E14. Calculation of effective bending strength under applied axial tension

Transverse bending strength along diagonal yield-lines

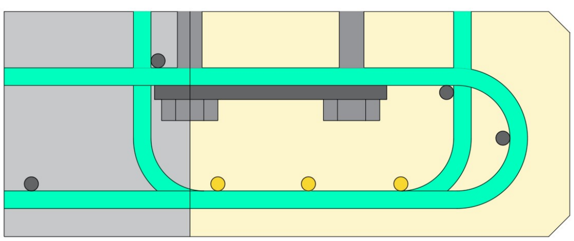
It was assumed that the entire lateral tension was taken by the steel along the longitudinal yield-line. Therefore, along the diagonal yield-lines, the transverse bending strength is:

$$M_{stA} = 28.9\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

Longitudinal bending strength along diagonal yield-lines

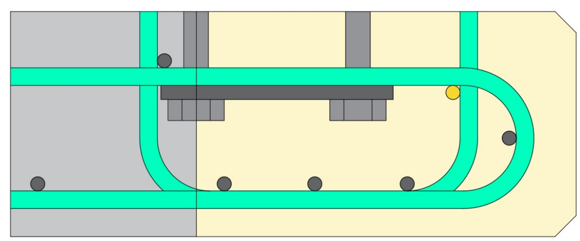
The longitudinal bending strength is asymmetric. Along the diagonal yield-lines, the top surface of the slab is in tension; along the transverse yield-lines on the bottom slab surface, the bottom surface is in tension. As the diagonal and transverse yield-lines are rotated through equal angles, the effective longitudinal bending strength of the slab can be taken as the average of the positive and negative bending strengths. As the extent to which the curb and slab act as a composite beam is unknown, the longitudinal bending strength of the curb is not included in this evaluation.

The effective edge beam area for longitudinal bending is shown below. For positive and negative bending, bars assumed to contribute to the respective bending strength are highlighted.



$$M_{slp} := 18.8\ \text{kip} \cdot \text{ft}$$

Figure E12. Areas and bars considered in calculation of longitudinal bending strength



$$M_{sln} := 5.4\ \text{kip} \cdot \text{ft}$$

This value is zero if preceding diagonal damage check is failed

The effective longitudinal bending strength is taken as the average of the positive and negative bending strengths:

$$M_{sl} := \frac{1}{2}\left(M_{slp} + M_{sln}\right) = 12.1\ \text{kip} \cdot \text{ft}$$

The basic slab bending capacities are summarized below.

$$M_{strA} = 25.0\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$ *Longitudinal yield line*

$$M_{stA} = 28.9\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$ *Diagonal yield-lines*

$$M_{sl} = 12.1\ \text{kip} \cdot \text{ft}$$ *Diagonal yield-lines*

Yield-line mechanism

The critical length of the yield-line mechanism is:

$$L_{cs} := L_{curb} + \sqrt{\frac{8 \cdot M_{sl} \cdot X_A}{M_{stA}}} = 4.3 \text{ ft} \qquad (13.10.2.4.3\text{-}14)$$

The maximum post moment able to be supported by the slab is:

$$M_{posteff} := \frac{M_{post} \cdot \left(Y - h_{curb}\right)}{C_p \cdot Y} \cdot \left(\frac{X_A}{X_A - e_b - e_c}\right) \left(M_{strA} \cdot \frac{L_{curb}}{X_A} + M_{stA} \cdot \frac{L_{cs} - L_{curb}}{X_A} + M_{sl} \cdot \frac{8}{L_{cs} - L_{curb}}\right) = 80.2 \text{ kip} \cdot \text{ft}$$

The utility ratio for the slab yield-line capacity is: (13.10.2.4.3-13)

$$\frac{M_{post}}{M_{posteff}} = 0.86$$

Therefore, the yield-line capacity of the slab is adequate to develop the full strength of the post, and Mposteff equals Mpost.

---

6. Evaluation of slab for distributed demands at exterior girder

---

After the local capacity of the slab under the post is evaluated, the second overhang critical section must be checked against distributed demands. The two evaluations that must be performed at Design Region B-B are:

Design Case 1: distributed tension and moment demands associated with plastic moment capacity of post

Design Case 2: distributed moment demand induced by vertical loading applied to railing

In both design cases, the system is evaluated in its post-overlay state. Using this state maximizes the Design Case 1 design moment at Design Region B-B due to the additional weight and has no effect on the impact moment, as it is determined using M_{post}, not F_t. Further, for Design Case 2, using the post-overlay state maximizes the system weight and TL-4 vertical design load, as F_v increases with decreasing railing height due to increased vehicle roll.

Design Case 1 evaluation

The total moment demand acting on the slab at the ultimate strength of of the post is:

$$P_{post} \cdot \left(Y + 0.5\ t_s\right) = 106.3 \text{ kip} \cdot \text{ft}$$

Design Case 1 moments are assumed to distribute longitudinally according to the pattern shown in Figure E13.

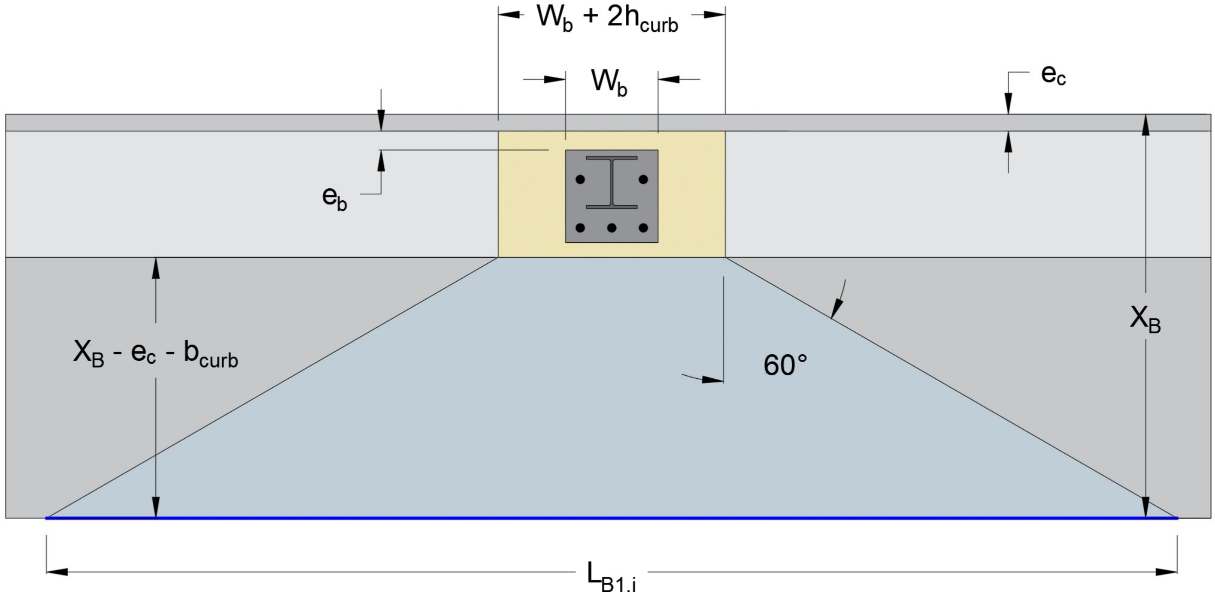


Figure E13. Distribution of post moment demand through slab

The effective distribution length for the post moment demand at Design Region B-B is:

$$L_{1B} := W_b + 2\ h_{curb} + 2\ \left(X_B - e_c - b_{curb}\right)\ \tan\left(60\ \text{deg}\right) = 11.2\ \text{ft} \qquad (13.10.2.4.3\text{-}17)$$

The slab and curb self-weight moment at Design Region B-B is:

$$M_{DC} := 150\ \text{pcf} \cdot 12\ \frac{\text{in}}{\text{ft}} \cdot \left(t_s \cdot \frac{{X_B}^2}{2} + h_{curb} \cdot b_{curb} \cdot \left(X_B - e_c - 0.5\ b_{curb}\right)\right) = 1.2\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

Assuming the 3-in. wearing surface is poured flush with the traffic face of the curb, the wearing surface moment at Design Region B-B is:

$$H_{wear} := 3\ \text{in}$$

$$M_{DW} := 140\ \text{pcf} \cdot \left(12\ \frac{\text{in}}{\text{ft}}\right) \cdot H_{wear} \cdot \frac{1}{2}\ \left(X_B - e_b - b_{curb}\right)^2 = 0.11\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

Therefore, the Design Case 1 moment demand at Design Region B-B, which includes the distributed post moment, self-weight, and wearing surface, is:

$$M_{swB} := M_{DC} + M_{DW} = 1.3\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

$$M_{1B} := \frac{P_{post} \cdot \left(Y + 0.5\ t_s\right)}{L_{1B}} + M_{swB} = 10.8\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.4.3\text{-}20)$$

The distributed tension at Design Region B-B can conservatively be assumed to be equal to the tensile demand at A-A:

$$N = 16.7\ \frac{\text{kip}}{\text{ft}} \qquad (13.10.2.4.3\text{-}1)$$

Therefore, the tension-reduced bending strength of the slab at Design Region B-B is:

$$M_{strB} := M_{strA} = 25\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

and the utility ratio for Design Case 1 at Design Region B-B is:

$$\frac{M_{1B}}{M_{strB}} = 0.43 \qquad (13.10.2.4.3\text{-}19)$$

Design Case 2 evaluation

The vertical impact load for a 39-in. tall, TL-4 railing with a 3-in. overlay is:

$$H := 39\ \text{in}$$

$$F_v := 101\ \text{kip} - 1.75\ \frac{\text{kip}}{\text{in}} \cdot \left(H - H_{wear}\right) = 38\ \text{kip} \qquad (\text{Table } 13.7.2\text{-}1)$$

applied over a length of:

$$L_v := 18\ \text{ft} \qquad (\text{Table } 13.7.2\text{-}1)$$

The portion of the total vertical load acting on a single post can be taken as:

$$\frac{L}{L_v}\ F_v = 16.9\ \text{kip}$$

which is conservatively applied at the back face of the railing. Therefore, the total moment at Design Region B-B is:

$$\frac{L}{L_v}\ F_v \cdot \left(X_B - e_c - e_b - e_p\right) = 60.5\ \text{kip} \cdot \text{ft}$$

which is distributed over a distance calculated using the same pattern shown above, but with a 45-degree angle, rather than a 60-degree angle:

$$L_{2B} := W_b + 2\ h_{curb} + 2\ \left(X_B - e_c - b_{curb}\right) = 7.4\ \text{ft} \qquad (13.10.2.4.3\text{-}21)$$

Therefore, the Design Case 2 moment demand at Design Region B-B, which includes the distributed vertical load moment, self-weight moment, and wearing surface moment, is:

$$M_{2B} := \frac{L \cdot F_v \cdot \left(X_B - e_c - e_b\right)}{L_v \cdot L_{2B}} + M_{swB} = 9.6\ \frac{\text{kip} \cdot \text{ft}}{\text{ft}} \qquad (13.10.2.4.3\text{-}25)$$

The utility ratio of this demand when compared to the bending strength at Design Region B-B (with no tension penalty) is:

$M_{stB} := M_{stA} = 28.9 \text{ kip}$

$$\frac{M_{2B}}{M_{stB}} = 0.33 \qquad (13.10.2.4.3\text{-}24)$$

Due to the low magnitude of the vertical post load, punching shear for Design Case 2 does not need to be evaluated.

As the overhang's local yield-line capacity is sufficient to develop the full capacity of the post, and Design Region B-B is sufficient to withstand distributed lateral and vertical design loads, the overhang design is **adequate** for the attached steel post-and-beam railing.