## APPENDIX B

## Statistical Design of Material Combinations for Mortar Tests

## Statistical Design of Material Combinations for Mortar Tests

The following parameters are considered:
Four types of clinkers 4
Four types of processing additions and a non addition $4+1$
Three levels of dosages most of the four additions and clinkers $11+7+(2 \times 3 \times 4)+4 \quad 46$
Three supplementary cementitious materials and none 4
Total number of combinations 184
Of the 184 combinations, 99 will be selected. Because of the disjointed nature of the processing additions and their dosage levels (see Table A) the selection of design points was done in two parts.

## Table A. Processing addition by dosage

|  |  | Dosage |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Frequency | 0 | 1 | 2 | 3 | Total |
| Processing | 0 | 16 | 0 | 0 | 0 | 16 |
| addition | 1 | 0 | 12 | 16 | 16 | 44 |
|  | 2 | 0 | 0 | 12 | 16 | 28 |
|  | 3 | 0 | 16 | 16 | 16 | 48 |
|  | 4 | 0 | 16 | 16 | 16 | 48 |
|  | Total | 16 | 44 | 60 | 64 | 184 |

To generate the design, the 184 combination points were divided into 16 combination points without processing additions and 168 combination points with processing additions. Since $8.7 \%$ $(16 / 184)$ of the combination points were without processing additions, $9(\cong 8.6 \%$ of 99$)$ of the 99 design points were selected from the 16 combination points without processing additions and 91 ( $\cong 91.4 \%$ of 99 ) of the 99 design points were selected from the 168 combination points with processing additions.

For the 9 design points without processing additions only one type of design is reasonable with respect to balance. In Table B are the frequency tables of clinker by supplementing material.

Table B. Clinker by supplementing material

| Freq | 1 | 2 | $3 \mid$ | 4 | Total |
| ---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 1 | 0 | 2 |
| 2 | 1 | 1 | 0 | 0 | 2 |
| 3 | 1 | 0 | 0 | 1 | 2 |
| 4 | 0 | 1 | 1 | 1 | 3 |
| Total | 3 | 2 | 2 | 2 | 9 |

To obtain the remaining 91 data points with processing additions at various dose levels a design was generated with all different combinations and as balanced as possible.

To generate such a design a fractional factorial design was used as the base design. The initial design was generated as if it was a full factorial of 4 levels of clinker, 4 levels of processing addition at 3 dosages each, and 4 levels of supplementary material. The full factorial would contain 192 combinations, 24 more combinations than in the original matrix.

From this design a half fractional factorial using 8 factors with 2 levels each was created. This makes a total of $128(=256 / 2)$ combinations, of which those not in the original design were discarded leaving 84 combinations. There are no repeated combinations in this design. As can be seen in Table C the combinations of factors are evenly spread over the levels of each factor and matches the original matrix frequencies except for minor differences for supplementary materials.

Table C. Number of observations of each combination in the design and original matrix

| Clinker | Frequency | Percent | Frequency | Percent |
| ---: | ---: | ---: | :---: | :---: |
| 1 | 22 | 26.19 | 44 | 26.19 |
| 2 | 22 | 26.19 | 44 | 26.19 |
| 3 | 22 | 26.19 | 44 | 26.19 |
| 4 | 18 | 21.43 | 36 | 21.43 |


| Proc_add | Frequency | Percent | Frequency | Percent |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 22 | 26.19 | 44 | 26.19 |
| 2 | 14 | 16.67 | 28 | 16.67 |
| 3 | 24 | 28.57 | 48 | 28.57 |
| 4 | 24 | 28.57 | 48 | 28.57 |


| Dosage | Frequency | Percent | Frequency | Percent |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 22 | 26.19 | 44 | 26.19 |
| 2 | 30 | 35.71 | 60 | 35.71 |
| 3 | 32 | 38.10 | 64 | 38.10 |


| Supp_mat | Frequency | Percent | Frequenc | Percent |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 20 | 23.81 | 42 | 25.00 |
| 2 | 22 | 26.19 | 42 | 25.00 |
| 3 | 22 | 26.19 | 42 | 25.00 |
| 4 | 20 | 23.81 | 42 | 25.00 |

Table Da. Supp_mat by proc_add Controlling for clinker=1
Supp_mat proc_add

| Freq |  | 1 | 2 | 3 | 4 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | ---: |
| 1 | 1 | 2 | 2 | 1 | 6 |  |
| 2 | 2 | 0 | 1 | 2 | 5 |  |
| 3 | 2 | 0 | 1 | 2 | 5 |  |
| 4 | 1 | 2 | 2 | 1 | 6 |  |
| Total | 6 | 4 | 6 | 6 | 22 |  |

Table Dc. Supp_mat by proc_add
Controlling for clinker=2
Supp_mat proc_add

| Freq |  | 1 | 2 | 3 | 4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 0 | Total |  |  |  |
| 2 | 1 | 2 | 1 | 2 | 1 | 5 |
| 3 | 1 | 2 | 2 | 1 | 6 |  |
| 4 | 2 | 0 | 1 | 2 | 6 |  |
|  |  |  |  |  |  |  |
| Total | 6 | 4 | 6 | 6 | 22 |  |

Table De. Supp_mat by proc_add
Controlling for clinker=3
Supp_mat proc_add

| Freq |  | 1 | 2 | 2 |  | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | ---: |
| 1 | Total |  |  |  |  |  |
| 2 | 2 | 0 | 1 | 2 | 5 |  |
| 3 | 1 | 2 | 2 | 1 | 6 |  |
| 3 | 1 | 2 | 2 | 1 | 6 |  |
| 4 | 2 | 0 | 1 | 2 | 5 |  |
| Total | 6 | 4 | 6 | 6 | 22 |  |

Table Dg. Supp_mat by proc_add
Controlling for clinker=4
Supp_mat proc_add

| Freq |  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | 2 | 1 | 4 |
| 2 | 2 | 0 | 1 | 2 | 5 |
| 3 | 2 | 0 | 1 | 2 | 5 |
| 4 | 0 | 1 | 2 | 1 | 4 |
| Total | 4 | 2 | 6 | 6 | 18 |

Table Db. Supp_mat by dosage
Controlling for clinker=1
Supp_mat dosage

| Freq |  | 1 | 2 | 2 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 2 | 2 | 6 |  |
| 2 | 1 | 2 | 2 | 5 |  |
| 3 | 1 | 2 | 2 | 5 |  |
| 4 | 2 | 2 | 2 | 6 |  |
| Total | 6 | 8 | 8 | 22 |  |

Table Dd. Supp_mat by dosage
Controlling for clinker=2
Supp_mat dosage

| Freq |  | 1 | 2 | 3 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 2 | 2 | 5 |  |
| 2 | 2 | 2 | 2 | 6 |  |
| 3 | 2 | 2 | 2 | 6 |  |
| 4 | 1 | 2 | 2 | 5 |  |
| Total | 6 | 8 | 8 | 22 |  |

Table Df. Supp_mat by dosage
Controlling for clinker=3
Supp_mat dosage

| Freq |  | 1 | 2 | 3 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 2 | 2 | 5 |  |
| 2 | 2 | 2 | 2 | 6 |  |
| 3 | 2 | 2 | 2 | 6 |  |
| 4 | 1 | 2 | 2 | 5 |  |
| Total | 6 | 8 | 8 | 22 |  |

Table Dh. Supp_mat by dosage
Controlling for clinker=4
Supp_mat dosage

| Freq |  | 1 | 2 | 3 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 1 | 2 | 4 |  |
| 2 | 1 | 2 | 2 | 5 |  |
| 3 | 1 | 2 | 2 | 5 |  |
| 4 | 1 | 1 | 2 | 4 |  |
| Total | 4 | 6 | 8 | 18 |  |

The combinations of factors were spread well as can be seen in the following Tables Da to Dh : The total number of observations was $93(=84+9)$. By adding 6 more observations almost
balances the design for the counts of each factor. Together with the 9 non-addition combinations the frequencies for each factor are as presented in Table E.

Table E. Frequency table for the combined set of 99 design points

| Clinker | Frequency | Percent | Frequency | Percent |
| ---: | ---: | :---: | :---: | :---: |
| 1 | 25 | 25.25 | 48 | 26.09 |
| 2 | 26 | 26.26 | 48 | 26.09 |
| 3 | 26 | 26.26 | 48 | 26.09 |
| 4 | 22 | 22.22 | 40 | 21.74 |
|  |  |  |  |  |
| Proc_add | Frequency | Percent | Frequency | Percent |
| 0 | 9 | 9.09 | 16 | 8.70 |
| 1 | 24 | 24.24 | 44 | 23.91 |
| 2 | 14 | 14.14 | 28 | 15.22 |
| 3 | 26 | 26.26 | 48 | 26.09 |
| 4 | 26 | 26.26 | 48 | 26.09 |
|  |  |  |  |  |
| Dosage | Frequency | Percent | Frequency | Percent |
| 0 | 9 | 9.09 | 16 | 8.70 |
| 1 | 23 | 23.23 | 44 | 23.91 |
| 2 | 32 | 32.32 | 60 | 32.61 |
| 3 | 35 | 35.35 | 64 | 34.78 |
|  |  |  |  |  |
| Supp_mat | Frequency | Percent | Frequency | Percent |
| 1 | 24 | 24.24 | 46 | 25.00 |
| 2 | 25 | 25.25 | 46 | 25.00 |
| 3 | 25 | 25.25 | 46 | 25.00 |
| 4 | 25 | 25.25 | 46 | 25.00 |

The 99 design points are shown in Table 4.
To ensure that analysis results will be readily available a simulated data set was run with a model containing only main effects and several models that also include two-factor interactions. In models with just main effects or includes main effects and clinker by supplemental material the least squares means can be estimated.

Modeling the three-factor interaction of clinker, processing addition and supplementary materials is possible with this design. However, while least squares means can be estimated for all the clinker, processing addition and supplementary combinations not all the least squares means for dosages can be estimated. This would be over-modeling the data to some extent, but with only 2-factor interactions, not all least squares means for the two factor effects can be estimated. . In the case with two factor interactions dosages can be estimated.

Between these different models, it should be possible to determine the best combinations of the factors and get an idea of the general trends in the factors.

Table 4: List of Cement Mixes to be Tested
Obs clinker proc_add dosage Supp_mat


