Attachment P.

Design Example with Lightweight Concrete Beam and Deck

With ϕ = 0.85 and λ = 1.0

SUMMARY OF RESULTS

<div style="border:1px solid green">BEAM SATISFIES ALL DESIGN REQUIREMENTS</div>

Project Name: C - LWC Slab & Girder, $\varphi = 0.85$, $\lambda = 1.00$

Bridge Cross Section

Overall Deck Width:	51.0	ft
Design Span:	120.0	ft
Number of Beams:	6	
Beam Spacing:	9.00	ft
Deck Slab Thickness:	8.00	in
Relative Humidity:	70	%

Cast-in-Place Deck Slab

Concrete Type:	Sand-LWC	
Concrete Unit Weight:	0.125	kcf
28-day Strength:	4.0	ksi
Effective Slab Width:	108.0	in

Prestressed Beams   AASHTO-PCI BT-72

Concrete Type:	Sand-LWC	
Resist. Factor for Shear, ϕ	0.85	
Modif. Factor for LWC, λ:	1.00	
Concrete Unit Weight:	0.125	kcf
Strength at Transfer:	5.8	ksi
28-day Strength:	6.5	ksi
Modulus of Elasticity, E_c:	3,718	ksi
Effective Haunch Depth:	0.50	in

Loads

Non-Composite DL:	1.434	kip/ft
Composite DL:	0.300	kip/ft
LLDF for Moment:	0.732	lanes/beam
LLDF for Shear:	0.884	lanes/beam

Prestressing Strands 1/2 in. dia. 7-wire low-relaxation

Total Number of Strands:	40	
# Harped Strands:	6	
Top Row of Harp Strands:	70.0	in
e of Strands at Midspan:	31.70	in
e of Strands at Beam End:	23.00	in

Prestress Losses

Prestress Loss at Transfer:	21.50	ksi =	10.6%
Total Prestress Loss at Service			
(including loss at transfer):	46.36	ksi =	23%

Deflections

Camber:	4.54	in	↑
At Erection Before Deck:	5.95	in	↑
After Deck:	4.18	in	↑
Due to LL & Impact:	1.03	in	↓

Stress Limits at Transfer

Compression:	3.480	ksi
Tension (w/o bonded reinf):	-0.200	ksi
Tension (w/ bonded reinf):	-0.578	ksi

Stress Limits at Service

Compression - Permanent Loads (deck):	1.800	ksi
Compression - Permanent Loads (beam):	2.925	ksi
Compression - Total Loads (deck):	2.400	ksi
Compression - Total Loads (beam):	3.900	ksi
Tension:	-0.484	ksi

Calculated Stresses at Transfer

	Top of Beam f_t (ksi)		Bottom of Beam f_b (ksi)	
	Stress	Limit	Stress	Limit
Transfer Length (2.50 ft)	-0.163	[-0.578]	3.108	[3.480]
Harp Point (48.50 ft)	0.078	[3.480]	2.858	[3.480]
Midspan	0.115	[3.480]	2.819	[3.480]

Calculated Stresses at Service (at midspan)

	Top of Deck Service I (ksi)		Top of Beam Service I (ksi)		Bottom of Beam Service III (ksi)	
	Stress	Limit	Stress	Limit	Stress	Limit
Permanent Loads	0.117	[1.800]	1.391	[2.925]	--	--
Total Loads	0.694	[2.400]	1.893	[3.900]	-0.399	[-0.484]

Flexural Strength at Midspan

Factored Resistance, M_r:	9,843	kip-ft
Ultimate Moment, M_u:	8,670	kip-ft
Strength for Min Reinf:	6,535	kip-ft

Nominal Shear Resistance:

Vertical Shear, V_c:	210.8	kips
Interface Shear, V_{ni}:	153.14	kips/ft

Required Shear Reinforcement

Vertical Shear:	#4 stirrups w/ 2 legs at 24.0"
Interface Shear:	#4 stirrups w/ 2 legs at 24.0"

Longitudinal Reinforcement Req'd at Face of Bearing

Add'l Reinf Req'd: (6) #7 bars

Anchorage Zone Reinforcement

Req'd Reinf: (4) #5 stirrups w/ 2 legs at 5.0"

Fatigue of Reinforcement

Fact'd Fatigue Stress:	5.141	ksi
Threshold Stress:	18.000	ksi

GEOMETRY AND DESIGN PARAMETERS INPUT

Project Name: C - LWC Slab & Girder, φ = 0.85, λ = 1.00

Bridge Cross Section:

Overall Deck Width =	51.00	ft	
Roadway Width =	48.00	ft	
Barrier/Rail Width =	1.50	ft	Width of One Barrier
Overhang Width =	3.00	ft	Assumed Symmetrical
Beam Spacing, S =	9.00	ft	
Number of Beams, N_b =	6		
Curvature =	0.00	degrees	
Are Beams Parallel and Have Same Stiffness?	yes		(yes,no)
Cross Section Type =	k		AASHTO Table 4.6.2.2.1-1
Are AASHTO Curvature Requirements Met?	yes		AASHTO Art. 4.6.2.2.1

Cast-in-Place Deck Slab:

Actual Thickness, t_s =	8.0	in	
Structural Thickness =	7.5	in	
Concrete Type =	Sand-LWC		Normal Weight, Sand-Lightweight, All-Lightweight 2
28-day Concrete Strength, f'_c =	4.0	ksi	
Slab Concrete Unit Weight =	0.125	kcf	See AASHTO Table 3.5.1-1 and C3.5.1
Is Deck Width Constant?	yes		(yes,no)

Precast Beams:

Beam Type =	AASHTO-PCI BT-72		Update Figure 2-1 according to beam type
Resistance Factor for Shear, φ =	0.85		AASHTO Art. 5.5.4.2.1 (0.90 for normal weight, 0.70 for lightweight)
Modification Factor for LWC, λ =	1.00		AASHTO Art. 5.8.2.2 (1.00 for NWC, 0.75 for all LWC, 0.85 for sand LWC)
Concrete Type =	Sand-LWC		Normal Weight, Sand-Lightweight, All-Lightweight 2
Concrete Strength at Transfer, f'_{ci} =	5.8	ksi	
28-day Concrete Strength, f'_c =	6.5	ksi	
Beam Concrete Unit Weight, w_c =	0.125	kcf	See AASHTO Table 3.5.1-1 and C3.5.1
Design Span, L =	120.0	ft	Measured from CL Bearing to CL Bearing
Overall Beam Length =	121.0	ft	
Effective Haunch Depth, H_{eff} =	0.50	in	
Age of Concrete at Prestress Transfer, t_i =	1	days	Typical TxDOT Values: t_i = 1 day
Age of Concrete at Deck Placement, t_d =	90	days	t_d = 120 days
Final Age of Concrete, t_f =	20,000	days	t_f = 2,000 days
Time between Strand Prestressing and Casting, t_p =	1	days	
Is LWC Splitting Tensile Strength Specified?	no		(yes, no)
Cohesion and Friction Case =	1		AASHTO Art. 5.8.4.3:

Cohesion Factor, c =	0.280
Friction Factor, μ =	1.0
K_1 =	0.30
K_2 =	1.3

1. For a cast-in-place concrete slab on clean concrete girder surfaces, free of laitance with surface roughened to an amplitude of 0.25 in.
2. For normal weight concrete place monolithically.
3. For lightweight concrete placed monolithically, or nonmonolithically, against a clean concrete surface, free of laitance with surface intentionally roughened to an amplitude of 0.25 in.
4. For normal weight concrete placed against a clean concrete surface, free of laitance, with surface intentionally roughened to an amplitude of 0.25 in.
5. For concrete placed against a clean concrete surface, free of laitance, but not intentionally roughened.
6. For concrete anchored to as-rolled structural steel by headed studs or by reinforcing bars where all steel in contact with concrete is clean and free of paint.

BRIDGE PRESTRESSED CONCRETE BEAM DESIGN

Prestressing Strands:

Type =	1/2 in. dia. 7-wire low-relaxation	
Area of One Strand, A_p =	0.153	in^2
Ultimate Strength, f_{pu} =	270.0	ksi
Modulus of Elasticity, E_p =	28,500	ksi
Assumed Percentage of Final Prestress Loss =	23	%

Use an initial estimate of 25% and see Section 6.2.4 for the calculated value. If different, adjust input value here until it is equal to the calculated value.

Link to Section 6.2.4

Reinforcing Bars:

Yield Strength, f_y =	60	ksi
Modulus of Elasticity, E_s =	29,000	ksi

Future Wearing Surface (Overlay):

Thickness =	2.0	in
Unit Weight =	0.150	kcf

Rail / Barrier:

Type =	New Jersey	
Unit Weight =	0.300	kip/ft/side

Environmental Factors:

Aggregate Source Correction Factor, K_1 =	1.0	See AASHTO Eq. 5.4.2.4-1
Relative Humidity, H =	70	%
Corrosion Conditions =	Moderate Corrosion	

Prestressed Strand Pattern:

Number of Harped Strands =	6	
Dist. from Beam Bottom to Top Row of Harped Strands at End of Beam =	70.0	in

If no strands need to be harped (Section 7.2), enter zero for both values. If harped strands are required, adjust number of harped strands and/or distance to the top row of harped strands until the stresses in Section 7.2 satisfy the stress limits. The spacing between rows of harped strands at the beam end is the same as the spacing between rows of strands at midspan. Only the middle two strands of each row can be harped. See total number of strands in Section 5.4 to set the maximum number of strands that can be harped.

Link to Section 5.4

Link to Section 7.2

Transverse Shear Reinforcement:

Stirrup Size =	#4	See Sections 11.4.2 and 12.3 for required spacing.
Number of Legs =	2	Link to Section 11.4.2
		Link to Section 12.3

Transverse Anchorage Zone Reinforcement:

Stirrup Size =	#5	See Section 14.1 for required spacing.
Number of Legs =	2	Link to Section 14.1

Additional Longitudinal Reinforcement Due to Shear:

Bar Size =	#7	See Section 13.1 for required additional longitudinal reinforcement.
Number of Bars =	6	Link to Section 13.1

1 INTRODUCTION

In this document, the detailed design of a 120-ft single span AASHTO-PCI BT-72 beam bridge is presented. Analysis and design at the critical sections in positive flexure, shear and deflection due to prestress, dead loads and live load are shown for a typical interior beam. The superstructure consists of 6 beams spaced at 9'-0" on center, as shown in Figure 1-1. Beams are designed to act compositely with the 8.0-in. cast-in-place concrete deck to resist superimposed dead loads, live loads and impact. This example applies to prestressed beams with only bonded tendons and with no externally applied axial load. The design is performed in accordance with the AASHTO LRFD Bridge Design Specifications, 5th Edition, 2010 (henceforth referred to as AASHTO Specifications).

Reference to AASHTO articles, tables and equations is shown along the right margin of the document using square brackets. For all other equations, not from AASHTO, parentheses are used.
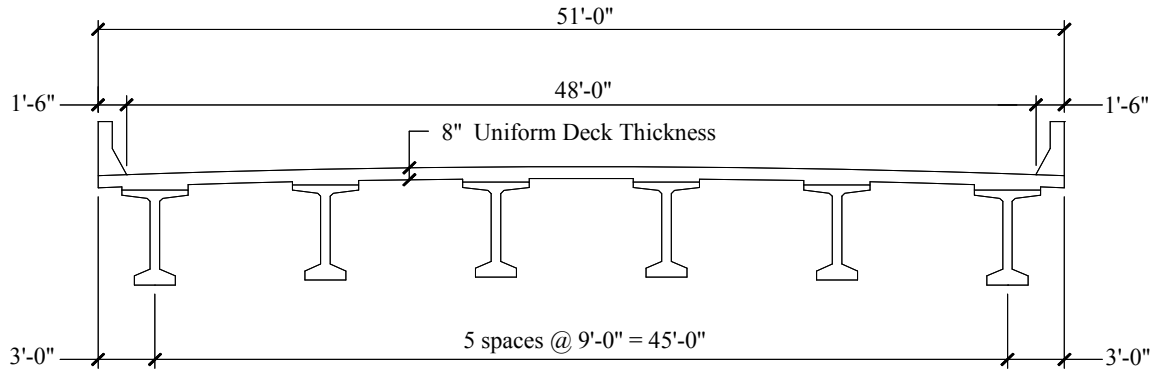
Figure 1-1: Bridge Cross-Section

2 MATERIALS

Cast-in-place slab (sand lightweight concrete):

$$\text{Actual thickness, } t_s = \quad 8.0 \quad \text{in}$$

$$\text{Structural thickness, } t_{s,str} = \quad 7.5 \quad \text{in}$$

$$\text{Concrete strength at 28 days, } f'_c = \quad 4.0 \quad \text{ksi}$$

$$\text{Slab concrete unit weight, } w_c = \quad 0.125 \quad \text{kcf}$$

Note that the top 0.50 in. of the slab is considered to be a wearing surface.

Precast beams − AASHTO-PCI BT-72 (sand lightweight concrete):

$$\text{Concrete strength at transfer, } f'_{ci} = \quad 5.8 \quad \text{ksi}$$

$$\text{Concrete strength at 28 days, } f'_c = \quad 6.5 \quad \text{ksi}$$

$$\text{Beam concrete unit weight, } w_c = \quad 0.125 \quad \text{kcf}$$

$$\text{Design span, } L = \quad 120.0 \quad \text{ft}$$

$$\text{Overall beam length} = \quad 121.0 \quad \text{ft}$$

$$\text{Number of beams, } N_b = \quad 6$$

$$\text{Beam spacing, } S = \quad 9.00 \quad \text{ft}$$

$$\text{Effective haunch depth, } H_{eff} = \quad 0.50 \quad \text{in}$$

The effective haunch depth is an equivalent uniform depth, calculated assuming the actual haunch is parabolic in shape.
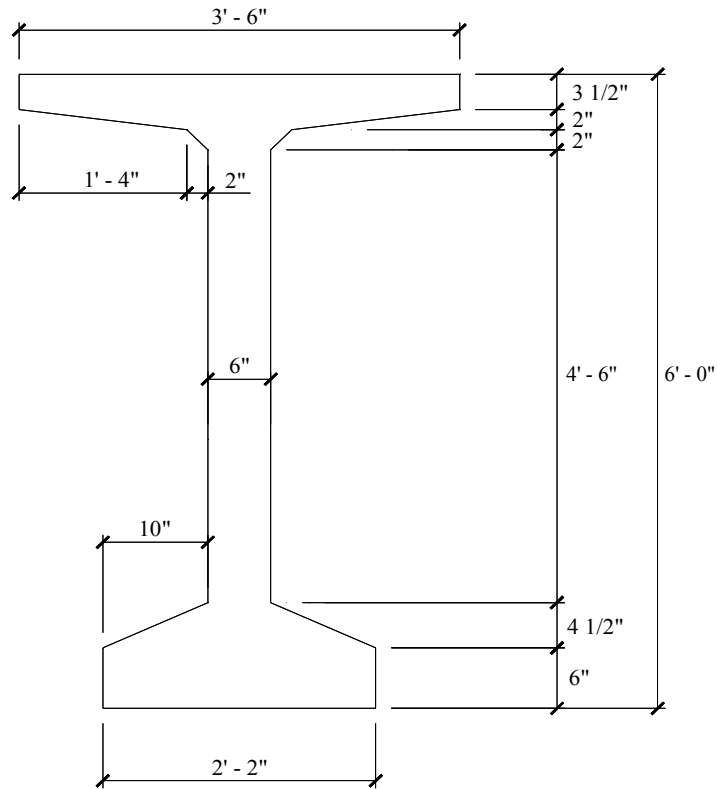
Figure 2-1: AASHTO-PCI BT-72 Dimensions

Prestressing strands: 1/2 in. dia. 7-wire low-relaxation:

$$\text{Area of one strand, } A_p = \quad 0.153 \quad \text{in}^2$$

$$\text{Ultimate strength, } f_{pu} = \quad 270.0 \quad \text{ksi}$$

$$\text{Yield strength, } f_{py} = 0.9f_{pu} = \quad 243.0 \quad \text{ksi} \qquad \text{[AASHTO Table 5.4.4.1-1]}$$

$$\text{Modulus of elasticity, } E_p = \quad 28{,}500 \quad \text{ksi} \qquad \text{[AASHTO Art. 5.4.4.2]}$$

Stress limits for low relaxation prestressing strands: [AASHTO Table 5.9.3-1]

$$\text{before transfer, } f_{pi} \le 0.75f_{pu} = \quad 202.5 \quad \text{ksi}$$

$$\text{at service limit state (after all losses), } f_{pe} \le 0.80f_{py} = \quad 194.4 \quad \text{ksi}$$

Reinforcing bars:

$$\text{Yield strength, } f_y = \quad 60 \quad \text{ksi}$$

$$\text{Modulus of elasticity, } E_s = \quad 29{,}000 \quad \text{ksi} \qquad \text{[AASHTO Art. 5.4.3.2]}$$

Future wearing surface (overlay):

$$\text{Thickness, } t_{fws} = \quad 2.0 \quad \text{in}$$

$$\text{Unit weight, } w_{fws} = \quad 0.150 \quad \text{kcf}$$

Rail / Barrier:

$$\text{Type} = \quad \text{New Jersey}$$

$$\text{Unit weight, } w_r = \quad 0.300 \quad \text{kip/ft/side}$$

3 CROSS-SECTION PROPERTIES FOR A TYPICAL INTERIOR BEAM

3.1 Non-Composite Section

$$\text{Cross sectional area, A} = \quad 767 \quad \text{in}^2$$

$$\text{Overall beam depth, h} = \quad 72.0 \quad \text{in}$$

$$\text{Moment of inertia, I} = 545{,}894 \quad \text{in}^4$$

$$\text{Distance from centroid to extreme bottom fiber, } y_b = \quad 36.60 \quad \text{in}$$

$$\text{Distance from centroid to extreme top fiber, } y_t = \quad 35.40 \quad \text{in}$$

$$\text{Section modulus for extreme bottom fiber, } S_b = I/y_b = \quad 14{,}915 \quad \text{in}^3$$

$$\text{Section modulus for extreme top fiber, } S_t = I/y_t = \quad 15{,}421 \quad \text{in}^3$$

$$\text{Unit weight of beam, } w_b = \quad 0.666 \quad \text{kip/ft}$$

$$\text{Modulus of Elasticity, } E_c = 33{,}000(K_1)(w_c)^{1.5}\sqrt{f'_c} \qquad \text{[AASHTO Eq. 5.4.2.4-1]}$$

$$\text{Aggregate correction factor, } K_1 = \quad 1.0$$

$$w_c = \text{concrete unit weight (kcf) for:}$$

$$\text{slab, } w_c = \quad 0.125 \quad \text{kcf}$$

$$\text{beams, } w_c = \quad 0.125 \quad \text{kcf}$$

$$f'_c = \text{specified strength of concrete (ksi) for:}$$

$$\text{slab, } f'_c = \quad 4.0 \quad \text{ksi}$$

$$\text{beam at transfer, } f'_{ci} = \quad 5.8 \quad \text{ksi}$$

$$\text{beam at service, } f'_c = \quad 6.5 \quad \text{ksi}$$

Therefore, the modulus of elasticity for:

$$\text{slab, } E_c = 33{,}000(1.0)(0.125)^{1.5}\sqrt{4.0} = \quad 2{,}917 \quad \text{ksi}$$

$$\text{beam at transfer, } E_{ci} = 33{,}000(1.0)(0.125)^{1.5}\sqrt{5.8} = \quad 3{,}512 \quad \text{ksi}$$

$$\text{beam at service, } E_c = 33{,}000(1.0)(0.125)^{1.5}\sqrt{6.5} = \quad 3{,}718 \quad \text{ksi}$$

3.2 Composite Section

3.2.1 Effective Flange Width   [AASHTO Art. 4.6.2.6.1]

Effective flange width may be taken as one-half the distance to the adjacent girder on each side of the component:

$$b_{eff} = \text{beam spacing} = (9.0 \times 12) = \quad 108 \quad \text{in}$$

3.2.2 Modular Ratio between Slab and Beam Materials

Modular ratio between slab and beam materials:

$$n = \frac{E_c(\text{slab})}{E_c(\text{beam})} = \frac{2{,}917}{3{,}718} = \quad 0.7846$$

3.2.3 Transformed Section Properties

$$\text{Transformed flange width} = n(b_{eff}) \qquad = (0.7846)(108) = \quad 84.74 \quad \text{in}$$

Transformed flange area = n(Effective flange width)(t_s)

$$= (0.7846)(108)(7.5) = \quad 635.5 \quad in^2$$

Note: Only the structural thickness of the deck, 7.5 in., is considered.

Due to camber of the precast, prestressed beam, a haunch thickness of 0.5 in. at midspan is considered in the structural properties of the composite section. Also, the width of the haunch must be transformed.

Transformed width of haunch = n(Top flange width)

$$= (0.7846)(42) = \quad 32.95 \quad in$$

Transformed area of haunch = n(Top flange width)(H_{eff})

$$= (0.7846)(42)(0.5) = \quad 16.48 \quad in^2$$

where H_{eff} is the effective haunch depth

Note that the haunch should only be considered to contribute to section properties if it is required to be provided in the completed structure. Therefore, some designers neglect its contribution to the section properties.
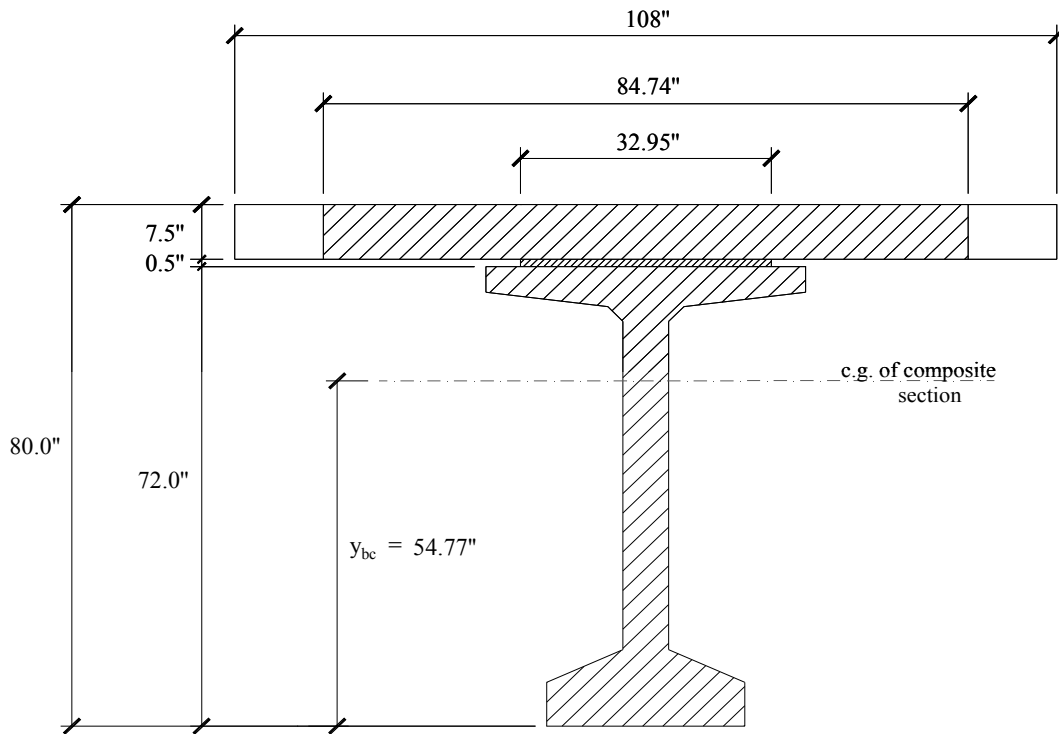
Figure 3.2.3-1: Dimensions of the Composite Section

Table 3-1: Properties of the Composite Section

	Area, in^2	y_b, in	Ay_b, in^3	$A(y_{bc} - y_b)^2$, in^4	I, in^4	$I+A(y_{bc} - y_b)^2$, in^4
Beam	767.00	36.60	28,072.20	253,224.21	545,894.00	799,118.21
Haunch	16.48	72.25	1,190.68	5,035.47	0.34	5,035.81
Deck	635.50	76.25	48,456.88	293,213.60	2,979.14	296,192.74
Σ	1,418.98		77,719.76			1,100,346.76

A_c = total area of composite section = $\qquad$ 1,419 $\ \text{in}^2$

h_c = overall depth of composite section = $\qquad$ 80.0 $\ \text{in}$

I_c = moment of inertia of composite section = $\qquad$ 1,100,347 $\ \text{in}^4$

y_{bc} = distance from the centroid of the composite section to the beam extreme bottom fiber
$$= 77{,}719.76\ /\ 1{,}418.98 =\ 54.77\ \text{in}$$

y_{tg} = distance from the centroid of the composite section to the beam extreme top fiber
$$= 72.0 - 54.77 =\ 17.23\ \text{in}$$

y_{tc} = distance from the centroid of the composite section to the deck extreme top fiber
$$= 80.0 - 54.77 =\ 25.23\ \text{in}$$

S_{bc} = composite section modulus for the beam extreme bottom fiber
$$= \frac{I_c}{y_{bc}} = \frac{1{,}100{,}347}{54.77} =\ 20{,}090\ \text{in}^3$$

S_{tg} = composite section modulus for the beam extreme top fiber
$$= \frac{I_c}{y_{tg}} = \frac{1{,}100{,}347}{17.23} =\ 63{,}862\ \text{in}^3$$

S_{tc} = composite section modulus for the deck extreme top fiber
$$= \left(\frac{1}{n}\right)\left(\frac{I_c}{y_{tc}}\right) = \frac{1{,}100{,}347}{0.7846 \times 25.23} = 55{,}586\ \text{in}^3$$

4 SHEAR FORCES AND BENDING MOMENTS

The self-weight of the beam and the weight of the deck and haunch act on the non-composite, simple-span structure, while the weight of rails/barriers, future wearing surface (overlay), and live loads with impact act on the composite, simple-span structure. A summary of the unfactored load values, calculated below, is presented in Table 4-1 at the end of this section.

4.1 Shear Forces and Bending Moments Due to Dead Loads

4.1.1 Dead Loads [AASHTO Art. 3.3.2]

DC = Dead load of structural components and non-structural attachments

DW = Dead load of future wearing surface (overlay) and utilities

Dead loads acting on the non-composite structure:

$$\text{Beam self weight} =\ 0.666\ \text{kip/ft}$$
$$\text{Deck weight} = (8.0/12\ \text{ft})(9.0\ \text{ft})(0.125\ \text{kcf}) =\ 0.750\ \text{kip/ft}$$
$$\text{Haunch weigth} = (0.50\ \text{in})(42\ \text{in})(0.125\ \text{kcf})\ /\ 144 =\ 0.018\ \text{kip/ft}$$
$$DC = 0.666 + 0.750 + 0.018 =\ 1.434\ \text{kip/ft}$$

Notes:

1. Actual deck thickness is used for computing dead load.

2. The effective haunch depth, which is considered to be constant along the length of the beam, is used for computing dead load.

3. The weight of cross-diaphragms is ignored since most agencies are changing from cast-in-place concrete diaphragms to lightweight steel diaphragms.

Dead loads acting on the composite structure:

AASHTO Article 4.6.2.2.1 states that permanent loads (curbs and future wearing surface) may be distributed uniformly among all beams if the following conditions are met:

- Width of deck is constant **OK**

- Number of beams is not less than four ($N_b = 6$) **OK**

- Beams are parallel and have approximately the same stiffness **OK**

- The roadway part of the overhang, $d_e \leq 3.0$ ft $= 3.0 - 1.5 = 1.5$ ft **OK**

- Curvature in plan is less than specified limit (curvature $= 0.0°$) **OK**

- Cross-section of the bridge is consistent with one of the cross-sections given in AASHTO Table 4.6.2.2.1-1 (bridge is Type k) **OK**

Since these criteria are satisfied, the rail/barrier and wearing surface loads may be equally distributed among the 6 beams.

DC = Rail/barrier weight = (2)(0.300 kip/ft) / (6 beams) =   0.100   kip/ft

DW = (2.0 in/12)(0.150 kcf)(48.0 ft) / (6 beams) =   0.200   kip/ft

4.1.2 Unfactored Shear Forces and Bending Moments

For a simply supported beam with span (L) loaded with a uniformly distributed load (w), the shear force (V_x) and bending moment (M_x) at any distance (x) from the support are given by:

$$V_x = w(0.5L - x)$$ (Eq. 4.1.2-1)

$$M_x = 0.5wx(L - x)$$ (Eq. 4.1.2-2)

Using the above equations, values of shear forces and bending moments for a typical interior beam due to the self-weight of the beam, weight of the slab and haunch, weight of rails/barriers and future wearing surface are computed as shown in Table 4-1. For these calculations, the span length (L) is the design span, 120.0 ft. However, for calculations of stresses and deformation at the time prestress is released, the overall length of the precast member, 121.0 ft, is used as illustrated later in this example.

4.2 Shear Forces and Bending Moments Due to Live Loads

4.2.1 Live Loads [AASHTO Art. 3.6.1.2]

Design live load is HL-93 which consists of a combination of:

1. Design truck or design tandem with dynamic allowance
   The design truck consists of one 8.0-kip axle and two rear 32.0-kip axles. The front 8.0-kip axle and the first 32.0-kip axle are 14.0 ft apart. The spacing between the two 32.0-ft axles shall be varied between 14.0 ft and 30.0 ft to produce extreme force effects. The design tandem consists of a pair of 25.0-kip axles spaced at 4.0 ft apart with the transverse spacing of the wheels equal to 6.0 ft. The design tandem generally controls for short spans (L < 40 ft).

2. Design lane load of 0.64 kip/ft assumed to be distributed over a 10.0-ft width without dynamic allowance

4.2.2 Live Load Distribution Factor for Typical Interior Beam

The live load bending moments and shear forces are determined by using the simplified distribution factor formulas (AASHTO Art. 4.6.2.2). To use the simplified live load distribution factor formulas, the following conditions must be met (AASHTO Art. 4.6.2.2.1):

• Width of deck is constant			**OK**
• Number of beams is not less than four ($N_b = 6$)			**OK**
• Beams are parallel and have approximately the same stiffness			**OK**
• The roadway part of the overhang, $d_e \leq 3.0$ ft $= 3.0 - 1.5 = 1.5$ ft			**OK**
• Curvature in plan is less than specified limit (curvature $= 0.0°$)			**OK**

• Cross-section of the bridge is consistent with one of the cross-sections given
  in AASHTO Table 4.6.2.2.1-1 (bridge is Type k)   **OK**

Since these criteria are satisfied, the simplified live load distribution factor formulas may be used.

The number of design lanes is computed as: [AASHTO Art. 3.6.1.1.1]

    Number of design lanes = the integer part of the ratio of (w/12) where (w) is the clear roadway width, in ft, between
                        the curbs and/or rails/barriers

    From Figure 1-1, w = 48.0 ft

    Number of design lanes = the integer part of (48.0/12) = 4 lanes

4.2.2.1 Distribution Factor for Bending Moment

The following formulas for live load distribution factors apply to interior beams in bridges of Type (a), (e) or (k), and also (i) or (j) if sufficiently connected to act as a unit (bridge types from AASHTO Table 4.6.2.2.1-1). Different formulas, found in AASHTO Table 4.6.2.2.2b-1, should be used for bridges of different types.

 • For all limit states except fatigue limit state:

    For two or more lanes loaded:

$$DFM = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \left(\frac{S}{L}\right)^{0.2} \left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1} \qquad \text{[AASHTO Table 4.6.2.2.2b-1]}$$

provided that:

$3.5 \leq S \leq 16.0$;	$S =$	9.00	ft	**OK**	
$4.5 \leq t_s \leq 12.0$;	$t_s =$	7.5	in	**OK**	
$20 \leq L \leq 240$;	$L =$	120.0	ft	**OK**	
$N_b \geq 4$;	$N_b =$	6		**OK**	
$10{,}000 \leq K_g \leq 7{,}000{,}000$	$K_g =$	2,232,732		**OK**	(see calculations below)

where: DFM = distribution factor for moment for interior beam

        S = beam spacing, ft

        t_s = depth of concrete slab, in

        L = beam span, ft

        N_b = number of beams

        K_g = longitudinal stiffness parameter, in^4, $= n(I + Ae_g^2)$ [AASHTO Eq. 4.6.2.2.1-1]

        where: n = modular ratio between beam and slab materials

$$= \frac{E_c(beam)}{E_c(slab)} = \frac{3{,}718}{2{,}917} = 1.2746$$

         A = cross-sectional area of the beam (non-composite section) (in^2)

         I = moment of inertia of the beam (non-composite section) (in^4)

e_g = distance between the centers of gravity of the beam and slab (in)

$= (t_s/2 + H_{eff} + y_t) = (7.5/2 + 0.5 + 35.40) = \quad 39.65 \quad$ in

$K_g = 1.2746 \, [\, 545{,}894 + 767 \, (39.65)^2 \,] = \quad\quad 2{,}232{,}732 \quad$ in^4

Therefore,

$$\text{DFM} = 0.075 + \left(\frac{9.00}{9.5}\right)^{0.6} \left(\frac{9.00}{120.0}\right)^{0.2} \left(\frac{2{,}232{,}732}{12.0(\,120.0\,)(\,7.5\,)^3}\right)^{0.1}$$

$= 0.075 + (0.968)\,(0.596)\,(1.139) = \quad 0.732 \quad$ lanes/beam

For one design lane loaded:

$$\text{DFM} = 0.06 + \left(\frac{S}{14}\right)^{0.4} \left(\frac{S}{L}\right)^{0.3} \left(\frac{K_g}{12.0 L t_s^3}\right)^{0.1} \qquad\qquad \text{[AASHTO Table 4.6.2.2.2b-1]}$$

$$= 0.06 + \left(\frac{9.00}{14}\right)^{0.4} \left(\frac{9.00}{120.0}\right)^{0.3} \left(\frac{2{,}232{,}732}{12.0(120.0)(\,7.5\,)^3}\right)^{0.1}$$

$= 0.06 + (0.838)\,(0.460)\,(1.139) = \quad 0.499 \quad$ lanes/beam

Thus, the case of two or more lanes loaded controls.   DFM = $\quad 0.732 \quad$ lanes/beam

• For fatigue limit state:

> The Commentary of Article 3.4.1 in the AASHTO Specifications states that for fatigue limit state, a single design truck should be used. However, live load distribution factors given in AASHTO Article 4.6.2.2 take into consideration the multiple presence factor, m. AASHTO Article 3.6.1.1.2 states that the multiple presence factor, m, for one design lane loaded is 1.2. Therefore, the distribution factor for one design lane loaded with the multiple presence factor removed should be used.
> The distribution factor for fatigue limit state is: 0.499 / 1.2 = 0.416 lanes/beam

4.2.2.2 Distribution Factor for Shear Force

The following formulas for live load distribution factors apply to interior beams in bridges of Type (a), (e) or (k), and also (i) or (j) if sufficiently connected to act as a unit (bridge types from AASHTO Table 4.6.2.2.1-1). Different formulas, found in AASHTO Table 4.6.2.2.3a-1, should be used for bridges of different types.

For two or more lanes loaded:

$$\text{DFV} = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^{2.0} \qquad\qquad \text{[AASHTO Table 4.6.2.2.3a-1]}$$

provided that: $3.5 \le S \le 16.0;$ S =   9.00   ft   **OK**

                 $4.5 \le t_s \le 12.0;$ t_s =   7.5   in   **OK**

                 $20 \le L \le 240;$ L =   120.0   ft   **OK**

                 $N_b \ge 4;$ N_b =   6   **OK**

where: DFV = distribution factor for shear for interior beam

        S = beam spacing, ft

Therefore,

$$\text{DFV} = 0.2 + \left(\frac{9.00}{12}\right) - \left(\frac{9.00}{35}\right)^{2.0}$$

$= 0.2 + (0.750) - (0.066) = \quad 0.884 \quad$ lanes/beam

For one design lane loaded:

$$DFV = 0.36 + \left(\frac{S}{25.0}\right) \qquad \text{[AASHTO Table 4.6.2.2.3a-1]}$$

$$= 0.36 + \left(\frac{9.00}{25.0}\right)$$

$$= 0.36 + (0.360) = 0.720 \quad \text{lanes/beam}$$

Thus, the case of two or more lanes loaded controls and DFV = 0.884 lanes/beam

4.2.3 Dynamic Load Allowance [AASHTO Art. 3.6.2.1]

IM = dynamic load allowance (applied to truck only)

IM for fatigue = 15% [AASHTO Table 3.6.2.1-1]

IM for all other limit states = 33%

4.2.4 Unfactored Shear Forces and Bending Moments

4.2.4.1 Due to Truck Load, V_{LT} and M_{LT}

Live loads must be placed on the span so as to create the maximum force effects. The following formulas (from Section 8.11 of the PCI Bridge Design Manual, 2003) calculate the maximum bending moment and shear force per lane at any point on a span due to the design truck loading (the design truck load controls over the design tandem load for spans greater than 40 ft.).

- For Truck Load with Impact:

$$V_x = \frac{72\,[(L - x) - 9.33]}{L} \quad \text{for } 0 < x/L < 0.5 \qquad \text{(Eq. 4.2.4.1-1)}$$

$$M_x = \frac{72\,(x)\,[(L - x) - 9.33]}{L} \quad \text{for } 0 < x/L < 0.333 \qquad \text{(Eq. 4.2.4.1-2)}$$

$$M_x = \left[\frac{72\,(x)\,[(L - x) - 4.67]}{L} - 112\right] \quad \text{for } 0.333 < x/L < 0.5 \quad \text{and} \quad x > 14 \qquad \text{(Eq. 4.2.4.1-3)}$$

- For Fatigue Truck with Impact:

$$M_x = \frac{72\,(x)\,[(L - x) - 18.22]}{L} \quad \text{for } 0 < x/L < 0.241 \qquad \text{(Eq. 4.2.4.1-4)}$$

$$M_x = \left[\frac{72\,(x)\,[(L - x) - 11.78]}{L} - 112\right] \quad \text{for } 0.241 < x/L < 0.5 \quad \text{and} \quad x > 14 \qquad \text{(Eq. 4.2.4.1-5)}$$

- For all limit states except fatigue limit state:

Truck load shear force and bending moments per beam are:

V_{LT} = (shear force per lane)(DFV)(1 + IM)

= (shear force per lane)(0.884)(1 + 0.33)

= (shear force per lane)(1.176) kips

M_{LT} = (bending moment per lane)(DFM)(1 + IM)

= (bending moment per lane)(0.732)(1 + 0.33)

= (bending moment per lane)(0.974) ft-kips

Values of V_{LT} and M_{LT} are calculated at tenth-points along the span and are presented in Table 4-1.

• For fatigue limit state:

Art. 3.6.1.4.1 in the AASHTO Specifications states that the fatigue load is a single design truck which has the same axle weight used in all other limit states but with a constant spacing of 30.0 ft between the 32.0-kip axles.

Therefore, the bending moment of the fatigue truck load is:

M_f = (bending moment per lane)(DFM)(1 + IM)

     = (bending moment per lane)(0.416)(1 + 0.15)

     = (bending moment per lane)(0.478) ft-kips

Values of M_f are calculated at tenth-points along the span and are presented in Table 4-1.

4.2.4.2 Due to Design Lane Load, V_{LL} and M_{LL}

To obtain the maximum shear force at a section located at a distance (x) from the left support under a uniformly distributed load of 0.64 kip/ft, load the member to the right of the section under consideration as shown in Figure 4.2.4.2-1. Therefore, the maximum shear force per lane is:

$$V_x = \frac{w(L-x)^2}{2L} \text{ for } x \leq 0.5L \tag{Eq. 4.2.4.2-1}$$

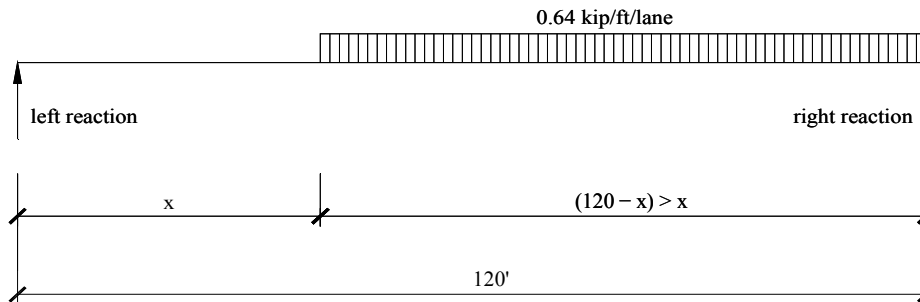where V_x is in kip/lane, L and x are in ft, and w=0.64 kip/ft

Figure 4.2.4.2-1: Maximum Shear Force Due to Design Lane Load

To calculate the maximum bending moment at any section, use Eq. 4.1.2-2:

$$M_x = \frac{wx(L-x)}{2} \tag{Eq. 4.1.2-2.}$$

where M_x is in kip-ft/lane, L and x are in ft, and w=0.64 kip/ft

Lane load shear force and bending moment per typical interior beam are as follows:

V_{LL} = (lane load shear force)(DFV)

     = (lane load shear force)(0.884) kips

For all limit states except for fatigue limit state:

M_{LL} = (lane load bending moment)(DFM)

     = (lane load bending moment)(0.732) ft-kips

Note that the dynamic allowance is not applied to the design lane loading.

Values of V_{LL} and M_{LL} are calculated at tenth-points along the span and are presented in Table 4-1.

4.3 Load Combinations [AASHTO Art. 3.4]

The total factored load shall be taken as:

$Q = \Sigma\eta_i\gamma_iQ_i$ [AASHTO Eq. 3.4.1-1]

where: η_i =   factors relating to ductility, redundancy and operational classification [AASHTO Art. 1.3.2]
(Here, η_i are considered to be 1.0)

      γ_i =   load factors [AASHTO Table 3.4.1-1]

      Q_i =   force effects (from specified loads)

Investigating different limit states given in AASHTO Article 3.4.1, the following limit states are applicable:

• Service I: check compressive stresses in prestressed concrete components:

$Q = 1.00(DC + DW) + 1.00(LL + IM)$ [AASHTO Table 3.4.1-1]

This load combination is the general combination for service limit state stress checks and applies to all conditions other than Service III.

• Service III: check tensile stresses in prestressed concrete components:

$Q = 1.00(DC + DW) + 0.80(LL + IM)$ [AASHTO Table 3.4.1-1]

This load combination is a special combination for service limit state stress checks that applies only to tension in prestressed concrete structures to control cracks.

• Strength I: check ultimate strength:

Maximum $Q = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)$ [AASHTO Tables 3.4.1-1 & 2]

Minimum $Q = 0.90(DC) + 0.65(DW) + 1.75(LL + IM)$ [AASHTO Tables 3.4.1-1 & 2]

This load combination is the general load combination for strength limit state design.

Note: For simple-span bridges, the maximum load factors produce maximum effects. However, use minimum load factors for dead load (DC), and wearing surface (DW) when dead load and wearing surface stresses are opposite to those of live load.

• Fatigue I: check stress range in strands:

$Q = 1.50(LL + IM)$ [AASHTO Table 3.4.1-1]

This load combination is a special load combination to check the tensile stress range in the strands due to live load and dynamic allowance.

Table 4-1: Unfactored Shear Forces and Bending Moments for a Typical Interior Beam

Distance x	Section x / L	Beam Weight		(Slab + Haunch) Weight		Rail / Barrier Weight		Wearing Surface	
		Shear	Moment M_g	Shear	Moment M_s	Shear	Moment M_b	Shear	Moment M_{ws}
ft		kips	kip-ft	kips	kip-ft	kips	kip-ft	kips	kip-ft
0	0	40.0	0.0	46.1	0.0	6.0	0.0	12.0	0.0
*6.15	0.05	35.9	233.2	41.4	268.9	5.4	35.0	10.8	70.0
12	0.1	32.0	431.6	36.9	497.7	4.8	64.8	9.6	129.6
24	0.2	24.0	767.2	27.6	884.7	3.6	115.2	7.2	230.4
36	0.3	16.0	1,007.0	18.4	1,161.2	2.4	151.2	4.8	302.4
48	0.4	8.0	1,150.8	9.2	1,327.1	1.2	172.8	2.4	345.6
60	0.5	0.0	1,198.8	0.0	1,382.4	0.0	180.0	0.0	360.0

* Critical section for shear (see Section 11)

Table 4-1 continued

Distance x	Section x / L	Truck Load with Impact		Lane Load		Fatigue Truck with Impact
		Shear V_{LT}	Moment M_{LT}	Shear V_{LL}	Moment M_{LL}	Moment M_f
ft		kips	kip-ft	kips	kip-ft	kip-ft
0	0	78.1	0.0	33.9	0.0	0.0
*6.15	0.05	73.7	375.5	30.6	164.0	168.8
12	0.1	69.6	691.6	27.5	303.6	309.2
24	0.2	61.1	1,215.0	21.7	539.7	535.8
36	0.3	52.7	1,570.2	16.6	708.3	692.7
48	0.4	44.2	1,778.8	12.2	809.5	776.1
60	0.5	35.7	1,830.2	8.5	843.3	776.9

* Critical section for shear (see Section 11)

5 ESTIMATE REQUIRED PRESTRESS

The required number of strands is usually governed by concrete tensile stresses at the bottom fiber for load combination Service III at the section of maximum moment or at the harp points. For estimating the number of strands, only the stresses at midspan are considered.

5.1 Service Load Stresses at Midspan

Bottom tensile stress due to applied dead and live loads using load combination Service III is:

$$f_b = \frac{M_g + M_s}{S_b} + \frac{M_b + M_{ws} + (0.8)(M_{LT} + M_{LL})}{S_{bc}}$$

where: f_b = bottom tensile stress, ksi

M_g = unfactored bending moment due to beam self-weight (kip-ft)

M_s = unfactored bending moment due to slab and haunch weights (kip-ft)

M_b = unfactored bending moment due to rail/barrier weight (kip-ft)

M_{ws} = unfactored bending moment due to future wearing surface weight (kip-ft)

M_{LT} = unfactored bending moment due to truck load (kip-ft)

M_{LL} = unfactored bending moment due to lane load (kip-ft)

S_b = section modulus for extreme bottom fiber of non-composite section (in^3)

S_{bc} = composite section modulus for extreme bottom fiber of precast beam (in^3)

Using values of bending moments from Table 4-1 and section properties derived in Sections 3.1 and 3.2, the bottom tensile stress at midspan is:

$$f_b = \frac{(1,198.8 + 1,382.4)}{14,915}(12) + \frac{(180.0 + 360.0) + (0.8)(1,830.2 + 843.3)}{20,090}(12)$$

$$= 2.077 + 1.600 = 3.677 \text{ ksi}$$

5.2 Stress Limits for Concrete

The tensile stress limit at service loads = $0.19\sqrt{f'_c}$ [AASHTO Table 5.9.4.2.2-1]

where: f'_c = specified 28-day concrete strength of beam, ksi

Concrete tensile stress limit = $0.19\sqrt{6.5}$ = -0.484 ksi

5.3 Required Number of Strands

The required pre-compressive stress at the bottom fiber of the beam is the difference the between bottom tensile stress due to the applied loads and the concrete tensile stress limits:

Required pre-compressive stress at bottom fiber, f_{pb} = (3.677 − 0.484) = 3.193 ksi

The location of the strand center of gravity at midspan ranges from 5 to 15% of the beam depth measured from the bottom of the beam. A value of 5% is appropriate for newer efficient sections like the bulb-tee beams and 15% for less efficient AASHTO standard shapes.

Assume the distance between the center of gravity of the bottom strands and the bottom fiber of the beam:

y_{bs} = 0.05 h = 0.05 (72) =   3.60 in   Use y_{bs} =   4.0 in

Therefore, the strand eccentricity at midspan, e_c = (y_b − y_{bs}) = (36.6 − 4.0) = 32.6 in

If P_{pe} is the total prestressing force, the stress at the bottom fiber due to prestress is:

$$f_{pb} = \frac{P_{pe}}{A} + \frac{P_{pe}e_c}{S_b} \text{ , or, } 3.193 \ = \frac{P_{pe}}{767} + \frac{P_{pe}(32.6)}{14{,}915}$$

Solving for P_{pe}, the required P_{pe} =   915.0   kips

Final prestress force per strand, f_{pf} = (area of strand) (f_{pi}) (1 − losses, %)

where: f_{pi} = initial stress before transfer, ksi (see Section 2)   = 202.5 ksi

An initial assumption of the percent loss in prestress must be made in order to determine the required number of strands. Once the number of strands has been determined, the actual prestress loss can be calculated. The assumed prestress loss shall be compared to the calculated loss and iterated until they are equal to determine the correct number of required strands.

After iterating, the final loss in prestress was determined to be 23% of f_{pi}.

Therefore, the prestress force per strand after all losses, f_{pf}, is:

f_{pf} = (0.153) (202.5) (1 − 0.23) =   23.9 kips

The number of strands required is equal to (P_{pe}/f_{pf}) = (915.0/23.9) =   38.3 strands

As an initial trial, set number of strands, n_p, equal to: 40

The center of gravity of the strands, c.g.s. = ($\Sigma n_i y_i$) / N

where: n_i = number of strands in row i

   y_i = distance to center of row i from bottom of beam section

   N = total number of strands

c.g.s = [(2 x 12)+(2 x 10)+(4 x 8)+(8 x 6)+(12 x 4)+(12 x 2)] / 40 = 4.90 in

The center of gravity of 40 strands at midspan is 4.90 in. from the bottom of the concrete, which is higher than the assumed value of 4.0 in. Thus, iterations must be performed using the new value of strand eccentricity until the values converge:

Iteration =	1
Assumed number of strands =	40
c.g.s. of assumed strands, y_{bs} (in) =	4.90
Midspan eccentricity, e_c (in) =	31.70
Required prestressing force, P_{pe} (kips) =	931.1
Number of required strands =	40
c.g.s. of required strands, y_{bs} (in) =	4.90
Assumed y_{bs} = Required y_{bs}?	yes

5.4 Strand Pattern

The required number of strands = 40

The strand pattern at midspan for 40 strands is shown in Figure 5.4-1.

Midspan center of gravity of strands, y_{bs} = 4.90 in

Midspan strand eccentricity, $e_c = (y_b - y_{bs}) = (36.60 - 4.90) = 31.70$ in

No. Strands	Distance from bottom (in)
2	12.00
2	10.00
4	8.00
8	6.00
12	4.00
12	2.00

Row Spacing: 2.00 in
Column Spacing: 2.00 in
Side Cover: 2.00 in
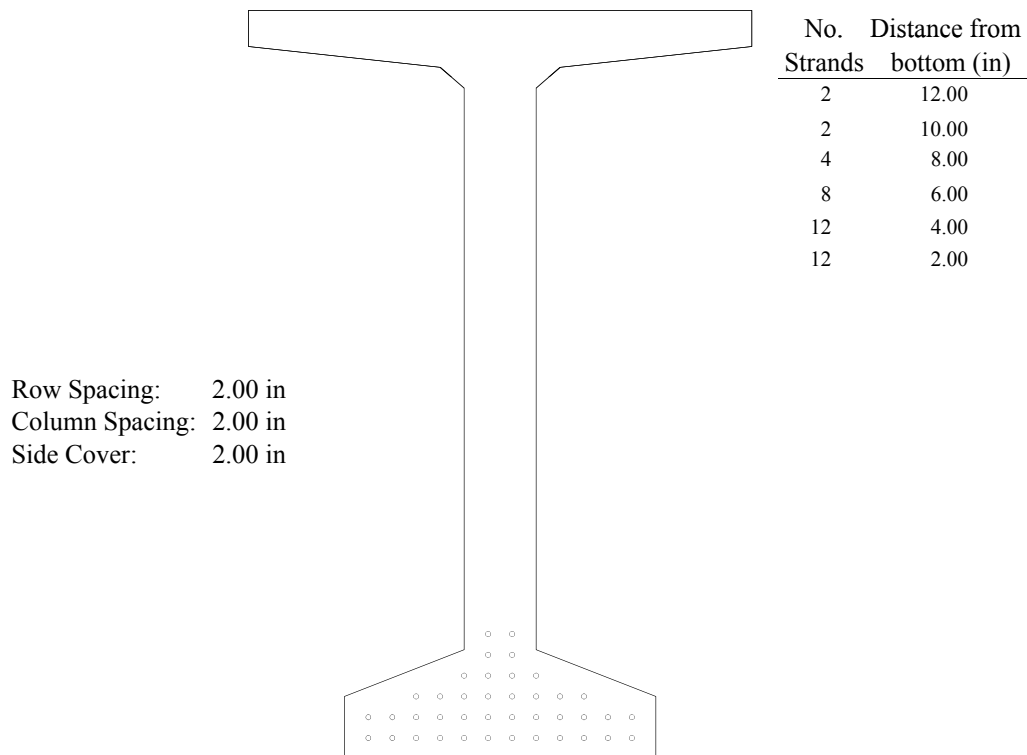
Figure 5.4-1: Assumed Strand Pattern at Midspan

6 PRESTRESS LOSSES

Total prestress loss:

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} \qquad\qquad \text{[AASHTO Eq. 5.9.5.1-1]}$$

where: Δf_{pES} = sum of all losses or gains due to elastic shortening or extension at time of application of prestress and/ or external loads (ksi)

Δf_{pLT} = losses due to long-term shrinkage and creep of concrete, and relaxation of the steel (ksi)

6.1 Elastic Shortening

When the prestressing force is transferred from the prestressing strands to the concrete member, the force causes elastic shortening of the member as it cambers upward. This results in a loss of the initial prestress of the strands. However, some of that loss is gained back due to the self-weight of the member which creates tension in the strands.

$$\Delta f_{pES} = \frac{E_p}{E_{ct}} f_{cgp} \qquad\qquad \text{[AASHTO Eq. 5.9.5.2.3a-1]}$$

where: E_p = modulus of elasticity of prestressing steel (ksi) =   28,500 ksi

E_{ct} = modulus of elasticity of the concrete at transfer or time of load application (ksi) = E_{ci} = 3,512 ksi

f_{cgp} = the concrete stress at the center of gravity of prestressing tendons due to the prestressing force immediately after transfer and the self-weight of the member at sections of maximum moment (ksi)

AASHTO Article C5.9.5.2.3a states that to calculate the prestress after transfer, an initial estimate of prestress loss is assumed and iterated until acceptable accuracy is achieved. In this example, an initial estimate of 10% is assumed.

Force per strand at transfer = (area of strand)(prestress before transfer)(1 − % loss)

$$= (0.153)(202.5)(1 - 0.10) = 27.9 \text{ kips}$$

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c{}^2}{I} - \frac{M_g e_c}{I}$$

where: e_c = eccentricity of strands measured from the center of gravity of the precast beam at midspan

P_i = total prestressing force at release =   (40 strands) (27.9) =   1,116.0   kips

M_g = moment due to beam self-weight at midspan (should be calculated using the overall beam length = 121 ft)

Note: Because the elastic shortening loss is part of the total loss, f_{cgp} is conservatively computed based on M_g using the design span length of 120 ft.

$$f_{cgp} = \frac{1,116.0}{767} + \frac{(1,116.0)(31.70)^2}{545,894} - \frac{[(1,198.8)(12)](31.70)}{545,894}$$

$$= 1.455 + 2.054 - 0.835 = 2.674 \quad \text{ksi}$$

Therefore, loss due to elastic shortening:

$$\Delta f_{pES} = \frac{28,500}{3,512} (2.674) = 21.700 \text{ ksi}$$

Actual percent loss due to elastic shortening = (21.700 / 202.5) (100) = 10.7 %

Since the calculated loss of 10.7% is not equal to the initial estimate of 10%, further iterations must be performed.

Iteration =	1	2
Initial estimate of prestress loss (%) =	10.7	10.6
Force per strand at transfer (kips) =	27.7	27.7
Total prestress force at release, P_i (kips) =	1,108.0	1,108.0
Concrete stress due to prestress, f_{cgp} (ksi) =	2.649	2.649
Prestress loss due to elastic shortening, Δf_{pES} (ksi) =	21.497	21.497
Actual percent loss of prestress (%) =	10.6	10.6
Initial loss estimate = Actual loss ?	no	yes

Therefore, the percent loss due to elastic shortening = 10.6 %

The concrete stress due to prestress, f_{cgp} = 2.649 ksi

The prestress loss due to elastic shortening, Δf_{pES} = 21.497 ksi

The force per strand at transfer = 27.7 kips

6.2 Long Term Losses

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})_{df} \qquad \text{[AASHTO Eq. 5.9.5.4.1-1]}$$

where: Δf_{pSR} = prestress loss due to shrinkage of girder concrete between time of transfer and deck placement (ksi)

Δf_{pCR} = prestress loss due to creep of girder concrete between time of transfer and deck placement (ksi)

Δf_{pR1} = prestress loss due to relaxation of prestressing strands between time of transfer and deck placement (ksi)

Δf_{pSD} = prestress loss due to shrinkage of girder concrete between time of deck placement and final time (ksi)

Δf_{pCD} = prestress loss due to creep of girder concrete between time of deck placement and final time (ksi)

Δf_{pR2} = prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time (ksi)

Δf_{pSS} = prestress gain due to shrinkage of deck in composite section (ksi)

$(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id}$ = sum of time-dependent prestress losses between time of transfer and deck placement (ksi)

$(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})_{df}$ = sum of time-dependent prestress losses after deck placement (ksi)

6.2.1 Prestress Losses: Time of Transfer to Time of Deck Placement

6.2.1.1 Shrinkage of Girder Concrete

$$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id} \qquad \text{[AASHTO Eq. 5.9.5.4.2a-1]}$$

where: ε_{bid} = shrinkage strain of girder between the time of transfer and deck placement

$= k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3}$ [AASHTO Eq. 5.4.2.3.3-1]

where: k_s = factor for the effect of the volume-to-surface ratio of the component

$= 1.45 - 0.13 \, (V/S) \geq 1.0$ [AASHTO Eq. 5.4.2.3.2-2]

where: (V/S) = volume-to-surface ratio (in) = (Area / Perimeter)

Area = 767 in²

Perimeter = $2 \, [(42.0/2) + 3.5 + \sqrt{(2.0^2 + 16.0^2)} + \sqrt{(2.0^2 + 2.0^2)} + 54.0 +$

$\sqrt{(4.5^2 + 10.0^2)} + 6.0 + (26.0/2)] = 254.8$ in

(V/S) = (Area / Perimeter) = 767 / 254.8 = 3.01 in

$= 1.45 - 0.13 \, (3.01) = 1.06 \qquad > 1.0$

k_{hs} = humidity factor for shrinkage

= $2.00 - 0.014\,H$ [AASHTO Eq. 5.4.2.3.3-2]

where: H = relative humidity (%) = 70 %

= $2.00 - 0.014\,(70) =$ 1.02

k_f = factor for the effect of concrete strength

= $\dfrac{5}{1 + f'_{ci}} = \dfrac{5}{1 + 5.8} =$ 0.735   [AASHTO Eq. 5.4.2.3.2-4]

k_{td} = time development factor

= $\dfrac{t}{61 - 4f'_{ci} + t}$ [AASHTO Eq. 5.4.2.3.2-5]

where: t = maturity of concrete (days)

= $t_d - t_i$ for time between transfer and deck placement

where: t_d = age of concrete at time of deck placement (days) = 90 days

t_i = age of concrete at time of prestress transfer (days) = 1 day

= $\dfrac{90 - 1}{61 - 4(5.8) + (90 - 1)} =$ 0.702

ε_{bid} = $(1.06)(1.02)(0.735)(0.702)(0.00048) =$ 0.000268

K_{id} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between transfer and deck placement

= $\dfrac{1}{1 + \dfrac{E_p}{E_{ci}}\dfrac{A_{ps}}{A_g}\left(1 + \dfrac{A_g e_{pg}^2}{I_g}\right)[1 + 0.7\Psi_b(t_f,t_i)]}$ [AASHTO Eq. 5.9.5.4.2a-2]

where: e_{pg} = eccentricity of prestressing force with respect to centroid of girder (in); positive in common construction where force is below centroid = e_c = 31.70 in

A_{ps} = area of the prestressing steel (in²) = A_p (no. of strands) = 6.120 in²

A_g = gross area of non-composite concrete section (in²) = A = 767 in²

I_g = moment of inertia of the gross concrete section (in⁴) = I = 545,894 in⁴

$\Psi_b(t_f,t_i)$ = girder creep coefficient at final time due to loading introduced at transfer

= $1.9\,k_s\,k_{hc}\,k_f\,k_{td}\,t_i^{-0.118}$ [AASHTO Eq. 5.4.2.3.2-1]

where: k_{hc} = humidity factor for creep

= $1.56 - 0.008\,H$ [AASHTO Eq. 5.4.2.3.2-3]

= $1.56 - 0.008\,(70) =$ 1.00

k_{td} = time development factor

= $\dfrac{t}{61 - 4f'_{ci} + t}$ [AASHTO Eq. 5.4.2.3.2-5]

where: t = maturity of concrete (days)

= $t_f - t_i$ for time between transfer and final time

where: t_f = final age of concrete (days) = 20000 days

$$= \frac{20,000 - 1}{61 - 4(5.8) + (20,000 - 1)} = 0.998$$

$$= 1.9(1.06)(1.00)(0.735)(0.998)(1.000) = 1.477$$

$$K_{id} = \frac{1}{1 + \dfrac{28,500}{3,512} \dfrac{6.120}{767} \left(1 + \dfrac{(767)(31.70)^2}{545,894}\right) [1 + 0.7(1.477)]} = 0.759$$

Therefore, $\Delta f_{pSR} = (0.000268)(28,500)(0.759) = 5.797$ ksi

6.2.1.2 Creep of Girder Concrete

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \Psi_b(t_d,t_i) K_{id} \qquad \text{[AASHTO Eq. 5.9.5.4.2b-1]}$$

where: $\Psi_b(t_d,t_i)$ = girder creep coefficient at time of deck placement due to loading introduced at transfer

$$= 1.9 \, k_s \, k_{hc} \, k_f \, k_{td} \, t_i^{-0.118} \qquad \text{[AASHTO Eq. 5.4.2.3.2-1]}$$

where: k_{td} = time development factor

$$= \frac{t}{61 - 4f'_{ci} + t} \qquad \text{[AASHTO Eq. 5.4.2.3.2-5]}$$

where: t = maturity of concrete (days)

$= t_d - t_i$ for time between transfer and deck placement

$= 0.702$ (as calculated above)

$$= 1.9(1.06)(1.00)(0.735)(0.702)(1.000) = 1.039$$

Therefore, $\Delta f_{pCR} = \dfrac{28,500}{3,512}(2.649)(1.039)(0.759) = 16.952$ ksi

6.2.1.3 Relaxation of Prestressing Strands

$$\Delta f_{pR1} = \left[\frac{f_{pt}}{K'_L} \frac{\log(24t)}{\log(24t_i)} \left(\frac{f_{pt}}{f_{py}} - 0.55\right)\right]\left[1 - \frac{3(\Delta f_{pSR} + \Delta f_{pCR})}{f_{pt}}\right] K_{id} \qquad \text{[AASHTO Eq. C5.9.5.4.2c-1]}$$

where: f_{pt} = stress in prestressing strands immediately after transfer, taken not less than $0.55f_{py}$

$= f_{pi} - \Delta f_{pES} = 202.5 - 21.497 = 181.003$ ksi $>$ $0.55f_{py} = 0.55(243.0) = 133.650$ ksi

use $f_{pt} = 181.003$ ksi

K'_L = 45 for low relaxation steel and 10 for stress relieved steel = 45

t = time between strand prestressing and deck placement (days)

$= t_p + t_d$

where: t_p = time between strand prestressing and concrete casting (days) = 1 day

$= 1 + 90 = 91$ days

Therefore, $\Delta f_{pR1} = \left[\dfrac{(181.003)}{(45)} \dfrac{\log(24 \times 91)}{\log(24 \times 1)} \left(\dfrac{181.003}{243.0} - 0.55\right)\right]\left[1 - \dfrac{3(5.797 + 16.952)}{181.003}\right](0.759) = 0.897$ ksi

6.2.2 Prestress Losses: Time of Deck Placement to Final Time

6.2.2.1 Shrinkage of Girder Concrete

$\Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df}$ [AASHTO Eq. 5.9.5.4.3a-1]

where: ε_{bdf} = shrinkage strain of girder between the time of deck placement and final time

$= \varepsilon_{bif} - \varepsilon_{bid}$

where: ε_{bif} = $k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3}$ [AASHTO Eq. 5.4.2.3.3-1]

where: k_{td} = time development factor

$= \dfrac{t}{61 - 4f'_{ci} + t}$ [AASHTO Eq. 5.4.2.3.2-5]

where: t = maturity of concrete (days)

$= t_f - t_i$ for time between transfer and final time

= 0.998 (as calculated above)

$= (1.06)(1.02)(0.735)(0.998)(0.00048) = $ 0.000381

$= 0.000381 - 0.000268 = $ 0.000113

K_{df} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between deck placement and final time

$$= \frac{1}{1 + \dfrac{E_p}{E_{ci}} \dfrac{A_{ps}}{A_c} \left(1 + \dfrac{A_c e_{pc}^2}{I_c}\right)[1 + 0.7\Psi_b(t_f,t_i)]}$$ [AASHTO Eq. 5.9.5.4.3a-2]

where: e_{pc} = eccentricity of prestressing force with respect to centroid of composite section (in); positive in common construction where force is below centroid

$= y_{bc} - y_{bs} = 54.77 - 4.90 = $ 49.87 in

A_c = area of section calculated using the gross composite concrete section properties of the girder and the deck, and the deck-to-girder modular ratio = 1,419 in²
(See Section 3.2 for calculation)

I_c = moment of inertia calculated using gross composite concrete properties of the girder and the deck, and the deck-to-girder modular ratio at service = 1,100,347 in⁴
(See Section 3.2 for calculation)

$$K_{df} = \frac{1}{1 + \dfrac{28,500}{3,512} \dfrac{6.120}{1,419} \left(1 + \dfrac{(1,419)(49.87)^2}{1,100,347}\right)[1 + 0.7(1.477)]} = 0.770$$

Therefore, $\Delta f_{pSD} = (0.000113)(28,500)(0.770) = $ 2.480 ksi

6.2.2.2 Creep of Girder Concrete

$\Delta f_{pCD} = \dfrac{E_p}{E_{ci}} f_{cgp}[\Psi_b(t_f,t_i) - \Psi_b(t_d,t_i)]K_{df} + \dfrac{E_p}{E_c} \Delta f_{cd} \Psi_b(t_f,t_d)K_{df}$ [AASHTO Eq. 5.9.5.4.3b-1]

where: $\Psi_b(t_f,t_d)$ = girder creep coefficient at final time due to loading at deck placement

$= 1.9 k_s k_{hc} k_f k_{td} t_d^{-0.118}$ [AASHTO Eq. 5.4.2.3.2-1]

where: k_{td} = time development factor

$$= \frac{t}{61 - 4f'_{ci} + t}$$ [AASHTO Eq. 5.4.2.3.2-5]

where: t = maturity of concrete (days)

= $t_f - t_d$ for time between deck placement and final time

$$= \frac{20,000 - 90}{61 - 4(5.8) + (20,000 - 90)} = 0.998$$

= 1.9(1.06)(1.00)(0.735)(0.998)(0.588) = 0.869

Δf_{cd} = change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement, combined with deck weight on the non-composite transformed section, and superimposed loads on the composite transformed section (ksi)

$$= -\left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right)\frac{A_{ps}}{A_g}\left(1 + \frac{A_g e_{pg}^2}{I_g}\right) - \left[\frac{M_s e_{ptf}}{I_{tf}} + \frac{(M_b + M_{ws})e_{ptc}}{I_{tc}}\right]$$

where: e_{ptf} = eccentricity of the prestressing force with respect to the centroid of the non-composite transformed section = 30.10 in. (see Table 6.3-4 for transformed section properties)

e_{ptc} = eccentricity of the prestressing force with respect to the centroid of the composite transformed section = 48.48 in. (see Table 6.3-4 for transformed section properties)

I_{tf} = moment of inertia of the non-composite transformed section = 584,816 in⁴ (see Table 6.3-4 for transformed section properties)

I_{tc} = moment of inertia of the composite transformed section = 1,198,963 in⁴ (see Table 6.3-4 for transformed section properties)

$$= -(5.797 + 16.952 + 0.897)\frac{6.120}{767}\left(1 + \frac{(767)(31.70)^2}{545,894}\right)$$

$$- \left[\frac{12(1,382.4)(30.10)}{584,816} + \frac{(12 \times 180.0 + 12 \times 360.0)(48.48)}{1,198,963}\right] = -1.571 \text{ ksi}$$

$$\Delta f_{pCD} = \frac{28,500}{3,512}(2.649)[(1.477) - (1.039)](0.770) + \frac{28,500}{3,718}(-1.571)(0.869)(0.770) = -0.808 \text{ ksi}$$

A negative number indicates a gain in prestress.

6.2.2.3 Relaxation of Prestressing Strands

$\Delta f_{pR2} = \Delta f_{pR1} = 0.897$ ksi   (See Section 6.2.1.3 for calculation)   [AASHTO Eq. 5.9.5.4.3c-1]

6.2.2.4 Shrinkage of Deck Concrete

$$\Delta f_{pSS} = \frac{E_p}{E_c}\Delta f_{cdf} K_{df} [1 + 0.7 \Psi_b(t_f,t_d)]$$ [AASHTO Eq. 5.9.5.4.3d-1]

where: Δf_{cdf} = change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete (ksi)

$$= \frac{\varepsilon_{ddf} A_d E_{cd}}{[1 + 0.7 \Psi_d(t_f,t_d)]}\left(\frac{1}{A_c} - \frac{e_{pc} e_d}{I_c}\right)$$ [AASHTO Eq. 5.9.5.4.3d-2]

where: A_d = area of the deck concrete (in²) = (108.0)(7.5) = 810.0 in²

E_{cd} = modulus of elasticity of the deck concrete (ksi) = 2,917 ksi

e_d = eccentricity of deck with respect to the gross composite section, positive in typical construction where deck is above girder (in)

= $y_{bdeck} - y_{bc} = 76.25 - 54.77 = 21.48$ in

ε_{ddf} = shrinkage strain of deck concrete between placement and final time

= $k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3}$ [AASHTO Eq. 5.4.2.3.3-1]

These factors need to be recalculated for the deck:

k_s = factor for the effect of the volume-to-surface ratio

= $1.45 - 0.13 \, (V/S) \geq 1.0$ [AASHTO Eq. 5.4.2.3.2-2]

where: (V/S) = volume-to-surface ratio of the deck (in)

= Area / Perimeter (excluding edges)

= $(b_{eff})(t_{s,str}) / (b_{eff})(2)$

= $(108)(7.5) / (108)(2)$

= 3.750 in

= $1.45 - 0.13 \, (3.750) = 0.96$ <   1.00   use $k_s = 1.00$

k_f = factor for the effect of concrete strength

= $\dfrac{5}{1 + f'_{ci}}$ [AASHTO Eq. 5.4.2.3.2-4]

where: f'_{ci} = specified compressive strength of deck concrete at time of initial loading

may be taken as $0.80 f'_c$ [AASHTO Art. 5.4.2.3.2]

= 0.80 (4.0) =   3.20   ksi

= $\dfrac{5}{1 + 3.20} = $ 1.190

k_{td} = time development factor

= $\dfrac{t}{61 - 4f'_{ci} + t}$ [AASHTO Eq. 5.4.2.3.2-5]

where: t = maturity of concrete (days)

= $t_f - t_d$ for time between deck placement and final time

= $\dfrac{20,000 - 90}{61 - 4(3.20) + (20,000 - 90)} = $ 0.998

ε_{ddf} = $(1.00)(1.02)(1.190)(0.998)(0.00048) = $ 0.000581

$\Psi_d(t_f, t_d)$ = deck creep coefficient at final time due to loading at deck placement

= $1.9 \, k_s k_{hc} k_f k_{td} t_i^{-0.118}$ [AASHTO Eq. 5.4.2.3.2-1]

= $1.9(1.00)(1.00)(1.190)(0.998)(1.000) = $ 2.256

$$\Delta f_{cdf} = \frac{(0.000581)(810.0)(2,917)}{[1 + 0.7(2.256)]} \left(\frac{1}{1,419} - \frac{(49.87)(21.48)}{1,100,347} \right) = \ \text{-0.143 ksi}$$

$$\Delta f_{pSS} = \frac{28,500}{3,718}(\text{-0.143})(0.770)[1 + 0.7(0.869)] = \ \text{-1.357 ksi}$$

6.2.3 Total Prestress Losses at Transfer

The prestress loss due to elastic shortening:

$$\Delta f_{pES} = 21.497 \text{ ksi}$$

Stress in tendons after transfer, $f_{pt} = f_{pi} - \Delta f_{pES} = 202.500 - 21.497 = 181.003 \text{ ksi}$

Force per strand after transfer $= (f_{pt})(\text{area of strand}) = (181.003)(0.153) = 27.7 \text{ kips}$

Initial loss, $\% = (\text{Total losses at transfer}) / (f_{pi}) = (21.497) / (202.500) = 10.6\%$

6.2.4 Total Prestress Losses at Service

The sum of time-dependent prestress losses between time of transfer and deck placement:

$$(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} = 5.797 + 16.952 + 0.897 = 23.646 \text{ ksi}$$

The sum of time-dependent prestress losses after deck placement:

$$(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})_{df} = 2.480 + -0.808 + 0.897 - 1.357 = 1.212 \text{ ksi}$$

The total time-dependent prestress losses:

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})_{df} = (23.646) + (1.212) = 24.858 \text{ ksi}$$

The total prestress loss at service:

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} = 21.497 + 24.858 = 46.355 \text{ ksi}$$

Stress in strands after all losses, $f_{pe} = f_{pi} - \Delta f_{pT} = 202.500 - 46.355 = 156.145 \text{ ksi}$

Check prestressing stress limit at service limit state: [AASHTO Table 5.9.3-1]

$$f_{pe} \leq 0.8 f_{py} = 0.8(243.0) = 194.400 \text{ ksi} > 156.145 \text{ ksi} \qquad \textbf{OK}$$

Force per strand $= (f_{pe})(\text{area of strand}) = (156.1)(0.153) = 23.9 \text{ kips}$

Therefore, the total prestressing force after all losses, $P_{pe} = (23.9)(40) = 0,956.0 \text{ ksi}$

Final loss, $\% = (\text{total losses at service}) / (f_{pi}) = (46.355) / (202.500) (100) = 23\%$

6.3 Transformed Section Properties

Summary of Transformed Section Properties:

Modulus of elasticity of beam at transfer, $E_{ci} =$	3,512	ksi
Modulus of elasticity of beam at service, $E_c =$	3,718	ksi
Modulus of elasticity of prestressing strands, $E_p =$	28,500	ksi
Modular ratio of strands to beam at transfer, $n_{pi} = E_p/E_{ci} =$	8.12	
Modular ratio of strands to beam at service, $n_p = E_p/E_c =$	7.67	
Total area of prestressing strands, $A_{ps} =$	6.120	in^2
Transformed area of strands at transfer, $A_{ps,tt} = A_{ps}(n_{pi} - 1) =$	43.54	in^2
Transformed area of strands at service, $A_{ps,ts} = A_{ps}(n_p - 1) =$	40.79	in^2

Table 6.3-1: Non-Composite Transformed Section Properties at Transfer

	Area, in^2	y_b, in	Ay_b, in^3	$A(y_{b,tt} - y_b)^2$, in^4	I, in^4	$I+A(y_{bc} - y_b)^2$, in^4
Beam	767	36.60	28,072	2,224	545,894	548,118
Strands	43.54	4.90	213	39,182	0	39,182
Σ	811		28,286			587,300

Distance from centroid of section to bottom fiber, $y_{b,tt}$ = Σ Ay_b / Σ A = 34.90 in

Table 6.3-2: Non-Composite Transformed Section Properties at Service

	Area, in^2	y_b, in	Ay_b, in^3	$A(y_{b,ts} - y_b)^2$, in^4	I, in^4	$I+A(y_{bc} - y_b)^2$, in^4
Beam	767	36.60	28,072	1,965	545,894	547,859
Strands	40.8	4.90	200	36,956	0	36,956
Σ	808		28,272			584,816

Distance from centroid of section to bottom fiber, $y_{b,ts}$ = Σ Ay_b / Σ A = 35.00 in

Table 6.3-3: Composite Transformed Section Properties at Service

	Area, in^2	y_b, in	Ay_b, in^3	$A(y_{bc,ts} - y_b)^2$, in^4	I, in^4	$I+A(y_{bc} - y_b)^2$, in^4
Beam	1,419	54.77	77,719	2,756	1,100,347	1,103,103
Strands	40.8	4.90	200	95,861	0	95,861
Σ	1,460		77,919			1,198,963

Distance from centroid of section to bottom fiber, $y_{bc,ts}$ = Σ Ay_b / Σ A = 53.38 in

Table 6.3-4: Summary of Section Properties

	Non-Composite Section			Composite Section	
	Gross	Transformed at Transfer	Transformed at Service	Gross	Transformed
Area of section, A (in^2)	767	811	808	1,419	1,460
Distance from centroid to bottom fiber, y_b (in)	36.60	34.90	35.00	54.77	53.38
Distance from centroid to top fiber, y_t (in)	35.40	37.10	37.00	17.23	18.62
Moment of inertia of the section, I (in^4)	545,894	587,300	584,816	1,100,347	1,198,963
Strand eccentricity with respect to centroid, e_p (in)	31.70	30.00	30.10	49.87	48.48

7 STRESSES AT TRANSFER

Force per strand after initial losses = 27.7 kips   (See Section 6.1 for calculation)

Therefore, the total prestressing force after transfer = (27.7) (40) =   1,108.0 kips

In the following calculations, compressive stresses will be taken as positive numbers and tensile stresses will be taken as negative numbers.

7.1 Stress Limits for Concrete

Compression Limit: [AASHTO Art. 5.9.4.1.1]

$0.6f'_{ci} = 0.6(5.8) = +3.480$ ksi

where f'_{ci} = concrete strength at release = 5.80 ksi

Tension Limit: [AASHTO Table 5.9.4.1.2-1]

• Without bonded reinforcement

$0.0948\sqrt{f'_{ci}} \leq 0.200$ ksi; $-0.0948\sqrt{5.8} = -0.228$ ksi > -0.200

Therefore, tension limit = -0.200 ksi

• With bonded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force in the concrete computed assuming an uncracked section where reinforcement is proportioned using a stress of $0.5f_y$, not to exceed 30 ksi.

$0.24\sqrt{f'_{ci}} = -0.24\sqrt{5.8} = -0.578$ ksi

If the tensile stress is between these two limits, the tensile force at the location being considered must be computed following the procedure in AASHTO Art. C5.9.4.1.2. The required area of reinforcement is computed by dividing tensile force by the permitted stress in the reinforcement ($0.5f_y \leq 30$ ksi).

7.2 Stresses at Transfer Length Section

Stresses at this location need only be checked at release since this stage almost always governs. Also, losses with time will reduce the concrete stresses making them less critical.

Transfer length = 60(strand diameter) = 60(0.5) = 30 in. = 2.5 ft [AASHTO Art. 5.11.4.1]

Due to camber of the beam at release, the beam self-weight acts on the overall beam length of 121 ft. Therefore, values for the bending moment given in Table 4-1 cannot be used because they are based on the design span of 120 ft. Using statics, the bending moment at the transfer length due to beam self-weight is:

$M_g = 0.5wx(L - x)$

where: w = self-weight of beam

x = transfer length

= 0.5(0.666 kip/ft)(2.5 ft)(121 ft − 2.5 ft) = 98.7 kip-ft

Compute the stress in the top of beam:

$$f_t = \frac{P_i}{A} - \frac{P_i e}{S_t} + \frac{M_g}{S_t} = \frac{1,108.0}{767} - \frac{(1,108.0)(31.70)}{15,421} + \frac{(98.7)(12)}{15,421}$$

= 1.445 − 2.278 + 0.077 = -0.756 ksi

Tensile stress limits for concrete =   -0.200 ksi (without bonded reinforcement) < -0.756 ksi   **NG**

-0.578 ksi (with bonded reinforcement) < -0.756 ksi   **NG**

Compute the stress in the bottom of beam:

$$f_b = \frac{P_i}{A} + \frac{P_i e}{S_b} - \frac{M_g}{S_b} = \frac{1,108.0}{767} + \frac{(1,108.0)(31.70)}{14,915} - \frac{(98.7)(12)}{14,915}$$

$$= 1.445 + 2.355 - 0.079 = 3.721 \text{ ksi}$$

Compressive stress limit for concrete = 3.480 ksi < 3.721 ksi **NG**

Since stresses at the top and bottom exceed the stress limits, harp strands to satisfy the specified limits.

Number of harped strands: 6

Height of top most row of harped strands at end of beam = 70.0 in

Harp strands at 0.4L points of the girder as shown in Figure 7.2-1, where L is the overall beam length and 0.4L = 0.4(121.0) =

= 48.50 ft
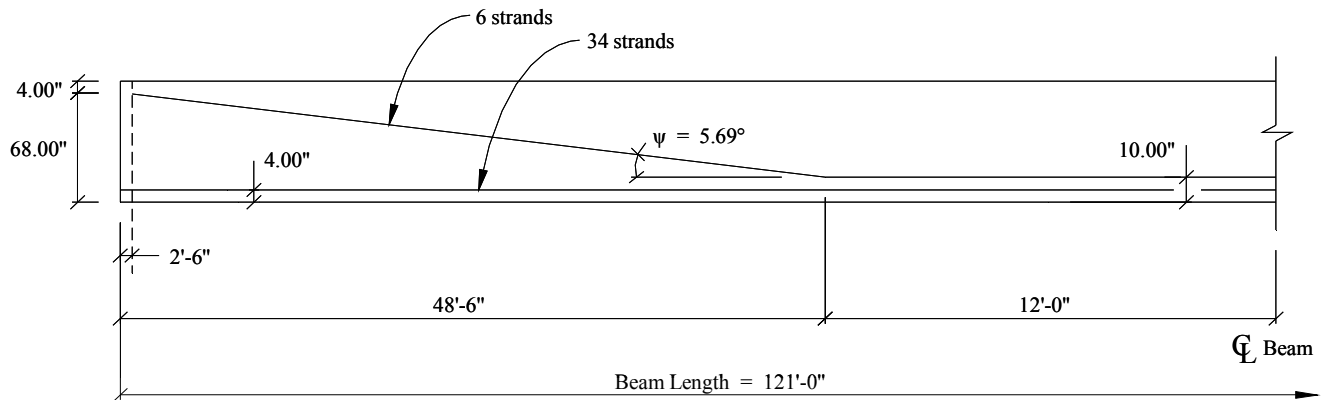
Figure 7.2-1: Longitudinal Strand Profile for Beams with Harped Strands

Compute the center of gravity of the prestressing strands at the transfer length section using the harped strand pattern.

The distance between the top fiber of the beam and the top row of harped strands:

= 72 in − 70.0 in = 2.00 in

The distance between the center of gravity of the 6 harped strands at the end of the beam and the top fiber of the beam:

= [2(2) + 2(4) + 2(6)] / 6

= 4.00 in

The distance between the center of gravity of the 6 harped strands at the harp point and the bottom fiber of the beam:

= [2(12) + 2(10) + 2(8)] / 6

= 10.00 in

The distance between the center of gravity of the 6 harped strands and the top fiber of the beam at the transfer length section:

$\qquad$ = 4.00 in + [(72 in − 10.00 in − 4.00 in) (2.5 ft) / (48.50 ft)]

$\qquad$ = 6.99 in

The distance between the center of gravity of the bottom straight 34 strands and the extreme bottom fiber of the beam:

$\qquad$ = [12(2) + 12(4) + 8(6) + 2(8)] / 34

$\qquad$ = 4.00 in

Therefore, the distance between the center of gravity of the total number of strands measured to the bottom of the precast beam at transfer length is:

$\qquad$ = [34(4.00 in) + 6(72 in − 6.99 in)] / 40

$\qquad$ = 13.15 in

Eccentricity of the strand group at transfer length, $e_{tr} = y_b − 13.15 = 36.60 − 13.15 = 23.45$ in

The center of gravity of all prestressing strand with respect to the extreme bottom fiber at the end of the beam, y_{bs}, is:

$\qquad$ = [34(4.00 in) + 6(72 in − 4.00 in)] / 40

$\qquad$ = 13.60 in

Recompute the stresses at the transfer length section with harped strands:

Concrete stress in top of beam:

$$f_t = \frac{P_i}{A} - \frac{P_i e}{S_t} + \frac{M_g}{S_t} = \frac{1,108.0}{767} - \frac{(1,108.0)(23.45)}{15,421} + \frac{(98.7)(12)}{15,421}$$

$\qquad$ = 1.445 − 1.685 + 0.077 = -0.163 ksi

Tensile stress limits for concrete = $\qquad$ -0.200 ksi (without bonded reinforcement) > -0.163 ksi $\qquad$ **OK**

$\qquad$ -0.578 ksi (with bonded reinforcement) > -0.163 ksi $\qquad$ **OK**

Concrete stress in bottom of beam:

$$f_b = \frac{P_i}{A} + \frac{P_i e}{S_b} - \frac{M_g}{S_b} = \frac{1,108.0}{767} + \frac{(1,108.0)(23.45)}{14,915} - \frac{(98.7)(12)}{14,915}$$

$\qquad$ = 1.445 + 1.742 − 0.079 = 3.108 ksi

Compressive stress limit for concrete = $\qquad$ 3.480 ksi > 3.108 ksi $\qquad$ **OK**

If any of the resulting stresses above are tensile and between the two tensile stress limits calculated in Section 7.1, -0.200 and -0.578 ksi, the required amount of reinforcement needs to be calculated and checked against the provided amount to determine if additional reinforcement is required. $\qquad$ [AASHTO Art. C5.9.4.1.2]

Tensile stress is below limit, this check does not apply.

$\qquad$ Stress in top fiber of beam, $f_{ci\,top}$ = -0.163 ksi

$\qquad$ Stress in bottom fiber of beam, $f_{ci\,bot}$ = 3.108 ksi

<div align="right">

Depth of the tensile zone, x =   3.6   in

Tensile force, T = -12.3   kip

Required area of reinforcement, A_s = 0.41   in^2

Provided area of reinforcement, A_{ps} = 6.12   in^2

Is $A_{ps} > A_s$?   yes
</div>

Depth of tensile zone is greater than thickness of top flange!

(area of reinforcement in compression zone)

The provided amount of reinforcement is sufficient. No additional reinforcement is required.

7.3 Stresses at Harp Points

The strand eccentricity at harp points is the same as at midspan, $e_c = 31.70$ in

The bending moment at the harp points (0.4L) due to the self-weight of the beam is:

0.5wx(L − x)

where: w   = self-weight of beam =   0.666   kip/ft

             x   = distance to harp point = 0.4L =   48.50   ft

= 0.5 (0.666 kip/ft) (48.50 ft) (121 ft − 48.50 ft) =   1,170.9 kip-ft

Concrete stress in top of beam:

$$f_t = \frac{P_i}{A} - \frac{P_i e}{S_t} + \frac{M_g}{S_t} = \frac{1,108.0}{767} - \frac{(1,108.0)(31.70)}{15,421} + \frac{(1,170.9)(12)}{15,421}$$

= 1.445 − 2.278 + 0.911 =   0.078   ksi

Compressive stress limit for concrete =   3.480   ksi > 0.078 ksi   **OK**

Concrete stress in bottom of beam:

$$f_b = \frac{P_i}{A} + \frac{P_i e}{S_b} - \frac{M_g}{S_b} = \frac{1,108.0}{767} + \frac{(1,108.0)(31.70)}{14,915} - \frac{(1,170.9)(12)}{14,915}$$

= 1.445 + 2.355 − 0.942 =   2.858   ksi

Compressive stress limit for concrete =   3.480   ksi > 2.858 ksi   **OK**

7.4 Stresses at Midspan

The bending moment at midspan due to the self-weight of the beam is:

$M_g = 0.5wx(L − x)$

where: w   = self-weight of beam

             x   = distance to midspan

= 0.5 (0.666 kip/ft) (60.5 ft) (121 ft − 60.5 ft) =   1,218.9 kip-ft

Concrete stress in top of beam:

$$f_t = \frac{P_i}{A} - \frac{P_i e}{S_t} + \frac{M_g}{S_t} = \frac{1,108.0}{767} - \frac{(1,108.0)(31.70)}{15,421} + \frac{(1,218.9)(12)}{15,421}$$

= 1.445 − 2.278 + 0.948 =   0.115   ksi

Compressive stress limit for concrete =   3.480   ksi > 0.115 ksi   **OK**

Concrete stress in bottom of beam:

$$f_b = \frac{P_i}{A} + \frac{P_i e}{S_b} - \frac{M_g}{S_b} = \frac{1,108.0}{767} + \frac{(1,108.0)(31.70)}{14,915} - \frac{(1,218.9)(12)}{14,915}$$

$$= 1.445 + 2.355 - 0.981 = 2.819 \text{ ksi}$$

Compressive stress limit for concrete = 3.480 ksi > 2.819 ksi **OK**

7.5 Hold-Down Forces

Assume that the stress in the strand at the time of prestressing, before seating losses, is:

$0.80 \, f_{pu} = 0.80(270.0) = 216.000 \text{ ksi}$

Thus, the prestress force per strand before seating losses is:

$(0.153)(216) = 33.0 \text{ kips}$

From Figure 7.2-1, the harp angle is:

$$\psi = \tan^{-1} \left(\frac{72.0 \text{ in} - 4.00 \text{ in} - 10.00 \text{ in}}{48.50 \, (12)} \right) = 5.69°$$

PCI limit for ψ = 9.0° **OK**

Therefore, the hold-down force per strand is:

1.05 (force per strand) (sin ψ) = 1.05 (33.0) (sin 5.69°) = 3.44 kips/strand

Note the factor of 1.05 is applied to account for friction.

The total hold-down force = 6 strands (3.44 kips/strand) = 20.6 kips

The hold-down force and the harp angle should be checked against maximum limits for local practices.

7.6 Summary of Stresses at Transfer

Table 7.6-1: Summary of Stresses at Transfer

	Top Stresses f_t (ksi)	Bottom Stresses f_b (ksi)
At transfer length section	-0.163	3.108
At harp points	0.078	2.858
At midspan	0.115	2.819

8 STRESSES AT SERVICE LOADS

The total prestressing force after all losses is P_{pe} = 956.0 kips   (See Section 6.2.4 for calculation)

8.1 Stress Limits for Concrete

Compression Limit: [AASHTO Art. 5.9.4.2.1]

- Due to the sum of effective prestress and permanent loads (i.e. beam self-weight, weight of slab and haunch, weight of future wearing surface, and weight of barriers) for the Load Combination Service 1:

    for precast beam: $0.45f'_c = 0.45(6.5) = +2.925$ ksi

    for deck: $0.45f'_c = 0.45(4.0) = +1.800$ ksi

- Due to the sum of effective prestress, permanent loads, and transient loads as well as during shipping and handling for the Load Combination Service 1:

    for precast beam: $0.60\phi_w f'_c = 0.60(1.0)(6.5) = +3.900$ ksi

    for deck: $0.60\phi_w f'_c = 0.60(1.0)(4.0) = +2.400$ ksi

Tension Limit: [AASHTO Art. 5.9.4.2.2]

- For components with bonded prestressing tendons or reinforcement that are subjected to not worse than moderate corrosion conditions for Load Combination Service III:

    for precast beam: $0.19\sqrt{f'_c} = -0.19\sqrt{6.5} = -0.484$ ksi

- For components with bonded prestressing tendons or reinforcement that are subjected to severe corrosive conditions

    for precast beam: $0.0948\sqrt{f'_c} = -0.0948\sqrt{6.5} = -0.242$ ksi

- For components with unbonded prestressing tendons − not allowed

8.2 Stresses at Midspan

- Concrete stress at top fiber of the beam

    To check top stresses, two cases are checked:

    1. Under permanent loads, Service I:

    Using the bending moment values given in Table 4-1, compute the top fiber stresses:

$$f_{tg} = \frac{P_{pe}}{A} - \frac{P_{pe}e_c}{S_t} + \frac{M_g + M_s}{S_t} + \frac{M_{ws} + M_b}{S_{tg}} \qquad \text{(See Section 3.2 for calculation of } S_{tg})$$

$$= \frac{956.0}{767} - \frac{(956.0)(31.70)}{15,421} + \frac{(1,198.8 + 1,382.4)(12)}{15,421} + \frac{(360.0 + 180.0)(12)}{63,862}$$

$$= 1.246 - 1.965 + 2.009 + 0.101 = 1.391 \text{ ksi}$$

Compressive stress limit for concrete =   2.925 ksi > 1.391 ksi   **OK**

    2. Under permanent and transient loads, Service I:

$$f_{tg} = 1.391 + \frac{M_{LT} + M_{LL}}{S_{tg}} = 1.391 + \frac{(1,830.2 + 843.3)(12)}{63,862}$$

$$= 1.391 + 0.502 = 1.893 \text{ ksi}$$

Compressive stress limit for concrete =   3.900 ksi > 1.893 ksi   **OK**

- Concrete stress at top fiber of the deck

    Note: Compressive stress in the deck slab at service never controls the design for typical applications. The calculations shown below are for illustration purposes and may not be necessary in most practical applications.

1. Under permanent loads, Service I:

$$f_{tc} = \frac{M_{ws} + M_b}{S_{tc}} = \frac{(360.0 + 180.0)(12)}{55,586} = 0.117 \text{ ksi}$$ (See Section 3.2 for calculation of S_{tc})

   Compressive stress limit for concrete = 1.800 ksi > 0.117 ksi   **OK**

2. Under permanent and transient loads, Service I:

$$f_{tc} = 0.117 + \frac{M_{LT} + M_{LL}}{S_{tc}} = 0.117 + \frac{(1,830.2 + 843.3)(12)}{55,586}$$

$$= 0.117 + 0.577 = 0.694 \text{ ksi}$$

   Compressive stress limit for concrete = 2.400 ksi > 0.694 ksi   **OK**

• Concrete stress at bottom fiber of beam under permanent and transient loads, Service III:

$$f_b = \frac{P_{pe}}{A} + \frac{P_{pe}e_c}{S_b} - \frac{M_g + M_s}{S_b} - \frac{(M_{ws} + M_b) + 0.8(M_{LT} + M_{LL})}{S_{bc}}$$ (See Section 3.2 for calculation of S_{bc})

$$= \frac{956.0}{767} + \frac{(956.0)(31.70)}{14,915} - \frac{(1,198.8 + 1,382.4)(12)}{14,915}$$

$$- \frac{(360.0 + 180.0)(12) + 0.8(1,830.2 + 843.3)(12)}{20,090}$$

$$= 1.246 + 2.032 - 2.077 - 1.600 = \text{-0.399 ksi}$$

   Tensile stress limit for concrete = -0.484 ksi > -0.399 ksi   **OK**

8.3 Fatigue Stress Limit

In regions of compressive stress due to permanent loads and prestress in reinforced and partially prestressed concrete components, fatigue shall be considered only if this compressive stress is less than the maximum tensile live load stress resulting from the Fatigue I load combination. However, the AASHTO Specifications do not state where this stress is computed: at the bottom fiber of concrete or at the lower strand level. In this example, for convenience, the calculations are done at the bottom fiber of concrete.

Fatigue of reinforcement need not be checked for fully prestressed components designed to have extreme fiber tensile stress within the limit specified in Section 8.1. However, for fully prestressed components in other than segmentally constructed bridges, the compressive stress due to the Fatigue I load combination and one half the sum of the effective prestress and permanent loads shall not exceed $0.40f'_c$ after losses.   [AASHTO Art. 5.5.3.1]

Partially prestressed concrete components are defined as concrete with a combination of prestressing steel and reinforcing bars. Fully prestressed concrete components are defined as those components in which stresses satisfy the tensile stress limits at Service Limit State specified in Section 8.1 and are assumed to remain uncracked (see AASHTO Art. 5.2). Since all stress limits in Section 8.1 are met, the section is considered to be Fully Prestressed. While fatigue of the reinforcement need not be considered, it will be checked here for illustration purposes. The compressive stress check for fully prestressed components is also performed.

At midspan, the bottom compressive stress due to permanent loads and prestress is:

$$\frac{P_{pe}}{A} + \frac{P_{pe}e_c}{S_b} - \frac{M_g + M_s}{S_b} - \frac{M_{ws} + M_b}{S_{bc}}$$

$$= \frac{956.0}{767} + \frac{(956.0)(31.70)}{14,915} - \frac{(1,198.8 + 1,382.4)(12)}{14,915} - \frac{(360.0 + 180.0)(12)}{20,090}$$

$$= 1.246 + 2.032 - 2.077 - 0.323 = 0.878 \text{ ksi}$$

From Table 4-1, the unfactored bending moment at midspan due to the fatigue truck, M_f = 776.9 kip-ft

Thus, the tensile stress at the bottom fiber of the beam due to Fatigue I load combination is:

$$-\frac{1.50\,M_f}{S_{bc}} = -\frac{1.50\,(\,776.9\,)(12)}{20{,}090} = \text{-0.696 ksi}$$

Comparing absolute values of the stresses, the compressive stress due to permanent loads and prestress, 0.878 ksi, is more than the maximum tensile live load stress from the Fatigue I load combination, 0.696 ksi, resulting in net compression in the bottom of the beam. Therefore, fatigue need not be considered, but is checked below for illustration purposes.

Concrete members shall satisfy:

$\gamma(\Delta f) \le (\Delta F)_{TH}$ [AASHTO Eq. 5.5.3.1-1]

where: γ = load factor for Fatigue I load combination = 1.50

Δf = force effect, live load stress range due to the passage of the fatigue load

$(\Delta F)_{TH}$ = constant-amplitude fatigue threshold for prestressing strands = 18.0 ksi for radii of curvature in excess of 30.0 ft and 10.0 ksi for radii of curvature not exceeding 12.0 ft (using linear interpolation for radii in between 12.0 and 30.0 ft)

The section properties for fatigue investigations shall be based on cracked sections where the sum of stresses, due to unfactored permanent loads and prestress, and the Fatigue I load combination is tensile and exceeds $0.095\sqrt{f'_c}$.

Stress at bottom of beam due to unfactored permanent loads and prestress:

0.878 ksi   (calculated above)

Stress at bottom of beam due to the Fatigue I load combination:

-0.696 ksi   (calculated above)

Sum of stresses = 0.878 + -0.696 = 0.182 ksi

Since the sum of the stresses is positive (compressive), uncracked section properties may be used.

$$\Delta f = \frac{M_f\,(y_{bc} - 2\text{ in})}{I_c}\,\frac{E_p}{E_c} = \frac{(\,776.9\,)(12)\,(\,54.77\,-2\text{ in})}{1{,}100{,}347}\,\frac{28{,}500}{3{,}718} = 3.427 \text{ ksi}$$

$\gamma(\Delta f) = $ 1.50(3.427) = 5.141 ksi   < $(\Delta F)_{TH}$ = 18.000 ksi   **OK**

Compressive stress check for fully prestressed components

• Check stress at bottom of beam

Stress at bottom of beam due to the Fatigue I load combination:

-0.696 ksi

One-half of the stress at bottom of beam due to unfactored permanent loads and prestress:

= 0.5(0.878) = 0.439 ksi

Sum of stresses = -0.696 + 0.439 = -0.257 ksi

Since the sum of the stresses is negative (tensile), the check does not apply.

• Check stress at top of beam

Stress at top of beam due to the Fatigue I load combination:

$$\frac{1.50\,M_f}{S_{tg}} = \frac{1.50\,(\;776.9\;\;)(12)}{63,862} = \quad 0.219 \text{ ksi}$$

One-half of the stress at top of beam due to unfactored permanent loads and prestress:

$= 0.5(1.391) = \quad 0.696 \quad$ ksi   (calculated in Section 8.2)

Sum of stresses $= 0.219 + 0.696 = 0.915$ ksi $\quad < \quad 0.40 f'_c = \; 2.600 \quad$ ksi

The sum of the stresses does not exceed the limit.

8.4 Summary of Midspan Stresses at Service Loads

Table 8.4-1: Summary of Midspan Stresses at Service Loads

Top of Deck (ksi) Service I		Top of Beam (ksi) Service I		Bottom of Beam (ksi) Service III
Permanent Loads	Total Loads	Permanent Loads	Total Loads	Total Loads
0.117	0.694	1.391	1.893	-0.399

8.5 Summary of Stress Components

Table 8.5-1: Summary of Individual Stress Components (ksi)

		Top of Deck	Top of Beam	Bot. of Beam
Prestressing	Prestress	0.000	1.246	1.246
	Prestress Eccentricity	0.000	-1.965	2.032
	Σ Prestressing Stresses	0.000	-0.719	3.278
Permanent Loads	Girder	0.000	0.933	-0.965
	Slab	0.000	1.076	-1.112
	Wearing Surface	0.078	0.068	-0.215
	Barrier	0.039	0.034	-0.108
	Σ Permanent Load Stresses	0.117	2.110	-2.399
	Σ Permanent & Prestress Stresses	0.117	1.391	0.879
Transient Loads	Design Truck	0.395	0.344	-1.093
	Lane Load	0.182	0.158	-0.504
	Σ Transient Load Stresses	0.577	0.502	-1.597
	Σ Perm., Trans. & PS Stresses	0.694	1.894	-0.718
Fatigue Load	Fatigue I LC (factored)	0.252	0.219	-0.696
	Σ Fatigue, Perm. & PS Stresses	0.368	1.610	0.183
	Σ Fatigue, 1/2(Perm. & PS) Stresses	0.310	0.915	-0.257

9 STRENGTH LIMIT STATE

The total ultimate bending moment for Strength I is:

$M_u = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)$

Using the values of unfactored bending moment given in Table 4-1, the ultimate bending moment at midspan is:

$$
\begin{aligned}
M_u &= 1.25 \, (M_g + M_s + M_b) \; + \; 1.50 \, (M_{ws}) \; + \; 1.75 \, (M_{LT} + M_{LL}) \\
&= 1.25 \, (1{,}198.8 + 1{,}382.4 + 180.0) \; + \; 1.50 \, (360.0) \; + \; 1.75 \, (1{,}830.2 + 843.3) \\
&= 8{,}670.1 \text{ kip-ft}
\end{aligned}
$$

Average stress in prestressing steel when $f_{pe} \geq 0.5 f_{pu}$:

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) \qquad\qquad \text{[AASHTO Eq. 5.7.3.1.1-1]}$$

Check: $f_{pe} \geq 0.5 f_{pu}$

f_{pe} = stress in strands after all losses = 156.145 ksi (See Section 6.2.4 for calculation)

f_{pu} = specified tensile strength of prestressing steel = 270.000 ksi

$0.5 f_{pu} = 135.000$ ksi

$156.145 \geq 135.000$ OK

where: f_{ps} = average stress in prestressing steel

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 2\left(1.04 - \frac{243.0}{270.0}\right) = 0.28 \qquad\qquad \text{[AASHTO Eq. 5.7.3.1.1-2]}$$

d_p = distance from extreme compression fiber of the composite section to the centroid of the prestressing tendons (in) = $h - y_{bs}$

= $80.00 - 4.90 = 75.10$ in (See Section 5.3 for calculation of y_{bs})

c = distance between the neutral axis and the compression face

To compute c, assume rectangular section behavior and check if the depth of the equivalent compression stress block, c, is less than or equal to t_s. If c is larger than t_s, c will need to be recalculated assuming T-section behavior.

For rectangular section behavior:

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{0.85 f'_c \beta_1 b + k A_{ps}\dfrac{f_{pu}}{d_p}} \qquad\qquad \text{[AASHTO Eq. 5.7.3.1.1-4]}$$

where: A_{ps} = area of prestressing steel = $40(0.153) = 6.120$ in²

A_s = area of mild steel tension reinforcement = 0 in²

f_s = stress in mild steel tension reinforcement at nominal flexural resistance = 0 ksi

A'_s = area of compression reinforcement = 0 in²

f'_s = stress in mild steel compression reinforcement at nominal flexural resistance = 0 ksi

f'_c = compressive strength of deck concrete = 4.0 ksi

β_1 = stress block factor [AASHTO Art. 5.7.2.2]

= 0.85 for $f'_c \leq 4.0$ ksi, $0.85 - 0.05(f'_c - 4.0) \geq 0.65$ for $f'_c > 4.0$ ksi

= 0.85

b = effective width of compression flange = 108 in

$$c = \frac{(6.120)(270.0) + 0 - 0}{0.85(4.0)(0.85)(108) + (0.28)(6.120)\frac{270.0}{75.10}} = 5.19 \text{ in}$$

c = 5.19 in. is less than or equal to the slab thickness, 7.5 in. Therefore, the rectangular section behavior is valid. Skip recalculation of c based on T-section behavior.

For T-section behavior:

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s - 0.85f'_c(b - b_w)h_f}{0.85f'_c\beta_1 b_w + kA_{ps}\frac{f_{pu}}{d_p}} \qquad \text{[AASHTO Eq. 5.7.3.1.1-3]}$$

where: b_w = width of web = 6.00 in

h_f = depth of compression flange = 7.50 in

$$= \frac{(6.120)(270.0) + 0 - 0 - 0.85(4.0)(108 - 6.00)(7.50)}{0.85(4.0)(0.85)(6.00) + (0.28)(6.120)\frac{270.0}{75.10}} = \text{Does not apply}$$

c = 5.19 in

$$f_{ps} = 270.0\left(1 - 0.28\frac{5.19}{75.10}\right) = 264.775 \text{ ksi}$$

Nominal flexural resistance, M_n:

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right) - A'_s f'_s\left(d'_s - \frac{a}{2}\right) + 0.85\,f'_c(b - b_w)h_f\left(\frac{a}{2} - \frac{h_f}{2}\right) \qquad \text{[AASHTO Eq. 5.7.3.2.2-1]}$$

where: b_w = width of web (in) for T-sections where compression flange depth is less than a = $\beta_1 c$

= b for rectangular sections where the compression flange is not less than a = $\beta_1 c$ [AASHTO Art. 5.7.3.2.3]

a = (0.85)(5.19) = 4.41 in < t_s = 7.5 in

Therefore, b_w = 108 in

$$M_n = \left[(6.120)(264.775)\left(75.10 - \frac{4.41}{2}\right) + 0 - 0 + 0.85(4.0)(108 - 108)(7.50)\left(\frac{4.41}{2} - \frac{7.50}{2}\right)\right]/12$$

= 9,843.4 kip-ft

Factored flexural resistance, M_r:

$M_r = \phi M_n$ [AASHTO Eq. 5.7.3.2.1-1]

where: ϕ = resistance factor   [AASHTO Art. 5.5.4.2.1]

= 1.00 for tension-controlled prestressed concrete sections

= 0.75 for compression-controlled concrete sections

= $0.75 \leq 0.583 + 0.25(d_t/c - 1) \geq 1.0$ for sections in transition between tension and compression controlled

where: d_t = distance from extreme compression fiber to the extreme tension steel (in)

= 80.00 − 2.00 = 78.00 in

Check if section is tension or compression controlled:

ε_p = net tensile strain in the extreme tension steel

= $0.003(d_t / c) - 0.003$

= $0.003(78.00 / 5.19) - 0.003$ = $0.042 \geq$ tension-controlled limit of 0.005

Section is tension-controlled. Use resistance factor equal to 1.00.

M_r = $(1.00)(9,843.4)$ = $9,843.4$ kip-ft $>$ M_u = $8,670.1$ kip-ft **OK**

10 MINIMUM REINFORCEMENT [AASHTO Art. 5.7.3.3.2]

At any section of a flexural component, the amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance, M_r, at least equal to the lesser of:

- 1.2 times the cracking moment, M_{cr}, determined on the basis of elastic stress distribution and the modulus of rupture, f_r, of the concrete
- 1.33 times the factored moment required by the applicable strength load combinations

Check at midspan:

f_r = $0.20 \sqrt{f'_c}$ = $0.20 \sqrt{6.5}$ = 0.510 ksi [AASHTO Art. 5.4.2.6]

$M_{cr} = S_c(f_r + f_{cpe}) - M_{dnc}\left(\dfrac{S_c}{S_{nc}} - 1\right) \geq S_c f_r$ [AASHTO Eq. 5.7.3.3.2-1]

where: f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

$= \dfrac{P_{pe}}{A} + \dfrac{P_{pe}e_c}{S_b} = \dfrac{956.0}{767} + \dfrac{(956.0)(31.70)}{14,915} = 1.246 + 2.032 = 3.278$ ksi

M_{dnc} = total unfactored dead load moment acting on the monolithic or noncomposite section

= $M_g + M_s$ = $1,198.8 + 1,382.4$ = $2,581.2$ kip-ft (See Table 4-1)

S_c = section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads (in^3)

= S_{bc} = $20,090$ in^3 (See Section 3.2.3 for calculation)

S_{nc} = section modulus for the extreme fiber of the monolithic or noncomposite section where tensile stress is caused by externally applied loads (in^3)

= S_b = $14,915$ in^3 (See Section 3.1 for calculation)

M_{cr} = $\dfrac{20,090}{12}(0.510 + 3.278) - 2,581.2\left(\dfrac{20,090}{14,915} - 1\right)$ = $5,446.2$ kip-ft

$> S_c f_r$ = $(20,090)(0.510) / 12$ = 853.8 kip-ft

$1.2 M_{cr}$ = $1.2 (5,446.2)$ = $6,535.4$ kip-ft

At midspan, the factored moment required by the Strength I load combination is:

M_u = $8,670.1$ kip-ft (See Section 9 for calculation)

$1.33 M_u$ = $1.33 (8,670.1)$ = $11,531.2$ kip-ft

$1.2M_{cr} = 6,535.4$ kip-ft $<$ $1.33M_u = 11,531.2$ kip-ft

Therefore, the 6,535.4 kip-ft limit controls.

$M_r = 9,843.4$ kip-ft $>$ 6,535.4 kip-ft **OK**

Section meets minimum reinforcement requirements at midspan. Similar calculations shall be conducted at every section along beam.

Shrinkage and temperature reinforcement must be provided as specified in AASHTO Article 5.10.8.

11 SHEAR DESIGN

The area and spacing of shear reinforcement must be determined at regular intervals along the entire length of the beam. Transverse shear design procedures are demonstrated below by determining these values at the critical section for shear near the supports.

Transverse shear reinforcement shall be provided where:

$V_u > 0.5\phi(V_c + V_p)$ [AASHTO Eq. 5.8.2.4-1]

where: V_u = factored shear force (kips)

V_c = nominal shear resistance provided by tensile stresses in the concrete (kips)

V_p = component of prestressing force in the direction of the shear force (kips)

ϕ = resistance factor for shear = 0.85 [AASHTO Art. 5.5.4.2.1]

11.1 Critical Section for Shear [AASHTO Art. 5.8.3.2]

The location of the critical section for shear shall be taken as d_v from the internal face of the support.

where: d_v = effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure; it need not be taken to be less than the greater of $0.9d_e$ or $0.72h$ (in) [AASHTO Art. 5.8.2.9]

$= d_e - a/2$

a = depth of compression block = 4.41 in (See Section 9 for calculation)

h = overall depth of composite section = 80.0 in

d_e = effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (in)

$= \dfrac{A_{ps}f_{ps}d_p + A_s f_y d_s}{A_{ps}f_{ps} + A_s f_y}$ = d_p for sections with no mild reinforcement [AASHTO Eq. 5.8.2.9-2]

The effective depth, d_e, may vary along the length of the beam if some of the strands are harped. However, the value of d_e must be calculated at the critical section in shear which is not yet determined; therefore, for the first iteration, d_e is calculated based on the center of gravity of the tensile reinforcement at the end of the beam, y_{bs}.

$= h - y_{bs}$ at end of beam $= 80.00 - 4.00 = 76.00$ in

$= 76.00 - 4.41/2 = 73.80$ in

$0.9d_e = 0.9 (76.00) = 68.40$ in $<$ 73.80 in

$0.72h = 0.72 (80.00) = 57.60$ in $<$ 73.80 in

Therefore, d_v = 73.80 in

Since the width of the bearing is not yet determined, it is conservatively assumed to be zero. Therefore, the critical section in shear is located at a distance of:

73.80 in = 6.15 ft   from the centerline of support

(x / L) = (6.15 / 120) = 0.05L

The effective depth, d_e, and the position of the critical section in shear may be refined based on the position of the critical section calculated above. However, the difference is small and on the conservative side. Thus, no more refinement is performed.

11.2 Compute Shear Stress

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v}$$

[AASHTO Eq. 5.8.2.9-1]

where: v_u = shear stress in concrete

ϕ = resistance factor for shear = 0.85   [AASHTO Art. 5.5.4.2.1]

V_u = factored shear force at specified section at Strength I limit state
(See Table 4-1 for shears at 6.15 ft)

= 1.25(35.9 + 41.4 + 5.4) + 1.50(10.8) + 1.75(73.7 + 30.6)

= 302.1   kips

V_p = component of the effective prestressing force in direction of the applied shear (positive if resisting applied shear) (kip)

= (force per strand)(number of harped strands)(sin ψ)

= (23.9)(6)(sin 5.69°)   (See Section 6.2.4 for force per strand calculation)

= 14.2   kips

b_v = effective web width taken as the minimum web width between the resultants of the tensile and compressive forces due to flexure (in)

= b_w = 6.00 in

d_v = 73.80 in

$$v_u = \frac{|\ 302.1\ -\ (0.85)(\ 14.2\ )|}{(0.85)(\ 6.00\ )(\ 73.80\ )} =\ 0.771\ \text{ksi}$$

11.3 Contribution of Concrete to Nominal Shear Resistance

The contribution of the concrete to the nominal shear resistance is:

V_c = 0.0316 β $\sqrt{(f'_c)}$ b_v d_v [AASHTO Eq. 5.8.3.3-3]

$\sqrt{(f'_c)}$ shall be replaced by 1.00 $\sqrt{(f'_c)}$ for sand-lightweight concrete   [AASHTO Art. 5.8.2.2]

where: b_v = effective web width taken as the minimum web width, measured parallel to the neutral axis, between the resultants of the tensile and compressive forces due to flexure (in)

= b_w = 6.00 in

β = factor indicating the ability of diagonally cracked concrete to transmit tension and shear

= $\dfrac{4.8}{(1 + 750\varepsilon_s)}$ for sections containing at least the minimum amount of transverse shear reinforcement (calculated in Section 11.3.4)   [AASHTO Eq. 5.8.3.4.2-1]

where: ε_s = strain in longitudinal tension reinforcement

11.3.1 Strain in Flexural Tension Reinforcement

ε_s is the net longitudinal tensile strain in the section at the centroid of the tension reinforcement, and may be determined as:

$$\varepsilon_s = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po}\right)}{E_s A_s + E_p A_{ps}}$$

[AASHTO Eq. 5.8.3.4.2-4]

where: N_u = factored axial force, taken as positive if tensile and negative if compressive (kip) = 0 kip

V_u = 302.1 kips

V_p = 14.2 kips

$|M_u|$ = applied factored bending moment at specified section, not to be taken less than $|V_u - V_p| d_v$ (kip-in)
(See Table 4-1 for moments at 6.15 ft)

= 1.25(233.2 + 268.9 + 35.0) + 1.50(70.0) + 1.75(375.5 + 164.0)

= 1,720.5 kip-ft = 20,646.0 kip-in

check if greater than $|V_u - V_p| d_v$ = |302.1 − 14.2| (73.80) = 21,247.0 kip-in

use M_u = 21,247.0 kip-in

A_{ps} = area of prestressing steel on the flexural tension side of the member (in^2)

= (number of non-harped strands)(strand area) = (34)(0.153) = 5.202 in^2

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi)
For the usual levels of prestressing, a value of $0.7f_{pu}$ can be used

= 0.7 (270.0) = 189.0 ksi

A_s = area of nonprestressed steel on the flexural tension side of the member at the section under consideration (in^2) = 0 in^2

$$\varepsilon_s = \frac{\left(\frac{21,247.0}{73.80} + 0 + (302.1 - 14.2) - (5.202)(189.0)\right)}{0 + (28,500)(5.202)} = \text{-2.75x10}^{-3}$$

11.3.1.1 Calculation for Negative Strain

If the value of ε_s is negative, ε_s can conservatively be taken as zero, or needs to be recalculated with the denominator of the equation replaced by:

$(E_s A_s + E_p A_{ps} + E_c A_{ct})$

However, ε_s should not be taken as less than $-0.40\text{x}10^{-3}$ or greater than $6.0\text{x}10^{-3}$. [AASHTO Art. 5.8.3.4.2]

In this case, the strain is negative and is recalculated with the new denominator below.

$$\varepsilon_s = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po}\right)}{E_s A_s + E_p A_{ps} + E_c A_{ct}}$$

where: A_{ct} = area of concrete on the flexural tension side of the member, where the flexural tension side of the member shall be taken as the half-depth containing the flexural tension zone (in²)

= area of concrete below h/2 = 80.00/2 = 40.00 in (as shown in Figure 11.2.1.1-1)

= [(26)(6) + (2)(0.5)(10)(4.5) + (6)(34.00)] = 405.0 in²

$$\varepsilon_s = \frac{\left(\dfrac{21{,}247.0}{73.80} + 0 + (\ 302.1\ -\ 14.2\ ) - (\ 5.202\ )(\ 189.0\ )\right)}{0 + (\ 28{,}500\ )(\ 5.202\ ) + (\ 3{,}718\ )(\ 405.0\ )} = \text{-0.25x10}^{-3}$$

Note that the sign of ε_s should be maintained.

Check that ε_s is greater than -0.40x10^{-3} and less than 6.0x10^{-3}:

-0.25x10⁻³	>	-0.40 x10⁻³	**OK**
-0.25x10⁻³	<	6.0 x10⁻³	**OK**

Check that ε_s is greater than -0.40x10^{-3} and less than 6.0x10^{-3}: $\text{-0.25x10}^{-3} > \text{-0.40 x10}^{-3}$ **OK**

$\text{-0.25x10}^{-3} < 6.0\text{ x10}^{-3}$ **OK**

Therefore, $\varepsilon_s = \text{-0.25 x10}^{-3}$
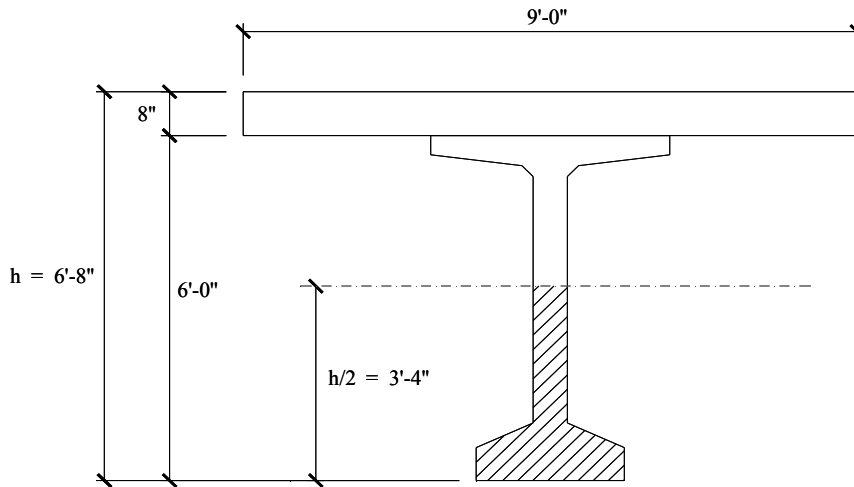
Figure 11.2.1.1-1: Illustration of A$_{ct}$

11.3.2 Determine Values of β and θ

β = factor indicating the ability of diagonally cracked concrete to transmit tension and shear

$$= \frac{4.8}{(1 + 750\varepsilon_s)} = \frac{4.8}{\left(1 + 750(\text{-0.25x10}^{-3})\right)} = 5.91 \qquad \text{[AASHTO Eq. 5.8.3.4.2-1]}$$

Note that to use this equation, the section must contain at least the minimum amount of transverse reinforcement (calculated in Section 11.3.4). If the section does not contain the minimum amount, AASHTO Eq. 5.8.3.4.2-2 must be used to find β.

θ = angle of inclination of diagonal compressive stresses (degrees)

$$= 29 + 3500\varepsilon_s = 29 + 3500(\text{-0.25 x10}^{-3}) = 28.1° \qquad \text{[AASHTO Eq. 5.8.3.4.2-3]}$$

11.3.3 Compute Concrete Contribution

The nominal shear resisted by the concrete is:

$$V_c = 0.0316\,\beta\,\sqrt{(f'_c)}\,b_v\,d_v \qquad \text{[AASHTO Eq. 5.8.3.3-3]}$$

$$= 0.0316(5.91)(1.00)\,\sqrt{(6.5)}(6.00)(73.80) = 210.8\ \text{kips} \qquad \text{(includes 1.00 factor for sand-lightweight concrete)}$$

11.4 Contribution of Reinforcement to Nominal Shear Resistance

11.4.1 Requirement for Reinforcement

Transverse shear reinforcement shall be provided where:

$V_u > 0.5\phi(V_c + V_p)$ [AASHTO Eq. 5.8.2.4-1]

where: V_u = 302.1 kips

          ϕ = resistance factor for shear = 0.85

          V_c = 210.8 kips

          V_p = 14.2 kips

   V_u = 302.1 kips > $0.5\phi(V_c + V_p)$ = 0.5 (0.85) (210.8 + 14.2) = 95.6 kips

Therefore, transverse shear reinforcement shall be provided.

11.4.2 Required Area of Shear Reinforcement

$V_u / \phi \leq V_n = V_c + V_s + V_p$ [AASHTO Eq. 5.8.3.3-1]

where: V_s = shear resistance provided by shear reinforcement (kips)

            = $(V_u / \phi) - V_c - V_p$

            = (302.1 / 0.85) − 210.8 − 14.2 = 130.4 kips

    V_s = $\dfrac{A_v f_y d_v (\cot\theta + \cot\alpha) \sin\alpha}{s}$ = 130.4 kips   [AASHTO Eq. 5.8.3.3-4]

      where: A_v = area of shear reinforcement (in²)

               = (0.20 in²)(2)   for #4 stirrup with 2 legs

               = 0.40 in²

          α = angle of inclination of transverse reinforcement to longitudinal axis (°)

               = 90°   for vertical stirrups

                $\cot(\alpha) = \cot(90°) \approx 0$

                $\sin(\alpha) = \sin(90°) = 1$

        s = spacing of stirrups (in)

        f_y = yield strength of shear reinforcement (ksi) = 60 ksi

Solving Eq. 5.8.3.3-4 for the required area of shear reinforcement per unit length:

$A_v / s = V_s / (f_y d_v \cot\theta)$

  = 130.4 / [(60)(73.80)cot(28.1)]   = 0.0157 in²/in

For #4 stirrups with 2 legs, provided spacing shall not exceed: $s = A_v / (A_v / s) = 0.40 / 0.0157 = $ 25.48 in

11.4.3 Spacing Requirements of Reinforcement   [AASHTO Art. 5.8.2.7]

The spacing of the transverse reinforcement shall not exceed the maximum permitted spacing, s_{max}, determined as:

  If $v_u < 0.125 f'_c$: $s_{max} = 0.8 d_v \leq 24.0$ in   [AASHTO Eq. 5.8.2.7-1]

  If $v_u \geq 0.125 f'_c$: $s_{max} = 0.4 d_v \leq 12.0$ in   [AASHTO Eq. 5.8.2.7-2]

$v_u = 0.771 \text{ ksi} \quad < \quad 0.125 \, f'_c = 0.813 \text{ ksi}$ (See Section 11.2 for calculation of v_u)

$s_{max} = 0.8(73.80) = 59.04 \text{ in} \geq 24.0 \text{ in}$

$s_{max} = 24.0 \text{ in} < \text{ required spacing} = 25.48 \text{ in}$

Use spacing, $s = 24.0$ in $\qquad A_v/s = (0.40 \text{ in}^2) / (24.0 \text{ in}) = 0.0167 \text{ in}^2/\text{in} > 0.0157 \text{ in}^2/\text{in}$

$$V_s = \frac{(0.40)(60)(73.80)[\cot(28.1°) + \cot(90)]\sin(90)}{24.0} = 138.2 \text{ kips}$$

11.4.4 Minimum Reinforcement Required

The area of transverse reinforcement should not be less than: [AASHTO Eq. 5.8.2.5-1]

$$A_v = 0.0316\sqrt{f'_c}\frac{b_v s}{f_y} = 0.0316\sqrt{6.5}\frac{(6.00)(24.0)}{60} = 0.19 \text{ in}^2 < \text{provided area} = 0.40 \text{ in}^2 \quad \textbf{OK}$$

AASHTO Article 5.8.2.2 states that the term $0.75\sqrt{(f'_c)}$ for all lightweight concrete and $0.85\sqrt{(f'_c)}$ for sand-lightweight concrete shall be substituted for $\sqrt{(f'_c)}$ in the expressions given in Articles 5.8.2 and 5.8.3. However, applying this factor to the minimum reinforcement expression would produce a smaller, less conservative limit for the amount of transverse reinforcement required. Therefore, the factor is not included in this design example.

Therefore, #4 stirrups with 2 legs shall be provided at 24.0 in.

11.5 Maximum Nominal Shear Resistance

In order to ensure that the concrete in the web of the beam will not crush prior to yielding of the transverse reinforcement, the AASHTO Specifications give an upper limit of V_n as follows:

$V_n = 0.25 f'_c b_v d_v + V_p$ [AASHTO Eq. 5.8.3.3-2]

Comparing this equation with Eq. 5.8.3.3-1, it can be concluded that:

$V_c + V_s \leq 0.25 f'_c b_v d_v$

$210.8 + 138.2 = 349.0 \text{ kips} \leq 0.25(6.5)(6.00)(73.80) = 719.6 \text{ kips} \qquad \textbf{OK}$

At the critical section for shear (see Section 11.1), the shear stress is limited by the provisions of AASHTO Art. 5.8.3.2 to a maximum of $0.18 f'_c$. If this limit is exceeded, AASHTO permits the end region of the beam to be designed using the strut-and-tie model (specified in AASHTO Art. 5.6.3). Here, however, the stress limit will be enforced.

Shear stress at critical section, $v_u = 0.771$ ksi (See Section 11.2 for calculation)

Shear stress upper limit at critical section $= 0.18 f'_c = 1.17 \text{ ksi} > 0.771 \text{ ksi} \qquad \textbf{OK}$

Using the foregoing procedures, the transverse reinforcement can be determined at increments along the entire length of the beam.

12 INTERFACE SHEAR TRANSFER   [AASHTO Art. 5.8.4]

12.1 Factored Interface Shear

To calculate the factored interface shear between the girder and slab, the procedure in the commentary of AASHTO Art. 5.8.2 will be used. This procedure calculates the factored interface shear force per unit length.

At the Strength I Limit State, the factored interface shear force, V_{hi}, at a section on a per unit basis is:

$V_{hi} = V_1 / d_v$ [AASHTO Eq. C5.8.4.2-7]

where: V_1 = factored shear force at specified section due to total load (noncomposite and composite loads)

                     The AASHTO Specifications does not identify the location of the critical section. For convenience, it will be assumed here to be the same location as the critical section for vertical shear, at point 0.05L.

                 = V_u = 302.1   kips   (See Section 11.3.1 for calculation)

           d_v = 73.80   in

    V_{hi} = 302.1 / 73.80 =   4.09   kips/in

12.2 Required Nominal Interface Shear Resistance

The required nominal interface shear resistance (per unit length) is:

$V_{ni} = V_{ri} / \phi$ [AASHTO Eq. 5.8.4.1-1]

where: $V_{ri} \geq V_{ui}$ [AASHTO Eq. 5.8.4.1-2]

        where: $V_{ui} = V_{hi}$ = 4.09   kips/in

Therefore, $V_{ni} = V_{ui} / \phi$ =   4.09 / 0.85   =   4.81   kips/in   =   57.72   kips/ft

12.3 Required Interface Shear Reinforcement

The nominal shear resistance of the interface surface (per unit length) is:

$V_{ni} = c A_{cv} + \mu (A_{vf} f_y + P_c)$ [AASHTO Eq. 5.8.4.1-3]

where: A_{cv} = area of concrete considered to be engaged in interface shear transfer (in²)

                = $12 b_{vi}$ (to calculate A_{vf} per foot of girder length)   [AASHTO Art. C5.8.4.1]

                where: b_{vi} = interface width engaged in shear transfer = width of top flange = 42.0 in

                = (12 in/ft) (42.0 in)   =   504.0   in²/ft

          A_{vf} = area of interface shear reinforcement crossing the shear plane within the area A_{cv} (in²/ft)

          f_y = yield stress of reinforcement but design value not to exceed 60 (ksi)   =   60 ksi

          P_c = permanent net compressive force normal to the shear plane; if force is tensile, $P_c = 0.0$ (kip)

                It is conservative to neglect P_c if it is compressive.   [AASHTO Art. C5.8.4.1]

         For a cast-in-place concrete slab on clean concrete girder surfaces, free of laitance with surface roughened to an amplitude of 0.25 in.

         c   = cohesion factor = 0.280 ksi   [AASHTO Art. 5.8.4.3]

         μ = friction factor = 1.0

AASHTO Eq. 5.8.4.1-3 can be solved for A_{vf}:

$A_{vf} = [V_{ni} - c A_{cv} - \mu P_c] / \mu f_y$

    = [57.72 − 0.280(504.0) − 1.0(0.0)] / 1.0(60)   =   -1.39   in²/ft   =   - 0.1158 in²/in

The resistance provided by cohesion is higher than the applied force. Provide the minimum required interface reinforcement.

From vertical shear design (Sect. 11.4.3), the provided shear reinforcement, $A_v/s = 0.0167$ in²/in > -0.1158 in²/in   **OK**

The provided reinforcement for vertical shear is sufficient to resist interface shear.

The required spacing for shear reinforcement =   (0.20 in²) (2 legs) / 0.0167 in²/in $=$ 24.00 in

Use spacing, s $=$ 24.0 in

The provided amount of interface shear reinforcement, $A_{vf} = $ (0.20 in²) (2 legs) / (24.0 in) $=$ 0.0167 in²/in

12.3.1 Minimum Interface Shear Reinforcement

The cross-sectional area of the interface shear reinforcement, A_{vf}, crossing the interface area, A_{cv}, shall satisfy:

$A_{vf} \geq 0.05A_{cv}/f_y = 0.05(504.0) / 60 = 0.42$ in²/ft $= 0.0350$ in²/in   [AASHTO Eq. 5.8.4.4-1]

The minimum interface shear reinforcement, A_{vf}, need not exceed the lesser of the amount determined in AASHTO Equation 5.8.4.4-1 and the amount needed to resist $1.33V_{ui}/\phi$ as determined using Eq. 5.8.4.1-3.   [AASHTO Art. 5.8.4.4]

$1.33V_{ui}/\phi = 1.33(57.72) / 0.85 = 90.31$ kips/ft $= V_{ni}$

$A_{vf} = [V_{ni} - cA_{cv} - \mu P_c] / \mu f_y$

   $= [90.31 - 0.280(504.0) - 1.0(0.0)] / 1.0(60) =$ $- 0.85$ in²/ft $=$ $- 0.0708$ in²/in $<$ 0.0350 in²/in

Therefore, the minimum amount of interface shear reinforcement, $A_{vf} =$ -0.0708 in²/in

The provided amount of interface shear reinforcement, $A_{vf} = 0.0167$ in²/in $>$ -0.0708 in²/in   **OK**

12.4 Maximum Nominal Shear Resistance

The provided nominal shear resistance, $V_{ni} = 0.280(504.0) + 1.0 [(0.0167)(12)(60) + 0] = 153.14$ kips/ft
<div align="right">[AASHTO Eq. 5.8.4.1-3]</div>

The nominal shear resistance, V_{ni}, used in design shall not be greater than the lesser of:

$V_{ni} \leq K_1 f'_c A_{cv}$ or   [AASHTO Eq. 5.8.4.1-4]

$V_{ni} \leq K_2 A_{cv}$ [AASHTO Eq. 5.8.4.1-5]

where: For a cast-in-place concrete slab on clean concrete girder surfaces, free of laitance with surface roughened to an amplitude of 0.25 in.

   K_1 = fraction of concrete strength available to resist interface shear $= 0.30$ [AASHTO Art. 5.8.4.3]

   K_2 = limiting interface shear resistance $= 1.3$ ksi

   f'_c = specified 28-day compressive strength of the weaker concrete on either side of the interface (ksi)

$K_1 f'_c A_{cv} = 0.30(4.0)(504.0) = 604.80$ kip/ft $>$ 153.14 kip/ft   **OK**

$K_2 A_{cv} = 1.3(504.0) = 655.20$ kip/ft $>$ 153.14 kip/ft   **OK**

13 MINIMUM LONGITUDINAL REINFORCEMENT DUE TO SHEAR

$$A_{ps}f_{ps} + A_sf_y \geq \frac{|M_u|}{d_v\phi_f} + 0.5\frac{N_u}{\phi_c} + \left[\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right]\cot\theta$$

[AASHTO Eq. 5.8.3.5-1]

where: A_{ps} =   area of prestressing steel on the flexural tension side of the member at section under consideration (in^2)

=   (number of non-harped strands)(area of strand) = (34)(0.153)

=   5.202   in^2

f_{ps} =   average stress in prestressing steel at the time for which the nominal resistance is required (ksi)

M_u =   factored bending moment at the section corresponding to the factored shear force (kip-ft)

V_u =   factored shear force at section under consideration (kip)

V_p =   component of the effective prestressing force in direction of the applied shear (kip) = 14.2 kips

V_s =   shear resistance provided by the transverse reinforcement at the section under investigation as given by Eq. 5.8.3.3-4, except that V_s shall not be taken as greater than V_u/ϕ (kip)

ϕ_f =   resistance factor for flexure

ϕ_c =   resistance factor for axial resistance

ϕ_v =   resistance factor for shear

θ =   angle of inclination of diagonal compressive stresses used in determining the nominal shear resistance of the section under investigation as determined by Art. 5.8.3.4 (degrees)   = 28.1°

13.1 Required Reinforcement at Face of Bearing

Because the width of the bearing is not yet determined, it is assumed to be zero. This assumption is more conservative for these calculations. Thus, the failure crack assumed for this analysis radiates from the centerline of the bearing, 6 in. from the end of the beam.

As 6 in. is very close to the end of the beam, shear and moment values at the end of the beam will be used from Table 4-1.

V_u =   1.25(40.0 + 46.1 + 6.0) + 1.50(12.0) + 1.75(78.1 + 33.9)

=   329.1   kips

M_u =   0.0   kip-ft

$$V_s = \frac{A_vf_yd_v(\cot\theta + \cot\alpha)\sin\alpha}{s}$$

[AASHTO Eq. 5.8.3.3-4]

V_s was calculated in Section 11.4.3 and is repeated here for clarity.

where: s =   24.0   in

A_v =   0.40   in²

$$= \frac{(0.40)(60)(73.80)[\cot(28.1°) + \cot(90)]\sin(90)}{24.0} = 138.2 \text{ kips}$$

Check if V_s is greater than V_u/ϕ = 329.1 / 0.85 = 387.2 kips   >   138.2 kips

Use V_s = 138.2 kips

$$\frac{|M_u|}{d_v\phi_f} + 0.5\frac{N_u}{\phi_c} + \left[\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right]\cot\theta = 0 + 0 + \left[\left|\frac{329.1}{0.85} - 14.2\right| - 0.5(138.2)\right]\cot(28.1°) = 569.2 \text{ kips}$$

As shown in Figure 13.1-1, the assumed crack plane crosses the centroid of the 34 straight strands at a distance of [6 + 4.00cot(28.1°)] = 13.49 in. from the end of the beam. Since the transfer length is 30 in. (2.5 ft) from the end of the

beam, the available prestress from the 34 straight strands is a fraction of the effective prestress in these strands. The 6 harped strands do not contribute to the tensile capacity since they are not on the flexural tension side of the member.

Thus, f_{ps} = f_{pe} (13.49 / 30) = 156.145 (13.49 / 30) = 70.213 ksi   (See Section 6.2.4 for calculation of f_{pe})

$A_{ps} f_{ps} + A_s f_y$ = (5.202) (70.213) + 0 =   365.2 kips   <   569.2   kips   **NG**

Provide (569.2 − 365.2) / 60.0 = 3.40 in² of additional flexural tension reinforcement at face of bearing.

(6) #7 bars are provided for an area of 3.60 in²   >   3.40 in²   **OK**
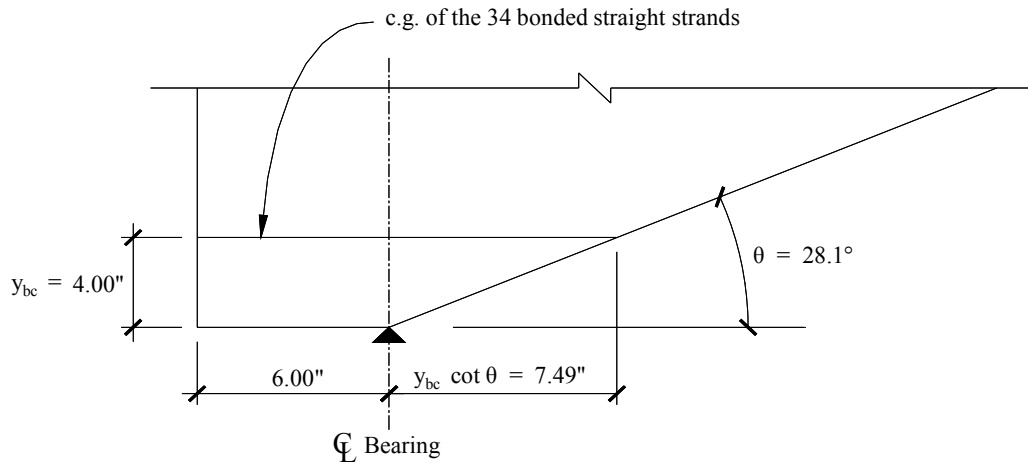
Figure 13.1-1: Assumed Failure Crack

14 PRETENSIONED ANCHORAGE ZONE   [AASHTO Art. 5.10.10]

14.1 Anchorage Zone Reinforcement   [AASHTO Art. 5.10.10.1]

Design of the anchorage zone reinforcement is computed using the force in the strands just prior to transfer:

Force in the strands before transfer = F_{pi} = (number of strands)(area of one strand)(f_{pi}) =   40(0.153)(202.5)   = 1,239.3 kips

(See Section 2 for calculation of f_{pi})

The splitting resistance, P_r, should not be less than 4% of F_{pi}:

$P_r = f_s A_s \geq 0.04 F_{pi} = 0.04 (1,239.3) = 49.6$ kips

where: A_s =   total area of reinforcement located within the distance h/4 from the end of the beam (in²)

For pretensioned I-beams and bulb tees, A_s shall be taken as the total area of the vertical reinforcement located within a distance h/4 from the end of the member, where h is the overall height of the member (in)

f_s =   stress in the steel not to exceed 20 ksi

Solving for the required area of steel, $A_s = P_r / f_s = 49.6 / 20 = 2.48$ in²

At least 2.48 in² of vertical transverse reinforcement should be provided within a distance of h/4 from the end of beam.

h/4 = 72/4 = 18.0 in

The area of a #5 stirrup with 2 legs is: (2)(0.31 in²) =   0.62 in²

The required number of stirrups is: (2.48 in²) / (0.62 in²) =   4

The required spacing for 4 stirrups over a distance of 18.0 in. starting 2 in. from the end of the beam is:

$(18.0 - 2) / (4 - 1) =$ 5.0 in

Use (4) #5 stirrups with 2 legs at 5.0 in. spacing starting at 2 in. from the end of the beam.

The provided A_s = (4)(2)(0.31 in²) = 2.48 in²   >   2.48 in²   **OK**

14.2 Confinement Reinforcement   [AASHTO Art. 5.10.10.2]

For the distance of 1.5d = 1.5(72) = 108.0 in. from the end of the beam, reinforcement shall be placed to confine the prestressing steel in the bottom flange. The reinforcement shall not be less than #3 deformed bars, with spacing not exceeding 6.0 in. and shaped to enclose the strands.

15 DEFLECTION AND CAMBER   [AASHTO Art. 5.7.3.6.2]

15.1 Deflection Due to Prestressing Force at Transfer

The force per strand at transfer =   27.7 kips   (See Section 6.2.3 for calculation)

$$\Delta_p = \frac{P_i}{E_{ci}I}\left(\frac{e_c L^2}{8} - \frac{e' a^2}{6}\right)$$ (Eq. 15.1-1)

where: P_i =   total prestressing force after transfer = (40)(27.7) =   1,108.0 kips

e_c =   eccentricity of prestressing steel at midspan = 31.70 in   (See Section 5.4 for calculation)

e' =   difference between the eccentricity of the prestressing steel at midspan and at the end of the beam

= $e_c - e_e$ = 31.70 − (36.60 −13.60) =   8.70   in   (See Sections 3.1 and 7.2 for calculations)

a   =   distance from the end of the beam to the harp point = 48.50 ft   (See Section 7.2 for calculation)

L   =   overall length of beam = 121.0 ft

E_{ci} =   modulus of elasticity of precast beam at transfer = 3,512 ksi   (See Section 3.1 for calculation)

I   =   gross moment of inertia of the precast beam = 545,894 in⁴

$$\Delta_p = \frac{1,108.0}{(3,512)(545,894)}\left(\frac{(31.70)(121.0 \text{ x } 12)^2}{8} - \frac{(8.70)(48.50 \text{ x } 12)^2}{6}\right) =   4.54 \text{ in}   \uparrow$$

15.2 Deflection Due to Beam Self-Weight

$$\Delta_g = \frac{5wL^4}{384E_{ci}I}$$ (Eq. 15.2-1)

where: w   =   beam self weight = 0.666 kip/ft

L   =   beam length = 121.0 ft at transfer = 120.0 ft at erection

E_{ci} =   modulus of elasticity of precast beam at transfer = 3,512 ksi

I   =   gross moment of inertia of the precast beam = 545,894 in⁴

Deflection due to beam self-weight at transfer:

$$\Delta_g = \frac{5\left(\frac{0.666}{12}\right)(121.0 \text{ x } 12)^4}{384(\,3,512\,)(\,545,894\,)} = \quad 1.68 \text{ in} \qquad \downarrow$$

Deflection due to beam self-weight used to compute deflection at erection:

$$\Delta_g = \frac{5\left(\frac{0.666}{12}\right)(120.0 \text{ x } 12)^4}{384(\,3,512\,)(\,545,894\,)} = \quad 1.62 \text{ in} \qquad \downarrow$$

15.3 Deflection Due to Slab and Haunch Weights

$$\Delta_s = \frac{5wL^4}{384E_cI} \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(Eq. 15.3-1)}$$

where: w =   slab and haunch weight $= 0.750 + 0.018 = \quad 0.768 \quad$ kip/ft   (See Section 4.1.1 for calculation)

L =   design span length $= 120.0$ ft

E_c =   modulus of elasticity of precast beam at service loads $= 3,718$ ksi   (See Section 3.1 for calculation)

I   =   gross moment of inertia of the precast beam $= 545,894$ in^4

$$\Delta_s = \frac{5\left(\frac{0.768}{12}\right)(120.0 \text{ x } 12)^4}{384(\,3,718\,)(\,545,894\,)} = \quad 1.77 \text{ in} \qquad \downarrow$$

15.4 Deflection Due to Rail/Barrier and Future Wearing Surface (Overlay)

$$\Delta_{b+ws} = \frac{5wL^4}{384E_cI_c} \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(Eq. 15.4-1)}$$

where: w =   barrier and future wearing surface weight $= 0.100 + 0.200 = \quad 0.300 \quad$ kip/ft   (See Section 4.1.1)

L =   design span length $= 120.0$ ft

E_c =   modulus of elasticity of precast beam at service loads $= 3,718$ ksi

I_c =   gross moment of inertia of the composite section $= 1,100,347$ in^4 (See Section 3.2.3 for calculation)

$$\Delta_{b+ws} = \frac{5\left(\frac{0.300}{12}\right)(120.0 \text{ x } 12)^4}{384(\,3,718\,)(1,100,347\,)} = \quad 0.34 \text{ in} \qquad \downarrow$$

15.5 Deflection and Camber Summary

At transfer, $(\Delta_p + \Delta_g)$ = 4.54 + -1.68 = 2.86 in   ↑

The deflection at erection can be calculated by using the multipliers given in Table 8.7.2-1 of the PCI Bridge Design Manual:

Table 15.5-1: Time-Dependent Multipliers for Deflections at Time of Erection

Load Condition	Formula	Multiplier	Deflection	Deflection at Erection
Initial Prestress	$1 + C_a$	2.04	4.54 in ↑	9.26 in ↑
Self-Weight	$1 + C_a$	2.04	1.62 in ↓	3.30 in ↓
Dead Load on Plain Beam	1.00	1.00	1.77 in ↓	1.77 in ↓
Dead Load on Composite Beam	1.00	1.00	0.34 in ↓	0.34 in ↓

Where C_a = creep coefficient for deflections at time of erection due to loads applied at release = $\Psi_b(t_d, t_i)$ = 1.04

Total deflection at erection before deck placement = 9.26 − 3.30 = 5.95 in   ↑

Total deflection after deck placement = 9.26 − 3.30 − 1.77 = 4.18 in   ↑

Total deflection due to dead loads on composite section = 9.26 − 3.30 − 1.77 − 0.34 = 3.84 in   ↑

15.6 Deflection Due to Live Load and Impact

Live load deflection limit (optional) = Span / 800   [AASHTO Art. 2.5.2.6.2]

   = (120.0 x 12) / 800 = 1.80 in

If the owner invokes the optional live load deflection criteria specified in AASHTO Article 2.5.2.6.2, the deflection is the greater of:

   • That resulting from the design truck alone, or   [AASHTO Art. 3.6.1.3.2]

   • That resulting from 25% of the design truck taken together with the design lane load.

The live load portion of Load Combination Service I should be used, including the dynamic load allowance, IM.

When investigating the maximum absolute deflection, all design lanes should be loaded, and all supporting components should be assumed to deflect equally.   [AASHTO Art. 2.5.2.6.2]

Therefore, the distribution factor for deflection, DFD, is calculated as follows:

 DFD =   (number of lanes / number of beams)

   = (4 / 6) = 0.667 lanes/beam

However, it is more conservative to use the distribution factor for moment, DFM = 0.732 (See Section 4.2.2.1 for calculation)

Deflection due to Lane Load:

   Design lane load, w = 0.64 kips/ft (DFM) = (0.64)(0.732) = 0.468 kips/ft/beam

$$\Delta_{LL} = \frac{5wL^4}{384E_cI_c}$$

where: w = design lane load = 0.468 kips/ft/beam

L = design span length = 120.0 ft

E_c = modulus of elasticity of precast beam at service loads = 3,718 ksi

I_c = gross moment of inertia of the composite section = 1,100,347 in^4

$$\Delta_{LL} = \frac{5\left(\frac{0.468}{12}\right)(120.0 \text{ x } 12)^4}{384(\,3,718\,)(\,1,100,347\,)} = 0.53 \text{ in} \qquad \downarrow$$

Deflection due to Design Truck Load and Impact:

To obtain maximum moment and deflection at midspan due to the truck load, set the spacing between the rear axles to 14 ft, and let the centerline of the beam coincide with the middle point of the distance between the inner 32-kip axle and the resultant of the truck load, as shown in Figure 15.6-1.
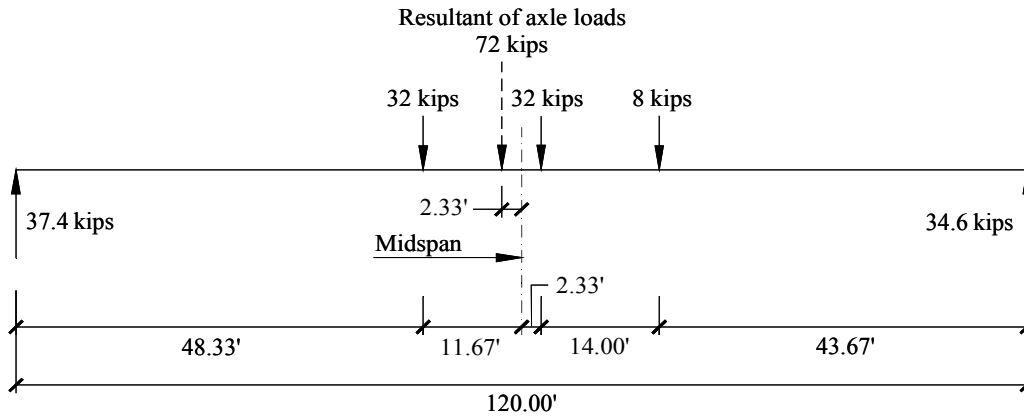
Figure 15.6-1: Design Truck Axle Load Position for Maximum Bending Moment

The deflection at point x due to a point load at point a is given by the following equations:

$$\Delta = \frac{Pbx}{6E_cI_cL}(L^2 - b^2 - x^2) \text{ for } x < a \qquad \Delta = \frac{Pb}{6E_cI_cL}\left[(x-a)^3\frac{L}{b} + (L^2 - b^2)x - x^3)\right] \text{ for } x > a$$

where: P = point load

L = span length = 120.00 ft

x = location at which deflection is to be determined = midspan = 60.00 ft

a = distance from end of beam to location of point load

b = L − a

E_c = modulus of elasticity of precast beam at service loads = 3,718 ksi

I_c = gross moment of inertia of the composite section = 1,100,347 in^4

Using superposition, the deflection at midspan can be determined by adding the deflections due to the individual axle loads:

Table 15.6-1: Deflections due to Axle Loads

P (kips)	a (ft)	b (ft)	x (ft)	Δ (in)
32	48.33	71.67	60.00	0.461
32	62.33	57.67	60.00	0.485
8	76.33	43.67	60.00	0.109

The total deflection $=0.461 + 0.485 + 0.109 = 1.055$ in

Including impact and the distribution factor, the deflection at midspan due to the design truck load is:

$\Delta_{LT} = $ (1.055 in)(IM)(DFM) $=$ (1.055 in)(1.33)(0.732) $= 1.03$ in $\quad\downarrow$

Therefore, the live load deflection is the greater of:

$\Delta_{LT} = 1.03$ in

or

$0.25\Delta_{LT} + \Delta_{LL} = 0.25(1.03) + 0.53 = 0.79$ in

Therefore, the live load deflection is 1.03 in. $<$ allowable deflection $= 1.80$ in. **OK**