APPENDIX B

RELIABILITY MODEL FOR SCOUR ANALYSIS

Scour is typically assessed using the methodologies presented in FHWA's Hydraulic Engineering Circular No. 18 (HEC-18) (Richardson and Davis, 1995), although many other methods exist. The equations in HEC-18 are deterministic; they do not account for uncertainties in the models, the model parameters, or the hydraulic and hydrologic variables. Pier scour is given in HEC-18 by

$$\frac{y_s}{y_1} = 2.0K_1K_2K_3K_4\left(\frac{b^{-}}{y_1}\right)^{0.65} Fr_1^{0.43} \qquad (1)$$

where

y_s = scour depth,
y_1 = the upstream flow depth,
bN = the effective pier width,
Fr = the Froude number, and
K_1, K_2, K_3, and K_4 = correction factors for the pier shape, angle of attack, bed forms, and sediment gradation, respectively.

Adjustments to the pier width are given for the case of piles and pile caps or footings. Each of the four correction factors is given in tables and equations provided in HEC-18.

Monte Carlo simulation can be used to generate random samples of the parameters in Equation 1 based on the specified coefficients of variation and distributions. Equation 1 can be modified by a model correction factor λ to account for uncertainty in the model form and coefficients (Ang and Tang, 1984) as follows:

$$\frac{y_s}{y_1} = 2.0\lambda K_1K_2K_3K_4\left(\frac{b^{-}}{y_1}\right)^{0.65} Fr_1^{0.43} \qquad (2)$$

The selection of coefficients of variation and distributions are dependent on each bridge and on difficulties in assessing parameters at that specific bridge. To obtain a probabilistic scour depth, the following steps are followed:

1. Values for each of the random variables in Equation 2, including the model correction factor, are generated from their respective distributions for each of N simulation cycles.
2. Pier scour is calculated from Equation 2 based on the generated random variables.
3. This process is repeated for N simulation cycles.
4. The mean and standard deviation are calculated for the N values of scour depth.
5. A distribution is determined based on the N values of scour depth.

The following example, taken from HEC-18, is used to generate probabilistic scour depths for 5-, 10-, 25-, 50-, and 100-year hydrologic events. The means and coefficients of variations for the variables in Equation 2 are given in Table 1 for the range of hydrologic events (in terms of T-year return periods). The coefficient of variation for flow velocity was calculated based on Manning's resistance equation and assuming that n and S were the only significant sources of uncertainty. Manning's equation is given by

$$V = \frac{\phi}{n} R^{2/3} S^{1/2} \qquad (3)$$

where

n = Manning's roughness coefficient,
R = hydraulic radius, and
S = slope.

The uncertainty in V based on the uncertainty in n and S is given as follows (Mays and Tung, 1992):

$$\Omega_V^2 = \Omega_n^2 + 0.25\Omega_S^2 \qquad (4)$$

where Ω is the coefficient of variation. Assuming that flow depth is determined from the standard step method, the uncertainty in flow depth can be calculated based on the results of uncertainty analyses conducted by the U.S. Army Corps of Engineers (Hydraulic Engineering Center, 1986):

$$\Omega_y = 0.76y^{0.6} S^{0.11} (5N_r)^{0.65} \qquad (5)$$

where N_r is the reliability estimate for n, $0 \le N_r \le 1$. In this example, it is assumed that $N_r = 0.5$ (moderate reliability). Uncertainty in K_3 is assumed. Uncertainty in λ is based on comparisons of observed scour depths for 515 sites around the world with calculated values from Equation 1 (Johnson, 1995).

Using the procedure outlined above, the data in Table 1, and 1,000 simulation cycles, probabilistic scour depths were calculated. Table 2 provides the deterministic scour depths computed for each return period based on the HEC-18 equation (Equation 1). The 1,000 scour depths for each return period yield a normal distribution (Johnson and Dock, 1998) with mean $\overline{y_s}$ and standard deviation S_{ys}, given in Table 2.

TABLE 1 Parameter estimates, coefficients of variation, and distributions for a hypothetical bridge and a range of return periods

Variable	Probability Distribution	5-year		10-year		25-year		50-year		100-year	
		Mean	Ω	Mean	Ω	Mean	Ω	Mean	Ω	Mean	Ω
b (m)	N/A	1.52	0.000	1.52	0.000	1.52	0.000	1.52	0.000	1.52	0.000
V (m/s)	symmetrical triangular	2.04	0.280	2.29	0.28	2.68	0.28	2.91	0.28	3.73	0.28
y (m)	symmetrical triangular	1.14	0.075	1.37	0.084	1.73	0.097	1.95	0.10	2.84	0.13
λ	asymmetrical triangular	0.55	0.520	0.55	0.520	0.55	0.520	0.55	0.520	0.55	0.520
K_1	N/A	1.0	0.000	1.0	0.000	1.0	0.000	1.0	0.000	1.0	0.000
K_2	N/A	1.0	0.000	1.0	0.000	1.0	0.000	1.0	0.000	1.0	0.000
K_3	uniform	1.1	0.050	1.1	0.050	1.1	0.050	1.1	0.050	1.1	1.000
K_4	N/A	1.0	0.000	1.0	0.000	1.0	0.000	1.0	0.000	1.0	0.000

TABLE 2 Computed scour depths using HEC-18 equation (Equation 1), simulated scour depths (including correction factor λ), and standard deviations

Return Period (years)	Computed Scour Depth (m) from Equation 1	Simulated Mean Scour Depth (m) from Equation 2	Standard Deviation (m)
5	2.53	1.39	0.46
10	2.71	1.49	0.48
25	3.00	1.65	0.49
50	3.11	1.71	0.47
100	3.58	2.07	0.48

Thus, for each return period the probability of exceeding scour depths ranging from 0.5 m to 5.5 m can be calculated as follows:

$$p(y_s \geq k) = p\left(z \geq \frac{k - \bar{y}_s}{S_{y_s}}\right) \tag{6}$$

where

p = probability,
y_s = scour depth,
k = selected scour depth,
z = standard normal variate,
$\bar{y}_s$ = mean scour depth, and
S_{ys} = standard deviation of scour depth.

Table 3 shows these results. From Tables 2 and 3, the probability of exceeding the scour depths given by HEC-18 can be estimated. For example, for the 100-year return period, the HEC-18 equation (Equation 1) yields a value of 3.6 m. The probability of exceeding this scour depth is approximately 0.0013. Such a low exceedance probability is to be expected given that the HEC-18 equation is intended to predict a conservative, maximum scour depth.

TABLE 3 Exceedance probabilities for selected scour depths

Selected Scour Depth, *k* (m)	5-year	10-year	25-year	50-year	100-year
0.5	0.9735	0.9804	0.9905	0.9950	0.9993
1	0.8017	0.8463	0.9077	0.9346	0.9848
1.5	0.4055	0.4917	0.6202	0.6725	0.8705
2	0.0924	0.1440	0.2375	0.2686	0.5364
2.5	0.0079	0.0177	0.0414	0.0464	0.1721
3	2.327E-04	0.0008	0.0029	0.0030	0.0237
3.5	2.251E-06	1.411E-05	7.986E-05	6.992E-05	0.0013
4	6.998E-09	8.530E-08	8.106E-07	5.521E-07	2.475E-5
4.5	6.897E-12	1.803E-10	3.017E-09	1.463E-09	1.743E-7
5	2.109E-15	1.321E-13	4.075E-12	1.288E-12	4.335E-10
5.5	0	0	0	0	3.773E-13

REFERENCES FOR APPENDIX B

Ang, A.H-S., and Tang, W.H. (1984). *Probability Concepts in Engineering Planning and Design, Volume II.* Wiley and Sons, NY.

Hydraulic Engineering Center (1986). *Accuracy of Computed Water Surface Profiles*. U.S. Army Corps of Engineers, Davis, CA.

Johnson, P.A. (1995). Comparison of Pier Scour Equations Using Field Data. *Journal of Hydraulic Engineering,* ASCE, 121(8); pp. 626–629.

Johnson, P.A., and Dock, D.A. (1998). Probabilistic Bridge Scour Estimates. *Journal of Hydraulic Engineering,* ASCE, 124(7); pp. 750–755.

Mays, L.W., and Tung, Y.K. (1992). *Hydrosystems Engineering and Management*. McGraw-Hill, Inc.

Richardson, E.V., and Davis, S.R. (1995). *Evaluating Scour at Bridges*, 3rd edition. Report No. FHWA-IP-90-017, Hydraulic Engineering Circular No. 18, Federal Highway Administration, Washington, DC.