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**LRFD DESIGN SPECIFICATIONS FOR SHALLOW FOUNDATIONS**

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**APPENDIX A  
ALTERNATIVE MODEL BACKGROUND**

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Aloys Kisse  
Institute of Soil Mechanics and Foundation Engineering  
Department of Civil Engineering  
University of Duisburg-Essen, Germany



# **A Consistent Description of the Serviceability and Ultimate Limit States of Foundations<sup>1</sup>**

Dipl. - Ing. Aloys Kisse  
Institute of Soil Mechanics and Foundation Engineering  
University of Duisburg-Essen  
Essen, Germany

## **1. INTRODUCTION**

A thorough understanding of the structure-soil interaction is the basis for a safe and economical design. Because different inconsistent limit states and serviceability states have to be distinguished, the interaction is usually unpredictable. Therefore, it is necessary to ensure that the foundation of a structure, e.g. a shallow foundation, will not fail for a prescribed safety and allowable displacement.

The traditional analytical methods are not applicable for the probabilistic safety assessment of structures (Eurocode 7). In this context the failure of a system means that it loses its ultimate bearing capacity and serviceability capacity.

If it is possible to describe the system behavior in a consistent way, i.e. the relationship between loading up to failure load and corresponding displacements and rotations of the foundation, the distinction between different limit states would no longer be necessary. Such a relationship is similar to the constitutive models of soil. An Ongoing research project at the Institute of Soil Mechanics and Foundation Engineering (SM&FE) at the University of Duisburg-Essen aims to develop this systematic formulation. The research is sponsored by the German Society of Research (DFG) and directed by Prof. Dr.-Ing. W. Richwien.

At present, the experimental results obtained from tests on small scale model foundations on sand carried out at the Institute of SM&FE, UDE by Perau (1995) are being reanalyzed. The analysis was required in order to examine the test results' suitability from a similarity theory point of view. Following this verification, the system's behavior in various loading cases can be analyzed.

## **2. EXPERIMENTAL STUDIES**

The development of a systematic model to describe the deformation behavior requires at first the determination of the related parameters. For this purpose the test results by Perau (1995) will be analyzed. The main attention will be paid to the system behavior in various loading cases. The tests conducted by Perau (1995) were small scale in nature (model tests). Because it is usually seldom possible to conduct prototype tests to study the behavior of the system, model tests are commonly adopted in engineering science.

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<sup>1</sup> A paper published in proceedings of the XIV European Young Geotechnical Engineers Conference, Plovdiv, Bulgaria, 2001.

The study of the system behavior by small scale model tests is based on the principle of similarity, which is derived from the rules of physics. Different systems will show similar behavior only if they are mechanically similar. According to Walz (1982), two systems are mechanically equivalent, if for the problem under study the related dimensionless parameters in both, model system (M) and prototype system (P), are correspondingly identical. For the forces  $F$  applied on the systems, this rule means:

$$\left( \frac{F}{\gamma \cdot b^3} \right)_P = \left( \frac{F}{\gamma \cdot b^3} \right)_M \quad (1)$$

The model is formed by reducing the scale of the prototype by a factor  $\lambda$ , which results from the geometrical similarity for length  $l$ :

$$\left( \frac{l_i}{l_k} \right)_P = \left( \frac{l_i}{l_k} \right)_M \Leftrightarrow \frac{l_{iP}}{l_{iM}} = \lambda = \frac{l_{kPi}}{l_{kMi}} \Leftrightarrow l_{iP} = \lambda \cdot l_{iM} \quad (2)$$

The scales of all geometrical parameters must be reduced according to this factor. In soil mechanics, usually the same soil (in this case sand) for model tests and prototype tests is used. Therefore the grain size is not reduced according to factor  $\lambda$ . This is also not expected because, for instance, sand must be substituted by silt in order to meet the geometrical similarity if the factor  $\lambda$  equals 20. De Beer (1961) proved by comparing many failure cases of foundations with different sizes on sand that if the characteristic size of a foundation is large enough ( $> 10$  cm) the scale effects due to the same grain size are negligible. Otherwise this influence has to be kept in mind while analyzing the deformation process up to failure.

Additionally, the initial stress states in the model and in the prototype must be identical. In prototype scale this initial stress state depends on the weight of the soil and the sedimentation process (earth pressure at rest). In model tests such a stress state can be achieved by homogeneous falling of the sand into the box in which the tests are conducted. Nevertheless, the stress level under loading is dependent of the size of the footing. So, absolute displacement values of the models cannot be transferred to the prototype. But the system behavior, i.e. the load-displacement relations, in model scale is similar to prototype scale and can therefore be transferred (Franke and Muth, 1987).

Based on a great number of model experimental results Franke and Muth (1987) proposed that there exists a potent relationship between the applied load and the characteristic displacement:

$$\frac{u}{l} = a \cdot \left( \frac{F}{F_0} \right)^\alpha \quad (3)$$

where  $l$  is the characteristic size and  $F_0$  is a reference load. The factor 'a' reflects the influence of other parameters (e.g. the embedment) on the displacement. If  $\alpha = \text{const.}$ , equation (3) can be written in the form below:

$$\log\left(\frac{u}{l}\right) = \log a + \alpha \cdot \log\left(\frac{F}{F_0}\right) \quad (4)$$

An advantage of this form is that it can be represented by straight lines in a double-logarithmic coordinate system. The influence of the input parameters can then be recognized by plotting the test results in these coordinates. According to Hettler (1981), this will be reflected by the exponent  $\alpha$ , which is dependent on the type of soil. But  $\alpha$  is independent either of the porosity of soil or of the form of the foundation. If more model tests are conducted with the same sand for different densities and for different side ratios of the foundation, the value of  $\alpha$  must be kept constant. Figure 1 and Figure 2 represent the results of tests from Perau (1995). The tests were conducted under the side ratios  $b_2/b_3 = 1, 0.33, \text{ and } 0.20$  for different porosities  $n$ .

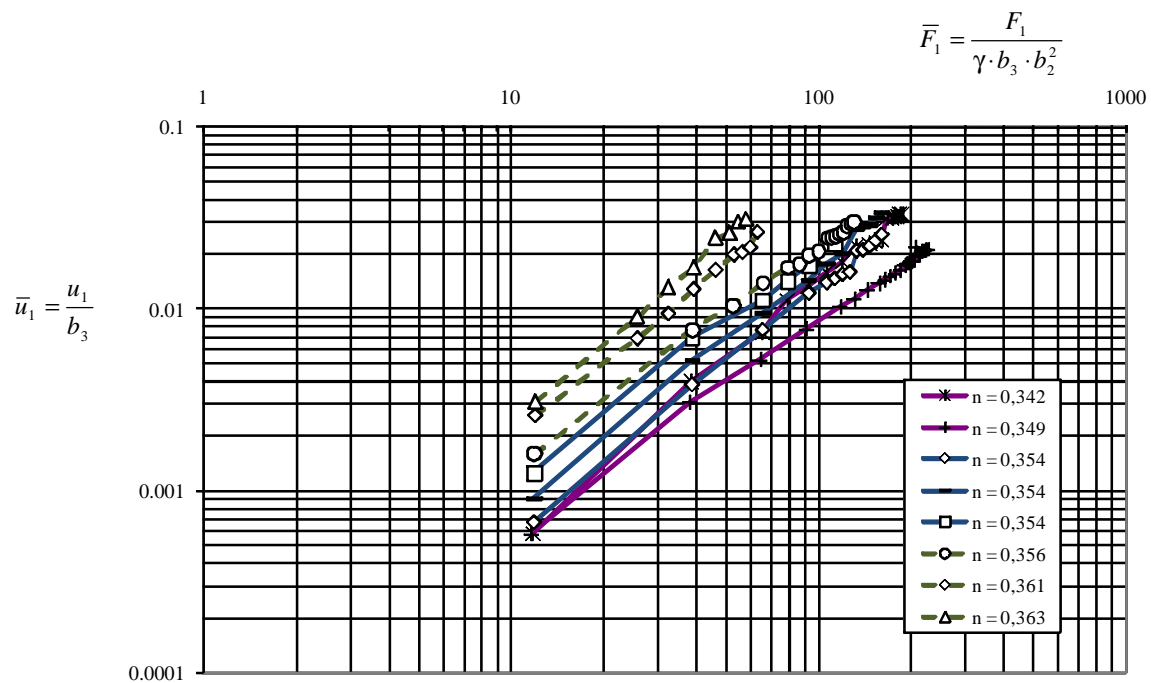


Figure 1: Relationship between dimensionless force  $F_1$  and displacement  $u_1$  for different porosities  $n$  ( $b_2/b_3 = 0.5$ )

The straight lines in Figure 1 have approximately the same slope  $\alpha$ . This implies that exponent  $\alpha$  is independent of the density of the soil. Figure 2 represents the relationship between dimensionless force and displacement for constant soil density but different side ratios. It can be seen from Figure 2 that all straight lines have the same slope as mentioned before. Thus, the exponent  $\alpha$  is also independent of the foundation form.

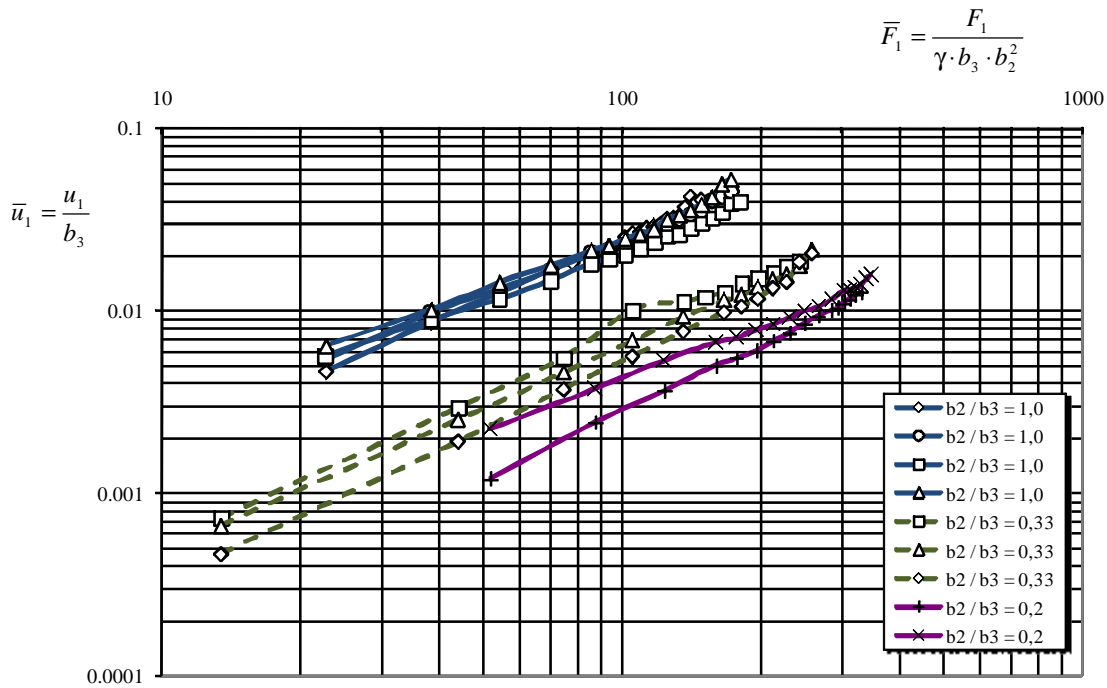


Figure 2: Relationship between dimensionless force  $F_1$  and displacement  $u_1$  for different side ratios ( $n = 0.34$ )

The distance between the lines in Figure 1, described by quantity  $a$  in equation (3), reflects the influence of the porosity on the dimensionless displacement. If it had no influence, all result points would be located on one line.

Similarly, Figure 2 shows the influence of the side ratio. If the side ratio had no influence on the structure-soil interaction, all test results would be located on one line. These properties cannot clearly be observed if all quantities are left with dimensions, as shown for instance in Figure 3, in which the same test results as in Figure 2 are represented. It can be seen in Figure 3 that all lines are located in a very narrow range without a clear distance as in Figure 2. The influences of the geometry of the foundation are concealed

It is easy to notice the great advantage of the dimensionless presentation, i.e., the physical relations are revealed.

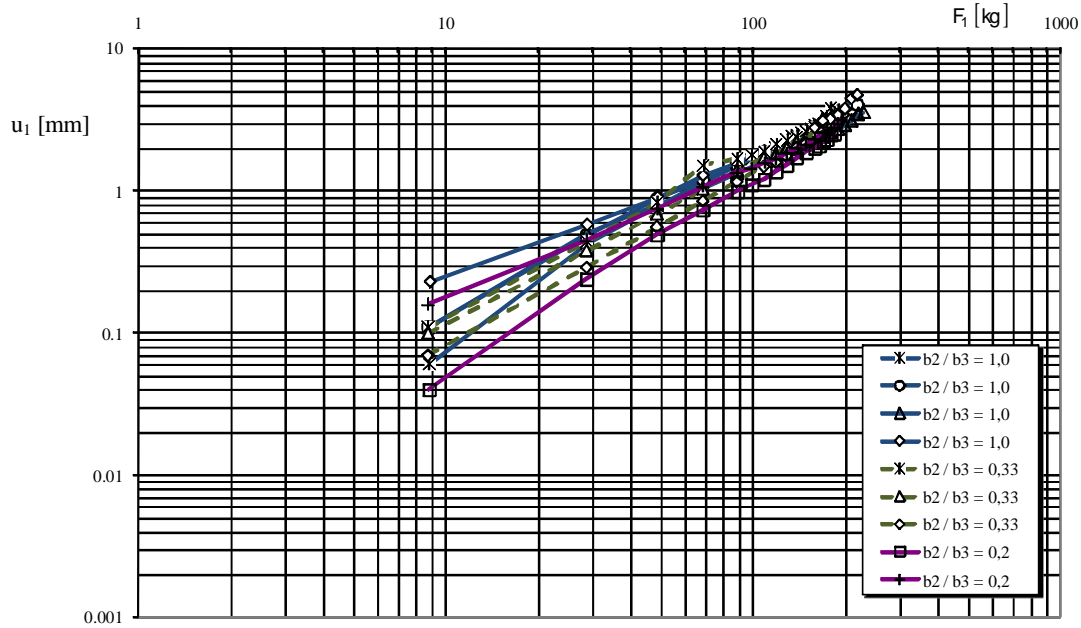


Figure 3: Relationship between force  $F_1$  and displacement  $u_1$  (with dimension) for different side ratios ( $n = 0.34$ )

### 3. DISPLACEMENT RULE

The so called displacement rule to be developed should reflect the complete load-displacement relation before the system reaches its ultimate limit state. In the three dimensional case the relationship between the dimensionless load components

$$\bar{L} = [\bar{F}_1, \bar{F}_2, \bar{F}_3, \bar{M}_1, \bar{M}_2, \bar{M}_3]^T$$

and the corresponding dimensionless displacement and rotation components of the system

$$\bar{u} = [\bar{u}_1, \bar{u}_2, \bar{u}_3, \theta_1, \theta_2, \theta_3]^T$$

is expressed generally as  $\bar{u} = f(\bar{L})$

Figure 4 shows the load-displacement relationship of a foundation under a vertical load applied at the center of the structure ( $b_2/b_3 = 0.2$ ;  $n = 0.340 \sim 0.364$  and  $D = 0.7 \sim 0.9$ ). If the load is small, the load-displacement relation is linear. With the increase in load the relation becomes non-linear. In principle, three phases of foundation behavior can be distinguished.

The first phase corresponds to the linear load-displacement behavior which can be described by a modulus depending on the load and the porosity  $n$  (Figures 1 and 4). The analysis of the test results has revealed that the geometry of the foundation influences the load-displacement relationship (Figure 2) as well. When the load approaches the limit, plastic zones in the soil

beneath the foundation appear and expand, and finally become the failure body. The modulus describing this load-displacement phase includes the influencing parameters mentioned before and the shear strength parameters  $\phi'$  and  $c'$  of the soil.

Thus, for the case of a vertically and centrally loaded footing without embedment the displacement rule can be generally formulated as

$$\bar{u} = f\left(\frac{b_2}{b_3}, \gamma, n, \tan \phi', \bar{F}_1\right)$$

The third phase starts when the load reaches its limit, which is determined by an additional failure condition (Lesny, 2001). The limit loads are not clearly reflected in Figure 4, because the tests were load-controlled, i.e. the tests were terminated when the loads reached the limit.

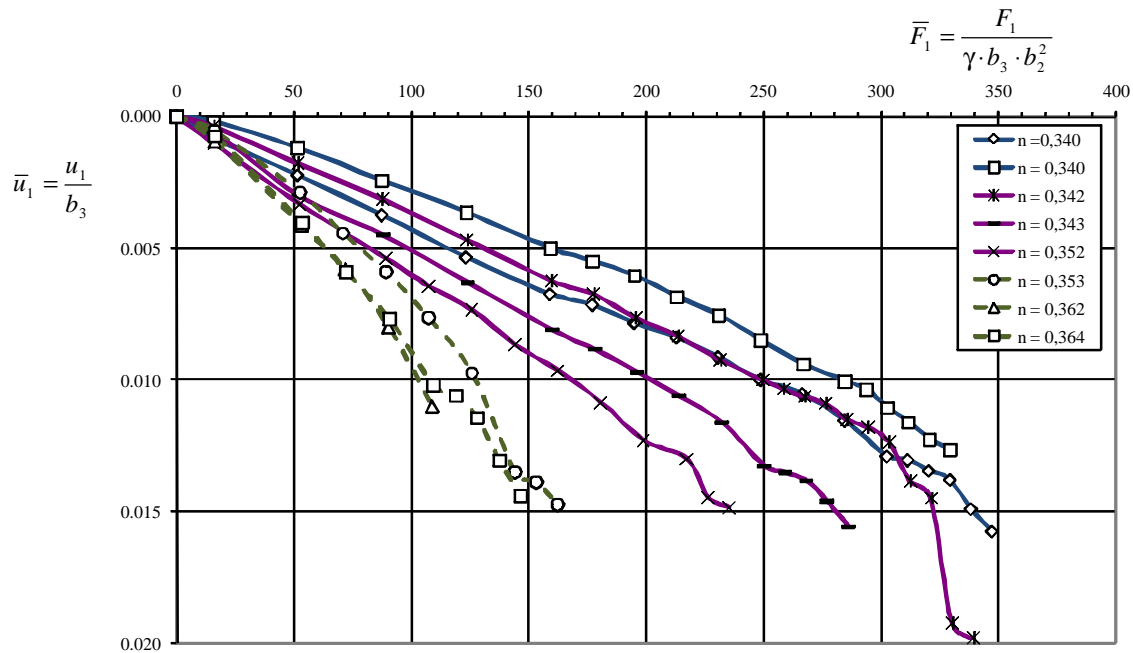


Figure 4: Dimensionless force  $F_1$  and displacement  $u_1$  for different porosities  $n$  ( $b_2/b_3=0.2$ )

#### 4. SUMMARY

The usefulness of the small scale model tests conducted by Perau (1995) to describe the load displacement behavior of shallow foundations has been demonstrated by using the model theory. The analysis has proved that the dimensionless representation of the test results is necessary to reveal the physical essence.

The evaluation of the test results has also shown that the displacement rule should be able to describe the three phases of system behavior. Further analysis of these test results is necessary especially for determining the dependence of the individual modulus on the load components.



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