Appendix C

Strain Limits for Tension-Controlled/Compression-Controlled and Strains to Allow Negative Moment Redistribution

C.1 Introduction

As shown in Figure C1, the strength reduction factor for flexural members varies depending on the expected behavior. The strain in the extreme tension reinforcement (ε_t) controls whether the behavior will be tension-controlled, compression-controlled, or in transition zone.



Figure C1 Strength Reduction Factors for Flexural Members

The current steel strain limits of 0.005 defining the lower bound of tension-controlled behavior and 0.002 or less defining compression-controlled behavior are based on having an adequate change in steel strain from service load to nominal strength. The current strain limit of 0.005 defining tension-controlled behavior is based on having an adequate change in steel strain from service load to nominal strength. This concept is shown in Figure C2 for Grade 60 reinforcement with and without a well-defined yield point. The additional strain beyond service load provides warning at high load levels, which is the basic characteristic of tension-controlled flexural members.





(b) No well-defined yield point



Nonetheless, the strain limits have been calibrated based on the expected performance of flexural members reinforced with Grade 60 longitudinal bars. Considering that A1035 bars could be subjected to larger service level strains and have different stress-strain relationships, the strain limits defining tension-controlled and compression-controlled behaviors need to be reevaluated.

C.2 Formulation

Proposed strain limits for A1035 reinforcing steel were developed as follows.

1. For a given value of f'_c , ρ , ρ' , and d'/d, calculate the tensile steel strain (ε_s) required to maintain the equilibrium of forces shown in C3.



Figure C3 Strains, Stresses, and Forces

Consistent with the basis of current definitions of tension-controlled and compressioncontrolled behaviors (Mast 1992), an elastic-perfectly plastic model is used for ASTM A615 bars, i.e.,

$$if \ \varepsilon_{s} \le \varepsilon_{y} \ f_{s} = E_{s}\varepsilon_{s} \left(\varepsilon_{s}' \le \varepsilon_{y} \ f_{s}' = E_{s}\varepsilon_{s}'\right)$$
$$if \ \varepsilon_{s} > \varepsilon_{y} \ f_{s} = f_{y} \left(\varepsilon_{s}' > \varepsilon_{y} \ f_{s}' = f_{y}\right)$$

Using basic principles, the following equations are obtained to maintain equilibrium:

If the top bars are compressive:
$$\gamma f'_c \beta_1 \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_s} = \rho f_s - \left(\frac{d'}{d}\right) \rho' \left(f'_s - \gamma f'_c\right)$$
 (C-1)

If the top bars are tensile:
$$\gamma f'_c \beta_1 \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_s} = \rho f_s + \left(\frac{d'}{d}\right) \rho' f'_s$$
 (C-2)

Note that
$$c = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_s} d$$
 and $\varepsilon_{s'} = \varepsilon_c \left(1 - \frac{d'}{d}\right) - \left(\frac{d'}{d}\right) \varepsilon_s$

According to NCHRP 12-64 (Rizkalla et al., 2007), use the following concrete parameters:

$$\beta_{1} = \begin{cases} 0.85 & \text{for } f'_{c} \leq 4ksi \\ 0.85 - 0.05(f'_{c} - 4) \geq 0.65 & \text{for } f'_{c} > 4ksi \end{cases}$$

$$\gamma = \begin{cases} 0.85 & \text{for } f'_{c} \leq 10ksi \\ 0.85 - 0.02(f'_{c} - 10) \geq 0.75 & \text{for } f'_{c} > 10ksi \end{cases}$$

$$\varepsilon = 0.003$$

Obtain the value of ε_s required to satisfy Eq. (C-1) or (C-2).

- 2. Repeat Step (1) for $f'_c = 4$ to 15 ksi (in 1 ksi increments), $\rho(0.1\%$ to 6% in 0.06% increments), ρ' (taken as 0, 0.5 ρ , and ρ), and d'/d = 0 or 0.1.
- 3. For each of the cases discussed in Step (2), obtain an implied curvature ductility with

reference to Figures C3 and C4 from
$$\mu_{\phi} = \frac{\phi_{design}}{\phi_{service}} \frac{\frac{\varepsilon_c}{c}}{\frac{\varepsilon_s}{d-kd}} = \frac{\left(\frac{\varepsilon_c}{\varepsilon_c d}\right)}{\left(\frac{f_s}{\varepsilon_s}\right)}}{\frac{\left(\frac{f_s}{\varepsilon_s}\right)}{d(1-k)}} = \frac{\left(\varepsilon_c + \varepsilon_s\right)(1-k)}{\frac{f_s}{\varepsilon_s}}$$
 in which ε_s = strain computed in Step (2); and $k = \sqrt{\left(\rho + \rho'\right)^2 n^2 + 2\left(\rho + \rho'\frac{d'}{d}\right)n - \left(\rho + \rho'\right)n}$

ε

where f_s is service load stress, taken as $0.60f_y$ (to be consistent with the original development of strain limits for tension-controlled and compression-controlled responses), $n = E_s / E_c$, and $E_c = 310,000 K_1 (W_c)^{2.5} (f'_c)^{0.33}$ with $K_1 = 1, W_c = 0.15 \ kcf (f'_c = ksi \ units)$ according to NCHRP 12-64.



Figure C4 Strain Distribution under Service Loads

- 4. Establish the implied values of curvature ductility at $\varepsilon_s = 0.005$ and 0.002.
- 5. Repeat Steps (1) to (4) for ASTM A1035 bars with the following modifications: Use Mast equation (Mast, 2006) to model steel stress-strain relationship, i.e.,

$$if \ \varepsilon_{s} \le 0.00241 \ f_{s} = E_{s}\varepsilon_{s} \left(\varepsilon_{s'} \le 0.00241 \ f_{s'} = E_{s}\varepsilon_{s'}\right)$$
$$if \ \varepsilon_{s} > 0.00241 \ f_{s} = 170 - \frac{0.43}{\varepsilon_{s} + 0.00188} \left(\varepsilon_{s'} > 0.00241 \ f_{s'} = 170 - \frac{0.43}{\varepsilon_{s'} + 0.00188}\right)$$

In addition, assume $f_y = 100$ ksi; hence, $f_s = 60$ ksi (instead of 36 ksi used in Step 3).

As evident from Figure C5, Mast equation provides a lower-bound estimate of the measured stress-strain diagrams of all A1035 bars tested as part of this part.

6. Using the curvature ductility-strain relationships obtained in Step 5 and Step 6, determine the steel strains corresponding to the implied curvature ductility calculated in Step 4.



Figure C5 Comparison of Measured and Calculated Stress-Strain Relationships

The relationship between the strain levels for A615 and A1035 reinforcing bars is illustrated in Figure C6 for one of the 144 cases considered. For this example, a singly reinforced member having $f_c = 4$ ksi, the strain in the A1035 bars needs to be 0.00793 in order to achieve the same implied ductility of the same tension-controlled member reinforced with A615 bars.



Figure C6 Example for $f'_c = 4 \text{ ksi}$, $\rho' = 0$, d'/d = 0, target $\varepsilon_t = 0.005$ in ASTM A615

The complete set of results is shown in Figure C7. Each data point in this figure was established based on the methodology illustrated in Figure C6. As expected, the addition of compression bars (i.e., $\rho' > 0$) increases the strain in the tension reinforcement, which improves the ductility. As the concrete compressive strength increases, the tension reinforcement strain drops, which is an indication of reduced ductility.



Figure C7 Equivalent Strains for Tension-Controlled and Compression-Controlled Members Reinforced with ASTM A1035

C.3 Recommendations

The original research (Mast, 1992) used as the basis of defining strains for tension and compression controlled members did not account for the benefits of compression reinforcement. A similar conservative approach was followed herein. Based on the results shown in Figure C7, the following strains are recommended to define tension-controlled and compression-controlled members that use ASTM A1035 with the service load stresses limited to 60 ksi or less.

Tension-Controlled:	$\varepsilon_t \ge 0.008$
Compression-Controlled:	$\varepsilon_t \leq 0.004$

C.4 Moment Redistribution

AASHTO Section 5.7.3.5 allows redistribution of negative moments at the internal supports of continuous reinforced concrete beams. Redistribution is allowed only when ε_t is equal to or greater than 0.0075. For such cases, negative moments calculated based on elastic theory may be increased or decreased by not more than $1000\varepsilon_t$ percent, with a maximum of 20 percent. Prior to introduction of unified design provisions, moment redistribution was allowed for members with reinforcement ratio ($\rho = A_s/bd$) limited to one half of the balanced reinforcement ratio. The current strain limit of 0.0075 is derived in Mast (1992), which is represented differently in the following.

1. Using the shown strain distribution and basic equations of equilibrium, the area of steel needed to produce balanced failure (A_{sb}) is



2. For tension-controlled beams, the area of steel can be related to tensile strain at the extreme tension reinforcement (ϵ_t) based on basic principles of equilibrium and strain distribution shown below.



3. Dividing Eq. (C-3) into Eq. (C-4) and simplifying, the following expression is obtained.

$$\varepsilon_{t} = \frac{0.003 + \varepsilon_{y}}{\frac{A_{s}}{A_{sb}}} - 0.003 \qquad (C-5)$$
4. If $\frac{A_{s}}{A_{sb}}$ is taken as ¹/₂, which is another method of indicating $\frac{\rho}{\rho_{b}} = 0.5$, Eq. (C-5) is simplified to $\varepsilon_{t} = 0.003 + 2\varepsilon_{y}$ (C-6)

Using Eq. (C-6), it is possible to compute the value of strain at the extreme tension reinforcement (ε_t) corresponding to cases for which the area of steel is one half of the steel area to produce balanced failure. In other words, this equation can be used to compute the value of ε_t for which moment redistribution is permitted. For Gr. 60 reinforcement, the yield strain (ε_y) is 0.0021; hence, ε_t from Eq. (C-6) becomes 0.0072. This strain is essentially the same as 0.0075, which is the strain beyond which moment redistribution is permitted.

In case of A1035 reinforcement, the yield strain is higher than that for Gr. 60 reinforcement – see Figure C5. As discussed previously, Mast's equation⁺ provides a very good lower-bound estimate of A1035 stress-strain relationship. Based on Mast's equation, the strain at 100 ksi is 0.0043. Substituting this strain as the yield strain (ε_y) in Eq. (C-6), the value of ε_t is 0.0115 or approximately 0.012. Therefore, 0.012 is proposed as the strain limit for which moment redistribution is allowed for members reinforced with A1035 reinforcement.

According to current AASHTO Specifications, strain in the extreme tension reinforcement (ϵ_t) has to exceed 0.0075 in order to be able to redistribute moments. This strain is 1.5 times the current strain limit of 0.005 that defines tension-controlled. The proposed strain limit of 0.012 is also 1.5 times the proposed tension-controlled strain limit of 0.008.

$$if \ \varepsilon_{s} \le 0.00241 \ f_{s} = E_{s}\varepsilon_{s} \left(\varepsilon_{s'} \le 0.00241 \ f_{s'} = E_{s}\varepsilon_{s'}\right)$$
$$^{+} if \ \varepsilon_{s} > 0.00241 \ f_{s} = 170 - \frac{0.43}{\varepsilon_{s} + 0.00188} \left(\varepsilon_{s'} > 0.00241 \ f_{s'} = 170 - \frac{0.43}{\varepsilon_{s'} + 0.00188}\right)$$