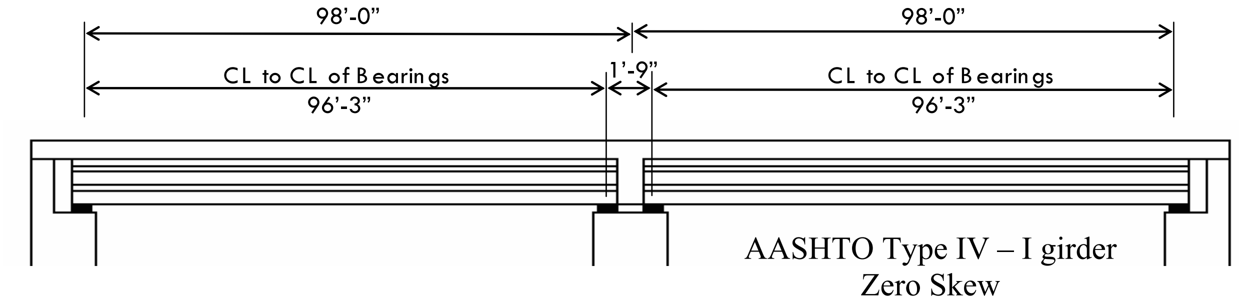
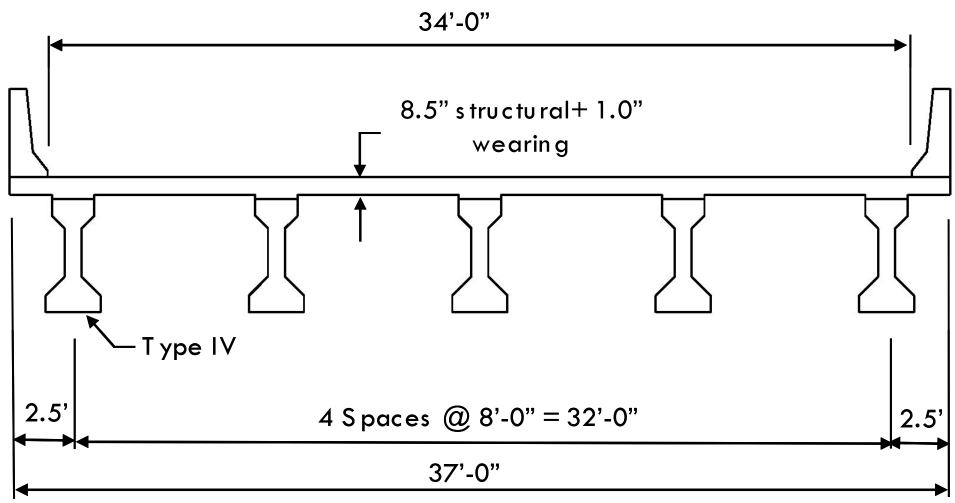
Appendix K
Design Examples

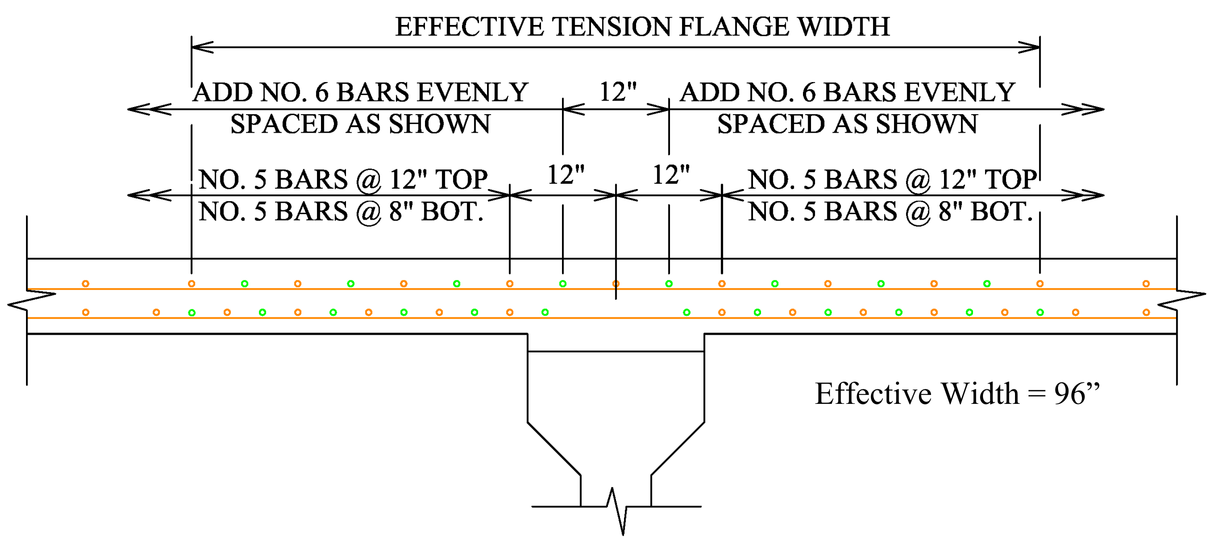
Example 1[*] Two-Span I-Girder Bridge Continuous for Live Loads





(a) Bridge Deck

The bridge deck reinforcement using A615 rebars is shown below.



[*] Based on an example prepared as part of "Training Classes on AASHTO LRFD Bridge Specifications," ODOT, R.A. Miller

Redesign by using A1035 bars with f_y = 100 ksi.

$$3{,}483(12) = 0.90A_s(100)\left(58.25 - \frac{A_s(100)}{1.7(7.0)(26)}\right); \quad A_s = 8.36in^2$$

Provide 28 # 5 bars (14 on the top with 2.5" cover and 14 on the bottom with 2 5/8" cover). The bars will be at 8" o.c. Note: A1035 bars are not epoxy coated; hence, the cover is 2.5".

The centroid of the bars from the top is

$$\bar{x} = \frac{14\times(2.5+0.5\times5/8)+14(8.5-(2+5/8+0.5\times5/8))}{28} = 4.12"$$

Crack control reinforcement:

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

γ_e = 0.75

f_s =Tensile stress in steel reinforcement at the service limit state.

$$d = 62.5 - 4.12 = 58.4"$$

$$\rho = \frac{A_s}{bd} = \frac{28\times0.31}{(26)(58.4)} = 0.0057$$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n = \sqrt{2(0.0057)(5.718) + (0.0057\times5.718)^2} - 0.0057\times5.718 = 0.225$$

$$j = 1 - k/3 = 1 - 0.225/3 = 0.925$$

$$f_s = \frac{M_{sl}}{A_s jd} = \frac{2{,}141(12)}{28\times0.31(0.925\times58.4)} = 56.6ksi \le 60ksi$$

$$\therefore f_s = 56.6ksi$$

$$d_c = 2.5 + 0.5\times5/8 = 2.81$$

$$\beta_s = 1 + \frac{d_c}{0.7(h-d_c)} = 1 + \frac{2.81}{0.7(62.5-2.81)} = 1.067$$

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c = \frac{700\times0.75}{1.067\times56.6} - 2\times2.81 = 3.07"$$

This spacing is too small. Aim for 6" spacing, which is more realistic.

Try: 17 top bars: # 5 @ 12" alternating with # 6 @ 12"
17 bottom bars: # 5 @ 12" alternating with # 6 @ 12"

Distances to centroid of bars:

Top Bars: $\dfrac{(2.5+0.5\times 5/8)+(2.5+0.5\times 3/4)}{2}=2.84"$

Bottom Bars: $\dfrac{(8.5-(2+5/8+0.5\times 5/8))+(8.5-(2+5/8+0.5\times 3/4))}{2}=5.53"$

$$\bar{x}=\frac{17\times 2.84+17\times 5.53}{34}=4.19"$$

$$d=62.5-4.19=58.31"$$

$$\rho=\frac{A_s}{bd}=\frac{17\times 0.31+17\times 0.44}{(26)(58.31)}=\frac{12.75}{(26)(58.31)}=0.0084$$

$$k=\sqrt{2\rho n+(\rho n)^2}-\rho n=\sqrt{2(0.0084)(5.718)+(0.0084\times 5.718)^2}-0.0084\times 5.718=0.266$$

$$j=1-k/3=1-0.266/3=0.911$$

$$f_s=\frac{M_{sl}}{A_s j\, d}=\frac{2{,}141(12)}{12.75(0.911\times 58.3)}=37.9ksi$$

$$d_c=2.84375$$

$$\beta_s=1+\frac{d_c}{0.7(h-d_c)}=1+\frac{2.84375}{0.7(62.5-2.84375)}=1.068$$

$$s\le\frac{700\gamma_e}{\beta_s f_s}-2d_c=\frac{700\times 0.75}{1.068\times 37.9}-2\times 2.84375=7.28">6\text{” O.K.}$$

(b) <u>Shear Reinforcement</u>

If prestressing steel is ignored, #4 A615 stirrups @ 4” o.c. will be needed.

Redesign by using A1035 U shaped # 4 stirrups.

$$s=\frac{A_v f_y d_v \cot\theta}{V_s}=\frac{0.4(100)(55.63)\cot 45}{284.6}=7.8"$$ Controls, say 7”

§5.8.2.5 $A_v\ge 0.0316\sqrt{f'_c}\,\dfrac{b_v s}{f_y};\quad 0.4\ge 0.0316\sqrt{7}\,\dfrac{8s}{124};\quad s\le 74"$

§5.8.2.9 $v_u=\dfrac{|V_u-\phi V_p|}{\phi b_v d_v}=\dfrac{323.1}{0.9\times 8\times 55.63}=0.81ksi$

Note that s < 74”; hence, the simplified procedure with β = 2 and θ = 45° may be used.

$$v_u=0.81ksi<0.125f'_c=0.125\times 8=1ksi$$

§5.8.2.7 $\therefore s_{\max}\le smaller\ of\begin{cases}0.8d_v=0.8\times 55.63=44.5"\\ 24"\quad Controls\end{cases}$

Provide A1035 U shaped #4 stirrups @ 7” o.c.

Interface Shear Reinforcement

Factored horizontal shear, V_u = 323 kips

$$V_n = cA_{cv} + \mu[A_{vf} f_y + P_c]; \quad c = 0.28; \quad \mu = 1.0$$

Although shear reinforcement spacing was previously calculated to be 7" o.c., calculate the spacing of A1035 interface shear reinforcement with f_y limited to 60 ksi.

$$V_u = \phi V_{ni} = \phi\left[cA_{cv} + \mu\left(A_{vf} f_y + P_c\right)\right]; \quad 323 = 0.9\left[0.28(20 \times s) + 1.0\left(0.4 \times 100 + 0\right)\right]; \quad s = 57"$$

Say 55"

5.8.4.1-2 & 5.8.4.1-3 $V_n \leq \max of \begin{cases} 0.2 f'_c A_{cv} = 0.2 \times 7(52 \times 20) = 1456 kips \\ 0.8 A_{cv} = 0.8 \times 7(52 \times 20) = 5824 kips \end{cases}$

Hence, V_n = 323/0.9 = 359 kips as used is o.k.

$$A_{vf} \geq \frac{0.05 b_v}{f_y} = \frac{0.05 \times 20}{60} = 0.017 \ in^2 / length$$

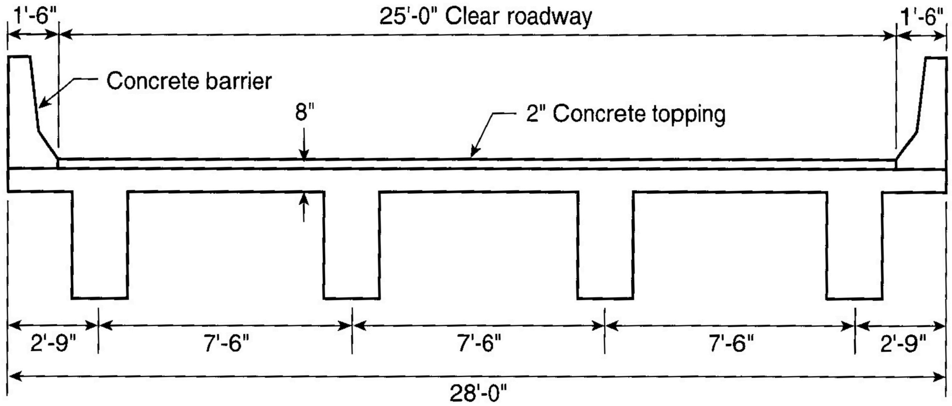
5.8.4.10-4 $\quad Actual\ A_{vf} = 0.40 / 57 = 0.0070\ in^2 / length \quad \therefore N.G., Redcue\ the\ spacing$

$$0.40 / s = 0.017; \quad s = 23.5" \quad say\, 23"$$

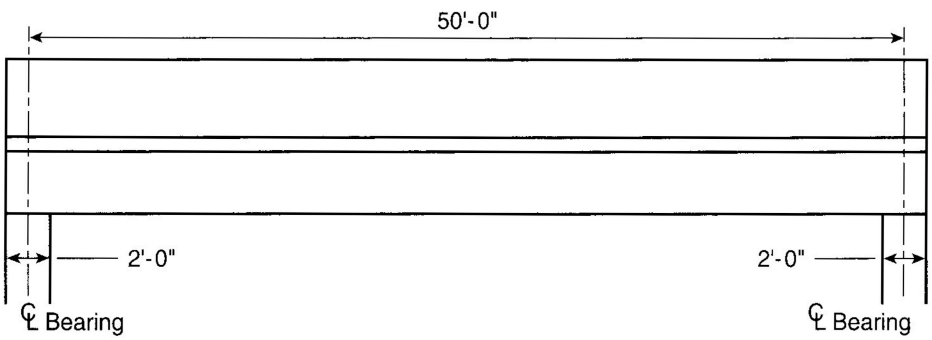
Use #4 U shaped A1035 interface reinforcement at s = 23".

From a durability point of view corrosion resistant A1035 interface shear reinforcement provides advantages. However, practical issues may arise from a placement point of view as the spacing of girder shear reinforcement and that for interface reinforcement are significantly different.

Example 2* Simple Span T-Beam

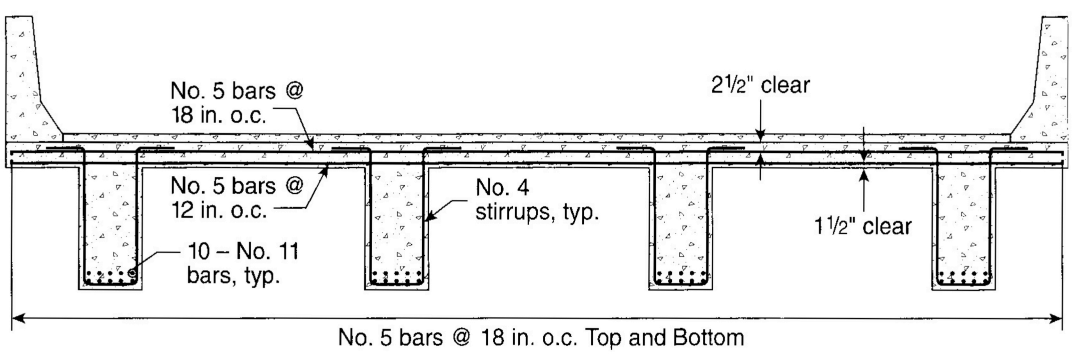


Cross Section



Elevation

The reinforcement using A615 rebars is shown below.



The stirrups are symmetrically spaced at 10" o.c. up to 5' from the bearing center line and then @ 16" o.c. up to 25'.

* Based on an example from "LRFD Design of Cast-in-Place Concrete Bridges," 1st Ed., Schneider, E.F. and Bhide, S.B. Portland Cement Association, Skokie, IL, 2006, 156 pages.

Redesign by using A1035 bars with a specified yield strength of 100 ksi.

1. Girder – Flexure

Capacity

M_u = 2052 k-ft.

With 10 # 8 bars in 2 layers, which fit within 22” , ϕM_n = 2204 k-ft. O.K.

Maximum Reinforcement Requirement (§5.7.2.1)

$$\frac{c}{d_t} = \frac{3.04}{39.5} = 0.077 < \frac{3}{8} \quad \therefore Tension - controlled, O.K.$$

Minimum Reinforcement Requirement (§5.7.3.3.2)

$1.2M_{cr}$ = 1.2 (4691/12) = 469 k-ft Controls

$1.33M_u$ = 1.33 (2052) = 2729 k-ft

ϕM_n = 2204 k-ft > $1.2M_{cr}$ Hence, minimum reinforcement requirements are met.

Maximum Spacing of Tension Reinforcement (§5.7.3.4)

I_g = 262874 in.^{4}

$$f_c = \frac{My}{I} = \frac{(1336 \times 12)(42 - 15.42)}{262874} = 1.62ksi$$

§5.4.2.6: $f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{4} = 0.48ksi$

f_c>80% of f_r; hence, §5.7.3.4 needs to be checked.

$M_{service}$ = 1336 k-ft. > M_{cr} = 391 k-ft. Use cracked transformed section properties

I_{cr} = 63200 in.^{4}

y^- = 6.21 in. (Measured from compression face)

$$d_c = 1.5 + 0.5 + 1/2(1) = 2.5"$$

$$f_s = n\frac{My}{I} = \frac{29000}{57\sqrt{4000}} \times \frac{(1336 \times 12)(42 - 2.5 - 6.21)}{63200} = 67.9ksi$$ (Approximately $0.6f_y = 0.6 \times 100$ = 60 ksi)

γ_e = 0.75

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{2.5}{0.7(42 - 2.5)} = 1.09$$

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c = \frac{700 \times 0.75}{1.09 \times 67.9} - 2 \times 2.5 = 2.09"$$

The actual spacing of 5 # 8 bars, in each layer, is $1+\dfrac{22-2(1.5+0.5)-5\times 1}{4} = 4.25" > 2.09"$,

which is not acceptable.

Revise the design by using 12 # 8 bars in 2 layers.

<u>Capacity</u>

$M_u = 2052$ k-ft.

$\phi M_n = 2672$ k-ft. O.K.

<u>Maximum Reinforcement Requirement (§5.7.2.1)</u>

$$\frac{c}{d_t} = \frac{3.64}{39.5} = 0.109 < \frac{3}{8} \quad \therefore Tension-controlled, O.K.$$

<u>Minimum Reinforcement Requirement (§5.7.3.3.2)</u>

$1.2M_{cr} = 1.2\ (4803/12) = 480$ k-ft Controls

$1.33M_u = 1.33\ (2052) = 2729$ k-ft

$\phi M_n = 2672$ k-ft $> 1.2M_{cr}$ Hence, minimum reinforcement requirements are met.

<u>Maximum Spacing of Tension Reinforcement (§5.7.3.4)</u>

$I_g = 267731$ in.^{4}

$$f_c = \frac{My}{I} = \frac{(1336\times 12)(42-15.56)}{267731} = 1.58 ksi$$

§5.4.2.6: $f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{4} = 0.48 ksi$

$f_c > 80\%$ of f_r; hence, §5.7.3.4 needs to be checked.

$M_{service} = 1336$ k-ft. $> M_{cr} = 400$ k-ft. Use cracked transformed section properties

$I_{cr} = 74218$ in.^{4}

$y^- = 6.74$ in. (Measured from compression face)

$d_c = 1.5 + 0.5 + 1/2(1) = 2.5"$

$f_s = n\dfrac{My}{I} = \dfrac{29000}{57\sqrt{4000}} \times \dfrac{(1336\times 12)(42-2.5-6.74)}{74218} = 56.9 ksi$ (Approximately $0.6f_y = 0.6\times 100$ = 60 ksi)

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

$\gamma_e = 0.75$

$$\beta_s = 1 + \frac{d_c}{0.7(h-d_c)} = 1 + \frac{2.5}{0.7(42-2.5)} = 1.09$$

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c = \frac{700\times 0.75}{1.09\times 56.9} - 2\times 2.5 = 3.46"$$

The actual spacing of 12 # 8 bars, in each layer, is $1+\dfrac{22-2(1.5+0.5)-6\times 1}{5} = 3.4" < 3.46$”, which is acceptable.

Skin Reinforcement (§5.7.3.4)

d_e = 39.6” > 36”; skin reinforcement needs to be provided. However, for consistency with the original example, skin reinforcement is not provided in the redesign with A1035 reinforcing bars.

Fatigue Limit State

M_f = 278 k-ft

Cracked transformed section properties: I_{cr} =74218 in.^{4} and y^- = 6.74 in.

$$f_s = n\frac{My}{I_{cr}} = 8\frac{(278\times 12)(42-2.5-6.74)}{74218} = 11.8ksi$$

$$f_f = 21 - 0.33 f_{\min} + 8(r/h)$$

f_{min} = stress under dead load moment, which is 597 k-ft. > M_{cr} = 400 k-ft.

$$f_{\min} = n\frac{My}{I_{cr}} = 8\frac{(597\times 12)(42-2.5-6.74)}{74218} = 25.3ksi$$

$$f_f = 21 - 0.33 f_{\min} + 8(r/h) = 21 - 0.33\times 25.3 + 8\times 0.3 = 15.1ksi$$

11.8 ksi < 15.1 ksi O.K.

Summary: From a strength point of view, 10 #8 A1035 bars (A_s = 7.9 in.^{2}) provide adequate flexural capacity. However, the requirements related to spacing of mild reinforcement (§5.7.3.4) result in additional area of steel. The use of A1035 bars as flexural reinforcement saves about 49% in terms of the amount of steel (10 #11 A615 (A_s = 18.72 in.^{2}) vs. 12 #8 A1035 (A_s = 9.48 in.^{2})).

Girder – Shear

h	42.0	in.
$d_s = d_e$	38.5	in.
a	3.09	in.
d_v=d_s-a/2	37.0	in.
0.72h	30.2	in.
0.9d_e	34.7	in.
Final d_v	37.0	in.

Assume 1'-4" wide support.

The critical section is at x = 37 + 8 =45"=3.75'

Distance (ft.) x	Point along span x/L	V_u kips
0	0	189
0.67	0.0134	185
3.75	0.07	168
5	0.1	160
10	0.2	132
15	0.3	103
20	0.4	75
25	0.5	46

Use U shaped # 4 A1035 stirrups

$$s_{max} = \frac{A_v f_y}{0.0316\sqrt{f'_c}\, b_v}$$

A_v	0.4	in.^{2}
f'_c	4	ksi
f_y	100	ksi
b_v	22	in.
s_{max}	28.8	in.

The simplified procedure for the determination of β and θ may be used if the spacing of the stirrups does not exceed s_{max} = 28.8 in. For the simplified procedure β = 2.0 and θ = 45°.

Distance (ft.) x	v_u ksi	Is v_u <0.125f'c?	s_{max} (in.)
0	0.258	Yes	24
0.67	0.253	Yes	24
3.75	0.229	Yes	24
5	0.219	Yes	24
10	0.180	Yes	24
15	0.141	Yes	24
20	0.102	Yes	24
25	0.063	Yes	24

The maximum shear resistance, V_n, is given by the lesser of $V_n=V_c+V_s+V_p$ and $V_n=0.25f'_c b_v d_v$.

$V_n=0.25f'_c b_v d_v$ = 0.25(4)(22)(37) = 814 kips

V_u @ the critical section = 168 <ϕV_n = 0.9(814) = 733 kips

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v = 0.0316(2)\sqrt{4}(22)(37) = 103kips$$

$$V_s = \frac{A_v f_y d_v(\cot\theta + \cot\alpha)(\sin\alpha)}{s} = \frac{A_v f_y d_v(\cot 45 + \cot 90)(\sin 90)}{s} = \frac{A_v f_y d_v}{s}$$

At the critical section, V_u = 168 kips

$$V_u = \phi(V_c + V_p + V_s)$$
$$168 = 0.9\left(103 + 0 + \frac{A_v f_y d_v}{s}\right) = 0.9\left(103 + 0 + \frac{0.4\times100\times37}{s}\right); s = 17.69"$$

Stirrup layout (symmetrically placed) of U-shaped #4 A1035 stirrups:

- Start the first stirrup at 9" from the support.
- Provide 3 spaces @ 17" o.c.
- Provide 10 spaces @ 24" o.c.

Tensile Capacity of Longitudinal Reinforcement (§5.8.3.5)

The following equation needs to be satisfied.

$$A_{ps}f_{ps} + A_s f_y \geq \frac{|M_u|}{\phi d_v} + 0.5\frac{N_u}{\phi} + \left(\left|\frac{V_u}{\phi} - V_p\right| - 0.5V_s\right)\cot\theta \qquad 5.8.3.5\text{-}1$$

For this case, the above equation is simplified to $A_s f_y \geq \frac{M_u}{\phi d_v} + \left[\left(\frac{V_u}{\phi}\right) - 0.5V_s\right]\cot\theta$

(i) <u>Critical Section</u>

M_u = 633 k-ft & V_u = 168 kips

For # 4 stirrups @ 17" o.c., $V_s = \frac{A_v f_y d_v}{s} = \frac{0.4\times100\times37}{17} = 87.1kips$

V_s in Eq. 5.8.3.5-1 cannot be V_u/ϕ = 168/0.9 = 187 kips Hence, use V_s = 87.1 kips.

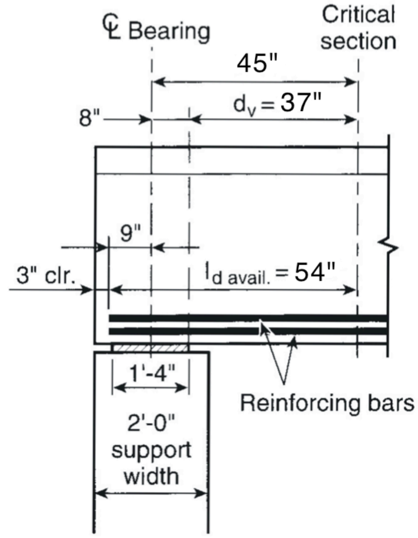
$$T = \frac{M_u}{\phi d_v} + \left[\left(\frac{V_u}{\phi}\right) - 0.5V_s\right]\cot\theta = \frac{633\times12}{0.9\times37} + \left[\left(\frac{168}{0.9}\right) - 0.5\times87.1\right]\cot(45) = 371kips$$

$$l_{db} = \max\left(\frac{1.25A_b f_y}{\sqrt{f'_c}}, 0.4d_b f_y\right) = \max\left(\frac{1.25\times0.60\times100}{\sqrt{4}}, 0.4\times0.875\times100\right) = 37.5"$$

No modification factors are necessary; hence, $l_d = l_{db} = 37.5" > 12" \therefore l_d = 38"$

As seen below, the available distance to develop the bar is 55", which is larger than $l_d = 38"$.

Therefore, $f_{sx} = f_y$



$$T_{provided} = A_s f_y = (16 \times 0.79)100 = 1264 kips > T = 371 kips \quad O.K.$$

(ii) <u>Midspan</u>

M_u = 2052 k-ft and V_u = 46 kips

For # 4 stirrups @ 24" o.c., $V_s = \dfrac{A_v f_y d_v}{s} = \dfrac{0.4 \times 100 \times 37}{24} = 61.7 kips$

V_s in Eq. 5.8.3.5-1 cannot be taken greater than V_u/ϕ = 46/0.9 = 51 kips; hence, use V_s = 51 kips.

$$T = \frac{M_u}{\phi d_v} + \left[\left(\frac{V_u}{\phi}\right) - 0.5V_s\right]\cot\theta = \frac{2052 \times 12}{0.9 \times 37} + \left[\left(\frac{46}{0.9}\right) - 0.5 \times 51\right]\cot(45) = 765 kips$$

At midspan, there is no concern about development length; hence, f_{sx}=f_y.

$$T_{provided} = A_s f_y = (16 \times 0.79)100 = 1264 kips > T = 765 kips \quad O.K.$$

(iii) <u>Face of Bearing</u>

M_u = 0 and V_u = 168 kips *(Note: V_u is taken as the shear at d_v from the face of support.)*

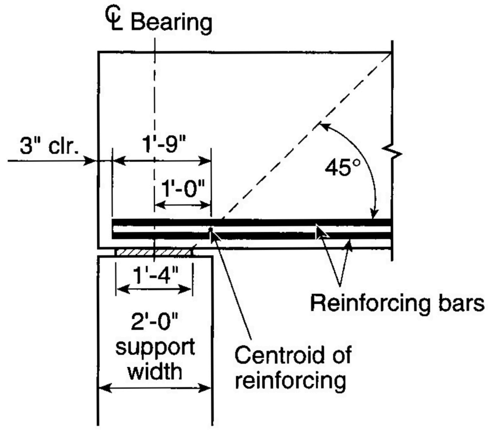
For # 4 stirrups @ 17" o.c., $V_s = \dfrac{A_v f_y d_v}{s} = \dfrac{0.4 \times 100 \times 37}{17} = 87.1 kips$

V_s in Eq. 5.8.3.5-1 cannot be taken greater than V_u/ϕ = 168/0.9 = 187 kips; hence, use V_s = 87.1 kips.

$$T = \frac{M_u}{\phi d_v} + \left[\left(\frac{V_u}{\phi}\right) - 0.5V_s\right]\cot\theta = 0 + \left[\left(\frac{168}{0.9}\right) - 0.5 \times 87.1\right]\cot(45) = 143 kips$$

$l_d = 38"$

As seen below, the available distance to develop the bar is 21".



$$f_{sx} = \left(\frac{l_{d,available}}{l_d}\right) f_y = \left(\frac{21}{38}\right) 100 = 55.2ksi$$

$$T_{provided} = A_s f_{sx} = (16 \times 0.79)55.2 = 698kips > T = 143kips \quad O.K.$$