Appendix I

Crack Control

I.1 Introduction

Current design codes address flexural crack width by specifying the spacing of longitudinal reinforcement. The spacing requirements for A615 and A1035 longitudinal and their effect on crack widths are examined in this appendix.

I.2 Analytical Evaluation of Flexural Crack Width Requirements

The current provision for control of cracking by distribution of longitudinal reinforcement is based on a physical crack model proposed by Frosch (2001). Using classical theory, Frosch developed Eq. (I-1).

$$s = 2\sqrt{\left(\frac{w_c E_s}{2 f_s \beta}\right)^2 - d_c^{\,2}} \qquad \text{(I-1)}$$

in which s = maximum permissible bar spacing, w_c = limiting crack width, E_s = reinforcement modulus of elasticity, f_s = reinforcing bar stress, β = factor to account for amplification of strain calculated at the bar level to that at the surface due to strain gradient, and d_c = bottom cover measured from the center of the lowest bar.

The value of f_s can reasonably be approximated as $0.6f_y$ where f_y = reinforcement yield strength. The value of β is approximately $1.0+0.08d_c$ (Frosch, 2001). Hence, Eq. (I-1) can be written in the form of Eq. (I-2).

$$s \approx 2\sqrt{\left(\frac{w_c E_s}{2\left(0.6 f_y\right)(1.0 + 0.08 d_c)}\right)^2 - d_c^{\,2}} \qquad \text{(I-2)}$$

The value of permissible bar spacing varies as a function of acceptable crack width; stress in the reinforcing bars, which is indirectly related to the yield strength; and the cover. From this equation it can be understood that as the yield strength of the reinforcement increases, the allowable spacing of the reinforcement decreases for a given acceptable crack width and cover.

Crack width of members reinforced with A1035 has emerged as a critical issue. The higher strength of A1035 reinforcement allows a designer to use less steel; however, the crack width spacing provision requires the designer to put more steel than necessary to meet the requirement. Analytical studies reported in this appendix were conducted to examine crack width of members reinforced with high-strength bars.

I.2.1 Analytical Models

Two formulations were used to evaluate crack width of concrete members reinforced with A1035 and A615 longitudinal bars.

Model 1: Frosch's equation (Eq. I-3), which is the original equation that forms the basis of AASHTO §5.7.3.4, is used for this method.

$$s = 2\sqrt{\left(\frac{w_c E_s}{2 f_s \beta}\right)^2 - d_c^{\,2}} \approx 2\sqrt{\left(\frac{w_c E_s}{2\left(\alpha f_y\right)(1.0 + 0.08 d_c)}\right)^2 - d_c^{\,2}} \qquad \text{(I-3)}$$

in which f_s = steel stress under service load (ksi), which is taken as αf_y where α is a factor between 0 and 1 and f_y = yield strength (ksi).

Model 2: This method is based on Gergly-Lutz (1968) equation, which is shown in Eq. (I-4).

$$w_{\max} = 0.076 \times 10^{-6} \left(\sqrt[3]{t_b A} \, \frac{h_2}{h_1} f_s \right) \qquad \text{(I-4)}$$

in which w_{max} = maximum crack width (in inches) at the extreme tension fiber, t_b = distance from the extreme tension fiber to the center of the closest bar (in inches), A = average effective area of concrete in tension around each reinforcing bar (in^2), i.e., A_e/n where $A_e = 2Yb_w$ (see Figure I1) and n is the number of bars, h_2 = distance (in inches) from the extreme tension fiber to the neutral axis determined assuming a steel stress of f_s, h_1 = distance from the centroid of tension bars to the neutral axis (in inches) (note that for most beams h_2/h_1 is about 1.2), f_s = steel stress under service load, taken in this case as αf_y (in psi) α is a factor between 0 and 1.

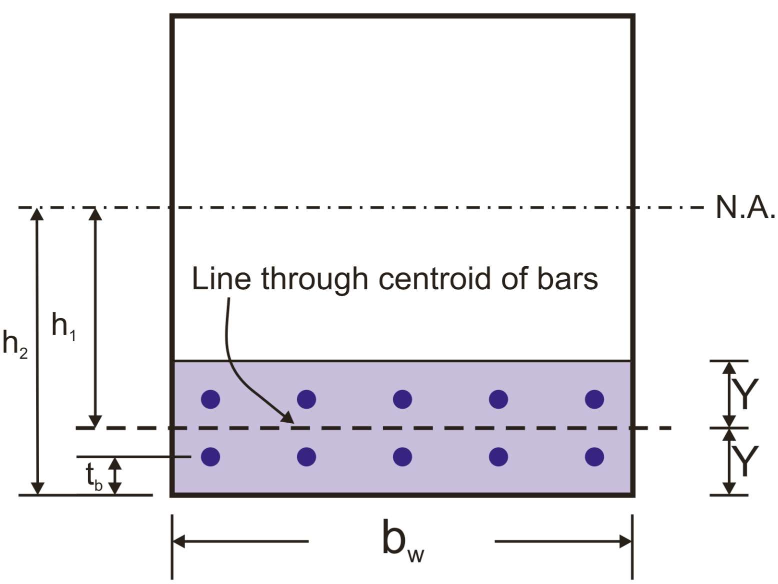

Figure I1 Terms in Gergly-Lutz's Equation

Improved crack control is obtained when the steel reinforcement is well distributed over the zone of maximum concrete tension. Several bars at moderate spacing are more effective in controlling cracking than one or two larger bars of equivalent area. However, only a limited number of bars can fit within a given beam width. Assuming #4 stirrups and 1.5 in. clear cover to the stirrups, the maximum number of bars that can be placed in a single layer was calculated according to AASHTO §5.10.3, and is summarized in Table I1.

Table I1 Maximum Numbers in a Single Layer According to AASHTO §5.10.3

Bar size	Beam Width (inches)											
	10	12	14	16	18	20	22	24	26	28	30	32
3	4	5	6	7	8	9	10	11	12	13	14	15
4	3	4	5	6	7	8	9	10	11	12	13	14
5	3	4	5	6	7	8	9	10	11	12	12	13
6	3	4	5	6	6	7	8	9	10	11	12	13
7	3	4	4	5	6	7	8	9	9	10	11	12
8	3	3	4	5	6	7	7	8	9	10	11	11
9	2	3	4	4	5	6	6	7	8	9	9	10
10	2	3	3	4	5	5	6	6	7	8	8	9
11	2	2	3	4	4	5	5	6	6	7	7	8

I2.2 Results and Discussions

The aforementioned models were used to evaluate whether it is possible to satisfy the current crack width requirements for Class 1 and Class 2 exposures (0.017 in. and 0.01275 in., respectively) if the maximum number of bars allowed per Table I1 is placed. Both A615 and A1035 bars with yield strengths of 60 ksi and 100 ksi, respectively, were considered. The value of α in Eq. (I-2) and Eq. (I-3) was taken as 0.60, h_2/h_1 in Eq. (I-4) was set equal to 1.2, and #4 stirrups with 1.5 in. clear cover were assumed in the calculations. The reported parametric studies considered cases with a single layer of longitudinal reinforcement.

The results are summarized in Tables I2 to I5. In these tables, "GL+F" indicates that the crack width computed from both models meets the requirements, "F" means that the crack width from only Model 1 (which is based on the Frosch (2001) model) meets the requirements, "GL" implies that the crack width from only Model 2 (which is based on the Gergely & Lutz (1968) model) meets the requirements, and "X" means that the crack width from neither of the two models meets the requirements.

With the exception of one case (#11 bars in a 12 in. wide beam, see Table I4), A615 bars meet Class 1 and Class 2 requirements. In case of A1035 bars, Class 1 requirement is generally met with the exception of using large bars (#9, #10, #11) particularly in relatively narrow beams (10 in., 12 in., or 14 in.), which is impractical (refer to Table I3). A large number of cases using A1035 bars do not meet Class 2 requirements as evident from Table I5.

Table I2 Results for Flexural Crack Width Analysis: A615 – Class 1; f_s=0.60f_y

Bar	b_w (in.)											
	10	12	14	16	18	20	22	24	26	28	30	32
#3	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#4	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#5	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#6	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#7	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#8	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#9	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#10	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#11	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F

Table I3 Results for Flexural Crack Width Analysis: A1035 – Class 1; f_s=0.60f_y

Bar	b_w (in.)											
	10	12	14	16	18	20	22	24	26	28	30	32
#3	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#4	F	F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#5	F	F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#6	F	F	F	F	F	F	F	F	GL+F	GL+F	GL+F	GL+F
#7	F	F	F	F	F	F	F	F	F	F	F	F
#8	F	F	F	F	F	F	F	F	F	F	F	F
#9	X	F	F	F	F	F	F	F	F	F	F	F
#10	X	F	X	F	F	F	F	F	F	F	F	F
#11	X	X	X	F*	X*	F	X	F	X	F	X	F

- Note: As indicated in Table I1, four #11 bars fit in 16 in. and 18 in. wide beams. The spacing between the bars in an 18-inch wide is larger than that for a 16-inch wide beam. Therefore, crack width requirement is satisfied for a 16-inch wide and not for an 18-inch wide. A similar argument can be made for 24 in. and 26 in. wide beams; or 28 in. and 30 in. wide beams.

Table I4 Results for Flexural Crack Width Analysis: A615 – Class 2; f_s=0.60f_y

Bar	b_w (in.)											
	10	12	14	16	18	20	22	24	26	28	30	32
#3	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#4	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#5	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#6	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#7	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#8	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#9	F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#10	F	GL+F	F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#11	F	X	F	GL+F	F	GL+F	F	GL+F	F	GL+F	F	GL+F

Table I5 Results for Flexural Crack Width Analysis: A1035 – Class 2; f_s=0.60f_y

Bar	b_w (in.)											
	10	12	14	16	18	20	22	24	26	28	30	32
#3	F	F	F	F	F	F	F	F	F	F	F	F
#4	X	F	F	F	F	F	F	F	F	F	F	F
#5	X	X	F	F	F	F	F	F	F	F	F	F
#6	X	X	X	X	X	X	X	X	X	X	X	X
#7	X	X	X	X	X	X	X	X	X	X	X	X
#8	X	X	X	X	X	X	X	X	X	X	X	X
#9	X	X	X	X	X	X	X	X	X	X	X	X
#10	X	X	X	X	X	X	X	X	X	X	X	X
#11	X	X	X	X	X	X	X	X	X	X	X	X

If the steel stress under service load is limited to 0.5f_y (= 50 ksi), i.e., by setting α = 0.5 in Eq. (I-3) or Eq. (I-4), a large number of cases employing A1035 bars will meet Class 1 requirement. Table I6 suggests only one case (#11 bars in a 12 in. wide beam) will not meet Class 1 requirement from either Froch's equation or Gergely-Lutz equation. In comparison to Table I5, Table I7 shows a remarkable improvement in the number of cases that will meet Class 2 requirement. The cases for which A1035 bars do not meet Class 2 requirement involve #9, #10, and #11 bars. Note that a larger number of cases are satisfied if Frosch's equation is used.

Table I6 Results for Flexural Crack Width Analysis: A1035 – Class 1; f_s=0.50f_y

Bar	b_w (in.)											
	10	12	14	16	18	20	22	24	26	28	30	32
#3	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#4	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#5	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#6	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#7	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#8	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#9	F	F	GL+F	F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#10	F	F	F	F	GL+F	F	GL+F	F	GL+F	GL+F	F	GL+F
#11	F	X	F	F	F	F	F	F	F	F	F	F

Table I7 Results for Flexural Crack Width Analysis: A1035 – Class 2; f_s=0.50f_y

Bar	b_w (in.)											
	10	12	14	16	18	20	22	24	26	28	30	32
#3	F	F	F	F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#4	F	F	F	F	F	F	F	F	F	F	F	F
#5	F	F	F	F	F	F	F	F	F	F	F	F
#6	F	F	F	F	F	F	F	F	F	F	F	F
#7	F	F	F	F	F	F	F	F	F	F	F	F
#8	F	F	F	F	F	F	F	F	F	F	F	F
#9	X	X	F	X	F	F	X	F	F	F	F	F
#10	X	X	X	X	X	X	X	X	X	X	X	X
#11	X	X	X	X	X	X	X	X	X	X	X	X

Further improvement is possible if a slightly larger crack width is accepted for A1035 bars. For example, by using a crack width of 0.014 in. (which is approximately 0.001 in. larger than the crack width of 0.01275 in. for Class 2 exposure) and limiting the steel stress to 0.5f_y (i.e., 50 ksi), a relatively large number of cases will be satisfied, see Table I8.

Table I8 Results for Flexural Crack Width Analysis: A1035 – Class 3 (w_c = 0.014 in); f_s=0.50f_y

Bar	b_w (in.)											
	10	12	14	16	18	20	22	24	26	28	30	32
#3	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#4	F	F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#5	F	F	F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F	GL+F
#6	F	F	F	F	F	F	F	F	F	GL+F	GL+F	GL+F
#7	F	F	F	F	F	F	F	F	F	F	F	F
#8	F	F	F	F	F	F	F	F	F	F	F	F
#9	X	F	F	F	F	F	F	F	F	F	F	F
#10	X	F	X	F	F	F	F	F	F	F	F	F
#11	X	X	X	F*	X*	F	X	F	X	F	X	F

* Note: As indicated in Table I1, four #11 bars fit in 16 in. and 18 in. wide beams. The spacing between the bars in an 18-inch wide is larger than that for a 16-inch wide beam. Therefore, crack width requirement is satisfied for a 16-inch wide and not for an 18-inch wide. A similar argument can be made for 20 in. and 22 in. wide beams; 24 in. and 26 in. wide beams; or 28 in. and 30 in. wide beams.

I.3 Recommendations

AASHTO §C5.7.3.4 states that

The crack width is directly proportional to the γ_e factor, therefore, if the individual Authority with jurisdiction desires and alternate crack width, the γ_e factor can be adjusted directly. For example a γ_e factor of 0.5 will result in an approximate crack width of 0.0085.

Two alternatives for revising the current provisions are proposed herein. One alternative is based on allowing a larger crack width, as suggested by AASHTO §C5.7.3.4 for A1035 reinforcement.

Alternative #1

The results and discussions in Section 5.2.2 suggest that for A1035 bars, the stress under service load may be limited to 0.5f_y (i.e., 50 ksi) in order to meet the current Class 1 and Class 2 exposure crack width requirements for a relatively large number of cases. The use of a new class with an implied crack width of 0.014 in. further increases the number of cases that can meet crack width requirements. The use of a crack width of 0.014 in. appears to be a reasonable substitute for the current Class 2 exposure (which has an implied crack width of 0.01275 in.), is consistent with the commentary for §5.7.3.4, and is apparently logical in view of improved corrosion resistance of A1035 reinforcing bars.

Therefore, Eq. 5.7.3.4-1 in AASHTO §5.7.3.4 could be revised as follows (the revisions are **bold** and highlighted).

The spacing s of mild steel reinforcement in the layer closest to the tension face shall satisfy the following:

$$s \leq \frac{700\gamma_e}{\left(1+\frac{d_c}{0.7(h-d_c)}\right)f_{ss}} \qquad (5.7.3.4\text{-}1)$$

where:

γ_e = exposure factor

= 1.00 for Class 1 exposure condition

= 0.75 for Class 2 exposure condition

= **0.82 for Class 3 exposure condition**

d_c = thickness of concrete cover measured from extreme tension fiber to center of the flexural reinforcement located closest thereto (in.)

f_{ss} = tensile stress in steel reinforcement at the service limit state (ksi) **not to exceed 50 ksi**

h = overall thickness or depth of the component (in.)

Class 3 exposure condition would include decks and substructures reinforced with A1035 bars.

Alternative #2

Accepting the superior corrosion resistance of A1035 bars and considering the results discussed in Section 5.2.2, an alternative is to simply remove the Class 2 restriction when dealing with A1035 bars and apply an upper limit of 60 ksi to the value of f_{ss}. Thus, Eq. 5.7.3.4-1 in AASHTO §5.7.3.4 could be revised as follows (the revisions are **bold** and highlighted).

The spacing s of mild steel reinforcement in the layer closest to the tension face shall satisfy the following:

$$s \leq \frac{700\gamma_e}{\left(1+\frac{d_c}{0.7(h-d_c)}\right)f_{ss}} \qquad (5.7.3.4\text{-}1)$$

where:

γ_e = exposure factor

= 1.00 for Class 1 exposure condition

= 0.75 for Class 2 exposure condition**; except A1035 reinforcing bars, which only need satisfy Class 1 exposure requirements.**

d_c = thickness of concrete cover measured from extreme tension fiber to center of the flexural reinforcement located closest thereto (in.)

f_{ss} = tensile stress in steel reinforcement at the service limit state (ksi) **not to exceed 60 ksi**

h = overall thickness or depth of the component (in.)