APPENDIX I

VISCOELASTIC THERMAL STRESS COMPUTATION

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The model developed by Hiltunen and Roque (40) accounts for thermal viscoelastic material behavior through a generalized Maxwell model as illustrated in Figure I-1. The Hiltunen and Roque model for viscoelastic thermal stress is expressed in Equation (I-1):

$$\sigma(\xi) = \int_0^{\xi} \frac{E(\xi - \xi')d\varepsilon}{d\xi'} d\xi' \quad \text{(I-1)}$$

where $\sigma(\xi)$ is stress at the reduced time ξ;

$E(\xi - \xi')$ is relaxation modulus at the reduced time $\xi - \xi'$;

ε is strain at the reduced time ξ .

This viscoelastic thermal stress equation is expressed in terms of reduced time ξ which is defined in the process of time-temperature superposition.

$$\xi = \frac{t}{a_T}$$

where t is real time;

a_T is the time-temperature shift factor.

Considering the strain is viscoelasticity. This strain can be expressed as a function of reduced time ξ' and thermal coefficient α as in Equation (I-2).

$$\varepsilon = \alpha(T(\xi') - T_0) \quad \text{(I-2)}$$

where $T(\xi')$ is pavement temperature at the reduced time;

T_0 is pavement temperature at stress free temperature (20℃).

Therefore, equation (I-1) can be rewritten in real time instead of reduced time (I-3).

$$\sigma(t) = \int_0^t \frac{E\left(\xi(t) - \xi'(t)\right)\left(\partial\left(\alpha\left(T(\xi') - T_0\right)\right)\right)}{\partial \xi'} \qquad \text{(I-3)}$$

In order to calculate the viscoelastic thermal stress, this program uses the Collocation method to calculate the coefficients of a Prony series ($E(\xi - \xi')_i$). The Collocation method is summarized in the next part of this appendix. In addition, the calculation of the shift factor a_T is also shown in the next part.

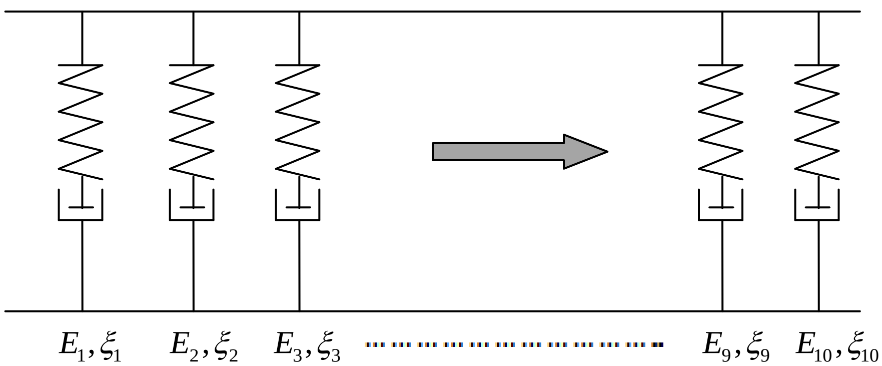

Figure I-1. Generalized Maxwell model for relaxation.

COLLOCATION MATRIX

The Collocation method is a method to approximate the computed number and actual number by using predetermined loading times, t_i and corresponding retardtion times, T_j. The Prony series coefficients, E_j , for the viscoelastic relaxation modulus, ($E(\xi - \xi')_i$), are then calculated with the collocation matrix shown below.

$$\begin{bmatrix} e^{\frac{-t_1}{T_1}} & \cdots & e^{\frac{-t_1}{T_{10}}} \\ \vdots & \ddots & \vdots \\ e^{\frac{-t_{11}}{T_1}} & \cdots & e^{\frac{-t_{11}}{T_{10}}} \end{bmatrix} \begin{bmatrix} E_1 \\ \vdots \\ E_{10} \end{bmatrix} = \begin{bmatrix} E(t_1) + E_\infty \\ \vdots \\ E(t_{11}) + E_\infty \end{bmatrix}$$

where

t_i are the 11 loading times we determined;

T_i are the 10 retardation times we determined;

E_i are the coefficients of Prony series;

$E(t_i$ are the relaxation moduli from the ANN 2006 model (3) at -10°C;

$$E_\infty = E_{FWD}(t,T) - E_{ANN}(t_{0.06}, T_{FWD}) \quad \text{IF} \quad E_\infty \le 0, \text{ set } \quad E_\infty = 0$$

SHIFT FACTORS

Shift factors were determined at three different temperatures which are -10°C, 0°C, and 10°C. The assumed reference temperature (shift factor $a_T = 1$) is -10°C. In order to evaluate the shift factors at 0°C and 10°C, the ANN (Artifical Neural Network) relaxation modulus program is used to calculate the modulus at three different loading times which are 360 seconds, 3600 seconds (1 hour), and 36000 (10 hours) seconds. The purpose of evaluating the moduli at different temperatures is to find the log-log slope of the line (m_{mix}) of relaxation modulus versus loading time as shown in Figure I-2. As shown in that figure, m_{mix} can be calculated at different temperatures as:

$$m_{mix(-10)} = \frac{\log E(360,-10) - \log E(36000,-10)}{\log(36000) - \log(360)}$$

$$m_{mix(0)} = \frac{\log E(360,0) - \log E(36000,0)}{\log(36000) - \log(360)}$$

$$m_{mix(10)} = \frac{\log E(360,10) - \log E(36000,10)}{\log(36000) - \log(360)}$$

Knowing the m_{mix} and relaxation modulus at different temperatures, the time-temperature shift factors for 0°C and 10°C can be determined from Equations (I-4) and (I-5).

$$\log\left(\frac{1}{a_T}\right)_{T=0} = \frac{\log E(3600,-10) - \log E(3600,0)}{m_{mix(0)}} \qquad \text{(I-4)}$$

$$\log\left(\frac{1}{a_T}\right)_{T=00} = \frac{\log E(3600,-10) - \log E(3600,0)}{m_{mix(00)}} \quad \text{(I-5)}$$

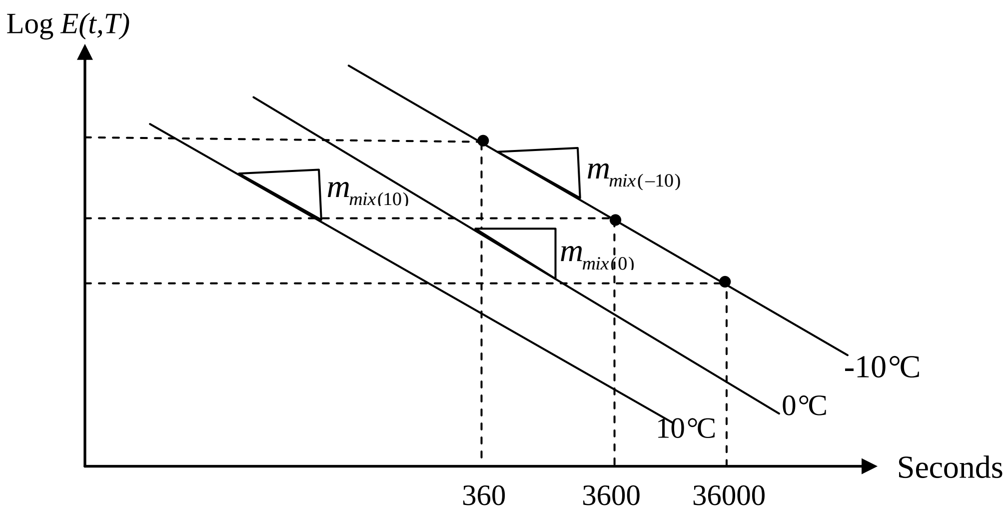

Figure I-2. m_{mix} at different temperatures.

A numerical version of Equation I-3 is used in a subroutine of the Calibration and Design programs to calculate the viscoelastic thermal stress at the tip of the growing crack for every hour of each day. This stress is used in turn in the ANN stress intensity factor subroutine to calculate the stress intensity factor at the tip of the crack and the largest one that occurs each day is used to calculate the growth of the crack due to thermal stresses for that day.