APPENDIX D

POTENTIAL PROCESSES TO DEVELOP AND CALIBRATE VEHICULAR DESIGN LOADS.

Background

Task 2 of this research project was focused on existing and potential processes to develop and calibrate vehicular loads for superstructure design, fatigue design, deck design, and overload permitting. WIM data should be used to obtain accurate nationwide statistics on truck headway (multiple presences) and overload (above legal levels) occurrences. Characterization of the live load effect requires the quantification of the expected or mean maximum value, uncertainty expressed by the standard deviation (or coefficient of variation equal to standard deviation divided by mean) and the determination of the probability distribution type (Normal, Lognormal, Gumbel etc.). Depending on the application, the live loading must be characterized as the maximum lifetime event (strength) or in terms of a histogram of repetitive loads (fatigue). Maximum live loading depends on the weights of the individual trucks on the span. Typically traffic conditions cause a large number of vehicle multiple- presences on the span, especially those due to side by side moving trucks. These multiple presence events will often control the distribution of the maximum loading event. With multiple-presence, the maximum bridge loading depends on the weight and axle configuration of each truck on the span and their relative spacing or headway (nose-to-nose separation of trucks).

This data is analyzed using processes discussed herein to project a maximum bridge-loading event (L_{max}) defined over a period of 75 years for design. One simplified calibration approach recommended for this project is to focus on the expected or mean maximum live load variable, L_{max} but update the load model or the load factor for current traffic conditions in a manner consistent with the LRFD calibration approach. The most direct way of using the new WIM data reflecting current truck loads in the various jurisdictions is to focus on L_{max} and compare it to the value utilized during the calibration of the existing LRFD code, as described in NCHRP Report 368. This assumes that the overall LRFD calibration and safety indices are adequate for the strength and load data then available. Another key assumption in this regard is that the site-to-site variability in L_{max} as measured by the COV is the same as that used during the AASHTO LRFD calibration. In AASHTO LRFD calibration, the overall live load COV was taken as 20%.

The maximum loading, L_{max}, is defined for a suite of representative bridge configurations and should be normalized by the maximum allowed loading in the jurisdiction where the WIM data is taken. L_{max} reflects both the multiple presence, which is input to any load model and the degree of weight overloading for various types of truck classifications and jurisdictions. The HL-93, and in fact all bridge loading models, are driven by the degree that truck weights exceed their allowable legal limits. In LRFD, a load model has been used in which 8% of trucks are overweight and the average of the top 20% of trucks is 95% of the legal limiting value with a 25% COV. This overweight value has a very important influence on the live load factors selected for bridge designs. With additional WIM data now available and consistent examination of the tails of these distributions the research team will expand the basis for truck weight modeling.

The availability of new WIM data should be used to check the fatigue truck developed in the 1980's and determine if the stress ranges under newer truck configuration and current traffic conditions typically exceed the ranges found from the 1980's WIM data. Fatigue is an averaging process, so small numbers of extreme loads are not relatively significant. Rather, the average fatigue "damage" per truck determines the design and assessment criteria. In this case, data from many WIM sites should be combined and compared to the data from the 1980's to determine the adequacy of the present AASHTO fatigue truck configuration and weight for current applications. The increasing presence of multi-axle trucks in the traffic stream, investigated in NCHRP 12-63, must also be accounted for in the fatigue load model. Also, the live load modeling in LRFD did not specifically address the increasing load effects on bridge decks from the heavier and more complex axle configurations of current truck traffic. Recent WIM data should be analyzed to ascertain overweight statistics specific to axle loads, just as it was done for truckloads. It is likely that axle overload probabilities may be significantly higher than the overload probabilities for truck gross weights.

Several procedures exist to estimate the maximum expected load effect on a highway bridge. These procedures, of various levels of complexity, explain how site-specific truck weight and traffic data can be used to obtain estimates of the maximum live load for the design life of a bridge, specified to be 75 years as per the AASHTO LRFD code, or the two-year return period to be used for the load capacity evaluation of existing bridges. The models require as input the WIM data collected at a site after being "scrubbed" and processed for quality assurance as described in the following chapter. The models described in this section include: 1) a probability convolution approach, 2) a Monte Carlo simulation approach, and 3) simplified methods. The convolution method uses numerical integrations of the collected WIM data histograms to obtain projections of the expected maximum load effect within a given return period (e.g. 2-years or 75-years). The Monte Carlo simulation uses random sampling from the collected data to obtain the maximum load effect.

Potential Processes for Estimating Maximum Lifetime Loading L_{max} from Traffic Statistics

Convolution Approach

This section presents a convolution (numerical integration) procedure for modeling the maximum live load effect on a highway bridge.

1. Assemble histogram for truck load effect

In this step, using the WIM data files, the moment effect of each truck in the WIM record is calculated by passing the trucks through the proper influence line. The moment of each truck is then normalized by dividing the calculated value by the moment of the HL-93 load model. The data is collected into a percent frequency histogram. This histogram will provide a discretized form of the probability density function (pdf) of the moment effects for the site. The histogram is designated as $H_x(X)$ while the pdf is designated as $f_x(X)$. The relation between $H_x(X)$ and $f_x(X)$ is given by:

$$H_x(X) = \int_{X_l}^{X_u} f_x(x)dx \qquad \text{(D-1)}$$

where X_l and X_u give the upper and lower bounds of the bin within which X lies. If the bin size is small, then $f_x(X)$ can be assumed to be constant within the range of X_l to X_u and Equation D-1 becomes:

$$H_x(X) = f_x(X)\Delta X = f_x(X)\left(X_u - X_l\right) \qquad \text{(D-2)}$$

where ΔX is the bin size.

Assemble histogram for the moment effects of side-by-side trucks.

The total moment effect when two side-by-side trucks are on a bridge is obtained from $x_s=x_1+x_2$ where x_1 is the effect of the truck in the drive lane and x_2 is the effect of the truck in the passing lane. Assuming independence between truck moments, the probability density function of the effect of side-by-side trucks $f_s(S)$ can be calculated using a convolution approach. The convolution equation is presented as:

$$f_{xs}(X_s) = \int_{-\infty}^{+\infty} f_{x2}(X_s - x_1) f_{x1}(x_1) dx_1 \qquad \text{(D-3)}$$

where: f_{xs} (…) is the probability distribution of the side-by-side effects
f_{x1}(…) is the probability distribution of the effects of trucks in lane 1
f_{x2}(…) is the probability distribution of the effects of trucks in lane 2

Equation D-3 can be interpreted as follows: Given that the effect of the truck in lane 1 is equal to x_1, then the probability that the effects of two side-by-side trucks will take a value X_s, is equal to the probability that the effect of the truck in lane 1 is x_1, times the probability that the effect of the truck in lane 2 is equal to $X_2=X_S-x_1$. This will lead to the following expression: $f_{x2}(X_s - x_1) f_{x1}(x_1)$. The integration is executed to cover all possible values of x_1. Equation D-3 gives the probability density function (pdf) for one particular value of X_s. Thus, Equation D-3 must be repeated for each possible value of X_s. Equation D-2 is used to convert the pdf's into equivalent histograms.

2. Assemble cumulative distribution functions.

Use the histograms and the probability density functions f_{x1}(…), f_{x2}(…), f_{xs}(…) and f_z(…) assembled above to obtain the cumulative distributions for the moment effects of single trucks in the drive lane and moment effects of two trucks side-by-side. Note, that the moment effect for the passing lane is not considered independently since it does not govern for the data collected at this site.

The cumulative distribution is assembled from the pdf using the equation:

$$F_z(Z) = \int_{-\infty}^{Z} f_z(y) dy \qquad \text{(D-4)}$$

In essence, Equation D-4 assembles all the bins of the histogram below a certain value Z into a single bin at Z. This is repeated for all possible values of Z. Thus, $F_z(Z)$ will give the probability that a loading event will produce a load effect less or equal to Z.

3. Calculate the cumulative probability function for the maximum load effect over a return period of time, t_{return}.

A bridge structure should be designed to withstand the maximum load effect expected over the design life of the bridge. The AASHTO LRFD code specifies a design life of 75 years. The LRFR bridge load rating also requires checking the capacity to resist the maximum load effects for a two-year return period. It is simply impossible to get enough data to determine the maximum load effect expected over 75 years of loading. Even getting sufficient data for the two-year return period would require several cycles of two-year data. Therefore, some form of statistical projection should be performed. The proposed calculation procedure calculates the cumulative distribution function for the loading event over a short return period typically less than 75 years (in some cases even less than two years) and then using statistical projections to obtain the information required for the longer two-year and 75-year return periods.

To find the cumulative distribution for the maximum loading event in a period of time t we have to start by assuming that N loading events occur during this period of time t. These events are designated as S_1, S_2, … S_N. The maximum of these N events, call it $s_{max,N}$, is defined as:

$$s_{max,N} = \max(s_1, s_2, \ldots s_N) \qquad \text{(D-5)}$$

We are interested in finding the cumulative probability distribution of $S_{max,N}$. This cumulative probability distribution, $F_{s\ max\ N}(S)$, gives the probability that $s_{max,N}$ is less than or equal to a value S. If $s_{max,N}$ is less than S, this implies that s_1 is less than s, and s_2 is less than s, … and S_N is less than s. Hence, assuming that the loading events are independent, the probability that $s_{max,N} \leq S$ can be calculated from:

$$F_{s_{max\ N}}(S) = F_{s_1}(S).F_{s_2}(S)...F_{s_N}(S) \qquad \text{(D-6)}$$

If s_1, s_2, ... s_N are independent random variables but they are drawn from the same probability distribution, then:

$$F_{s_1}(S) = F_{s_2}(S) = ... = F_{s_N}(S) = F_s(S) \qquad \text{(D-7)}$$

and Equation (D-6) reduces to

$$F_{s_{max\ N}}(S) = \lfloor F_s\ (S) \rfloor^N \qquad \text{(D-8)}$$

Note that Equation D-8 assumes that the number of events is a deterministic value. Sensitivity analysis, however, demonstrates that the results of Equation D-8 will not be highly sensitive to variations in N as N becomes large.

One way to project the results is by using the Normal fit of the tail of the histogram and substitute the Normal distribution into Equation D-8 rather than the WIM data cumulative distribution.

4. Determine the probability density function of maximum load effect in a return period t_{return}.

The probability density function and the histogram for the load effect of the maximum loading in different return periods can be calculated from:

$$f_{S\,max\ N}(S) = \frac{dF_{s_{max\ N}}(s)}{ds}\Big|_S \qquad \text{(D-9)}$$

5. Statistical projection of maximum load effects using Extreme Value probability principles.

Since it is not possible to collect enough data to estimate the maximum load effect for long return periods, statistical methods need to be used to obtain such estimates. Although statistical projections are based on some basic assumptions that should be justified by physical evidence and mathematical reasoning, there is no guarantee that the projections will exactly forecast the future. Statistical estimates of the confidence limits can only give some assurance that the projections are reasonable within the constraints of the assumptions. In the previous section, the statistical projection was executed assuming that the tail of the WIM data histogram approaches that of a Normal distribution whose mean and standard deviation are obtained from the regression fit of the data plotted on Normal curves. Another method described in this section, executes the statistical projection based on the principles of extreme value theory. The advantage of the approach described in this section lies in the fact that it would be valid even if the tail of the WIM histogram does not approach that of a Normal distribution.

The largest value for N loading repetitions approaches the double exponential distribution as the value of N increases (Ang & Tang (2007)). The double exponential distribution, also known as the Extreme Value Type I distribution or the Gumbel distribution, is the asymptotic distribution for the largest value if the original distribution, F_s (S), has an exponentially decaying tail. Thus, this approach is valid if the tail of the WIM histogram can be fitted by a Normal distribution but would also be valid if the tail can be fitted by any other exponentially decaying distribution. If the tail end of the WIM histogram can be fitted by a Lognormal distribution, then the Frechet Type II extreme value distribution can be used for the statistical projection. If the tail end of the WIM histogram has an upper limit, then the Type III Weibull distribution may be used. In this section, the process of using Extreme value distributions is illustrated using the Gumbel Type I distribution although the process is applicable for the other types as well. It should be noted that the Normal fit of the tail end of the WIM histogram is consistent with the extreme value fit of the projected data but the extreme value fit of the projected data is more general as it applies for non-normal distributions as well.

The cumulative Gumbel distribution has the form:

$$F_{s\max,k}(S) = e^{-e^{-\alpha_k(S-u_k)}} \tag{D-10}$$

where $F_{smax\ k}$ (S) is the cumulative distribution of the maximum of k events, u_k is the most probable value of $S_{max,\ k}$ and α_k is an inverse measure of the dispersion in $s_{max,\ k}$. The corresponding probability density function of the Gumbel distribution is then:

$$f_{s\max,k}(S) = \alpha_k e^{-\alpha_k(S-u_k)} e^{-e^{-\alpha_k(S-u_k)}} \tag{D-11}$$

The mean value for $s_{max\ k}$ can be calculated as:

$$\mu_{sk} = u_k + \frac{\gamma}{\alpha_k} \tag{D-12}$$

in which γ is the Euler number γ=0.577216. The standard deviation, $\sigma_{s,k}$ of $s_{max\ k}$ can be calculated from:

$$\sigma^2_{S,k} = \frac{\pi^2}{6\alpha_k^2} \quad \text{(D-13)}$$

To verify that the distribution of the maximum load effect for a return period t_{return} approaches the Gumbel distribution, one can use some form of probability plot. In this case, the plot is executed by taking the double natural logarithm of both sides of Equation D-10 so that:

$$\log n\left(-\log n\left(F_{s\max,k}\right)\right) = -\alpha_k s + \alpha_k u_k \quad \text{(D-14)}$$

which has the linear form:

$$y = m\,s + n \quad \text{(D-15)}$$

This indicates that if $F_{smax,k}$ follows a Gumbel distribution, the plot of the double log should follows a straight line with a slope = $-\alpha_k$ and an intercept at $\alpha_k u_k$

6. Projection of results to longer return periods

The design and safety evaluation of bridges requires the estimation of the maximum load effect over periods of 75 years and 2 years respectively. However, it is clear that the WIM data collected cannot reasonably be accurate enough in the tail of the distributions to obtain good estimates of the parameters for such extend return periods. Hence, the only possible means to obtain the parameters of the distributions of the maximum two-year and 75-year maximum load effect is by using statistical projections. As mentioned earlier, the probability distribution of the maximum value of a random variable will asymptotically approach an extreme value distribution as the number of repetition increases.

Although the Gumbel fit was executed on the tail of the one-week maximum, the properties of the Gumbel distribution allows for the projection of the results for any return period. For example, given the mean and standard deviation of the one week maximum for which the number of events is k=19x7 for the side-by-side events, we can calculate the mean and standard deviation for any return period during which a total of N events take place using the relationships:

$$\sigma_N = \sigma_k \quad \text{(D-16)}$$

$$\mu_N = \mu_k + \frac{\sqrt{6}}{\pi}\sigma_k \log n\left(\frac{N}{k}\right) \quad \text{(D-17)}$$

Equation D-16 indicates that the standard deviation of the Gumbel distribution is independent of the return period or the number of repetitions, while the mean value of the Gumbel is a linear function of the natural logarithm of the number of repetitions.

7. Summary of convolution approach

The convolution approach presented in this section shows a procedure that can be used to obtain the maximum load effect over different return periods. When the approach was applied using the original raw data, it was found to be insufficient for projecting the maximum load for the drive lane for large return periods. Hence, two statistical projections techniques were compared. The first statistical projection fitted a Normal distribution to the tail end of the raw WIM data histogram while the second approach fitted an extreme value distribution to the tail end of the projected one-week maximum load effect. The assumptions made in this section are:

- The WIM data can be used to generate histograms for the load effects of the trucks that are in the drive and passing lanes with good accuracy in the tail end of the histograms. Since it is not possible to gather sufficient data to have exact results for the maximum 2-year and 75-year return periods, statistical projection techniques must be used.
- The average number of loading events including single lane loadings and side-by-side cases is reasonably well known from the WIM data or other headway data available for the site. A sensitivity analysis to be performed in the next chapter will demonstrate that the results are not very sensitive to small errors in these values. However, a good estimate is still required.
- The trucks in the passing lane are independent and uncorrelated from those in the drive lane. This assumption has been verified by the analysis of the WIM data obtained for this project.
- The probability distribution of the maximum load effect approximately approaches an extreme value distribution as the return period increases. If the tail end of the original raw data has an exponential form (including a Normal distribution shape) then the Gumbel Type I extreme value distribution is adequate.
- If the tail end of the raw WIM data approaches the tail end of a known probability distribution (e.g. a Normal distribution), this known distribution can be directly used to obtain the maximum load effect for any projection period.

The next section of this chapter describes how Monte Carlo simulations can be executed to obtain the projections for the maximum load effect. Another section describes a number of simplified approximate procedures that would lead to similar results as those obtained from the convolution method and the Monte Carlo simulations.

Monte Carlo Simulation

An alternative to the convolution approach described above consists of using a Monte Carlo simulation to obtain the maximum load effect. In this approach, results of WIM data observed over a short period can be used as a basis for projections over longer periods of time. A Monte Carlo simulation requires the performance of an analysis a large number of times and then assembling the results of the analysis into a histogram that will describe the scatter in the final results. Each iteration is often referred to as a cycle. The process can be executed for the single lane-loading situation or the side-by-side loading. A step-by-step procedure for Monte Carlo Simulation is described in Step 12.3 of the Draft protocols, in the next chapter.

Like the convolution approach, the Monte Carlo simulation will not be accurate for large projections periods because of the limitations in the originally collected data. Also, the Monte Carlo simulation will be extremely slow when the number of repetitions K is very high. Furthermore, one should make sure not to exceed the random number generation limits of the software used, otherwise the generated numbers will not be independent and the final results will be erroneous. Hence, statistical projections must be made to estimate the maximum load effects for long return periods. For example, the same steps 7 through 9 used in the convolution approach can be used to fit the one-week data into cumulative distribution functions that are plotted on Gumbel scales from which the dispersion coefficients are extracted and used to find the mean values and standard deviations for different return periods. Alternatively, one can used a smoothed tail end of the WIM histogram by fitting the tail with a known probability distribution function (such as the Normal distribution) and use that fitted distribution with the Monte Carlo simulation. It should be emphasized however, that the Monte Carlo simulation would still be very inefficient for projections over long return periods.

Simplified Procedures for Estimating Maximum Load Effects

The advantages of accurate convolution and simulation approaches for Lmax is that it allows a more accurate calculation of failure probability in those applications where target risks should be known in quantitative form. For most applications, such as code calibration and comparative risk assessments, simplified models for L_{max} may be acceptable. Several reasons support this approach:

1. There may never be sufficient data to fully support an actuarial approach to risk calculation for a bridge structure.

2. Specifications by their very nature represent simplification and compromises that allow designers to perform calculations in a deterministic manner which is transparent to the risk analysis process.

3. There are many variables affecting risk which are beyond the control of designers for which accurate statistics are not available. These variables include growth in future traffic, human errors which are the cause of most structural failures and limitations in the data base of variables other than the external live load including structural analysis uncertainties, fabrication variabilities, costs of failure which affect the optimal target risk, future changes in truck types and horsepower which affect multiple presence probabilities, etc.

Two simplified procedures are proposed for estimating the maximum load effect in a given return period. The two approaches are based on the assumption that the tail end of the moment load effect for the original population of trucks as assembled from the WIM data approaches a Normal distribution. The first method uses the properties of the extreme value distribution and the second is based on the properties of the Normal distribution.

Simplified Extreme Value Model

Let us assume that the tail end of the parent moment effect histogram approaches a Normal distribution. If this assumption is accepted, it will provide a powerful tool to use the results of the Normal probability fit to obtain projections of the maximum load effect for different return periods without the need to execute the convolution or the Monte Carlo simulation. The approach is based on the following known concepts as provided by Ang and Tang (2006) and stated earlier in this section but repeated herein for emphasis: "if the parent distribution of the initial variable x has a general Normal distribution with mean μ_x and standard deviation σ_x, then the maximum value of x after K repetitions approaches asymptotically an Extreme Value Type I (Gumbel) distribution with a dispersion α_K given by:

$$\alpha_K = \frac{\sqrt{2\ln(K)}}{\sigma_X} \qquad \text{(D-18)}$$

and a most probable value u_K given by:

$$u_K = \mu_x + \sigma_x \left(\sqrt{2\ln(K)} - \frac{\log(\log(K)) + \log(4\pi)}{2\sqrt{2\log(K)}} \right) \tag{D-19}$$

These values for α_K and u_K can then be used in Equation D-10 and D-11 to find the probability density functions and the histogram for any return period with K load repetitions. Also, α_K and u_K can be used in Equations D-12 and D-13 to find the mean and standard deviation for the maximum load effect for any return period having K repetitions.

Simplified Normal Distribution Model

A final comparison is performed between the projected results obtained using the procedures described above and the simplified method used by Fred Moses in previous NCHRP projects on load modeling and LRFR code calibration. This procedure starts by fitting a Normal distribution through the tail end of the moment histogram (Fig D-1). The approach can be used for trucks in the drive lane and for side-by-side trucks.

The expected maximum two-year side-by-side event can be estimated to be:

$$\mu_{2year} = \mu_{s'} + t\,\sigma_{s'} \tag{D-20}$$

where t = standard deviate corresponding to a probability of exceedance

It is noted that the results of this simplified Normal approach and the simplified Gumbel approach are less than 7.5% for the worst case scenario. The reason for the larger differences for short return periods is due to the fact that the maximum loading approaches the Gumbel distributions (i.e Equations D-18 and D-19) are exact only in an asymptotic sense i.e. only as the number of repetitions becomes very high (approaches infinity). Unlike the simplified Gumbel approach and the Convolution and Monet Carlo simulation methods that provide the whole probability distribution of the maximum load effect as well as the mean values and the standard deviations, the simplified Normal approach only gives the expected value but cannot calculate the standard deviation of the maximum load effect.

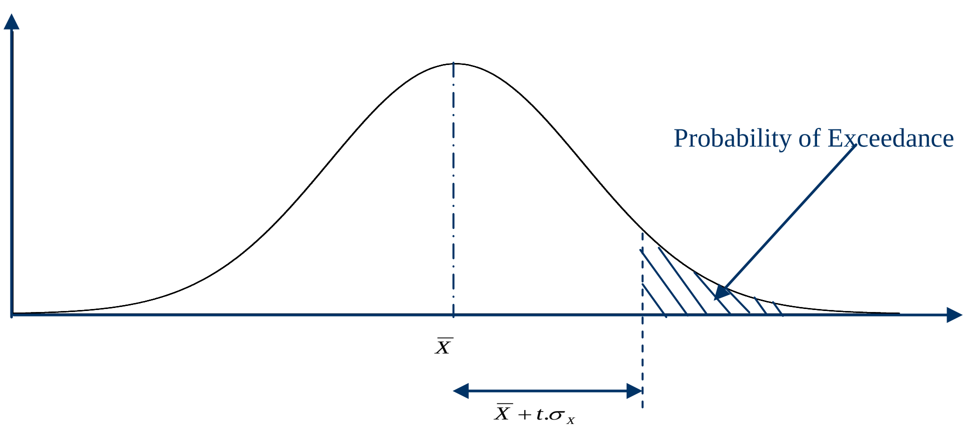

Figure D -1 Illustration of the Simplified Normal distribution approach

Summary

This chapter presented several methods for projecting the data from load effect histograms to obtain the maximum load effect for different return periods. The projections assume that the collected WIM data histogram extends beyond the largest value observed during the data collection period. The methods are illustrated for two cases: 1) the tail end of the parent histogram follows a Normal distribution or 2) the tail of the maximum load effect for a short return period follows a Gumbel distribution. The two assumptions are not necessarily contradictory since assumption 1 leads to assumption 2. But, assumption 2 is more general because it is valid when the tail of the parent histogram follows any type of exponentially decaying distribution. For whichever assumption is used, the results show that all the proposed methods asymptotically approach the same values as the number of loading repetitions increases. The convolution and Monte Carlo approaches are however valid even if the tail of the original WIM histogram was found to fit other than Normal distributions. If the fitted distribution is not exponentially decaying, then the Gumbel may not be valid and other types of extreme value distributions such as the Frechet or the Weibull distributions may be more adequate. In any case, the general steps for the analysis process remain the same.