APPENDIX D – SUMMARY OF EXISTING SIMPLIFIED METHODS

An extensive literature search revealed many methods for the calculation of live load distribution factors. This appendix will discuss, in detail, the procedures outlined in the AASHTO LRFD Specifications (6) since the accuracy of most newly developed methods are compared to this method. Other simple and/or more accurate methods available will also be discussed in detail. Extensive research has shown that these methods compare favorably with refined methods of analysis. Detailed calculations for each method presented in this appendix are performed on a simple bridge and compared to finite element analysis. These calculations can be found in Appendix E.

AASHTO LRFD Method

The AASHTO LRFD presents a set of equations for calculation of live load distribution factors for interior and exterior beams for both moment and shear. In addition to these equations, skew reduction and increase factor equations are presented for moment and shear, respectively. Most variables in each set of equations contain ranges of applicability. These variables include span length, beam spacing, slab thickness, beam stiffness, and number of beams. When the variables are within their range of applicability, the distribution factors calculated from these equations are considered as accurate. However, the equations become less accurate when the ranges of applicability are exceeded. The AASHTO LRFD Specifications state that a refined analysis should be pursued for the distribution factors of bridge beams when the requirements and/or ranges of applicability of the equations are not met.

Like any simplified method, the AASHTO LRFD method outlines several conditions that a bridge must conform to in order to sustain a reasonable level of accuracy. Section 4.6.2.2 of the specifications lists the conditions that must be met in

addition to any condition identified in the tables of distribution factors. The conditions are as follows:

- Width of deck is constant;
- Unless otherwise specified, the number of beams is not less than four;
- Beams are parallel and have approximately the same stiffness;
- Unless otherwise specified, the roadway part of the overhang, d_e, does not exceed 3.0 ft;
- Curvature in plan is less than the limit specified in Article 4.6.1.2, which states: segments of horizontally curved superstructures with torsionally stiff closed sections whose central angle subtended by a curved span or portion thereof is less than 12.0° may be analyzed as if the segment were straight; and
- Cross-section is consistent with one of the cross-sections shown in Table 4.6.2.2.1-1 of the specifications.

Distribution of Live Load Moment

The live load moment distribution factor equations for selected bridges are shown in Table D-1 and Table D-2 for interior and exterior beams, respectively. The equations for slab bridges are shown in Table D-3. The range of applicability of each equation is also included in these tables. The corresponding skew correction factors for each bridge type are shown in Table D-4.

Table D-1. Distribution of Live Loads per Lane for Moment in Interior Beams

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Distribution Factors	Range of Applicability
Concrete Deck, Filled Grid, or Partially Filled Grid on Steel or Concrete Beams; Concrete T-Beams, T- and Double T- Sections	a, e, k and also i, j if sufficiently connected to act as a unit	One Design Lane Loaded: $0.06+\left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$ Two or More Design Lanes Loaded: $0.075+\left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$	$3.5 \le S \le 16.0$ $4.5 \le t_s \le 12.0$ $20 \le L \le 240$ $N_b \ge 4$
		use lesser of the values obtained from the equation above with $N_b = 3$ or the lever rule	$N_b = 3$
Multicell Concrete Box Beam	d	One Design Lane Loaded: $\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}$ Two or More Design Lanes Loaded: $\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}$	$7.0 \le S \le 13.0$ $60 \le L \le 240$ $N_c \ge 3$ If $N_c > 8$ use $N_c = 8$
Concrete Deck on Concrete Spread Box Beams	b, c	One Design Lane Loaded: $\left(\frac{S}{3.0}\right)^{0.35}\left(\frac{Sd}{12.0L^2}\right)^{0.25}$ Two or More Design Lanes Loaded: $\left(\frac{S}{6.3}\right)^{0.6}\left(\frac{Sd}{12.0L^2}\right)^{0.125}$	$6.0 \le S \le 11.5$ $20 \le L \le 140$ $18 \le d \le 65$ $N_c \ge 3$
		Use Lever Rule	$S > 11.5$
Concrete Deck on Multiple Steel Box Girders	b, c	Regardless of Number of Loaded Lanes: $0.05+0.85\frac{N_L}{N_b}+\frac{0.425}{N_L}$	$0.5 \le \frac{N_L}{N_b} \le 1.5$

Table D-2: Distribution of Live Loads per Lane for Moment in Exterior Beams

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Concrete Deck, Filled Grid, or Partially Filled Grid on Steel or Concrete Beams; Concrete T-Beams, T- and Double T- Sections	a, e, k and also i, j if sufficiently connected to act as a unit	Lever Rule	$g = eg_{\text{interior}}$ $e = 0.77 + \frac{d_e}{9.1}$	$-1.0 \le d_e \le 5.5$
			use lesser of the values obtained from the equation above with N_b = 3 or the lever rule	N_b = 3
Multicell Concrete Box Beam	d	$g = \frac{W_e}{14}$	$g = \frac{W_e}{14}$	$W_e \le S$
Concrete Deck on Concrete Spread Box Beams	b, c	Lever Rule	$g = eg_{\text{interior}}$ $e = 0.97 + \frac{d_e}{28.5}$	$0 \le d_e \le 4.5$ $6.0 < S \le 11.5$
			Use Lever Rule	S > 11.5
Concrete Deck on Multiple Steel Box Girders	b, c	As Specified in Table 4.6.2.2.2b-1		

Table D-3: Equivalent Width of Longitudinal Strips per Lane for Moment

Type of Superstructure	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Cast-in-Place Concrete Slab or Voided Slab	$10.0 + 5.0\sqrt{L_1 W_1}$	$84.0 + 1.44\sqrt{L_1 W_1} \le \frac{12.0W}{N_L}$	N/A

Table D-4: Reduction of Load Distribution Factors for Moment in Longitudinal Beams on Skewed Supports

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Any Number of Design Lanes Loaded	Range of Applicability
Concrete Deck, Filled Grid, or Partially Filled Grid on Steel or Concrete Beams; Concrete T-Beams, T- and Double T- Sections	a, e, k and also i, j if sufficiently connected to act as a unit	$1 - c_1(\tan\theta)^{1.5}$ $c_1 = 0.25\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.25}\left(\frac{S}{L}\right)^{0.5}$ If $\theta < 30°$ then $c_1 = 0.0$ If $\theta > 60°$ use $\theta = 60°$	$30° \le \theta \le 60°$ $3.5 \le S \le 16.0$ $20 \le L \le 240$ $N_b \ge 4$
Concrete Deck on Concrete Spread Box Beams, Concrete Box Beams, and Double T-Sections used in Multibeam Decks	b, c, f, g	$1.05 - 0.25\tan\theta \le 1.0$ If $\theta > 60°$ use $\theta = 60°$	$0 \le \theta \le 60°$
Cast-in-Place Concrete Slab or Voided Slab	N/A	$1.05 - 0.25\tan\theta \le 1.0$	N/A

The multiple presence factors shown in Table D-5 have already been incorporated into the tabulated distribution factor equations previously shown.

Because diaphragms were not considered in the models, the specifications include an interim provision that states:

> *"In beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam shall not be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section."*

The procedure is the same as the conventional approximation for loads on piles. The distribution factor is determined by the following equation:

$$R = \frac{N_L}{N_b} + \frac{X_{ext} \sum^{N_L} e}{\sum^{N_b} x^2} \tag{D-1}$$

where,

R = reaction on exterior beam in terms of lanes;

N_L = number of loaded lanes under consideration;

Table D-5: Multiple Presence Factors

Number of Loaded Lanes	Multiple Presence Factor "m"
1	1.20
2	1.00
3	0.85
4 or more	0.65

e = eccentricity of a design truck or a design lane load from the center of gravity of the pattern of girders (ft);

x = horizontal distance from the center of gravity of the pattern of girders to each girder (ft);

X_{ext} = horizontal distance from the center of gravity of the pattern of girders to the exterior girder (ft); and

N_b = number of beams.

Distribution of Live Load Shear

The live load shear distribution factor equations for selected bridges are shown in Table D-6 and Table D-7 for interior and exterior beams, respectively. The equations for

slab bridges are the same as shown in Table D-3. The range of applicability of each equation is also included in these tables. The corresponding skew correction factors for each bridge type are shown in Table D-8.

Table D-6: Distribution of Live Loads per Lane for Shear in Interior Beams

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Concrete Deck, Filled Grid, or Partially Filled Grid on Steel or Concrete Beams; Concrete T-Beams, T- and Double T- Sections	a, e, k and also i, j if sufficiently connected to act as a unit	$0.36 + \frac{S}{25.0}$	$0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0}$	$3.5 \le S \le 16.0$ $4.5 \le t_s \le 12.0$ $20 \le L \le 240$ $10{,}000 \le K_g \le 7{,}000{,}000$ $N_b \ge 4$
		Lever Rule	Lever Rule	$N_b = 3$
Multicell Concrete Box Beam	d	$\left(\frac{S}{9.5}\right)^{0.6}\left(\frac{d}{12.0L}\right)^{0.1}$	$\left(\frac{S}{7.3}\right)^{0.9}\left(\frac{d}{12.0L}\right)^{0.1}$	$6.0 \le S \le 13.0$ $20 \le L \le 240$ $35 \le d \le 110$ $N_c \ge 3$
Concrete Deck on Concrete Spread Box Beams	b, c	$\left(\frac{S}{10}\right)^{0.6}\left(\frac{d}{12.0L}\right)^{0.1}$	$\left(\frac{S}{7.4}\right)^{0.8}\left(\frac{d}{12.0L}\right)^{0.1}$	$6.0 \le S \le 11.5$ $20 \le L \le 140$ $18 \le d \le 65$ $N_c \ge 3$
		Lever Rule	Lever Rule	$S > 11.5$
Concrete Deck on Multiple Steel Box Girders	b, c	As specified in Table 4.6.2.2.2b-1		

Table D-7: Distribution of Live Loads per Lane for Shear in Exterior Beams

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Concrete Deck, Filled Grid, or Partially Filled Grid on Steel or Concrete Beams; Concrete T-Beams, T- and Double T- Sections	a, e, k and also i, j if sufficiently connected to act as a unit	Lever Rule	$g = eg_{\text{interior}}$ $e = 0.6 + \frac{d_e}{10}$	$-1.0 \le d_e \le 5.5$
Multicell Concrete Box Beam	d	Lever Rule	$g = eg_{\text{interior}}$ $e = 0.64 + \frac{d_e}{12.5}$	$-2.0 \le d_e \le 5.0$
Concrete Deck on Concrete Spread Box Beams	b, c	Lever Rule	$g = eg_{\text{interior}}$ $e = 0.8 + \frac{d_e}{10}$	$0 \le d_e \le 4.5$
			Lever Rule	$S > 11.5$
Concrete Deck on Multiple Steel Box Girders	b, c	As Specified in Table 4.6.2.2.2b-1		

Table D-8: Correction Factors for Load Distribution Factors for Support Shear of the Obtuse Corner

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Two or More Design Lanes Loaded	Range of Applicability
Concrete Deck, Filled Grid, or Partially Filled Grid on Steel or Concrete Beams; Concrete T-Beams, T- and Double T- Sections	a, e, k and also i, j if sufficiently connected to act as a unit	$1.0 + 0.20\left(\frac{12.0Lt_s^3}{K_g}\right)^{0.3} \tan\theta$	$0° \le \theta \le 60°$ $3.5 \le S \le 16.0$ $20 \le L \le 240$ $N_b \ge 4$
Multicell Concrete Box Beam	d	$1.0 + \left[0.25 + \left(\frac{12.0L}{70d}\right)\right] \tan\theta$	$0° < \theta \le 60°$ $6.0 < S \le 13.0$ $20 \le L \le 240$ $35 \le d \le 110$ $N_c \ge 3$
Concrete Deck on Concrete Spread Box Beams	b, c	$1.0 + \frac{\sqrt{\frac{Ld}{12.0}}}{6S} \tan\theta$	$0° < \theta \le 60°$ $6.0 < S \le 11.5$ $20 \le L \le 140$ $18 \le d \le 65$ $N_b \ge 3$

The variables in Table D-1 through Table D-4 and Table D-6 through Table D-8 are defined as follows:

A = area of beam (in^2);

d = depth of beam (in);

d_e = distance from exterior web of exterior beam and the interior edge of curb or traffic barrier (ft);

e = correction factor;

e_g = distance between the centers of gravity of the basic beam and deck (in.);

g = distribution factor;

I = moment of inertia of beam (in^4);

K_g = $n(I + Ae_g^2)$ = longitudinal stiffness parameter (in^4);

L = span length (ft);

L_1 = modified span length taken equal to the lesser of the actual span or 60.0 ft;

n = modular ratio;

N_b = number of beams;

N_c = number of cells in a concrete box girder;

N_L = number of design lanes;

S = beam spacing (ft);

t_s = slab thickness (in);

θ = skew angle (deg);

W = physical edge-to-edge width of bridge (ft);

W_1 = modified edge-to-edge width of bridge taken equal to the lesser of the actual width or 60.0 ft for multilane loading, or 30.0 ft for single-lane loading (ft); and

W_e = half the web spacing, plus the total overhang (ft).

Canadian Highway Bridge Design Code Method

The simplified method presented in the Canadian Highway Bridge Design Code (CHBDC) (27) for live load analysis is applicable, provided that the bridge in question satisfies the following conditions:

- The width is constant;
- The support conditions are closely equivalent to line support, both at the ends of the bridge and, in the case of multispan bridges, at intermediate supports;
- For slab bridges and slab-on-girder bridges with skew, the following provisions are met:
    - For solid and voided slab bridges, the skew parameter

$$\varepsilon = \frac{(\text{Bridge Width})[\tan(\text{Skew Angle})]}{\text{Span Length}} \leq \frac{1}{6}$$

    - For slab-on-girder bridges, the skew parameter

$$\varepsilon = \frac{(\text{Girder Spacing})[\tan(\text{Skew Angle})]}{\text{Span Length}} \leq \frac{1}{18};$$

- For bridges that are curved in plan, the radius of curvature, span, and width satisfy the following provisions:
    - $$\frac{(\text{Span Length})^2}{0.5(\text{Bridge Width})(\text{Radius of Curvature})} \leq 1.0$$
    - There are at least two intermediate diaphragms per span;

- A solid or voided slab is of substantially uniform depth across a transverse section, or tapered in the vicinity of a free edge provided that the length of the taper in the transverse direction does not exceed 2.5 m;

- For slab-on-girder bridges, there shall be at least three longitudinal girders that are of equal flexural rigidity and equally spaced, or with variations from the mean of not more than 10% in each case;

- For a bridge having longitudinal girders and an overhanging deck slab, the overhang does not exceed 60% of the mean spacing between the longitudinal girders or the spacing of the two outermost adjacent webs for box girder bridges and, also, is not more than 1.80 m;

- For a continuous span bridge, the assumed points of inflexion shown in Figure D-1 shall apply;

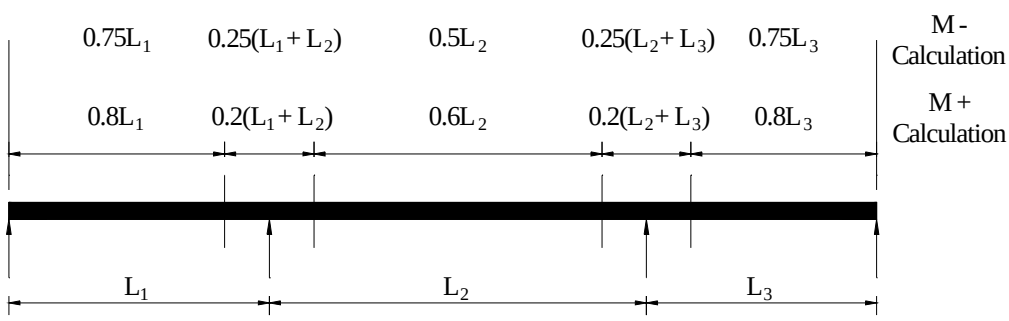

Figure D-1: Assumed Points of Inflexion Under Dead Loads

- In the case of multispine bridges, each spine has only two webs; and

- If the above conditions are not fully met, the code states that engineering judgment shall be exercised as to whether the bridge meets them sufficiently for the simplified methods to be applicable.

Superstructure types are grouped into two categories, which include shallow superstructure and multispine bridges. Shallow superstructures include the following bridge types:

- Slab;
- Voided Slab, including multicell box girders meeting diaphragm requirements;
- Slab-on-girder;
- Steel grid deck-on-girder;
- Wood deck-on-girder;
- Wood deck on longitudinal wood beam;
- Stress-laminated wood deck bridges spanning longitudinally;
- Longitudinal nail-laminated wood deck bridges;
- Longitudinal laminates of wood-concrete composite decks; and
- Shear-connected beam bridges in which the interconnection of adjacent beams is such as to provide continuity of transverse flexural rigidity across the cross-section.

Multispine bridges include:

- Box girder bridges in which the boxes are connected by only the deck slab and transverse diaphragms, if present; and

- Shear-connected beam bridges in which the interconnection of adjacent beams is such as not to provide continuity of transverse flexural rigidity across the cross-section.

Longitudinal Bending Moments for Ultimate and Serviceability Limit States

Girder-Type Bridges. The longitudinal moment per girder, Mg, is determined by using the following equations:

$$M_{g\,avg} = \frac{nM_T R_L}{N} \tag{D-2}$$

$$F_m = \frac{SN}{F\left(1 + \frac{\mu C_f}{100}\right)} \geq 1.05 \tag{D-3}$$

$$M_g = F_m M_{g\,avg} \tag{D-4}$$

where,

$M_{g\,avg}$ = the average moment per girder determined by sharing equally the total moment on the bridge cross-section among all girders in the cross-section;

M_T = the maximum moment per design lane at the point of the span under consideration;

n = the number of design lanes;

R_L = the modification factor for multilane loading equal to 1.0 for 1 lane loaded, 0.90 for two lanes loaded, 0.80 for three lanes loaded, 0.70 for four lanes loaded, 0.60 for five lanes loaded, and 0.55 for six or more lanes loaded;

N = the number of girders;

F_m = an amplification factor to account for the transverse variation in maximum longitudinal moment intensity, as compared to the average longitudinal moment intensity;

μ = $\frac{W_e - 3.3}{0.6} \leq 1.0$;

W_e = width of a design lane (m);

S = center-to-center girder spacing (m);

C_f = a correction factor (%); and

F = a width dimension that characterizes load distribution for a bridge.

The values of F and C_f are shown in Table D-9.

Slab and Voided Slab Bridges.

$$m_{avg} = \frac{nM_T R_L}{B_e} \tag{D-5}$$

$$F_m = \frac{B}{F\left(1 + \frac{\mu C_f}{100}\right)} \geq 1.05 \tag{D-6}$$

$$m = F_m m_{avg} \tag{D-7}$$

where,

B_e = effective width of the bridge found by reducing the total width B for the effects of tapered edges, if present; and

B = total width of bridge, regardless of whether tapered edges are present.

The values of F and C_f are shown in Table D-9.

Table D-9: Expressions for F and C_f for Longitudinal Bending Moment Corresponding to the Ultimate and Serviceability Limit State

Type of Bridge	No. of Design Lanes	Location	F, m		C_f, %
			for $3 < L \leq 10$ m	for $L > 10$ m	
Slab & Voided Slab	1	external	3.80 + 0.04L	4.20	16 - (36/L)
		internal	4.00 + 0.04L	4.40	16 - (36/L)
	2	external	7.10	7.10	20 - (40/L)
		internal	7.60 - (6/L)	7.30 - (3/L)	20 - (40/L)
	3	external	7.90 + 0.21L	10.80 - (8/L)	16 - (30/L)
		internal	5.90 + 0.41L	10.80 - (8/L)	16 - (30/L)
	4	external	10.10 + 0.26L	14.30 - (16/L)	16 - (30/L)
		internal	7.40 + 0.56L	14.00 - (10/L)	16 - (30/L)
Slab-on-girder	1	external	3.30	3.50 - (2/L)	5 - (12/L)
		internal	3.30 + 0.05L	4.40 - (6/L)	5 - (12/L)
	2	external	6.50	6.80 - (3/L)	5 - (15/L)
		internal	4.80 + 0.10L	7.20 - (14/L)	5 - (15/L)
	3	external	8.30	8.70 - (4/L)	10 - (25/L)
		internal	6.70 + 0.08L	9.60 - (21/L)	10 - (25/L)
	4	external	9.50	10.00 - (5/L)	10 - (25/L)
		internal	7.60 + 0.14L	11.20 - (22/L)	10 - (25/L)

Longitudinal Bending Moments for Fatigue Limit State

Girder-Type Bridges.

$$M_{g\,avg} = \frac{M_T}{N} \tag{D-8}$$

$$F_m = \frac{SN}{F\left(1 + \frac{\mu C_f}{100} + \frac{C_e}{100}\right)} \geq 1.05 \tag{D-9}$$

$$M_g = F_m M_{g\,avg} \tag{D-10}$$

where,

C_e = a correction factor for vehicle edge distance (%).

The values of F and C_f are shown in Table D-10.

The value of F may be modified for interior girders of bridges with two or more lanes. The modification accounts for the variation of F with girder spacing. The modification is as follows:

For $10\,\text{m} \le L \le 50\,\text{m}$

$$F = F_{tab}\left[1.00 + (0.29S - 0.35)\left(\frac{L - 10}{40}\right)\right] \tag{D-11}$$

For L > 50 m

$$F = F_{tab}(0.29S + 0.65) \tag{D-12}$$

where,

F_{tab} = value of F found for the internal girders from Table D-10. The girder spacing S is limited to $1.2\,\text{m} \le S \le 3.6\,\text{m}$. A value of S equal to 3.6 m may be used if S exceeds 3.6 m.

Slab and Voided Slab Bridges.

$$m_{avg} = \frac{M_T}{B_e} \tag{D-13}$$

$$F_m = \frac{B}{F\left(1 + \frac{\mu C_f}{100}\right)} \ge 1.05 \tag{D-14}$$

$$m = F_m m_{avg} \tag{D-15}$$

The values of F and C_f are found in Table D-10.

Table D-10: Expressions for F and C_f for Longitudinal Bending Moments Corresponding to the Fatigue Limit State

Type of Bridge	No. of Design Lanes	Location	F, m		C_f, %
			for $3 < L \leq 10$ m	for $L > 10$ m	
Slab & Voided Slab	1	external	3.80 + 0.04L	4.20	16 - (36/L)
		internal	4.00 + 0.04L	4.40	16 - (36/L)
	2	external	3.60 + 0.26L	7.10	16 - (36/L)
		internal	3.20 + 0.30L	7.30 - (3/L)	16 - (36/L)
	3	external	3.30 + 0.30L	10.80 - (8/L)	16 - (36/L)
		internal	3.00 + 0.40L	10.80 - (8/L)	16 - (36/L)
	4 or more	external	3.40 + 0.30L	14.30 - (16/L)	10 - (30/L)
		internal	3.00 + 0.44L	14.00 - (10/L)	10 - (30/L)
Slab-on-girder	1	external	3.30	3.50 - (2/L)	5 - (12/L)
		internal	3.30 + 0.05L	4.40 - (6/L)	5 - (12/L)
	2	external	3.60	3.80 - (2/L)	5 - (15/L)
		internal	2.80 + 0.12L	4.60 - (6/L)	5 - (15/L)
	3	external	3.60 + 0.01L	$3.70 + \frac{(L - 10)}{140}$	0.0
		internal	2.80 + 0.12L	4.80 - (8/L)	0.0
	4 or more	external	3.80	$3.80 + \frac{(L - 10)}{140}$	0.0
		internal	2.80 + 0.12L	5.00 - (10/L)	0.0

3.2.3 Longitudinal Bending Moments in Multispine Bridges

The procedure for the calculation of bending moment in multispine bridges is similar to that shown in Section 3.3.1 and 3.3.2 for the ultimate and serviceability limit state and fatigue limit state, respectively. However, an addition equation is introduced, which is used in conjunction with Table D-11 to determine a value for F and C_f. The equation is given by:

$$\beta = \pi \left(\frac{B}{L} \right) \left(\frac{D_x}{D_{xy}} \right)^{0.5} \tag{D-16}$$

where,

B = width of bridge for ULS and SLS, but no greater than three times the spine spacing S for FLS;

D_x = total bending stiffness, EI, of the bridge cross-section divided by the width of the bridge; and

D_{xy} = total torsional stiffness, GJ, of the cross-section divided by the width of the bridge.

Table D-11: Expressions for F and C_f for Longitudinal Moments in Multispine Bridges

Limit State	Number of Design Lanes	F, m	C_f, %
ULS or SLS	2	8.5 - 0.3β	16 - 2β
	3	11.5 - 0.5β	16 - 2β
	4	14.5 - 0.7β	16 - 2β
FLS	2 or more	8.5 - 0.9β	16 - 2β

3.2.4 Longitudinal Vertical Shear for Ultimate and Serviceability Limit States

Girder Type Bridges. The longitudinal shear per girder, Vg, is determined by using the following equations:

$$V_{g\,avg} = \frac{nV_T R_L}{N} \tag{D-17}$$

$$F_v = \frac{SN}{F} \tag{D-18}$$

$$V_g = F_v V_{g\,avg} \tag{D-19}$$

where,

$V_{g\,avg}$ = the average shear per girder determined by sharing equally the total shear on the bridge cross-section among all girders in the cross-section;

V_T = the maximum shear per lane at the points of the span under consideration; and

Fv = an amplification factor to account for the transverse variation in maximum longitudinal vertical shear intensity, as compared to the average longitudinal vertical shear intensity.

The values of F are found in Table D-12. When the girder spacing is less than 2.00 m, the value of F shall be multiplied by the following reduction factor:

$$\left[\frac{S}{2.00}\right]^{0.25} \tag{D-20}$$

Slab and Voided Slab Bridges. The longitudinal vertical shear per meter of width is calculated from the following:

$$v_{avg} = \frac{nV_T R_L}{B_e} \tag{D-21}$$

$$F_v = \frac{B}{F} \geq 1.05 \tag{D-22}$$

$$v = F_v v_{avg} \tag{D-23}$$

The values of F are found in Table D-12. For voided slab bridges where the center-to-center spacing, S, of longitudinal web lines is less than 2.00 m, the value of F shall be multiplied by the following reduction factor:

$$\left[\frac{S}{2.00}\right]^{0.25} \tag{B-24}$$

Table D-12: Values of *F* for Longitudinal Vertical Shear for Ultimate and Serviceability Limit States

Type of Bridge	Number of Design Lanes			
	1	2	3	4 or more
Slab	$2.60 + 0.45\sqrt{L}$	$4.20 + 0.66\sqrt{L}$	$5.60 + 1.05\sqrt{L}$	$6.50 + 1.44\sqrt{L}$
Voided Slab	3.60	6.30	8.40	9.80
Slab-on-girder	3.50	6.10	8.20	9.50

Longitudinal Vertical Shear for Fatigue Limit State

For the fatigue limit state, the same procedure is followed as previously outlined except that the values of F are obtained from Table D-13 and V_T is calculated using a single truck on the bridge, in one lane only, such that n = 1 and R_L = 1.00.

Longitudinal Vertical Shear in Multispine Bridges

Use the same procedure as before except F is obtained from Table D-14.

Table D-13: Values of *F* for Longitudinal Vertical Shear for Fatigue Limit State

Type of Bridge	Number of Design Lanes			
	1	2	3	4 or more
Slab	$2.60 + 0.45\sqrt{L}$	$3.20 + 0.13\sqrt{L}$	$3.20 + 0.16\sqrt{L}$	$3.60 + 0.14\sqrt{L}$
Voided Slab	3.60	3.60	3.80	3.90
Slab-on-girder	3.50	3.60	3.60	3.70

Table D-14: Expressions for *F* for Longitudinal Vertical Shear in Multispine Bridges

Limit State	Number of Design Lanes	F, m
ULS or SLS	2	7.2
	3	9.6
	4	11.2
FLS	2 or more	4.25

Henry's Equal Distribution Factor Method

Henry's equal distribution factor (EDF) method is by far the simplest of all methods previously discussed. Because the EDF method requires only the width of the roadway, number of traffic lanes, number of beam lines, and the multiple-presence factor of the bridge, it can be applied without difficulty to different types of superstructures and beam arrangements. The procedure for the calculation of distribution factors is as follows (60):

Step 1: Reinforced Concrete I-beams, Reinforced Concrete Box Beams, Precast Box Beams:

(a) Divide roadway width by 10 ft to determine the fractional number of traffic lanes.

(b) Reduce the value from (a) by a factor obtained from a linear interpolation for intensity to determine the total number of traffic lanes considered for carrying live load on bridge. Table D-15 shows the multiple-presence factor for a specific number of loaded lanes.

(c) Divide the total number of lanes by the number of beams to determine the number of lanes of live load per beam, or the distribution factor of lane load per beam.

Step 2: Steel and Prestressed I-beams:

(d) Proceed with steps (a) through (c) above.

(e) Multiply the value from (c) by a ratio of 6/5.5 or 1.09 to determine the

distribution factor of lane load per beam

Table D-15: Multiple Presence Factors

Number of Loaded Lanes	Multiple Presence Factor "m"
2	1.00
3	0.90
4 or more	0.75

Modified Henry's Method

Modification factors to Henry's method were developed based on a comparison and evaluation study conducted by Huo et al. (59). Distribution factors from Henry's method were compared to the AASHTO Standard and AASHTO LRFD methods in addition to finite element analysis. Two sets of modification factors for moment and shear distribution were recommended. The first set includes moment modification factors based on superstructure type along with a single shear factor applicable to all structure types. The second set of modification factors includes separate sets of factors for moment and shear. The effects of skew and span length are included in the second set of modification factors.

Modification Factors for Live Load Moment and Shear (Set 1). The procedure for the calculation of moment and shear distribution factors using set 1 modification factors is as follows:

Step 1: Basic Equal Distribution Factor - follow (a) through (c) in Section 3.3.

Step 2: Superstructure Type Modification - Moment Distribution Factors

(d) Multiply the value from (c) by the appropriate moment modification factor from Table D-16 to determine the moment distribution factor.

Step 3: Superstructure Type Modification – Shear Distribution Factors

(e) Multiply the value from (d) by the shear factor in Table D-16 to determine the shear distribution factor.

Modification Factors for Live Load Moment and Shear (Set 2). The procedure for the calculation of moment and shear distribution factors using set 2 modification factors is as follows:

Step 1: Basic Equal Distribution Factor - follow (a) through (c) in Section 3.3.

For Moment Distribution Factor:

Step 2: Superstructure Type Modification

(d) Multiply the value from (c) by the appropriate structure type modification factor from Table D-17.

Step 3: Skew Angle and Span Length Modification

(a) If applicable, multiply the value from (d) by the skew angle and/or span length modification factor from Table D-17 to determine the final moment distribution factor.

For Shear Distribution Factor:

Step 2: Superstructure Type Modification

(e) Multiply the value from (c) by the appropriate structure type modification factor from Table D-18.

Step 3: Skew Angle Modification

(b) If applicable, multiply the value from (d) by the skew angle modification factor from Table D-18 to determine the final shear distribution factor.

Table D-16: Structure Type Modification Factors (Set 1)

Structure Type	AASHTO LRFD Cross Section Type	Modification Factor	
		Moment	Shear
Precast Spread Box Beam	b	1.00	1.15
Precast Concrete I-Sections	k	1.10	
CIP Concrete T-Beam	e	1.05	
CIP Concrete Box Beam	d	1.05	
Steel I-Beam	a	1.10	
Steel Open Box Beam	c	1.00	

Table D-17: Modification Factors for Live Load Moment (Set 2)

Structure Type	AASHTO LRFD Cross Section Type	Modification Factor				
		Structure Type	Skew Angle		Span Length	
			$\theta < 30^o$	$\theta > 30^o$	L < 100 ft	L > 100 ft
Precast Spread Box Beam	b	1.00	1.00	0.94	1.00	0.94
Precast Concrete I-Sections	k	1.15				
CIP Concrete T-Beam	e	1.10				
CIP Concrete Box Beam	d	1.10				
Steel I-Beam	a	1.15				
Steel Open Box Beam	c	1.10				

Table D-18: Modification Factors for Live Load Shear (Set 2)

Structure Type	AASHTO LRFD Cross Section Type	Modification Factor	
		Structure Type	Skew Angle
Precast Spread Box Beam	b	1.05	$1.0 + 0.2\tan\theta$
Precast Concrete I-Sections	k	1.20	
CIP Concrete T-Beam	e	1.05	
CIP Concrete Box Beam	d	1.20	
Steel I-Beam	a	1.15	
Steel Open Box Beam	c	1.15	

Sanders and Elleby Method

The method proposed by Sanders and Elleby (106) follows the simple *S/D* approach, which has been utilized in the AASHTO Standard Specifications for many years. The exception is the calculation of the *D* constant. The value of *D* is determined by the following relationships:

For $C \leq 3$

$$D = 5 + \frac{N_L}{10} + \left(3 - \frac{2N_L}{7}\right)\left(1 - \frac{C}{3}\right)^2 \tag{D-25}$$

For $C > 3$

$$D = 5 + \frac{N_L}{10} \tag{D-26}$$

where,

N_L = total number of design traffic lanes; and

C = a stiffness parameter that depends on the type of bridge, bridge and beam properties, and material properties.

The stiffness parameter, *C* is calculated as follows, for beam and slab and multi-beam bridges:

$$C = \frac{W}{L}\left[\frac{E}{2G}\frac{I_1}{J_1 + J_t}\right]^{\frac{1}{2}} \tag{D-27}$$

For concrete box girder bridges:

$$C = \frac{W}{2L}\left(1 + N_g\sqrt{\frac{d}{W}}\right)\left[\frac{E}{2G(1+N_d)}\right]^{\frac{1}{2}} \tag{D-28}$$

where,

W = overall bridge width (ft);

L = span length (distance between live load points of inflection for continuous span) (ft);

E = modulus of elasticity of the transformed beam section;

G = modulus of rigidity of the transformed beam section;

I_1 = flexural moment of inertia of the transformed beam section per unit width;

J_1 = torsional moment of inertia of the transformed beam section per unit width;

J_t = ½ of the torsional moment of inertia of a unit width of bridge deck slab;

d = depth of the bridge from center of top slab to center of bottom slab;

N_g = number of girder stems; and

N_d = number of interior diaphragms.

For preliminary designs of beam and slab bridges, the parameter C can be approximated by using the values given in Table D-19.

Table D-19: Values of K to be used in C = K(W/L)

Bridge Type	Beam Type and Deck Material	*K*
Beam and slab (includes concrete slab bridge)	Concrete deck:	
	Noncomposite steel I-beams	3.0
	Composite steel I-beams	4.8
	Nonvoided concrete beams (prestressed or reinforced)	3.5
	Separated concrete box-beams	1.8
	Concrete slab bridge	0.6

The following two parameters were considered the most important regarding lateral distribution of a wheel line:

$$\theta = \frac{W}{2L}\sqrt[4]{\frac{D_x}{D_y}} \tag{D-29}$$

$$\alpha = \frac{1}{2}\left(\frac{D_{xy} + D_{yx}}{\sqrt{D_x D_y}}\right) \tag{D-30}$$

where,

θ = flexural parameter;

α = torsional parameter;

L = span length (ft);

W = bridge width (ft);

D_{xy} = torsional stiffness in the longitudinal direction (lb-in^2/ft);

D_{yx} = torsional stiffness in the transverse direction (lb-in^2/ft);

D_x = flexural stiffness in the longitudinal direction (lb-in^2/ft); and

D_y = flexural stiffness in the transverse direction (lb-in^2/ft).

For simplification, these two parameters were combined into one parameter defined by:

$$C = \frac{\theta}{\sqrt{\alpha}} = \frac{\sqrt{2}}{2}\frac{W}{L}\sqrt{\frac{D_x}{D_{xy} + D_{yx}}} \tag{D-31}$$

The calculation of this parameter is based on the following assumptions:

(1) A typical interior beam or diaphragm shall include a portion of the deck slab equal to the beam or diaphragm spacing.

(2) Full transverse flexural and torsional continuity of the diaphragms is assumed only when they are rigidly connected to the longitudinal beams.

(3) The torsional rigidity of the steel beams or diaphragms is ignored.

(4) For flexural and torsional rigidity calculations of steel beam-concrete deck bridge types, the steel cross-sectional area should be expressed as an equivalent area of concrete.

(5) The uncracked gross area of the concrete cross section may be used for rigidity calculations involving prestressed or reinforced concrete structural members.

(6) Standard engineering procedures are used for computing the torsional and flexural rigidities of typical bridge systems.

To consider the effect of diaphragms, the torsional stiffness in the transverse direction, D_{yx}, should be increased by the torsional stiffness of the diaphragm divided by its spacing.

Alternate Distribution Factor Method for Moments

None of the simplified methods listed above performed well for moment distribution for an interior girder with one lane loaded. An effort was made to develop a simplified method to address this problem. A parametric equation was developed for use in this case. In the interest of being thorough, the method was applied to all loading and girder position cases, and it was discovered that it performed well for several cases.

The equation takes the form of

$$g = \left(\frac{S}{D}\right)^{Exp1} \left(\frac{S}{L}\right)^{Exp2} \left(\frac{1}{N_g}\right)^{Exp3}$$

where *Exp1*, *Exp2*, *Exp3*, and *D* are constants that vary with bridge type,

S = the girder spacing in feet,

L = the span length in feet,

and N_g = the number of girders or number of cells + 1 for the bridge.

This parametric equation was developed by combining terms for the one lane loaded interior girder moment distribution factor equations for I section and cast-in-place box girder bridges. The terms that were dependent on the stiffness of the section were dropped. By varying the values of the three exponents and the *D* constant, this equation form produced reasonable results for the various bridge types. In cases where one of the exponents was determined to be near zero, the term was dropped (exponent was set equal to zero) in order to simplify the application of this equation as much as possible