APPENDIX L

CALIBRATION OF THE REFLECTION CRACKING AMOUNT AND SEVERITY MODEL

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INTRODUCTION

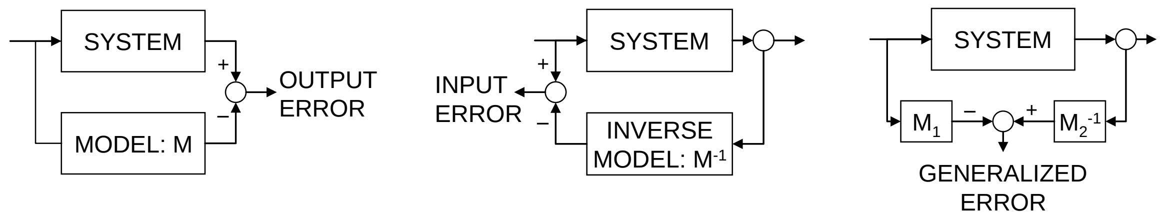
One of objectives of the study is to calibrate the reflection cracking amount and severity model with observed reflection crack data in the field. The calibration refers to the mathematical process through which the total error or difference between observed and predicted values of distress is minimized. The process used to achieve the calibration, which determines ρ and β in the reflection cracking model, was conducted using available field reflection cracking data described in the previous appendix and an iterative System Identification process.

System Identification Process

The reflection cracking amount and severity model at a given severity level was considered to have been calibrated when the error between observed and predicted crack lengths was minimized in some sense. Since the predicted number of days is calculated by the mechanistic crack growth model at each test section, a solution method was required to determine the parameters, ρ and β, in the empirical s-shaped amount and severity model. In this study, the method of solving for the parameters is by use of the system identification process.

The purpose of the system identification process is to develop a mathematical model which describes the behavior of a system (real physical process) in a rationally satisfying method. The actual system and the mathematical model are identified when the error between them is minimized or satisfies the error criteria; otherwise, the model is adjusted until the error is reduced sufficiently (30). There are three different error minimization models in system identification process depending on the choice or residuals combined with the model: forward model, inverse model, and generalized model shown in Figure L-1 The forward approach employs output error between the model and the system to minimize them using same input. In the inverse approach, the input error is used to be minimized based on same output. The generalized model is a combination of the forward and inverse approach when the model is invertible (30).

As in the calibration process in this study, when the system output is fixed because it is observed or obtained from an actual system, the output from the model must be refined to calibrate the mathematical model by adjusting the parameters. That is, the reflection cracking amount and severity model (mathematical model) is calibrated based on observed reflection crack data (actual system output) to produce predicted crack data (model output) which is close to the observed crack data. Therefore, the forward model system identification process was used for calibrating the reflection cracking amount and severity model since it is easier to compute the model output.



(a) Forward Model **(a) Inverse Model** **(c) Generalized Model**

Figure L-1. Methods for system identification process (30).

When the output error between the system and the model is small enough to meet an error criterion, it is assumed that an optimal model for the system is obtained. However, if the error does not meet the criterion, the parameters in the mathematical model should be corrected by a parameter adjustment and adaptation algorithm. The correction process is performed iteratively until the error becomes small enough using the algorithm. Figure L-2 depicts the scheme of a system identification process based on the forward model and parameter adjustment and adaptation algorithm for the reflection cracking amount and severity model calibration.

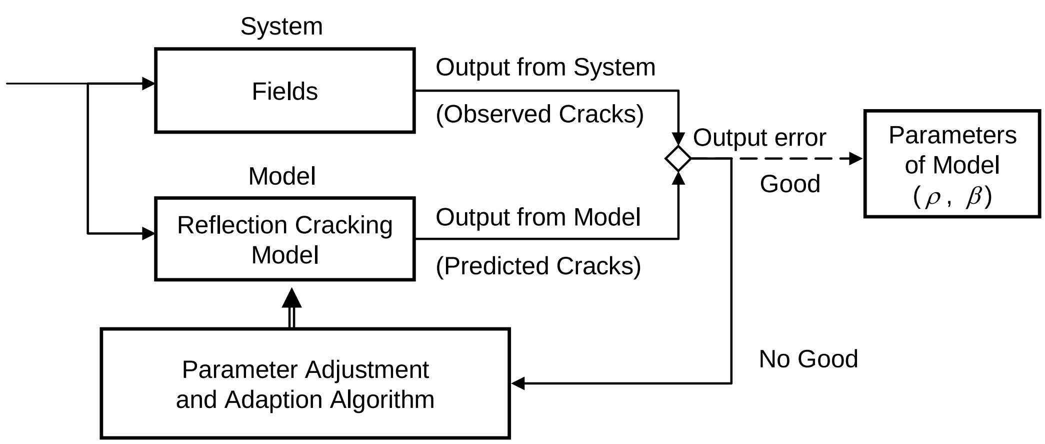


Figure L-2. Scheme of system identification process.

Parameter Adjustment and Adaption Algorithm

A parameter adjustment and adaption algorithm was developed based on the Taylor series expansion as follows (31).

$$\left[F_{ki}\right]\left\{\alpha_i\right\}=\left\{r_k\right\} \qquad \text{(L-1)}$$

where

$[F_{ki}]$ = sensitivity matrix = $\sum_{k=1}^{m}\sum_{i=1}^{n}\frac{\partial f_k}{\partial p_i}\frac{p_i}{f_k}$ ($m \times n$ matrix),

m, n = number of output data and model parameters, respectively,

f_k = mathematical model,

p_i = model parameters,

$\{\alpha_i\}$ = change vector (relative change of parameters) = $[\alpha_1\ \alpha_2\ \ldots\ \alpha_n]^T$, and

$\{r_k\}$ = residual vector (error between system and model outputs) = $[r_1\ \ r_2\ \ldots\ \ r_m]^T$

The minimization of error contained within the residual vector $\{r_k\}$is analogous to the reduction of error employed in least squared error analysis. The squared error between the actual output and the predicted output is calculated by using a mathematical model to determine the sensitivity of the weighting parameters for allocating the squared error. It is possible to adjust the model parameters until there is no squared error remaining; however, because of the presence of random error, the values in the residual matrix $\{r_k\}$ should not be forced to zero (32). Since the elements in the residual vector $\{r_k\}$ which represents errors between the actual and model outputs are determined based on model parameters, p_i, assumed at each iteration process, they are known values. The sensitivity matrix $[F_{ki}]$ which reflects the sensitivity of the output from mathematical model, f_k, to the assumed parameters, p_i, is also a known value. Therefore, the unknown change vector $\{\alpha_i\}$ presents the relative changes of the model parameters and is the target matrix to be determined in the process. Equation L-1 can be rewritten as:

$$\left\{\alpha_i\right\}=\left[F_{ki}^T F_{ki}\right]^{-1}\left[F_{ki}\right]^T\left\{r_k\right\} \qquad \text{(L-2)}$$

As soon as change vector $\{\alpha_i\}$ is obtained using an initial assumption of parameters, a new set of parameters is determined as

$$p_i^{j+1} = p_i^j \left(1 + 0.6\alpha_i\right) \qquad \text{(L-3)}$$

where

j = iteration count

By minimizing the change vector $\{\alpha_i\}$, solutions for the parameters in the model are found. In order to achieve the solution, the iteration process using Equation L-3 was continued until there is no squared error remaining or the desired convergence was reached. In this study, the convergence criterion was set to 1.0 percent; that is, the iteration should be repeated until the elements in the change vector $\{\alpha_i\}$ are less than 0.01.

Calibrating Reflection Cracking Model of Test Sections

Based on the system identification and the parameter adjustment algorithm addressed previously, the reflection cracking models were calibrated using the data obtained from LTPP, New York City, and Texas asphalt overlay test sections. The process was used to fit the predicted crack length to the measured crack length by iteration. The parameter adjustment algorithm of Equation L-1 can be expressed for determining the parameters in the reflection cracking model as follows:

$$[F]\{\alpha\} = \{r\} \qquad \text{(L-4)}$$

where

$\overline{D}(N_i)$ = crack length at N_i, calculated using ρ^j and β^j,

$D(N_i)$ = measured crack length at N_i, and

The parameters ρ and β in the model were determined when the relative changes of adjusted parameters were minimized and so the elements in the change vector were less than 0.01.

$$\begin{bmatrix} \frac{\partial \overline{D}(N_1)}{\partial \rho^j} \frac{\rho^j}{\overline{D}(N_1)} & \frac{\partial \overline{D}(N_1)}{\partial \beta^j} \frac{\beta^j}{\overline{D}(N_1)} \\ \frac{\partial \overline{D}(N_2)}{\partial \rho^j} \frac{\rho^j}{\overline{D}(N_2)} & \frac{\partial \overline{D}(N_2)}{\partial \beta^j} \frac{\beta^j}{\overline{D}(N_2)} \\ \vdots & \vdots \\ \frac{\partial \overline{D}(N_i)}{\partial \rho^j} \frac{\rho^j}{\overline{D}(N_i)} & \frac{\partial \overline{D}(N_i)}{\partial \beta^j} \frac{\beta^j}{\overline{D}(N_i)} \end{bmatrix} \begin{bmatrix} \frac{\rho^{j+1} + \rho^j}{\rho^j} \\ \frac{\beta^{j+1} + \beta^j}{\beta^j} \end{bmatrix} = \begin{bmatrix} \frac{D(N_1) - \overline{D}(N_1)}{\overline{D}(N_1)} \\ \frac{D(N_2) - \overline{D}(N_2)}{\overline{D}(N_2)} \\ \vdots \\ \frac{D(N_i) - \overline{D}(N_i)}{\overline{D}(N_i)} \end{bmatrix} \qquad \text{(L-5)}$$

The percent crack length at each of the pavement ages was used to develop the model parameters ρ and β in the reflection cracking amount and severity model along with the system identification process. Table L-2 presents an example of the predicted percent of reflective cracking development of all severity level of four LTPP sections, which were calculated based on reflection crack data in Table L-1. Table L-3 shows the developed model parameters, and Figure L-3 to L-6 present the plots of the calibrated model corresponding to the measured data for the LTPP sections. The results presented good data fitting along with satisfying the convergence criterion. The calibrated parameters for whole asphalt overlay test sections in the LTPP, New York City and Texas data bases are listed in Appendix M.

Table L-1. Collected reflection crack information of LTPP test sections.

Section No.	Survey Date	Days after Overlay	Observed Crack Length (meter)		
			L + M + H	M + H	H
340503	7/27/1992	0	88.20*	-	-
	11/9/1995	1,200	0.00	0.00	0.00
	8/27/1996	1,492	0.00	0.00	0.00
	10/27/1998	2,283	8.70	0.00	0.00
	10/19/1999	2,640	7.30	0.00	0.00
	10/17/2000	3,004	18.00	0.70	0.00
	10/15/2001	3,367	24.80	4.20	0.00
	11/9/2002	3,757	26.10	0.00	0.00
	11/8/2003	4,121	40.50	7.90	0.00
	3/13/2004	4,247	49.20	11.90	0.00
270506	9/15/1990	0	177.70*	-	-
	11/6/1990	52	0.00	0.00	0.00
	6/17/1992	641	51.10	3.60	0.00
	9/29/1993	1,110	54.80	40.20	0.00
	8/23/1995	1,803	82.80	46.80	10.80
	10/23/1997	2,595	114.70	95.90	37.00
	6/3/1999	3,183	119.10	111.00	29.60
	7/24/2000	3,600	118.10	114.50	84.10
	8/20/2001	3,992	117.40	113.80	109.80
240563	6/10/1992	0	40.30*	-	-
	10/19/1995	1,226	8.30	0.00	0.00
	5/14/1997	1,799	26.50	0.00	0.00
	7/14/1999	2,590	28.40	0.00	0.00
	8/17/2000	2,990	26.10	0.00	0.00
	9/6/2001	3,375	33.00	16.60	0.00
	11/7/2002	3,802	35.70	22.60	0.00
	6/5/2003	4,012	37.10	25.90	0.00
	6/22/2004	4,395	37.60	22.20	0.00
55B901	7/1/1992	0	125.40*	-	-
	10/23/1992	114	4.90	0.00	0.00
	6/24/1993	358	46.80	14.40	0.00
	11/17/1994	869	94.60	18.00	0.00
	5/5/1999	2,499	117.70	3.70	0.00
	10/17/2002	3,760	119.30	61.70	0.00
	7/28/2004	4,410	124.30	115.90	28.80

***** Observed transverse crack length before asphalt overlay obtained from LTPP database

Table L-2. Predicted reflective cracking development of L+M+H severity for LTPP test sections.

Section No.	Overlay Type	Number of Days after Overlay	% Crack Length
340503	AC/AC OL	0	0
		1,200	0
		1,492	0
		2,283	9.86
		2,640	8.28
		3,004	20.41
		3,367	28.12
		3,757	29.59
		4,121	45.92
		4,247	55.78
270506	AC/Mill/AC OL	0	0
		52	0
		641	28.76
		1,110	30.84
		1,803	46.60
		2,595	64.55
		3,183	67.02
		3,600	66.46
		3,992	66.07
240563	AC/FC/AC OL	0	0
		1226	20.60
		1799	65.76
		2590	70.47
		2990	64.76
		3375	81.89
		3802	88.59
		4012	92.06
		4395	93.30
55B901	JRC/AC OL	0	0
		114	3.91
		358	37.32
		869	75.44
		2,499	93.86
		3,760	95.14
		4,410	99.12

Table L-3. Calibrated model parameters of LTPP sections.

LTPP Section No.	Overlay Type	Model Parameters (L+M+H)	
		β	ρ
340503	AC/AC	2.365	3,617.12
270506	AC/Mill/AC	0.702	1,004.85
240563	AC/FC/AC	2.276	1461.25
55B901	JRC/AC	1.159	329.42

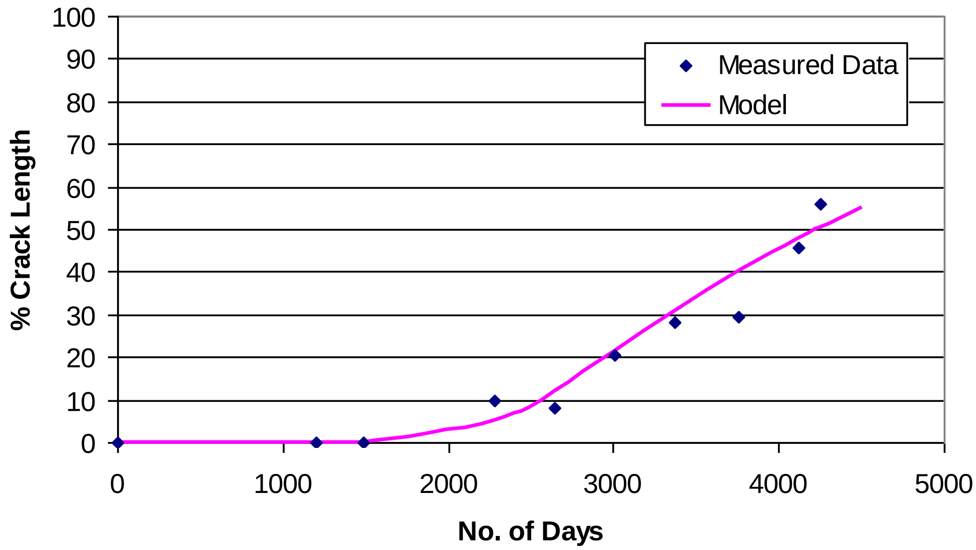


Figure L-3 Calibrated model on measured reflective crack for LTPP section 340503.

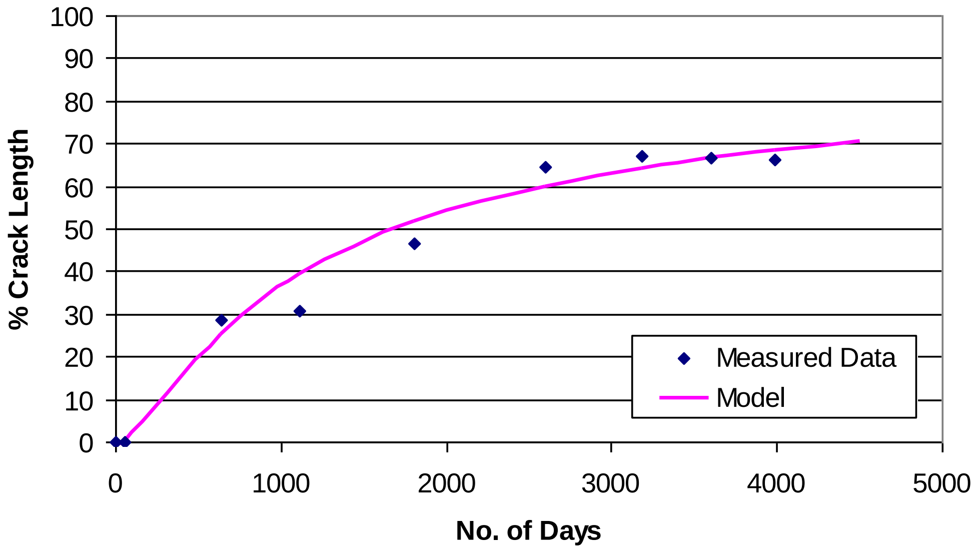


Figure L-4. Calibrated model on measured reflective crack for LTPP section 270506.

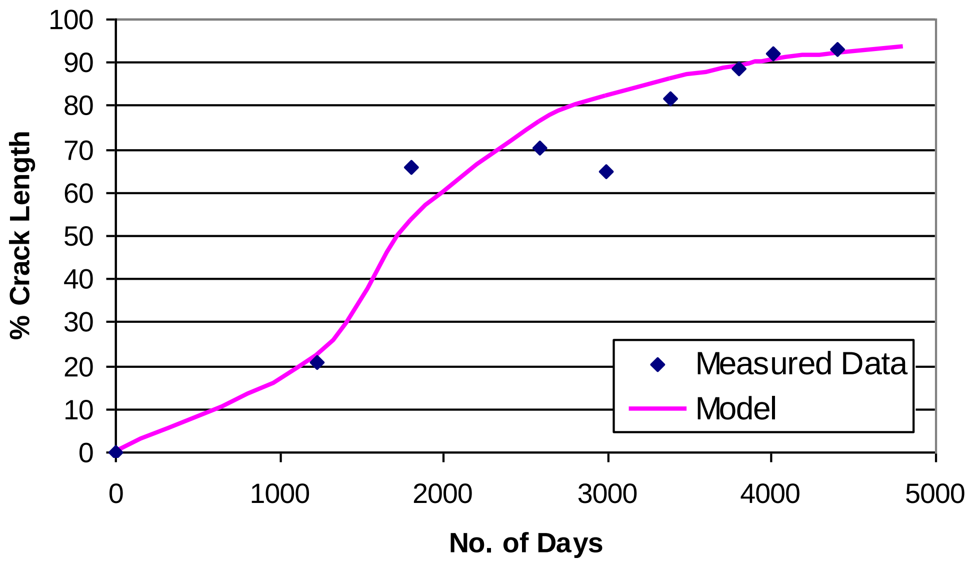


Figure L-5. Calibrated model on measured reflective crack for LTPP section 240563.

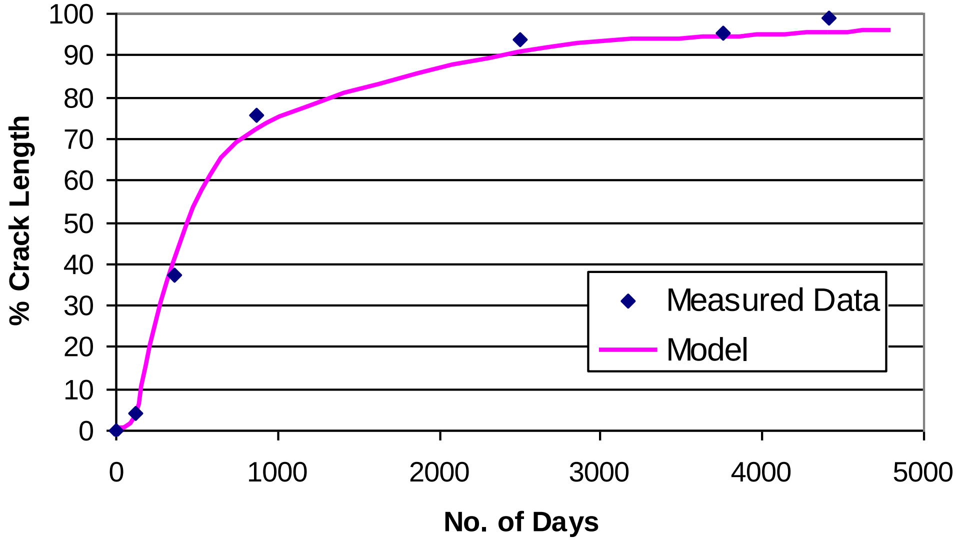


Figure L-6. Calibrated model on measured reflective crack for LTPP section 55B901.