Appendix E
Refined Analysis Guidelines

National Cooperative Highway Research Program
Project 15-29

CNA Consulting Engineers
Simpson, Gumpertz & Heger

April 2009

Table of Contents

List of Tables

List of Figures

1 Introduction

This project has conducted extensive 3D modeling of the transfer of surface live loads to buried culverts. From the results, we have proposed Simplified Design Equations that permit culvert design without modeling. However, many design situations have conditions not covered by the Simplified Design Equations. In these situations, 2D and 3D modeling may be used for design. The following two sections provide guidelines for conducting 2D and 3D modeling. The guidelines are developed from the work of NCHRP 15-29. The 2D guidelines provide a means for selecting the surface load intensity to be applied to a 2D model. The 3D guidelines address software, live load application, representing the pavement, representing the soil, model dimensions, element size, symmetry and boundary conditions, representing culvert structures, and the soil-culvert interface.

2 Guidelines for Two-Dimensional Analysis

This section addresses the question of what surface load intensity should be applied in a 2D model, in order to correctly compute the structural response of the culvert.

2.1 Longitudinal and Transverse Subsurface Spreading

Two-dimensional computer models have an inherent limitation when computing the effect of surface live loads. Because the models are 2D, the load spreading that occurs in the longitudinal direction, parallel to the axis of the culvert, cannot be correctly computed. The model represents a single, vertical slice through the real-world geometry.

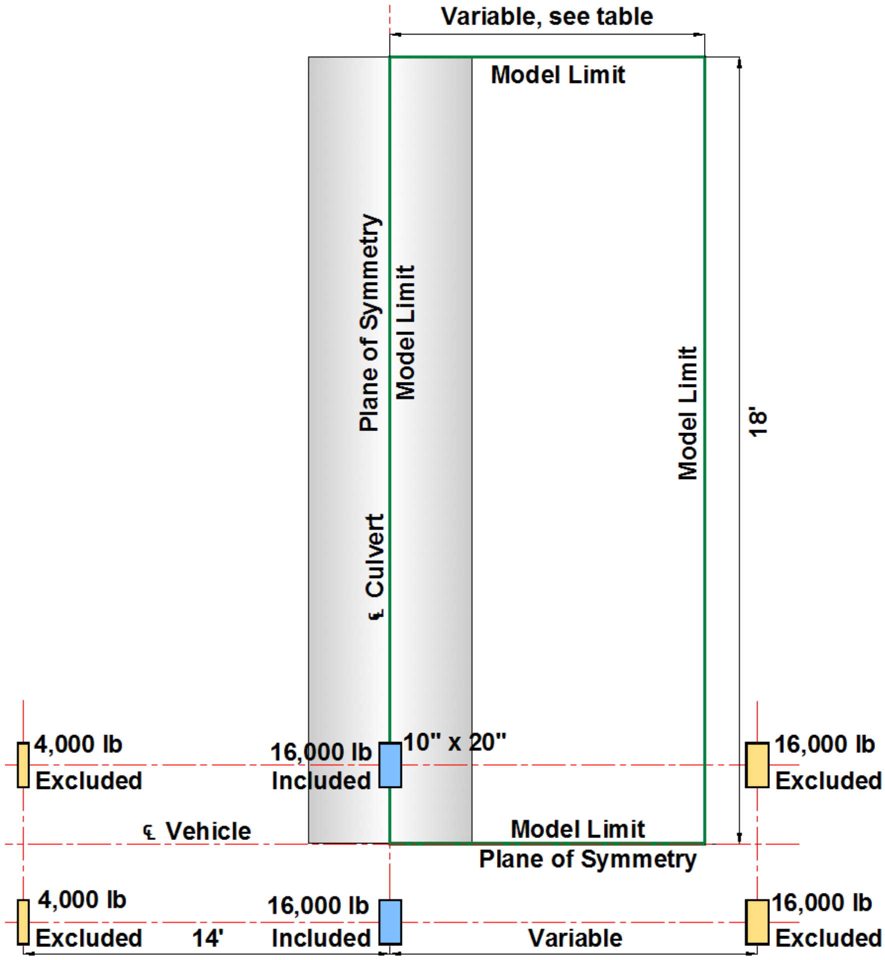

Figure 1—Location and Intensity of Live Load, Before Factoring

Figure 1 illustrates the location and intensity of the live load. The vehicle centerline is in the left-right plane of symmetry and the culvert centerline is in the up-down plane of symmetry. This section calls the left-right direction the transverse direction and the up-down direction the longitudinal direction.

The fundamental equation in all live load spread calculations is that the total force at depth H is equal to the total force at the surface:

$$F_S = F_H \tag{1}$$

The surface pressure is:

$$P_S = \frac{LL}{w_t \cdot l_t} \tag{2}$$

For 3D spreading, the live load pressure at depth is the force divided by the area:

$$P_H = \frac{LL}{(w_t + LLDF \cdot H)(l_t + LLDF \cdot H)} \tag{3}$$

where:

P_H is the vertical pressure at depth H

LL is the live load force at the surface, 16,000 lb unfactored

w_t is the transverse dimension of the tire patch, typically 10 inches

l_t is the longitudinal dimension of the tire patch, typically 20 inches

$LLDF$ is the live load distribution factor, 1.75 for Standard, for LRFD: 1.15 for granular fill, 1.0 for other fills

In the case of 2D modeling, models correctly determine the load spread in the transverse direction (in the plane of the model). In the longitudinal direction, 2D models do not compute load spreading. Hence, the live load must be factored (or "spread") to achieve the spreading that cannot be modeled.

Assuming that the transverse live load spread will be computed by the model, the vertical pressure at depth is:

$$P_H^{2D} = \frac{LL}{(l_t + LLDF \cdot H)} \tag{4}$$

and at the surface:

$$P_S^{2D} = \frac{LL}{l_t} \tag{5}$$

Hence, the ratio of the live load pressure at depth to the surface live load pressure is:

$$\frac{P_H^{2D}}{P_S^{2D}} = \frac{l_t}{(l_t + LLDF \cdot H)} \tag{6}$$

2.2 2D and 3D Modeling

We have previously reported the results of 3D modeling of a range of culvert types, soils and depths. All were done with service live loads. Selected 3D models, with Mohr-Coulomb soil behavior were rerun with 16,000 lb live load, for comparison with analogous 2D modeling. The culvert types, sizes, depths and soils were as follows:

1. Materials: Concrete Pipe (RCP), Corrugated Metal Pipe (CMP), Profile Wall Pipe (PW)
2. Size:
    a. RCP using 24"dia. 48" dia. and 96" dia.
    b. CMP using 12" dia. 24" dia. 48" dia. and 96" dia.
    c. PW using 12" dia. 24" dia. 48" dia. and 60" dia.
3. Soil Type: SW85
4. Cover Depth: 12", 24", 48" and 96"

The 2D models were all run using FLAC 3D (with a 2D geometry), unfactored loads and elastic soil behavior.

Peak thrust and crown moment were compared by computing the following ratios of 3D structural response to 2D structural response:

$$TDRR_i^T = \frac{T_{p,3d}}{T_{p,2d}} \quad \textbf{(7)}$$

$$TDRR_i^{MC} = \frac{M_{c,3d}}{M_{c,2d}} \quad \textbf{(8)}$$

where:

$TDRR_i^T$ is the Two-Dimensional Response Ratio for peak thrust, and

$TDRR_i^{MC}$ is the Two-Dimensional Response Ratio for crown moment.

Figure 2, Figure 3 and Figure 4 illustrate the peak thrust two-dimensional response ratio. Note that the peak thrust generally does not occur at the crown of the culvert. Each figure includes the curve resulting from Eqn **(3)**, with f equal to 1.15.

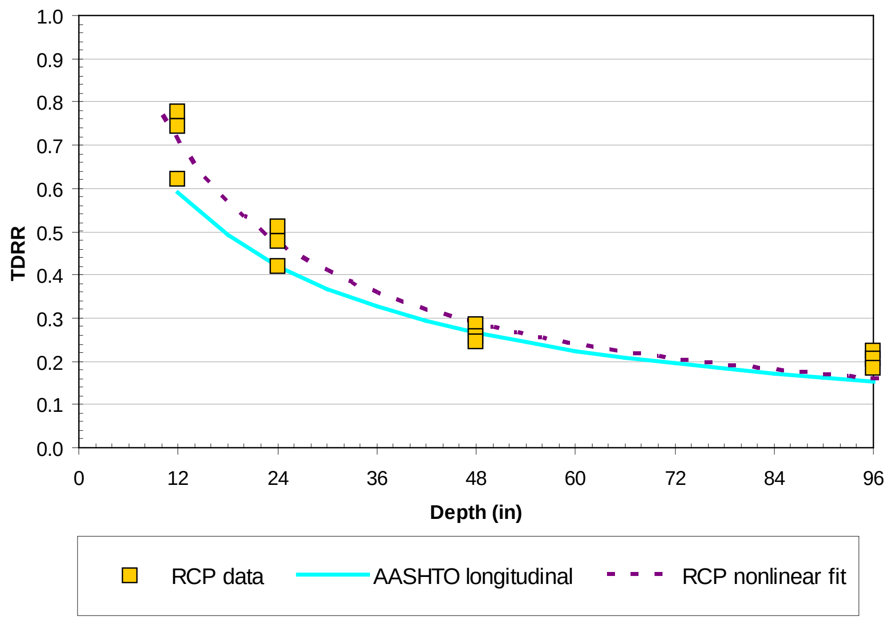

Figure 2—Peak Thrust Two-Dimensional Response Ratio for RCP (AASHTO refers to Eqn (3)**)**

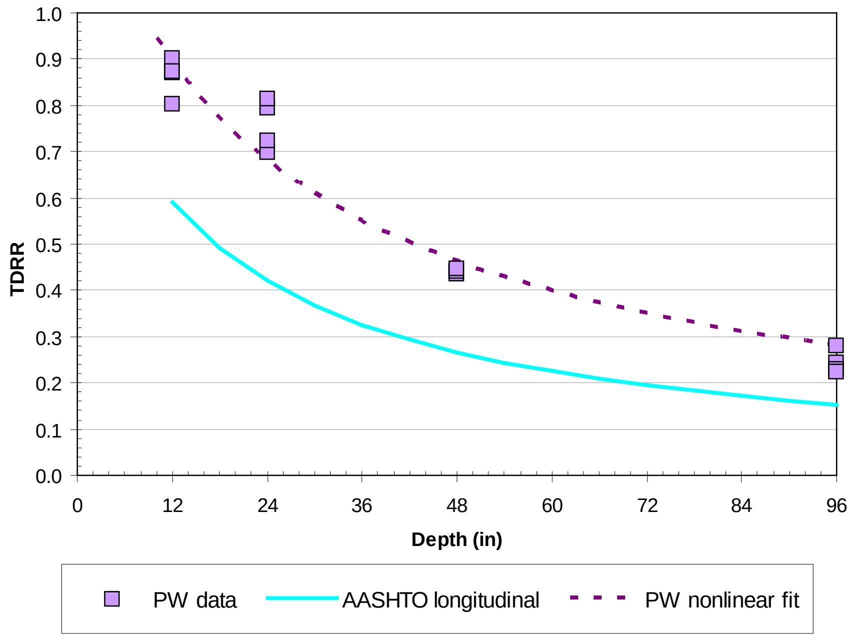

Figure 3—Peak Thrust Two-Dimensional Response Ratio for PW (AASHTO refers to Eqn (3)**)**

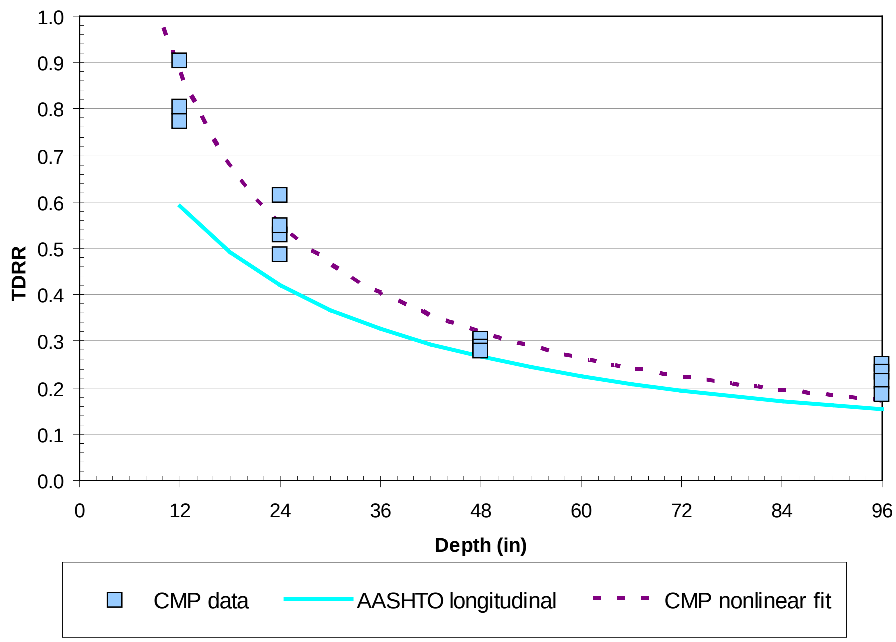

Figure 4—Peak Thrust Two-Dimensional Response Ratio for CMP (AASHTO refers to Eqn (3)**)**

The peak thrust figures show that the TDDR is strongly influenced by culvert type. Figure 2, illustrating RCP results, shows that peak thrust response is very close to the longitudinal spread equation (Eqn **(3)**). In comparison, Figure 3 shows that the profile wall response is about 1.5 times greater than Eqn **(3)**. Figure 4 shows that the CMP response is similar to Eqn **(3)** at depths greater than 48 inches, but significantly higher at shallower depths.

Figure 5, Figure 6 and Figure 7 illustrate the crown moment two-dimensional response ratio. Each figure includes the curve resulting from Eqn **(3)**, with $LLDF$ equal to 1.15. Like the results for peak thrust, the crown moment results are strongly influenced by culvert type. For RCP (Figure 5), the model response is less at shallow depths and greater at 96 inches. The profile wall data (Figure 6)shows significant variation due to culvert diameters, and is also significantly greater than Eqn **(3)** for all depths. CMP results (Figure 7) are about the same as Eqn **(3)** at 12 inches, but increase with increasing depth.

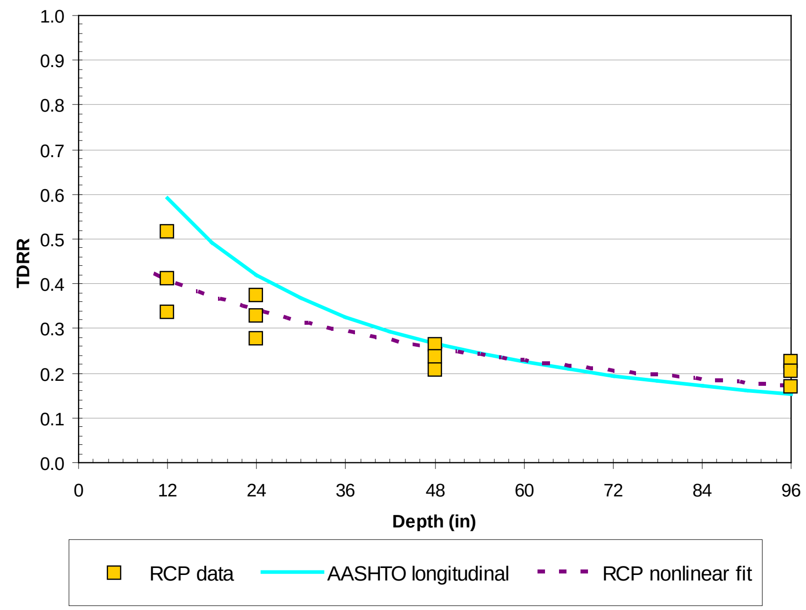

Figure 5—Crown Moment Two-Dimensional Response Ratio for RCP (AASHTO refers to Eqn (3)**)**

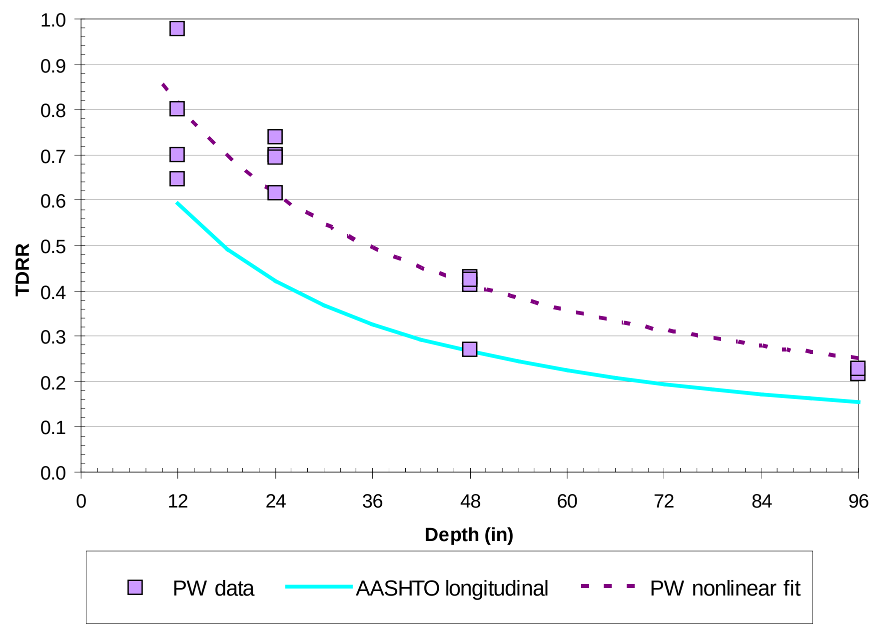

Figure 6—Crown Moment Two-Dimensional Response Ratio for PW (AASHTO refers to Eqn (3)**)**

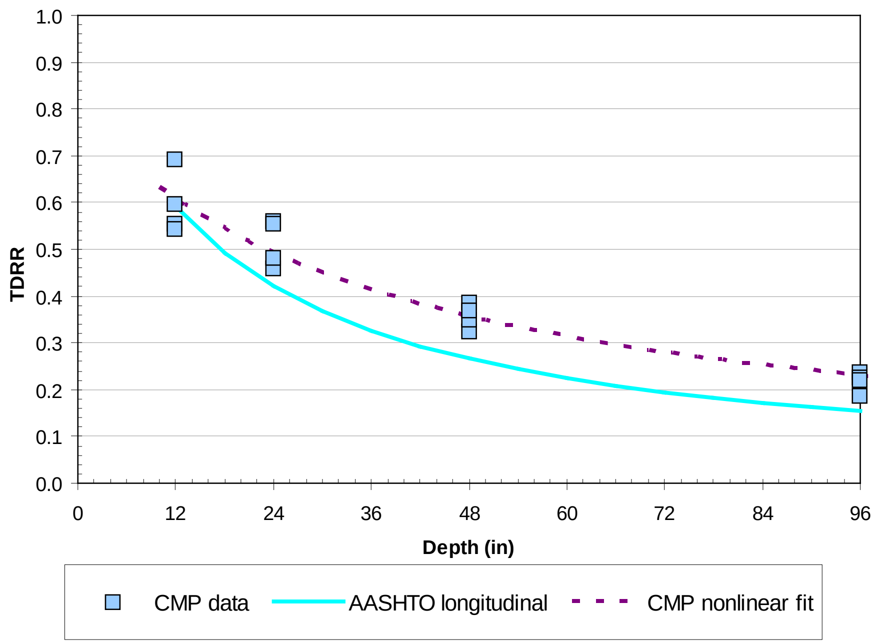

Figure 7—Crown Moment Two-Dimensional Response Ratio for CMP (AASHTO refers to Eqn (3)**)**

2.3 Guideline

In order to characterize the variations illustrated in the figures of the previous section, nonlinear curve fitting was used to select parameters for a variant of Eqn **(3)**, with one additional parameter:

$$\frac{P_H^{2D}}{P_S^{2D}} = TDRR = \frac{a \cdot l_t}{(l_t + b \cdot H)} \quad \textbf{(9)}$$

where a is the additional parameter and $LLDF$ is replaced by b. These parameters are to be selected from curve fitting. Microsoft Excel's Solver function was used to select values for these parameters, but minimizing the sum of the square differences between the function in Eqn (9) and data points. The nonlinear fit curves plotted on each figure illustrate that the curves fit the data relatively well.

Table 1 illustrates the resulting parameter values for Eqn (9); and Figure 8 and Figure 9 illustrate the composite graphs (all data and curves) for peak thrust and crown moment, respectively.

Table 1—Two-Dimensional Response Ratio Equation Parameters

Culvert Type	Structural Response	Constant a	Constant b
RCP	Peak Thrust	1.387	1.595
RCP	Crown Moment	0.509	0.411
PW	Peak Thrust	1.303	0.757
PW	Crown Moment	1.195	0.787
CMP	Peak Thrust	2.132	2.379
CMP	Crown Moment	0.794	0.511

Figure 8 illustrates that nearly all data shows a TDDR greater than the longitudinal load spread from Eqn **(3)**, and that there is a significant variation in the data depending upon culvert type. Hence, the fitted curves also have significant variations. Figure 9 illustrates similar results for crown moment, except that the RCP data is less than Eqn **(3)** for depths of 48 inches or less.

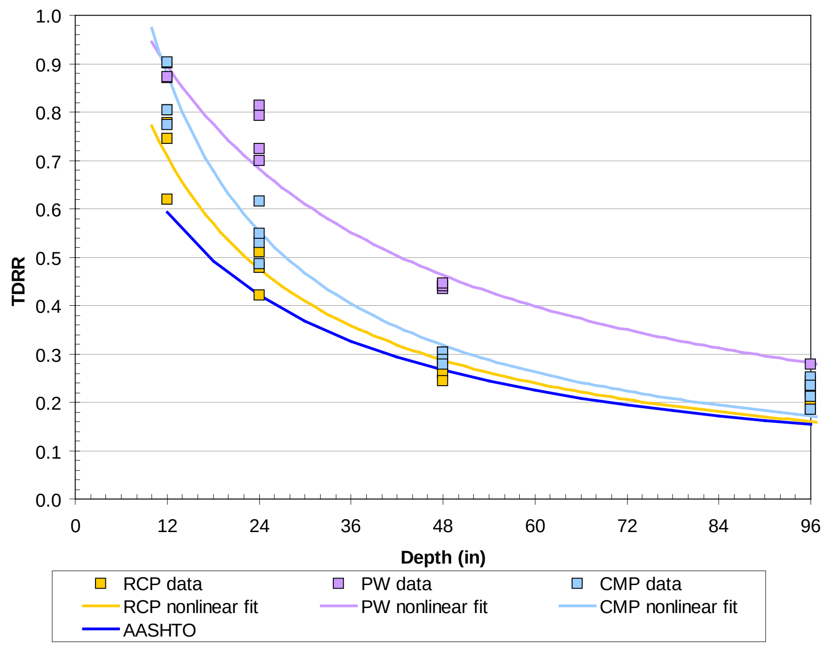

Figure 8—Composite Graph for Peak Thrust Two-Dimensional Response Ratio (AASHTO refers to Eqn (3)**)**

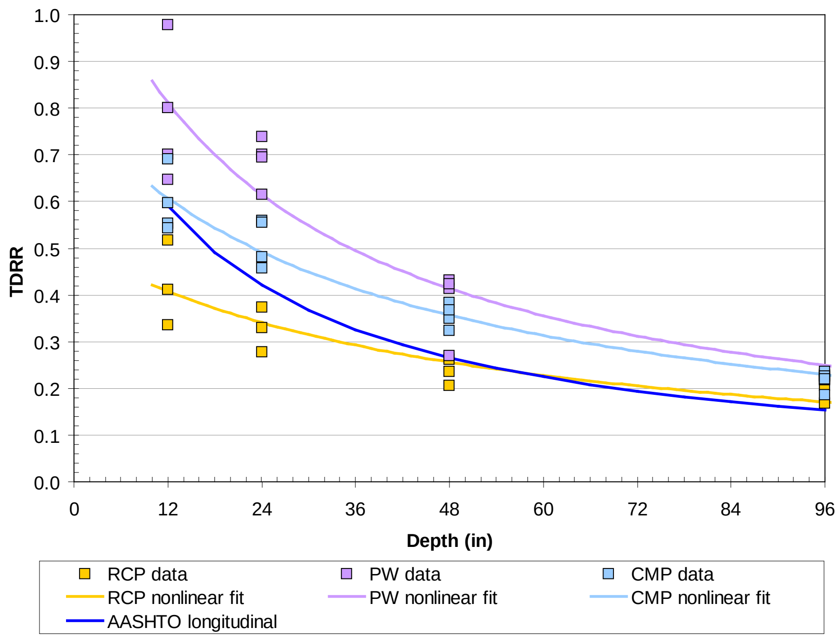

Figure 9—Composite Graph for Crown Moment Two-Dimensional Response Ratio (AASHTO refers to Eqn (3)**)**

The resulting guideline for the surface pressure to be used for conducting 2D analyses is:

$$P_S^{2D} = \frac{P_S^{3D} \cdot a \cdot l_t}{(l_t + b \cdot H)} \qquad \textbf{(10)}$$

where:

P_S^{2D} is the 2D surface pressure,

P_S^{3D} is the 3D surface pressure,

and a and b are parameters from Table 1.

Note that the parameters for peak thrust and crown moment are sufficiently different that separate analyses should be conducted for each.

3 Guidelines for Three-Dimensional Analysis

3.1 Software

3.1.1 Software Used in NCHRP 15-29

Most analyses were done with FLAC3D, with supporting or quality control calculations done using SAP 2000, PLAXIS, ANSYS and FLAC2D.

FLAC3D is a continuum code that is used in analysis, testing, and design by geotechnical, civil, and mining engineers. FLAC3D is designed to accommodate any kind of geo-technical engineering project where continuum analysis is necessary.

FLAC3D is an accurate and efficient geotechnical analysis tool that utilizes an explicit finite difference formulation. The formulation can accommodate large displacements and strains and non-linear material behavior, even if yield or failure occurs over a large area or if total collapse occurs. FLAC3D can model a number of complex behaviors not readily suited to many codes, such as: problems that consist of several stages, large displacements and strains, non-linear material behavior and unstable systems (even cases of yield/failure over large areas, or total collapse).

Built-in soil models include elastic, Mohr-Coulomb, Cam-Clay and others, but all analyses reported here used the Mohr-Coulomb soil model (except for an elastic layer representing pavement, used to prevent the live load from causing excessive failure of the soil surface).

A variety of structural elements may be modeled, but all culvert structures modeled here used shell structural elements, described as follows in the FLAC3D manual (Itasca Consulting Group, 2005):

> Shell structural elements (shellSELs) are three-noded, flat finite elements. Five finite-element types (2 membrane elements, 1 plate-bending element and 2 shell elements) are available. A physical shell (i.e., an arbitrarily curved, shell structure of either isotropic or orthotropic material) can be modeled as a faceted surface comprised of a collection of shellSELs. The structural response of the shell is controlled by the finite element type (to resist membrane loading only, bending loading only, or both membrane and bending loading). Each shellSEL behaves as an isotropic or orthotropic, linearly elastic material with no failure limit; however, one can introduce a plastic-hinge line (across which a discontinuity in rotation may develop) along the edges between shellSELs, using the same double-node procedure as applied to beams. ShellSELs may be rigidly connected to the grid such that stresses develop within the shell as the grid deforms, and they may be loaded by point loads or surface pressures. ShellSELs are used to model the structural support provided by any thin-shell structure in which the displacements caused by transverse-shearing deformations can be neglected.

Isotropic shell elements were used for concrete, PVC, FRP and smooth metal culverts, and orthotropic shell elements were used for corrugated metal and profile wall culverts. Limitations on orthotropic properties discussed in previous reports was eliminated in a new version of FLAC3D used for all orthotropic culverts. See Section 3.8 for additional details about the structural elements employed.

3.1.2 General Software Guidelines

Following are general software guidelines for conducting 3D analyses of live loads on culverts:

1. 3D elements, geometry, boundary conditions, etc.
2. Shell structural elements (see also Section 3.8):
    a. Isotropic shell elements for isotropic culvert materials
    b. Orthotropic shell elements for orthotropic culvert materials
3. Ability to model live loads placed on the soil surface
4. At least the following constitutive models
    a. Elastic (for pavement)
    b. Mohr-Coulomb (for soil)
5. Soil-culvert interface logic that permits arbitrary interface strength and stiffness

3.2 Live Load

3.2.1 Magnitude, Contact Area, Location

Analyses were conducted for dead load (soil loading only) and combined dead plus live load. The dead load response was subtracted from the combined response to determine the live only response. Dead loads, that is soil loads, were not factored. Live loads were applied and factored as follows:

$$LL = m_{mpf}\left[1+\frac{IM}{100}\right]P \tag{11}$$

where:

m_{mpf} is the multiple presence factor (1.2)

P is the wheel load magnitude (16,000 lb)

IM is the dynamic load allowance $33\left[1-\frac{0.125\cdot H}{12}\right],\ H\leq 8$

H is the depth of cover from road surface to top of culvert, in.

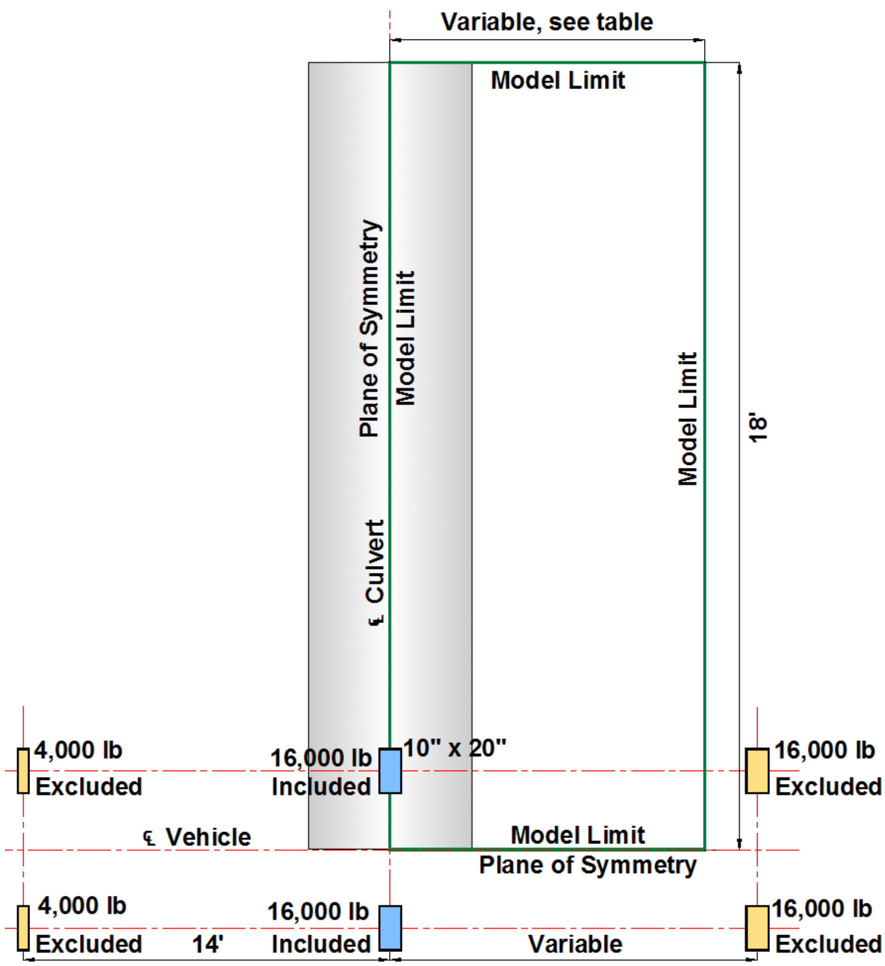

Figure 10—Location and Intensity of Live Load, Before Factoring

Note in Figure 10 that two surface load patches are included in the models, either explicitly or via symmetry, while the other load patches at the front and rear of the vehicle are not included. For some conditions, it may be necessary to include the other load patches.

3.2.2 Factored versus Unfactored Live Loads

The culvert community is divided on the issue of modeling using factored versus unfactored live loads. As a result, the study included a comparison of structural responses to unfactored and factored live loads.

If the structure and surrounding soil have linear-elastic material properties, structural responses to the factored live loads will differ from those to the unfactored live loads by a load factor. However, backfill surrounding the structure is nonlinear, and the ratio of structural response to the factored load to the response to the unfactored live load will not be exactly equal to the load factor. To examine the effect of soil nonlinearity, soil-structure interaction analyses were performed and compared for culverts subjected to factored and unfactored live loads. The analyses were for a variety of 2D and 3D conditions, structure types, soil behavior and software.

Based on the cases we examined, structural responses to the factored live load can be estimated by scaling unfactored live load responses by the load factor. The exceptions are thrusts for shallow burial.

3.3 Representing the Pavement

During the study, the number of models with and without concrete pavement was approximately equal. In models with concrete pavement, the pavement was represented by a single layer of zones with the elastic behavior and properties suitable for concrete. In models without pavement, live loads produced excessive localized bearing failure of the soil. As a result, the surface layer of zones was modeled using the same properties as the underlying zones, but with elastic rather than Mohr-Coulomb behavior.

Results showed that pavement spreads the load and shields the culvert. Because of the significant affect of this load spreading and shielding, and since live loads are possible prior to paving or during roadway rehabilitation, we concluded that unpaved is the controlling case. The influence of pavement is greater for shallow culvert cover depth, and for flexible culverts; and is smaller for stiffer culverts and deeper burial. For example, the ratios of unpaved to paved response were 1.0 to 3.3, for a 48-inch RCP culvert, with 2 ft of cover and SW85 soil. In contrast, the ratios of unpaved to paved response were 0.85 to more than 30, for a 48-in profile wall culvert with 2 ft of cover and SW85 soil. Ratios were computed for crown and invert moment, crown and springline thrust, and crown and invert displacement.

3.4 Representing the Soil

3.4.1 Soil Constitutive Models

In this study, we found that a linearly-elastic, perfectly-plastic model with a Mohr-Coulomb failure criterion was appropriate. This selection offers the best mix of capturing the important aspects of soil behavior in transmitting live loads to structures. In addition, select elastic soil properties based on depth of fill. The Mohr-Coulomb constitutive model does not offer all of the benefits of the Duncan-Selig/Hardening Soil models in capturing stress-dependent stiffness behavior of soil, but for the purposes of a live load study, it appears to provide sufficient accuracy.

3.4.2 Soil Properties

All models used one of four soil materials: well-graded or gravelly sand at 85 percent standard compaction (SW85), well-graded or gravelly sand at 95 percent standard compaction (SW95), inorganic silts and fine sands at 85 percent standard compaction (ML85), and inorganic clays at 85 percent standard compaction (CL85). We recommend that angles of friction at a reference confinement of 14.7 psi from Selig's parameters (1988) be used at any depth in the 3D analysis instead of variable angles of friction calculated from the stress state before the live load application. Mohr-Coulomb soil parameters for SW95, SW85, ML85, and CL85 are listed in the following tables.

Table 2—Parameters for Linear-Elastic and Mohr-Coulomb Models for SW85

Depth (ft)	Modulus of Elasticity E (psi)	Poisson's Ratio ν	Angle of Friction ϕ (deg)	Dilatation Angle ψ (deg)	Cohesion c (psi)
0 to 1	1,300	0.26	38.0	8.0	0.001
1 to 6	2,100	0.21	38.0	8.0	0.001
6 to 11	2,600	0.19	38.0	8.0	0.001
11 to 18	3,300	0.19	38.0	8.0	0.001

Table 3—Parameters for Linear-Elastic and Mohr-Coulomb Models for SW95

Depth (ft)	Modulus of Elasticity E (psi)	Poisson's Ratio ν	Angle of Friction ϕ (deg)	Dilatation Angle ψ (deg)	Cohesion c (psi)
0 to 1	1,600	0.40	48.0	18.0	0.001
1 to 5	4,100	0.29	48.0	18.0	0.001
5 to 10	6,000	0.24	48.0	18.0	0.001
10 to 18	8,600	0.23	48.0	18.0	0.001

Table 4—Parameters for Linear-Elastic and Mohr-Coulomb Models for ML85

Depth (ft)	Modulus of Elasticity E (psi)	Poisson's Ratio ν	Angle of Friction ϕ (deg)	Dilatation Angle ψ (deg)	Cohesion c (psi)
0 to 1	600	0.25	30.0	0.0	3.0
1 to 6	700	0.24	30.0	0.0	3.0
6 to 13	800	0.23	30.0	0.0	3.0
13 to 18	850	0.3	30.0	0.0	3.0

Table 5—Parameters for Linear-Elastic and Mohr-Coulomb Models for CL85

Depth (ft)	Modulus of Elasticity E (psi)	Poisson's Ratio ν	Angle of Friction ϕ (deg)	Dilatation Angle ψ (deg)	Cohesion c (psi)
0 to 1	100	0.33	18.0	0.0	6.0
1 to 7	250	0.29	18.0	0.0	6.0
7 to 14	400	0.28	18.0	0.0	6.0
14 to 18	600	0.25	18.0	0.0	6.0

If site-specific soil properties are known, these values should be used instead of the values presented in the preceding paragraphs.

Near surface soil modulus measurements using the Humboldt GeoGauge, lightweight deflectometer and dynamic cone penetrometer produce near surface values significantly higher than the values presented in the preceding paragraphs. The modulus values for SW85 and SW95 are lower bounds for DCP data from one site. Many DCP values are two to five times greater.

Because the focus of NCHRP 15-29 was live load effects, inhomogeneous culvert bedding was not modeled, so we cannot offer any guidelines for modeling culvert bedding.

3.5 Model Dimensions

In general, model dimensions were larger for increasing cover depth and increasing culvert diameter (span). In some instances, we initially used smaller model widths, observed results indicating that the models were too narrow, and rerun the models using greater width. Table 6 summarizes the model dimensions.

3.6 Element Size

The size of continuum elements used in the study varied depending upon the size of the culvert and the location of the element in the model. In general, smaller elements were used for smaller culvert diameters and larger elements were used for larger culvert diameters. Element size also increased with distance away from the live load—the largest elements were typically at the bottom corner of the model at the end farthest from the live load.

Table 7 lists the continuum and structural element sizes for nine selected models. Since the continuum element sizes near the culvert are wedge-shaped, the inner and outer width are listed.

Table 6—Summary of Model Dimensions

Culvert Type	Culvert Dimensions	Cover Depth (inches)	Model Width (inches)	Model Height (inches)
Round Pipe	12”	12 24 48 96	36 54 108 216	42 54 78 126
Round Pipe	24”	12 24 48 96 144	72 72 108 216 72	73 84 108 156 204
Round Pipe	48”	12 24 48 96 144	144 144 144 144 144	138 150 174 198 222
Round Pipe	96”	12 24 48 96 144	288 288 288 288 288	252 264 288 336 384
Concrete Box	48” x 48”	12 24 48 96	144 144 144 144	138 150 174 222
Concrete Box	96” x 96”	12 24 48 96	288 288 288 288	252 264 288 336
Corrugated Metal Arch	20.1 ft x 9.1 ft	12 48 96	240 240 240	229 265 313
Corrugated Metal Arch	30.1 ft x 18.1 ft	12 48 96	360 360 360	444 456 528
Concrete Arch	25.4 ft x 10 ft	12 48 96	300 300 300	252 288 336
Concrete Arch	43.11 ft x 13.8 ft	12 48 96	528 528 528	352 364 436

Table 7—Continuum and Structural Element Sizes for Selected Models

Case	Minimum Continuum Element Size (inches)	Maximum Continuum Element Size(inches)	Minimum Structural Element Size(inches)	Maximum Structural Element Size(inches)
12-inch round	1.6-3.8 x 6 x 6	6 x 6 x 11	1.6 x 6	1.6 x 11
24-inch round	2.4-4.1 x 6 x 6	6 x 10 x 11	2.4 x 6	2.4 x 11
48-inch round	2.4-3.2 x 6 x 6	9 x 11 x 11	2.4 x 6	2.4 x 11
96-inch round	3.8-4.5 x 6 x 6	12 x 12 x 11	3.8 x 6	3.8 x 11
120-inch round	3.4-3.8 x 3.9 x 6	23 x 24 x 11	3.4 x 6	3.4 x 11
25.4 ft x 10 ft conc. arch	12 x 6 x 6	12 x 12 x 18	6 x 6	6 x 18
43.1 ft x 13.8 ft conc. arch	3 x 7 x 6	18 x 18 x 18	7 x 6	7 x 18
20.1 ft x 9.1 ft metal arch	3 x 6 x 6	17 x 12 x 18	6 x 6	6 x 18
30.1 ft x 18 ft metal arch	3 x 8 x 6	17 x 18 x 18	8 x 6	8 x 18

Figure 11 illustrates how continuum element volumes varied for different round culvert diameters. The smallest element in each model was approximately constant, ranging from about 85 cubic inches to 150 cubic inches. The largest element in each model increases from about 400 cubic inches to 12,000 cubic inches, a thirty-fold increase.

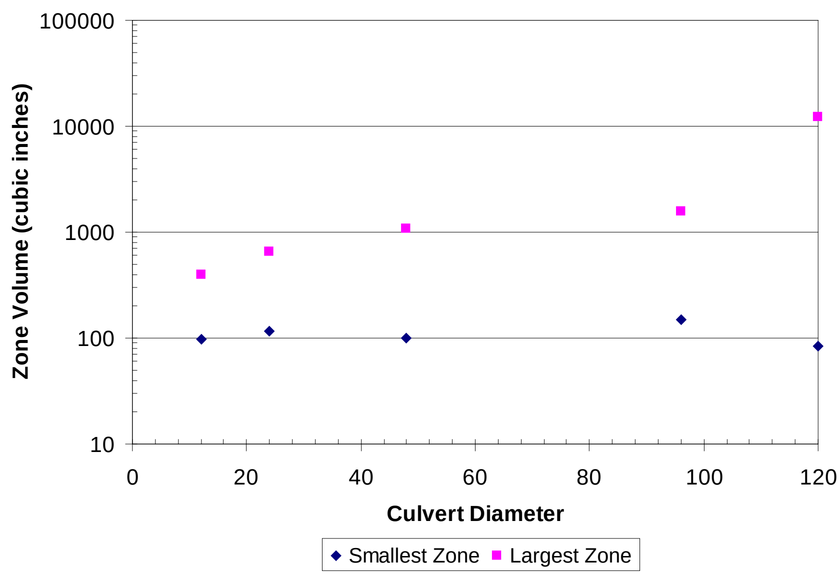

Figure 11—Element Volume as a Function of Culvert Diameter

During the course of the study, we found the element sizes were sufficiently small to produce good results. It may have been possible to use larger minimum element sizes for the large diameter culverts.

3.7 Symmetry and Boundary Conditions

Planes of symmetry may be used to reduce the size of and simplify models. All culvert structures modeled were symmetric about a vertical plane of symmetry through the culvert axis. The culvert structures were also of uniform cross section, so planes of symmetry may be used to reduce the length of the models.

The live load magnitudes and geometry illustrated in Figure 10 are symmetric about the centerline of the vehicle (which is perpendicular to the centerline of the culvert). The live load is not symmetric about the centerline of the culvert, if all three axles are included. However, for a single, relatively shallow culvert, the front and rear axles do not significantly affect culvert loads. As a result, we neglected the live loads from the front and rear axles. The result is a live load distribution that is symmetric about the culvert axis. Nearly all models in this study employed two planes of symmetry to reduce model size. A few analyses did not employ the plane of symmetry through the culvert axis, as a means to check the analysis results.

Boundary conditions for the continuum (soil) parts of the models are straightforward—live loads were applied as pressures on the model top surface and all other surfaces had free or fixed displacements. The conditions are:

1. Model top—The top is free, with a 10-inch by 20-inch patch of live load applied as shown in Figure 10.
2. Model bottom—The top is fixed in the vertical direction and free otherwise.
3. Model ends—The model ends, where the ends of the culvert is exposed, are fixed in the direction parallel to the culvert axis and free otherwise.
4. Model sides—The model side that contains the culvert centerline is fixed in the horizontal direction perpendicular to the culvert axis and free otherwise. The model side opposite the culvert centerline has the same boundary condition, meaning that it is also a plane of symmetry.

Boundary conditions for the culvert were similar:

1. Culvert ends—The culvert ends, where the end of the culvert is exposed, were fixed in translation in the direction of the culvert axis, and were fixed in rotation about the vertical and transverse direction. All other degrees of freedom were free.
2. Culvert crown and invert—The culvert crown and invert, where cut by the plane of symmetry, were fixed in translation perpendicular to the culvert axis, and were fixed in rotation about the longitudinal and vertical direction. All other degrees of freedom were free.

During the study, we became concerned that translational fixity of the culvert ends was increasing the stiffness of the overall culvert structure. A few analyses were rerun with no translation fixity of the culvert ends. The results were only slightly different, confirming that this boundary condition was not affecting the results.

3.8 Representing Culvert Structures

3.8.1 Structure Representation

Culverts structures may be represented as:

1. Continuum elements, where the structure is built up as a series of continuum elements across the thickness of the structure. This method was used in 2D analyses to model box culvert haunch behavior, as a basis for selecting structural element properties.
2. Multiple structural elements, where the culvert is built up of structural elements. This method was used to model the complex interior structure of profile wall pipe for comparison with orthotropic structural elements.
3. Single structural elements, where a single element (of zero thickness) is used to represent a segment of the culvert. This method was used for most analyses.

In all cases, the structures were linear elastic.

The three methods have advantages and disadvantages, but in general, the built up methods, either continuum elements or structural elements were only used in special cases where a single, zero-thickness structural element is not adequate. In this study, single structural elements were used to represent the culverts in all production analyses

In 3D, structural elements for representing culverts must accommodate both bending action and membrane action. As a result, shell elements are necessary and were used for all production analyses. The formulation of shell elements does not permit the calculation of transverse shear forces.

3.8.2 Iso- and Orthotropic Structural Elements

Culverts composed of solid material and regular geometry may be represented by isotropic structural elements, meaning that bending and membrane properties are the same in all directions. This category includes concrete boxes and concrete pipe, smooth steel pipe and smooth thermoplastic pipe.

Both plastic and metal culvert products use cross sectional shapes that are orthotropic, meaning the structural properties vary by direction. These culvert shapes typically have much higher circumferential bending stiffness than longitudinal bending stiffness. In addition, the circumferential membrane stiffness is much higher than the longitudinal membrane stiffness. In plane shear stiffness is reduced from that of flat plate of the same thickness. In order to accurately model buried pipes with such properties, we must have accurate and well-behaved three-dimensional, orthotropic structural elements that permit specification of different stiffnesses for bending and membrane behavior.

We found during the study, however, that the current version of ABAQUS, while providing for orthotropic shells, do not permit specification of the necessary stiffnesses. (PLAXIS3D permits only isotropic shell elements.)

Before discussing modeling with three-dimensional shells, we will describe the analogous issue in two dimensions. A two-dimensional beam formulation permits specification of the following properties:

1. Material Young's modulus E
2. Member area, A, used to calculate the axial stiffness EA of the member
3. Member moment of inertia, I, used to calculate the bending stiffness EI of the member

If the beam were of solid cross section, A and I may be calculated from the beam width and thickness. However, commonly used beams are not of solid cross section and hence the area A and moment of inertia I must be specified separately.

Modeling of a three-dimensional shell has a similar issue. The current formulation of shell elements in ABAQUS is based on a "solid" representation of the shell. For profile wall pipes, which are not solid, and for corrugated metal culverts, which are not "solid" due to the corrugations, two bending stiffnesses and two membrane stiffnesses must be specified to capture the structural behavior. ABAQUS permits only three of the four stiffness pairs ($EA_{transverse}$, $EA_{longitudinal}$, $EI_{transverse}$ and $EI_{longitudinal}$) to be specified independently. At the start of the study, FLAC3D also had this restriction, but Itasca Consulting Group modified the software to permit the four stiffness pairs to be input independently. Results were confirmed by several culvert and non-culvert test cases.

Two guidelines result. When modeling orthotropic culverts:

1. Use orthotropic shell elements for all culvert types that are orthotropic, and

2. Confirm via simple demonstration analyses that the 3D analysis software correctly models orthotropic materials. We found that a model of a plywood plate with 2x4 stiffeners in one direction was effective in confirming model behavior.

3.8.3 Element Size

Structural element sizes are provided in Section 3.6.

3.8.4 Culvert Joints

In distributing live loads through fill onto buried structures, past practice has been to ignore the presence of joints in a pipe. There are two potential issues:

1. The discontinuity created by a pipe joint will prevent load spreading though the pipe resulting in an overstress.
2. A joint loaded on one side but not the other will undergo differential deflection resulting in a joint leak.

Parameters that could affect this condition include:

1. Pipe bell and spigot joints are often heavier and stronger than the barrel, providing more strength to resist the live load. Bells are typically thicker than pipe barrels, and the spigots, which may not be thicker than the barrels are contained within the bell which provides additional confinement.
2. Pipe joints completed by wrap around couplings provide a mechanical connector to two adjacent lengths of pipe that likely provides shear transfer.
3. Unlike box culvert slabs, buried pipes are not assumed to have any inherent load distribution capability – i.e. in a box culvert under less than 2 ft of fill, a live load is distributed over a width about 4 ft wider than the actual loaded width while in pipe the loaded length is typically assumed to carry the entire load.
4. Most thermoplastics pipe are required to pass joint shear tests that require imposition of an unbalanced load without causing leakage. Concrete pipes have a joint shear test but drainage pipe are not typically subjected to it. Metal pipes do not currently have a joint shear test.
5. Most pipes have excess structural capacity at the minimum depths of fill allowed by specifications. Minimum depths of fill are set to control road surface performance and are virtually always, if not always, set at depths where the pipe has extra capacity to carry unanticipated loads.

The research team is not aware of any definitive studies on the above issues, thus, we cannot state with certainty that the presence of a joint can be ignored when distributing live loads through earth fills; however, we find the lack of any problems associated with this matter to be very powerful. It is well known that pipes can be subjected to severe abuse during installation and are often installed in backfill conditions that do not comply with specifications, or subjected to large construction loads that exceed design loads. In spite of this, problems are limited and often only result from extreme loading conditions.

Neither the RFP nor our proposal foresaw the need to include culvert joints in the computer models. From a practical standpoint, investigating the issue of load distribution onto jointed buried pipes would entail a significant study. Each culvert type has one or more different joint types, each with different behavior. Our current full scale models use shell elements to represent the culverts, In order to provide the basis for incorporating joints in full scale models, large, complex models using continuum elements would need to be developed, tested and analyzed for each joint type. The macro structural properties of each joint could then be incorporated in a structural element model. Consideration of gasket pressures would also be required.

In summary, we believe that the technical points outlined above are sufficiently compelling that, when combined with the practical considerations, support a decision to address jointed culverts in the commentary.

3.9 Soil-Culvert Interface

The soil-culvert interface connects the continuum elements representing the soil to the structural elements representing the culvert. Historically, it was common to model soil-culvert interaction with no interface—the soil and culvert structure were bonded and in fact had common nodes.

Now, in typical formulations, the interface has stiffness and strength properties, which vary depending upon compression or tension loading. If modeled in this manner, nonlinear behavior may occur in the interface between the culvert and the soil, or in the soil.

The influence of the soil-culvert interface stiffness on culvert response was not investigated, so we offer no guidelines.

For preliminary 2D analyses, the interface strength was 50 percent of the soil shear strength. To examine the effect of interface strength on structural response, we analyzed the concrete and thermoplastic pipe with backfill modeled by the Mohr-Coulomb constitutive model with the interface strength equal to 100 percent of the soil shear strength. Structural responses to live loads did not change significantly when the interface strength was changed from 50 percent of the soil shear strength to 100 percent although the cases with the 100 percent strength showed slightly larger peak responses than those with the 50 percent strength except for moments of the thermoplastic pipe with 2 ft cover. A change in the Interface strength affected thrusts more than moments. Structural responses of the thermoplastic pipe were affected more by a change of interface strength than those of the concrete pipe. Structural responses of the 6 ft cover cases were affected more by a change of interface strength than those of the 2 ft cover cases; however, it should be noted that responses of the 6 ft cover cases were much smaller than those for the 2 ft cover cases.

For 3D analyses, the influence of soil-culvert interface strength was investigated, for a small number of culvert types, sizes and depths, by varying the interface strength. Four interface strengths were considered:

1. Fully bonded—No relative deformation was permitted between the soil and the culvert
2. 100-percent soil strength—Interface strength was 100 percent of the soil friction angle, and 100 percent of the soil cohesion.
3. 50-percent soil strength—Interface strength was 50 percent of the soil friction angle, and 50 percent of the soil cohesion.
4. Unbonded—Interface friction and cohesion were zero.

Interfaces had a Mohr-Coulomb failure criterion.

Most production analyses were conducted for interfaces with strength properties of 100 percent of the soil strength. Like the 2D results, the 3D results show that the reasonable interface strengths do not have significant influence on the structural response.

3.10 Modeling Sequence

Three states of the model were analyzed and saved for each analysis conducted:

- State 1 is the soil mass in equilibrium, with no culvert or live load. State 1 was achieved by creating the model grid, applying material properties to the soil materials and placing stresses in the grid.
- State 2 (dead load) is the soil mass plus the culvert, in equilibrium. This State was achieved by excavating the soil (with no cycling of the model), installing the culvert in the soil, then cycling to equilibrium.
- State 3 (dead load plus live load) is State 2 plus application of the live load defined above.

While saving and reviewing States 1 and 2 is not necessary in order to find State 3, we recommend conducting analyses in this manner.

4 References

Itasca Consulting Group, Inc. (2005), “FLAC3D Fast Lagrangian Analysis of Continua in 3 Dimensions, Version 3.”