ATTACHMENT B

Design Examples

The following five design examples illustrate the use of the design guide specifications prepared in this project and subsequently published by AASHTO (AASHTO Guide Specifications, 2018):

Example B-1: Design of a rectangular beam pretensioned with straight CFRP cables

Example B-2: Design of a Decked AASHTO pretensioned girder with straight CFRP cables

Example B-3: Design of a Decked AASHTO pretensioned girder with harped CFRP cables

Example B-4: Design of a rectangular beam post-tensioned with straight CFRP cables

Example B-5: Design of a Decked AASHTO post-tensioned girder with draped CFRP cables

REFERENCES

AASHTO-LRFD (2017). AASHTO LRFD Bridge Design Specifications, 8th edition, Washington, DC, USA.

AASHTO Guide Specifications (2018). Guide Specifications for the Design of Concrete Bridge Beams Prestressed with Carbon Fiber-Reinforced Polymer (CFRP) Systems, 1st edition, Washington, DC, USA.

Adil, M., Adnan, M., Hueste, M., and Keating, P. (2007). Impact of LRFD Specifications on Design of Texas Bridges Volume 2: Prestressed Concrete Bridge Girder Design Examples.

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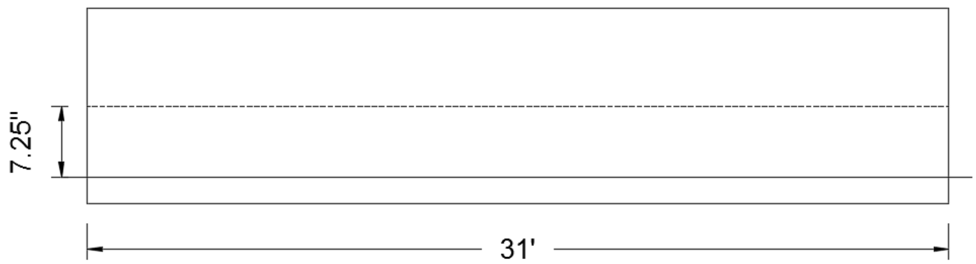
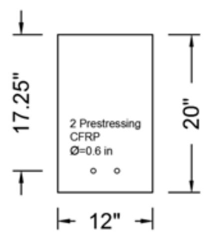
Cousins, T. E., Roberts-Wollmann, C. L., and Brown, M. C. (2013). "NCHRP Report 733: High performance/high-strength lightweight concrete for bridge girders and decks." *Transportation Research Board of the National Academies, Washington, DC.*

Wassef, W., Smith, C., Clancy, C., and Smith, M. (2003). Comprehensive design example for prestressed concrete (PSC) girder superstructure bridge with commentary. *Federal Highway Administration report no. FHWA NHI-04-043, grant no. DTFH61-02-D-63006. Washington, DC.*

Example B-1: Design of a rectangular beam pretensioned with straight CFRP cables

The following example illustrates the analysis of rectangular beam pretensioned with two prestressing cables of 0.6 inch diameter and a jacking stress of $0.70 \cdot f_{pu}$. The beam is $31\ ft$ in length and carries a superimposed dead load of 20% of it's self-weight and the live load of $0.35\ \frac{kip}{ft}$ in addition to its own weight. The analysis includes checking all applicable service and strength limit states according to AASHTO-LRFD (2017) and AASHTO Guide Specifications (2018). They are referred in the following example as AASHTO and AASHTO-CFRP respectively. The analysis also includes the computations of deflection corresponding to the moment of $130.0\ ft \cdot kip$.





Overall beam Length $L_{span} := 31\ ft$

Design Span $L_{design} := 30\ ft$

Concrete Properties

Concrete strength at release, $f'_{ci} := 5.5\ ksi$

Concrete strength at 28 days, $f'_c := 9.0\ ksi$

Unit weight of concrete $\gamma_c := 150\ pcf$

Prestressing CFRP

Diameter of one prestressing CFRP cable $d_b := 0.6\ in$

Area of one prestressing CFRP cable $A_{pf} := 0.18\ in^2$

The design tensile load is the characteristics value of the tensile test data conducted as a part of NCHRP 12-97 project and computed according to ASTM D7290 as recommended by the proposed material guide specification. The design tensile stress is obtained as follows:

Design tensile stress $f_{pu} := \dfrac{64.14\ kip}{A_{pf}} = 356.33\ ksi$

Modulus of elasticity (AASHTO-CFRP Art. 1.4.1.3) $E_f := 22500\ ksi$

Design tensile strain $\varepsilon_{pu} := \dfrac{f_{pu}}{E_f} = 0.016$

Stress limitation for prestressing CFRP (AASHTO-CFRP Art. 1.9.1)

Before transfer $f_{pi} := 0.70 \cdot f_{pu} = 249.43\ ksi$

At service, after all losses $f_{pe} := 0.65 \cdot f_{pu} = 231.62\ ksi$

Nonprestressed Reinforcement:

Yield strength $f_y := 60\ ksi$

Modulus of elasticity (AASHTO Art. 5.4.3.2) $E_s := 29000\ ksi$

Beam Section Properties

Width of beam $b := 12 \cdot in$

Height of beam $h := 20 \cdot in$

Cross-section area of beam $A := b \cdot h = 240\ in^2$

Distance from centroid to the extreme bottom fiber of the non-composite precast girder $y_b := \dfrac{h}{2} = 10\ in$

Distance from centroid to the extreme top fiber of the non-composite precast girder $y_t := h - y_b = 10\ in$

Moment of inertia of deck about it centriod $I := \dfrac{b \cdot h^3}{12} = \left(8 \cdot 10^3\right)\ in^4$

Section modulus referenced to the extreme bottom fiber of the non-composite precast girder $S_c := \dfrac{I}{y_b} = 800\ in^3$

Section modulus referenced to the extreme top fiber of the non-composite precast girder $S_{ct} := \dfrac{I}{y_t} = 800\ in^3$

Weight of the beam $w := (b \cdot h) \cdot \gamma_c = 0.25\ \frac{kip}{ft}$

Material Properties for Girder and Deck Concrete:

Modulus of elasticity of concrete (AASHTO Art. 5.4.2.4) $E(f'_c) := 12 \cdot \left(\frac{\gamma_c}{pcf}\right)^{2.0} \left(\frac{f'_c}{psi}\right)^{0.33} \cdot psi$

At release $E_{ci} := E(f'_{ci}) = (4.63 \cdot 10^3)\ ksi$

At 28 days (Girder) $E_c := E(f'_c) = (5.45 \cdot 10^3)\ ksi$

Modulus of rupture of concrete (AASHTO Art 5.4.2.6) $f_{mr}(f'_c) := 0.24 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi$

At release $f_{ri} := f_{mr}(f'_{ci}) = 0.56\ ksi$

At 28 days (Girder) $f_r := f_{mr}(f'_c) = 0.72\ ksi$

Number of Strands and Strand Arrangement:

Total number of prestressing CFRP $n_p := 2$

Concrete cover $cc := 2.75\ in$

Depth of prestressing CFRP from the top fiber of the beam $d_p := h - cc = 17.25\ in$

Eccentricity of prestressing CFRP $e_c := d_p - y_t = 7.25\ in$

Load and Moment on Beam:

Unit weight due to superimposed load $w_{SD} := 0.2 \cdot w = 0.05\ \frac{kip}{ft}$

Unit weight due to live load $w_L := 0.35\ \frac{kip}{ft}$

M_b=unfactored bending moment due to beam self-weight, k-ft

$$M_b := \frac{w \cdot {L_{design}}^2}{8} = 28.13\ ft \cdot kip$$

M_{SD}=unfactored bending moment due to superimposed dead load, k-ft

$$M_{SD} := \frac{w_{SD} \cdot {L_{design}}^2}{8} = 5.63\ ft \cdot kip$$

M_L=unfactored bending moment due to live load, k-ft

$$M_L := \frac{w_L \cdot L_{design}^2}{8} = 39.38\ ft \cdot kip$$

Moment at service III [AASHTO Table 3.4.1-1]

$$M_s := M_b + M_{SD} + 0.8 \cdot M_L = 65.25\ ft \cdot kip$$

Moment at Strength I [AASHTO Table 3.4.1-1]

$$M_u := 1.25\ M_b + 1.5\ M_{SD} + 1.75\ M_L = 112.5\ ft \cdot kip$$

Prestressing Loss

Prestressing CFRP stress before transfer $f_{pi} := 0.70 \cdot f_{pu} = 249.43\ ksi$

Elastic Shortening

$$\Delta f_{pES} = \frac{E_f}{E_{ct}} \cdot f_{cgp}$$ [AASHTO-CFRP Eq. (1.9.2.2.3a-1)]

Where E_f=modulus of elasticity of prestressing CFRP (ksi)

E_{ct}=modulus of elasticity of the concrete at transfer or time of load application (ksi)=E_{ci}

f_{cgp}=the concrete stress at the center of gravity of CFRP due to the prestressing force immediately after transfer and the self-weight of the member at sections of maximum moment (ksi)

AASHTO Article C5.9.5.2.3a states that to calculate the prestress after transfer, an initial estimate of prestress loss is assumed and iterated until acceptable accuracy is achieved. In this example, an initial estimate of 10% is assumed.

$eloss := 10\%$

Force per strand at transfer

$$f_{cgp} = \frac{P_i}{A_g} + \frac{P_i \cdot e_c^2}{I_g} - \frac{M_G \cdot e_c}{I_g}$$

Where, P_i=total prestressing force at release=$n_p * p$
e_c=eccentricity of strands measured from the center of gravity of the precast beam at midspan

M_G=moment due to beam self-weight at midspan (should be calculated using the overall beam length)

$$M_G := \frac{w \cdot L_{span}^{\ 2}}{8} = 30.03 \ ft \cdot kip$$

Guess Values

$$eloss := 10\%$$

Constraints

$$eloss = \frac{E_f}{f_{pi} \cdot E_{ci}} \cdot \left(\frac{n_p \cdot A_{pf} \cdot f_{pi} \cdot (1 - eloss)}{A} + \frac{(n_p \cdot A_{pf} \cdot f_{pi} \cdot (1 - eloss)) \cdot e_c^{\ 2}}{I} - \frac{M_G \cdot e_c}{I} \right)$$

Solver

$$eloss := \mathbf{find}(eloss) = 0.01$$

Therefore, the loss due to elastic shortening= $eloss = 0.01$

The stress at transfer= $f_{pt} := f_{pi} \cdot (1 - eloss) = 246.39 \ ksi$

The force per strand at transfer= $p_t := A_{pf} \cdot f_{pi} \cdot (1 - eloss) = 44.35 \ kip$

The concrete stress due to prestress=

$$f_{cgp} := \frac{n_p \cdot p_t}{A} + \frac{n_p \cdot p_t \cdot e_c^{\ 2}}{I} - \frac{M_b \cdot e_c}{I} = 646.53 \ psi$$

The prestress loss due to elastic shortening= $\Delta f_{pES} := \frac{E_f}{E_{ci}} \cdot f_{cgp} = 3.14 \ ksi$

Total prestressing force at release $P_t := n_p \cdot p_t = 88.7 \ kip$

Final prestressing loss including Elastic Shortening

Assume a total prestress loss of 18% [This assumption is based on the average of all cases in the design space considered in the reliability study]

$$ploss := 18\%$$

$$f_{pe} := f_{pi} \cdot (1 - ploss) = 204.54 \ ksi$$

Force per strand at service $p_e := f_{pe} \cdot A_{pf} = 36.82 \ kip$

Check prestressing stress limit at service limit state: [AASHTO-CFRP Table 1.9.1-1]

$$\begin{array}{l} \text{if } f_{pe} \le 0.6 \cdot f_{pu} \\ \quad \| \text{"Stress limit satisfied"} \\ \text{else} \\ \quad \| \text{"Stress limit not satisfied"} \end{array} = \text{"Stress limit satisfied"}$$

Check Stress at Transfer and Service:

Stresses at Transfer

Total prestressing force after transfer $P_t := n_p \cdot p_t = 88.7\ kip$

Stress Limits for Concrete

Compression Limit: [AASHTO Art. 5.9.2.3.1a]

$0.6 \cdot f'_{ci} = 3.3\ ksi$

Where, f'_{ci}=concrete strength at release=5.5 ksi

Tension Limit: [AASHTO Art. 5.9.2.3.1b]

Without bonded reinforcement

$-0.0948 \cdot \sqrt{\frac{f'_{ci}}{ksi}}\ ksi = -0.22\ ksi$ ≤ -0.2 ksi

Therefore, tension limit, $\blacksquare = -0.2\ ksi$

With bonded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force in the concrete computed assuming an uncracked section where reinforcement is proportioned using a stress of $0.5 f_y$, not to exceed 30 ksi.

$-0.24 \cdot \sqrt{\frac{f'_{ci}}{ksi}}\ ksi = -0.56\ ksi$

If the tensile stress is between these two limits, the tensile force at the location being considered must be computed following the procedure in AASHTO Art. C5.9.4.1.2. The required area of reinforcement is computed by dividing tensile force by the permitted stress in the reinforcement ($0.5 f_y \le 30$ ksi)

Stresses at Transfer Length Section

Stresses at this location need only be checked at release since this stage almost always governs. Also, losses with time will reduce the concrete stresses making them less critical.

Transfer length $l_t = \dfrac{f_{pi} \cdot d_b}{\alpha_t \cdot f'_{ci}{}^{0.67}}$ [AASHTO-CFRP Eq. 1.9.3.2.1-1]

Where, d_b =prestressing CFRP diameter (in.)

α_t =coefficient related to transfer length taken as 1.3 for cable

Also can be estimated as $l_t := 50 \cdot d_b = 30\ in$

Moment due to self-weight of the beam at transfer length

$$M_{bt} := 0.5 \cdot w \cdot l_t \cdot \left(L_{design} - l_t\right) = 8.59\ ft \cdot kip$$

Stress in the top of beam:

$$f_t := \frac{P_t}{A} - \frac{P_t \cdot e_c}{S_{ct}} + \frac{M_{bt}}{S_{ct}} = -0.31\ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [NOT OK]

-0.56 ksi with bonded reinforcement [OK]

stress in the bottom of the beam:

$$f_b := \frac{P_t}{A} + \frac{P_t \cdot e_c}{S_c} - \frac{M_{bt}}{S_c} = 1.04\ ksi$$

Compressive stress limit for concrete = 3.3 ksi [OK]

Stresses at midspan

Stress in the top of beam:

$$f_t := \frac{P_t}{A} - \frac{P_t \cdot e_c}{S_{ct}} + \frac{M_b}{S_{ct}} = -0.01\ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [NOT OK]

-0.56 ksi with bonded reinforcement [OK]

stress in the bottom of the beam:

$$f_b := \frac{P_t}{A} + \frac{P_t \cdot e_c}{S_c} - \frac{M_b}{S_c} = 0.75\ ksi$$

Compressive stress limit for concrete = 3.3 ksi [OK]

Stresses at Service Loads

Total prestressing force after all losses $P_e := n_p \cdot p_e = 73.63\ kip$

Concrete Stress Limits: [AASHTO Art. 5.9.2.3.2a]
Due to the sum of effective prestress and permanent loads (i.e. beam self-weight, weight of future wearing surface, and weight of barriers) for the Load Combination Service 1:

for precast beam $0.45 \cdot f'_c = 4.05\ ksi$

Due to the sum of effective prestress, permanent loads, and transient loads as well as during shipping and handling for the Load Combination Service 1:

for precast beam $0.60 \cdot f'_c = 5.4\ ksi$

Tension Limit: [AASHTO Art. 5.9.2.3.2b]
For components with bonded prestressing tendons or reinforcement that are subjected to not worse than moderate corrosion conditions for Load Combination Service III

for precast beam $-0.19 \cdot \sqrt{\frac{f'_c}{ksi}}\ ksi = -0.57\ ksi$

Stresses at Midspan
Concrete stress at top fiber of the beam

To check top stresses, two cases are checked:

Under permanent loads, Service I:

$$f_{tg} := \frac{P_e}{A} - \frac{P_e \cdot e_c}{S_{ct}} + \frac{M_b + M_{SD}}{S_{ct}} = 0.15\ ksi$$ $< 4.05\ ksi$ [OK]

Under permanent and transient loads, Service I:

$$f_{tg} := f_{tg} + \frac{M_L}{S_{ct}} = 0.74\ ksi \qquad < 5.4\ ksi \qquad \text{[OK]}$$

Concrete stress at bottom fiber of beam under permanent and transient loads, Service III:

$$f_b := \frac{P_e}{A} + \frac{P_e \cdot e_c}{S_c} - \frac{M_b + M_{SD} + (0.8) \cdot (M_L)}{S_c} = -4.65 \cdot 10^{-3}\ ksi$$

$$> -0.57\ ksi \qquad \text{[OK]}$$

If the tensile stress is between these two limits, the tensile force at the location being considered must be computed following the procedure in AASHTO Art. C5.9.2.3.1b. The required area of reinforcement is computed by dividing tensile force by the permitted stress in the reinforcement ($0.5 f_y \leq 30$ ksi)

Stregth Limit State

Effective prestressing strain $\varepsilon_{pe} := \frac{f_{pe}}{E_f} = 9.09 \cdot 10^{-3}$

If $\varepsilon_{cc} \leq 0.003$, the stress block factors are given by

$$\varepsilon_{co} := \left(\left(\frac{f'_c}{11\ ksi}\right) + 1.6\right) \cdot 10^{-3} = 0.0024$$

$$\beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) := \max\left(0.65, \frac{4 - \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)}{6 - 2 \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)} \cdot \left(-\left(\frac{f'_c}{50\ ksi}\right) + 1.1\right)\right) \qquad \text{[AASHTO-CFRP Eq. C1.7.2.1-3]}$$

$$\alpha_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) := \frac{\left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right) - \frac{1}{3} \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)^2}{\beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right)} \cdot \left(-\left(\frac{f'_c}{60\ ksi}\right) + 1\right) \qquad \text{[AASHTO-CFRP Eq. C1.7.2.1-4]}$$

By using equillibrium and compatibility, the depth of the neutral axis (c) and the strain at top fiber of the beam can be found using following

Guess Values

$c := 8\ in$ $\qquad d_p = 17.25\ in$ $\qquad \varepsilon_{pu} = 0.0158$

$\varepsilon_{cc} := 0.0015$ $\qquad \varepsilon_{co} = 0.0024$

Constraints

$\varepsilon_{cc} \le 0.003$

$$\varepsilon_{pe} + \frac{d_p - c}{c} \cdot \varepsilon_{cc} = \varepsilon_{pu}$$

$$\alpha_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot f'_c \cdot \beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot b \cdot c = n_p\ A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \frac{d_p - c}{c} \cdot \varepsilon_{cc}\right)$$

Solver

$$\begin{bmatrix} c \\ \varepsilon_{cc} \end{bmatrix} := \textbf{find}\left(c, \varepsilon_{cc}\right) = \begin{bmatrix} 0.2482\ ft \\ 0.0014 \end{bmatrix}$$

$\varepsilon_{cc} = 0.0014$

$$\beta_1 := \max\left(0.65, \frac{4 - \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)}{6 - 2 \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)} \cdot \left(-\left(\frac{f'_c}{50\ ksi}\right) + 1.1\right)\right) = 0.65$$

$$\alpha_1 := \frac{\left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right) - \frac{1}{3} \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)^2}{\beta_1} \cdot \left(-\left(\frac{f'_c}{60\ ksi}\right) + 1\right) = 0.61$$

Depth of neutral axis, $\qquad c = 2.98\ in$

Strain at prestressing CFRP at ultimate $\qquad \varepsilon_f := \frac{d_p - c}{c} \cdot \varepsilon_{cc} = 0.0067$

Total tension force $\qquad T_f := n_p \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_f + \varepsilon_{pe}\right) = 128.28\ kip$

Total compression force $\qquad C_c := \alpha_1 \cdot f'_c \cdot \beta_1 \cdot b \cdot c = 128.28\ kip$

Check for equillibrium $\qquad T_f - C_c = \left(2.54 \cdot 10^{-12}\right)\ kip$

The capacity of the section is:

$$M_n := T_f \cdot (d_p - c) + C_c \cdot \left(c - \frac{\beta_1 \cdot c}{2}\right) = 174.05\ ft \cdot kip$$

Selection of strength resistance factor:

$\phi := 0.75$ [for CFRP prestressed beams] [AASHTO-CFRP Art. 1.5.3.2]

Check for capacity $\phi \cdot M_n = 130.54\ ft \cdot kip$ $M_u = 112.5\ ft \cdot kip$

if $\phi \cdot M_n > M_u$
  "Section capacity is adequate"
else
  "Section capacity is NOT adequate"

= "Section capacity is adequate"

Minimum Reinforcement

There is a on-going NCHRP project for revising the minimum reinforcement provisions for prestressed beams. Therefore, the outcome of the NCHRP 12-94 may also influence the requirements for CFRP prestressed beams.

At any section of a flexural component, the amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance, M_r, at least equal to the lesser of:

1.33 times the factored moment required by the applicable strength load combinations

$$M_{cr} = \gamma_3 \left((\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}) \cdot S_c - M_{dnc} \cdot \left(\frac{Sc}{S_{nc}} - 1 \right) \right)$$ [AASHTO-CFRP 1.7.3.3.1-1]

Where, γ_1 = 1.6 flexural cracking variability factor
γ_2 =1.1 prestress variability factor
γ_3 =1.0 prestressed concrete structures

$\gamma_1 := 1.6$ $\gamma_2 := 1.1$ $\gamma_3 := 1.0$

$$f_{cpe} := \frac{P_e}{A} + \frac{P_e \cdot e_c}{S_c} - \frac{M_b}{S_c} = 0.55\ ksi$$ $$f_r := 0.20 \cdot \sqrt{\frac{f'_c}{ksi}}\ ksi = 0.6\ ksi$$

$$M_{cr} := \gamma_3 \cdot (\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}) \cdot S_c = 104.5\ ft \cdot kip$$

Check for governing moment:

$$gov_{moment} := \text{if } M_{cr} < 1.33 \cdot M_u \;\; \| \; M_{cr} \;\; \text{else} \;\; \| \; 1.33 \cdot M_u \quad = 104.5 \; (ft \cdot kip)$$

Check for minimum reinforcement requirement

$$\text{if } gov_{moment} < \phi \cdot M_n \;\; \| \; \text{“Minimum reinf. requirement OK”} \;\; \text{else} \;\; \| \; \text{“Minimum reinf. requirement NOT OK”} \quad = \text{“Minimum reinf. requirement OK”}$$

<u>Deflection and Camber</u> [Upward deflection is negative]

Deflection due to Prestressing Force at Transfer

$$\Delta_{pt} := \frac{-P_t}{E_{ci} \cdot I} \frac{e_c \cdot \left(L_{span}\right)^2}{8} = -0.3 \; in$$

Deflection due to Beam Self-Weight

$$\Delta_b = \frac{5 \cdot w \cdot \left(L_{span}\right)^4}{384 \cdot E_{ci} \cdot I}$$

Deflection due to beam self-weight at transfer:

$$\Delta_{bt} := \frac{5 \cdot w \cdot \left(L_{span}\right)^4}{384 \cdot E_{ci} \cdot I} = 0.14 \; in$$

Deflection due to beam self-weight used to compute deflection at erection:

$$\Delta_{be} := \frac{5 \cdot w \cdot \left(L_{design}\right)^4}{384 \cdot E_{ci} \cdot I} = 0.12 \; in$$

Deflection due to Superimposed Dead Load

$$\Delta_{SD} := \frac{5 \cdot w_{SD} \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I} = 0.02 \; in$$

Deflection due to Live Load

$$\Delta_L := \frac{5 \cdot w_L \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I} = 0.15 \; in$$

Using ACI 440 multipliers for long-term deflections

Immidiate camber at transfer $\delta_t := \Delta_{pt} + \Delta_{bt} = -0.16\ in$

Camber at erection $\delta_e := 1.80 \cdot \Delta_{pt} + 1.85\ \Delta_{bt} = -0.28\ in$

Deflection at final $\delta_f := 1 \cdot \Delta_{pt} + 2.70\ \Delta_{bt} + 4.10 \cdot \Delta_{SD} + \Delta_L = 0.31\ in$

Deflection due to Live Load when the Section is Cracked (i.e, for an moment of 160 ft-kip)

Stress at bottom fiber due to the effect of prestress only

$$f_{cpe} := \frac{P_e}{A} + \frac{P_e \cdot e_c}{S_c} = 0.97\ ksi$$

Tensile strength of concrete

$$f_r := 0.24 \cdot \sqrt{\frac{f'_c}{ksi}}\ ksi = 0.72\ ksi$$

Cracking moment of the beam can be computed as:

$$M_{cr} := \left(f_r + f_{cpe}\right) \cdot S_c = 112.94\ ft \cdot kip$$

Factor to soften effective moment of inertia (because of the use of prestressing CFRP)

$$\beta_d := 0.5\ \left(\frac{E_f}{E_s} + 1\right) = 0.89$$ [AASHTO-CFRP Eq. 1.7.3.4.2-2]

Modular ratio

$$n := \frac{E_f}{E_c} = 4.13$$

Cracked moment of inertia [AASHTO-CFRP Eq. 1.7.3.4.2-3]

$$I_{cr} := \frac{b \cdot c^3}{12} + b \cdot c \cdot \left(c - 0.5 \cdot c\right)^2 + n \cdot A_{pf} \cdot \left(d_p - c\right)^2 = 257.07\ in^4$$

Moment at which deflection is computed, $M_a := 130\ ft \cdot kip$

Effective moment of inertia, [AASHTO-CFRP Eq.1.7.3.4.2-1]

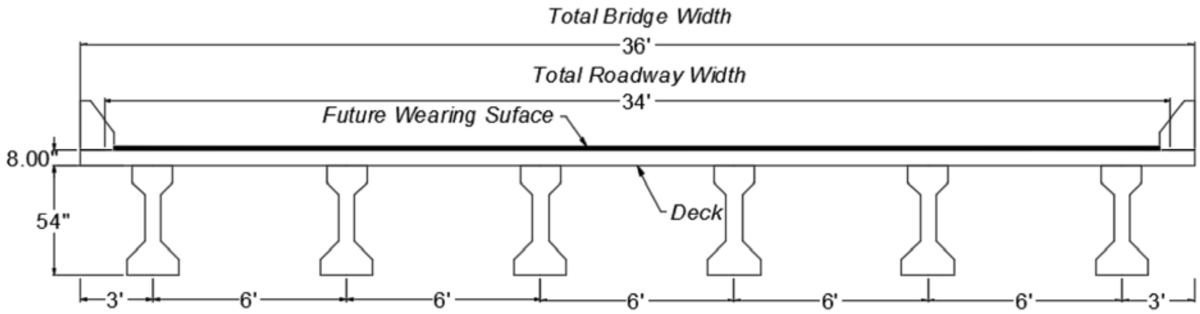
$$I_e := \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) \cdot I_{cr} = \left(4.75 \cdot 10^3\right)\ in^4$$

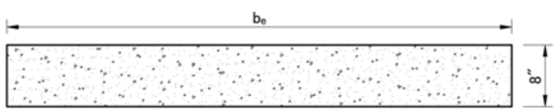
Deflection due to live load producing a moment of 130 ft-kip

$$\Delta_L := \frac{5 \cdot M_a \cdot \left(L_{design}\right)^2}{48 \cdot E_c \cdot I_e} = 0.81\ in$$

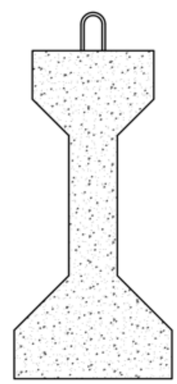
Example B-2: Design of a Decked AASHTO girder pretensioned with straight CFRP cables

The bridge considered for this design example has a span length of 90 ft. (center-to-center (c/c) pier distance), a total width of 36 ft., and total roadway width of 34 ft. The bridge superstructure consists of six AASHTO Type IV girders spaced 6 ft. center-to-center, designed to act compositely with an 8 in. thick cast-in-place (CIP) concrete deck. The wearing surface thickness is 2.0 in., which includes the thickness of any future wearing surface. T501 type rails are considered in the design. HL-93 is the design live load. A relative humidity (RH) of 60 percent is considered in the design. The design is performed for an interior girder based on service and strength limit states according to AASHTO-LRFD (2017) and AASHTO Guide Specifications (2018). They are referred in the following example as AASHTO and AASHTO-CFRP respectively.





8"-Deck



AASHTO Type-IV

Overall beam Length	$L_{span} := 91\ ft$
Design Span	$L_{design} := 90\ ft$
Girder spacing	$g_{spacing} := 6\ ft$
Number of beams	$N_{beams} := 6$
Total roadway width	$w_{roadway} := 36\ ft$

<u>Cast in Place Deck:</u>

Structural thickness, (effective)	$h_d := 7.5\ in$

Actual thickness, (for dead load calculation) $t_s := 8\ in$

Concrete strength at 28 days, $f'_{cDeck} := 6.0\ ksi$

Thickness of asphalt-wearing surface (including any future wearing surface) $h_{ws} := 2\ in$

Unit weight of concrete $\gamma_c := 150\ pcf$

Haunch thickness $h_h := 0.5\ in$

<u>Precast Girders: AASHTO Type IV</u>

Concrete strength at release, $f'_{ci} := 6.0\ ksi$

Concrete strength at 28 days, $f'_c := 9.0\ ksi$

Unit weight of concrete $\gamma_c := 150\ pcf$

<u>Prestressing CFRP</u>

Diameter of one prestressing CFRP cable $d_b := 0.6\ in$

Area of one prestressing CFRP cable $A_{pf} := 0.18\ in^2$

Design tensile stress $f_{pu} := \dfrac{64.14\ kip}{A_{pf}} = 356.33\ ksi$

Modulus of elasticity (AASHTO-CFRPArt. 1.4.1.3) $E_f := 22500\ ksi$

Design tensile strain $\varepsilon_{pu} := \dfrac{f_{pu}}{E_f} = 0.016$

Stress limitation for prestressing CFRP
(AASHTO-CFRP Art. 1.9.1)

Before transfer $f_{pi} := 0.70 \cdot f_{pu} = 249.43\ ksi$

At service, after all losses $f_{pe} := 0.65 \cdot f_{pu} = 231.62\ ksi$

<u>Nonprestressed Reinforcement:</u>

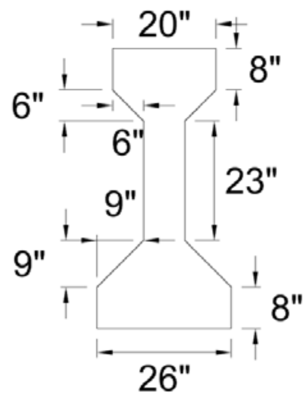
Yield strength $f_y := 60\ ksi$

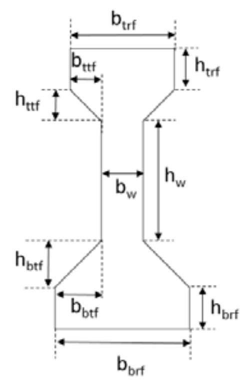
Modulus of elasticity (AASHTO Art. 5.4.4.2) $E_s := 29000\ ksi$

Unit weight of concrete $\gamma_{aws} := 150\ pcf$

T501 type barrier weight/side $\gamma_{bw} := 326\ plf$

<u>Section Properties of AASHTO Type IV Girder:</u>





Cross-section area of girder $A_g := 789\ in^2$

Moment of inertia of about the centroid of the noncomposite precast girder $I_g := 260730\ in^4$

Weight of the girder $w_g := 0.822\ \frac{kip}{ft}$

Height of girder $h_g := 54 \cdot in$

Width of bottom rectangular flange $b_{brf} := 26 \cdot in$

Height of bottom rectangular flange $h_{brf} := 8 \cdot in$

Width of bottom tapered flange $b_{btf} := 9 \cdot in$

Height of bottom tapered flange $h_{btf} := 9 \cdot in$

Width of web $b_w := 8 \cdot in$

Height of web $h_w := 23 \cdot in$

Width of top rectangular flange $b_{trf} := 20 \cdot in$

Height of top rectangular flange $h_{trf} := 8 \cdot in$

Width of top tapered flange $b_{ttf} := 6 \cdot in$

Height of top tapered flange $h_{ttf} := 6 \cdot in$

Distance from centroid to the extreme bottom fiber of the non-composite precast girder $y_{gbot} := 24.73\ in$

Distance from centroid to the extreme top fiber of the non-composite precast girder $y_{gtop} := h_g - y_{gbot} = 29.27\ in$

Section modulus referenced to the extreme bottom fiber of the non-composite precast girder $S_{gbot} := \frac{I_g}{y_{gbot}} = \left(1.05 \cdot 10^4\right)\ in^3$

Section modulus referenced to the extreme top fiber of the non-composite precast girder $S_{gtop} := \frac{I_g}{y_{gtop}} = \left(8.91 \cdot 10^3\right)\ in^3$

Effective flange width (AASHTO Art. 4.6.2.6.1)

$b_e := g_{spacing} = 72\ in$ Average spacing of adjacent girders

Material Properties for Girder and Deck Concrete:

Modulus of elasticity of concrete [AASHTO Art. 5.4.2.4] $E\left(f'_c\right) := 12 \cdot \left(\frac{\gamma_c}{pcf}\right)^{2.0} \left(\frac{f'_c}{psi}\right)^{0.33} \cdot psi$

At release $E_{ci} := E\left(f'_{ci}\right) = \left(4.77 \cdot 10^3\right)\ ksi$

At 28 days (Girder) $E_c := E\left(f'_c\right) = \left(5.45 \cdot 10^3\right)\ ksi$

At 28 days (Deck) $E_{cDeck} := E\left(f'_{cDeck}\right) = \left(4.77 \cdot 10^3\right)\ ksi$

Modulus of rupture of concrete [AASHTO Art 5.4.2.6] $f_{mr}\left(f'_c\right) := 0.24 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi$

At release $f_{ri} := f_{mr}\left(f'_{ci}\right) = 0.59\ ksi$

At 28 days (Girder) $f_r := f_{mr}\left(f'_c\right) = 0.72\ ksi$

At 28 days (Deck) $f_{rDeck} := f_{mr}\left(f'_{cDeck}\right) = 0.59\ ksi$

$n_1 := \frac{E_{cDeck}}{E_c} = 0.87$ [Modular ratio for transformed section]

Section Properties of Composite Deck:

Height of deck $h_d := 7.5 \cdot in$

Transformed width of deck $b_d := n_1 \cdot b_e = 62.98\ in$

Cross-section area of deck $A_d := h_d \cdot b_d = 472.37\ in^2$

Moment of inertia of deck about it centriod $I_d := \dfrac{b_d \cdot h_d^{\ 3}}{12} = \left(2.21 \cdot 10^3\right)\ in^4$

Weight of the deck $w_d := \left(b_e \cdot t_s\right) \cdot \gamma_c = 0.6\ \dfrac{kip}{ft}$

Due to camber of the precast, prestressed beam, a minimum haunch thickness of 1/2 in. at midspan is considered in the structural properties of the composite section. Also, the width of haunch must be transformed.

Height of haunch $h_h := 0.5\ in$

Width of haunch $b_h := b_{trf} = 20\ in$

Transformed width of haunch $b_{th} := n_1 \cdot b_h = 17.5\ in$

Area of haunch $A_h := h_h \cdot b_{th} = 8.75\ in^2$

Moment of inertia of haunch about it centriod $I_h := \dfrac{b_{th} \cdot h_h^{\ 3}}{12} = 0.18\ in^4$

Weight of the haunch $w_h := \left(b_h \cdot h_h\right) \cdot \gamma_c = 0.0104\ \dfrac{kip}{ft}$

Total height of composite beam $h_c := h_d + h_g + h_h = 62\ in$

Total area of composite beam $A_c := A_d + A_g + A_h = \left(1.27 \cdot 10^3\right)\ in^2$

Total weight of the composite beam $w_c := w_g + w_d + w_h = 1.43\ \dfrac{kip}{ft}$

Neutral axis location from bottom for composite beam

$$y_{cbot} := \frac{A_g \cdot y_{gbot} + A_d \cdot \left(h_c - \dfrac{h_d}{2}\right) + A_h \cdot \left(h_g + \dfrac{h_h}{2}\right)}{A_g + A_d + A_h} = 37.4\ in$$

Neutral axis location from top for composite beam $y_{ctop} := \left(h_c\right) - y_{cbot} = 24.6\ in$

Moment of inertia of composite beam

$$I_{comp} := I_g + I_d + I_h + A_g \cdot (y_{cbot} - y_{gbot})^2 + A_d \cdot \left(y_{ctop} - \frac{h_d}{2}\right)^2 + A_h \cdot \left(y_{ctop} - h_d - \frac{h_h}{2}\right)^2 = (5.97 \cdot 10^5)\ in^4$$

Shear Force and Bending Moment Due to Dead Loads

Dead loads:

Dead loads acting on the non-composite structure:

Self-weight of the girder $w_g := 0.822\ \frac{kip}{ft}$

Weight of cast-in-place deck on each interior girder $w_d = 0.6\ \frac{kip}{ft}$

Weight of haunch on each interior girder $w_h = 0.01\ \frac{kip}{ft}$

Total dead load on non-composite section $w_T := w_g + w_d + w_h = 1.43\ \frac{kip}{ft}$

Superimposed dead loads:

Dead and live load on the deck must be distributed to the precast, prestressed beams. AASHTO provides factors for the distribution of live load into the beams. The same factors can be used for dead loads if the following criteria is met [AASHTO Art. 4.6.2.2.1]:

- Width of deck is constant [OK]
- Number of beams is not less than four,

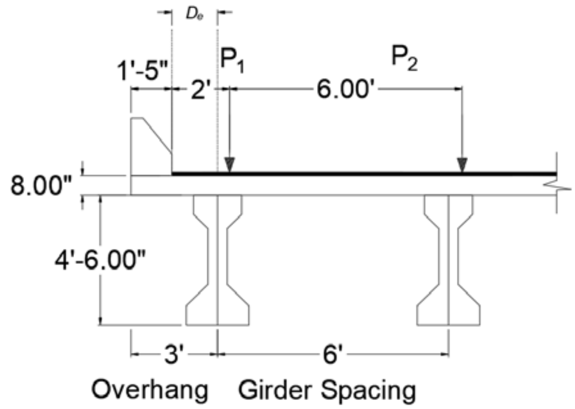
```
if N_beams < 4
    "NOT OK"
else
    "OK"
```
= "OK"

- Beams are parallel and have approximately the same stiffness
- The overhang minus the barrier width does not exceed 3.0 feet

$D_e = Overhang - 17\ in$

$Overhang := 3\ ft$

$D_e := Overhang - 17\ in = 19\ in$



$$\left\| \begin{array}{l} \text{if } D_e > 3\ ft \\ \quad \| \text{“NOT OK”} \\ \text{else} \\ \quad \| \text{“OK”} \end{array} \right\| = \text{“OK”}$$

- Curvature in plan is less than the limit specified in Article 4.6.1.2.4 [OK]

- Cross section of the bridge is consistent with one of the cross sections given in AASHTO Table 4.6.2.2.1-1 Precast concrete I sections are specified as Type k [OK]

Because all of the above criteria are satisfied, the barrier and wearing surface loads are equally distributed among the six girders.

Weight of T501 rails or barriers on each girder

$$w_b := 2 \cdot \left(\frac{\gamma_{bw}}{6} \right) = 0.11\ \frac{kip}{ft}$$

Weight of 2.0 in. wearing surface

$$w_{ws1} := \gamma_{aws} \cdot \left(h_{ws} \right) = 0.03\ \frac{kip}{ft^2}$$

This load is applied over the entire clear roadway width.Weight of wearing surface on each girder

$$w_{ws} := \frac{w_{ws1} \cdot w_{roadway}}{6} = 0.15\ \frac{kip}{ft}$$

Total superimposed dead load

$$w_{SD} := w_b + w_{ws} = 0.26\ \frac{kip}{ft}$$

Calculate modular ratio between girder and deck [AASHTO Eq. 4.6.2.2.1-2]

$$n := \frac{E_c}{E_{cDeck}} = 1.14$$

Calculate e_g, the distance between the center of gravity of the non-composite beam and the deck. Ignore the thickness of the haunch in determining e_g. It is also possible to ignore the integral wearing surface, i.e, use $h_d = 7.5\ in$. However, the difference in the distribution factor will be minimal.

$$e_g := y_{gtop} + \frac{h_d}{2} = 33.02\ in$$

Calculate K_g , the longitudinal stiffness parameter. [AASHTO Eq. 4.6.2.2.1-1]

$$K_g := n \cdot \left(I_g + A_g \cdot {e_g}^2\right) = \left(1.28 \cdot 10^6\right)\ in^4$$

Moment Distribution Factors

Interior girder type k [AASHTO Art. 4.6.2.2.2 b]

Distribution factor for moment when one design lane is loaded

$$D_{M.Interior} = 0.06 + \left(\frac{S}{14}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot \left(\frac{K_g}{12 \cdot L \cdot {t_s}^3}\right)^{0.1}$$

Using variables defined in this example

$$\left(\frac{g_{spacing}}{14\ ft}\right)^{0.4} = 0.71 \qquad \left(\frac{g_{spacing}}{L_{design}}\right)^{0.3} = 0.44 \qquad \left(\frac{K_g}{L_{design} \cdot {h_d}^3}\right)^{0.1} = 1.11 \qquad h_d = 7.5\ in$$

$$D_{M.Interior1} := 0.06 + \left(\frac{g_{spacing}}{14\ ft}\right)^{0.4} \cdot \left(\frac{g_{spacing}}{L_{design}}\right)^{0.3} \cdot \left(\frac{K_g}{L_{design} \cdot {h_d}^3}\right)^{0.1} = 0.41$$

Distribution factor for moment when two design lanes are loaded

$$D_{M.Interior} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \cdot \left(\frac{S}{L}\right)^{0.2} \left(\frac{K_g}{12 \cdot L \cdot {t_s}^3}\right)^{0.1}$$

Using variables defined in this example

$$D_{M.Interior2} := 0.075 + \left(\frac{g_{spacing}}{9.5\ ft}\right)^{0.6} \cdot \left(\frac{g_{spacing}}{L_{design}}\right)^{0.2} \cdot \left(\frac{K_g}{L_{design} \cdot {h_d}^3}\right)^{0.1} = 0.56$$

The greater distribution factor is selected for moment design of the beams.

$$D_{M.Interior} := \max\left(D_{M.Interior1}, D_{M.Interior2}\right) = 0.56$$

Check for range of applicability

$$D_{M.Interior} := \left\| \begin{array}{l} S \leftarrow (g_{spacing} \geq 3.5\ ft) \cdot (g_{spacing} \leq 16\ ft) \\ t_s \leftarrow (h_d \geq 4.5\ in) \cdot (h_d \leq 12\ in) \\ L \leftarrow (L_{design} \geq 20\ ft) \cdot (L_{design} \leq 240\ ft) \\ N_b \leftarrow (N_{beams} \geq 4) \\ K_g \leftarrow (K_g \geq 10000\ in^4) \cdot (K_g \leq 7000000\ in^4) \\ \text{if } (S \cdot t_s \cdot L \cdot N_b \cdot K_g) \\ \quad \| D_{M.Interior} \\ \text{else} \\ \quad \| \text{“Does not satisfy range of applicability”} \end{array} \right. = 0.56$$

Exterior girder [AASHTO Art. 4.6.2.2.2 d]

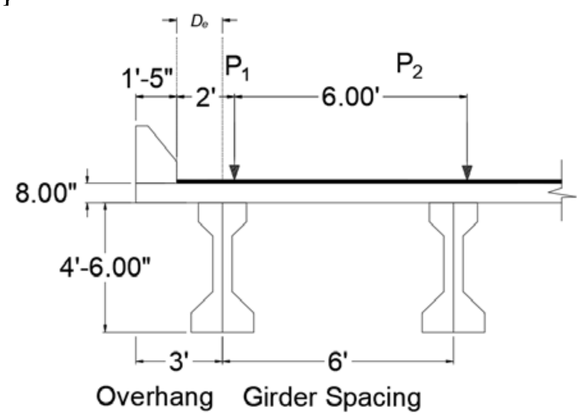
$$P_1 = \frac{D_e + S - 2\ ft}{S}$$

$$P_2 = \frac{D_e + S - 8\ ft}{S}$$

$$D_e = Overhang - 17\ in$$

$$Overhang := 3\ ft$$

$$D_e := Overhang - 17\ in = 19\ in$$



$$S := g_{spacing} = 6\ ft$$

The distribution factor for one design lanes loaded is based on the lever rule, which includes a 0.5 factor for converting the truck load to wheel loads and a 1.2 factor for multiple truck presence.

$$D_{M.Exterior1} := \mathbf{if}\left((2\ ft + 6\ ft) < (D_e + S), \frac{2 \cdot S + 2\ D_e - 8\ ft}{S} \cdot 0.5, \frac{S + D_e - 2\ ft}{S} \cdot 0.5\right) \cdot 1.2 = 0.56$$

The distribution factor for two design lane loaded

$$D_{M.Exterior} = D_{M.Interior} \cdot \left(0.77 + \frac{D_e}{9.1}\right)$$

Using variables defined in this example,

$$D_{M.Exterior2} := D_{M.Interior2} \cdot \left(0.77 + \frac{D_e}{9.1\ ft}\right) = 0.53$$

$D_{M.Exterior} := \max\left(D_{M.Exterior1}, D_{M.Exterior2}\right) = 0.56$

Range of applicability

$$D_{M.Exterior} := \left\| \begin{array}{l} d_e \leftarrow \left(D_e \geq -1\ ft\right) \cdot \left(D_e \leq 5.5\ ft\right) \\ \text{if } \left(d_e\right) \\ \quad \left\| D_{M.Exterior} \right. \\ \text{else} \\ \quad \left\| \text{“Does not satisfy range of applicability”} \right. \end{array} \right\| = 0.56$$

For fatigue limit state

The commentary of article 3.4.1 in the AASHTO LRFD specification states that for fatigue limit state a single design truck should be used. However, live load distribution factors given in AASHTO Art 4.6.2.2 take into consideration the multiple presence factor, m. AASHTO Art 3.6.1.1.2 states that the multiple presence factor, m, for one design lane loaded is 1.2. Therefore, the distribution factor for one design lane loaded with the multiple presence factor removed should be used.

Distribution factor for fatigue limit state $D_{MF.Interior} := \dfrac{D_{M.Interior1}}{1.2} = 0.34$

<u>Shear Distribution Factors</u>

Interior girder [AASHTO Art. 4.6.2.2.3 a]

Distribution factor for shear when one design lane is loaded

$$D_{S.Interior} = 0.36 + \left(\frac{S}{25}\right)$$

Using variables defined in this example

$$\left(\frac{g_{spacing}}{25\ ft}\right) = 0.24$$

$$D_{S.Interior1} := 0.36 + \left(\frac{g_{spacing}}{25\ ft}\right) = 0.6$$

Distribution factor for shear when two design lanes are loaded

$$D_{S.Interior} = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^{2.0}$$

Using variables defined in this example

$$D_{S.Interior2} := 0.075 + \left(\frac{g_{spacing}}{12\ ft}\right) - \left(\frac{g_{spacing}}{35\ ft}\right)^2 = 0.55$$

The greater distribution factor is selected for moment design of the beams.

$$D_{S.Interior} := \max\left(D_{S.Interior1}, D_{S.Interior2}\right) = 0.6$$

Check for range of applicability

$$D_{S.Interior} := \left\| \begin{array}{l} S \leftarrow \left(g_{spacing} \ge 3.5\ ft\right) \cdot \left(g_{spacing} \le 16\ ft\right) \\ t_s \leftarrow \left(h_d \ge 4.5\ in\right) \cdot \left(h_d \le 12\ in\right) \\ L \leftarrow \left(L_{design} \ge 20\ ft\right) \cdot \left(L_{design} \le 240\ ft\right) \\ N_b \leftarrow \left(N_{beams} \ge 4\right) \\ K_g \leftarrow \left(K_g \ge 10000\ in^4\right) \cdot \left(K_g \le 7000000\ in^4\right) \\ \text{if } \left(S \cdot t_s \cdot L \cdot N_b \cdot K_g\right) \\ \quad \left\| D_{S.Interior} \right. \\ \text{else} \\ \quad \left\| \text{“Does not satisfy range of applicability”} \right. \end{array} \right. = 0.6$$

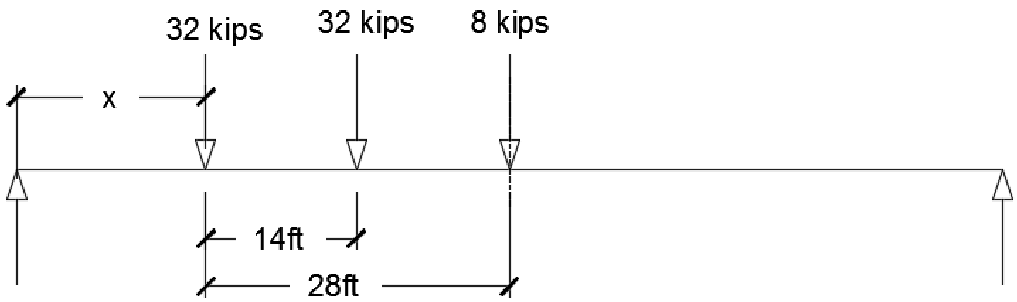
The AASHTO Specifications specify the dynamic load effects as a percentage of the static live load effects. AASHTO Table 3.6.2.1-1 specifies the dynamic allowance to be taken as 33 percent of the static load effects for all limit states, except the fatigue limit state, and 15 percent for the fatigue limit state. The factor to be applied to the static load shall be taken as:

(1 + IM/100)

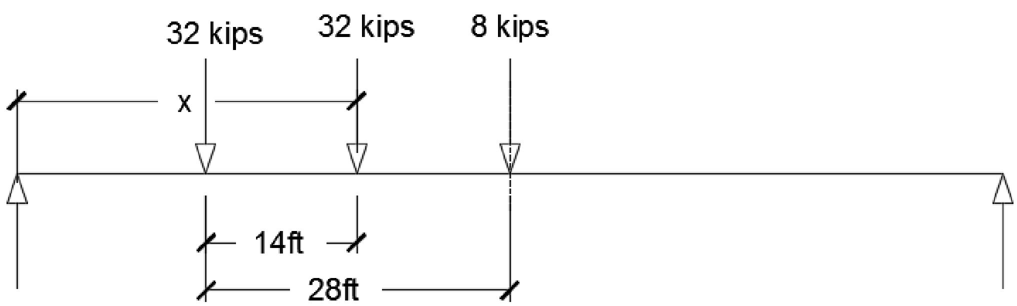
where:

IM = Dynamic load allowance, applied to truck load or tandem load only
= 33 for all limit states except the fatigue limit state
= 15 for fatigue limit state

The maximum shear forces and bending moments due to HS 20-44 truck loading for all limit states is calculated using the influence line approach. The live load moments and shear forces for the simple span is computed by positioning the axle load of HS-20 truck in following locations



Case I



Case II

Case I: HS-20 truck moment and shear

$P_1 := 32\ kip$ $\quad P_2 := 32\ kip$ $\quad P_3 := 8\ kip$ $\quad x := 5\ ft$ [Initialize value]

$$M_{truck1}(x) := P_1 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot x + P_2 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}} \cdot x + P_3 \cdot \frac{(L_{design} - x - 28\ ft)}{L_{design}} \cdot x$$

$$V_{truck1}(x) := P_1 \cdot \frac{(L_{design} - x)}{L_{design}} + P_2 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}} + P_3 \cdot \frac{(L_{design} - x - 28\ ft)}{L_{design}}$$

Case II: HS-20 truck moment and shear

$$M_{truck2}(x) := P_1 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot (x - 14\ ft) + P_2 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot x + P_3 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}} \cdot x$$

$$V_{truck2}(x) := P_1 \cdot \frac{-(x - 14\ ft)}{L_{design}} + P_2 \cdot \frac{(L_{design} - x)}{L_{design}} + P_3 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}}$$

$M_{truck1}\left(\mathbf{maximize}\left(M_{truck1}, x\right)\right) = \left(1.3 \cdot 10^3\right)\ ft{\cdot}kip$

$M_{truck2}\left(\mathbf{maximize}\left(M_{truck2}, x\right)\right) = \left(1.34 \cdot 10^3\right)\ ft{\cdot}kip$

Maximum bending moment due to HS 20-44 truck load

$M := \left(1.344 \cdot 10^3\right)\ ft{\cdot}kip$

The calculation of shear force is carried out later for the critical shear section.
Distributed bending moment due to truck load including dynamic load allowance (M_{LT}) is calculated as follows:
M_{LT} = (Moment per lane due to truck load)(*DFM*)(1+*IM*/100)

$IM := 33$

$D_{M.Interior} = 0.56$

$$M_{LT} := M \cdot D_{M.Interior} \cdot \left(1 + \frac{IM}{100}\right) = \left(1.01 \cdot 10^3\right)\ ft \cdot kip$$

The maximum bending moments (M_L) due to a uniformly distributed lane load of 0.64 klf are calculated using the following formulas given by the *PCI Design Manual* (PCI 2017).

Maximum bending moment, $M_x = 0.5(0.64)(x)(L - x)$

where:
x = Distance from centerline of bearing to section at which the bending moment or shear force is calculated, ft.
L = Design span length

At the section of maximum truck load $\quad \mathbf{maximize}\left(M_{truck2}, x\right) = 47.33\ ft$

$x := 47.333\ ft$

$$M_L := 0.5 \cdot 0.64\ \frac{kip}{ft} \cdot (x)\ \left(L_{design} - x\right) = 646.26\ ft \cdot kip$$

$M_{LL} := D_{M.Interior} \cdot M_L = 364.96\ ft \cdot kip$

For fatigue limit state:
Therefore, the bending moment of the fatigue truck load is:
Mf = (bending moment per lane)(DFM)(1 + IM)

$$M_f := M \cdot D_{MF.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 611.73\ ft \cdot kip$$

Shear forces and bending moments for the girder due to dead loads, superimposed dead loads at every tenth of the design span, and at critical sections (hold-down point or harp point and critical section for shear) are provided in this section. The bending moment (*M*) and shear force (*V*) due to uniform dead loads and uniform superimposed dead loads at any section at a distance *x* from the centerline of bearing are calculated using the following formulas, where the uniform load is denoted as *w*.

$$M = 0.5w\,x\,(L - x)$$
$$V = w(0.5L - x)$$

The critical section for shear is located at a distance $h_c/2$ from the face of the support. However, as the support dimensions are not specified in this project, the critical section is measured from the centerline of bearing. This yields a conservative estimate of the design shear force.

Inputs

$excel_{\text{“F1”}} := 3.28084\ (L_{design})$ $\quad$ $excel_{\text{“A13”}} := 0.5 \cdot 3.28084\ (L_{design})$

$excel_{\text{“B1”}} := 0.0000685217\ (w_g)$

$excel_{\text{“D1”}} := 0.0000685217\ (w_d + w_h)$

$excel_{\text{“B2”}} := 0.0000685217\ (w_b)$

w_g	0.822		0.61042		90.000		L		90.000
w_{SD}	0.109								

Distance (x)	Section (x/L)	Dead Load							
		Girder Weight		Slab Weight		Barrier weight		Total Dead Load	
		Shear	Moment	Shear	Moment	Shear	Moment	Shear	Moment
ft.		k	k-ft	k	k-ft	k	k-ft	k	k-ft
0	0.000	36.990	0.00	27.469	0.00	4.890	0.00	69.35	0.00
4.654	0.052	33.164	163.25	24.628	121.23	4.384	21.58	62.18	306.06
10.858	0.121	28.065	353.18	20.841	262.27	3.710	46.69	52.62	662.14
21.717	0.241	19.139	609.47	14.212	452.59	2.530	80.57	35.88	1142.64
32.575	0.362	10.213	768.82	7.584	570.93	1.350	101.64	19.15	1441.39
43.433	0.483	1.288	831.26	0.957	617.30	0.170	109.89	2.41	1558.45

Outputs

$V_g := excel_{\text{“C8”}} \cdot kip$ $\quad$ $V_s := excel_{\text{“E8”}} \cdot kip$ $\quad$ $V_b := excel_{\text{“G8”}} \cdot kip$

$M_{gv} := excel_{\text{“D8”}} \cdot ft \cdot kip$ $\quad$ $M_{sv} := excel_{\text{“F8”}} \cdot ft \cdot kip$ $\quad$ $M_{bv} := excel_{\text{“G8”}} \cdot ft \cdot kip$

The AASHTO design live load is designated as HL-93, which consists of a combination of:

- Design truck with dynamic allowance or design tandem with dynamic allowance, whichever produces greater moments and shears, and
- Design lane load without dynamic allowance. [AASHTO Art. 3.6.1.2]

The design truck is designated as HS 20-44 consisting of an 8 kip front axle and two 32 kip rear axles. [AASHTO Art. 3.6.1.2.2]
The design tandem consists of a pair of 25-kip axles spaced 4 ft. apart. However, for spans longer than 40 ft. the tandem loading does not govern, thus only the truck load is investigated in this example. [AASHTO Art. 3.6.1.2.3]
The lane load consists of a load of 0.64 klf uniformly distributed in the longitudinal direction. [AASHTO Art. 3.6.1.2.4]

This design example considers only the dead and vehicular live loads. The wind load and the extreme event loads, including earthquake and vehicle collision loads, are not included in the design. Various limit states and load combinations provided by AASHTO Art. 3.4.1 are investigated, and the following limit states are found to be applicable in present case:

Service I: This limit state is used for normal operational use of a bridge. This limit state provides the general load combination for service limit state stress checks and applies to all conditions except Service III limit state. For prestressed concrete components, this load combination is used to check for compressive stresses. The load combination is presented as follows[AASHTO Table 3.4.1-1]:
Q = 1.00 (DC + DW) + 1.00(LL + IM)

Service III: This limit state is a special load combination for service limit state stress checks that applies only to tension in prestressed concrete structures to control cracks. The load combination for this limit state is presented as follows [AASHTO Table 3.4.1-1]:
Q = 1.00(DC + DW) + 0.80(LL + IM)
(Subsequent revisions to the AASHTO specification have revise this load combination)

Strength I: This limit state is the general load combination for strength limit state design relating to the normal vehicular use of the bridge without wind. The load combination is presented as follows [AASHTO Table 3.4.1-1 and 2]:
$Q = \gamma P(DC) + \gamma P(DW) + 1.75(LL + IM)$

Type of Load	Load Factor, γ_P	
	Maximum	Minimum
DC: Structural components and non-structural attachments	1.25	0.90
DW: Wearing surface and utilities	1.50	0.65

The maximum and minimum load combinations for the Strength I limit state are presented as follows:
Maximum Q = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)
Minimum Q = 0.90(DC) + 0.65(DW) + 1.75(LL + IM)

Estimation of Required Prestress

The required number of strands is usually governed by concrete tensile stress at the bottom fiber of the girder at the midspan section. The load combination for the Service III limit state is used to evaluate the bottom fiber stresses at the midspan section. The calculation for compressive stress in the top fiber of the girder at midspan section under service loads is also shown in the following section. The compressive stress is evaluated using the load combination for the Service I limit state.

Service Load Stresses at Midspan

Bottom tensile stress due to applied dead and live loads using load combination Service III is:

$$f_b = \frac{M_g + M_d}{S_b} + \frac{M_b + M_{ws} + (0.8) \cdot (M_{LT} + M_{LL})}{S_{bc}}$$

f_b =Concrete stress at the bottom fiber of the girder, ksi

M_g =unfactored bending moment due to beam self-weight, k-ft

$$M_g := \frac{w_g \cdot L_{design}{}^2}{8} = 832.28 \ ft{\cdot}kip$$

M_d =unfactored bending moment due to deck self-weight and haunch, k-ft

$$M_d := \frac{\left(w_d + w_h\right) \cdot L_{design}{}^2}{8} = 618.05 \ ft{\cdot}kip$$

M_b =unfactored bending moment due to barrier self-weight, k-ft

$$M_b := \frac{w_b \cdot L_{design}{}^2}{8} = 110.03 \ ft{\cdot}kip$$

M_{ws} =unfactored bending moment due to future wearing , k-ft

$$M_{ws} := \frac{w_{ws} \cdot L_{design}{}^2}{8} = 151.88 \ ft{\cdot}kip$$

M_{LT} =unfactored bending moment due to truck load (kip-ft) $\qquad M_{LT} = \left(1.01 \cdot 10^3\right) \ ft \cdot kip$

M_{LL} =unfactored bending moment due to truck load (kip-ft) $\qquad M_{LL} = 364.96 \ ft \cdot kip$

S_{bc} =composite section modulus for extreme bottom fiber of precast beam (in^3)

Using the variables used in this example

$$S_{cbot} := \frac{I_{comp}}{y_{cbot}} = \left(1.6 \cdot 10^4\right) \ in^3$$

$$f_b := \frac{M_g + M_d}{S_{gbot}} + \frac{M_b + M_{ws} + (0.8) \cdot \left(M_{LT} + M_{LL}\right)}{S_{cbot}} = 2.67 \ ksi$$

Stress Limits for Concrete

The tensile stress limit at service load= $0.19 \cdot \sqrt{f'_c}$ $\qquad$ [AASHTO Table 5.9.2.3.2b-1]

where: f'c = specified 28-day concrete strength of beam, ksi

Concrete tensile stress limit = $\quad f_{tl} := 0.19 \cdot \sqrt{\frac{f'_c}{ksi}} \ ksi = 0.57 \ ksi$

Required Number of Strands

The required pre-compressive stress at the bottom fiber of the beam is the difference the between bottom tensile stress due to the applied loads and the concrete tensile stress limits:

Required pre-compressive stress at bottom fiber, $f_{pb} := f_b - f_{tl} = 2.1\ ksi$

Assume the distance between the center of gravity of the bottom strands and the bottom fiber of the beam:

$e_c := y_{gbot} = 24.73\ in$

If P_{pe} is the total prestressing force, the stress at the bottom fiber due to prestress is:

$$f_{pb} = \frac{P_{pe}}{A} + \frac{P_{pe} \cdot e_c}{S_b}$$

Using the variables in this example

$$P_{pe} := \frac{f_{pb}}{\left(\frac{1}{A_g} + \frac{e_c}{S_{gbot}}\right)} = 582.18\ kip$$

Final prestress force per strand, P_{pf} = (area of prestressing CFRP) (f_{pi}) (1 − $ploss$, %)

$ploss := 20\%$

$P_{pf} := A_{pf} \cdot f_{pi} \cdot (1 - ploss) = 35.92\ kip$ $\qquad n_p := \frac{P_{pe}}{P_{pf}} = 16.21$

The number of prestressing CFRP is equal to

$$n_p := \text{round}\left(\frac{(n_p)}{2}\right) \cdot 2 + 4 = 20$$

$n_{b1} := 12$ $\qquad d_{p1} := 51.25\ in$

$n_{b2} := 8$ $\qquad d_{p2} := 49.25\ in$

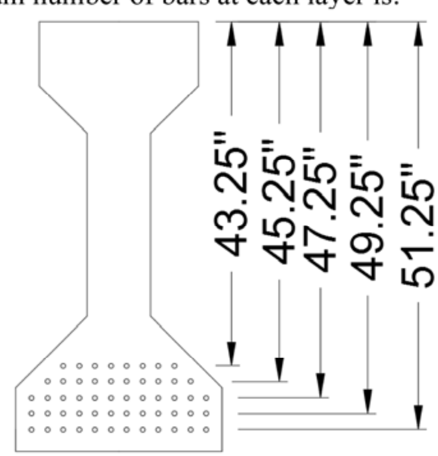
$n_{b3} := 0$ $\qquad d_{p3} := 47.25\ in$

$n_{b4} := 0$ $\qquad d_{p4} := 45.25\ in$

$n_{b5} := 0$ $\qquad d_{p5} := 43.25\ in$

Change the number of bars based on the value of n_p.
If no bars is needed at certain layer input 0.
The maximum number of bars at each layer is:

$n_{b1} = 12$
$n_{b2} = 12$
$n_{b3} = 12$
$n_{b4} = 10$
$n_{b5} = 8$



The center of gravity of the strands, c.g.s. = $\frac{\sum n_i y_i}{N}$

where: n_i = number of strands in row i

y_i = distance to center of row i from bottom of beam section

N = total number of strands

$$x_p := h_g - \frac{n_{b1} \cdot d_{p1} + n_{b2} \cdot d_{p2} + n_{b3} \cdot d_{p3} + n_{b4} \cdot d_{p4} + n_{b5} \cdot d_{p5}}{n_{b1} + n_{b2} + n_{b3} + n_{b4} + n_{b5}} = 3.55 \ in$$

$$e_c := y_{gbot} - x_p = 21.18 \ in$$

Using the variables in this example

$$P_{pe} := \frac{f_{pb}}{\left(\frac{1}{A_g} + \frac{e_c}{S_{gbot}}\right)} = 642.01 \ kip$$

Final prestress force per strand, P_{pf} = (area of prestressing CFRP) (f_{pi}) (1 − $ploss$, %)

$$ploss := 20\%$$

$$P_{pf} := A_{pf} \cdot f_{pi} \cdot (1 - ploss) = 35.92 \ kip$$

The number of prestressing CFRP is equal to $n_p := \frac{P_{pe}}{P_{pf}} = 17.87$

$$n_p := \text{round}\left(\frac{\langle n_p \rangle}{2}\right) \cdot 2 + 4 = 22$$

$n_{b1} := 12$ $\quad d_{p1} := 51.25 \ in$

$n_{b2} := 10$ $\quad d_{p2} := 49.25 \ in$

$n_{b3} := 0$ $\quad d_{p3} := 47.25 \ in$

$n_{b4} := 0$ $\quad d_{p4} := 45.25 \ in$

$n_{b5} := 0$ $\quad d_{p5} := 43.25 \ in$

Change the number of bars based on the value of n_p. If no bars is needed at certain layer input 0.
The maximum number of bars at each layer is:
$n_{b1} = 12$
$n_{b2} = 12$
$n_{b3} = 12$
$n_{b4} = 10$
$n_{b5} = 8$

$$x_p := h_g - \frac{n_{b1} \cdot d_{p1} + n_{b2} \cdot d_{p2} + n_{b3} \cdot d_{p3} + n_{b4} \cdot d_{p4} + n_{b5} \cdot d_{p5}}{n_{b1} + n_{b2} + n_{b3} + n_{b4} + n_{b5}} = 3.66 \ in$$

$$e_c := y_{gbot} - x_p = 21.07 \ in$$

Using the variables in this example

$$P_{pe} := \frac{f_{pb}}{\left(\frac{1}{A_g} + \frac{e_c}{S_{gbot}}\right)} = 644.05\ kip$$

Final prestress force per strand, P_{pf} = (area of prestressing CFRP) (f_{pi}) (1 − $ploss$, %)

$ploss := 20\%$

$P_{pf} := A_{pf} \cdot f_{pi} \cdot (1 - ploss) = 35.92\ kip$

The number of prestressing CFRP is equal to $\qquad n_p := \frac{P_{pe}}{P_{pf}} = 17.93$

$$n_p := \text{round}\left(\frac{\langle n_p \rangle}{2}\right) \cdot 2 + 4 = 22$$

<u>Strand Pattern</u>

$n_p = 22$

midspan center of gravity of prestressing CFRP $\qquad y_{bs} := x_p = 3.66\ in$

midspan prestressing CFRP eccentricity $\qquad e_c := y_{gbot} - y_{bs} = 21.07\ in$

<u>Prestress Losses</u> [AASHTO Art. 5.9.3]

Total prestress loss

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$$

Δf_{pES}=sum of all losses or gains due to elastic shortening or extension at time of application of prestress and/or external loads (ksi)

Δf_{pLT}=losses due to long-term shrinkage and creep of concrete, and relaxation of the prestressing CFRP (ksi)

Elastic Shortening

When the prestressing force is transferred from the prestressing strands to the concrete member, the force causes elastic shortening of the member as it cambers upward. This results in a loss of the initial prestress of the strands. However, some of that loss is gained back due to the self-weight of the member which creates tension in the strands.

$$\Delta f_{pES} = \frac{E_f}{E_{ct}} \cdot f_{cgp}$$ [AASHTO-CFRP Eq. 1.9.2.2.3a-1]

Where E_f=modulus of elasticity of prestressing CFRP (ksi)

E_{ct}=modulus of elasticity of the concrete at transfer or time of load application (ksi)=E_{ci}

f_{cgp}=the concrete stress at the center of gravity of CFRP due to the prestressing force immediately after transfer and the self-weight of the member at sections of maximum moment (ksi)

AASHTO Article C5.9.3.2.3a states that to calculate the prestress after transfer, an initial estimate of prestress loss is assumed and iterated until acceptable accuracy is achieved. In this example, an initial estimate of 10% is assumed.

$eloss := 10\%$

Force per strand at transfer

$$f_{cgp} = \frac{P_i}{A_g} + \frac{P_i \cdot e_c^{\ 2}}{I_g} - \frac{M_G \cdot e_c}{I_g}$$

Where, P_i=total prestressing force at release=$n_p * p$

e_c=eccentricity of strands measured from the center of gravity of the precast beam at midspan

M_G=moment due to beam self-weight at midspan (should be calculated using the overall beam length)

$$M_G := \frac{w_g \cdot \left(L_{span}\right)^2}{8} = 850.87\ ft \cdot kip$$

Guess Values

$eloss := 10\%$

Constraints

$$eloss = \frac{E_f}{f_{pi} \cdot E_{ci}} \cdot \left(\frac{n_p \cdot A_{pf} \cdot f_{pi} \cdot (1 - eloss)}{A_g} + \frac{\left(n_p \cdot A_{pf} \cdot f_{pi} \cdot (1 - eloss)\right) \cdot e_c^{\ 2}}{I_g} - \frac{M_G \cdot e_c}{I_g}\right)$$

Solver

$eloss := \mathbf{find}\left(\mathbf{eloss}\right) = 0.04$

Therefore, the loss due to elastic shortening= $eloss = 0.04$

The force per strand at transfer= $p := A_{pf} \cdot f_{pi} \cdot (1 - eloss) = 43.2\ kip$

The concrete stress due to prestress= $$f_{cgp} := \frac{n_p \cdot p}{A_g} + \frac{n_p \cdot p \cdot e_c^{\,2}}{I_g} - \frac{M_G \cdot e_c}{I_g} = 2\ ksi$$

The prestress loss due to elastic shortening= $$\Delta f_{pES} := \frac{E_f}{E_{ci}} \cdot f_{cgp} = 9.43\ ksi$$

Total prestressing force at release $$P_i := n_p \cdot p = 950.41\ kip$$

Long Term Losses

$$\Delta f_{pLT} = \left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) + \left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS}\right)$$

Δf_{pSR}=prestress loss due to shrinkage of girder concrete between time of transfer and deck placement (ksi)
Δf_{pCR}=prestress loss due to creep of girder concrete between time of transfer and deck placement (ksi)
Δf_{pR1}=prestress loss due to relaxation of prestressing strands between time of transfer and deck placement ksi)
Δf_{pSD}=prestress loss due to shrinkage of girder concrete between time of deck placement and final time (ksi)
Δf_{pCD}=prestress loss due to creep of girder concrete between time of deck placement and final time (ksi)
Δf_{pR2}=prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time (ksi)
Δf_{pSS}=prestress gain due to shrinkage of deck in composite section (ksi)
$\left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right)$ =sum of time-dependent prestress losses between time of transfer and deck placement (ksi)
$\left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS}\right)$ =sum of time-dependent prestress losses after deck placement (ksi)

Prestress Losses: Time of Transfer to Time of Deck Placement
Shrinkage of Girder Concrete

$$\Delta f_{pSR} = \varepsilon_{bid} \cdot E_{pc} \cdot K_{id}$$ [AASHTO Eq. 5.9.3.4.2a-1]

where, ε_{bid}=shrinkage strain of girder between the time of transfer and deck placement

$$= k_s \cdot k_{hs} \cdot k_f \cdot k_{td} \cdot 0.48 \cdot 10^{-3}$$ [AASHTO Eq. 5.4.2.3.3-1]

and, k_s=factor for the effect of volume to surface ratio of the component
k_s=1.45-0.13(V/S) [AASHTO Eq. 5.4.2.3.2-2]
where (V/S)=volume to surface ratio=(Area/Perimeter)

Perimeter

$$P_g := 2\left(\frac{b_{brf}}{2} + h_{brf} + \sqrt{b_{btf}^{\ 2} + h_{btf}^{\ 2}} + h_w + \sqrt{b_{ttf}^{\ 2} + h_{ttf}^{\ 2}} + h_{trf} + \frac{b_{trf}}{2}\right) = 166.43\ in$$

$$k_s := 1.45 - 0.13 \cdot \frac{1}{in}\left(\frac{A_g}{P_g}\right) = 0.83 \qquad k_s := \begin{Vmatrix} \text{if } k_s \le 1 \\ \quad \Vert\ 1 \\ \text{else} \\ \quad \Vert\ k_s \end{Vmatrix} = 1$$

k_{hs}=humidity factor for shrinkage=2.00-0.014H [AASHTO Eq. 5.4.2.3.3-2]

$H := 70$

$k_{hs} := 2.00 - 0.014 \cdot H = 1.02$

k_f=factor for the effect of concrete strength [AASHTO Eq. 5.4.2.3.2-4]

$$= \frac{5}{1 + f'_{ci}}$$

$$k_f := \frac{5}{1 + \frac{f'_{ci}}{ksi}} = 0.71$$

k_{td}=time developement factor

$$= \frac{t}{61 - 4 \cdot f'_{ci} + t}$$

[AASHTO Eq. 5.4.2.3.2-5]

$t_d := 90 \qquad t_i := 1$

$$k_{td}(t, t_i) := \frac{t - t_i}{61 - 4 \cdot \frac{f'_{ci}}{ksi} + t - t_i}$$

$k_{td}(t_d, t_i) = 0.71$

$$\varepsilon_{bid} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}(t_d, t_i) \cdot 0.48 \cdot 10^{-3} = 2.47 \cdot 10^{-4}$$

K_{id}=transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between transfer and deck placement

$$=\frac{1}{1+\frac{E_{pc}}{E_{ci}}\cdot\frac{A_{psc}}{A_g}\cdot\left(1+\frac{A_g\cdot {e_{pg}}^2}{I_{comp}}\right)\left(1+0.7\ \Psi_b\left(t_f,t_i\right)\right)}$$ [AASHTO Eq. 5.9.3.4.2a-2]

where, $\Psi_b\left(t_f,t_i\right)=1.9\cdot k_s\cdot k_{hc}\cdot k_f\cdot k_{td}\cdot {t_i}^{-0.118}$ [AASHTO Eq. 5.4.2.3.2-1]

k_{hc}=humidity factor for creep=1.56-0.008H [AASHTO Eq. 5.4.2.3.2-3]

$e_{pg}:=e_c=21.07\ in$

$k_{hc}:=1.56-0.008\ H=1$

$t_f:=20000$ $\quad t_i:=1$

$k_{td}\left(t_f,t_i\right)=1$

$\Psi_b\left(t,t_i\right):=1.9\cdot k_s\cdot k_{hc}\cdot k_f\cdot k_{td}\left(t,t_i\right)\cdot\left(t_i\right)^{-0.118}$

$\Psi_b\left(t_f,t_i\right)=1.35$

$$K_{id}:=\frac{1}{1+\frac{E_f}{E_{ci}}\cdot\frac{A_{pf}}{A_g}\cdot\left(1+\frac{A_g\cdot {e_c}^2}{I_g}\right)\left(1+0.7\cdot\Psi_b\left(t_f,t_i\right)\right)}=1$$

$\Delta f_{pSR}:=\varepsilon_{bid}\cdot E_f\cdot K_{id}=5.53\ ksi$

Creep of Girder Concrete

$$\Delta f_{pCR}=\frac{E_{pc}}{E_{ciACI}}\cdot f_{cgp}\cdot\Psi_b\left(t_d,t_i\right)\cdot K_{id}$$ [AASHTO Eq. 5.9.3.4.2b-1]

Where, $\Psi_b\left(t_d,t_i\right)$=girder creep coefficient at time of deck placement due to loading introduced at transfer

$=1.9\cdot k_s\cdot k_{hc}\cdot k_f\cdot k_{td}\cdot\left(t_i\right)^{-0.118}$

$$\Delta f_{pCR}:=\frac{E_f}{E_{ci}}\cdot f_{cgp}\cdot\Psi_b\left(t_d,t_i\right)\cdot K_{id}=9\ ksi$$

Relaxation of Prestressing Strands

$$\Delta f_{pR1} = \left(0.019 \cdot \left(\frac{f_{pt}}{f_{pu}}\right) - 0.0066\right) \log\left(t \cdot 24\right) \cdot f_{pu}$$ [AASHTO-CFRP Eq. 1.9.2.5.2-2]

Where, f_{pt}=stress in prestressing strands immediately after transfer,

$$f_{pt} := f_{pi} - \Delta f_{pES} = 240\ ksi$$

t=time between strand prestressing and deck placement (days)

$$t := t_i + t_d = 91$$

Therefore,

$$\Delta f_{pR1} := \left(0.019 \cdot \left(\frac{f_{pt}}{f_{pu}}\right) - 0.0066\right) \log\left(t \cdot 24\right) \cdot f_{pu} = 7.37\ ksi$$

Prestress Losses: Time of Deck Placement to Final Time

Shrinkage of Girder Concrete

$$\Delta f_{pSD} = \varepsilon_{bdf} \cdot E_f \cdot K_{df}$$

where ε_{bdf}=shrinkage strain of girder between the time of deck placement and final time $=\varepsilon_{bif} - \varepsilon_{bid}$

$$\varepsilon_{bif} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}\left(t_f, t_i\right) \cdot 0.48 \cdot 10^{-3} = 3.49 \cdot 10^{-4}$$

$$\varepsilon_{bdf} := \varepsilon_{bif} - \varepsilon_{bid} = 1.02 \cdot 10^{-4}$$

K_{df}=transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between deck placement and final time

$$= \frac{1}{1 + \frac{E_{pc}}{E_{ci}} \cdot \frac{A_{psc}}{A_c} \cdot \left(1 + \frac{A_c \cdot {e_{pc}}^2}{I_c}\right)\left(1 + 0.7\ \Psi_b\left(t_f, t_i\right)\right)}$$ [AASHTO Eq. 5.9.3.4.3a-2]

where, e_{pc}=eccentricity of prestressing force with respect to centroid of composite section (in); positive in common construction where force is below centroid $= y_{cbot} - y_{bs}$

A_c=area of section calculated using the gross composite concrete section properties of the girder and the deck, and the deck-to-girder modular ratio

I_c =moment of inertia calculated using gross composite concrete properties of the girder and the deck, and the deck-to-girder modular ratio at service=I_{comp}

$$e_{pc} := y_{cbot} - y_{bs} = 33.74\ in$$

$$\Psi_b\left(t_f, t_i\right) = 1.35$$

$$K_{df} := \frac{1}{1 + \frac{E_f}{E_{ci}} \cdot \frac{A_{pf}}{A_c} \cdot \left(1 + \frac{A_c \cdot {e_{pc}}^2}{I_{comp}}\right) \left(1 + 0.7 \cdot \Psi_b\left(t_f, t_i\right)\right)} = 1$$

$$\Delta f_{pSD} := \varepsilon_{bdf} \cdot E_f \cdot K_{df} = 2.29\ ksi$$

Creep of Girder Concrete

$$\Delta f_{pCD} = \frac{E_f}{E_{ci}} \cdot f_{cgp} \cdot \left(\Psi_b\left(t_f, t_i\right) - \Psi_b\left(t_d, t_i\right)\right) \cdot K_{df} + \frac{E_f}{E_c} \cdot \Delta f_{cd} \cdot \Psi_b\left(t_f, t_d\right) \cdot K_{df}$$

[AASHTO Eq. 5.9.3.4.3b-1]

Where, $\Psi_b\left(t_f, t_d\right)$ =girder creep coefficient at final time due to loading at deck placement

$$= 1.9 \cdot k_s \cdot k_{hc} \cdot k_f \cdot k_{td}\left(t_f, t_d\right) \cdot \left(t_i\right)^{-0.118}$$

$$\Psi_b\left(t_f, t_d\right) = 0.8$$

Δf_{cd} =change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement, combined with deck weight on the non-composite transformed section, and superimposed loads on the composite transformed section (ksi)

$$= -\left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) \cdot \frac{A_{pf}}{A_g} \cdot \left(1 + \frac{A_g \cdot {e_c}^2}{I_g}\right) - \left(\frac{M_S \cdot e_{ptf}}{I_{tf}} + \frac{\left(M_b + M_{ws}\right) \cdot e_{ptc}}{I_{tc}}\right)$$

Where e_{ptf} =eccentricity of the prestressing force with respect to the centroid of the non-composite transformed section

e_{ptc} =eccentricity of the prestressing force with respect to the centroid of the non-composite transformed section

I_{tf} =moment of inertia of the non-composite transformed section

I_{tc} =moment of inertia of the composite transformed section

To perform the calculations, it is necessary to calculate the non-composite and composite transformed section properties

$$n_{ci} := \frac{E_f}{E_{ci}} = 4.72 \qquad n_c := \frac{E_f}{E_c} = 4.13$$

Inputs

$excel_{\text{"B3"}} := A_g \cdot 39.3701^2$ $excel_{\text{"C3"}} := y_{gbot} \cdot 39.3701$ $excel_{\text{"G3"}} := I_g \cdot 39.3701^4$

$excel_{\text{"B4"}} := n_{ci} \cdot n_p \cdot A_{pf} \cdot 39.3701^2$ $excel_{\text{"C4"}} := y_{bs} \cdot 39.3701$

Non-composite transformed section properties at transfer							
	Area (in^2)	y_b (in)	A y_b (in^3)	$y_{b,tt}$ (in)	A *($y_{b,tt}$-y_b)^2 (in^4)	I (in^4)	I+A *($y_{b,tt}$-y_b)^2 (in^4)
Beam	789.00	24.73	19512.00	24.24	187.67	260730.56	260918.23
Pre. CFRP	18.69	3.66	68.41		7920.47	0.00	7920.47
	807.70		19580.41			I (in^4)	268838.70

Outputs

Inputs

$excel_{\text{"B3"}} := A_g \cdot 39.3701^2$ $excel_{\text{"C3"}} := y_{gbot} \cdot 39.3701$ $excel_{\text{"G3"}} := I_g \cdot 39.3701^4$

$excel_{\text{"B4"}} := n_c \cdot n_p \cdot A_{pf} \cdot 39.3701^2$ $excel_{\text{"C4"}} := y_{bs} \cdot 39.3701$

Non-composite transformed section properties at service							
	Area (in^2)	y_b (in)	A y_b (in^3)	$y_{b,tt}$ (in)	A *($y_{b,tt}$-y_b)^2 (in^4)	I (in^4)	I+A *($y_{b,tt}$-y_b)^2 (in^4)
Beam	789.00	24.73	19512.00	24.30	144.44	260730.56	260875.01
Pre. CFRP	16.35	3.66	59.84		6968.87	0.00	6968.87
	805.35		19571.84			I (in^4)	267843.87

Outputs

$e_{ptf} := excel_{\text{"E3"}} \cdot in$ $I_{tf} := excel_{\text{"H5"}} \cdot in^4$

Inputs

$excel_{\text{"B3"}} := A_c \cdot 39.3701^2$ $excel_{\text{"C3"}} := y_{cbot} \cdot 39.3701$ $excel_{\text{"G3"}} := I_{comp} \cdot 39.3701^4$

$excel_{\text{"B4"}} := n_c \cdot n_p \cdot A_{pf} \cdot 39.3701^2$ $excel_{\text{"C4"}} := y_{bs} \cdot 39.3701$

Composite transformed section properties at service							
	Area (in^2)	y_b (in)	A y_b (in^3)	$y_{b,tt}$ (in)	A *($y_{b,tt}$-y_b)^2 (in^4)	I (in^4)	I+A *($y_{b,tt}$-y_b)^2 (in^4)
Beam and Slab	1270.12	37.40	47502.24	36.97	233.66	597437.16	597670.82
Pre. CFRP	16.35	3.66	59.84		18147.23	0.00	18147.23
	1286.47		47562.08			I (in^4)	615818.05

Outputs

$e_{ptc} := excel_{\text{"E3"}} \cdot in$ $I_{tc} := excel_{\text{"H5"}} \cdot in^4$

From table above $e_{ptf}=24.3\ in$ $e_{ptc}=36.97\ in$

$I_{tf}=\left(2.68\cdot 10^5\right)\ in^4$ $I_{tc}=\left(6.16\cdot 10^5\right)\ in^4$

$$\Delta f_{cd}:=-\left(\Delta f_{pSR}+\Delta f_{pCR}+\Delta f_{pR1}\right)\cdot\frac{A_{pf}}{A_g}\cdot\left(1+\frac{A_g\cdot {e_c}^2}{I_g}\right)-\left(\frac{M_d\cdot e_{ptf}}{I_{tf}}+\frac{\left(M_b+M_{ws}\right)\cdot e_{ptc}}{I_{tc}}\right)=-0.87\ ksi$$

$$\Delta f_{pCD}:=\frac{E_f}{E_{ci}}\cdot f_{cgp}\cdot\left(\Psi_b\left(t_f,t_i\right)-\Psi_b\left(t_d,t_i\right)\right)\cdot K_{df}+\frac{E_f}{E_c}\cdot\Delta f_{cd}\cdot\Psi_b\left(t_f,t_d\right)\cdot K_{df}=0.86\ ksi$$

Relaxation of Prestressing Strands

$$\Delta f_{pR2}=\Delta f_{pRf}-\Delta f_{pR1}$$

$$\Delta f_{pRf}=\left(0.019\cdot\left(\frac{f_{pt}}{f_{pu}}\right)-0.0066\right)\ log\left(t\cdot 24\right)\cdot f_{pu}$$ [AASHTO-CFRP Eq. 1.9.2.5.2-2]

Where, f_{pt}=stress in prestressing strands immediately after transfer,

$$f_{pt}:=f_{pi}-\Delta f_{pES}=240\ ksi$$

t=time between strand prestressing and final (days)

$$t:=t_i+t_f=2\cdot 10^4$$

Therefore,

$$\Delta f_{pRf}:=\left(0.019\cdot\left(\frac{f_{pt}}{f_{pu}}\right)-0.0066\right)\ \log\left(t\cdot 24\right)\cdot f_{pu}=12.55\ ksi$$

$$\Delta f_{pR2}:=\Delta f_{pRf}-\Delta f_{pR1}=5.17\ ksi$$

Shrinkage of Deck Concrete

$$\Delta f_{pSS}=\frac{E_f}{E_c}\cdot\Delta f_{cdf}\cdot K_{df}\cdot\left(1+0.7\cdot\Psi_b\left(t_f,t_d\right)\right)$$ [AASHTO Eq. 5.9.3.4.3d-1]

Where, Δf_{cdf}=change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete (ksi)

$$=\frac{\varepsilon_{ddf}\cdot A_d\cdot E_{cd}}{\left(1+0.7\cdot\Psi_b\left(t_f,t_d\right)\right)}\cdot\left(\frac{1}{A_c}-\frac{e_{pc}\cdot e_d}{I_c}\right)$$

Where, A_d=area of deck concrete

E_{cd}=modulus of elascity of deck concrete

e_d=eccentricity of deck with respect to the gross composite section, positive in typical construction where deck is above girder (in)

$$y_{cDeck} := h_c - 0.5 \cdot h_d = 58.25 \ in$$

$$e_d := y_{cDeck} - y_{cbot} = 20.85 \ in$$

ε_{ddf}=shrinkage strain of deck concrete between placement and final time
$= k_s \cdot k_{hs} \cdot k_f \cdot k_{td} \left(t_f, t_i\right) \cdot 0.48 \cdot 10^{-3}$

and, k_s=factor for the effect of volume to surface ratio of the component (this has to be recalculated for deck)

k_s=1.45-0.13(V/S)
where (V/S)=volume to surface ratio of deck (in)
=Area/Perimeter (excluding edges)

$$P_d := b_e \cdot 2 = 144 \ in$$ [AASHTO Eq. 5.4.2.3.2-2]

$$k_s := 1.45 - 0.13 \cdot \frac{1}{in} \left(\frac{A_d}{P_d}\right) = 1.02$$

$$k_s := \begin{Vmatrix} \text{if } k_s \le 1 \\ \quad \Vert\, 1 \\ \text{else} \\ \quad \Vert\, k_s \end{Vmatrix} = 1.02$$

$$k_f = \frac{5}{1 + \frac{f'_{ci}}{ksi}}$$

f'_{ci}=specified compressive strength of deck concrete at time of initial loading may be taken as $0.80 f'_{cDeck}$

$$k_f := \frac{5}{1 + \frac{0.8 \cdot f'_{cDeck}}{ksi}} = 0.86$$

$$k_{td}\left(t, t_i\right) := \frac{t - t_i}{61 - 4 \cdot \dfrac{0.8\ f'_{cDeck}}{ksi} + t - t_i}$$

$$\varepsilon_{ddf} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}\left(t_f, t_d\right) \cdot 0.48 \cdot 10^{-3} = 4.31 \cdot 10^{-4}$$

$$\Delta f_{cdf} := \frac{\varepsilon_{ddf} \cdot A_d \cdot E_{cDeck}}{\left(1 + 0.7 \cdot \Psi_b\left(t_f, t_d\right)\right)} \cdot \left(\frac{1}{A_c} - \frac{e_{pc} \cdot e_d}{I_{comp}}\right) = -0.24\ ksi$$

$$\Delta f_{pSS} := \frac{E_f}{E_c} \cdot \Delta f_{cdf} \cdot K_{df} \cdot \left(1 + 0.7 \cdot \Psi_b\left(t_f, t_d\right)\right) = -1.56\ ksi$$

Total Prestress Losses at Transfer

The prestress loss due to elastic shortening:

$$\Delta f_{pES} = 9.43\ ksi$$

Stress in tendons after transfer $f_{pt} := f_{pi} - \Delta f_{pES} = 240\ ksi$

Force per strand after transfer $p_t := f_{pt} \cdot A_{pf} = 43.2\ kip$

Initial loss, % $eloss := \dfrac{\Delta f_{pES} \cdot 100}{f_{pi}} = 3.78$

Total Prestress Losses at Service

The sum of time-dependent prestress losses between time of transfer and deck placement:

$$\left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) = 21.9\ ksi$$

The sum of time-dependent prestress losses after deck placement:

$$\left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS}\right) = 6.76\ ksi$$

The total time-dependent prestress losses:

$$\Delta f_{pLT} := \left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) + \left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS}\right) = 28.66\ ksi$$

The total prestress loss at service:

$$\Delta f_{pT} := \Delta f_{pES} + \Delta f_{pLT} = 38.09\ ksi$$

Stress in strands after all losses,

$f_{pe} := f_{pi} - \Delta f_{pT} = 211.34\ ksi$

Check prestressing stress limit at service limit state: [AASHTO-CFRP Table 1.9.1-1]

$$\begin{array}{l} \text{if } f_{pe} \le 0.65 \cdot f_{pu} \\ \quad \| \text{“Stress limit satisfied”} \\ \text{else} \\ \quad \| \text{“Stress limit not satisfied”} \end{array} = \text{“Stress limit satisfied”}$$

Force per strand after all losses $p_e := f_{pe} \cdot A_{pf} = 38.04\ kip$ $n_p = 22$

Therefore, the total prestressing force after all losses $P_e := n_p \cdot p_e = 836.91\ kip$

Final loss, % $ploss := \dfrac{\Delta f_{pT} \cdot 100}{f_{pi}} = 15.27$

<u>Stresses at Transfer</u>

Total prestressing force after transfer $P_t := n_p \cdot p_t = 950.41\ kip$

Stress Limits for Concrete

Compression Limit: [AASHTO Art. 5.9.2.3.1a]

$0.6 \cdot f'_{ci} = 3.6\ ksi$

Where, f'_{ci}=concrete strength at release=6 ksi

Tension Limit: [AASHTO Art. 5.9.2.3.1b]

Without bonded reinforcement

$-0.0948 \cdot \sqrt{\dfrac{f'_{ci}}{ksi}}\ ksi = -0.23\ ksi$ ≤ -0.2 ksi

Therefore, tension limit, $= -0.2\ ksi$

With bonded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force in the concrete computed assuming an uncracked section where reinforcement is proportioned using a stress of $0.5 f_y$, not to exceed 30 ksi.

$$-0.24 \cdot \sqrt{\frac{f'_{ci}}{ksi}}\ ksi = -0.59\ ksi$$

If the tensile stress is between these two limits, the tensile force at the location being considered must be computed following the procedure in AASHTO Art. C5.9.2.3.1b. The required area of reinforcement is computed by dividing tensile force by the permitted stress in the reinforcement ($0.5 f_y \leq 30$ ksi)

Stresses at Transfer Length Section

Stresses at this location need only be checked at release since this stage almost always governs. Also, losses with time will reduce the concrete stresses making them less critical.

Transfer length $$l_t = \frac{f_{pi} \cdot d_b}{\alpha_t \cdot f'_{ci}{}^{0.67}}$$ [AASHTO-CFRP Eq. 1.9.3.2.1-1]

Where, d_b=prestressing CFRP diameter (in.)
α_t=coefficient related to transfer length taken as 1.3 for cable

Also can be estimated as $l_t := 50 \cdot d_b = 30\ in$

Moment due to self-weight of the beam at transfer length

$$M_{gt} := 0.5 \cdot w_g \cdot l_t \cdot \left(L_{design} - l_t\right) = 89.91\ ft \cdot kip$$

Stress in the top of beam:

$$f_t := \frac{P_t}{A_g} - \frac{P_t \cdot e_c}{S_{gtop}} + \frac{M_{gt}}{S_{gtop}} = -0.92\ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [NOT OK]

-0.588 ksi with bonded reinforcement [NOT OK]

stress in the bottom of the beam:

$$f_b := \frac{P_t}{A_g} + \frac{P_t \cdot e_c}{S_{gbot}} - \frac{M_{gt}}{S_{gbot}} = 3\ ksi$$

Compressive stress limit for concrete = 3.6 ksi [NOT OK]

Since stresses at the top and bottom exceed the stress limits, debond strands to satisfy the specified limits. Total of 4 strands were debonded at the length of 15 ft and 30 ft.

At a distance of 15 ft $dl := 15\ ft$ $n_{pd1} := 2$

$x := dl + l_t = 210\ in$ $P_t := \left(n_p - n_{pd1}\right) \cdot p_t = 864.01\ kip$

Since the 2 strands are debonded at the end, the CG of the strands need to be revised

$$x_{pe} := h_g - \frac{\left(n_{b1} - n_{pd1}\right) \cdot d_{p1} + n_{b2} \cdot d_{p2} + n_{b3} \cdot d_{p3} + n_{b4} \cdot d_{p4} + n_{b5} \cdot d_{p5}}{\left(n_{b1} - n_{pd1}\right) + n_{b2} + n_{b3} + n_{b4} + n_{b5}} = 3.75\ in$$

$e_{ce} := y_{gbot} - x_{pe} = 20.98\ in$

Moment due to self-weight of the beam at transfer length

$$M_{gdl} := 0.5 \cdot w_g \cdot x \cdot \left(L_{design} - x\right) = 521.46\ ft \cdot kip$$

Stress in the top of beam:

$$f_t := \frac{P_t}{A_g} - \frac{P_t \cdot e_{ce}}{S_{gtop}} + \frac{M_{gdl}}{S_{gtop}} = -0.24\ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [NOT OK]

-0.588 ksi with bonded reinforcement [OK]

stress in the bottom of the beam:

$$f_b := \frac{P_t}{A_g} + \frac{P_t \cdot e_{ce}}{S_{gbot}} - \frac{M_{gdl}}{S_{gbot}} = 2.22\ ksi$$

Compressive stress limit for concrete = 3.6 ksi [OK]

Stresses at Midspan

$x := L_{design} \cdot 0.5 = 45\ ft$ $P_t := n_p \cdot p_t = 950.41\ kip$

Moment due to self-weight of the beam at mid-span

$$M_g := 0.5 \cdot w_g \cdot x \cdot \left(L_{design} - x\right) = 832.28\ ft \cdot kip$$

Stress in the top of beam:

$$f_t := \frac{P_i}{A_g} - \frac{P_i \cdot e_c}{S_{gtop}} + \frac{M_g}{S_{gtop}} = 0.08\ ksi$$

Tensile stress limits for concrete=	-0.2 ksi without bonded reinforcement	[OK]
	-0.588 ksi with bonded reinforcement	[OK]

stress in the bottom of the beam:

$$f_b := \frac{P_i}{A_g} + \frac{P_i \cdot e_c}{S_{gbot}} - \frac{M_g}{S_{gbot}} = 2.16\ ksi$$

Compressive stress limit for concrete = 3.6 ksi [OK]

Stresses at Service Loads

Total prestressing force after all losses $P_e = 836.91\ kip$

Compression Limit: [AASHTO Art. 5.9.2.3.2a]

Due to the sum of effective prestress and permanent loads (i.e. beam self-weight, weight of slab and haunch, weight of future wearing surface, and weight of barriers) for the Load Combination Service 1:

for precast beam $0.45 \cdot f'_c = 4.05\ ksi$

for deck $0.45 \cdot f'_{cDeck} = 2.7\ ksi$

Due to the sum of effective prestress, permanent loads, and transient loads as well as during shipping and handling for the Load Combination Service 1:

for precast beam $0.60 \cdot f'_c = 5.4\ ksi$

for deck $0.60 \cdot f'_{cDeck} = 3.6\ ksi$

Tension Limit: [AASHTO Art. 5.9.2.3.2b]

For components with bonded prestressing tendons or reinforcement that are subjected to not worse than moderate corrosion conditions for Load Combination Service III

for precast beam $-0.19 \cdot \sqrt{\frac{f'_c}{ksi}}\ ksi = -0.57\ ksi$

Stresses at Midspan

Concrete stress at top fiber of the beam

To check top stresses, two cases are checked:

Under permanent loads, Service I:

$$S_{cgtop} := \frac{I_{comp}}{y_{ctop} - h_d} = \left(3.49 \cdot 10^4\right)\ in^3$$

$$f_{tg} := \frac{P_e}{A_g} - \frac{P_e \cdot e_c}{S_{gtop}} + \frac{M_g + M_d}{S_{gtop}} + \frac{M_{ws} + M_b}{S_{cgtop}} = 1.12\ ksi \qquad < 4.05\ ksi \qquad \text{[OK]}$$

Under permanent and transient loads, Service I:

$$f_{tg} := f_{tg} + \frac{M_{LT} + M_{LL}}{S_{cgtop}} = 1.6\ ksi \qquad < 5.4\ ksi \qquad \text{[OK]}$$

Concrete stress at bottom fiber of beam under permanent and transient loads, Service III:

$$f_b := \frac{P_e}{A_g} + \frac{P_e \cdot e_c}{S_{gtop}} - \frac{M_g + M_d}{S_{gbot}} - \frac{M_b + M_{ws} + (0.8) \cdot \left(M_{LT} + M_{LL}\right)}{S_{cbot}} = 0.37\ ksi \qquad < 5.4\ ksi \qquad \text{[OK]}$$

Fatigue Limit State

AASHTO Art. 5.5.3 specifies that the check for fatigue of the prestressing strands is not required for fully prestressed components that are designed to have extreme fiber tensile stress due to the Service III limit state within the specified limit of $0.19 f'_c$. The AASHTO Type IV girder in this design example is designed as a fully prestressed member, and the tensile stress due to Service III limit state is less than $0.19 f'_c$. Hence, the fatigue check for the prestressing strands is not required.

Strength Limit State

The total ultimate bending moment for Strength I is:

$M_u = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)$ [AASHTO Art. 3.4.1]

Using the values of unfactored bending moment used in this example

$$M_u := 1.25\ \left(M_g + M_d + M_b\right) + 1.50 \cdot M_{ws} + 1.75 \cdot \left(M_{LT} + M_{LL}\right) = \left(4.58 \cdot 10^3\right)\ ft \cdot kip$$

$\varepsilon_{pu} = 0.016$ $\qquad \varepsilon_{pe} := \frac{f_{pe}}{E_f} = 9.39 \cdot 10^{-3}$ $\qquad \varepsilon_{cu} := 0.003$

$d_p := \left(h_g + h_d - 2.75\ in\right) = 58.75\ in$

$d_{p1} := \left(h_g + h_d - 4.75\ in\right) = 56.75\ in$

$d_{p2} := \left(h_g + h_d - 6.75\ in\right) = 54.75\ in$

$d_{p3} := \left(h_g + h_d - 8.75\ in\right) = 52.75\ in$

$d_{p4} := \left(h_g + h_d - 10.75\ in\right) = 50.75\ in$

$$\begin{bmatrix} n_{b1} \\ n_{b2} \\ n_{b3} \\ n_{b4} \\ n_{b5} \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$c_b := \dfrac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{pu} - \varepsilon_{pe}} \cdot d_p = 18.66\ in$ $\qquad \beta_1 := 0.85$

$b_d := b_e = 6\ ft$

Total compressive force

$$C_c := \begin{cases} \text{if } \beta_1 \cdot c_b \le h_d \\ \quad 0.85 \cdot f'_{cDeck} \cdot \beta_1 \cdot b_d \cdot c \\ \text{else if } h_d \le \beta_1 \cdot c_b \le \left(h_{trf} + h_d\right) \\ \quad 0.85 \cdot f'_{cDeck} \cdot \left(b_d \cdot h_d + b_{trf} \cdot \left(\beta_1 \cdot c_b - h_d\right)\right) \\ \text{else} \\ \quad 0.85 \cdot f'_{cDeck} \cdot \left(b_d \cdot h_d + b_{trf} \cdot h_{trf} + \left(b_{ttf} + \left(h_{ttf} - \left(\beta_1 \cdot c_b - \left(h_{trf} + h_d\right)\right)\right) + b_w\right) \cdot \left(\beta_1 \cdot c_b - \left(h_{trf} + h_d\right)\right)\right) \end{cases} = \left(3.61 \cdot 10^{3}\right.$$

Area of prestressing reinforcement for balance condition $\qquad A_{pfb} := \dfrac{C_c}{f_{pu}} = 10.12\ in^2$

Number of cables required for balance condition $\qquad n_{pb} := \dfrac{A_{pfb}}{A_{pf}} = 56.23$

Check for

$$\begin{cases} \text{if } n_p < n_{pb} \\ \quad \text{“Section is tension controlled”} \\ \text{else} \\ \quad \text{“Section is compression controlled”} \end{cases} = \text{“Section is tension controlled”}$$

$\varepsilon_{co} := \left(\left(\dfrac{f'_{cDeck}}{11\ ksi}\right) + 1.6\right) \cdot 10^{-3} = 0.0021$

$$\beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) := \max\left(0.65, \left(\dfrac{4 - \left(\dfrac{\varepsilon_{cc}}{\varepsilon_{co}}\right)}{6 - 2 \cdot \left(\dfrac{\varepsilon_{cc}}{\varepsilon_{co}}\right)} \cdot \left(-\left(\dfrac{f'_{cDeck}}{50\ ksi}\right) + 1.1\right)\right)\right)$$ [AASHTO-CFRP Eq. C1.7.2.1-3]

$$\alpha_1(\varepsilon_{cc}, \varepsilon_{co}) := \frac{\left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right) - \frac{1}{3} \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)^2}{\beta_1(\varepsilon_{cc}, \varepsilon_{co})} \cdot \left(-\left(\frac{f'_{cDeck}}{60\ ksi}\right) + 1\right)$$ [AASHTO-CFRP Eq. C1.7.2.1-4]

Guess Values

$c := 9\ in$ $\quad d_p := 58.75\ in$ $\quad \varepsilon_{pu} = 0.02$ $\quad \varepsilon_{cc} := 0.0025$ $\quad \varepsilon_{co} := 0.002$

Constraints

$$\varepsilon_{pe} + \frac{d_p - c}{c} \cdot \varepsilon_{cc} = \varepsilon_{pu}$$ [AASHTO-CFRP Eq. 1.7.3.1.1-3]

$$\begin{array}{l} \text{if } \beta_1(\varepsilon_{cc}, \varepsilon_{co}) \cdot c \le h_d \\ \quad \alpha_1(\varepsilon_{cc}, \varepsilon_{co}) \cdot f'_{cDeck} \cdot \beta_1(\varepsilon_{cc}, \varepsilon_{co}) \cdot b_d \cdot c \\ \text{else if } h_d \le \beta_1(\varepsilon_{cc}, \varepsilon_{co}) \cdot c \le (h_{trf} + h_d) \\ \quad \alpha_1(\varepsilon_{cc}, \varepsilon_{co}) \cdot f'_{cDeck} \cdot (b_d \cdot h_d + b_{trf} \cdot (\beta_1(\varepsilon_{cc}, \varepsilon_{co}) \cdot c - h_d)) \\ \text{else} \\ \quad \alpha_1(\varepsilon_{cc}, \varepsilon_{co}) \cdot f'_{cDeck} \cdot (b_d \cdot h_d + b_{trf} \cdot h_{trf} + (b_{ttf} + (h_{ttf} - (\beta_1(\varepsilon_{cc}, \varepsilon_{co}) \cdot c - (h_{trf} + h_d))) + b_w) \cdot (\beta_1(\varepsilon_{cc}, \varepsilon_{co}) \cdot c - (h_{trf} + \end{array}$$

Solver

$$\begin{bmatrix} c \\ \varepsilon_{cc} \end{bmatrix} := \mathbf{find}(c, \varepsilon_{cc}) = \begin{bmatrix} 0.6897\ ft \\ 0.0011 \end{bmatrix}$$

$\varepsilon_{cc} = 0.0011$

$$\beta_1 := \max\left(0.65, \left(\frac{4 - \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)}{6 - 2 \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)} \cdot \left(-\left(\frac{f'_{cDeck}}{50\ ksi}\right) + 1.1\right)\right)\right) = 0.69$$

$$\alpha_1 := \frac{\left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right) - \frac{1}{3} \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)^2}{\beta_1} \cdot \left(-\left(\frac{f'_{cDeck}}{60\ ksi}\right) + 1\right) = 0.54$$

$c = 8.28\ in$

$$\varepsilon_f := \frac{d_p - c}{c} \cdot \varepsilon_{cc} = 0.0064$$

$$\varepsilon_{f1} := \frac{d_p - 2\ in - c}{c} \cdot \varepsilon_{cc} = 0.0062$$

$$\varepsilon_{f3} := \frac{d_p - 6\ in - c}{c} \cdot \varepsilon_{cc} = 0.0057$$

$$\varepsilon_{f2} := \frac{d_p - 4\ in - c}{c} \cdot \varepsilon_{cc} = 0.0059$$

$$\varepsilon_{f4} := \frac{d_p - 8\ in - c}{c} \cdot \varepsilon_{cc} = 0.0054$$

$$T_f := n_{b1}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_f + \varepsilon_{pe}\right) = 769.68\ kip$$

$$T_{f1} := n_{b2}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_{f1} + \varepsilon_{pe}\right) = 631.06\ kip$$

$$T_{f3} := n_{b4}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_{f3} + \varepsilon_{pe}\right) = 0\ kip$$

$$T_{f2} := n_{b3}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_{f2} + \varepsilon_{pe}\right) = 0\ kip$$

$$T_{f4} := n_{b5}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_{f4} + \varepsilon_{pe}\right) = 0\ kip$$

$$h_{trf} + h_d = 15.5\ in$$

$$C_c := \left\| \begin{array}{l} \text{if } \beta_1 \cdot c \le h_d \\ \quad \left\| \alpha_1 \cdot f'_{cDeck} \cdot \beta_1 \cdot b_d \cdot c \right. \\ \text{else} \\ \quad \left\| \alpha_1 \cdot f'_{cDeck} \cdot \left(b_d \cdot h_d + b_{trf} \cdot \left(\beta_1 \cdot c - h_d\right)\right) \right. \end{array} \right\| = \left(1.32 \cdot 10^3\right)\ kip$$

$$M_n := T_f \cdot \left(d_p - c\right) + T_{f1} \cdot \left(d_{p1} - c\right) + T_{f2} \cdot \left(d_{p2} - c\right) + C_c \cdot \left(c - \frac{\beta_1 \cdot c}{2}\right) + T_{f3} \cdot \left(d_{p3} - c\right) + T_{f4} \cdot \left(d_{p4} - c\right) = \left(6.39 \cdot 10^3\right)\ ft \cdot kip$$

$M_n = \left(6.39 \cdot 10^3\right)\ ft \cdot kip$ $\qquad$ $M_u = \left(4.58 \cdot 10^3\right)\ ft \cdot kip$

$\phi := 0.75$ $\qquad$ [for CFRP prestressed beams] $\qquad$ [AASHTO-CFRP Art. 1.5.3.2]

$\phi \cdot M_n = \left(4.79 \cdot 10^3\right)\ ft \cdot kip$ $\qquad$ $> M_u$ $\qquad$ [OK]

Minimum Reinforcement

There is a on-going NCHRP project 12-94 for revising the minimum reinforcement provisions for prestressed beams. Therefore, the outcome of the NCHRP 12-94 may also influence the requirements for CFRP prestressed beams.

At any section of a flexural component, the amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance, Mr, at least equal to the lesser of:

- 1.33 times the factored moment required by the applicable strength load combinations

and

- $M_{cr} = \gamma_3\left(\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_c - M_{dnc} \cdot \left(\frac{S_c}{S_{nc}} - 1\right)\right)$ [AASHTO-CFRP 1.7.3.3.1-1]

Where, γ_1 = 1.6 flexural cracking variability factor
γ_2 =1.1 prestress variability factor
γ_3 =1.0 prestressed concrete structures

$\gamma_1 := 1.6 \qquad \gamma_2 := 1.1 \qquad \gamma_3 := 1.0$

$$f_{cpe} := \frac{P_e}{A_g} + \frac{P_e \cdot e_c}{S_{gbot}} = 2.73\ ksi \qquad f_r := 0.20 \cdot \sqrt{\frac{f'_c}{ksi}}\ ksi = 0.6\ ksi$$

$$M_{dnc} := M_g + M_d = \left(1.45 \cdot 10^3\right)\ ft \cdot kip$$

Using the variables defined in this design example, $S_c = S_{cbot}$ and $S_{nc} = S_{gbot}$

$$M_{cr} := \gamma_3 \cdot \left(\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_{cbot} - M_{dnc} \cdot \left(\frac{S_{cbot}}{S_{gbot}} - 1\right)\right) = \left(4.53 \cdot 10^3\right)\ ft \cdot kip$$

```
gov_moment := if M_cr < 1.33 • M_u        = (4.53 • 10^3) (ft • kip)
                  ‖ M_cr
              else
                  ‖ 1.33 • M_u
```

```
if gov_moment < ϕ • M_n                        = "Minimum reinf. requirement OK"
  ‖ "Minimum reinf. requirement OK"
else
  ‖ "Minimum reinf. requirement NOT OK"
```

Shear Design

Transverse shear reinforcement will be provided where [AASHTO Eq. 5.7.2.3-1]

$$V_u > 0.5\ \phi \cdot (V_c + V_p)$$

Where,

V_u=factored shear force (kip)

V_c=nominal shear resistance provided by tensile stresses in the concrete (kip)

V_p=component of prestressing in the direction of shear force (kip)

ϕ=0.90=resistance factor for shear [AASHTO Art. 5.5.4.2]

Critical Section for Shear [AASHTO Art. 5.7.3.2]

The location of the critical section for shear shall be taken as d_v from the internal face of the support.

Where, d_v= effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultant of the tensile and compressive force due to flexure. It need not to be taken less than the greater of $0.9\,d_e$ or $0.72\,h$ (in).

$$= d_e - \frac{a}{2}$$ [AASHTO Art. 5.7.2.8]

Where,

a = depth of compression block

h = overall depth of composite section

d_e= Effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (For harped and draped configuration, this values varies along the length). For CFRP prestressed beams, this value can be taken as the centroid of prestressing CFRP at that location (substituting f_{ps} by f_{pu} and A_s@3#in AASHTO Eq. 5.8.2.9-2)

$$y_{bse} := x_{pe} = 3.75\ in$$

$$d_e := h_g + h_d - y_{bse} = 57.75\ in \qquad a := \beta_1 \cdot c = 5.67\ in$$

Effective Shear Depth

$$d_v := d_e - \frac{a}{2} = 54.91\ in$$

$$0.9 \cdot d_e = 51.98\ in \qquad 0.72 \cdot (h_g + h_d) = 44.28\ in$$

$$d_1 := \max(0.9 \cdot d_e, 0.72 \cdot (h_g + h_d)) = 51.98\ in$$

$$d_v := \max(d_1, d_v) = 54.91\ in$$

The bearing width is yet to be determined. It is conservatively assumed zero and the critical section for shear is locatied at the distance of

$$x_c := d_v = 54.91\ in \qquad \frac{x_c}{L_{design}} = 0.05$$

(0.049L) from the centerline of the bearing, where L is the design span length.
The value of d_e is calculated at the girder end, which can be refined based on the critical section location. However, it is conservative not to refine the value of d_e based on the critical section 0.049L. The value, if refined, will have a small difference (PCI 2017).

Shear Stress

Shear stress in the concrete (v_u) is given as:

$$v_u = \frac{V_u - \phi V_p}{\phi \cdot b_v \cdot d_v}$$ [AASHTO Eq. 5.7.2.8-1]

Where,

β = A factor indicating the ability of diagonally cracked concrete to transmit tension
ϕ = resistance factor for shear
$\phi := 0.9$
b_v = effective web width (in)
$b_v := b_w = 8\ in$
$d_v = 54.91\ in$
V_u =factored shear force at specified section at Strength Limit I state

Using the equation to calculate shear force due to the design truck $x := x_c = 54.91\ in$

$$V_{truck1}(x) := P_1 \cdot \frac{(L_{design} - x)}{L_{design}} + P_2 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}} + P_3 \cdot \frac{(L_{design} - x - 28\ ft)}{L_{design}}$$

$$V_{truck2}(x) := P_1 \cdot \frac{-(x - 14\ ft)}{L_{design}} + P_2 \cdot \frac{(L_{design} - x)}{L_{design}} + P_3 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}}$$

$$V_{truck1}(x) = 60.87\ kip$$

$$V_{truck2}(x) = 40.07\ kip$$

$$V := \max\left(V_{truck1}(x), V_{truck2}(x)\right) = 60.87\ kip$$

Distributed bending shear due to truck load including dynamic load allowance (V_{LT}) is calculated as follows:
V_{LT} = (Moment per lane due to truck load)(DFS)(1+IM/100)
$IM := 33$
$D_{S.Interior} = 0.6$

$$V_{LT} := V \cdot D_{S.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 48.58\ kip$$

The maximum shear force (V_L) due to a uniformly distributed lane load of 0.64 klf are calculated using the following formulas given by the *PCI Design Manual* (PCI 2017).

Maximum bending moment, $V_x = 0.64 \dfrac{kip}{ft} \dfrac{\left(L_{design} - x\right)^2}{2\ L_{design}}$

where:
x = Distance from centerline of bearing to section at which the shear force is calculated, ft.
L = Design span length

$$V_L := 0.64 \frac{kip}{ft} \frac{\left(L_{design} - x\right)^2}{2\ L_{design}} = 25.95\ kip$$

$$V_{LL} := D_{S.Interior} \cdot V_L = 15.57\ kip$$

$$V_{ws} := w_{ws} \cdot \left(0.5 \cdot L_{design} - x\right) = 6.06\ kip$$

$$V_u := 1.25\ \left(V_g + V_s + V_b\right) + 1.5 \cdot V_{ws} + 1.75 \cdot \left(V_{LT} + V_{LL}\right) = 199.07\ kip$$

V_p = Component of the effective prestressing force in the direction of the applied shear, kips
= (force per strand)(number of harped strands) $\left(sin\left(\Psi\right)\right)$

$$V_p := 0\ kip$$

Therefore,

$$v_u := \frac{V_u - \phi \cdot V_p}{\phi \cdot b_v \cdot d_v} = 0.5\ ksi$$

Contribution of Concrete to Nominal Shear Resistance [AASHTO Art. 5.7.3.3]

The contribution of the concrete to the nominal shear resistance is given as:
[AASHTO Eq. 5.7.3.3-3]

$$V_c = 0.0316\ \beta \cdot \sqrt{f'_{cGirder}} \cdot b_v \cdot d_v$$

where:
β = A factor indicating the ability of diagonally cracked concrete to transmit tension
$f'_{cGirder}$ = Compressive strength of concrete at service
b_v = Effective web width taken as the minimum web width within the depth d_v,
d_v = Effective shear depth

Strain in Flexural Tension Reinforcement

The θ and β values are determined based on the strain in the flexural tension reinforcement. The strain in the reinforcement, ε_f, is determined assuming that the section contains at least the minimum transverse reinforcement as specified in AASHTO-CFRP Eq. 1.8.3.2-1

$$\varepsilon_f = \frac{\frac{M_u}{d_v} + 0.5 \cdot N_u + 0.5 \cdot \left(V_u - V_p\right) - A_{pf} \cdot f_{po}}{E_p \cdot A_{pf}}$$

M_u=Applied factored bending moment at specified section.

$$M_{wsv} := w_{ws} \cdot x \cdot \left(0.5 \cdot L_{design} - x\right) = 27.75\ ft \cdot kip$$

$$M := \max\left(M_{truck1}(x), M_{truck2}(x)\right) = 278.56\ ft \cdot kip$$

$$M_L := 0.5 \cdot 0.64\ \frac{kip}{ft} \cdot (x)\ \left(L_{design} - x\right) = 125.09\ ft \cdot kip$$

$$M_{LLv} := D_{M.Interior} \cdot M_L = 70.64\ ft \cdot kip$$

$$M_{LTv} := D_{M.Interior} \cdot \left(1 + \frac{IM}{100}\right) \cdot M = 209.22\ ft \cdot kip$$

$$M_{uv} := 1.25\ \left(M_{gv} + M_{sv} + M_{bv}\right) + 1.50 \cdot M_{wsv} + 1.75 \cdot \left(M_{LTv} + M_{LLv}\right) = 892.46\ ft \cdot kip$$

$$M_u := \max\left(M_{uv}, V_u \cdot d_v\right) = 910.96\ ft \cdot kip$$

N_u = Applied factored normal force at the specified section, 0.049*L* = 0 kips

$$N_u := 0$$

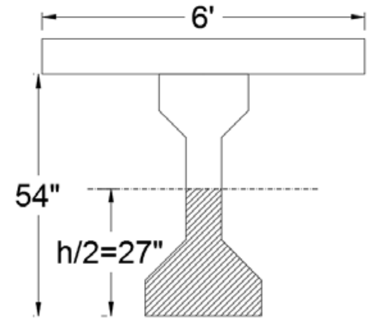
f_{po} = Parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi). For pretensioned members, AASHTO Art. C5.7.3.4.2 indicates that f_{po} can be taken as the stress in strands when the concrete is cast around them, which is jacking stress f_{pj}, or f_{pu}.

$f_{po} := f_{pi} = 249.43\ ksi$ $\quad$ $A_s := 0\ in^2$ $\quad$ $E_f = \left(2.25 \cdot 10^7\right)\ psi$ $\quad$ $A_{pf} = 0.18\ in^2$

$$\varepsilon_f := \frac{\frac{M_u}{d_v} + 0.5\ kip \cdot N_u + \left(V_u - V_p\right) - \left(n_p - n_{pd1}\right) \cdot A_{pf} \cdot f_{po}}{E_f \cdot \left(n_p - n_{pd1}\right) \cdot A_{pf}} = -6.17 \cdot 10^{-3}$$

Since this value is negative, ε_s should be recalculated using AASHTO Eq. 5.7.3.4.2-4 replacing the denominator by $\left(E_c \cdot A_{ct} + E_s \cdot A_s + E_f \cdot A_{pft}\right)$

A_{ct} = Area of the concrete on the flexural tension side below $h/2$



$$A_{ct} := \left(\frac{h_g + h_d}{2} - h_{btf} - h_{brf}\right) \cdot b_w + h_{btf} \cdot b_{btf} + h_{brf} \cdot b_{brf} = 399\ in^2$$

$$\varepsilon_f := \frac{\frac{M_u}{d_v} + 0.5\ kip \cdot N_u + \left(V_u - V_p\right) - \left(n_p - n_{pd1}\right) \cdot A_{pf} \cdot f_{po}}{\left(E_c \cdot A_{ct} + E_f \cdot \left(n_p - n_{pd1}\right) \cdot A_{pf}\right)} = -2.22 \cdot 10^{-4}$$

$$\varepsilon_f := \max\left(\varepsilon_f, -0.40 \cdot 10^{-3}\right) = -2.22 \cdot 10^{-4}$$

Therefore, β, factor indicating the ability of diagonally cracked concrete to transmit tension and shear can be calculated as:

$$\beta := \frac{4.8}{1 + 750 \cdot \varepsilon_f} = 5.76$$ [AASHTO Eq. 5.7.3.4.2-1]

And, θ, angle of inclination of diagonal compressive stress can be calculated as:

$$\theta := 29 + 3500 \cdot \varepsilon_f = 28.22$$ [AASHTO Eq. 5.7.3.4.2-3]

$$\theta := 28.2\ deg$$

Computation of Concrete Contribution

The contribution of the concrete to the nominal shear resistance is given as:

$$V_c := 0.0316\ \beta \cdot \sqrt{\frac{f'_c}{ksi}} \cdot b_v \cdot d_v \cdot ksi = 239.76\ kip$$

Contribution of Reinforcement to Nominal Shear Resistance

$$\begin{array}{l} \text{if } V_u < \phi \cdot \dfrac{\left(V_c + V_p\right)}{2} \\ \quad \Vert \text{“Transverse reinforcement not provided”} \\ \text{else} \\ \quad \Vert \text{“Transverse shear reinforcement provided”} \end{array} = \text{“Transverse shear reinforcement provided”}$$

[AASHTO Eq. 5.7.2.3-1]

Required Area of Shear Reinforcement

The required area of transverse shear reinforcement is: $\dfrac{v_u}{f'_c} = 0.06$

$\dfrac{V_u}{\phi} \le V_n$ $\qquad$ $V_n = V_c + V_p + V_s$

Where,

V_s =Shear force carried by transverse reinforcement

$V_s := \dfrac{V_u}{\phi} - V_c - V_p = -18.58\ kip$ $\qquad$ [Minimum Shear Reinforcement shall be provided]

Determine Spacing of Reinforcement [AASHTO Art. 5.7.2.6]

Check for maximum spacing of transverse reinforcement

check if $v_u < 0.125\ f'_c$

or $\quad v_u \ge 0.125\ f'_c$

$$s_{max} := \left\Vert \begin{array}{l} \text{if } v_u < 0.125\ f'_c \\ \quad \Vert\ min\left(0.8 \cdot d_v, 24\ in\right) \\ \text{else} \\ \quad \Vert\ min\left(0.4 \cdot d_v, 12\ in\right) \end{array} \right\Vert = 24\ in$$

Use $\qquad s := 22\ in$

$$\begin{array}{l} \text{if } s < s_{max} \\ \quad \Vert \text{“transverse reinforcement spacing OK”} \\ \text{else} \\ \quad \Vert \text{“transverse reinforcement spacing NOT OK”} \end{array} = \text{“transverse reinforcement spacing OK”}$$

Minimum Reinforcement Requirement [AASHTO Eq. 5.7.2.5-1]

The area of transverse reinforcement should not be less than:

$$0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot \frac{b_v \cdot s}{f_y}$$

$$A_{vmin} := 0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot \frac{b_v \cdot s}{f_y} \ ksi = 0.28 \ in^2$$

Use #4 bar double-legged stirrups at 12 in. c/c,

$$A_{vprov} := 2 \cdot \left(0.20 \ in^2\right) = 0.4 \ in^2$$

$$V_s := \frac{A_{vprov} \cdot f_y \cdot d_v \cdot \cot(\theta)}{s} = 111.72 \ kip \qquad V_{sprov} := V_s$$

if $A_{vprov} > A_{vmin}$ = "Minimum shear reinforcement criteria met"
"Minimum shear reinforcement criteria met"
else
"Minimum shear reinforcement criteria not met"

Therefore, #4 stirrups with 2 legs shall be provided at 22 in spacing

Maximum Nominal Shear Resistance

In order to ensure that the concrete in the web of the girder will not crush prior to yielding of the transverse reinforcement, the AASHTO Specifications give an upper limit for V_n as follows:

$$V_n = 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p$$ [AASHTO Eq. 5.7.3.3-2]

Comparing the above equation with AASHTO Eq. 5.7.3.3-1

$$V_c + V_s \le 0.25 \cdot f'_c \cdot b_v \cdot d_v = 1$$

$$V_c + V_s = 351.49 \ kip$$

$$0.25 \cdot f'_c \cdot b_v \cdot d_v = 988.44 \ kip$$

This is a sample calculation for determining the transverse reinforcement requirement at the critical section. This procedure can be followed to find the transverse reinforcement requirement at increments along the length of the girder.

Interface Shear Transfer [AASHTO Art. 5.7.4]

Factored Interface Shear

To calculate the factored interface shear between the girder and slab, the procedure in the commentary of AASHTO Art. 5.7.4.5 will be used. This procedure calculates the factored interface shear force per unit length.

At the Strength I Limit State, the factored interface shear force, V_{hi}, at a section on a per unit basis is:

$$V_{hi} = \frac{V_1}{d_v}$$ [AASHTO Eq. C5.7.4.5-7]

where: V_1 = factored shear force at specified section due to total load (noncomposite and composite loads)

The AASHTO Specifications does not identify the location of the critical section. For convenience, it will be assumed here to be the same location as the critical section for vertical shear, at point 0.049L.

$$V_u = 199.07\ kip$$

$$V_1 := V_u = 199.07\ kip$$

$$V_{hi} := \frac{V_1}{d_v} = 3.63\ \frac{kip}{in}$$

Required Nominal Interface Shear Resistance

The required nominal interface shear resistance (per unit length) is:

$$V_{ni} = \frac{V_{ri}}{\phi}$$ [AASHTO Eq. 5.7.4.3-1]

where: $V_{ri} \geq V_{ui}$ [AASHTO Eq. 5.7.4.3-2]

where, $V_{ui} := V_{hi} = 3.63\ \frac{kip}{in}$

Therefore, $V_{ni} = \frac{V_{ui}}{\phi}$

$$V_{ni} := \frac{V_{ui}}{\phi} = 4.03\ \frac{kip}{in}$$

Required Interface Shear Reinforcement

The nominal shear resistance of the interface surface (per unit length) is:

$$V_{ni} = c_1 A_{cv} + \mu(A_{vf}\ f_y + P_c)$$ [AASHTO Eq. 5.7.4.3-3]

where:

c_1 = Cohesion factor [AASHTO Art. 5.7.4.4]

μ = Friction factor [AASHTO Art. 5.7.4.4]

A_{cv} = Area of concrete engaged in shear transfer, in.2

A_{vf} = Area of shear reinforcement crossing the shear plane, in.2

P_c = Permanent net compressive force normal to the shear plane, kips

f_y = Shear reinforcement yield strength, ksi

For concrete normal-weight concrete placed against a clean concrete surface, free of laitance, with surface intentionally roughened to an amplitude of 0.25 in: [AASHTO Art. 5.7.4.4]

$$c_1 := 0.28\ ksi$$

$$\mu := 1$$

The actual contact width, *bv*, between the slab and the girder is 20 in.

$$A_{cv} := b_{trf} = 240\ \frac{in^2}{ft} \qquad d_v = 54.91\ in$$

$$P_c := 0\ kip$$

The AASHTO Eq. 5.7.4.3-3 can be solved for A_{vf} as follows:

$$A_{vf} := \frac{V_{ni} - c_1 \cdot A_{cv} - \dfrac{\mu \cdot P_c}{in}}{\mu \cdot f_y} = -0.31\ \frac{in^2}{ft}$$

The provided vertical shear reinforcement $\qquad \dfrac{A_{vprov}}{s} = 0.22\ \dfrac{in^2}{ft}$

Since, $\dfrac{A_{vprov}}{s} > A_{vf}$,

The provided reinforcement for vertical shear is sufficient to resist interface shear.

$$A_{vfprov} := \frac{A_{vprov}}{s} = 0.22\ \frac{in^2}{ft}$$

Minimum Interface Shear Reinforcement

The cross-sectional area of the interface shear reinforcement, A_{vf}, croossing the interface are, A_{cv}, shall satisfy

Minimum $A_{vf} \geq \dfrac{0.05 \cdot A_{cv}}{f_y}$ [AASHTO Eq. 5.7.4.2-1]

$$A_{vf1} := \frac{0.05 \cdot A_{cv}}{\dfrac{f_y}{ksi}} = 0.2\ \frac{in^2}{ft}$$

The minimum interface shear reinforcement, A_{vf}, need not exceed the lesser of the amount determined using Eq. 5.7.4.2-1 and the amount needed to resist $1.33 \frac{V_{ui}}{\phi}$ as determined using Eq. 5.7.4.3-1.

$$A_{vf2} := \frac{1.33 \frac{V_{ui}}{\phi} - c_1 \cdot A_{cv} - \frac{\mu \cdot P_c}{in}}{\mu \cdot f_y} = -0.05 \ \frac{in^2}{ft}$$

Therefore, minimum amount of shear reinforcement

$$A_{vfmin} := min\left(A_{vf1}, A_{vf2}\right) = -0.05 \ \frac{in^2}{ft}$$

if $A_{vfprov} > A_{vfmin}$
  "Minm. Interface shear reinforcement OK"
else
  "Minm. Interfaceshear reinforcement NOT OK"

= "Minm. Interface shear reinforcement OK"

Maximum Nominal Shear Resistance

$$V_{niprov} := c_1 \cdot A_{cv} + \mu \cdot A_{vf} \cdot f_y = 48.33 \ \frac{kip}{ft}$$ [AASHTO Eq. 5.7.4.3-3]

The nominal shear resistance, V_{ni}, used in the design shall not be greater than the lesser of

$$V_{ni} \le k_1 \cdot f'_c \cdot A_{cv}$$ [AASHTO Eq. 5.7.4.3-4]

$$V_{ni} \le k_2 \cdot A_{cv}$$ [AASHTO Eq. 5.7.4.3-5]

Where: For a cast-in-place concrete slab on clean concrete girder surfaces, free of laitance with surface roughened to an amplitude of 0.25 in.

$$k_1 := 0.30 \qquad k_2 := 1.8 \ ksi$$

$$k_1 \cdot f'_c \cdot A_{cv} = 648 \ \frac{1}{ft} \cdot kip$$

$$k_2 \cdot A_{cv} = 432 \ \frac{1}{ft} \cdot kip$$

$V_{niprov} < k_1 \cdot f'_c \cdot A_{cv} = 1$ [1=OK]

$V_{niprov} < k_2 \cdot A_{cv} = 1$ [1=OK]

Minimum Longitudinal Reinforcement Requirement [AASHTO CFRP Art. 1.8.3.3]

Longitudinal reinforcement should be proportioned so that at each section the following equation is satisfied:

$$\sum_{x=1}^{n} A_{pf} \cdot f_{pu} \geq \frac{M_u}{d_v \cdot \phi_f} + 0.5 \cdot \frac{N_u}{\phi_n} + \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p \right) cot(\theta)$$

[AASHTO-CFRP Eq. 1.8.3.3-1]

where: $n \cdot A_{pf}$ = area of prestressing steel on the flexural tension side of the member at section under consideration (in)

f_{pu} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi) conservatively taken as effective prestress

M_u = factored bending moment at the section corresponding to the factored shear force (kip-ft)

V_u = factored shear force at section under consideration (kip)

V_p = component of the effective prestressing force in direction of the applied shear (kip) = 0

V_s = shear resistance provided by the transverse reinforcement at the section under investigation as given by Eq. 5.7.3.3-4, except that Vs shall not be taken as greater than Vu /φ (kip)

ϕ_f = resistance factor for flexure

ϕ_n = resistance factor for axial resistance

ϕ_v = resistance factor for shear

θ = angle of inclination of diagonal compressive stresses used in determining the nominal shear resistance of the section under investigation as determined by Art. 5.7.3.4 (degrees)

Required Reinforcement at Face of Bearing

Width of the bearing is assumed to be zero. This assumption is more conservative for these calculations. Thus, the failure crack assumed for this analysis radiates from the centerline of the bearing, 6 in. from the end of the beam.

As 6 in. is very close to the end of the beam, shear and moment values at the end of the beam are used

Inputs

$$excel_{\text{“F1”}} := 3.28084\ \left(L_{design}\right) \qquad excel_{\text{“A13”}} := 0.5 \cdot 3.28084\ \left(L_{design}\right)$$

$$excel_{\text{“B1”}} := 0.0000685217\ \left(w_g\right)$$

$$excel_{\text{“D1”}} := 0.0000685217\ \left(w_d + w_h\right)$$

$$excel_{\text{“B2”}} := 0.0000685217\ \left(w_b\right)$$

w_g	0.822		0.61042		90.000		L	
w_{SD}	0.109							
			Dead Load					
Distance (x)	Section (x/L)	Girder Weight		Slab Weight		Barrier weight		Total D
		Shear	Moment	Shear	Moment	Shear	Moment	Shear
ft.		k	k-ft	k	k-ft	k	k-ft	k
0	0.000	36.990	0.00	27.469	0.00	4.890	0.00	69.35
4.58	0.051	33.225	160.79	24.673	119.40	4.392	21.26	62.29
10.858	0.121	28.065	353.18	20.841	262.27	3.710	46.69	52.62
21.717	0.241	19.139	609.47	14.212	452.59	2.530	80.57	35.88
32.575	0.362	10.213	768.82	7.584	570.93	1.350	101.64	19.15
43.433	0.483	1.288	831.26	0.957	617.30	0.170	109.89	2.41
45.000	0.500	0.000	832.27	0.000	618.05	0.000	110.02	0.00

Outputs

$$V_g := excel_{\text{“C7”}} \cdot kip \qquad V_s := excel_{\text{“E7”}} \cdot kip \qquad V_b := excel_{\text{“G7”}} \cdot kip$$

$$M_{gv} := excel_{\text{“D8”}} \cdot ft \cdot kip \qquad M_{sv} := excel_{\text{“F8”}} \cdot ft \cdot kip \qquad M_{bv} := excel_{\text{“G8”}} \cdot ft \cdot kip$$

$$x := 0\ ft$$

$$V_{truck1}(x) = 64.53\ kip \qquad V_{truck2}(x) = 43.73\ kip$$

$$V := \max\left(V_{truck1}(x), V_{truck2}(x)\right) = 64.53\ kip$$

$$V_{LT} := V \cdot D_{S.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 51.5\ kip$$

$$V_L := 0.64\ \frac{kip}{ft}\ \frac{\left(L_{design} - x\right)^2}{2\ L_{design}} = 28.8\ kip$$

$$V_{LL} := D_{S.Interior} \cdot V_L = 17.28\ kip$$

$$V_{ws} := w_{ws} \cdot (0.5 \cdot L_{design} - x) = 6.75\ kip$$

$$V_u := 1.25\ (V_g + V_s + V_b) + 1.5 \cdot V_{ws} + 1.75 \cdot (V_{LT} + V_{LL}) = 217.17\ kip$$

$$V_s = 27.47\ kip$$

$$V_u := 1.25\ (V_g + V_s + V_b) + 1.5 \cdot V_{ws} + 1.75 \cdot (V_{LT} + V_{LL}) = 217.17\ kip$$

$\phi_v := 0.9$ $\qquad V_s := V_{sprov}$

$M_u := 0$ $\qquad N_u := 0$ $\qquad$ Therefore,

$$\left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right) \cot(\theta) = 345.84\ kip$$

The crack plane crosses the centroid of the 18 straight strands at a distance of

$$x_c := 6 + 3.659 \cdot \cot(\theta) = 12.82$$

in. from the girder end. Because the transfer length is 30 in., the available prestress from 18 straight strands is a fraction of the effective prestress, f_{pe}, in these strands. Since the crack plane is assumed to radiate from the center line of the bearing (i.e., 6 in from the end of the beam, the moment and axial force can be taken as 0.

$$n_p \cdot A_{pf} \cdot f_{pe} \cdot \frac{x_c \cdot in}{30\ in} = 357.75\ kip \qquad \text{[AASHTO Art. 5.7.3.5]}$$

$$n_p \cdot A_{pf} \cdot f_{pe} \cdot \frac{x_c \cdot in}{24\ in} \ge \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right) \cot(\theta) \qquad \text{[AASHTO-CFRP Eq. 1.8.3.3-1]}$$

Therefore, no additional longitudinal reinforcement is required

Pretensioned Anchorage Zone [AASHTO Art. 5.9.4.4]

Splitting Reinforcement [AASHTO Art. 5.9.4.4-1]

Design of the anchorage zone splitting reinforcement is computed using the force in the strands just prior to transfer:

Force in the strands before transfer

$$P_i := n_p \cdot A_{pf} \cdot f_{pi} = 987.76\ kip \qquad \text{[AASHTO Eq. 5.9.4.4-1]}$$

The splitting resistance, P_r, should not be less than 4% of P_i

$P_r = f_s \cdot A_s > 0.04 \cdot P_i$ $\qquad 0.04 \cdot P_i = 39.51\ kip$

where: A_s = total area of reinforcement located within the distance h/4 from the end of the beam (in²) For pretensioned I-beams and bulb tees, As shall be taken as the total area of the vertical reinforcement located within a distance h/4 from the end of the member, where h is the overall height of the member (in)

f_s = stress in the steel not to exceed 20 ksi

$$f_s := 20\ ksi$$

$$A_s := 0.04 \cdot \frac{P_i}{f_s} = 1.98\ in^2$$

At least 1.98 in² of vertical transverse reinforcement should be provided within a distance of h/4 from the end of beam.

$$\frac{h_g + h_d}{4} = 15.38\ in$$

The area of a #4 stirrup with 2 legs is: $A_v := 2 \cdot 0.2\ in^2 = 0.4\ in^2$

The required number of stirrups is $\text{round}\left(\frac{A_s}{A_v}, 0\right) + 1 = 6$

The required spacing for 6 stirrups over a distance of 15.4 in. starting 2 in. from the end of the beam is:

$$\frac{(15.4 - 2)}{(6 - 1)} = 2.68$$

Use (6) #4 stirrups with 2 legs at 2.5 in. spacing starting at 2 in. from the end of the beam.

The provided $A_{sprov} := 6 \cdot 2 \cdot 0.2\ in^2 = 2.4\ in^2$

$A_{sprov} > A_s = 1$ [1= OK]

Confinement Reinforcement [AASHTO Art. 5.9.4.4.2]

For the distance of 1.5d = 1.5(72) = 108.0 in. from the end of the beam, reinforcement shall be placed to confine the prestressing steel in the bottom flange. The reinforcement shall not be less than #3 deformed bars, with spacing not exceeding 6.0 in. and shaped to enclose the strands.

<u>Deflection and Camber</u> [Upward deflection is negative]

Deflection Due to Prestressing Force at Transfer $n_{pd} := 4$

For fully bonded strands

$$P_{t1} := \left(n_p - n_{pd}\right) \cdot p = 777.61\ kip$$

$$\Delta_{pt1} := \frac{-P_{t1} \cdot e_c \cdot \left(L_{design}\right)^2}{8 \cdot E_{ci} \cdot I_g} = -1.92\ in$$

For partially debonded strands

$$P_{t2} := n_{pd} \cdot p = 172.8\ kip$$

$$\Delta_{pt2} := \frac{-P_{t2} \cdot e_c \cdot \left(L_{design} - 2 \cdot dl\right)^2}{8 \cdot E_{ci} \cdot I_g} = -0.19\ in$$

Deflection Due to Beam Self-Weight

$$\Delta_g = \frac{5 \cdot w_g \cdot \left(L_{girder}\right)^4}{384 \cdot E_{ci} \cdot I_g}$$

Deflection due to beam self-weight at transfer:

$$\Delta_{gt} := \frac{5 \cdot w_g \cdot \left(L_{design}\right)^4}{384 \cdot E_{ci} \cdot I_g} = 0.98\ in$$

Deflection due to beam self-weight used to compute deflection at erection:

$$\Delta_{ge} := \frac{5 \cdot w_g \cdot \left(L_{design}\right)^4}{384 \cdot E_{ci} \cdot I_g} = 0.98\ in$$

Deflection Due to Slab and Haunch Weights

$$\Delta_{gd} := \frac{5 \cdot w_d \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I_g} = 0.62\ in$$

Deflection Due to Rail/Barrier and Future Wearing Surface (Overlay)

$$\Delta_{bws} := \frac{5 \cdot \left(w_b + w_{ws}\right) \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I_{comp}} = 0.12\ in$$

$C_a := \Psi_b\left(t_d, t_i\right) = 0.96$ [From previous calculation of the creep of concrete]

Camber at transfer $\Delta_t := \Delta_{pt1} + \Delta_{pt2} + \Delta_{gt} = -1.14\ in$

Total deflection before deck placement

$$\Delta_{d1} := \left(\Delta_{pt1} + \Delta_{pt2} + \Delta_{gt}\right) \left(1 + C_a\right) = -2.22 \ in$$

Total deflection after deck placement

$$\Delta_{d2} := \left(\Delta_{pt1} + \Delta_{pt2} + \Delta_{gt}\right) \left(1 + C_a\right) + \Delta_{gd} = -1.6 \ in$$

Total deflection on composite section

$$\Delta := \left(\Delta_{pt1} + \Delta_{pt2} + \Delta_{gt}\right) \left(1 + C_a\right) + \Delta_{gd} + \Delta_{bws} = -1.48 \ in$$

The deflection criteria in S2.5.2.6.2 (live load deflection check) is considered optional. The bridge owner may select to invoke this criteria if desired.

Deflection Due to Live Load and Impact

Live load deflection limit (optional) = Span / 800 [AASHTO Art. 2.5.2.6.2]

$$\Delta_{Ll} := \frac{L_{design}}{800} = 1.35 \ in$$

If the owner invokes the optional live load deflection criteria specified in AASHTO Article 2.5.2.6.2, the deflection is the greater of:

- That resulting from the design truck alone, or[AASHTO Art. 3.6.1.3.2]
- That resulting from 25% of the design truck taken together with the design lane load.

Therefore, the distribution factor for deflection, DFD, is calculated as follows:

$$DFD := \frac{4}{N_{beams}} = 0.67$$

However, it is more conservative to use the distribution factor for moment

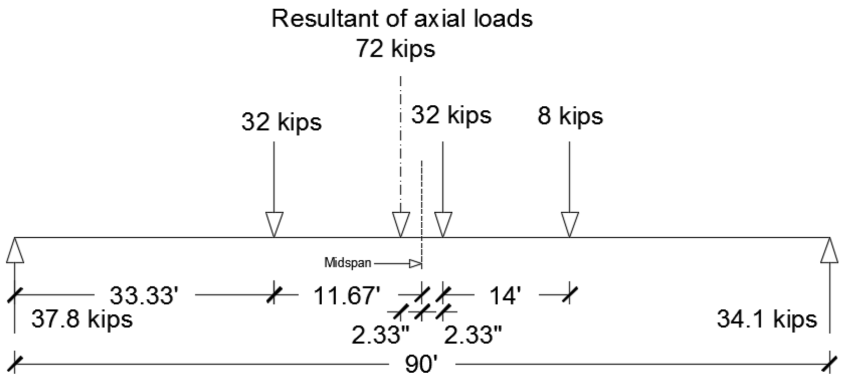
Deflection due to Lane Load:

Design lane load, $w_{LL} := 0.64 \cdot \frac{kip}{ft} \cdot DFD = 0.43 \ \frac{kip}{ft}$

$$\Delta_{LL} := \frac{5 \cdot w_{LL} \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I_{comp}} = 0.19 \ in$$

Deflection due to Design Truck Load and Impact:

To obtain maximum moment and deflection at midspan due to the truck load, set the spacing between the rear axles to 14 ft, and let the centerline of the beam coincide with the middle point of the distance between the inner 32-kip axle and the resultant of the truck load, as shown in figure below



The deflection at point x due to a point load at point a is given by the following equations:

$$\Delta = \frac{P \cdot b \cdot x}{6 \cdot E_c \cdot I_{comp} \cdot L} \left(L^2 - b^2 - x^2\right) \qquad \text{for } x<a$$

$$\Delta = \frac{P \cdot b}{6 \cdot E_c \cdot I_{comp} \cdot L} \left(\left(x-a\right)^3 \cdot \frac{L}{b} + \left(L^2 - b\right)^2 \cdot x - x^3\right) \qquad \text{for } x>a$$

where: P = point load

L = span length

x = location at which deflection is to determined

b = $L - a$

E_c = modulus of elasticity of precast beam at service loads

I_{comp} = gross moment of inertia of the composite section

$$E_c = \left(5.45 \cdot 10^3\right)\ ksi$$

Inputs

$excel_{\text{"D1"}} := 3.28084\ (L_{design})$ $excel_{\text{"B2"}} := (I_{comp}) \cdot (39.3701^4)$

$excel_{\text{"B1"}} := 1.45038 \cdot 10^{-7}\ E_c$

E_c	5448.345	L_{design}	90	
I_{comp}	597437.2			
Axle load	a	b	x	Δ
P(kips)	(ft)	(ft)	(ft)	in.
32	33.33	56.6	45	0.2300763
32	47.33	42.67	45	0.2569467
8	61.33	28.67	45	0.0502184

Outputs

$\delta_1 := excel_{\text{"E5"}} \cdot in$ $\delta_2 := excel_{\text{"E6"}} \cdot in$ $\delta_3 := excel_{\text{"E7"}} \cdot in$

The total deflection =

$$\Delta_{LT} := \delta_1 + \delta_2 + \delta_3 = 0.54\ in$$

Including impact and the distribution factor, the deflection at midspan due to the design truck load is:

$$\Delta_{LT} := \Delta_{LT} \cdot D_{M.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 0.4\ in$$

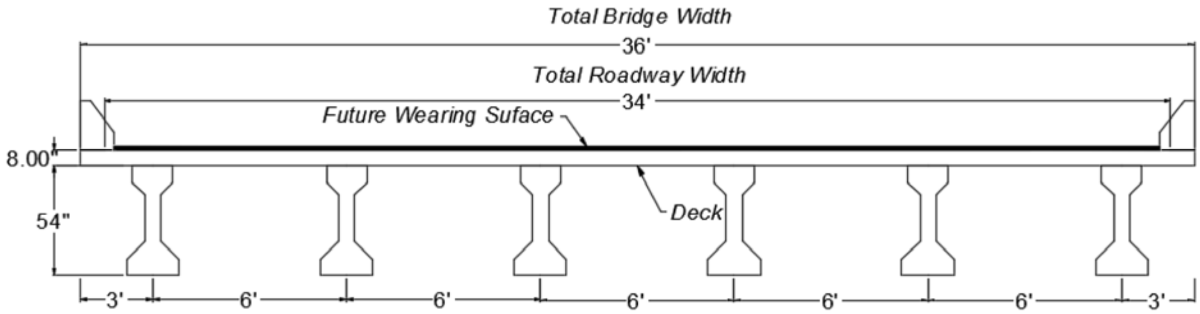
Therefore, the live load deflection is the greater of:

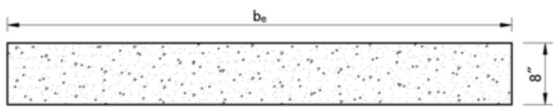
$$\Delta_L := \text{if } \Delta_{LT} > 0.25 \cdot \Delta_{LT} + \Delta_{LL}\ \ \| \Delta_{LT}\ \ \text{else}\ \ \| 0.25 \cdot \Delta_{LT} + \Delta_{LL} = 0.4\ in$$

if $\Delta_{Ll} > \Delta_L$
‖ "Deflection Limit Satisfied"
else
‖ "Deflection Limit Not Satisfied"
= "Deflection Limit Satisfied"

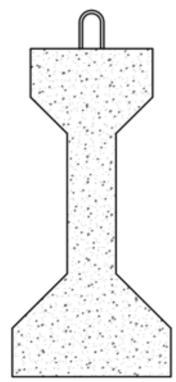
Example B-3: Design of a Decked AASHTO pretensioned girder with harped CFRP cables

The bridge considered for this design example has a span length of 90 ft. (center-to-center (c/c) pier distance), a total width of 36 ft., and total roadway width of 34 ft. The bridge superstructure consists of six AASHTO Type IV girders spaced 6 ft. center-to-center, designed to act compositely with an 8 in. thick cast-in-place (CIP) concrete deck. The wearing surface thickness is 2.0 in., which includes the thickness of any future wearing surface. T501 type rails are considered in the design. HL-93 is the design live load. A relative humidity (RH) of 60 percent is considered in the design. The design is performed for an interior girder based on service and strength limit states according to AASHTO-LRFD (2017) and AASHTO Guide Specifications (2018). They are referred in the following example as AASHTO and AASHTO-CFRP respectively.





8"-Deck



AASHTO Type-IV

Overall beam Length	$L_{span} := 91\ ft$
Design Span	$L_{design} := 90\ ft$
Girder spacing	$g_{spacing} := 6\ ft$
Number of beams	$N_{beams} := 6$
Total roadway width	$w_{roadway} := 36\ ft$

Cast in Place Deck:

Structural thickness, (effective)	$h_d := 7.5\ in$
Actual thickness, (for dead load calculation)	$t_s := 8\ in$

Concrete strength at 28 days, $f'_{cDeck} := 6.00\ ksi$

Thickness of asphalt-wearing surface (including any future wearing surface) $h_{ws} := 2\ in$

Unit weight of concrete $\gamma_c := 150\ pcf$

Haunch thickness $h_h := 0.5\ in$

Precast Girders: AASHTO Type IV

Concrete strength at release, $f'_{ci} := 6.00\ ksi$

Concrete strength at 28 days, $f'_c := 9.00\ ksi$

Unit weight of concrete $\gamma_c := 150\ pcf$

Prestressing CFRP

Diameter of one prestressing CFRP cable $d_b := 0.6\ in$

Area of one prestressing CFRP cable $A_{pf} := 0.18\ in^2$

Design tensile stress $f_{pu} := \dfrac{64.14\ kip}{A_{pf}} = 356.33\ ksi$

Modulus of elasticity (AASHTO-CFRP Art. 1.4.1.3) $E_f := 22500\ ksi$

Design tensile strain $\varepsilon_{pu} := \dfrac{f_{pu}}{E_f} = 0.02$

Stress limitation for prestressing CFRP
(AASHTO-CFRP Art. 1.9.1)
Before transfer $f_{pi} := 0.70 \cdot f_{pu} = 249.43\ ksi$

At service, after all losses $f_{pe} := 0.65 \cdot f_{pu} = 231.62\ ksi$

Nonprestressed Reinforcement:

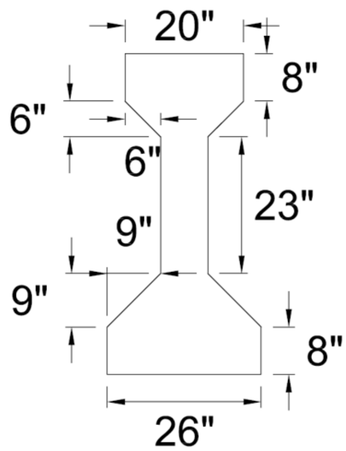
Yield strength $f_y := 60\ ksi$

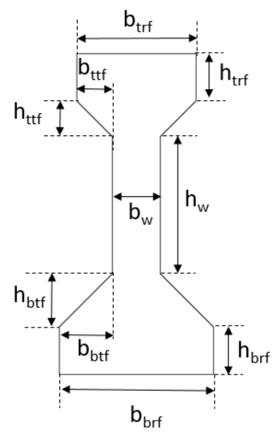
Modulus of elasticity (AASHTO Art. 5.4.3.2) $E_s := 29000\ ksi$

Unit weight of concrete $\gamma_{aws} := 150\ pcf$

T501 type barrier weight/side $\gamma_{bw} := 326\ plf$

Section Properties of AASHTO Type IV Girder:





Cross-section area of girder $A_g := 789\ in^2$

Moment of inertia of about the centroid of the noncomposite precast girder $I_g := 260730\ in^4$

Weight of the girder $w_g := 0.822\ \dfrac{kip}{ft}$

Height of girder $h_g := 54 \cdot in$

Width of bottom rectangular flange $b_{brf} := 26 \cdot in$

Height of bottom rectangular flange $h_{brf} := 8 \cdot in$

Width of bottom tapered flange $b_{btf} := 9 \cdot in$

Height of bottom tapered flange $h_{btf} := 9 \cdot in$

Width of web $b_w := 8 \cdot in$

Height of web $h_w := 23 \cdot in$

Width of top rectangular flange $b_{trf} := 20 \cdot in$

Height of top rectangular flange $h_{trf} := 8 \cdot in$

Width of top tapered flange $b_{ttf} := 6 \cdot in$

Height of top tapered flange $h_{ttf} := 6 \cdot in$

Distance from centroid to the extreme bottom fiber of the non-composite precast girder $y_{gbot} := 24.73\ in$

Distance from centroid to the extreme top fiber of the non-composite precast girder $y_{gtop} := h_g - y_{gbot} = 29.27\ in$

Section modulus referenced to the extreme bottom fiber of the non-composite precast girder $S_{gbot} := \dfrac{I_g}{y_{gbot}} = \left(1.05 \cdot 10^4\right)\ in^3$

Section modulus referenced to the extreme top fiber of the non-composite precast girder $S_{gtop} := \dfrac{I_g}{y_{gtop}} = \left(8.91 \cdot 10^3\right)\ in^3$

Effective flange width (AASHTO Art. 4.6.2.6.1)

$b_e := g_{spacing} = 72\ in$ Average spacing of adjacent girders

<u>Material Properties for Girder and Deck Concrete:</u>

Modulus of elasticity of concrete (AASHTO Art. 5.4.2.4) $E\left(f'_c\right) := 12 \cdot \left(\dfrac{\gamma_c}{pcf}\right)^{2.0} \left(\dfrac{f'_c}{psi}\right)^{0.33} \cdot psi$

At release $E_{ci} := E\left(f'_{ci}\right) = \left(4.77 \cdot 10^3\right)\ ksi$

At 28 days (Girder) $E_c := E\left(f'_c\right) = \left(5.45 \cdot 10^3\right)\ ksi$

At 28 days (Deck) $E_{cDeck} := E\left(f'_{cDeck}\right) = \left(4.77 \cdot 10^3\right)\ ksi$

Modulus of rupture of concrete (AASHTO Art 5.4.2.6) $f_{mr}\left(f'_c\right) := 0.24 \cdot \sqrt{\dfrac{f'_c}{ksi}} \cdot ksi$

At release $f_{ri} := f_{mr}\left(f'_{ci}\right) = 0.59\ ksi$

At 28 days (Girder) $f_r := f_{mr}\left(f'_c\right) = 0.72\ ksi$

At 28 days (Deck) $f_{rDeck} := f_{mr}\left(f'_{cDeck}\right) = 0.59\ ksi$

$n_1 := \dfrac{E_{cDeck}}{E_c} = 0.87$ [Modular ratio for transformed section]

Section properties composite deck:

Height of deck $h_d := 7.5 \cdot in$

Transformed width of deck $b_d := n_1 \cdot b_e = 62.98\ in$

Cross-section area of deck $A_d := h_d \cdot b_d = 472.37\ in^2$

Moment of inertia of deck about it centriod $I_d := \dfrac{b_d \cdot h_d^{\,3}}{12} = \left(2.21 \cdot 10^3\right)\ in^4$

Weight of the deck $w_d := \left(b_e \cdot t_s\right) \cdot \gamma_c = 0.6\ \dfrac{kip}{ft}$

Due to camber of the precast, prestressed beam, a minimum haunch thickness of 1/2 in. at midspan is considered in the structural properties of the composite section. Also, the width of haunch must be transformed.

Height of haunch $h_h := 0.5\ in$

Width of haunch $b_h := b_{trf} = 20\ in$

Transformed width of haunch $b_{th} := n_1 \cdot b_h = 17.5\ in$

Area of haunch $A_h := h_h \cdot b_{th} = 8.75\ in^2$

Moment of inertia of haunch about it centriod $I_h := \dfrac{b_{th} \cdot h_h^{\,3}}{12} = 0.18\ in^4$

Weight of the haunch $w_h := \left(b_h \cdot h_h\right) \cdot \gamma_c = 0.0104\ \dfrac{kip}{ft}$

Total height of composite beam $h_c := h_d + h_g + h_h = 62\ in$

Total area of composite beam $A_c := A_d + A_g + A_h = \left(1.27 \cdot 10^3\right)\ in^2$

Total weight of the composite beam $w_c := w_g + w_d + w_h = 1.43\ \dfrac{kip}{ft}$

Neutral axis location from bottom for composite beam

$$y_{cbot} := \frac{A_g \cdot y_{gbot} + A_d \cdot \left(h_c - \dfrac{h_d}{2}\right) + A_h \cdot \left(h_g + \dfrac{h_h}{2}\right)}{A_g + A_d + A_h} = 37.4\ in$$

Neutral axis location from top for composite beam $$y_{ctop} := (h_c) - y_{cbot} = 24.6\ in$$

Moment of inertia of composite beam

$$I_{comp} := I_g + I_d + I_h + A_g \cdot (y_{cbot} - y_{gbot})^2 + A_d \cdot \left(y_{ctop} - \frac{h_d}{2}\right)^2 + A_h \cdot \left(y_{ctop} - h_d - \frac{h_h}{2}\right)^2 = \left(5.97 \cdot 10^5\right)\ in^4$$

<u>Shear Force and Bending Moment due to Dead Loads</u>

Dead Loads:

Dead loads acting on the non-composite structure:

Self-weight of the girder $$w_g := 0.822\ \frac{kip}{ft}$$

Weight of cast-in-place deck on each interior girder $$w_d = 0.6\ \frac{kip}{ft}$$

Weight of haunch on each interior girder $$w_h = 0.01\ \frac{kip}{ft}$$

Total dead load on non-composite section $$w_T := w_g + w_d + w_h = 1.43\ \frac{kip}{ft}$$

Superimposed Dead loads:

Dead and live load on the deck must be distributed to the precast, prestressed beams. AASHTO provides factors for the distribution of live load into the beams. The same factors can be used for the dead loads if the following criteria is met [AASHTO Art. 4.6.2.2.1]:

- Width of deck is constant [OK]
- Number of beams is not less than four,

$$\left\| \begin{array}{l} \text{if } N_{beams} < 4 \\ \quad \left\| \text{"NOT OK"} \right. \\ \text{else} \\ \quad \left\| \text{"OK"} \right. \end{array} \right\| = \text{"OK"}$$

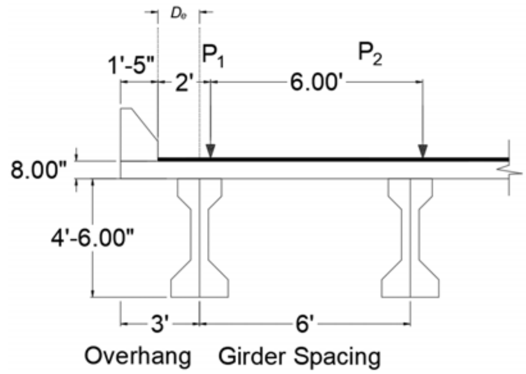
- Beams are parallel and have approximately the same stiffness
- The overhang minus the barrier width does not exceed 3.0 feet

$$D_e = Overhang - 17\ in$$

$Overhang := 3\ ft$

$D_e := Overhang - 17\ in = 19\ in$

$$\left\| \begin{array}{l} \text{if } D_e > 3\ ft \\ \quad \left\| \text{“NOT OK”} \right. \\ \text{else} \\ \quad \left\| \text{“OK”} \right. \end{array} \right\| = \text{“OK”}$$



- Curvature in plan is less than the limit specified in Article 4.6.1.2.4 [OK]

- Cross section of the bridge is consistent with one of the cross sections given in AASHTO Table 4.6.2.2.1-1 Precast concrete I sections are specified as Type k [OK]

Because all of the above criteria are satisfied, the barrier and wearing surface loads are equally distributed among the six girders.

Weight of T501 rails or barriers on each girder

$$w_b := 2 \cdot \left(\frac{\gamma_{bw}}{6}\right) = 0.11\ \frac{kip}{ft}$$

Weight of 2.0 in. wearing surface

$$w_{ws1} := \gamma_{aws} \cdot \left(h_{ws}\right) = 0.03\ \frac{kip}{ft^2}$$

This load is applied over the entire clear roadway width. Weight of wearing surface on each girder

$$w_{ws} := \frac{w_{ws1} \cdot w_{roadway}}{6} = 0.15\ \frac{kip}{ft}$$

Total superimposed dead load

$$w_{SD} := w_b + w_{ws} = 0.26\ \frac{kip}{ft}$$

Calculate modular ratio between girder and deck (AASHTO Eq. 4.6.2.2.1-2)

$$n := \frac{E_c}{E_{cDeck}} = 1.14$$

Calculate e_g, the distance between the center of gravity of the non-composite beam and the deck. Ignore the thickness of the haunch in determining e_g. It is also possible to ignore the integral wearing surface, i.e, use $h_d = 7.5\ in$. However, the difference in the distribution factor will be minimal.

$$e_g := y_{gtop} + \frac{h_d}{2} = 33.02\ in$$

Calculate K_g , the longitudinal stiffness parameter. (AASHTO Eq. 4.6.2.2.1-1)

$$K_g := n \cdot \left(I_g + A_g \cdot {e_g}^2\right) = \left(1.28 \cdot 10^6\right)\ in^4$$

Moment Distribution Factors

Interior Girder (AASHTO Art. 4.6.2.2.2 b)

Distribution factor for moment when one design lane is loaded

$$D_{M.Interior} = 0.06 + \left(\frac{S}{14}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot \left(\frac{K_g}{12 \cdot L \cdot {t_s}^3}\right)^{0.1}$$

Using variables defined in this example

$$\left(\frac{g_{spacing}}{14\ ft}\right)^{0.4} = 0.71 \qquad \left(\frac{g_{spacing}}{L_{design}}\right)^{0.3} = 0.44 \qquad \left(\frac{K_g}{L_{design} \cdot {h_d}^3}\right)^{0.1} = 1.11 \qquad h_d = 7.5\ in$$

$$D_{M.Interior1} := 0.06 + \left(\frac{g_{spacing}}{14\ ft}\right)^{0.4} \cdot \left(\frac{g_{spacing}}{L_{design}}\right)^{0.3} \cdot \left(\frac{K_g}{L_{design} \cdot {h_d}^3}\right)^{0.1} = 0.41$$

Distribution factor for moment when two design lanes are loaded

$$D_{M.Interior} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \cdot \left(\frac{S}{L}\right)^{0.2} \left(\frac{K_g}{12 \cdot L \cdot {t_s}^3}\right)^{0.1}$$

Using variables defined in this example

$$D_{M.Interior2} := 0.075 + \left(\frac{g_{spacing}}{9.5\ ft}\right)^{0.6} \cdot \left(\frac{g_{spacing}}{L_{design}}\right)^{0.2} \cdot \left(\frac{K_g}{L_{design} \cdot {h_d}^3}\right)^{0.1} = 0.56$$

The greater distribution factor is selected for moment design of the beams.

$$D_{M.Interior} := \max\left(D_{M.Interior1}, D_{M.Interior2}\right) = 0.56$$

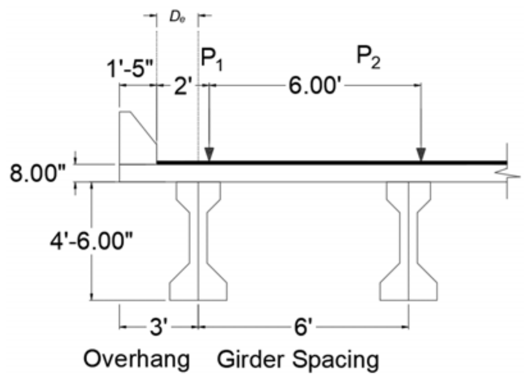
Check for range of applicability

$$D_{M.Interior} := \left\| \begin{array}{l} S \leftarrow (g_{spacing} \geq 3.5\ ft) \cdot (g_{spacing} \leq 16\ ft) \\ t_s \leftarrow (h_d \geq 4.5\ in) \cdot (h_d \leq 12\ in) \\ L \leftarrow (L_{design} \geq 20\ ft) \cdot (L_{design} \leq 240\ ft) \\ N_b \leftarrow (N_{beams} \geq 4) \\ K_g \leftarrow (K_g \geq 10000\ in^4) \cdot (K_g \leq 7000000\ in^4) \\ \text{if } (S \cdot t_s \cdot L \cdot N_b \cdot K_g) \\ \quad \| D_{M.Interior} \\ \text{else} \\ \quad \| \text{“Does not satisfy range of applicability”} \end{array} \right| = 0.56$$

Exterior Girder (AASHTO Art. 4.6.2.2.2 d)

$$P_1 = \frac{D_e + S - 2\ ft}{S}$$

$$P_2 = \frac{D_e + S - 8\ ft}{S}$$



$$D_e = Overhang - 17\ in$$

$$Overhang := 3\ ft$$

$$D_e := Overhang - 17\ in = 19\ in$$

$$S := g_{spacing} = 6\ ft$$

The distribution factor for one design lane loaded is based on the lever rule, which includes a 0.5 factor for converting the truck load to wheel loads and a 1.2 factor for multiple truck presence.

$$D_{M.Exterior1} := \mathbf{if}\left((2\ ft + 6\ ft) < (D_e + S), \frac{2 \cdot S + 2\ D_e - 8\ ft}{S} \cdot 0.5, \frac{S + D_e - 2\ ft}{S} \cdot 0.5\right) \cdot 1.2 = 0.56$$

The distribution factor for two design lane loaded

$$D_{M.Exterior} = D_{M.Interior} \cdot \left(0.77 + \frac{D_e}{9.1}\right)$$

Using variables defined in this example,

$$D_{M.Exterior2} := D_{M.Interior2} \cdot \left(0.77 + \frac{D_e}{9.1\ ft}\right) = 0.53$$

$$D_{M.Exterior} := \max\left(D_{M.Exterior1}, D_{M.Exterior2}\right) = 0.56$$

Range of applicability

$$D_{M.Exterior} := \left\| \begin{array}{l} d_e \leftarrow (D_e \geq -1\ ft) \cdot (D_e \leq 5.5\ ft) \\ \text{if } (d_e) \\ \quad \| D_{M.Exterior} \\ \text{else} \\ \quad \| \text{“Does not satisfy range of applicability”} \end{array} \right| = 0.56$$

For fatigue limit state

The commentary of article 3.4.1 in the AASHTO LRFD specification states that for fatigue limit state a single design truck should be used. However, live load distribution factors given in AASHTO Art 4.6.2.2 take into consideration the multiple presence factor, m. AASHTO Art 3.6.1.1.2 states that the multiple presence factor, m, for one design lane loaded is 1.2. Therefore, the distribution factor for one design lane loaded with the multiple presence factor removed should be used.

Distribution factor for fatigue limit state $D_{MF.Interior} := \dfrac{D_{M.Interior1}}{1.2} = 0.34$

<u>Shear Distribution Factors</u>

Interior Girder [AASHTO Art. 4.6.2.2.3 a]

Distribution factor for shear when one design lane is loaded

$$D_{S.Interior} = 0.36 + \left(\frac{S}{25}\right)$$

Using variables defined in this example

$$\left(\frac{g_{spacing}}{25\ ft}\right) = 0.24$$

$$D_{S.Interior1} := 0.36 + \left(\frac{g_{spacing}}{25\ ft}\right) = 0.6$$

Distribution factor for shear when two design lanes are loaded

$$D_{S.Interior} = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^{2.0}$$

Using variables defined in this example

$$D_{S.Interior2} := 0.075 + \left(\frac{g_{spacing}}{12\ ft}\right) - \left(\frac{g_{spacing}}{35\ ft}\right)^2 = 0.55$$

The greater distribution factor is selected for moment design of the beams.

$$D_{S.Interior} := \max\left(D_{S.Interior1}, D_{S.Interior2}\right) = 0.6$$

Check for range of applicability

$$D_{S.Interior} := \left\| \begin{array}{l} S \leftarrow \left(g_{spacing} \geq 3.5\ ft\right) \cdot \left(g_{spacing} \leq 16\ ft\right) \\ t_s \leftarrow \left(h_d \geq 4.5\ in\right) \cdot \left(h_d \leq 12\ in\right) \\ L \leftarrow \left(L_{design} \geq 20\ ft\right) \cdot \left(L_{design} \leq 240\ ft\right) \\ N_b \leftarrow \left(N_{beams} \geq 4\right) \\ K_g \leftarrow \left(K_g \geq 10000\ in^4\right) \cdot \left(K_g \leq 7000000\ in^4\right) \\ \text{if } \left(S \cdot t_s \cdot L \cdot N_b \cdot K_g\right) \\ \quad \left\| D_{S.Interior} \right. \\ \text{else} \\ \quad \left\| \text{“Does not satisfy range of applicability”} \right. \end{array} \right. = 0.6$$

The AASHTO Specifications specify the dynamic load effects as a percentage of the static live load effects. AASHTO Table 3.6.2.1-1 specifies the dynamic allowance to be taken as 33 percent of the static load effects for all limit states, except the fatigue limit state, and 15 percent for the fatigue limit state. The factor to be applied to the static load shall be taken as:

$$(1 + IM/100)$$

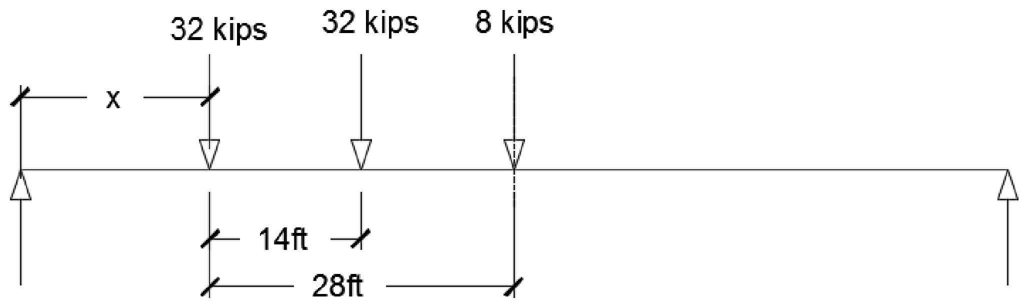
where:

IM = Dynamic load allowance, applied to truck load or tandem load only

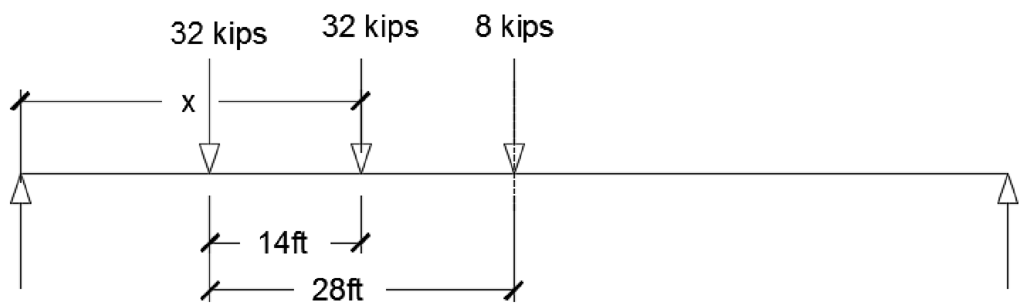
= 33 for all limit states except the fatigue limit state

= 15 for fatigue limit state

The maximum shear forces and bending moments due to HS 20-44 truck loading for all limit states is calculated using the influence line approach. The live load moments and shear forces for the simple span is computed by positioning the axle load of HS-20 truck in following locations



Case I



Case II

Case I: HS-20 truck moment and shear

$P_1 := 32\ kip \qquad P_2 := 32\ kip \qquad P_3 := 8\ kip \qquad x := 5\ ft$

$$M_{truck1}(x) := P_1 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot x + P_2 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}} \cdot x + P_3 \cdot \frac{(L_{design} - x - 28\ ft)}{L_{design}} \cdot x$$

$$V_{truck1}(x) := P_1 \cdot \frac{(L_{design} - x)}{L_{design}} + P_2 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}} + P_3 \cdot \frac{(L_{design} - x - 28\ ft)}{L_{design}}$$

Case II: HS-20 truck moment and shear

$$M_{truck2}(x) := P_1 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot (x - 14\ ft) + P_2 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot x + P_3 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}} \cdot x$$

$$V_{truck2}(x) := P_1 \cdot \frac{-(x - 14\ ft)}{L_{design}} + P_2 \cdot \frac{(L_{design} - x)}{L_{design}} + P_3 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}}$$

$M_{truck1}\left(\textbf{maximize}\left(M_{truck1}, x\right)\right) = \left(1.3 \cdot 10^{3}\right)\ ft \cdot kip$

$M_{truck2}\left(\textbf{maximize}\left(M_{truck2}, x\right)\right) = \left(1.34 \cdot 10^{3}\right)\ ft \cdot kip$

Maximum bending moment due to HS 20-44 truck load

$M := \left(1.344 \cdot 10^{3}\right)\ ft \cdot kip$

The calculation of shear force is carried out later for the critical shear section.
Distributed bending moment due to truck load including dynamic load allowance (M_{LT}) is calculated as follows:

M_{LT} = (Moment per lane due to truck load)(*DFM*)(1+*IM*/100)

$IM := 33$

$D_{M.Interior} = 0.56$

$$M_{LT} := M \cdot D_{M.Interior} \cdot \left(1 + \frac{IM}{100}\right) = \left(1.01 \cdot 10^{3}\right)\ ft \cdot kip$$

The maximum bending moments (M_L) due to a uniformly distributed lane load of 0.64 klf are calculated using the following formulas given by the *PCI Design Manual* (PCI 2017).

Maximum bending moment, $M_x = 0.5(0.64)(x)(L - x)$

where:
x = Distance from centerline of bearing to section at which the bending moment or shear force is calculated, ft.
L = Design span length

At the section of maximum truck load $\textbf{maximize}\left(M_{truck2}, x\right) = 47.33\ ft$

$x := 47.33\ ft$

$$M_L := 0.5 \cdot 0.64\ \frac{kip}{ft} \cdot (x)\ \left(L_{design} - x\right) = 646.26\ ft \cdot kip$$

$M_{LL} := D_{M.Interior} \cdot M_L = 364.96\ ft \cdot kip$

For fatigue limit state:
Therefore, the bending moment of the fatigue truck load is:
Mf = (bending moment per lane)(DFM)(1 + IM)

$$M_f := M \cdot D_{MF.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 611.73\ ft \cdot kip$$

Shear forces and bending moments for the girder due to dead loads, superimposed dead loads at every tenth of the design span, and at critical sections (hold-down point or harp point and critical section for shear) are provided in this section. The bending moment (*M*) and shear force (*V*) due to uniform dead loads and uniform superimposed dead loads at any section at a distance *x* from the centerline of bearing are calculated using the following formulas, where the uniform load is denoted as *w*.

$$M = 0.5w\,x\,(L - x)$$
$$V = w(0.5L - x)$$

The critical section for shear is located at a distance $h_c/2$ from the face of the support. However, as the support dimensions are not specified in this project, the critical section is measured from the centerline of bearing. This yields a conservative estimate of the design shear force.

Inputs

$excel_{\text{"F1"}} := 3.28084\ (L_{design})$ $excel_{\text{"A13"}} := 0.5 \cdot 3.28084\ (L_{design})$

$excel_{\text{"B1"}} := 0.0000685217\ (w_g)$

$excel_{\text{"D1"}} := 0.0000685217\ (w_d + w_h)$

$excel_{\text{"B2"}} := 0.0000685217\ (w_b)$

w_g	0.822		0.61042		90.000	L			90.000
w_{SD}	0.109								

Distance (x)	Section (x/L)	Dead Load							
		Girder Weight		Slab Weight		Barrier weight		Total Dead Load	
		Shear	Moment	Shear	Moment	Shear	Moment	Shear	Moment
ft.		k	k-ft	k	k-ft	k	k-ft	k	k-ft
0	0.000	36.990	0.00	27.469	0.00	4.890	0.00	69.35	0.00
4.654	0.052	33.164	163.25	24.628	121.23	4.384	21.58	62.18	306.06
10.858	0.121	28.065	353.18	20.841	262.27	3.710	46.69	52.62	662.14
21.717	0.241	19.139	609.47	14.212	452.59	2.530	80.57	35.88	1142.64
32.575	0.362	10.213	768.82	7.584	570.93	1.350	101.64	19.15	1441.39
43.433	0.483	1.288	831.26	0.957	617.30	0.170	109.89	2.41	1558.45
45.000	0.500	0.000	832.27	0.000	618.05	0.000	110.02	0.00	1560.35

Outputs

$V_g := excel_{\text{"C8"}} \cdot kip$ $V_s := excel_{\text{"E8"}} \cdot kip$ $V_b := excel_{\text{"G8"}} \cdot kip$

$M_{gv} := excel_{\text{"D8"}} \cdot ft \cdot kip$ $M_{sv} := excel_{\text{"F8"}} \cdot ft \cdot kip$ $M_{bv} := excel_{\text{"G8"}} \cdot ft \cdot kip$

The AASHTO design live load is designated as HL-93, which consists of a combination of:

- Design truck with dynamic allowance or design tandem with dynamic allowance, whichever produces greater moments and shears, and
- Design lane load without dynamic allowance. [AASHTO Art. 3.6.1.2]

The design truck is designated as HS 20-44 consisting of an 8 kip front axle and two 32 kip rear axles. [AASHTO Art. 3.6.1.2.2]
The design tandem consists of a pair of 25-kip axles spaced 4 ft. apart. However, for spans longer than 40 ft. the tandem loading does not govern, thus only the truck load is investigated in this example. [AASHTO Art. 3.6.1.2.3]
The lane load consists of a load of 0.64 klf uniformly distributed in the longitudinal direction. [AASHTO Art. 3.6.1.2.4]

This design example considers only the dead and vehicular live loads. The wind load and the extreme event loads, including earthquake and vehicle collision loads, are not included in the design. Various limit states and load combinations provided by AASHTO Art. 3.4.1 are investigated, and the following limit states are found to be applicable in present case:

Service I: This limit state is used for normal operational use of a bridge. This limit state provides the general load combination for service limit state stress checks and applies to all conditions except Service III limit state. For prestressed concrete components, this load combination is used to check for compressive stresses. The load combination is presented as follows[AASHTO Table 3.4.1-1]:
$Q = 1.00\ (DC + DW) + 1.00(LL + IM)$

Service III: This limit state is a special load combination for service limit state stress checks that applies only to tension in prestressed concrete structures to control cracks. The load combination for this limit state is presented as follows [AASHTO Table 3.4.1-1]:
$Q = 1.00(DC + DW) + 0.80(LL + IM)$
(Subsequent revisions to the AASHTO specification have revise this load combination)

Strength I: This limit state is the general load combination for strength limit state design relating to the normal vehicular use of the bridge without wind. The load combination is presented as follows [AASHTO Table 3.4.1-1 and 2]:
$Q = \gamma P(DC) + \gamma P(DW) + 1.75(LL + IM)$

Type of Load	Load Factor, γ_P	
	Maximum	Minimum
DC: Structural components and non-structural attachments	1.25	0.90
DW: Wearing surface and utilities	1.50	0.65

The maximum and minimum load combinations for the Strength I limit state are presented as follows:
Maximum $Q = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)$
Minimum $Q = 0.90(DC) + 0.65(DW) + 1.75(LL + IM)$

Estimation of Required Prestress

The required number of strands is usually governed by concrete tensile stress at the bottom fiber of the girder at the midspan section. The load combination for the Service III limit state is used to evaluate the bottom fiber stresses at the midspan section. The calculation for compressive stress in the top fiber of the girder at midspan section under service loads is also shown in the following section. The compressive stress is evaluated using the load combination for the Service I limit state.

Service Load Stresses at Midspan

Bottom tensile stress due to applied dead and live loads using load combination Service III is:

$$f_b = \frac{M_g + M_d}{S_b} + \frac{M_b + M_{ws} + (0.8) \cdot (M_{LT} + M_{LL})}{S_{bc}}$$

f_b=Concrete stress at the bottom fiber of the girder, ksi

M_g=unfactored bending moment due to beam self-weight, k-ft

$$M_g := \frac{w_g \cdot L_{design}^{\ 2}}{8} = 832.28\ ft \cdot kip$$

M_d=unfactored bending moment due to deck self-weight and haunch, k-ft

$$M_d := \frac{\left(w_d + w_h\right) \cdot L_{design}^{\ 2}}{8} = 618.05\ ft \cdot kip$$

M_b=unfactored bending moment due to barrier self-weight, k-ft

$$M_b := \frac{w_b \cdot L_{design}^{\ 2}}{8} = 110.03\ ft \cdot kip$$

M_{ws}=unfactored bending moment due to future wearing , k-ft

$$M_{ws} := \frac{w_{ws} \cdot L_{design}^{\ 2}}{8} = 151.88\ ft \cdot kip$$

M_{LT}=unfactored bending moment due to truck load (kip-ft) $M_{LT} = \left(1.01 \cdot 10^3\right)\ ft \cdot kip$

M_{LL}=unfactored bending moment due to truck load (kip-ft) $M_{LL} = 364.96\ ft \cdot kip$

S_{bc}=composite section modulus for extreme bottom fiber of precast beam (in^3)

Using the variables used in this example

$$S_{cbot} := \frac{I_{comp}}{y_{cbot}} = \left(1.6 \cdot 10^4\right)\ in^3$$

$$f_b := \frac{M_g + M_d}{S_{gbot}} + \frac{M_b + M_{ws} + (0.8) \cdot \left(M_{LT} + M_{LL}\right)}{S_{cbot}} = 2.67\ ksi$$

Stress Limits for Concrete

The tensile stress limit at service load= $0.19 \cdot \sqrt{f'_c}$ [AASHTO Table 5.9.2.3.2b-1]

where: f'c = specified 28-day concrete strength of beam, ksi

Concrete tensile stress limit = $f_{tl} := 0.19 \cdot \sqrt{\frac{f'_c}{ksi}}\ ksi = 0.57\ ksi$

Required Number of Strands

The required pre-compressive stress at the bottom fiber of the beam is the difference the between bottom tensile stress due to the applied loads and the concrete tensile stress limits:

Required pre-compressive stress at bottom fiber, $f_{pb} := f_b - f_{tl} = 2.1\ ksi$

Assume the distance between the center of gravity of the bottom strands and the bottom fiber of the beam:

$e_c := y_{gbot} = 24.73\ in$

If P_{pe} is the total prestressing force, the stress at the bottom fiber due to prestress is:

$$f_{pb} = \frac{P_{pe}}{A} + \frac{P_{pe} \cdot e_c}{S_b}$$

Using the variables in this example

$$P_{pe} := \frac{f_{pb}}{\left(\frac{1}{A_g} + \frac{e_c}{S_{gbot}}\right)} = 582.18\ kip$$

Final prestress force per strand, P_{pf} = (area of prestressing CFRP) (f_{pi}) (1 − $ploss$, %)

$ploss := 20\%$

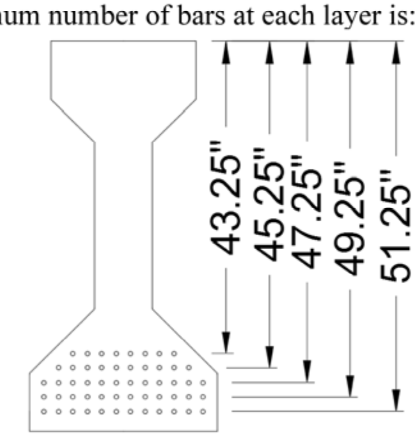
$P_{pf} := A_{pf} \cdot f_{pi} \cdot (1 - ploss) = 35.92\ kip$

The number of prestressing CFRP is equal to $n_p := \frac{P_{pe}}{P_{pf}} = 16.21$

$n_p := \text{round}\left(\frac{(n_p)}{2}\right) \cdot 2 + 4 = 20$

Change the number of bars based on the value of n_p.
If no bars is needed at certain layer input 0.
The maximum number of bars at each layer is:

$n_{b1} = 12$
$n_{b2} = 12$
$n_{b3} = 12$
$n_{b4} = 10$
$n_{b5} = 8$



$n_{b1} := 12$ $d_{p1} := 51.25\ in$

$n_{b2} := 8$ $d_{p2} := 49.25\ in$

$n_{b3} := 0$ $d_{p3} := 47.25\ in$

$n_{b4} := 0$ $d_{p4} := 45.25\ in$

$n_{b5} := 0$ $d_{p5} := 43.25\ in$

The center of gravity of the strands, c.g.s. = $\dfrac{\sum n_i y_i}{N}$

where: n_i = number of strands in row i

y_i = distance to center of row i from bottom of beam section

N = total number of strands

$$x_p := h_g - \frac{n_{b1} \cdot d_{p1} + n_{b2} \cdot d_{p2} + n_{b3} \cdot d_{p3} + n_{b4} \cdot d_{p4} + n_{b5} \cdot d_{p5}}{n_{b1} + n_{b2} + n_{b3} + n_{b4} + n_{b5}} = 3.55 \ in$$

$$e_c := y_{gbot} - x_p = 21.18 \ in$$

Using the variables in this example

$$P_{pe} := \frac{f_{pb}}{\left(\frac{1}{A_g} + \frac{e_c}{S_{gbot}}\right)} = 642.01 \ kip$$

Final prestress force per strand, P_{pf} = (area of prestressing CFRP) (f_{pi}) (1 − $ploss$, %)

$$ploss := 20\%$$

$$P_{pf} := A_{pf} \cdot f_{pi} \cdot (1 - ploss) = 35.92 \ kip$$

The number of prestressing CFRP is equal to $n_p := \dfrac{P_{pe}}{P_{pf}} = 17.87$

$$n_p := \text{round}\left(\frac{\langle n_p \rangle}{2}\right) \cdot 2 + 4 = 22$$

$n_{b1} := 12$ $\quad d_{p1} := 51.25 \ in$

$n_{b2} := 10$ $\quad d_{p2} := 49.25 \ in$

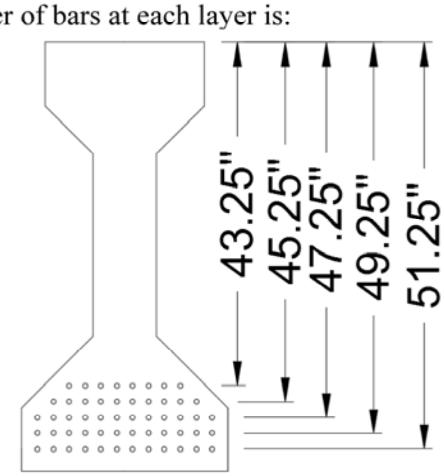
$n_{b3} := 0$ $\quad d_{p3} := 47.25 \ in$

$n_{b4} := 0$ $\quad d_{p4} := 45.25 \ in$

$n_{b5} := 0$ $\quad d_{p5} := 43.25 \ in$

Change the number of bars based on the value of n_p. If no bars is needed at certain layer input 0.
The maximum number of bars at each layer is:

$n_{b1} = 12$
$n_{b2} = 12$
$n_{b3} = 12$
$n_{b4} = 10$
$n_{b5} = 8$



$$x_p := h_g - \frac{n_{b1} \cdot d_{p1} + n_{b2} \cdot d_{p2} + n_{b3} \cdot d_{p3} + n_{b4} \cdot d_{p4} + n_{b5} \cdot d_{p5}}{n_{b1} + n_{b2} + n_{b3} + n_{b4} + n_{b5}} = 3.66\ in$$

$$e_c := y_{gbot} - x_p = 21.07\ in$$

Using the variables in this example

$$P_{pe} := \frac{f_{pb}}{\left(\frac{1}{A_g} + \frac{e_c}{S_{gbot}}\right)} = 644.05\ kip$$

Final prestress force per strand, P_{pf} = (area of prestressing CFRP) (f_{pi}) (1 − $ploss$, %)

$$ploss := 20\%$$

$$P_{pf} := A_{pf} \cdot f_{pi} \cdot (1 - ploss) = 35.92\ kip$$

The number of prestressing CFRP is equal to $n_p := \frac{P_{pe}}{P_{pf}} = 17.93$

$$n_p := \text{round}\left(\frac{\langle n_p \rangle}{2}\right) \cdot 2 + 4 = 22$$

Strand Pattern

$n_p = 22$ [To satisfy strength limit state only]

midspan center of gravity of prestressing CFRP $y_{bs} := x_p = 3.66\ in$

midspan prestressing CFRP eccentricity $e_c := y_{gbot} - y_{bs} = 21.07\ in$

Prestress Losses [AASHTO Art. 5.9.3]

Total prestress loss

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$$

Δf_{pES}=sum of all losses or gains due to elastic shortening or extension at time of application of prestress and/or external loads (ksi)

Δf_{pLT}=losses due to long-term shrinkage and creep of concrete, and relaxation of the prestressing CFRP (ksi)

Elastic Shortening

When the prestressing force is transferred from the prestressing strands to the concrete member, the force causes elastic shortening of the member as it cambers upward. This results in a loss of the initial prestress of the strands. However, some of that loss is gained back due to the self-weight of the member which creates tension in the strands.

$$\Delta f_{pES} = \frac{E_f}{E_{ct}} \cdot f_{cgp}$$ [AASHTO-CFRP Eq. 1.9.2.2.3a-1]

Where E_f=modulus of elasticity of prestressing CFRP (ksi)

E_{ct}=modulus of elasticity of the concrete at transfer or time of load application (ksi)=E_{ci}

f_{cgp}=the concrete stress at the center of gravity of CFRP due to the prestressing force immediately after transfer and the self-weight of the member at sections of maximum moment (ksi)

AASHTO Article C5.9.3.2.3a states that to calculate the prestress after transfer, an initial estimate of prestress loss is assumed and iterated until acceptable accuracy is achieved. In this example, an initial estimate of 10% is assumed.

$$eloss := 10\%$$

Force per strand at transfer

$$f_{cgp} = \frac{P_i}{A_g} + \frac{P_i \cdot {e_c}^2}{I_g} - \frac{M_G \cdot e_c}{I_g}$$

Where, P_i=total prestressing force at release=$n_p * p$

e_c=eccentricity of strands measured from the center of gravity of the precast beam at midspan

M_G=moment due to beam self-weight at midspan (should be calculated using the overall beam length)

$$M_G := \frac{w_g \cdot \left(L_{span}\right)^2}{8} = 850.87\ ft \cdot kip$$

Guess Values

$$eloss := 10\%$$

Constraints

$$eloss = \frac{E_f}{f_{pi} \cdot E_{ci}} \cdot \left(\frac{n_p \cdot A_{pf} \cdot f_{pi} \cdot (1 - eloss)}{A_g} + \frac{\left(n_p \cdot A_{pf} \cdot f_{pi} \cdot (1 - eloss)\right) \cdot {e_c}^2}{I_g} - \frac{M_G \cdot e_c}{I_g}\right)$$

Solver

$$eloss := \mathbf{find}\left(\mathbf{eloss}\right) = 0.04$$

Therefore, the loss due to elastic shortening= $eloss = 0.04$

The force per strand at transfer= $p := A_{pf} \cdot f_{pi} \cdot (1 - eloss) = 43.2\ kip$

The concrete stress due to prestress= $f_{cgp} := \frac{n_p \cdot p}{A_g} + \frac{n_p \cdot p \cdot {e_c}^2}{I_g} - \frac{M_G \cdot e_c}{I_g} = 2\ ksi$

The prestress loss due to elastic shortening= $\Delta f_{pES} := \frac{E_f}{E_{ci}} \cdot f_{cgp} = 9.43\ ksi$

Total prestressing force at release $P_i := n_p \cdot p = 950.41\ kip$

Long Term Losses

$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}) + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})$

Δf_{pSR}=prestress loss due to shrinkage of girder concrete between time of transfer and deck placement (ksi)

Δf_{pCR}=prestress loss due to creep of girder concrete between time of transfer and deck placement (ksi)

Δf_{pR1}=prestress loss due to relaxation of prestressing strands between time of transfer and deck placement ksi)

Δf_{pSD}=prestress loss due to shrinkage of girder concrete between time of deck placement and final time (ksi)

Δf_{pCD}=prestress loss due to creep of girder concrete between time of deck placement and final time (ksi)

Δf_{pR2}=prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time (ksi)

Δf_{pSS}=prestress gain due to shrinkage of deck in composite section (ksi)

$(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})$ =sum of time-dependent prestress losses between time of transfer and deck placement (ksi)

$(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})$ =sum of time-dependent prestress losses after deck placement (ksi)

Prestress Losses: Time of Transfer to Time of Deck Placement

Shrinkage of Girder Concrete

$\Delta f_{pSR} = \varepsilon_{bid} \cdot E_{pc} \cdot K_{id}$ [AASHTO Eq. 5.9.3.4.2a-1]

where, ε_{bid}=shrinkage strain of girder between the time of transfer and deck placement

$= k_s \cdot k_{hs} \cdot k_f \cdot k_{td} \cdot 0.48 \cdot 10^{-3}$ [AASHTO Eq. 5.4.2.3.3-1]

and, k_s=factor for the effect of volume to surface ratio of the component

k_s=1.45-0.13(V/S) [AASHTO Eq. 5.4.2.3.2-2]

where (V/S)=volume to surface ratio=(Area/Perimeter)

Perimeter

$$P_g := 2\left(\frac{b_{brf}}{2} + h_{brf} + \sqrt{b_{btf}^{\ 2} + h_{btf}^{\ 2}} + h_w + \sqrt{b_{ttf}^{\ 2} + h_{ttf}^{\ 2}} + h_{trf} + \frac{b_{trf}}{2}\right) = 166.43\ in$$

$$k_s := 1.45 - 0.13 \cdot \frac{1}{in}\left(\frac{A_g}{P_g}\right) = 0.83 \qquad k_s := \begin{Vmatrix} \text{if } k_s \le 1 \\ \quad \| 1 \\ \text{else} \\ \quad \| k_s \end{Vmatrix} = 1$$

k_{hs}=humidity factor for shrinkage=2.00-0.014H [AASHTO Eq. 5.4.2.3.3-2]

$H := 70$

$k_{hs} := 2.00 - 0.014 \cdot H = 1.02$

k_f=factor for the effect of concrete strength [AASHTO Eq. 5.4.2.3.2-4]

$$= \frac{5}{1 + f'_{ci}}$$

$$k_f := \frac{5}{1 + \frac{f'_{ci}}{ksi}} = 0.71$$

k_{td}=time developement factor

$$= \frac{t}{61 - 4 \cdot f'_{ci} + t}$$

[AASHTO Eq. 5.4.2.3.2-5]

$t_d := 90$ $\qquad$ $t_i := 1$

$$k_{td}(t, t_i) := \frac{t - t_i}{61 - 4 \cdot \frac{f'_{ci}}{ksi} + t - t_i}$$

$k_{td}(t_d, t_i) = 0.71$

$$\varepsilon_{bid} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}(t_d, t_i) \cdot 0.48 \cdot 10^{-3} = 2.47 \cdot 10^{-4}$$

K_{id}=transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between transfer and deck placement

$$=\frac{1}{1+\frac{E_{pc}}{E_{ci}}\cdot\frac{A_{psc}}{A_g}\cdot\left(1+\frac{A_g\cdot e_{pg}{}^2}{I_{comp}}\right)\left(1+0.7\ \Psi_b\left(t_f,t_i\right)\right)}$$ [AASHTO Eq. 5.9.3.4.2a-2]

where, $\Psi_b\left(t_f,t_i\right)=1.9\cdot k_s\cdot k_{hc}\cdot k_f\cdot k_{td}\cdot t_i{}^{-0.118}$ [AASHTO Eq. 5.4.2.3.2-1]

k_{hc}=humidity factor for creep=1.56-0.008H [AASHTO Eq. 5.4.2.3.2-3]

$$e_{pg}:=e_c=21.07\ in$$

$$k_{hc}:=1.56-0.008\ H=1$$

$$t_f:=20000 \qquad t_i:=1$$

$$k_{td}\left(t_f,t_i\right)=1$$

$$\Psi_b\left(t,t_i\right):=1.9\cdot k_s\cdot k_{hc}\cdot k_f\cdot k_{td}\left(t,t_i\right)\cdot\left(t_i\right)^{-0.118}$$

$$\Psi_b\left(t_f,t_i\right)=1.35$$

$$K_{id}:=\frac{1}{1+\frac{E_f}{E_{ci}}\cdot\frac{A_{pf}}{A_g}\cdot\left(1+\frac{A_g\cdot e_c{}^2}{I_g}\right)\left(1+0.7\cdot\Psi_b\left(t_f,t_i\right)\right)}=1$$

$$\Delta f_{pSR}:=\varepsilon_{bid}\cdot E_f\cdot K_{id}=5.53\ ksi$$

Creep of Girder Concrete

$$\Delta f_{pCR}=\frac{E_{pc}}{E_{ciACI}}\cdot f_{cgp}\cdot\Psi_b\left(t_d,t_i\right)\cdot K_{id}$$ [AASHTO Eq. 5.9.3.4.2b-1]

Where, $\Psi_b\left(t_d,t_i\right)$=girder creep coefficient at time of deck placement due to loading introduced at transfer

$$=1.9\cdot k_s\cdot k_{hc}\cdot k_f\cdot k_{td}\cdot\left(t_i\right)^{-0.118}$$

$$\Delta f_{pCR}:=\frac{E_f}{E_{ci}}\cdot f_{cgp}\cdot\Psi_b\left(t_d,t_i\right)\cdot K_{id}=9\ ksi$$

Relaxation of Prestressing Strands

$$\Delta f_{pR1} = \left(0.019 \cdot \left(\frac{f_{pt}}{f_{pu}}\right) - 0.0066\right) \; log\left(t \cdot 24\right) \cdot f_{pu}$$ [AASHTO-CFRP Eq. 1.9.2.5.2-2]

Where, f_{pt}=stress in prestressing strands immediately after transfer,

$$f_{pt} := f_{pi} - \Delta f_{pES} = 240 \; ksi$$

t=time between strand prestressing and deck placement (days)

$$t := t_i + t_d = 91$$

Therefore,

$$\Delta f_{pR1} := \left(0.019 \cdot \left(\frac{f_{pt}}{f_{pu}}\right) - 0.0066\right) \; \log\left(t \cdot 24\right) \cdot f_{pu} = 7.37 \; ksi$$

Prestress Losses: Time of Deck Placement to Final Time

Shrinkage of Girder Concrete

$$\Delta f_{pSD} = \varepsilon_{bdf} \cdot E_f \cdot K_{df}$$

where ε_{bdf}=shrinkage strain of girder between the time of deck placement and final time $=\varepsilon_{bif}$-ε_{bid}

$$\varepsilon_{bif} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}\left(t_f, t_i\right) \cdot 0.48 \cdot 10^{-3} = 3.49 \cdot 10^{-4}$$

$$\varepsilon_{bdf} := \varepsilon_{bif} - \varepsilon_{bid} = 1.02 \cdot 10^{-4}$$

K_{df}=transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between deck placement and final time

$$= \frac{1}{1 + \frac{E_{pc}}{E_{ci}} \cdot \frac{A_{psc}}{A_c} \cdot \left(1 + \frac{A_c \cdot {e_{pc}}^2}{I_c}\right) \; \left(1 + 0.7 \; \Psi_b\left(t_f, t_i\right)\right)}$$ [AASHTO Eq. 5.9.3.4.3a-2]

where, e_{pc}=eccentricity of prestressing force with respect to centroid of composite section (in); positive in common construction where force is below centroid $= y_{cbot} - y_{bs}$

A_c=area of section calculated using the gross composite concrete section properties of the girder and the deck, and the deck-to-girder modular ratio

I_c=moment of inertia calculated using gross composite concrete properties of the girder and the deck, and the deck-to-girder modular ratio at service=I_{comp}

$$e_{pc} := y_{cbot} - y_{bs} = 33.74\ in$$

$$\Psi_b\left(t_f, t_i\right) = 1.35$$

$$K_{df} := \frac{1}{1 + \frac{E_f}{E_{ci}} \cdot \frac{A_{pf}}{A_c} \cdot \left(1 + \frac{A_c \cdot {e_{pc}}^2}{I_{comp}}\right) \left(1 + 0.7 \cdot \Psi_b\left(t_f, t_i\right)\right)} = 1$$

$$\Delta f_{pSD} := \varepsilon_{bdf} \cdot E_f \cdot K_{df} = 2.29\ ksi$$

Creep of Girder Concrete

$$\Delta f_{pCD} = \frac{E_f}{E_{ci}} \cdot f_{cgp} \cdot \left(\Psi_b\left(t_f, t_i\right) - \Psi_b\left(t_d, t_i\right)\right) \cdot K_{df} + \frac{E_f}{E_c} \cdot \Delta f_{cd} \cdot \Psi_b\left(t_f, t_d\right) \cdot K_{df}$$

[AASHTO Eq. 5.9.3.4.3b-1]

Where, $\Psi_b\left(t_f, t_d\right)$ =girder creep coefficient at final time due to loading at deck placement

$$= 1.9 \cdot k_s \cdot k_{hc} \cdot k_f \cdot k_{td}\left(t_f, t_d\right) \cdot \left(t_i\right)^{-0.118}$$

$$\Psi_b\left(t_f, t_d\right) = 0.8$$

Δf_{cd} =change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement, combined with deck weight on the non-composite transformed section, and superimposed loads on the composite transformed section (ksi)

$$= -\left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) \cdot \frac{A_{pf}}{A_g} \cdot \left(1 + \frac{A_g \cdot {e_c}^2}{I_g}\right) - \left(\frac{M_S \cdot e_{ptf}}{I_{tf}} + \frac{\left(M_b + M_{ws}\right) \cdot e_{ptc}}{I_{tc}}\right)$$

Where e_{ptf} =eccentricity of the prestressing force with respect to the centroid of the non-composite transformed section

e_{ptc} =eccentricity of the prestressing force with respect to the centroid of the non-composite transformed section

I_{tf} =moment of inertia of the non-composite transformed section

I_{tc} =moment of inertia of the composite transformed section

To perform the calculations, it is necessary to calculate the non-composite and composite transformed section properties

$$n_{ci} := \frac{E_f}{E_{ci}} = 4.72 \qquad n_c := \frac{E_f}{E_c} = 4.13$$

Inputs

$excel_{\text{"B3"}} := A_g \cdot 39.3701^2$ $excel_{\text{"C3"}} := y_{gbot} \cdot 39.3701$ $excel_{\text{"G3"}} := I_g \cdot 39.3701^4$

$excel_{\text{"B4"}} := n_{ci} \cdot n_p \cdot A_{pf} \cdot 39.3701^2$ $excel_{\text{"C4"}} := y_{bs} \cdot 39.3701$

Non-composite transformed section properties at transfer							
	Area (in^2)	y_b (in)	$A\,y_b$ (in^3)	$y_{b,tt}$ (in)	$A*(y_{b,tt}-y_b)^2$ (in^4)	I (in^4)	$I+A*(y_{b,tt}-y_b)^2$ (in^4)
Beam	789.00	24.73	19512.00	24.24	187.67	260730.56	260918.23
Pre. CFRP	18.69	3.66	68.41		7920.47	0.00	7920.47
	807.70		19580.41			I (in^4)	268838.70

Outputs

Inputs

$excel_{\text{"B3"}} := A_g \cdot 39.3701^2$ $excel_{\text{"C3"}} := y_{gbot} \cdot 39.3701$ $excel_{\text{"G3"}} := I_g \cdot 39.3701^4$

$excel_{\text{"B4"}} := n_c \cdot n_p \cdot A_{pf} \cdot 39.3701^2$ $excel_{\text{"C4"}} := y_{bs} \cdot 39.3701$

Non-composite transformed section properties at service							
	Area (in^2)	y_b (in)	$A\,y_b$ (in^3)	$y_{b,tt}$ (in)	$A*(y_{b,tt}-y_b)^2$ (in^4)	I (in^4)	$I+A*(y_{b,tt}-y_b)^2$ (in^4)
Beam	789.00	24.73	19512.00	24.30	144.44	260730.56	260875.01
Pre. CFRP	16.35	3.66	59.84		6968.87	0.00	6968.87
	805.35		19571.84			I (in^4)	267843.87

Outputs

$e_{ptf} := excel_{\text{"E3"}} \cdot in$ $I_{tf} := excel_{\text{"H5"}} \cdot in^4$

Inputs

$excel_{\text{"B3"}} := A_c \cdot 39.3701^2$ $excel_{\text{"C3"}} := y_{cbot} \cdot 39.3701$ $excel_{\text{"G3"}} := I_{comp} \cdot 39.3701^4$

$excel_{\text{"B4"}} := n_c \cdot n_p \cdot A_{pf} \cdot 39.3701^2$ $excel_{\text{"C4"}} := y_{bs} \cdot 39.3701$

Composite transformed section properties at service							
	Area (in^2)	y_b (in)	$A\,y_b$ (in^3)	$y_{b,tt}$ (in)	$A*(y_{b,tt}-y_b)^2$ (in^4)	I (in^4)	$I+A*(y_{b,tt}-y_b)^2$ (in^4)
Beam and slab	1270.12	37.40	47502.24	36.97	233.66	597437.16	597670.82
Pre. CFRP	16.35	3.66	59.84		18147.23	0.00	18147.23
	1286.47		47562.08			I (in^4)	615818.05

Outputs

$e_{ptc} := excel_{\text{"E3"}} \cdot in$ $I_{tc} := excel_{\text{"H5"}} \cdot in^4$

From table above $e_{ptf} := 20.02\ in$ $e_{ptc} := 33.15\ in$

$I_{tf} := 266584.66\ in^4$ $I_{tc} := 686823.24\ in^4$

$$\Delta f_{cd} := -\left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) \cdot \frac{A_{pf}}{A_g} \cdot \left(1 + \frac{A_g \cdot e_c^{\ 2}}{I_g}\right) - \left(\frac{M_d \cdot e_{ptf}}{I_{tf}} + \frac{\left(M_b + M_{ws}\right) \cdot e_{ptc}}{I_{tc}}\right) = -0.72\ ksi$$

$$\Delta f_{pCD} := \frac{E_f}{E_{ci}} \cdot f_{cgp} \cdot \left(\Psi_b\left(t_f, t_i\right) - \Psi_b\left(t_d, t_i\right)\right) \cdot K_{df} + \frac{E_f}{E_c} \cdot \Delta f_{cd} \cdot \Psi_b\left(t_f, t_d\right) \cdot K_{df} = 1.36\ ksi$$

Relaxation of Prestressing Strands

$$\Delta f_{pR2} = \Delta f_{pRf} - \Delta f_{pR1}$$

$$\Delta f_{pRf} = \left(0.019 \cdot \left(\frac{f_{pt}}{f_{pu}}\right) - 0.0066\right)\ log\left(t \cdot 24\right) \cdot f_{pu}$$ [AASHTO-CFRP Eq. 1.9.2.5.2-2]

Where, f_{pt}=stress in prestressing strands immediately after transfer,

$$f_{pt} := f_{pi} - \Delta f_{pES} = 240\ ksi$$

t=time between strand prestressing and final (days)

$$t := t_i + t_f = 2 \cdot 10^4$$

Therefore,

$$\Delta f_{pRf} := \left(0.019 \cdot \left(\frac{f_{pt}}{f_{pu}}\right) - 0.0066\right)\ \log\left(t \cdot 24\right) \cdot f_{pu} = 12.55\ ksi$$

$$\Delta f_{pR2} := \Delta f_{pRf} - \Delta f_{pR1} = 5.17\ ksi$$

Shrinkage of Deck Concrete

$$\Delta f_{pSS} = \frac{E_f}{E_c} \cdot \Delta f_{cdf} \cdot K_{df} \cdot \left(1 + 0.7 \cdot \Psi_b\left(t_f, t_d\right)\right)$$ [AASHTO Eq. 5.9.3.4.3d-1]

Where, Δf_{cdf}=change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete (ksi)

$$= \frac{\varepsilon_{ddf} \cdot A_d \cdot E_{cd}}{\left(1 + 0.7 \cdot \Psi_b\left(t_f, t_d\right)\right)} \cdot \left(\frac{1}{A_c} - \frac{e_{pc} \cdot e_d}{I_c}\right)$$

Where, A_d=area of deck concrete
E_{cd}=modulus of elascity of deck concrete
e_d=eccentricity of deck with respect to the gross composite section, positive in typical construction where deck is above girder (in)

$$y_{cDeck} := h_c - 0.5 \cdot h_d = 58.25\ in$$

$$e_d := y_{cDeck} - y_{cbot} = 20.85\ in$$

ε_{ddf}=shrinkage strain of deck concrete between placement and final time
$= k_s \cdot k_{hs} \cdot k_f \cdot k_{td}\left(t_f, t_i\right) \cdot 0.48 \cdot 10^{-3}$

and, k_s=factor for the effect of volume to surface ratio of the component (this has to be recalculated for deck)

k_s=1.45-0.13(V/S) [AASHTO Eq. 5.4.2.3.2-2]
where (V/S)=volume to surface ratio of deck (in)
=Area/Perimeter (excluding edges)

$$P_d := b_e \cdot 2 = 144\ in$$

$$k_s := 1.45 - 0.13 \cdot \frac{1}{in}\left(\frac{A_d}{P_d}\right) = 1.02$$

$$k_s := \left\| \begin{array}{l} \text{if } k_s \le 1 \\ \quad \| \ 1 \\ \text{else} \\ \quad \| \ k_s \end{array} \right\| = 1.02$$

$$k_f = \frac{5}{1 + \frac{f'_{ci}}{ksi}}$$

f'_{ci}=specified compressive strength of deck concrete at time of initial loading may be taken as $0.80 f'_{cDeck}$

$$k_f := \frac{5}{1 + \frac{0.8 \cdot f'_{cDeck}}{ksi}} = 0.86$$

$$k_{td}\left(t, t_i\right) := \frac{t - t_i}{61 - 4 \cdot \frac{0.8\ f'_{cDeck}}{ksi} + t - t_i}$$

$$\varepsilon_{ddf} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}\left(t_f, t_d\right) \cdot 0.48 \cdot 10^{-3} = 4.31 \cdot 10^{-4}$$

$$\Delta f_{cdf} := \frac{\varepsilon_{ddf} \cdot A_d \cdot E_{cDeck}}{\left(1 + 0.7 \cdot \Psi_b\left(t_f, t_d\right)\right)} \cdot \left(\frac{1}{A_c} - \frac{e_{pc} \cdot e_d}{I_{comp}}\right) = -0.24 \ ksi$$

$$\Delta f_{pSS} := \frac{E_f}{E_c} \cdot \Delta f_{cdf} \cdot K_{df} \cdot \left(1 + 0.7 \cdot \Psi_b\left(t_f, t_d\right)\right) = -1.56 \ ksi$$

Total Prestress Losses at Transfer

The prestress loss due to elastic shortening:

$$\Delta f_{pES} = 9.43 \ ksi$$

Assuming a 5% loss due to friction (Based on the NCHRP 12-97 project tests). Only 4 prestressing CFRP were harped. Hence, assume a loss of 1% in all prestressing CFRP

Stress in tendons after transfer $\quad f_{pt} := 0.99 \cdot f_{pi} - \Delta f_{pES} = 237.51 \ ksi$

Force per strand after transfer $\quad p_t := f_{pt} \cdot A_{pf} = 42.75 \ kip$

Initial loss, % $\quad eloss := \dfrac{\Delta f_{pES} \cdot 100}{f_{pi}} = 3.78$

Total Prestress Losses at Service

The sum of time-dependent prestress losses between time of transfer and deck placement:

$$\left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) = 21.9 \ ksi$$

The sum of time-dependent prestress losses after deck placement:

$$\left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS}\right) = 7.26 \ ksi$$

The total time-dependent prestress losses:

$$\Delta f_{pLT} := \left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) + \left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS}\right) = 29.16 \ ksi$$

The total prestress loss at service:

$$\Delta f_{pT} := \Delta f_{pES} + \Delta f_{pLT} = 38.59 \ ksi$$

Stress in strands after all losses,

$$f_{pe} := 0.99 \cdot f_{pi} - \Delta f_{pT} = 208.35 \ ksi$$

Check prestressing stress limit at service limit state: [AASHTO-CFRP Table 1.9.1-1]

$$\begin{array}{l} \text{if } f_{pe} \le 0.65 \cdot f_{pu} \\ \quad \Vert \text{“Stress limit satisfied”} \\ \text{else} \\ \quad \Vert \text{“Stress limit not satisfied”} \end{array} = \text{“Stress limit satisfied”}$$

Force per strand after all losses $p_e := f_{pe} \cdot A_{pf} = 37.5\ kip$

Therefore, the total prestressing force after all losses $P_e := n_p \cdot p_e = 825.05\ kip$

Final loss, % $ploss := \dfrac{\Delta f_{pT} \cdot 100}{f_{pi}} = 15.47$

Stresses at Transfer

Total prestressing force after transfer $P_t := n_p \cdot p_t = 940.53\ kip$

Stress Limits for Concrete

Compression Limit: [AASHTO Art. 5.9.2.3.1a]

$0.6 \cdot f'_{ci} = 3.6\ ksi$

Where, f'_{ci}=concrete strength at release=6 ksi

Tension Limit: [AASHTO Art. 5.9.2.3.1b]

Without bonded reinforcement

$-0.0948 \cdot \sqrt{\dfrac{f'_{ci}}{ksi}}\ ksi = -0.23\ ksi$ $\le$ -0.2 ksi

Therefore, tension limit, $= -0.2\ ksi$

With bonded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force in the concrete computed assuming an uncracked section where reinforcement is proportioned using a stress of $0.5 f_y$, not to exceed 30 ksi.

$-0.24 \cdot \sqrt{\dfrac{f'_{ci}}{ksi}}\ ksi = -0.59\ ksi$

If the tensile stress is between these two limits, the tensile force at the location being considered must be computed following the procedure in AASHTO Art. C5.9.2.3.1b. The required area of reinforcement is computed by dividing tensile force by the permitted stress in the reinforcement ($0.5 f_y \le 30$ ksi)

Stresses at Transfer Length Section

Stresses at this location need only be checked at release since this stage almost always governs. Also, losses with time will reduce the concrete stresses making them less critical.

Transfer length $l_t = \dfrac{f_{pi} \cdot d_b}{\alpha_t \cdot f'_{ci}{}^{0.67}}$ AASHTO-CFRP Eq. 1.9.3.2.1-1

Where, d_b=prestressing CFRP diameter (in.)
α_t=coefficient related to transfer length taken as 1.1 for cable

Also can be estimated as $l_t := 50 \cdot d_b = 30\ in$

Moment due to self-weight of the beam at transfer length

$$M_{gt} := 0.5 \cdot w_g \cdot l_t \cdot \left(L_{design} - l_t\right) = 89.91\ ft \cdot kip$$

Stress in the top of beam:

$$f_t := \frac{P_t}{A_g} - \frac{P_t \cdot e_c}{S_{gtop}} + \frac{M_{gt}}{S_{gtop}} = -0.91\ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [NOT OK]

-0.588 ksi with bonded reinforcement [NOT OK]

stress in the bottom of the beam:

$$f_b := \frac{P_t}{A_g} + \frac{P_t \cdot e_c}{S_{gbot}} - \frac{M_{gt}}{S_{gbot}} = 2.97\ ksi$$

Compressive stress limit for concrete = 3.6 ksi [OK]

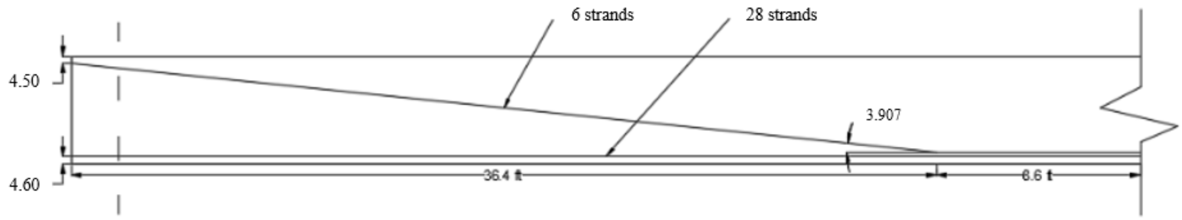
Since stresses at the top and bottom exceed the stress limits, harp prestressing CFRP to satisfy the specified limits

Number of harped strands $n_h := 4$

Height of top most row of harped strands at end of beam $d_h := h_g - 2.5\ in = 51.5\ in$

Harp strands at 0.4L points of the girder as shown in figure below where L is the overall beam length and

$$x_h := 0.4 \cdot L_{span} = 36.4\ ft$$



Compute the center of gravity of the prestressing strands at the transfer length section using the harped strand pattern.

The distance between the center of gravity of the 4 harped strands at the end of the beam and the top fiber of the beam:

$n_{h1} := 2$ $\quad d_{h1} := 2.5\ in$

$n_{h2} := 2$ $\quad d_{h2} := 4.5\ in$

$n_{h3} := 0$ $\quad d_{h3} := 6.5\ in$

$n_{h4} := 0$ $\quad d_{h4} := 8.5\ in$

$$y_{ht} := \frac{n_{h1} \cdot d_{h1} + n_{h2} \cdot d_{h2} + n_{h3} \cdot d_{h3} + n_{h4} \cdot d_{h4}}{n_{h1} + n_{h2} + n_{h3} + n_{h4}} = 3.5\ in$$

$$e_{ch} := y_{gtop} - y_{ht} = 25.77\ in$$

The distance between the center of gravity of the 4 harped strands at the harp point and the bottom fiber of the beam:

$n_{h1} := 2$

$n_{h2} := 2$

$n_{h3} := 0$

$n_{h4} := 0$

$$y_{hb} := h_g - \frac{n_{h1} \cdot d_{p3} + n_{h2} \cdot d_{p2} + n_{h3} \cdot d_{h3} + n_{h4} \cdot d_{h4}}{n_{h1} + n_{h2} + n_{h3} + n_{h4}} = 5.75\ in$$

The distance between the center of gravity of the 4 harped strands and the top fiber of the beam at the transfer length section:

$$y_t := y_{ht} + \left(h_g - y_{ht} - y_{hb}\right) \cdot \frac{l_t}{x_h} = 6.57\ in$$

The distance between the center of gravity of the bottom straight 18 strands and the extreme bottom fiber of the beam:

$$x_{pt} := h_g - \frac{\left(n_{b1} - n_{h3}\right) \cdot d_{p1} + \left(n_{b2} - n_{h2}\right) \cdot d_{p2} + \left(n_{b3} - n_{h1}\right) \cdot d_{p3}}{n_{b1} + n_{b2} + n_{b3} + n_{b4} - \left(n_{h1} + n_{h2} + n_{h3} + n_{h4}\right)} = 3.19\ in$$

$$e_{ct} := y_{gbot} - x_{pt} = 21.54\ in$$

Therefore, the distance between the center of gravity of the total number of strands measured to the bottom of the precast beam at transfer length is:

$$e_{ce} := y_{gbot} - \frac{\left(n_p - n_h\right) \cdot x_{pt} + n_h \cdot \left(h_g - y_t\right)}{n_p} = 13.49\ in$$

Recompute the stresses at the transfer length section with harped strands:

Provision for harped strands [AASHTO-CFRP 1.9.1.1]

The prestress force per strand before seating losses is: $P_j := A_{pf} \cdot f_{pi} = 44.9\ kip$

From above figure, the harp angle is: $\Psi := \operatorname{atan}\left(\frac{h_g - y_{ht} - y_{hb}}{x_h}\right) = 5.85\ deg$

```
if Ψ<9 deg                          = "PCI limit satisfied"
  ‖ "PCI limit satisfied"
else
  ‖ "PCI limit not satisfied"
```

Moment due to self-weight of the beam at transfer length

$$M_{gdl} := 0.5 \cdot w_g \cdot l_t \cdot \left(L_{design} - l_t\right) = 89.91\ ft \cdot kip$$

Stress in the top of beam:

$$f_t := \frac{P_t}{A_g} - \frac{P_t \cdot e_{ce}}{S_{gtop}} + \frac{M_{gdl}}{S_{gtop}} = -0.11\ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [OK]

-0.588 ksi with bonded reinforcement [OK]

Stress in the bottom of the beam:

$$f_b := \frac{P_t}{A_g} + \frac{P_t \cdot e_{ce}}{S_{gbot}} - \frac{M_{gdl}}{S_{gbot}} = 2.29\ ksi$$

Compressive stress limit for concrete = 3.6 ksi [OK]

Stresses at Harping Point

The strand eccentricity at harp points is the same as at midspan

$$x := x_h = 36.4\ ft$$

Moment due to self-weight of the beam at transfer length

$$M_g := 0.5 \cdot w_g \cdot x \cdot \left(L_{design} - x\right) = 801.88\ ft \cdot kip$$

Stress in the top of beam:

$$f_t := \frac{P_t}{A_g} - \frac{P_t \cdot e_c}{S_{gtop}} + \frac{M_g}{S_{gtop}} = 0.05\ ksi$$

Tensile stress limits for concrete=	-0.2 ksi without bonded reinforcement	[OK]
	-0.588 ksi with bonded reinforcement	[OK]

stress in the bottom of the beam:

$$f_b := \frac{P_t}{A_g} + \frac{P_t \cdot e_c}{S_{gbot}} - \frac{M_g}{S_{gbot}} = 2.16\ ksi$$

Compressive stress limit for concrete = 3.6 ksi [OK]

Stresses at Midspan

$$x := L_{design} \cdot 0.5 = 45\ ft$$

Moment due to self-weight of the beam at transfer length

$$M_g := 0.5 \cdot w_g \cdot x \cdot \left(L_{design} - x\right) = 832.28\ ft \cdot kip$$

Stress in the top of beam:

$$f_t := \frac{P_t}{A_g} - \frac{P_t \cdot e_c}{S_{gtop}} + \frac{M_g}{S_{gtop}} = 0.09\ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [OK]

-0.588 ksi with bonded reinforcement [OK]

stress in the bottom of the beam:

$$f_b := \frac{P_t}{A_g} + \frac{P_t \cdot e_c}{S_{gbot}} - \frac{M_g}{S_{gbot}} = 2.12\ ksi$$

Compressive stress limit for concrete = 3.6 ksi [OK]

Hold-Down Forces

Therefore, the hold-down force per strand is: $p_h := 1.05 \cdot P_j \cdot \sin(\Psi) = 4.8\ kip$

Note the factor of 1.05 is applied to account for friction.

The total hold-down force $P_h := n_h \cdot p_h = 19.22\ kip$

The hold-down force and the harp angle should be checked against maximum limits for local practices.

<u>Stresses at Service Loads</u>

Total prestressing force after all losses $P_e = 825.05\ kip$

Due to the sum of effective prestress and permanent loads (i.e. beam self-weight, weight of slab and haunch, weight of future wearing surface, and weight of barriers) for the Load Combination Service 1:

Compression Limit: [AASHTO Art. 5.9.2.3.2a]

for precast beam $0.45 \cdot f'_c = 4.05\ ksi$

for deck $0.45 \cdot f'_{cDeck} = 2.7\ ksi$

Due to the sum of effective prestress, permanent loads, and transient loads as well as during shipping and handling for the Load Combination Service 1:

for precast beam $0.60 \cdot f'_c = 5.4\ ksi$

for deck $0.60 \cdot f'_{cDeck} = 3.6\ ksi$

Tension Limit: [AASHTO Art. 5.9.2.3.2b]

For components with bonded prestressing tendons or reinforcement that are subjected to not worse than moderate corrosion conditions for Load Combination Service III

for precast beam $-0.19 \cdot \sqrt{\frac{f'_c}{ksi}}\ ksi = -0.57\ ksi$

Stresses at Midspan

Concrete stress at top fiber of the beam

To check top stresses, two cases are checked:

Under permanent loads, Service I:

$$S_{cgtop} := \frac{I_{comp}}{y_{ctop} - h_d} = \left(3.49 \cdot 10^4\right)\ in^3$$

$$f_{tg} := \frac{P_e}{A_g} - \frac{P_e \cdot e_c}{S_{gtop}} + \frac{M_g + M_d}{S_{gtop}} + \frac{M_{ws} + M_b}{S_{cgtop}} = 1.14\ ksi \qquad < 4.05\ ksi \qquad \text{[OK]}$$

Under permanent and transient loads, Service I:

$$f_{tg} := f_{tg} + \frac{M_{LT} + M_{LL}}{S_{cgtop}} = 1.61\ ksi \qquad < 5.4\ ksi \qquad \text{[OK]}$$

Concrete stress at bottom fiber of beam under permanent and transient loads, Service III:

$$f_b := \frac{P_e}{A_g} + \frac{P_e \cdot e_c}{S_{gbot}} - \frac{M_g + M_d}{S_{gbot}} - \frac{M_b + M_{ws} + (0.8) \cdot \left(M_{LT} + M_{LL}\right)}{S_{cbot}} = 0.02\ ksi \qquad < 5.4\ ksi \qquad \text{[OK]}$$

Fatigue Limit State

AASHTO Art. 5.5.3 specifies that the check for fatigue of the prestressing strands is not required for fully prestressed components that are designed to have extreme fiber tensile stress due to the Service III limit state within the specified limit of $0.19 f'_c$. The AASHTO Type IV girder in this design example is designed as a fully prestressed member, and the tensile stress due to Service III limit state is less than $0.19 f'_c$. Hence, the fatigue check for the prestressing strands is not required.

Stregth Limit State

The total ultimate bending moment for Strength I is:

M_u = 1.25(DC) + 1.50(DW) + 1.75(LL + IM) [AASHTO Art. 3.4.1]

Using the values of unfactored bending moment used in this example

$$M_u := 1.25\ (M_g + M_d + M_b) + 1.50 \cdot M_{ws} + 1.75 \cdot (M_{LT} + M_{LL}) = (4.58 \cdot 10^3)\ ft \cdot kip$$

$\varepsilon_{pu} = 0.016$ $\qquad \varepsilon_{pe} := \dfrac{f_{pe}}{E_f} = 9.26 \cdot 10^{-3}$ $\qquad \varepsilon_{cu} := 0.003$

$d_p := (h_g + h_d - 2.75\ in) = 58.75\ in$

$d_{p1} := (h_g + h_d - 4.75\ in) = 56.75\ in$

$d_{p2} := (h_g + h_d - 6.75\ in) = 54.75\ in$

$d_{p3} := (h_g + h_d - 8.75\ in) = 52.75\ in$

$d_{p4} := (h_g + h_d - 10.75\ in) = 50.75\ in$

$$\begin{bmatrix} n_{b1} \\ n_{b2} \\ n_{b3} \\ n_{b4} \\ n_{b5} \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$c_b := \dfrac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{pu} - \varepsilon_{pe}} \cdot d_p = 18.4\ in$ $\qquad \beta_1 := 0.85$

Total compressive force

$$C_c := \begin{array}{l} \text{if } \beta_1 \cdot c_b \le h_d \\ \quad 0.85 \cdot f'_{cDeck} \cdot \beta_1 \cdot b_d \cdot c \\ \text{else if } h_d \le \beta_1 \cdot c_b \le (h_{trf} + h_d) \\ \quad 0.85 \cdot f'_{cDeck} \cdot (b_d \cdot h_d + b_{trf} \cdot (\beta_1 \cdot c_b - h_d)) \\ \text{else} \\ \quad 0.85 \cdot f'_{cDeck} \cdot (b_d \cdot h_d + b_{trf} \cdot h_{trf} + (b_{ttf} + (h_{ttf} - (\beta_1 \cdot c_b - (h_{trf} + h_d))) + b_w) \cdot (\beta_1 \cdot c_b - (h_{trf} + h_d))) \end{array} = (3.24 \cdot 10^3$$

Area of prestressing reinforcement for balance condition $\qquad A_{pfb} := \dfrac{C_c}{f_{pu}} = 9.09\ in^2$

Number of cables required for balance condition $\qquad n_{pb} := \dfrac{A_{pfb}}{A_{pf}} = 50.51$

Check for

$$\begin{array}{l} \text{if } n_p < n_{pb} \\ \quad \text{“Section is tension controlled”} \\ \text{else} \\ \quad \text{“Section is compression controlled”} \end{array} = \text{“Section is tension controlled”}$$

$$\varepsilon_{co} := \left(\left(\frac{f'_{cDeck}}{11\ ksi} \right) + 1.6 \right) \cdot 10^{-3} = 0.0021$$

$$\beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) := \max\left(0.65, \left(\frac{4 - \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)}{6 - 2 \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)} \cdot \left(-\left(\frac{f'_{cDeck}}{50\ ksi} \right) + 1.1 \right) \right) \right)$$ [AASHTO-CFRP Eq. C1.7.2.1-3]

$$\alpha_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) := \frac{\left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right) - \frac{1}{3} \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)^2}{\beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right)} \cdot \left(-\left(\frac{f'_{cDeck}}{60\ ksi} \right) + 1 \right)$$ [AASHTO-CFRP Eq. C1.7.2.1-4]

Guess Values

$c := 9\ in$ $\quad d_p := 58.25\ in$ $\quad \varepsilon_{pu} = 0.02$ $\quad \varepsilon_{cc} := 0.0025$ $\quad \varepsilon_{co} := 0.002$

Constraints

$$\varepsilon_{pe} + \frac{d_p - c}{c} \cdot \varepsilon_{cc} \le \varepsilon_{pu}$$ [AASHTO-CFRP Eq. 1.7.3.1.1-3]

$$\varepsilon_{cc} \le 0.003$$

$$\begin{array}{l} \text{if } \beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot c \le h_d \\ \quad \alpha_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot f'_{cDeck} \cdot \beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot b_d \cdot c \\ \text{else if } h_d \le \beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot c \le \left(h_{trf} + h_d\right) \\ \quad \alpha_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot f'_{cDeck} \cdot \left(b_d \cdot h_d + b_{trf} \cdot \left(\beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot c - h_d\right)\right) \\ \text{else} \\ \quad \alpha_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot f'_{cDeck} \cdot \left(b_d \cdot h_d + b_{trf} \cdot h_{trf} + \left(b_{ttf} + \left(h_{ttf} - \left(\beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot c - \left(h_{trf} + h_d\right)\right)\right) + b_w\right) \cdot \left(\beta_1\left(\varepsilon_{cc}, \varepsilon_{co}\right) \cdot c - \left(h_{trf} + \right.\right.\right. \end{array}$$

Solver

$$\begin{bmatrix} c \\ \varepsilon_{cc} \end{bmatrix} := \mathbf{find}\left(c, \varepsilon_{cc}\right) = \begin{bmatrix} 0.7314\ ft \\ 0.0012 \end{bmatrix}$$

$$\varepsilon_{cc} = 0.0012$$

$$\beta_1 := \max\left(0.65, \left(\frac{4 - \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)}{6 - 2 \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)} \cdot \left(-\left(\frac{f'_{cDeck}}{50\ ksi}\right) + 1.1\right)\right)\right) = 0.69$$

$$\alpha_1 := \frac{\left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right) - \frac{1}{3} \cdot \left(\frac{\varepsilon_{cc}}{\varepsilon_{co}}\right)^2}{\beta_1} \cdot \left(-\left(\frac{f'_{cDeck}}{60\ ksi}\right) + 1\right) = 0.58$$

$$c = 8.78\ in$$

$$\varepsilon_f := \frac{d_p - c}{c} \cdot \varepsilon_{cc} = 0.0066$$

$$\varepsilon_{f1} := \frac{d_p - 2\ in - c}{c} \cdot \varepsilon_{cc} = 0.0064 \qquad \varepsilon_{f3} := \frac{d_p - 6\ in - c}{c} \cdot \varepsilon_{cc} = 0.0058$$

$$\varepsilon_{f2} := \frac{d_p - 4\ in - c}{c} \cdot \varepsilon_{cc} = 0.0061 \qquad \varepsilon_{f4} := \frac{d_p - 8\ in - c}{c} \cdot \varepsilon_{cc} = 0.0056$$

$$T_f := n_{b1}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_f + \varepsilon_{pe}\right) = 772.91\ kip$$

$$T_{f1} := n_{b2}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_{f1} + \varepsilon_{pe}\right) = 633.32\ kip \qquad T_{f3} := n_{b4}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_{f3} + \varepsilon_{pe}\right) = 0\ kip$$

$$T_{f2} := n_{b3}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_{f2} + \varepsilon_{pe}\right) = 0\ kip \qquad T_{f4} := n_{b5}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_{f4} + \varepsilon_{pe}\right) = 0\ kip$$

$$h_{trf} + h_d = 15.5\ in$$

$$C_c := \left\| \begin{array}{l} \text{if } \beta_1 \cdot c \le h_d \\ \quad \left\| \alpha_1 \cdot f'_{cDeck} \cdot \beta_1 \cdot b_d \cdot c \right. \\ \text{else} \\ \quad \left\| \alpha_1 \cdot f'_{cDeck} \cdot \left(b_d \cdot h_d + b_{trf} \cdot \left(\beta_1 \cdot c - h_d\right)\right) \right. \end{array} \right\| = \left(1.33 \cdot 10^3\right)\ kip$$

$$M_n := T_f \cdot \left(d_p - c\right) + T_{f1} \cdot \left(d_p - c - 2\ in\right) + T_{f2} \cdot \left(d_p - c - 4\ in\right) + C_c \cdot \left(c - \frac{\beta_1 \cdot c}{2}\right) + T_{f3} \cdot \left(d_p - c - 6\ in\right) + T_{f4} \cdot \left(d_p - c - 8\ in\right.$$

$$M_n = \left(6.39 \cdot 10^3\right)\ ft \cdot kip \qquad M_u = \left(4.58 \cdot 10^3\right)\ ft \cdot kip$$

$\phi := 0.75$ [for CFRP prestressed beams] [AASHTO-CFRP Art. 1.5.3.2]

$\phi \cdot M_n = \left(4.79 \cdot 10^3\right)\ ft \cdot kip$ $> M_u$ [OK]

Minimum Reinforcement

There is a on-going NCHRP project for revising the minimum reinforcement provisions for prestressed beams. Therefore, the outcome of the NCHRP 12-94 may also influence the requirements for CFRP prestressed beams.

At any section of a flexural component, the amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance, Mr, at least equal to the lesser of:

1.33 times the factored moment required by the applicable strength load combinations

and

$$M_{cr} = \gamma_3\left(\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_c - M_{dnc} \cdot \left(\frac{Sc}{S_{nc}} - 1\right)\right) \qquad \text{[AASHTO-CFRP 1.7.3.3.1-1]}$$

Where, γ_1= 1.6 flexural cracking variability factor

γ_2=1.1 prestress variability factor

γ_3=1.0 prestressed concrete structures

$\gamma_1 := 1.6 \qquad \gamma_2 := 1.1 \qquad \gamma_3 := 1.0$

$$f_{cpe} := \frac{P_e}{A_g} + \frac{P_e \cdot e_c}{S_{gbot}} = 2.69\ ksi \qquad f_r := 0.20 \cdot \sqrt{\frac{f'_c}{ksi}}\ ksi = 0.6\ ksi$$

$$M_{dnc} := M_g + M_d = \left(1.45 \cdot 10^3\right)\ ft \cdot kip$$

$$M_{cr} := \gamma_3 \cdot \left(\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_{cbot} - M_{dnc} \cdot \left(\frac{S_{cbot}}{S_{gbot}} - 1\right)\right) = \left(4.48 \cdot 10^3\right)\ ft \cdot kip$$

$$gov_{moment} := \text{if } M_{cr} < 1.33 \cdot M_u \;\|\; M_{cr} \;\text{else}\; \|\; 1.33 \cdot M_u \quad = \left(4.48 \cdot 10^3\right)\ (ft \cdot kip)$$

```
if gov_moment < ϕ • M_n
  ‖ "Minimum reinf. requirement OK"
else
  ‖ "Minimum reinf. requirement NOT OK"
```

= "Minimum reinf. requirement OK"

Shear Design

Transverse shear reinforcement will be provided where [AASHTO Eq. 5.7.2.3-1]

$$V_u > 0.5\ \phi \cdot \left(V_c + V_p\right)$$

Where,

V_u=factored shear force (kip)

V_c=nominal shear resistance provided by tensile stresses in the concrete (kip)

V_p=component of prestressing in the direction of shear force (kip)

ϕ=0.90=resistance factor for shear [AASHTO Art. 5.5.4.2]

Critical Section for Shear [AASHTO Art. 5.7.3.2]

The location of the critical section for shear shall be taken as d_v from the internal face of the support.

Where, d_v= effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultant of the tensile and compressive force due to flexure. It need not to be taken less than the greater of $0.9\,d_e$ or $0.72\,h$ (in).

$$= d_e - \frac{a}{2}$$ [AASHTO Art. 5.7.2.8]

Where,

a = depth of compression block

h = overall depth of composite section

d_e= Effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (For harped and draped configuration, this values varies along the length). For CFRP prestressed beams, this value can be taken as the centroid of prestressing CFRP at that location (substituting f_{ps} by f_{pu} and A_s@3#in AASHTO Eq. 5.8.2.9-2)

$$y_{bse} := x_{pt} = 3.19\ in$$

$$d_e := h_g + h_d - y_{bse} = 58.31\ in \qquad a := \beta_1 \cdot c = 6.05\ in$$

Effective Shear Depth

$$d_v := d_e - \frac{a}{2} = 55.28\ in$$

$$0.9 \cdot d_e = 52.48\ in \qquad 0.72 \cdot \left(h_g + h_d\right) = 44.28\ in$$

$$d_1 := \max\left(0.9 \cdot d_e, 0.72 \cdot \left(h_g + h_d\right)\right) = 52.48\ in$$

$$d_v := \max\left(d_1, d_v\right) = 55.28\ in$$

The bearing width is yet to be determined. It is conservatively assumed zero and the critical section for shear is locatied at the distance of

$$x_c := d_v = 55.28\ in \qquad \frac{x_c}{L_{design}} = 0.05$$

(0.049L) from the centerline of the bearing, where L is the design span length.
The value of d_e is calculated at the girder end, which can be refined based on the critical section location. However, it is conservative not to refine the value of d_e based on the critical section 0.049L. The value, if refined, will have a small difference (PCI 2017).

Shear Stress

Shear stress in the concrete (v_u) is given as:

$$v_u = \frac{V_u - \phi V_p}{\phi \cdot b_v \cdot d_v}$$ [AASHTO Eq. 5.7.2.8-1]

Where,

β = A factor indicating the ability of diagonally cracked concrete to transmit tension
ϕ = resistance factor for shear

$\phi := 0.9$

b_v = effective web width (in)

$b_v := b_w = 8\ in$

$d_v = 55.28\ in$
V_u =factored shear force at specified section at Strength Limit I state

Using the equation to calculate shear force due to the design truck $x := x_c = 55.28\ in$

$$V_{truck1}(x) := P_1 \cdot \frac{\left(L_{design} - x\right)}{L_{design}} + P_2 \cdot \frac{\left(L_{design} - x - 14\ ft\right)}{L_{design}} + P_3 \cdot \frac{\left(L_{design} - x - 28\ ft\right)}{L_{design}}$$

$$V_{truck2}(x) := P_1 \cdot \frac{-\left(x - 14\ ft\right)}{L_{design}} + P_2 \cdot \frac{\left(L_{design} - x\right)}{L_{design}} + P_3 \cdot \frac{\left(L_{design} - x - 14\ ft\right)}{L_{design}}$$

$$V_{truck1}(x) = 60.85\ kip$$

$$V_{truck2}(x) = 40.05\ kip$$

$$V := \max\left(V_{truck1}(x), V_{truck2}(x)\right) = 60.85\ kip$$

Distributed bending shear due to truck load including dynamic load allowance (V_{LT}) is calculated as follows:
V_{LT} = (Moment per lane due to truck load)(*DFS*)(1+*IM*/100)

$IM := 33$

$D_{S.Interior} = 0.6$

$$V_{LT} := V \cdot D_{S.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 48.56\ kip$$

The maximum shear force (V_L) due to a uniformly distributed lane load of 0.64 klf are calculated using the following formulas given by the *PCI Design Manual* (PCI 2017).

Maximum bending moment, $V_x = 0.64 \frac{kip}{ft} \frac{\left(L_{design} - x\right)^2}{2\ L_{design}}$

where:
x = Distance from centerline of bearing to section at which the shear force is calculated, ft.
L = Design span length

$$V_L := 0.64 \frac{kip}{ft} \frac{\left(L_{design} - x\right)^2}{2\ L_{design}} = 25.93\ kip$$

$$V_{LL} := D_{S.Interior} \cdot V_L = 15.56\ kip$$

$$V_{ws} := w_{ws} \cdot \left(0.5 \cdot L_{design} - x\right) = 6.06\ kip$$

$$V_u := 1.25\ \left(V_g + V_s + V_b\right) + 1.5 \cdot V_{ws} + 1.75 \cdot \left(V_{LT} + V_{LL}\right) = 199.01\ kip$$

V_p = Component of the effective prestressing force in the direction of the applied shear, kips
= (force per strand)(number of harped strands) $\left(sin\left(\Psi\right)\right)$

$$V_p := p_e \cdot n_h \cdot \sin\left(\Psi\right) = 15.29\ kip$$

Therefore,

$$v_u := \frac{V_u - \phi \cdot V_p}{\phi \cdot b_v \cdot d_v} = 0.47\ ksi$$

Contribution of Concrete to Nominal Shear Resistance [AASHTO Art. 5.7.3.3]

The contribution of the concrete to the nominal shear resistance is given as:
[AASHTO Eq. 5.7.3.3-3]

$$V_c = 0.0316\ \beta \cdot \sqrt{f'_{cGirder}} \cdot b_v \cdot d_v$$

where:
β = A factor indicating the ability of diagonally cracked concrete to transmit tension
$f'_{cGirder}$ = Compressive strength of concrete at service
b_v = Effective web width taken as the minimum web width within the depth d_v,
d_v = Effective shear depth

Strain in Flexural Tension Reinforcement

The θ and β values are determined based on the strain in the flexural tension reinforcement. The strain in the reinforcement, ε_f, is determined assuming that the section contains at least the minimum transverse reinforcement as specified in AASHTO-CFRP Eq. 1.8.3.2-1

$$\varepsilon_f = \frac{\frac{M_u}{d_v} + 0.5 \cdot N_u + 0.5 \cdot \left(V_u - V_p\right) - A_{pf} \cdot f_{po}}{E_p \cdot A_{pf}}$$

M_u =Applied factored bending moment at specified section.

$$M_{wsv} := w_{ws} \cdot x \cdot \left(0.5 \cdot L_{design} - x\right) = 27.91\ ft \cdot kip$$

$$M := \max\left(M_{truck1}(x), M_{truck2}(x)\right) = 280.3\ ft \cdot kip$$

$$M_L := 0.5 \cdot 0.64\ \frac{kip}{ft} \cdot (x)\ \left(L_{design} - x\right) = 125.88\ ft \cdot kip$$

$$M_{LLv} := D_{M.Interior} \cdot M_L = 71.09\ ft \cdot kip$$

$$M_{LTv} := D_{M.Interior} \cdot \left(1 + \frac{IM}{100}\right) \cdot M = 210.53\ ft \cdot kip$$

$$M_{uv} := 1.25\ \left(M_{gv} + M_{sv} + M_{bv}\right) + 1.50 \cdot M_{wsv} + 1.75 \cdot \left(M_{LTv} + M_{LLv}\right) = 895.78\ ft \cdot kip$$

$$M_u := \max\left(M_{uv}, V_u \cdot d_v\right) = 916.75\ ft \cdot kip$$

N_u = Applied factored normal force at the specified section, 0.049*L* = 0 kips

$$N_u := 0$$

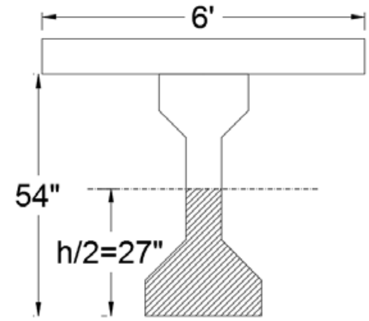
f_{po} = Parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi). For pretensioned members, AASHTO Art. C5.7.3.4.2 indicates that f_{po} can be taken as the stress in strands when the concrete is cast around them, which is jacking stress f_{pj}, or f_{pu}.

$f_{po} := f_{pi} = 249.43\ ksi$ $\quad A_s := 0\ in^2$ $\quad E_f = \left(2.25 \cdot 10^7\right)\ psi$ $\quad A_{pf} = 0.18\ in^2$

$$\varepsilon_f := \frac{\frac{M_u}{d_v} + 0.5\ kip \cdot N_u + \left(V_u - V_p\right) - \left(n_p - n_h\right) \cdot A_{pf} \cdot f_{po}}{E_f \cdot \left(n_p - n_h\right) \cdot A_{pf}} = -5.84 \cdot 10^{-3}$$

Since this value is negative, ε_s should be recalculated using AASHTO Eq. 5.7.3.4.2-4 replacing the denominator by $\left(E_c \cdot A_{ct} + E_f \cdot A_{pft}\right)$

A_{ct} = Area of the concrete on the flexural tension side below $h/2$



$$A_{ct} := \left(\frac{h_g + h_d}{2} - h_{btf} - h_{brf}\right) \cdot b_w + h_{btf} \cdot b_{btf} + h_{brf} \cdot b_{brf} = 399\ in^2$$

$$\varepsilon_f := \frac{\frac{M_u}{d_v} + 0.5\ kip \cdot N_u + \left(V_u - V_p\right) - \left(n_p - n_h\right) \cdot A_{pf} \cdot f_{po}}{\left(E_c \cdot A_{ct} + E_f \cdot \left(n_p - n_h\right) \cdot A_{pf}\right)} = -1.89 \cdot 10^{-4}$$

$$\varepsilon_f := \max\left(\varepsilon_f, -0.40 \cdot 10^{-3}\right) = -1.89 \cdot 10^{-4}$$

Therefore, β, factor indicating the ability of diagonally cracked concrete to transmit tension and shear can be calculated as:

$$\beta := \frac{4.8}{1 + 750 \cdot \varepsilon_f} = 5.59$$ [AASHTO Eq. 5.7.3.4.2-1]

And, θ, angle of inclination of diagonal compressive stress can be calculated as:

$$\theta := 29 + 3500 \cdot \varepsilon_f = 28.34$$ [AASHTO Eq. 5.7.3.4.2-3]

$$\theta := 28.34\ deg$$

Computation of Concrete Contribution

The contribution of the concrete to the nominal shear resistance is given as:

$$V_c := 0.0316\ \beta \cdot \sqrt{\frac{f'_c}{ksi}} \cdot b_v \cdot d_v \cdot ksi = 234.54\ kip$$

Contribution of Reinforcement to Nominal Shear Resistance

$$\begin{array}{l} \text{if } V_u < \phi \cdot \dfrac{\left(V_c + V_p\right)}{2} \\ \quad \big\| \text{“Transverse reinforcement not provided”} \\ \text{else} \\ \quad \big\| \text{“Transverse shear reinforcement provided”} \end{array} = \text{“Transverse shear reinforcement provided”}$$

[AASHTO Eq. 5.7.2.3-1]

Required Area of Shear Reinforcement

The required area of transverse shear reinforcement is:

$$\frac{V_u}{\phi} \le V_n \qquad V_n = V_c + V_p + V_s \qquad \frac{v_u}{f'_c} = 0.05$$

Where,
V_s=Shear force carried by transverse reinforcement

$$V_s := \frac{V_u}{\phi} - V_c - V_p = -28.71 \ kip$$ [Minimum Shear Reinforcement shall be provided]

Determine Spacing of Reinforcement [AASHTO Art. 5.7.2.6]

Check for maximum spacing of transverse reinforcement

check if $v_u < 0.125 \ f'_c$
or $v_u \ge 0.125 \ f'_c$

$$s_{max} := \left\| \begin{array}{l} \text{if } v_u < 0.125 \ f'_c \\ \quad \big\| min\left(0.8 \cdot d_v, 24 \ in\right) \\ \text{else} \\ \quad \big\| min\left(0.4 \cdot d_v, 12 \ in\right) \end{array} \right. = 24 \ in$$

Use $s := 22 \ in$

$$\begin{array}{l} \text{if } s < s_{max} \\ \quad \big\| \text{“transverse reinforcement spacing OK”} \\ \text{else} \\ \quad \big\| \text{“transverse reinforcement spacing NOT OK”} \end{array} = \text{“transverse reinforcement spacing OK”}$$

Minimum Reinforcement Requirement [AASHTO Eq. 5.7.2.5-1]

The area of transverse reinforcement should not be less than:

$$0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot \frac{b_v \cdot s}{f_y}$$

$$A_{vmin} := 0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot \frac{b_v \cdot s}{f_y} \; ksi = 0.28 \; in^2$$

Use #4 bar double-legged stirrups at 12 in. c/c,

$$A_{vprov} := 2 \cdot \left(0.20 \; in^2\right) = 0.4 \; in^2$$

$$V_s := \frac{A_{vprov} \cdot f_y \cdot d_v \cdot \cot(\theta)}{s} = 111.81 \; kip \qquad V_{sprov} := V_s$$

if $A_{vprov} > A_{vmin}$
  "Minimum shear reinforcement criteria met"
else
  "Minimum shear reinforcement criteria not met"

= "Minimum shear reinforcement criteria met"

Therefore, #4 stirrups with 2 legs shall be provided at 22 in spacing

Maximum Nominal Shear Resistance

In order to ensure that the concrete in the web of the girder will not crush prior to yielding of the transverse reinforcement, the AASHTO Specifications give an upper limit for V_n as follows:

$$V_n = 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p$$ [AASHTO Eq. 5.7.3.3-2]

Comparing the above equation with AASHTO Eq. 5.7.3.3-1

$$V_c + V_s \le 0.25 \cdot f'_c \cdot b_v \cdot d_v = 1$$

$$V_c + V_s = 346.36 \; kip$$

$$0.25 \cdot f'_c \cdot b_v \cdot d_v = 995.03 \; kip$$

This is a sample calculation for determining the transverse reinforcement requirement at the critical section. This procedure can be followed to find the transverse reinforcement requirement at increments along the length of the girder.

Interface Shear Transfer [AASHTO Art. 5.7.4]

Factored Interface Shear

To calculate the factored interface shear between the girder and slab, the procedure in the commentary of AASHTO Art. 5.7.4.5 will be used. This procedure calculates the factored interface shear force per unit length.

At the Strength I Limit State, the factored interface shear force, V_{hi}, at a section on a per unit basis is:

$$V_{hi} = \frac{V_1}{d_v}$$ [AASHTO Eq. C5.7.4.5-7]

where: V_1 = factored shear force at specified section due to total load (noncomposite and composite loads)

The AASHTO Specifications does not identify the location of the critical section. For convenience, it will be assumed here to be the same location as the critical section for vertical shear, at point 0.049L.

$$V_u = 199.01\ kip$$

$$V_1 := V_u = 199.01\ kip$$

$$V_{hi} := \frac{V_1}{d_v} = 3.6\ \frac{kip}{in}$$

Required Nominal Interface Shear Resistance

The required nominal interface shear resistance (per unit length) is:

$$V_{ni} = \frac{V_{ri}}{\phi}$$ [AASHTO Eq. 5.7.4.3-1]

where: $V_{ri} \geq V_{ui}$ [AASHTO Eq. 5.7.4.3-2]

where, $V_{ui} := V_{hi} = 3.6\ \frac{kip}{in}$

Therefore, $V_{ni} = \frac{V_{ui}}{\phi}$

$$V_{ni} := \frac{V_{ui}}{\phi} = 4\ \frac{kip}{in}$$

Required Interface Shear Reinforcement

The nominal shear resistance of the interface surface (per unit length) is:

$$V_{ni} = c_1 A_{cv} + \mu(A_{vf}\ f_y + P_c)$$ [AASHTO Eq. 5.7.4.3-3]

where:

c_1 = Cohesion factor [AASHTO Art. 5.7.4.4]
μ = Friction factor [AASHTO Art. 5.7.4.4]
A_{cv} = Area of concrete engaged in shear transfer, in.2
A_{vf} = Area of shear reinforcement crossing the shear plane, in.2
P_c = Permanent net compressive force normal to the shear plane, kips
f_y = Shear reinforcement yield strength, ksi

For concrete normal-weight concrete placed against a clean concrete surface, free of laitance, with surface intentionally roughened to an amplitude of 0.25 in: [AASHTO Art. 5.7.4.4]

$c_1 := 0.28 \ ksi$

$\mu := 1$

The actual contact width, *bv*, between the slab and the girder is 20 in.

$A_{cv} := b_{trf} = 240 \ \frac{in^2}{ft}$ $\qquad$ $d_v = 55.28 \ in$

$P_c := 0 \ kip$

The AASHTO Eq. 5.7.4.3-3 can be solved for A_{vf} as follows:

$$A_{vf} := \frac{V_{ni} - c_1 \cdot A_{cv} - \frac{\mu \cdot P_c}{in}}{\mu \cdot f_y} = -0.32 \ \frac{in^2}{ft}$$

The provided vertical shear reinforcement $\qquad \frac{A_{vprov}}{s} = 0.22 \ \frac{in^2}{ft}$

Since, $\frac{A_{vprov}}{s} > A_{vf}$,

The provided reinforcement for vertical shear is sufficient to resist interface shear.

$$A_{vfprov} := \frac{A_{vprov}}{s} = 0.22 \ \frac{in^2}{ft}$$

Minimum Interface Shear Reinforcement

The cross-sectional area of the interface shear reinforcement, A_{vf}, croossing the interface are, A_{cv}, shall satisfy

$$\text{Minimum } A_{vf} \geq \frac{0.05 \cdot A_{cv}}{f_y} \qquad \text{[AASHTO Eq. 5.7.4.2-1]}$$

$$A_{vf1} := \frac{0.05 \cdot A_{cv}}{\frac{f_y}{ksi}} = 0.2 \ \frac{in^2}{ft}$$

The minimum interface shear reinforcement, A_{vf}, need not exceed the lesser of the amount determined using Eq. 5.7.4.2-1 and the amount needed to resist $1.33\frac{V_{ui}}{\phi}$ as determined using Eq. 5.7.4.3-1.

$$A_{vf2} := \frac{1.33\,\frac{V_{ui}}{\phi} - c_1 \cdot A_{cv} - \frac{\mu \cdot P_c}{in}}{\mu \cdot f_y} = -0.06\ \frac{in^2}{ft}$$

Therefore, minimum amount of shear reinforcement

$$A_{vfmin} := min\left(A_{vf1}, A_{vf2}\right) = -0.06\ \frac{in^2}{ft}$$

if $A_{vfprov} > A_{vfmin}$
  "Minm. Interface shear reinforcement OK"
else
  "Minm. Interfaceshear reinforcement NOT OK"

= "Minm. Interface shear reinforcement OK"

Maximum Nominal Shear Resistance

$$V_{niprov} := c_1 \cdot A_{cv} + \mu \cdot A_{vf} \cdot f_y = 48\ \frac{kip}{ft}$$ [AASHTO Eq. 5.7.4.3-3]

The nominal shear resistance, V_{ni}, used in the design shall not be greater than the lesser of

$$V_{ni} \le k_1 \cdot f'_c \cdot A_{cv}$$ [AASHTO Eq. 5.7.4.3-4]

$$V_{ni} \le k_2 \cdot A_{cv}$$ [AASHTO Eq. 5.7.4.3-5]

Where: For a cast-in-place concrete slab on clean concrete girder surfaces, free of laitance with surface roughened to an amplitude of 0.25 in.

$$k_1 := 0.30 \qquad k_2 := 1.8\ ksi$$

$$k_1 \cdot f'_c \cdot A_{cv} = 648\ \frac{1}{ft} \cdot kip$$

$$k_2 \cdot A_{cv} = 432\ \frac{1}{ft} \cdot kip$$

$V_{niprov} < k_1 \cdot f'_c \cdot A_{cv} = 1$ [1= OK]

$V_{niprov} < k_2 \cdot A_{cv} = 1$ [1= OK]

Minimum Longitudinal Reinforcement Requirement [AASHTO-CFRP Art. 1.8.3.3]

Longitudinal reinforcement should be proportioned so that at each section the following equation is satisfied:

$$\sum_{x=1}^{n} A_{pf} \cdot f_{pu} \geq \frac{M_u}{d_v \cdot \phi_f} + 0.5 \cdot \frac{N_u}{\phi_n} + \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p \right) cot(\theta)$$

[AASHTO-CFRP Eq. 1.8.3.3-1]

where: $n_p \cdot A_{pf}$ = area of prestressing steel on the flexural tension side of the member at section under consideration (in)

f_{pu} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi) conservatively taken as effective prestress

M_u = factored bending moment at the section corresponding to the factored shear force (kip-ft)

V_u = factored shear force at section under consideration (kip)

V_p = component of the effective prestressing force in direction of the applied shear (kip) = 0

V_s = shear resistance provided by the transverse reinforcement at the section under investigation as given by Eq. 5.7.3.3-4, except that Vs shall not be taken as greater than Vu /φ (kip)

ϕ_f = resistance factor for flexure

ϕ_n = resistance factor for axial resistance

ϕ_v = resistance factor for shear

θ = angle of inclination of diagonal compressive stresses used in determining the nominal shear resistance of the section under investigation as determined by AASHTO Eq. 5.7.3.4.2-3 (degrees)

Required Reinforcement at Face of Bearing

Width of the bearing is assumed to be zero. This assumption is more conservative for these calculations. Thus, the failure crack assumed for this analysis radiates from the centerline of the bearing, 6 in. from the end of the beam.

As 6 in. is very close to the end of the beam, shear and moment values at the end of the beam are used

Inputs

$excel_{\text{"F1"}} := 3.28084\ (L_{design})$ $\qquad$ $excel_{\text{"A13"}} := 0.5 \cdot 3.28084\ (L_{design})$

$excel_{\text{"B1"}} := 0.0000685217\ (w_g)$

$excel_{\text{"D1"}} := 0.0000685217\ (w_d + w_h)$

$excel_{\text{"B2"}} := 0.0000685217\ (w_b)$

w_g	0.822		0.61042		90.000		L	
w_{SD}	0.109							
			Dead Load					
Distance (x)	Section (x/L)	Girder Weight		Slab Weight		Barrier weight		Total D
		Shear	Moment	Shear	Moment	Shear	Moment	Shear
ft.		k	k-ft	k	k-ft	k	k-ft	k
0	0.000	36.990	0.00	27.469	0.00	4.890	0.00	69.35
4.62	0.051	33.192	162.12	24.649	120.39	4.388	21.43	62.23
10.858	0.121	28.065	353.18	20.841	262.27	3.710	46.69	52.62
21.717	0.241	19.139	609.47	14.212	452.59	2.530	80.57	35.88
32.575	0.362	10.213	768.82	7.584	570.93	1.350	101.64	19.15
43.433	0.483	1.288	831.26	0.957	617.30	0.170	109.89	2.41
45.000	0.500	0.000	832.27	0.000	618.05	0.000	110.02	0.00

Outputs

$V_g := excel_{\text{"C7"}} \cdot kip \qquad V_s := excel_{\text{"E7"}} \cdot kip \qquad V_b := excel_{\text{"G7"}} \cdot kip$

$M_{gv} := excel_{\text{"D8"}} \cdot ft \cdot kip \qquad M_{sv} := excel_{\text{"F8"}} \cdot ft \cdot kip \qquad M_{bv} := excel_{\text{"G8"}} \cdot ft \cdot kip$

$x := 0\ ft$

$V_{truck1}(x) = 64.53\ kip \qquad V_{truck2}(x) = 43.73\ kip$

$V := \max\left(V_{truck1}(x), V_{truck2}(x)\right) = 64.53\ kip$

$$V_{LT} := V \cdot D_{S.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 51.5\ kip$$

$$V_L := 0.64\ \frac{kip}{ft}\ \frac{\left(L_{design} - x\right)^2}{2\ L_{design}} = 28.8\ kip$$

$V_{LL} := D_{S.Interior} \cdot V_L = 17.28\ kip$

$$V_{ws} := w_{ws} \cdot (0.5 \cdot L_{design} - x) = 6.75 \ kip$$

$$V_u := 1.25 \ (V_g + V_s + V_b) + 1.5 \cdot V_{ws} + 1.75 \cdot (V_{LT} + V_{LL}) = 217.17 \ kip$$

$\phi_v := 0.9$ $\qquad$ $V_s := V_{sprov}$

$M_u := 0$ $\qquad$ $N_u := 0$ $\qquad$ Therefore,

$$\left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right) \cot(\theta) = 315.4 \ kip$$

The crack plane crosses the centroid of the 18 straight strands at a distance of

$$x_c := 6 + 3.659 \cdot \cot(\theta) = 12.78$$

in. from the girder end. Because the transfer length is 24 in., the available prestress from 18 straight strands is a fraction of the effective prestress, f_{pe}, in these strands. The 4 harped strands do not contribute to the tensile capacity since they are not on the flexural tension side of the member.

$$(n_p - n_h) \cdot A_{pf} \cdot f_{pe} \cdot \frac{x_c \cdot in}{24 \ in} = 359.58 \ kip$$

$$(n_p - n_h) \cdot A_{pf} \cdot f_{pe} \cdot \frac{x_c \cdot in}{24 \ in} \geq \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right) \cot(\theta)$$ [AASHTO-CFRP Eq. 1.8.3.3-1]

Therefore, no additional longitudinal reinforcement is required

Pretensioned Anchorage Zone [AASHTO Art. 5.9.4.4]

Splitting Reinforcement [AASHTO Art. 5.9.4.4.1]

Design of the anchorage zone splitting reinforcement is computed using the force in the strands just prior to transfer:

Force in the strands before transfer

$$P_i := n_p \cdot A_{pf} \cdot f_{pi} = 987.76 \ kip$$ [AASHTO Eq. 5.9.4.4-1]

The splitting resistance, P_r , should not be less than 4% of P_i

$P_r = f_s \cdot A_s > 0.04 \cdot P_i$ $\qquad$ $0.04 \cdot P_i = 39.51 \ kip$

where: A_s = total area of reinforcement located within the distance h/4 from the end of the beam (in²) For pretensioned I-beams and bulb tees, As shall be taken as the total area of the vertical reinforcement located within a distance h/4 from the end of the member, where h is the overall height of the member (in)

$f_s := 20\ ksi$ $\qquad A_s := 0.04 \cdot \dfrac{P_i}{f_s} = 1.98\ in^2$

At least 1.98 in² of vertical transverse reinforcement should be provided within a distance of h/4 from the end of beam.

$$\frac{h_g + h_d}{4} = 15.38\ in$$

The area of a #4 stirrup with 2 legs is: $A_v := 2 \cdot 0.2\ in^2 = 0.4\ in^2$

The required number of stirrups is $\text{round}\left(\dfrac{A_s}{A_v}, 0\right) + 1 = 6$

The required spacing for 6 stirrups over a distance of 15.4 in. starting 2 in. from the end of the beam is:

$$\frac{(15.4 - 2)}{(6 - 1)} = 2.68$$

Use (6) #4 stirrups with 2 legs at 2.5 in. spacing starting at 2 in. from the end of the beam.

The provided $A_{sprov} := 5 \cdot 2 \cdot 0.2\ in^2 = 2\ in^2$

$A_{sprov} > A_s = 1$ [1=OK]

Confinement Reinforcement [AASHTO Art. 5.9.4.4.2]

For the distance of 1.5d = 1.5(72) = 108.0 in. from the end of the beam, reinforcement shall be placed to confine the prestressing steel in the bottom flange. The reinforcement shall not be less than #3 deformed bars, with spacing not exceeding 6.0 in. and shaped to enclose the strands.

Deflection and Camber

[Upward deflection is negative]

Deflection Due to Prestressing Force at Transfer

For fully bonded strands

$P_{t1} := n_p \cdot p = 950.41\ kip$

$e' := e_c - e_{ce} = 7.58\ in$ difference between the eccentricity of the prestressing steel at midspan and at the end of the beam

$a := x_h = 36.4\ ft$

$$\Delta_{pt} := \frac{-P_{t1}}{E_{ci} \cdot I_g} \left(\frac{e_c \cdot \left(L_{design}\right)^2}{8} - \frac{e' \cdot a^2}{6} \right) = -2.17\ in$$

Deflection Due to Beam Self-Weight

$$\Delta_g = \frac{5 \cdot w_g \cdot \left(L_{girder}\right)^4}{384 \cdot E_{ci} \cdot I_g}$$

Deflection due to beam self-weight at transfer:

$$\Delta_{gt} := \frac{5 \cdot w_g \cdot \left(L_{span}\right)^4}{384 \cdot E_{ci} \cdot I_g} = 1.02 \ in$$

Deflection due to beam self-weight used to compute deflection at erection:

$$\Delta_{ge} := \frac{5 \cdot w_g \cdot \left(L_{design}\right)^4}{384 \cdot E_{ci} \cdot I_g} = 0.98 \ in$$

Deflection Due to Slab and Haunch Weights

$$\Delta_{gd} := \frac{5 \cdot w_d \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I_{comp}} = 0.27 \ in$$

Deflection Due to Rail/Barrier and Future Wearing Surface (Overlay)

$$\Delta_{bws} := \frac{5 \cdot \left(w_b + w_{ws}\right) \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I_{comp}} = 0.12 \ in$$

$C_a := \Psi_b\left(t_d, t_i\right) = 0.96$ [From previous calculation of the creep of concrete]

Camber at transfer $\Delta_t := \Delta_{pt} + \Delta_{gt} = -1.14 \ in$

Total deflection before deck placement

$$\Delta_{d1} := \left(\Delta_{pt} + \Delta_{gt}\right) \ \left(1 + C_a\right) = -2.24 \ in$$

Total deflection after deck placement

$$\Delta_{d2} := \left(\Delta_{pt} + \Delta_{gt}\right) \ \left(1 + C_a\right) + \Delta_{gd} = -1.97 \ in$$

Total deflection on composite section

$$\Delta := \left(\Delta_{pt} + \Delta_{gt}\right) \ \left(1 + C_a\right) + \Delta_{gd} + \Delta_{bws} = -1.85 \ in$$

The deflection criteria in S2.5.2.6.2 (live load deflection check) is considered optional. The bridge owner may select to invoke this criteria if desired.

Deflection Due to Live Load and Impact

Live load deflection limit (optional) = Span / 800 [AASHTO Art. 2.5.2.6.2]

$$\Delta_{Ll} := \frac{L_{design}}{800} = 1.35\ in$$

If the owner invokes the optional live load deflection criteria specified in AASHTO Article 2.5.2.6.2, the deflection is the greater of:

- That resulting from the design truck alone, or[AASHTO Art. 3.6.1.3.2]
- That resulting from 25% of the design truck taken together with the design lane load.

Therefore, the distribution factor for deflection, DFD, is calculated as follows:

$$DFD := \frac{4}{N_{beams}} = 0.67$$

However, it is more conservative to use the distribution factor for moment

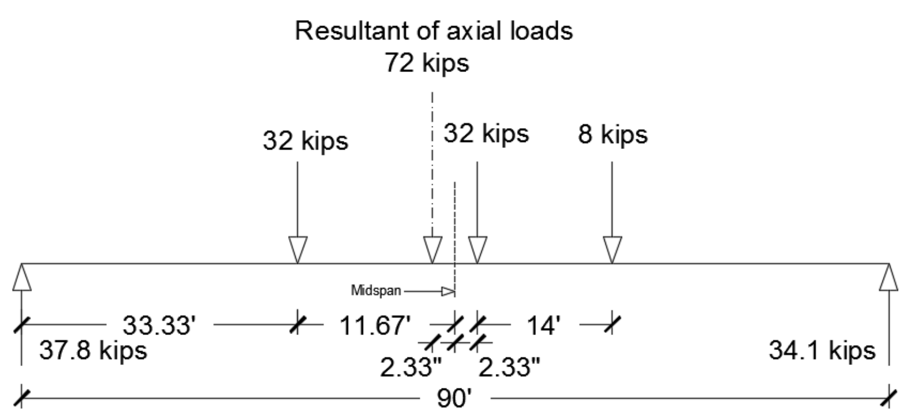
Deflection due to Lane Load:

Design lane load, $w_{LL} := 0.64 \cdot \frac{kip}{ft} \cdot DFD = 0.43\ \frac{kip}{ft}$

$$\Delta_{LL} := \frac{5 \cdot w_{LL} \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I_{comp}} = 0.19\ in$$

Deflection due to Design Truck Load and Impact:

To obtain maximum moment and deflection at midspan due to the truck load, set the spacing between the rear axles to 14 ft, and let the centerline of the beam coincide with the middle point of the distance between the inner 32-kip axle and the resultant of the truck load, as shown in figure below



The deflection at point x due to a point load at point a is given by the following equations:

$$\Delta = \frac{P \cdot b \cdot x}{6 \cdot E_c \cdot I_{comp} \cdot L} \left(L^2 - b^2 - x^2\right) \qquad \text{for } x<a$$

$$\Delta = \frac{P \cdot b}{6 \cdot E_c \cdot I_{comp} \cdot L} \left((x-a)^3 \cdot \frac{L}{b} + \left(L^2 - b\right)^2 \cdot x - x^3\right) \qquad \text{for } x>a$$

where:
- P = point load
- L = span length
- x = location at which deflection is to determined
- b = $L - a$
- E_c = modulus of elasticity of precast beam at service loads
- I_{comp} = gross moment of inertia of the composite section

$$E_c = \left(5.45 \cdot 10^6\right)\ psi$$

Inputs

$$excel_{\text{"D1"}} := 3.28084\ \left(L_{design}\right) \qquad excel_{\text{"B2"}} := \left(I_{comp}\right) \cdot \left(39.3701^4\right)$$

$$excel_{\text{"B1"}} := 1.45038 \cdot 10^{-7}\ E_c$$

E_c	5448.345	L_{design}	90	
I_{comp}	597437.2			
Axle load	a	b	x	Δ
P(kips)	(ft)	(ft)	(ft)	in.
32	33.33	56.6	45	0.2300763
32	47.33	42.67	45	0.2569467
8	61.33	28.67	45	0.0502184

Outputs

$$\delta_1 := excel_{\text{"E5"}} \cdot in \qquad \delta_2 := excel_{\text{"E6"}} \cdot in \qquad \delta_3 := excel_{\text{"E7"}} \cdot in$$

The total deflection =

$$\Delta_{LT} := \delta_1 + \delta_2 + \delta_3 = 0.54\ in$$

Including impact and the distribution factor, the deflection at midspan due to the design truck load is:

$$\Delta_{LT} := \Delta_{LT} \cdot D_{M.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 0.4\ in$$

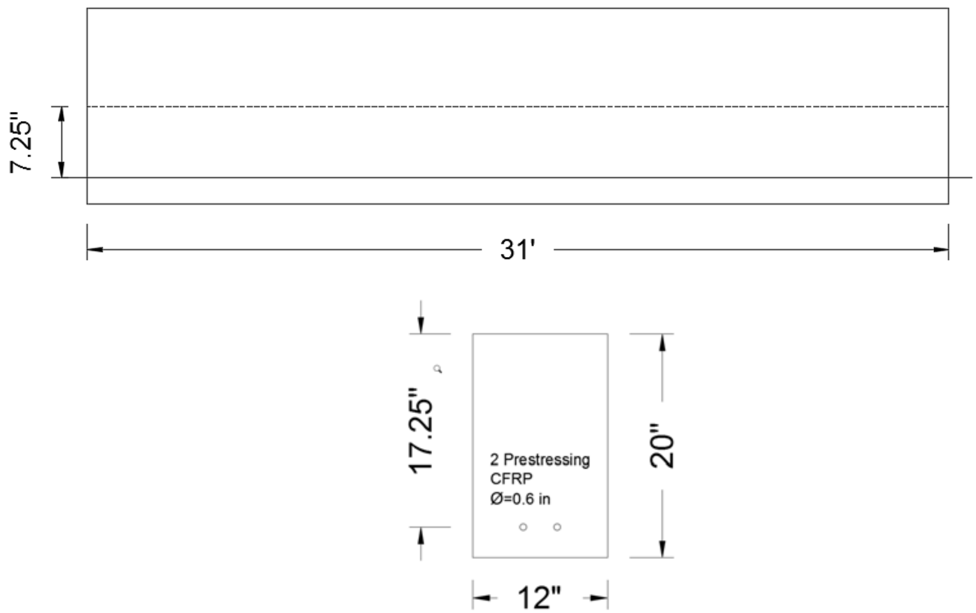
Therefore, the live load deflection is the greater of:

$$\Delta_L := \begin{array}{l} \text{if } \Delta_{LT} > 0.25 \cdot \Delta_{LT} + \Delta_{LL} \\ \quad \| \Delta_{LT} \\ \text{else} \\ \quad \| 0.25 \cdot \Delta_{LT} + \Delta_{LL} \end{array} = 0.4\ in$$

$$\begin{array}{l} \text{if } \Delta_{Ll} > \Delta_L \\ \quad \| \text{"Deflection Limit Satisfied"} \\ \text{else} \\ \quad \| \text{"Deflection Limit Not Satisfied"} \end{array} = \text{"Deflection Limit Satisfied"}$$

Example B-4: Design of a rectangular beam post-tensioned with straight CFRP cables

The following example illustrates the analysis of rectangular beam post-tensioned with two unbonded prestressing cables of 0.6 inch diameter and a jacking stress of $0.70 \cdot f_{pu}$. The beam is $31\ ft$ in length and carries a superimposed dead load of 20% of it's self-weight and the live load of $0.35\ \dfrac{kip}{ft}$ in addition to its own weight. The analysis includes checking all applicable service and strength limit states according to AASHTO-LRFD (2017) and AASHTO Guide Specifications (2018). They are referred in the following example as AASHTO and AASHTO-CFRP respectively. The analysis also includes the computations of deflection corresponding to the moment of $130.0\ ft \cdot kip$.



Overall beam Length $L_{span} := 31\ ft$

Design Span $L_{design} := 30\ ft$

<u>Concrete Properties</u>

Concrete strength at release, $f'_{ci} := 5.50\ ksi$

Concrete strength at 28 days, $f'_{c} := 9.00\ ksi$

Unit weight of concrete $\gamma_c := 150\ pcf$

<u>Prestressing CFRP</u>

Diameter of one prestressing CFRP cable $d_b := 0.6\ in$

Area of one prestressing CFRP cable $A_{pf} := 0.18\ in^2$

Design tensile stress $f_{pu} := \frac{64.14\ kip}{A_{pf}} = 356.33\ ksi$

Modulus of elasticity (AASHTO-CFRP Art. 1.4.1.3) $E_f := 22500\ ksi$

Design tensile strain $\varepsilon_{pu} := \frac{f_{pu}}{E_f} = 0.02$

Stress limitation for prestressing CFRP
(AASHTO-CFRP Art. 1.9.1)

Before transfer $f_{pi} := 0.70 \cdot f_{pu} = 249.43\ ksi$

At service, after all losses $f_{pe} := 0.65 \cdot f_{pu} = 231.62\ ksi$

Nonprestressed Reinforcement:

Yield strength $f_y := 60\ ksi$

Modulus of elasticity (AASHTO Art. 5.4.4.2) $E_s := 29000\ ksi$

Beam Section Properties

Width of beam $b := 12 \cdot in$

Height of beam $h := 20 \cdot in$

Cross-section area of beam $A := b \cdot h = 240\ in^2$

Distance from centroid to the extreme bottom fiber of the non-composite precast girder $y_b := \frac{h}{2} = 10\ in$

Distance from centroid to the extreme top fiber of the non-composite precast girder $y_t := h - y_b = 10\ in$

Moment of inertia of deck about it centriod $I := \frac{b \cdot h^3}{12} = \left(8 \cdot 10^3\right)\ in^4$

Section modulus referenced to the extreme bottom fiber of the non-composite precast girder $S_c := \frac{I}{y_b} = 800\ in^3$

Section modulus referenced to the extreme top fiber of the non-composite precast girder $S_{ct} := \frac{I}{y_t} = 800\ in^3$

Weight of the beam $w := (b \cdot h) \cdot \gamma_c = 0.25\ \frac{kip}{ft}$

Material Properties for Girder and Deck Concrete:

Modulus of elasticity of concrete (AASHTO Art. 5.4.2.4) $E\left(f'_c\right) := 12 \cdot \left(\frac{\gamma_c}{pcf}\right)^{2.0} \left(\frac{f'_c}{psi}\right)^{0.33} \cdot psi$

At release $E_{ci} := E\left(f'_{ci}\right) = \left(4.63 \cdot 10^3\right)\ ksi$

At 28 day (Girder) $E_c := E\left(f'_c\right) = \left(5.45 \cdot 10^3\right)\ ksi$

Modulus of rupture of concrete (AASHTO Art 5.4.2.6) $f_{mr}\left(f'_c\right) := 0.24 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi$

At release $f_{ri} := f_{mr}\left(f'_{ci}\right) = 0.56\ ksi$

At 28 day (Girder) $f_r := f_{mr}\left(f'_c\right) = 0.72\ ksi$

Number of Strands and Strand Arrangement:

Total number of prestressing CFRP $n_p := 2$

Concrete cover $cc := 2.75\ in$

Depth of prestressing CFRP from the top fiber of the beam $d_p := h - cc = 17.25\ in$

Eccentricity of prestressing CFRP $e_c := d_p - y_t = 7.25\ in$

Load and Moment on Beam:

Unit weight due to superimposed load $w_{SD} := 0.2 \cdot w = 0.05\ \frac{kip}{ft}$

Unit weight due to live load $w_L := 0.35\ \frac{kip}{ft}$

M_b=unfactored bending moment due to beam self-weight, k-ft

$$M_b := \frac{w \cdot L_{design}^{\ 2}}{8} = 28.13\ ft \cdot kip$$

M_{SD}=unfactored bending moment due to superimposed dead load, k-ft

$$M_{SD} := \frac{w_{SD} \cdot L_{design}^{\ 2}}{8} = 5.63\ ft \cdot kip$$

M_L=unfactored bending moment due to live load, k-ft

$$M_L := \frac{w_L \cdot L_{design}^{\ 2}}{8} = 39.38\ ft \cdot kip$$

Moment at service $M_s := M_b + M_{SD} + 0.8 \cdot M_L = 65.25\ ft \cdot kip$

Moment at ultimate $M_u := 1.25\ M_b + 1.5\ M_{SD} + 1.75\ M_L = 112.5\ ft \cdot kip$

Prestressing Loss

Prestressing CFRP stress before transfer $f_{pi} := 0.70 \cdot f_{pu} = 249.43\ ksi$

Elastic Shortening

$$\Delta f_{pES} = \frac{\left(N_p - 1\right)}{2 \cdot N_p} \cdot \frac{E_f}{E_{ct}} \cdot f_{cgp}$$ [AASHTO-CFRP Eq. 1.9.2.2.3a-1]

Where E_f=modulus of elasticity of prestressing CFRP (ksi)
E_{ct}=modulus of elasticity of the concrete at transfer or time of load application (ksi)=E_{ci}
f_{cgp}=the concrete stress at the center of gravity of CFRP due to the prestressing force immediately after transfer and the self-weight of the member at sections of maximum moment (ksi)
N_p= The number of identical prestressing CFRP

AASHTO Article C5.9.5.2.3a states that to calculate the prestress after transfer, an initial estimate of prestress loss is assumed and iterated until acceptable accuracy is achieved. In this example, an initial estimate of 10% is assumed.

$eloss := 10\%$

Force per strand at transfer

$$f_{cgp} = \frac{P_i}{A_g} + \frac{P_i \cdot e_c^{\ 2}}{I_g} - \frac{M_G \cdot e_c}{I_g}$$

Where, P_i=total prestressing force at release=$n_p * p$
e_c=eccentricity of strands measured from the center of gravity of the precast beam at midspan
M_G=moment due to beam self-weight at midspan (should be calculated using the overall beam length)

$$M_G := \frac{w \cdot L_{span}^2}{8} = 30.03\ ft \cdot kip$$

$eloss := 10\%$

$$eloss = \frac{E_f}{f_{pi} \cdot E_{ci}} \cdot \left(\frac{n_p \cdot A_{pf} \cdot f_{pi} \cdot (1 - eloss)}{A} + \frac{(n_p \cdot A_{pf} \cdot f_{pi} \cdot (1 - eloss)) \cdot e_c^2}{I} - \frac{M_G \cdot e_c}{I} \right) \cdot \frac{(n_p - 1)}{2 \cdot n_p}$$

$eloss := \mathbf{find}(eloss) = 3.09 \cdot 10^{-3}$

Therefore, the loss due to elastic shortening= $eloss = 3.09 \cdot 10^{-3}$

The stress at transfer= $f_{pt} := f_{pi} \cdot (1 - eloss) = 248.66\ ksi$

The force per strand at transfer= $p_t := A_{pf} \cdot f_{pi} \cdot (1 - eloss) = 44.76\ kip$

The concrete stress due to prestress= $f_{cgp} := \frac{n_p \cdot p_t}{A} + \frac{n_p \cdot p_t \cdot e_c^2}{I} - \frac{M_b \cdot e_c}{I} = 655.3\ psi$

The prestress loss due to elastic shortening= $\Delta f_{pES} := \frac{E_f}{E_{ci}} \cdot f_{cgp} = 3.18\ ksi$

Total prestressing force at release $P_i := n_p \cdot p_t = 89.52\ kip$

Final prestressing loss including Elastic Shortening

Assume a prestress loss of 18% [This assumption is based on the average of all cases in the design space considered in the reliability study]

$ploss := 18\%$

$f_{pe} := f_{pi} \cdot (1 - ploss) = 204.54\ ksi$

Force per strand at service $p_e := f_{pe} \cdot A_{pf} = 36.82\ kip$

Check prestressing stress limit at service limit state: [AASHTO-CFRP Table 1.9.1-1]

```
if f_pe ≤ 0.6 • f_pu                      = "Stress limit satisfied"
  ‖ "Stress limit satisfied"
else
  ‖ "Stress limit not satisfied"
```

Check Stress at Transfer and Service:

Stresses at transfer

Total prestressing force after transfer $P_t := n_p \cdot p_t = 89.52\ kip$

Stress Limits for Concrete

Compression Limit: [AASHTO Art. 5.9.2.3.1a]

$0.6 \cdot f'_{ci} = 3.3\ ksi$

Where, f'_{ci}=concrete strength at release=5.5 ksi

Tension Limit: [AASHTO Art. 5.9.2.3.1b]

No tension allowed.

Stresses at end zone

Stresses at this location need only be checked at release since this stage almost always governs. Also, losses with time will reduce the concrete stresses making them less critical.

Moment due to self-weight of the beam end zone

$M_{bt} := 0\ ft \cdot kip$

Stress in the top of beam:

$$f_t := \frac{P_t}{A} - \frac{P_t \cdot e_c}{S_{ct}} + \frac{M_{bt}}{S_{ct}} = -0.44\ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [NOT OK]

-0.48 ksi with bonded reinforcement [OK]

stress in the bottom of the beam:

$$f_b := \frac{P_t}{A} + \frac{P_t \cdot e_c}{S_c} - \frac{M_{bt}}{S_c} = 1.18\ ksi$$

Compressive stress limit for concrete = 3.3 ksi [OK]

Stresses at midspan

Stress in the top of beam:

$$f_t := \frac{P_t}{A} - \frac{P_t \cdot e_c}{S_{ct}} + \frac{M_b}{S_{ct}} = -0.02\ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [OK]

-0.48 ksi with bonded reinforcement [OK]

stress in the bottom of the beam:

$$f_b := \frac{P_t}{A} + \frac{P_t \cdot e_c}{S_c} - \frac{M_b}{S_c} = 0.76\ ksi$$

Compressive stress limit for concrete = 3.3 ksi [OK]

Stresses at Service Loads

Stress Limits for Concrete

Total prestressing force after all losses $P_e := n_p \cdot p_e = 73.63\ kip$

Concrete Stress Limits: [AASHTO Art. 5.9.2.3.2a]

Due to the sum of effective prestress and permanent loads (i.e. beam self-weight, weight of future wearing surface, and weight of barriers) for the Load Combination Service 1:

for precast beam $0.45 \cdot f'_c = 4.05\ ksi$

Due to the sum of effective prestress, permanent loads, and transient loads as well as during shipping and handling for the Load Combination Service 1:

for precast beam $0.60 \cdot f'_c = 5.4\ ksi$

Tension Limit: [AASHTO Art. 5.9.2.3.2b]

For components with un-bonded prestressing tendons, there should be no tensile stress at the bottom concrete fiber.

Stresses at Midspan

Concrete stress at top fiber of the beam

To check top stresses, two cases are checked:

Under permanent loads, Service I:

$$f_{tg} := \frac{P_e}{A} - \frac{P_e \cdot e_c}{S_{ct}} + \frac{M_b + M_{SD}}{S_{ct}} = 0.15\ ksi \qquad < 4.05\ ksi \qquad \text{[OK]}$$

Under permanent and transient loads, Service I:

$$f_{tg} := f_{tg} + \frac{M_L}{S_{ct}} = 0.74\ ksi \qquad < 5.4\ ksi \qquad \text{[OK]}$$

Concrete stress at bottom fiber of beam under permanent and transient loads, Service III:

$$f_b := \frac{P_e}{A} + \frac{P_e \cdot e_c}{S_c} - \frac{M_b + M_{SD} + (0.8) \cdot (M_L)}{S_c} = -4.7 \cdot 10^{-3}\ ksi \qquad \text{No Tension: [OK]}$$

If the tensile stress is between these two limits, the tensile force at the location being considered must be computed following the procedure in AASHTO Art. C5.9.2.3.1b. The required area of reinforcement is computed by dividing tensile force by the permitted stress in the reinforcement ($0.5 f_y \leq 30$ ksi)

<u>Stregth Limit State</u>

Effective prestressing strain $\qquad \varepsilon_{pe} := \frac{f_{pe}}{E_f} = 9.09 \cdot 10^{-3}$

$\beta_1 := 0.85 \qquad \alpha_1 := 0.85$

Strain reduction factors:

$$\Omega_{up} := \frac{3}{\left(\frac{L_{span}}{d_p}\right)} = 0.14 \qquad \text{[AASHTO-CFRP Eq. 1.7.3.1.2-6]}$$

By using equillibrium and compatibility, the depth of the neutral axis (c) and the strain at top fiber of the beam can be found using following

Guess Values

$c := 8\ in$ $\qquad d_p = 17.25\ in$ $\qquad \varepsilon_{pu} = 0.0158$ $\qquad \varepsilon_{cc} := 0.0015$

Constraints

$\varepsilon_{cc} = 0.003$

$\varepsilon_{pe} + \dfrac{d_p - c}{c} \cdot \varepsilon_{cc} \cdot \Omega_{up} \le \varepsilon_{pu}$

$\alpha_1 \cdot f'_c \cdot \beta_1 \cdot b \cdot c = n_p \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \dfrac{d_p - c}{c} \cdot \varepsilon_{cc} \cdot \Omega_{up}\right)$

Solver

$\begin{bmatrix} c \\ \varepsilon_{cc} \end{bmatrix} := \textbf{find}\left(c, \varepsilon_{cc}\right) = \begin{bmatrix} 0.1187\ ft \\ 0.003 \end{bmatrix}$

$\varepsilon_{cc} = 0.003$ $\qquad \varepsilon_{cu} := 0.003$

Depth of neutral axis, $\quad c = 1.42\ in$

Strain at prestressing CFRP at ultimate $\quad \varepsilon_f := \dfrac{d_p - c}{c} \cdot \varepsilon_{cc} \cdot \Omega_{up} = 0.0046$

Total tension force $\quad T_f := n_p \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_f + \varepsilon_{pe}\right) = 111.18\ kip$

Total compression force $\quad C_c := \alpha_1 \cdot f'_c \cdot \beta_1 \cdot b \cdot c = 111.18\ kip$

Check for equillibrium $\quad T_f - C_c = 0\ kip$

The capacity of the section is:

$$M_n := T_f \cdot \left(d_p - c\right) + C_c \cdot \left(c - \frac{\beta_1 \cdot c}{2}\right) = 154.21\ ft \cdot kip$$

Check for

```
if ε_cc = ε_cu
    "Section capacity is compression-controlled"
else
    "Section capacity is tension-controlled"
```
= "Section capacity is compression-controlled"

Selection of strength resistance factor:

$\phi := 0.75$ [for CFRP prestressed beams] [AASHTO-CFRP Art. 1.5.3.2]

Check for capacity

$$\begin{array}{l} \text{if } \phi \cdot M_n > M_u \\ \quad \text{“Section capacity is adequate”} \\ \text{else} \\ \quad \text{“Section capacity is NOT adequate”} \end{array} = \text{“Section capacity is adequate”}$$

<u>Deflection and Camber</u> [Upward deflection is negative]

Deflection due to Prestressing Force at Transfer

$$\Delta_{pt} := \frac{-P_t}{E_{ci} \cdot I} \frac{e_c \cdot \left(L_{span}\right)^2}{8} = -0.3 \ in$$

Deflection due to Beam Self-Weight

$$\Delta_b = \frac{5 \cdot w \cdot \left(L_{span}\right)^4}{384 \cdot E_{ci} \cdot I}$$

Deflection due to beam self-weight at transfer:

$$\Delta_{bt} := \frac{5 \cdot w \cdot \left(L_{span}\right)^4}{384 \cdot E_{ci} \cdot I} = 0.14 \ in$$

Deflection due to beam self-weight used to compute deflection at erection:

$$\Delta_{be} := \frac{5 \cdot w \cdot \left(L_{design}\right)^4}{384 \cdot E_{ci} \cdot I} = 0.12 \ in$$

Deflection due to Superimposed Dead Load

$$\Delta_{SD} := \frac{5 \cdot w_{SD} \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I} = 0.02 \ in$$

Deflection due to Live Load

$$\Delta_L := \frac{5 \cdot w_L \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I} = 0.15 \ in$$

Using ACI 440 multipliers for Long-term deflections

Immidiate camber at transfer $\delta_t := \Delta_{pt} + \Delta_{bt} = -0.16\ in$

Camber at erection $\delta_e := 1.80 \cdot \Delta_{pt} + 1.85\ \Delta_{bt} = -0.29\ in$

Deflection at final $\delta_f := 1 \cdot \Delta_{pt} + 2.70\ \Delta_{bt} + 4.10 \cdot \Delta_{SD} + \Delta_L = 0.31\ in$

Deflection due to Live Load when the Section is Cracked (i.e, for an moment of 130 ft-kip)

Stress at bottom fiber due to the effect of prestress only

$$f_{cpe} := \frac{P_e}{A} + \frac{P_e \cdot e_c}{S_c} = 0.97\ ksi$$

Tensile strength of concrete

$$f_r := 0.24 \cdot \sqrt{\frac{f'_c}{ksi}}\ ksi = 0.72\ ksi$$

Cracking moment of the beam can be computed as:

$$M_{cr} := \left(f_r + f_{cpe}\right) \cdot S_c = 112.94\ ft \cdot kip$$

Factor to soften effective moment of inertia (because of the use of prestressing CFRP)

$$\beta_d := 0.5\ \left(\frac{E_f}{E_s} + 1\right) = 0.89$$ [AASHTO-CFRP Eq. 1.7.3.4.2-2]

Modular ratio

$$n := \frac{E_f}{E_c} = 4.13$$

Cracked moment of inertia [AASHTO-CFRP Eq. 1.7.3.4.2-3]

$$I_{cr} := \frac{b \cdot c^3}{12} + b \cdot c \cdot \left(c - 0.5 \cdot c\right)^2 + n \cdot n_p \cdot A_{pf} \cdot \left(d_p - c\right)^2 = 383.89\ in^4$$

Moment at which deflection is computed at, $M_a := 130\ ft \cdot kip$

Effective moment of inertia, [AASHTO-CFRP Eq. 1.7.3.4.2-1]

$$I_e := \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) \cdot I_{cr} = \left(4.79 \cdot 10^3\right)\ in^4$$

Deflection due to live load producing a moment of 160 ft.kip

$$\Delta_L := \frac{5 \cdot M_a \cdot \left(L_{design}\right)^2}{48 \cdot E_c \cdot I_e} = 0.81\ in$$

Design of the Anchorage Zone [AASHTO Art. 5.9.5.6]

Stress at bottom fiber due to the effect of prestress only

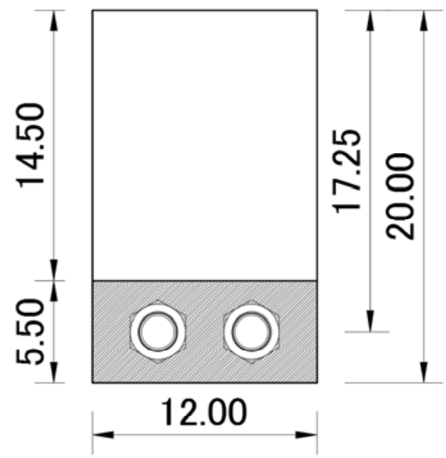


Plate dimensions

$b_x := 12 \cdot in$ $b_y := 5.5 \cdot in$ $t_p := 0.625 \cdot in$

$f_{cpi} := 0.5 \cdot f'_{ci} = 2.75\ ksi$

$P_j := 1.2\ f_{pu} \cdot A_{pf} \cdot n_p \cdot 0.75 = 115.45\ kip$

$f_{bi} := \dfrac{P_j}{b_x \cdot b_y} = 1.75\ ksi\ < f_{cpi}$ No reinforcement needed for the local zone

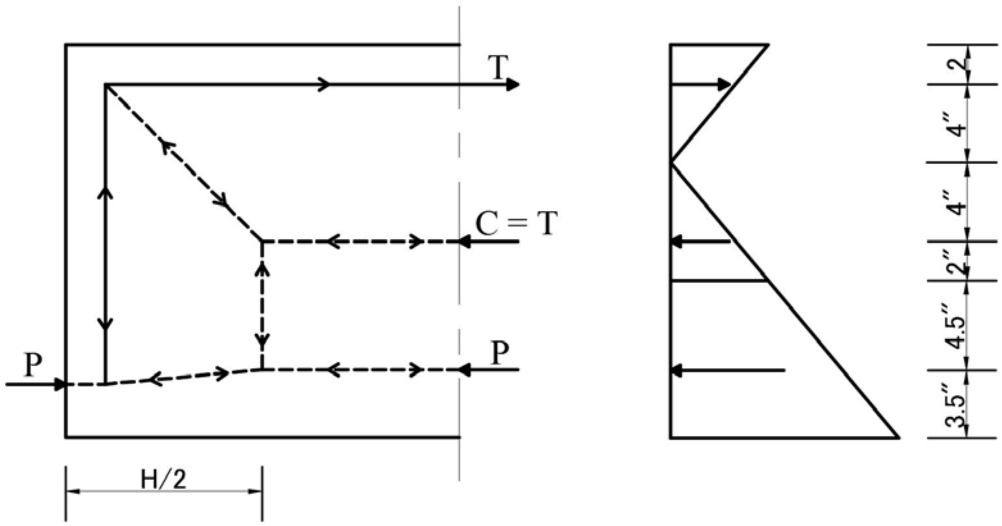
Check the plate thickness

$n := 2 \cdot in$ n: largest distance from edge of wedge plate to edge of the bearing plate l

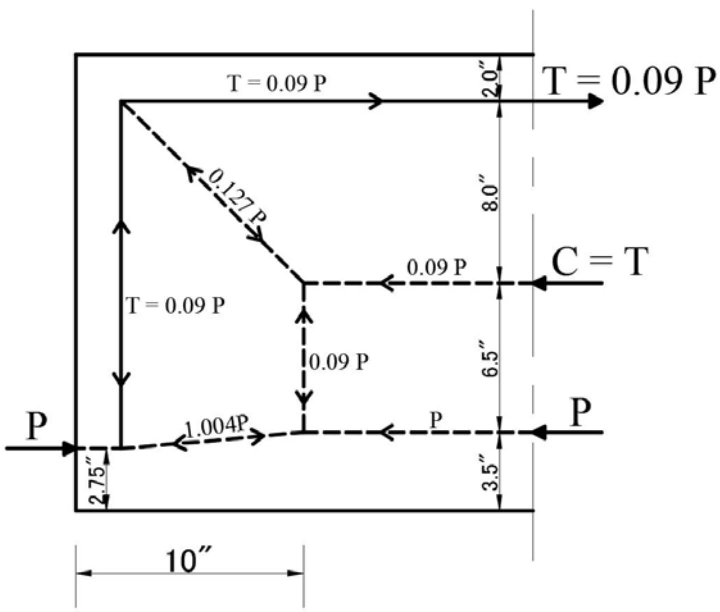
$f_s := 3 \cdot f_{bi} \cdot \left(\dfrac{n}{t_p}\right)^2 = 53.74\ ksi$ f_s: bending stress in bearing plate

$< 0.8\ f_y$ [OK]

Design of General Zone: Strut and Tie Model



Geometry and Forces distributions



Forces in Each element

Tie force

$$T := P_j \cdot 0.09 = 10.39\ kip$$

Try no. 3 steel rebar

$$n := \frac{T}{f_y \cdot 0.11 \cdot in^2} = 1.57$$ Use 2 # 3 U bar concentrated a head of the anchorage plates

Design for bursting force

$$T := P_j \cdot 0.09 = 10.39\ kip$$

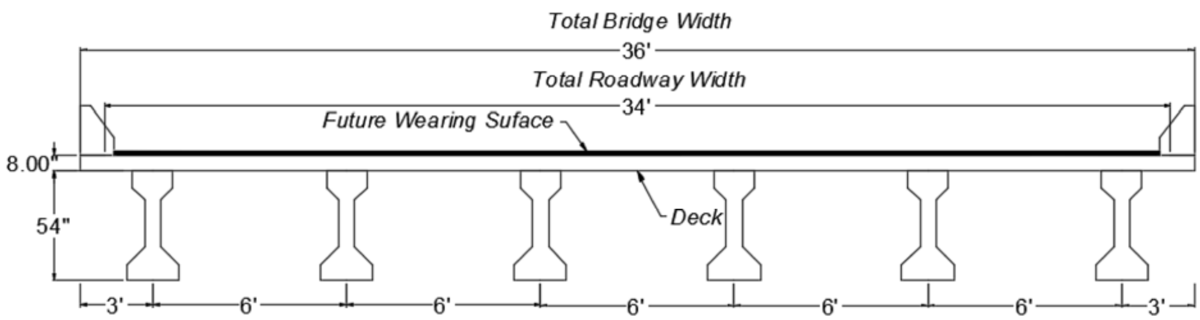
Try no. 3 steel rebar

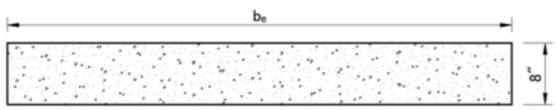
$$n := \frac{T}{f_y \cdot (2 \cdot 0.11) \cdot in^2} = 0.79$$

Use additional 1 closed ties stirrups @4"

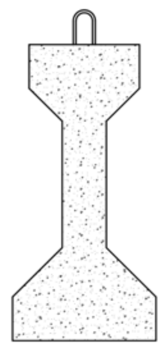
Example B-5: Design of a Decked AASHTO post-tensioned girder with draped CFRP cables

The bridge considered for this design example has a span length of 90 ft. (center-to-center (c/c) pier distance), a total width of 34 ft., and total roadway width of 36 ft. The bridge superstructure consists of six AASHTO Type IV girders spaced 6 ft. center-to-center, designed to act compositely with an 8 in. thick cast-in-place (CIP) concrete deck. The beams are fully post-tensioned before removal from the casting bed. The wearing surface thickness is 2.0 in., which includes the thickness of any future wearing surface. T501 type rails are considered in the design. HL-93 is the design live load. A relative humidity (RH) of 60 percent is considered in the design.The design is performed for an interior girder based on service and strength limit states according to AASHTO-LRFD (2017) and AASHTO Guide Specifications (2018). They are referred in the following example as AASHTO and AASHTO-CFRP respectively.





8"-Deck



AASHTO Type-IV

Overall beam Length $L_{span} := 91\ ft$

Design Span $L_{design} := 90\ ft$

Girder spacing $g_{spacing} := 6\ ft$

Number of beams $N_{beams} := 6$

Total roadway width $w_{roadway} := 36\ ft$

<u>Cast-in-Place Deck:</u>

Structural thickness, (effective) $h_d := 7.5\ in$

Actual thickness, (for dead load calculation) $t_s := 8\ in$

Concrete strength at 28 days, $f'_{cDeck} := 8.00\ ksi$

Thickness of asphalt-wearing surface (including any future wearing surface) $h_{ws} := 2\ in$

Unit weight of concrete $\gamma_c := 150\ pcf$

Precast Girders: AASHTO Type IV

Concrete strength at release, $f'_{ci} := 6.00\ ksi$

Concrete strength at 28 days, $f'_c := 9.00\ ksi$

Unit weight of concrete $\gamma_c := 150\ pcf$

Prestressing CFRP

Diameter of one prestressing CFRP cable $d_b := 0.76\ in$

Area of one prestressing CFRP cable $A_{pf} := 0.289 \cdot in^2$

Design tensile stress $f_{pu} := \dfrac{105.2\ kip}{A_{pf}} = 364\ ksi$

Modulus of elasticity (AASHTO-CFRP Art. 1.4.1.3) $E_f := 22700 \cdot ksi$

Design tensile strain $\varepsilon_{pu} := \dfrac{f_{pu}}{E_f} = 0.016$

Stress limitation for prestressing CFRP (AASHTO-CFRP Art. 1.9.1)
Before transfer $f_{pi} := 0.70 \cdot f_{pu} = 255\ ksi$

Maximum allowable jacking strain $\varepsilon_{pi} := min\left(\varepsilon_{pu} - 0.004, \dfrac{f_{pi}}{E_f}\right) = 0.01$

Jacking stress $f_{pi} := \varepsilon_{pi} \cdot E_f = 255\ ksi$

At service, after all losses $f_{pe} := \left(\dfrac{f_{pi}}{f_{pu}} - 0.05\right) \cdot f_{pu} = 237\ ksi$

Nonprestressed Reinforcement:

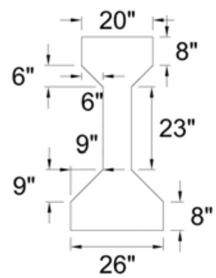
Yield strength $f_y := 60\ ksi$

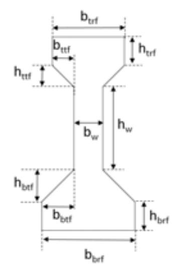
Modulus of elasticity (AASHTO Art. 5.4.3.2) $E_s := 29000\ ksi$

Unit weight of concrete $\gamma_{aws} := 150\ pcf$

T501 type barrier weight/side $\gamma_{bw} := 326\ plf$

<u>Section Properties of AASHTO Type IV Girder:</u>





Cross-section area of girder $A_g := 789\ in^2$

Moment of inertia of about the centroid of the noncomposite precast girder $I_g := 260730\ in^4$

Weight of the girder $w_g := 0.822\ \frac{kip}{ft}$

Height of girder $h_g := 54 \cdot in$

Width of bottom rectangular flange $b_{brf} := 26 \cdot in$

Height of bottom rectangular flange $h_{brf} := 8 \cdot in$

Width of bottom tapered flange $b_{btf} := 9 \cdot in$

Height of bottom tapered flange $h_{btf} := 9 \cdot in$

Width of web $b_w := 8 \cdot in$

Height of web $h_w := 23 \cdot in$

Width of top rectangular flange $b_{trf} := 20 \cdot in$

Height of top rectangular flange $h_{trf} := 8 \cdot in$

Width of top tapered flange $b_{ttf} := 6 \cdot in$

Height of top tapered flange $h_{ttf} := 6 \cdot in$

Distance from centroid to the extreme bottom fiber of the non-composite precast girder $y_{gbot} := 24.73\ in$

Distance from centroid to the extreme top fiber of the non-composite precast girder $y_{gtop} := h_g - y_{gbot} = 29.27\ in$

Section modulus referenced to the extreme bottom fiber of the non-composite precast girder $S_{gbot} := \dfrac{I_g}{y_{gbot}} = \left(1.05 \cdot 10^4\right)\ in^3$

Section modulus referenced to the extreme top fiber of the non-composite precast girder $S_{gtop} := \dfrac{I_g}{y_{gtop}} = \left(8.91 \cdot 10^3\right)\ in^3$

Effective flange width [AASHTO Art. 4.6.2.6.1]

$b_e := g_{spacing} = 72\ in$ Average spacing of adjacent girders

Material Properties for Girder and Deck Concrete:

Modulus of elasticity of concrete (AASHTO Art. 5.4.2.4) $E\left(f'_c\right) := 12 \cdot \left(\dfrac{\gamma_c}{pcf}\right)^{2.0} \left(\dfrac{f'_c}{psi}\right)^{0.33} \cdot psi$

At release $E_{ci} := E\left(f'_{ci}\right) = \left(4.77 \cdot 10^3\right)\ ksi$

At 28 days (Girder) $E_c := E\left(f'_c\right) = \left(5.45 \cdot 10^3\right)\ ksi$

At 28 days (Deck) $E_{cDeck} := E\left(f'_{cDeck}\right) = \left(5.24 \cdot 10^3\right)\ ksi$

Modulus of rupture of concrete (AASHTO Art 5.4.2.6) $f_{mr}\left(f'_c\right) := 0.24 \cdot \sqrt{\dfrac{f'_c}{ksi}} \cdot ksi$

At release $f_{ri} := f_{mr}\left(f'_{ci}\right) = 0.59\ ksi$

At 28 day (Girder) $f_r := f_{mr}\left(f'_c\right) = 0.72\ ksi$

At 28 day (Deck) $f_{rDeck} := f_{mr}\left(f'_{cDeck}\right) = 0.68\ ksi$

$n_1 := \dfrac{E_{cDeck}}{E_c} = 0.96$ [Modular ratio for transformed section]

Section Properties Composite Deck:

Height of deck $h_d := 7.5 \cdot in$

Width of deck $b_d := n_1 \cdot b_e = 69.26\ in$

Cross-section area of deck $A_d := h_d \cdot b_d = 519.41\ in^2$

Moment of inertia of deck about it centriod $I_d := \dfrac{b_d \cdot h_d^{\ 3}}{12} = \left(2.43 \cdot 10^3\right)\ in^4$

Weight of the deck $w_d := \left(b_e \cdot t_s\right) \cdot \gamma_c = 0.6\ \dfrac{kip}{ft}$

Due to camber of the precast, prestressed beam, a minimum haunch thickness of 1/2 in. at midspan is considered in the structural properties of the composite section. Also, the width of haunch must be transformed.

Height of haunch $h_h := 0.5\ in$

Width of haunch $b_h := b_{trf} = 20\ in$

Transformed width of haunch $b_{th} := n_1 \cdot b_h = 19.24\ in$

Area of haunch $A_h := h_h \cdot b_{th} = 9.62\ in^2$

Moment of inertia of haunch about it centriod $I_h := \dfrac{b_{th} \cdot h_h^{\ 3}}{12} = 0.2\ in^4$

Weight of the haunch $w_h := \left(b_h \cdot h_h\right) \cdot \gamma_c = 0.0104\ \dfrac{kip}{ft}$

Total height of composite beam $h_c := h_d + h_g + h_h = 62\ in$

Total area of composite beam $A_c := A_d + A_g + A_h = \left(1.32 \cdot 10^3\right)\ in^2$

Total weight of the composite beam $w_c := w_g + w_d + w_h = 1.43\ \dfrac{kip}{ft}$

Neutral axis location from bottom for composite beam

$$y_{cbot} := \frac{A_g \cdot y_{gbot} + A_d \cdot \left(h_c - \dfrac{h_d}{2}\right) + A_h \cdot \left(h_g + \dfrac{h_h}{2}\right)}{A_g + A_d + A_h} = 38.16\ in$$

Neutral axis location from top for composite beam $y_{ctop} := \left(h_c\right) - y_{cbot} = 23.84\ in$

Moment of inertia of composite beam

$$I_{comp} := I_g + I_d + I_h + A_g \cdot \left(y_{cbot} - y_{gbot}\right)^2 + A_d \cdot \left(y_{ctop} - \frac{h_d}{2}\right)^2 + A_h \cdot \left(y_{ctop} - h_d - \frac{h_h}{2}\right)^2 = \left(6.18 \cdot 10^5\right)\ in^4$$

Shear Force and Bending Moment due to Dead Loads

Dead Loads:

Dead loads acting on the non-composite structure:

Self-weight of the girder $w_g = 0.82 \ \dfrac{kip}{ft}$

Weight of cast-in-place deck on each interior girder $w_d = 0.6 \ \dfrac{kip}{ft}$

Weight of haunch on each interior girder $w_h = 0.01 \ \dfrac{kip}{ft}$

Total dead load on non-composite section $w_T := w_g + w_d + w_h = 1.43 \ \dfrac{kip}{ft}$

Superimposed Dead Loads:

Dead and live load on the deck must be distributed to the precast, prestressed beams. AASHTO provides factors for the distribution of live load into the beams. The same factors can be used for dead loads if the following criteria is met [AASHTO Art. 4.6.2.2.1]:

- Width of deck is constant [OK]
- Number of beams is not less than four,

$$\begin{array}{l} \text{if } N_{beams} < 4 \\ \quad \text{“NOT OK”} \\ \text{else} \\ \quad \text{“OK”} \end{array} = \text{“OK”}$$

- Beams are parallel and have approximately the same stiffness

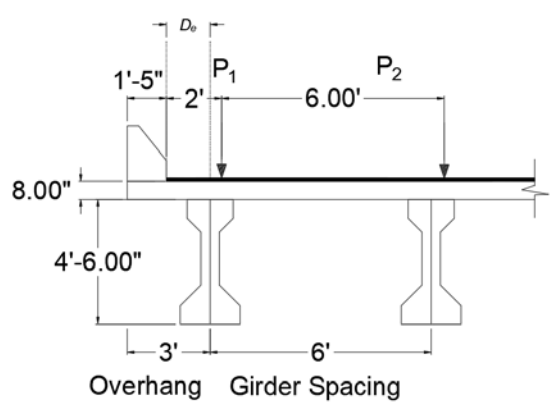
- The overhang minus the barrier width does not exceed 3.0 feet

$D_e = Overhang - 17 \ in$

$Overhang := 3 \ ft$

$D_e := Overhang - 17 \ in = 19 \ in$

$$\begin{array}{l} \text{if } D_e > 3 \ ft \\ \quad \text{“NOT OK”} \\ \text{else} \\ \quad \text{“OK”} \end{array} = \text{“OK”}$$



- Curvature in plan is less than the limit specified in Article 4.6.1.2.4 [OK]

- Cross section of the bridge is consistent with one of the cross sections given in AASHTO Table 4.6.2.2.1-1 Precast concrete I sections are specified as Type k [OK]

Because all of the above criteria are satisfied, the barrier and wearing surface loads are equally distributed among the six girders.

Weight of T501 rails or barriers on each girder $$w_b := 2 \cdot \left(\frac{\gamma_{bw}}{6}\right) = 0.11\ \frac{kip}{ft}$$

Weight of 2.0 in. wearing surface $$w_{ws1} := \gamma_{aws} \cdot \left(h_{ws}\right) = 0.03\ \frac{kip}{ft^2}$$

This load is applied over the entire clear roadway width.Weight of wearing surface on each girder $$w_{ws} := \frac{w_{ws1} \cdot w_{roadway}}{6} = 0.15\ \frac{kip}{ft}$$

Total superimposed dead load $$w_{SD} := w_b + w_{ws} = 0.26\ \frac{kip}{ft}$$

Calculate modular ratio between girder and deck (AASHTO Eq. 4.6.2.2.1-2) $$n := \frac{E_c}{E_{cDeck}} = 1.04$$

Calculate e_g, the distance between the center of gravity of the non-composite beam and the deck. Ignore the thickness of the haunch in determining e_g. It is also possible to ignore the integral wearing surface, i.e, use $h_d = 7.5\ in$. However, the difference in the distribution factor will be minimal.

$$e_g := y_{gtop} + \frac{h_d}{2} = 33.02\ in$$

Calculate K_g , the longitudinal stiffness parameter. (AASHTO Eq. 4.6.2.2.1-1)

$$K_g := n \cdot \left(I_g + A_g \cdot {e_g}^2\right) = \left(1.17 \cdot 10^6\right)\ in^4$$

Moment Distribution Factors

Interior girder typr k (AASHTO 4.6.2.2.2 b)

Distribution factor for moment when one design lane is loaded

$$D_{M.Interior1} := 0.06 + \left(\frac{g_{spacing}}{14\ ft}\right)^{0.4} \cdot \left(\frac{g_{spacing}}{L_{design}}\right)^{0.3} \cdot \left(\frac{K_g}{L_{design} \cdot {h_d}^3}\right)^{0.1} = 0.41$$

Distribution factor for moment when two design lanes are loaded

$$D_{M.Interior2} := 0.075 + \left(\frac{g_{spacing}}{9.5\ ft}\right)^{0.6} \cdot \left(\frac{g_{spacing}}{L_{design}}\right)^{0.2} \cdot \left(\frac{K_g}{L_{design} \cdot {h_d}^3}\right)^{0.1} = 0.56$$

The greater distribution factor is selected for moment design of the beams.

$$D_{M.Interior} := \max\left(D_{M.Interior1}, D_{M.Interior2}\right) = 0.56$$

Check for range of applicability

$$D_{M.Interior} := \left\| \begin{array}{l} S \leftarrow \left(g_{spacing} \geq 3.5\ ft\right) \cdot \left(g_{spacing} \leq 16\ ft\right) \\ t_s \leftarrow \left(h_d \geq 4.5\ in\right) \cdot \left(h_d \leq 12\ in\right) \\ L \leftarrow \left(L_{design} \geq 20\ ft\right) \cdot \left(L_{design} \leq 240\ ft\right) \\ N_b \leftarrow \left(N_{beams} \geq 4\right) \\ K_g \leftarrow \left(K_g \geq 10000\ in^4\right) \cdot \left(K_g \leq 7000000\ in^4\right) \\ \text{if } \left(S \cdot t_s \cdot L \cdot N_b \cdot K_g\right) \\ \quad \left\| D_{M.Interior} \right. \\ \text{else} \\ \quad \left\| \text{“Does not satisfy range of applicability”} \right. \end{array} \right. = 0.56$$

Exterior girder (AASHTO 4.6.2.2.2 d)

$$P_1 = \frac{D_e + S - 2\ ft}{S}$$

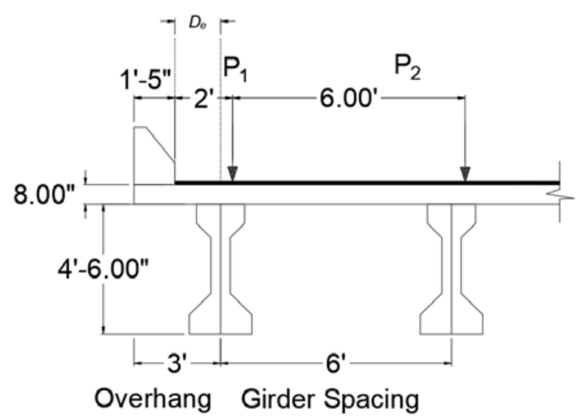
$$P_2 = \frac{D_e + S - 8\ ft}{S}$$

$$D_e = Overhang - 17\ in$$

$$Overhang := 3\ ft$$

$$D_e := Overhang - 17\ in = 19\ in$$

$$S := g_{spacing} = 6\ ft$$



The distribution factor for one design lane loaded is based on the lever rule, which includes a 0.5 factor for converting the truck load to wheel loads and a 1.2 factor for multiple truck presence.

$$D_{M.Exterior1} := \textbf{if}\left(\left(2\ ft + 6\ ft\right) < \left(D_e + S\right), \frac{2 \cdot S + 2\ D_e - 8\ ft}{S} \cdot 0.5, \frac{S + D_e - 2\ ft}{S} \cdot 0.5\right) \cdot 1.2 = 0.56$$

The distribution factor for moment when two design lanes are loaded

$$D_{M.Exterior} = D_{M.Interior} \cdot \left(0.77 + \frac{D_e}{9.1}\right)$$

Using variables defined in this example,

$$D_{M.Exterior2} := D_{M.Interior2} \cdot \left(0.77 + \frac{D_e}{9.1\ ft}\right) = 0.53$$

$$D_{M.Exterior} := \max\left(D_{M.Exterior1}, D_{M.Exterior2}\right) = 0.56$$

Range of applicability

$$D_{M.Exterior} := \left\| \begin{array}{l} d_e \leftarrow \left(D_e \geq -1\ ft\right) \cdot \left(D_e \leq 5.5\ ft\right) \\ \text{if } \left(d_e\right) \\ \quad \left\| D_{M.Exterior} \right. \\ \text{else} \\ \quad \left\| \text{“Does not satisfy range of applicability”} \right. \end{array} \right\| = 0.56$$

For fatigue limit state

The commentary of article 3.4.1 in the AASHTO LRFD specification states that for fatigue limit state a single design truck should be used. However, live load distribution factors given in AASHTO Art 4.6.2.2 take into consideration the multiple presence factor, m. AASHTO Art 3.6.1.1.2 states that the multiple presence factor, m,for one design lane loaded is 1.2. Therefore, the distribution factor for one design lane loaded with the multiple presence factor removed should be used.

Distribution factor for fatigue limit state $D_{MF.Interior} := \frac{D_{M.Interior1}}{1.2} = 0.34$

<u>Shear Distribution Factors</u>

Interior Girder [AASHTO Art. 4.6.2.2.3 a]

Distribution factor for shear when one design lane is loaded

$$D_{S.Interior} = 0.36 + \left(\frac{S}{25}\right)$$

Using variables defined in this example

$$\left(\frac{g_{spacing}}{25\ ft}\right) = 0.24$$

$$D_{S.Interior1} := 0.36 + \left(\frac{g_{spacing}}{25\ ft}\right) = 0.6$$

Distribution factor for shear when two design lanes are loaded

$$D_{S.Interior} = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^{2.0}$$

Using variables defined in this example

$$D_{S.Interior2} := 0.075 + \left(\frac{g_{spacing}}{12\ ft}\right) - \left(\frac{g_{spacing}}{35\ ft}\right)^{2} = 0.55$$

The greater distribution factor is selected for moment design of the beams.

$$D_{S.Interior} := \max\left(D_{S.Interior1}, D_{S.Interior2}\right) = 0.6$$

Check for range of applicability

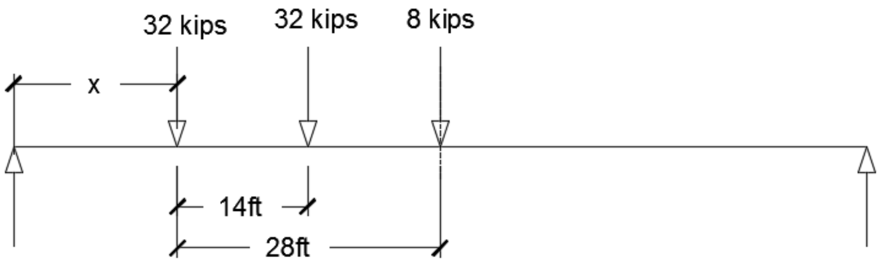
$$D_{S.Interior} := \left\| \begin{array}{l} S \leftarrow \left(g_{spacing} \geq 3.5\ ft\right) \cdot \left(g_{spacing} \leq 16\ ft\right) \\ t_s \leftarrow \left(h_d \geq 4.5\ in\right) \cdot \left(h_d \leq 12\ in\right) \\ L \leftarrow \left(L_{design} \geq 20\ ft\right) \cdot \left(L_{design} \leq 240\ ft\right) \\ N_b \leftarrow \left(N_{beams} \geq 4\right) \\ K_g \leftarrow \left(K_g \geq 10000\ in^4\right) \cdot \left(K_g \leq 7000000\ in^4\right) \\ \text{if } \left(S \cdot t_s \cdot L \cdot N_b \cdot K_g\right) \\ \quad \left\| D_{M.Interior} \right. \\ \text{else} \\ \quad \left\| \text{“Does not satisfy range of applicability”} \right. \end{array} \right. = 0.56$$

The AASHTO Specifications specify the dynamic load effects as a percentage of the static live load effects. AASHTO Table 3.6.2.1-1 specifies the dynamic allowance to be taken as 33 percent of the static load effects for all limit states, except the fatigue limit state, and 15 percent for the fatigue limit state. The factor to be applied to the static load shall be taken as:

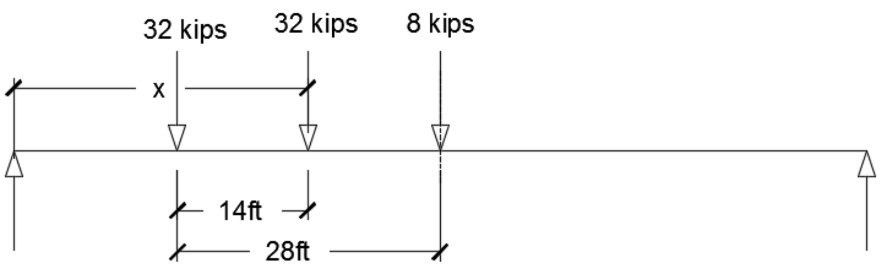
(1 + IM/100)

where:
IM = Dynamic load allowance, applied to truck load or tandem load only
= 33 for all limit states except the fatigue limit state
= 15 for fatigue limit state

The maximum shear forces and bending moments due to HS 20-44 truck loading for all limit states is calculated using the influence line approach. The live load moments and shear forces for the simple span is computed by positioning the axle load of HS-20 truck in following locations



Case I



Case II

Case I: HS-20 truck moment and shear

$P_1 := 32\ kip \qquad P_2 := 32\ kip \qquad P_3 := 8\ kip \qquad x := 5\ ft$

$$M_{truck1}(x) := P_1 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot x + P_2 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}} \cdot x + P_3 \cdot \frac{(L_{design} - x - 28\ ft)}{L_{design}} \cdot x$$

$$V_{truck1}(x) := P_1 \cdot \frac{(L_{design} - x)}{L_{design}} + P_2 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}} + P_3 \cdot \frac{(L_{design} - x - 28\ ft)}{L_{design}}$$

Case II: HS-20 truck moment and shear

$$M_{truck2}(x) := P_1 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot (x - 14\ ft) + P_2 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot x + P_3 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}} \cdot x$$

$$V_{truck2}(x) := P_1 \cdot \frac{-(x - 14\ ft)}{L_{design}} + P_2 \cdot \frac{(L_{design} - x)}{L_{design}} + P_3 \cdot \frac{(L_{design} - x - 14\ ft)}{L_{design}}$$

$$M_{truck1}(\mathbf{maximize}(M_{truck1}, x)) = (1.3 \cdot 10^3)\ ft \cdot kip$$

$M_{truck2}\left(\mathbf{maximize}\left(M_{truck2}, x\right)\right) = \left(1.34 \cdot 10^3\right)\ ft \cdot kip$

Maximum bending moment due to HS 20-44 truck load

$M := \left(1.344 \cdot 10^3\right)\ ft \cdot kip$

The calculation of shear force is carried out later for the critical shear section.

Distributed bending moment due to truck load including dynamic load allowance (M_{LT}) is calculated as follows:

M_{LT} = (Moment per lane due to truck load)(DFM)(1+IM/100)

$IM := 33$

$D_{M.Interior} = 0.56$

$$M_{LT} := M \cdot D_{M.Interior} \cdot \left(1 + \frac{IM}{100}\right) = \left(1 \cdot 10^3\right)\ ft \cdot kip$$

The maximum bending moments (M_L) due to a uniformly distributed lane load of 0.64 klf are calculated using the following formulas given by the *PCI Design Manual* (PCI 2017).

Maximum bending moment, $M_x = 0.5(0.64)(x)(L - x)$

where:
x = Distance from centerline of bearing to section at which the bending moment or shear force is calculated, ft.
L = Design span length

At the section of maximum truck load $\mathbf{maximize}\left(M_{truck2}, x\right) = 47.33\ ft$

$x := 47.333\ ft$

$$M_L := 0.5 \cdot 0.64\ \frac{kip}{ft} \cdot (x)\ \left(L_{design} - x\right) = 646.26\ ft \cdot kip$$

$M_{LL} := D_{M.Interior} \cdot M_L = 361.97\ ft \cdot kip$

For fatigue limit state:
Therefore, the bending moment of the fatigue truck load is:
Mf = (bending moment per lane)(DFM)(1 + IM)

$$M_f := M \cdot D_{MF.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 606.79\ ft \cdot kip$$

Shear forces and bending moments for the girder due to dead loads, superimposed dead loads at every tenth of the design span, and at critical sections (hold-down point or harp point and critical section for shear) are provided in this section. The bending moment (M) and shear force (V) due to uniform dead loads and uniform superimposed dead loads at any section at a distance x from the centerline of bearing are calculated using the following formulas, where the uniform load is denoted as w.

$$M = 0.5w\,x\,(L - x)$$
$$V = w(0.5L - x)$$

The critical section for shear is located at a distance $h_c/2$ from the face of the support. However, as the support dimensions are not specified in this project, the critical section is measured from the centerline of bearing. This yields a conservative estimate of the design shear force.

Inputs

$excel_{\text{“F1”}} := 3.28084\ (L_{design})$ $\qquad excel_{\text{“A13”}} := 0.5 \cdot 3.28084\ (L_{design})$

$excel_{\text{“B1”}} := 0.0000685217\ (w_g)$

$excel_{\text{“D1”}} := 0.0000685217\ (w_d + w_h)$

$excel_{\text{“B2”}} := 0.0000685217\ (w_b)$

w_g	0.822		0.61042		90.000		L		90.000
w_{SD}	0.109								
Distance (x)	Section (x/L)		Dead Load			Barrier weight		Total Dead Load	
		Girder Weight		Slab Weight					
		Shear	Moment	Shear	Moment	Shear	Moment	Shear	Moment
ft.		k	k-ft	k	k-ft	k	k-ft	k	k-ft
0	0.000	36.990	0.00	27.469	0.00	4.890	0.00	69.35	0.00
4.654	0.052	33.164	163.25	24.628	121.23	4.384	21.58	62.18	306.06
10.858	0.121	28.065	353.18	20.841	262.27	3.710	46.69	52.62	662.14
21.717	0.241	19.139	609.47	14.212	452.59	2.530	80.57	35.88	1142.64
32.575	0.362	10.213	768.82	7.584	570.93	1.350	101.64	19.15	1441.39
43.433	0.483	1.288	831.26	0.957	617.30	0.170	109.89	2.41	1558.45

Outputs

$V_g := excel_{\text{“C8”}} \cdot kip \qquad V_s := excel_{\text{“E8”}} \cdot kip \qquad V_b := excel_{\text{“G8”}} \cdot kip$

$M_{gv} := excel_{\text{“D8”}} \cdot ft \cdot kip \qquad M_{sv} := excel_{\text{“F8”}} \cdot ft \cdot kip \qquad M_{bv} := excel_{\text{“G8”}} \cdot ft \cdot kip$

The AASHTO design live load is designated as HL-93, which consists of a combination of:

- Design truck with dynamic allowance or design tandem with dynamic allowance, whichever produces greater moments and shears, and
- Design lane load without dynamic allowance. [AASHTO Art. 3.6.1.2]

The design truck is designated as HS 20-44 consisting of an 8 kip front axle and two 32 kip rear axles. [AASHTO Art. 3.6.1.2.2]

The design tandem consists of a pair of 25-kip axles spaced 4 ft. apart. However, for spans longer than 40 ft. the tandem loading does not govern, thus only the truck load is investigated in this example. [AASHTO Art. 3.6.1.2.3]

The lane load consists of a load of 0.64 klf uniformly distributed in the longitudinal direction. [AASHTO Art. 3.6.1.2.4]

This design example considers only the dead and vehicular live loads. The wind load and the extreme event loads, including earthquake and vehicle collision loads, are not included in the design. Various limit states and load combinations provided by AASHTO Art. 3.4.1 are investigated, and the following limit states are found to be applicable in present case:

Service I: This limit state is used for normal operational use of a bridge. This limit state provides the general load combination for service limit state stress checks and applies to all conditions except Service III limit state. For prestressed concrete components, this load combination is used to check for compressive stresses. The load combination is presented as follows[AASHTO Table 3.4.1-1]:
$Q = 1.00\,(DC + DW) + 1.00(LL + IM)$

Service III: This limit state is a special load combination for service limit state stress checks that applies only to tension in prestressed concrete structures to control cracks. The load combination for this limit state is presented as follows [AASHTOTable 3.4.1-1]:
$Q = 1.00(DC + DW) + 0.80(LL + IM)$
(Subsequent revisions to the AASHTO specification have revise this load combination)

Strength I: This limit state is the general load combination for strength limit state design relating to the normal vehicular use of the bridge without wind. The load combination is presented as follows [AASHTO Table 3.4.1-1 and 2]:
$Q = \gamma P(DC) + \gamma P(DW) + 1.75(LL + IM)$

Type of Load	Load Factor, γ_P	
	Maximum	Minimum
DC: Structural components and non-structural attachments	1.25	0.90
DW: Wearing surface and utilities	1.50	0.65

The maximum and minimum load combinations for the Strength I limit state are presented as follows:
Maximum $Q = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)$
Minimum $Q = 0.90(DC) + 0.65(DW) + 1.75(LL + IM)$

Estimation of Required Prestress

The required number of strands is usually governed by concrete tensile stress at the bottom fiber of the girder at the midspan section. The load combination for the Service III limit state is used to evaluate the bottom fiber stresses at the midspan section. The calculation for compressive stress in the top fiber of the girder at midspan section under service loads is also shown in the following section. The compressive stress is evaluated using the load combination for the Service I limit state.

Service Load Stresses at Midspan

Bottom tensile stress due to applied dead and live loads using load combination Service III is:

$$f_b = \frac{M_g + M_d}{S_b} + \frac{M_b + M_{ws} + (0.8)\cdot(M_{LT} + M_{LL})}{S_{bc}}$$

f_b=Concrete stress at the bottom fiber of the girder, ksi

M_g=unfactored bending moment due to beam self-weight, k-ft

$$M_g := \frac{w_g \cdot L_{design}^{\ 2}}{8} = 832.28\ ft \cdot kip$$

M_d=unfactored bending moment due to deck self-weight and haunch, k-ft

$$M_d := \frac{\left(w_d + w_h\right) \cdot L_{design}^{\ 2}}{8} = 618.05\ ft \cdot kip$$

M_b=unfactored bending moment due to barrier self-weight, k-ft

$$M_b := \frac{w_b \cdot L_{design}^{\ 2}}{8} = 110.03\ ft \cdot kip$$

M_{ws}=unfactored bending moment due to future wearing , k-ft

$$M_{ws} := \frac{w_{ws} \cdot L_{design}^{\ 2}}{8} = 151.88\ ft \cdot kip$$

M_{LT}=unfactored bending moment due to truck load (kip-ft) $M_{LT} = \left(1 \cdot 10^3\right)\ ft \cdot kip$

M_{LL}=unfactored bending moment due to truck load (kip-ft) $M_{LL} = 361.97\ ft \cdot kip$

S_{bc}=composite section modulus for extreme bottom fiber of precast beam (in^3)

Using the variables used in this example

$$S_{cbot} := \frac{I_{comp}}{y_{cbot}} = \left(1.62 \cdot 10^4\right)\ in^3$$

$$f_b := \frac{M_g + M_d}{S_{gbot}} + \frac{M_b + M_{ws} + (0.8) \cdot \left(M_{LT} + M_{LL}\right)}{S_{cbot}} = 2.65\ ksi$$

Stress Limits for Concrete

The tensile stress limit at service load = 0 [AASHTO Table 5.9.2.3.2b-1]

Concrete tensile stress limit = $f_{tl} := 0 \cdot ksi$

Required Number of Strands

The required pre-compressive stress at the bottom fiber of the beam is the difference the between bottom tensile stress due to the applied loads and the concrete tensile stress limits:

Required pre-compressive stress at bottom fiber, $f_{pb} := f_b - f_{tl} = 2.65\ ksi$

Assume the distance between the center of gravity of the bottom strands and the bottom fiber of the beam:

$e_c := y_{gbot} = 24.73\ in$

If Ppe is the total prestressing force, the stress at the bottom fiber due to prestress is:

$$f_{pb} = \frac{P_{pe}}{A} + \frac{P_{pe} \cdot e_c}{S_b}$$

Using the variables in this example

$$P_{pe} := \frac{f_{pb}}{\left(\frac{1}{A_g} + \frac{e_c}{S_{gbot}}\right)} = 734.38\ kip$$

Final prestress force per strand, P_{pf} = (area of prestressing CFRP) (f_{pi}) (1 − $ploss$, %)

$ploss := 20\%$

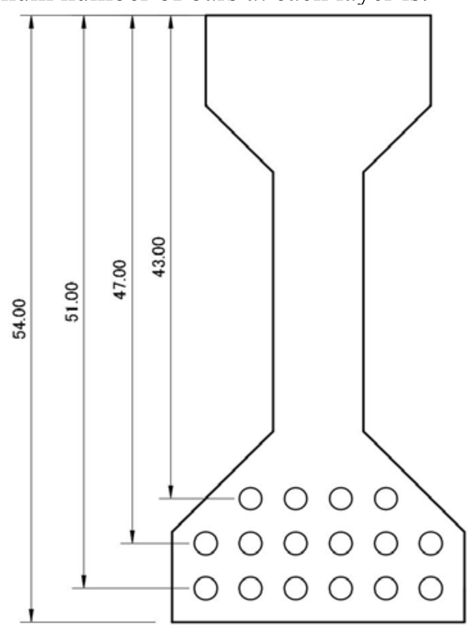
$P_{pf} := A_{pf} \cdot f_{pi} \cdot (1 - ploss) = 58.91\ kip$

The number of prestressing CFRP is equal to $n_p := \frac{P_{pe}}{P_{pf}} = 12.47$

$$n_p := \text{round}\left(\frac{(n_p)}{2}\right) \cdot 2 + 4 = 16$$

Change the number of bars based on the value of n_p.
If no bars is needed at certain layer input 0.
The maximum number of bars at each layer is:

$n_{b1}=6$
$n_{b2}=6$
$n_{b3}=4$
$n_{b4}=2$
$n_{b5}=1$
$n_{b6}=1$
$n_{b7}=1$



$n_{b1} := 6$ $\quad d_{p1} := 51\ in$

$n_{b2} := 6$ $\quad d_{p2} := 47\ in$

$n_{b3} := 4$ $\quad d_{p3} := 43\ in$

$n_{b4} := 0$ $\quad d_{p4} := 39\ in$

$n_{b5} := 0$ $\quad d_{p5} := 35\ in$

$n_{b6} := 0$ $\quad d_{p6} := 31\ in$

$n_{b7} := 0$ $\quad d_{p7} := 27\ in$

$n_{b8} := 0$ $\quad d_{p8} := 23\ in$

$n_{b9} := 0$ $\quad d_{p9} := 19\ in$

The center of gravity of the strands, c.g.s. = $\dfrac{\sum n_i y_i}{N}$

where: n_i = number of strands in row i
y_i = distance to center of row i from bottom of beam section
N = total number of strands

$$x_p := h_g - \frac{n_{b1} \cdot d_{p1} + n_{b2} \cdot d_{p2} + n_{b3} \cdot d_{p3} + n_{b4} \cdot d_{p4} + n_{b5} \cdot d_{p5} + n_{b6} \cdot d_{p6} + n_{b7} \cdot d_{p7} + n_{b8} \cdot d_{p8} + n_{b9} \cdot d_{p9}}{n_{b1} + n_{b2} + n_{b3} + n_{b4} + n_{b5} + n_{b6} + n_{b7} + n_{b8} + n_{b9}} = 6.5\ in$$

$e_c := y_{gbot} - x_p = 18.23\ in$ $\qquad y_{gbot} = 24.73\ in$

$n_p := n_{b1} + n_{b2} + n_{b3} + n_{b4} + n_{b5} + n_{b6} + n_{b7} + n_{b8} + n_{b9} = 16$

$P_{pe} := n_p \cdot P_{pf} = 942.59\ kip$ $\qquad h_g = 54\ in$

$$f_{pb} := P_{pe} \cdot \left(\frac{1}{A_g} + \frac{e_c}{S_{gbot}} \right) = 2.82\ ksi$$

midspan center of gravity of prestressing CFRP $y_{bs} := x_p = 6.5\ in$

midspan prestressing CFRP eccentricity $e_c := y_{gbot} - y_{bs} = 18.23\ in$

<u>Prestress Losses</u> [AASHTO Art. 5.9.3]

Total prestress loss "Δf_{pT}"

$$\Delta f_{pT} = \Delta f_{pF} + \Delta f_{pA} + \Delta f_{pES} + \Delta f_{pLT}$$

Δf_{pF} = loss due to friction (ksi)

Δf_{pA} = loss due to anchorage set (ksi)

Δf_{pES} = sum of all losses or gains due to elastic shortening or extension at time of application of prestress and/or external loads (ksi)

Δf_{pLT} = losses due to long-term shrinkage and creep of concrete, and relaxation of the prestressing CFRP (ksi)

Fricition Losses [AASHTO Art. 5.9.3.2.2b]

Losses due to friction between the internal prestressing tendons and the duct wall may be taken as:

$$\Delta f_{pF} = f_{pj} \cdot \left(1 - e^{-(K \cdot x + \mu \cdot \alpha)}\right)$$

f_{pj} = stress in the prestressing CFRP at jacking (ksi)
x = length of a prestressing CFRP from the jacking end to any point under consideration (ft)
K = wobble friction coefficient (per ft of tendon)
μ = coefficient of friction
α = sum of the absolute values of angular change of prestressing CFRP path from jacking end, or from the nearest jacking end if tensioning is done equally at both ends, to the point under investigation (rad.)
e = base of Napierian logarithms

Values of K and μ should be based on experimental data for the materials specified and shall be shown in the contract documents.

$K := 0.00026 \cdot \frac{1}{ft}$ $\mu := 0.3$ *Note: These numbers are based on the experiments done in AASHTO-CFRP*

$\alpha := 0 \cdot rad$ As a starting point, assume all cables are straight

Friciton losses at the non-jacking end

$$x := L_{design} = 90\ ft \qquad \Delta f_{pF} := f_{pi} \cdot \left(1 - e^{-(K \cdot x + \mu \cdot \alpha)}\right) = 5.89\ ksi$$

Fricton losses at the middle

$$x := \frac{L_{design}}{2} = 45\ ft \qquad \Delta f_{pF} := f_{pi} \cdot \left(1 - e^{-(K \cdot x + \mu \cdot \alpha)}\right) = 2.96\ ksi$$

Anchorage Set Losses

Based on experimental investigation in NCHRP Project 12-97, the anchorage set losses for Socket type anchor was measured for the full-scale post-tensioned beams and found to be less than 1.0 percent [AASHTO-CFRP *C1.9.2.2.1*]

$$\Delta f_{pA} := 0.01 \cdot f_{pi} = 2.55\ ksi$$

Elastic Shortening

When the prestressing force is transferred from the prestressing strands to the concrete member, the force causes elastic shortening of the member as it cambers upward. This results in a loss of the initial prestress of the strands. However, some of that loss is gained back due to the self-weight of the member which creates tension in the strands.

$$\Delta f_{pES} = \frac{\left(N_p - 1\right)}{2 \cdot N_p} \cdot \frac{E_f}{E_{ct}} \cdot f_{cgp} \qquad \text{[AASHTO-CFRP Eq. 1.9.2.2.3a-1]}$$

Where E_f=modulus of elasticity of prestressing CFRP (ksi)

E_{ct}=modulus of elasticity of the concrete at transfer or time of load application (ksi)=E_{ci}

f_{cgp}=the concrete stress at the center of gravity of CFRP due to the prestressing force immediately after transfer and the self-weight of the member at sections of maximum moment (ksi)

AASHTO Article C5.9.5.2.3a states that to calculate the prestress after transfer, an initial estimate of prestress loss is assumed and iterated until acceptable accuracy is achieved. In this example, an initial estimate of 10% is assumed.

where

N_p= The number of Identical Prestressing CFRP

Force per strand at transfer

$$f_{cgp} = \frac{P_i}{A_g} + \frac{P_i \cdot {e_c}^2}{I_g} - \frac{M_G \cdot e_c}{I_g}$$

Where, P_i=total prestressing force at release=$n_p * p$

e_c=eccentricity of strands measured from the center of gravity of the precast beam at midspan

M_G=moment due to beam self-weight at midspan (should be calculated using the overall beam length)

$$M_G := \frac{w_g \cdot (L_{span})^2}{8} = 850.87\ ft \cdot kip$$

Guess Values

$$eloss := 10\%$$

Constraints

$$eloss = \frac{E_f}{f_{pi} \cdot E_{ci}} \cdot \left(\frac{n_p \cdot A_{pf} \cdot f_{pi} \cdot (1 - eloss)}{A_g} + \frac{(n_p \cdot A_{pf} \cdot f_{pi} \cdot (1 - eloss)) \cdot e_c^{\ 2}}{I_g} - \frac{M_G \cdot e_c}{I_g} \right) \cdot \frac{(n_p - 1)}{2 \cdot n_p}$$

Solver

$$eloss := \mathbf{find}(eloss) = 0.0195$$

Therefore, the loss due to elastic shortening= $eloss = 0.02$

The force per strand at transfer= $p := A_{pf} \cdot f_{pi} \cdot (1 - eloss) = 72.21\ kip$

The concrete stress due to prestress=

$$f_{cgp} := \frac{n_p \cdot p}{A_g} + \frac{n_p \cdot p \cdot e_c^{\ 2}}{I_g} - \frac{M_G \cdot e_c}{I_g} = 2.22\ ksi$$

The prestress loss due to elastic shortening= $\Delta f_{pES} := \frac{(n_p - 1)}{2 \cdot n_p} \cdot \frac{E_f}{E_{ci}} \cdot f_{cgp} = 4.96\ ksi$

Total prestressing force at release

$$P_i := n_p \cdot p = (1.16 \cdot 10^3)\ kip$$

Long Term Losses

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}) + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})$$

Δf_{pSR}=prestress loss due to shrinkage of girder concrete between time of transfer and deck placement (ksi)

Δf_{pCR}=prestress loss due to creep of girder concrete between time of transfer and deck placement (ksi)

Δf_{pR1}=prestress loss due to relaxation of prestressing strands between time of transfer and deck placement ksi)

Δf_{pSD}=prestress loss due to shrinkage of girder concrete between time of deck placement and final time (ksi)
Δf_{pCD}=prestress loss due to creep of girder concrete between time of deck placement and final time (ksi)
Δf_{pR2}=prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time (ksi)
Δf_{pSS}=prestress gain due to shrinkage of deck in composite section (ksi)
$\left(\Delta f_{pSR}+\Delta f_{pCR}+\Delta f_{pR1}\right)$ =sum of time-dependent prestress losses between time of transfer and deck placement (ksi)
$\left(\Delta f_{pSD}+\Delta f_{pCD}+\Delta f_{pR2}-\Delta f_{pSS}\right)$ =sum of time-dependent prestress losses after deck placement (ksi)

Prestress Losses: Time of Transfer to Time of Deck Placement
Shrinkage of Girder Concrete

$\Delta f_{pSR}=\varepsilon_{bid}\cdot E_{pc}\cdot K_{id}$ [AASHTO Eq. 5.9.3.4.2a-1]

where, ε_{bid}=shrinkage strain of girder between the time of transfer and deck placement

$=k_s\cdot k_{hs}\cdot k_f\cdot k_{td}\cdot 0.48\cdot 10^{-3}$ [AASHTO Eq. 5.4.2.3.3-1]

and, k_s=factor for the effect of volume to surface ratio of the component
k_s=1.45-0.13(V/S)
where (V/S)=volume to surface ratio=(Area/Perimeter)

[AASHTO Eq. 5.4.2.3.2-2]

Perimeter

$$P_g:=2\left(\frac{b_{brf}}{2}+h_{brf}+\sqrt{{b_{btf}}^2+{h_{btf}}^2}+h_w+\sqrt{{b_{ttf}}^2+{h_{ttf}}^2}+h_{trf}+\frac{b_{trf}}{2}\right)=166.43\ in$$

$$k_s:=1.45-0.13\cdot\frac{1}{in}\left(\frac{A_g}{P_g}\right)=0.83 \qquad k_s:=\begin{Vmatrix}\text{if } k_s\le 1 \\ \quad\Vert\ 1 \\ \text{else} \\ \quad\Vert\ k_s\end{Vmatrix}=1$$

k_{hs}=humidity factor for shrinkage=2.00-0.014H [AASHTO Eq. 5.4.2.3.3-2]
$H:=70$

$k_{hs}:=2.00-0.014\cdot H=1.02$

k_f=factor for the effect of concrete strength [AASHTO Eq. 5.4.2.3.2-4]
5

$$=\frac{}{1+f'_{ci}}$$

$$k_f:=\frac{5}{1+\frac{f'_{ci}}{ksi}}=0.71$$

k_{td}=time developement factor

$$=\frac{t}{61-4\cdot f'_{ci}+t}$$ [AASHTO Eq. 5.4.2.3.2-5]

$t_d:=90$ $t_i:=1$

$$k_{td}\left(t,t_i\right):=\frac{t-t_i}{61-4\cdot\frac{f'_{ci}}{ksi}+t-t_i}$$

$$k_{td}\left(t_d,t_i\right)=0.71$$

$$\varepsilon_{bid}:=k_s\cdot k_{hs}\cdot k_f\cdot k_{td}\left(t_d,t_i\right)\cdot 0.48\cdot 10^{-3}=2.47\cdot 10^{-4}$$

K_{id}=transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between transfer and deck placement

$$=\frac{1}{1+\frac{E_{pc}}{E_{ci}}\cdot\frac{A_{psc}}{A_g}\cdot\left(1+\frac{A_g\cdot {e_{pg}}^2}{I_{comp}}\right)\left(1+0.7\ \Psi_b\left(t_f,t_i\right)\right)}$$ [AASHTO Eq. 5.9.3.4.2a-2]

where, $\Psi_b\left(t_f,t_i\right)=1.9\cdot k_s\cdot k_{hc}\cdot k_f\cdot k_{td}\cdot {t_i}^{-0.118}$ [AASHTO Eq. 5.4.2.3.2-1]

k_{hc}=humidity factor for creep=1.56-0.008H [AASHTO Eq. 5.4.2.3.2-3]

$e_{pg}:=e_c=18.23\ in$

$k_{hc}:=1.56-0.008\ H=1$

$t_f:=20000$ $t_i:=1$

$k_{td}\left(t_f,t_i\right)=1$

$$\Psi_b\left(t,t_i\right):=1.9\cdot k_s\cdot k_{hc}\cdot k_f\cdot k_{td}\left(t,t_i\right)\cdot\left(t_i\right)^{-0.118}$$

$$\Psi_b\left(t_f,t_i\right)=1.35$$

$$K_{id}:=\frac{1}{1+\frac{E_f}{E_{ci}}\cdot\frac{A_{pf}}{A_g}\cdot\left(1+\frac{A_g\cdot e_c^{\ 2}}{I_g}\right)\left(1+0.7\cdot\Psi_b\left(t_f,t_i\right)\right)}=0.99$$

$$\Delta f_{pSR}:=\varepsilon_{bid}\cdot E_f\cdot K_{id}=5.57\ ksi$$

Creep of Girder Concrete

$$\Delta f_{pCR}=\frac{E_{pc}}{E_{ciACI}}\cdot f_{cgp}\cdot\Psi_b\left(t_d,t_i\right)\cdot K_{id}$$ [AASHTO Eq. 5.9.3.4.2b-1]

Where, $\Psi_b\left(t_d,t_i\right)$ =girder creep coefficient at time of deck placement due to loading introduced at transfer

$$=1.9\cdot k_s\cdot k_{hc}\cdot k_f\cdot k_{td}\cdot\left(t_i\right)^{-0.118}$$

$$\Delta f_{pCR}:=\frac{E_f}{E_{ci}}\cdot f_{cgp}\cdot\Psi_b\left(t_d,t_i\right)\cdot K_{id}=10.08\ ksi$$

Relaxation of Prestressing Strands

$$\Delta f_{pR1}=\left(0.0215\cdot\left(\frac{f_{pt}}{f_{pu}}\right)-0.0076\right)\log\left(t\cdot 24\right)\cdot f_{pu}$$ [AASHTO-CFRP Eq. 1.9.2.5.2-2]

Where, f_{pt}=stress in prestressing strands immediately after transfer,

$$f_{pt}:=f_{pi}-\Delta f_{pES}=249.85\ ksi$$

t=time between strand prestressing and deck placement (days)

$$t:=t_i+t_d=91$$

Therefore,

$$\Delta f_{pR1}:=\left(0.0215\cdot\left(\frac{f_{pt}}{f_{pu}}\right)-0.0076\right)\log\left(t\cdot 24\right)\cdot f_{pu}=8.7\ ksi$$

Prestress Losses: Time of Deck Placement to Final Time

Shrinkage of Girder Concrete

$$\Delta f_{pSD}=\varepsilon_{bdf}\cdot E_f\cdot K_{df}$$

where ε_{bdf}=shrinkage strain of girder between the time of deck placement and final time

$= \varepsilon_{bif} - \varepsilon_{bid}$

$$\varepsilon_{bif} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}\left(t_f, t_i\right) \cdot 0.48 \cdot 10^{-3} = 3.49 \cdot 10^{-4}$$

$$\varepsilon_{bdf} := \varepsilon_{bif} - \varepsilon_{bid} = 1.02 \cdot 10^{-4}$$

K_{df}=transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between deck placement and final time

$$= \frac{1}{1 + \frac{E_{pc}}{E_{ci}} \cdot \frac{A_{psc}}{A_c} \cdot \left(1 + \frac{A_c \cdot e_{pc}^{\ 2}}{I_c}\right) \left(1 + 0.7\ \Psi_b\left(t_f, t_i\right)\right)}$$ [AASHTO Eq. 5.9.3.4.3a-2]

where, e_{pc}=eccentricity of prestressing force with respect to centroid of composite section (in); positive in common construction where force is below centroid

$= y_{cbot} - y_{bs}$

A_c=area of section calculated using the gross composite concrete section properties of the girder and the deck, and the deck-to-girder modular ratio

I_c =moment of inertia calculated using gross composite concrete properties of the girder and the deck, and the deck-to-girder modular ratio at service=I_{comp}

$e_{pc} := y_{cbot} - y_{bs} = 31.66\ in$ $\quad\quad$ $\Psi_b\left(t_f, t_i\right) = 1.35$

$$K_{df} := \frac{1}{1 + \frac{E_f}{E_{ci}} \cdot \frac{A_{pf}}{A_c} \cdot \left(1 + \frac{A_c \cdot e_{pc}^{\ 2}}{I_{comp}}\right) \left(1 + 0.7 \cdot \Psi_b\left(t_f, t_i\right)\right)} = 0.99$$

$$\Delta f_{pSD} := \varepsilon_{bdf} \cdot E_f \cdot K_{df} = 2.3\ ksi$$

Creep of Girder Concrete

$$\Delta f_{pCD} = \frac{E_f}{E_{ci}} \cdot f_{cgp} \cdot \left(\Psi_b\left(t_f, t_i\right) - \Psi_b\left(t_d, t_i\right)\right) \cdot K_{df} + \frac{E_f}{E_c} \cdot \Delta f_{cd} \cdot \Psi_b\left(t_f, t_d\right) \cdot K_{df}$$

[AASHTO Eq. 5.9.3.4.3b-1]

Where, $\Psi_b\left(t_f, t_d\right)$=girder creep coefficient at final time due to loading at deck placement

$$= 1.9 \cdot k_s \cdot k_{hc} \cdot k_f \cdot k_{td}\left(t_f, t_d\right) \cdot \left(t_i\right)^{-0.118}$$

$$\Psi_b\left(t_f, t_d\right) = 0.8$$

Δf_{cd}=change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement, combined with deck weight on the non-composite transformed section, and superimposed loads on the composite transformed section (ksi)

$$=-\left(\Delta f_{pSR}+\Delta f_{pCR}+\Delta f_{pR1}\right)\cdot\frac{A_{pf}}{A_g}\cdot\left(1+\frac{A_g\cdot e_c^{\,2}}{I_g}\right)-\left(\frac{M_S\cdot e_{ptf}}{I_{tf}}+\frac{\left(M_b+M_{ws}\right)\cdot e_{ptc}}{I_{tc}}\right)$$

Where e_{ptf}=eccentricity of the prestressing force with respect to the centroid of the non-composite transformed section

e_{ptc}=eccentricity of the prestressing force with respect to the centroid of the non-composite transformed section

I_{tf}=moment of inertia of the non-composite transformed section

I_{tc}=moment of inertia of the composite transformed section

To perform the calculations, it is necessary to calculate the non-composite and composite transformed section properties

$$n_{ci}:=\frac{E_f}{E_{ci}}=4.76 \qquad n_c:=\frac{E_f}{E_c}=4.17$$

Inputs

$$excel_{\text{“B3”}}:=A_g\cdot 39.3701^2 \qquad excel_{\text{“C3”}}:=y_{gbot}\cdot 39.3701 \qquad excel_{\text{“G3”}}:=I_g\cdot 39.3701^4$$

$$excel_{\text{“B4”}}:=n_{ci}\cdot n_p\cdot A_{pf}\cdot 39.3701^2 \qquad excel_{\text{“C4”}}:=y_{bs}\cdot 39.3701$$

Non-composite transformed section properties at transfer

	Area (in^2)	**y_b (in)**	**Ay_b (in^3)**	**$y_{b,tt}$ (in)**	**$A*(y_{b,tt}-y_b)^2$ (in^4)**	**I (in^4)**	**$I+A*(y_{b,tt}-y_b)^2$ (in^4)**
Beam	789.00	24.73	19512.00	24.23	193.36	260730.56	260923.92
Pre. CFRP	22.02	6.50	143.15		6927.10	0.00	6927.10
	811.02		19655.16			I (in^4)	267851.02

Outputs

Inputs

$excel_{\text{"B3"}} := A_g \cdot 39.3701^2$ $\quad excel_{\text{"C3"}} := y_{gbot} \cdot 39.3701$ $\quad excel_{\text{"G3"}} := I_g \cdot 39.3701^4$

$excel_{\text{"B4"}} := n_c \cdot n_p \cdot A_{pf} \cdot 39.3701^2$ $\quad excel_{\text{"C4"}} := y_{bs} \cdot 39.3701$

Non-composite transformed section properties at service

	Area (in^2)	y_b (in)	Ay_b (in^3)	$y_{b,tt}$ (in)	$A*(y_{b,tt}-y_b)^2$ (in^4)	I (in^4)	$I+A*(y_{b,tt}-y_b)^2$ (in^4)
Beam	789.00	24.73	19512.00	24.30	148.97	260730.56	260879.53
Pre. CFRP	19.27	6.50	125.23		6100.99	0.00	6100.99
	808.27		19637.23			I (in^4)	266980.52

Outputs

$e_{ptf} := excel_{\text{"E3"}} \cdot in$ $\quad I_{tf} := excel_{\text{"H5"}} \cdot in^4$

Inputs

$excel_{\text{"B3"}} := A_c \cdot 39.3701^2$ $\quad excel_{\text{"C3"}} := y_{cbot} \cdot 39.3701$ $\quad excel_{\text{"G3"}} := I_{comp} \cdot 39.3701^4$

$excel_{\text{"B4"}} := n_c \cdot n_p \cdot A_{pf} \cdot 39.3701^2$ $\quad excel_{\text{"C4"}} := y_{bs} \cdot 39.3701$

Composite transformed section properties at service

	Area (in^2)	y_b (in)	Ay_b (in^3)	$y_{b,tt}$ (in)	$A*(y_{b,tt}-y_b)^2$ (in^4)	I (in^4)	$I+A*(y_{b,tt}-y_b)^2$ (in^4)
Beam	1318.034	38.1551	50289.72	37.70	274.11	617603.94	617878.04
Pre. CFRP	19.27	6.500004	125.23		18752.69	0.00	18752.69
	1337.30		50414.95			I (in^4)	636630.73

Outputs

$e_{ptc} := excel_{\text{"E3"}} \cdot in$ $\quad I_{tc} := excel_{\text{"H5"}} \cdot in^4$

From table above $\quad e_{ptf} := 20.02\ in$ $\quad e_{ptc} := 33.15\ in$

$I_{tf} := 266584.66\ in^4$ $\quad I_{tc} := 686823.24\ in^4$

$$\Delta f_{cd} := -\left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) \cdot \frac{A_{pf}}{A_g} \cdot \left(1 + \frac{A_g \cdot e_c^{\ 2}}{I_g}\right) - \left(\frac{M_d \cdot e_{ptf}}{I_{tf}} + \frac{\left(M_b + M_{ws}\right) \cdot e_{ptc}}{I_{tc}}\right) = -0.73\ ksi$$

$$\Delta f_{pCD} := \frac{E_f}{E_{ci}} \cdot f_{cgp} \cdot \left(\Psi_b\left(t_f, t_i\right) - \Psi_b\left(t_d, t_i\right)\right) \cdot K_{df} + \frac{E_f}{E_c} \cdot \Delta f_{cd} \cdot \Psi_b\left(t_f, t_d\right) \cdot K_{df} = 1.77\ ksi$$

Relaxation of Prestressing Strands

$$\Delta f_{pR2} = \Delta f_{pRf} - \Delta f_{pR1}$$

$$\Delta f_{pRf} = \left(0.020 \cdot \left(\frac{f_{pt}}{f_{pu}}\right) - 0.0066\right) \log\left(t \cdot 24\right) \cdot f_{pu}$$ [AASHTO-CFRP Eq. 1.9.2.5.2-2]

Where, f_{pt}=stress in prestressing strands immediately after transfer,

$$f_{pt} := f_{pi} - \Delta f_{pES} = 249.85\ ksi$$

t=time between strand prestressing and final (days)

$$t := t_i + t_f = 2 \cdot 10^4$$

Therefore,

$$\Delta f_{pRf} := \left(0.020 \cdot \left(\frac{f_{pt}}{f_{pu}}\right) - 0.0066\right) \log\left(t \cdot 24\right) \cdot f_{pu} = 14.74\ ksi$$

$$\Delta f_{pR2} := \Delta f_{pRf} - \Delta f_{pR1} = 6.04\ ksi$$

Shrinkage of Deck Concrete

$$\Delta f_{pSS} = \frac{E_f}{E_c} \cdot \Delta f_{cdf} \cdot K_{df} \cdot \left(1 + 0.7 \cdot \Psi_b\left(t_f, t_d\right)\right)$$ [AASHTO Eq. 5.9.3.4.3d-1]

Where, Δf_{cdf}=change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete (ksi)

$$= \frac{\varepsilon_{ddf} \cdot A_d \cdot E_{cd}}{\left(1 + 0.7 \cdot \Psi_b\left(t_f, t_d\right)\right)} \cdot \left(\frac{1}{A_c} - \frac{e_{pc} \cdot e_d}{I_c}\right)$$

Where, A_d=area of deck concrete

E_{cd}=modulus of elascity of deck concrete

e_d=eccentricity of deck with respect to the gross

composite section, positive in typical construction where deck is above girder (in)

$$y_{cDeck} := h_c - 0.5 \cdot h_d = 58.25\ in$$

$$e_d := y_{cDeck} - y_{cbot} = 20.09\ in$$

ε_{ddf}=shrinkage strain of deck concrete between placement and final time
$= k_s \cdot k_{hs} \cdot k_f \cdot k_{td}\left(t_f, t_i\right) \cdot 0.48 \cdot 10^{-3}$

and, k_s=factor for the effect of volume to surface ratio of the component (this has to be recalculated for deck)

k_s=1.45-0.13(V/S)
where (V/S)=volume to surface ratio of deck (in)
=Area/Perimeter (excluding edges)

[AASHTO Eq. 5.4.2.3.2-2]

$$P_d := b_e \cdot 2 = 144\ in$$

$$k_s := 1.45 - 0.13 \cdot \frac{1}{in} \left(\frac{A_d}{P_d}\right) = 0.98$$

$$k_s := \begin{Vmatrix} \text{if } k_s \le 1 \\ \quad \Vert\ 1 \\ \text{else} \\ \quad \Vert\ k_s \end{Vmatrix} = 1$$

$$k_f = \frac{5}{1 + \frac{f'_{ci}}{ksi}}$$

f'_{ci}=specified compressive strength of deck concrete at time of initial loading may be taken as $0.80 f'_{cDeck}$

$$k_f := \frac{5}{1 + \frac{0.8 \cdot f'_{cDeck}}{ksi}} = 0.68$$

$$k_{td}\left(t, t_i\right) := \frac{t - t_i}{61 - 4 \cdot \frac{0.8\ f'_{cDeck}}{ksi} + t - t_i}$$

$$\varepsilon_{ddf} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}\left(t_f, t_d\right) \cdot 0.48 \cdot 10^{-3} = 3.3 \cdot 10^{-4}$$

$$\Delta f_{cdf} := \frac{\varepsilon_{ddf} \cdot A_d \cdot E_{cDeck}}{\left(1 + 0.7 \cdot \Psi_b\left(t_f, t_d\right)\right)} \cdot \left(\frac{1}{A_c} - \frac{e_{pc} \cdot e_d}{I_{comp}}\right) = -0.16\ ksi$$

$$\Delta f_{pSS} := \frac{E_f}{E_c} \cdot \Delta f_{cdf} \cdot K_{df} \cdot \left(1 + 0.7 \cdot \Psi_b \left(t_f, t_d\right)\right) = -1.01 \ ksi$$

Total Prestress Losses at Transfer

The prestress loss due to elastic shortening:

$$\Delta f_{pES} = 4.96 \ ksi$$

Stress in tendons after transfer $\quad f_{pt} := f_{pi} - \Delta f_{pES} = 249.85 \ ksi$

Force per strand after transfer $\quad p_t := f_{pt} \cdot A_{pf} = 72.21 \ kip$

Initial loss, % $\quad eloss := \dfrac{\Delta f_{pES} \cdot 100}{f_{pi}} = 1.95$

Total Prestress Losses at Service

The sum of time-dependent prestress losses between time of transfer and deck placement:

$$\left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) = 24.35 \ ksi$$

The sum of time-dependent prestress losses after deck placement:

$$\left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS}\right) = 9.1 \ ksi$$

The total time-dependent prestress losses:

$$\Delta f_{pLT} := \left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right) + \left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS}\right) = 33.45 \ ksi$$

The total prestress loss at service:

$$\Delta f_{pT} := \Delta f_{pF} + \Delta f_{pA} + \Delta f_{pES} + \Delta f_{pLT} = 43.93 \ ksi$$

Stress in strands after all losses,

$$f_{pe} := f_{pi} - \Delta f_{pT} = 210.88 \ ksi$$

Check prestressing stress limit at service limit state: [AASHTO-CFRP Table 1.9.1-1]

if $f_{pe} \le 0.6 \cdot f_{pu}$
  "Stress limit satisfied"
else
  "Stress limit not satisfied"

$= \text{"Stress limit satisfied"}$

Force per strand after all losses $p_e := f_{pe} \cdot A_{pf} = 60.95\ kip$

Therefore, the total prestressing force after all losses $P_e := n_p \cdot p_e = 975.12\ kip$

Final loss, % $ploss := \dfrac{\Delta f_{pT} \cdot 100}{f_{pi}} = 17.24$

<u>Stresses at Transfer</u>

Total prestressing force after transfer $P_t := n_p \cdot p_t = \left(1.16 \cdot 10^3\right)\ kip$

Stress Limits for Concrete

Compression Limit: [AASHTO Art. 5.9.2.3.1a]

$0.6 \cdot f'_{ci} = 3.6\ ksi$

Where, f'_{ci}=concrete strength at release = 5.5 ksi

Tension Limit: [AASHTO Art. 5.9.2.3.1b]

Without bonded reinforcement

$-0.0948 \cdot \sqrt{\dfrac{f'_{ci}}{ksi}}\ ksi = -0.23\ ksi$ ≤ -0.2 ksi

Therefore, tension limit, $= -0.2\ ksi$

With bonded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force in the concrete computed assuming an uncracked section where reinforcement is proportioned using a stress of $0.5 f_y$, not to exceed 30 ksi.

$-0.24 \cdot \sqrt{\dfrac{f'_{ci}}{ksi}}\ ksi = -0.59\ ksi$

If the tensile stress is between these two limits, the tensile force at the location being considered must be computed following the procedure in AASHTO Art. C5.9.4.1.2. The required area of reinforcement is computed by dividing tensile force by the permitted stress in the reinforcement ($0.5 f_y \leq 30$ ksi)

Stresses at end of the beam

Stress in the top of beam:

$$f_t := \frac{P_i}{A_g} - \frac{P_i \cdot e_c}{S_{gtop}} = -0.9 \ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [NOT OK]

-0.588 ksi with bonded reinforcement [NOT OK]

stress in the bottom of the beam:

$$f_b := \frac{P_i}{A_g} + \frac{P_i \cdot e_c}{S_{gbot}} = 3.46 \ ksi$$

Compressive stress limit for concrete =3.6 ksi [NOT OK]

$$e_c = 18.23 \ in$$

Eccentricity required to satisfy the tension limit

$$e_{c1} := \left(\frac{P_i}{A_g} + 0.48 \cdot ksi\right) \cdot \frac{S_{gtop}}{P_i} = 14.99 \ in$$

Since stresses at the top end do not satisfy the stress limits, one strand was draped

$y_{gbot} := 24.73 \cdot in$ $h_g := 54 \cdot in$

$n_{d1} := 6$ $d_{d1} := 51 \ in$

$n_{d2} := 4$ $d_{d2} := 47 \ in$

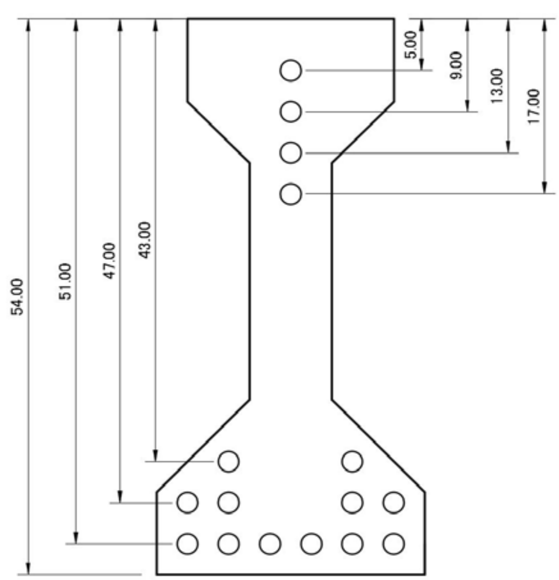
$n_{d3} := 2$ $d_{d3} := 43 \ in$

$n_{d4} := 1$ $d_{d4} := 17 \ in$

$n_{d5} := 1$ $d_{d5} := 13 \ in$

$n_{d6} := 1$ $d_{d6} := 9 \ in$

$n_{d7} := 1$ $d_{d7} := 5 \ in$



$n_{d8} := 0 \qquad d_{d8} := 19.25\ in$

$n_{d9} := 0 \qquad d_{d9} := 15.25\ in$

The center of gravity of the strands, c.g.s. = $\dfrac{\sum n_i y_i}{N}$

where: n_i = number of strands in row i

y_i = distance to center of row i from bottom of beam section

N = total number of strands

$$x_p := h_g - \frac{n_{d1} \cdot d_{d1} + n_{d2} \cdot d_{d2} + n_{d3} \cdot d_{d3} + n_{d4} \cdot d_{d4} + n_{d5} \cdot d_{d5} + n_{d6} \cdot d_{d6} + n_{d7} \cdot d_{d7} + n_{d8} \cdot d_{d8} + n_{d9} \cdot d_{d9}}{n_{d1} + n_{d2} + n_{d3} + n_{d4} + n_{d5} + n_{d6} + n_{d7} + n_{d8} + n_{d9}} = 15\ in$$

$e_{ce} := y_{gbot} - x_p = 9.73\ in$

Stress in the top of beam:

$$f_t := \frac{P_i}{A_g} - \frac{P_i \cdot e_{ce}}{S_{gtop}} = 0.2\ ksi$$

Tensile stress limits for concrete= -0.2 ksi without bonded reinforcement [OK]

-0.588 ksi with bonded reinforcement [OK]

Stress in the bottom of the beam:

$$f_b := \frac{P_i}{A_g} + \frac{P_i \cdot e_{ce}}{S_{gbot}} = 2.53\ ksi$$

Compressive stress limit for concrete = 3.6 ksi [OK]

Stresses at midspan

$x := L_{design} \cdot 0.5 = 45\ ft$

$$M_g := 0.5 \cdot w_g \cdot x \cdot \left(L_{design} - x\right) = 832.28\ ft \cdot kip$$

Stress in the top of beam:

$$f_t := \frac{P_i}{A_g} - \frac{P_i \cdot e_c}{S_{gtop}} + \frac{M_g}{S_{gtop}} = 0.22\ ksi$$

Compressive stress limit for concrete = 3.6 ksi [OK]

stress in the bottom of the beam:

$$f_b := \frac{P_i}{A_g} + \frac{P_i \cdot e_c}{S_{gbot}} - \frac{M_g}{S_{gbot}} = 2.51\ ksi$$

Compressive stress limit for concrete = 3.6 ksi [OK]

<u>Stresses at Service Loads</u>

Total prestressing force after all losses $P_e = 975.12\ kip$

Compression Limit: [AASHTO Art. 5.9.2.3.2a]

Due to the sum of effective prestress and permanent loads (i.e. beam self-weight, weight of slab and haunch, weight of future wearing surface, and weight of barriers) for the Load Combination Service 1:

for precast beam $0.45 \cdot f'_c = 4.05\ ksi$

for deck $0.45 \cdot f'_{cDeck} = 3.6\ ksi$

Due to the sum of effective prestress, permanent loads, and transient loads as well as during shipping and handling for the Load Combination Service 1:

for precast beam $0.60 \cdot f'_c = 5.4\ ksi$

for deck $0.60 \cdot f'_{cDeck} = 4.8\ ksi$

Tension Limit: [AASHTO Art. 5.9.2.3.2b]

Tension limit = 0 ksi For components with unbonded prestressing tendons

Stresses at Midspan

Concrete stress at top fiber of the beam

To check top stresses, two cases are checked:

Under permanent loads, Service I:

$$S_{cgtop} := \frac{I_{comp}}{y_{ctop} - h_d} = \left(3.78 \cdot 10^4\right)\ in^3$$

$$f_{tg} := \frac{P_e}{A_g} - \frac{P_e \cdot e_c}{S_{gtop}} + \frac{M_g + M_d}{S_{gtop}} + \frac{M_{ws} + M_b}{S_{cgtop}} = 1.28\ ksi$$

Under permanent and transient loads, Service I:

$$f_{tg} := f_{tg} + \frac{M_{LT} + M_{LL}}{S_{cgtop}} = 1.71\ ksi$$

Concrete stress at bottom fiber of beam under permanent and transient loads, Service III:

$$f_b := \frac{P_e}{A_g} + \frac{P_e \cdot e_c}{S_{gtop}} - \frac{M_g + M_d}{S_{gbot}} - \frac{M_b + M_{ws} + (0.8) \cdot (M_{LT} + M_{LL})}{S_{cbot}} = 0.58\ ksi \qquad \text{[OK]}$$

Strength Limit State

The total ultimate bending moment for Strength I is: [AASHTO Art. 3.4.1]
Mu = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)

Using the values of unfactored bending moment used in this example

$$M_u := 1.25\ (M_g + M_d + M_b) + 1.50 \cdot M_{ws} + 1.75 \cdot (M_{LT} + M_{LL}) = (4.56 \cdot 10^3)\ ft \cdot kip$$

$$\varepsilon_{pu} = 0.02 \qquad \varepsilon_{pe} := \frac{f_{pe}}{E_f} = 9.29 \cdot 10^{-3} \qquad \varepsilon_{cu} := 0.003$$

$$d_p := (h_g + h_d - 3.5\ in) = 58\ in$$

$$d_{p1} := (h_g + h_d - 8.25\ in) = 53.25\ in$$

$$d_{p2} := (h_g + h_d - 13\ in) = 48.5\ in$$

$$d_{p3} := (h_g + h_d - 17.75\ in) = 43.75\ in$$

$$d_{p4} := (h_g + h_d - 22.5\ in) = 39\ in$$

$$\begin{bmatrix} n_{b1} \\ n_{b2} \\ n_{b3} \\ n_{b4} \\ n_{b5} \\ n_{b6} \\ n_{b7} \\ n_{b8} \\ n_{b9} \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$h_d = 7.5\ in$$

$d_{p5} := \left(h_g + h_d - 23\ in\right) = 38.5\ in$

$d_{p6} := \left(h_g + h_d - 27\ in\right) = 34.5\ in$

$d_{p7} := \left(h_g + h_d - 31\ in\right) = 30.5\ in$

$d_{p8} := \left(h_g + h_d - 35\ in\right) = 26.5\ in$

$\beta_1 := 0.65 \qquad \alpha_1 := 0.85$

Strain reduction factors: [AASHTO-CFRP Eq. 1.7.3.1.2-6]

$\Omega_{up} := \dfrac{3}{\left(\dfrac{L_{span}}{d_p}\right)} = 0.16 \qquad \Omega_{up1} := \dfrac{3}{\left(\dfrac{L_{span}}{d_{p1}}\right)} = 0.15 \qquad \Omega_{up2} := \dfrac{3}{\left(\dfrac{L_{span}}{d_{p2}}\right)} = 0.13$

$\Omega_{up3} := \dfrac{3}{\left(\dfrac{L_{span}}{d_{p3}}\right)} = 0.12 \qquad \Omega_{up4} := \dfrac{3}{\left(\dfrac{L_{span}}{d_{p4}}\right)} = 0.11 \qquad \Omega_{up5} := \dfrac{3}{\left(\dfrac{L_{span}}{d_{p5}}\right)} = 0.11$

$\Omega_{up6} := \dfrac{3}{\left(\dfrac{L_{span}}{d_{p6}}\right)} = 0.09 \qquad \Omega_{up7} := \dfrac{3}{\left(\dfrac{L_{span}}{d_{p7}}\right)} = 0.08 \qquad \Omega_{up8} := \dfrac{3}{\left(\dfrac{L_{span}}{d_{p8}}\right)} = 0.07$

Guess Values

$c := 7\ in \qquad \varepsilon_{cu} := 0.002$

Constraints

AASHTO-CFRP C 1.7.2.1

$\varepsilon_{pe} + \dfrac{d_p - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up} \le \varepsilon_{pu} \qquad \varepsilon_{cu} = 0.003$

$$\begin{array}{l}
\text{if } \beta_1 \cdot c \le h_d \\
\quad \alpha_1 \cdot f'_{cDeck} \cdot \beta_1 \cdot b_d \cdot c \\
\text{else if } h_d \le \beta_1 \cdot c \le \left(h_{trf} + h_d\right) \\
\quad \alpha_1 \cdot f'_{cDeck} \cdot \left(b_d \cdot h_d + b_{trf} \cdot \left(\beta_1 \cdot c - h_d\right)\right) \\
\text{else} \\
\quad \alpha_1 \cdot f'_{cDeck} \cdot \left(b_d \cdot h_d + b_{trf} \cdot h_{trf} + \left(b_{ttf} + \left(h_{ttf} - \left(\beta_1 \cdot c - \left(h_{trf} + h_d\right)\right)\right) + b_w\right) \cdot \left(\beta_1 \cdot c - \left(h_{trf} + h_d\right)\right)\right)
\end{array} = \left(n_{b1} \cdot \left(\varepsilon_{pe} + \cdots\right.\right.$$

Solver

$\begin{bmatrix} c \\ \varepsilon_{cc} \end{bmatrix} := \textbf{find}\left(c, \varepsilon_{cu}\right) = \begin{bmatrix} 0.3968\ ft \\ 0.003 \end{bmatrix}$

$$c = 4.76\ in \qquad \varepsilon_{cu} = 3 \cdot 10^{-3} \qquad \varepsilon_{pe} + \frac{d_p - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up} = 0.01$$

$$C_c := \begin{Vmatrix} \text{if } \beta_1 \cdot c \le h_d \\ \quad \Vert\ \alpha_1 \cdot f'_{cDeck} \cdot \beta_1 \cdot b_d \cdot c \\ \text{else if } h_d \le \beta_1 \cdot c \le (h_{trf} + h_d) \\ \quad \Vert\ \alpha_1 \cdot f'_{cDeck} \cdot (b_d \cdot h_d + b_{trf} \cdot (\beta_1 \cdot c - h_d)) \\ \text{else} \\ \quad \Vert\ \alpha_1 \cdot f'_{cDeck} \cdot (b_d \cdot h_d + b_{trf} \cdot h_{trf} + (b_{ttf} + (h_{ttf} - (\beta_1 \cdot c - (h_{trf} + h_d))) + b_w) \cdot (\beta_1 \cdot c - (h_{trf} + h_d))) \end{Vmatrix} = (1.46 \cdot 10^3)\ kip$$

$$T_{f1} := n_{b1}\ A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \frac{d_p - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up}\right) = 576.02\ kip$$

$$T_{f2} := n_{b2} \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \frac{d_{p1} - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up1}\right) = 541.56\ kip$$

$$T_{f3} := n_{b3} \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \frac{d_{p2} - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up2}\right) = 340.12\ kip$$

$$T_{f4} := n_{b4} \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \frac{d_{p3} - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up3}\right) = 0\ kip$$

$$T_{f5} := n_{b5} \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \frac{d_{p4} - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up4}\right) = 0\ kip$$

$$T_{f6} := n_{b6} \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \frac{d_{p5} - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up5}\right) = 0\ kip$$

$$T_{f7} := n_{b7} \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \frac{d_{p6} - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up6}\right) = 0\ kip$$

$$T_{f8} := n_{b8} \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \frac{d_{p7} - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up7}\right) = 0\ kip$$

$$T_{f9} := n_{b9} \cdot A_{pf} \cdot E_f \cdot \left(\varepsilon_{pe} + \frac{d_{p8} - c}{c} \cdot \varepsilon_{cu} \cdot \Omega_{up8}\right) = 0\ kip \qquad T_{f9} \cdot (d_{p8} - c) = 0\ kip \cdot ft$$

$$T_{f1} + T_{f2} + T_{f3} + T_{f4} + T_{f5} + T_{f6} + T_{f7} + T_{f8} - C_c + T_{f9} = 0\ kip \qquad (d_{p8} - c) = 21.74\ in$$

$$M_n := T_{f1} \cdot (d_p - c) + T_{f2} \cdot (d_{p1} - c) + C_c \cdot \left(c - \frac{\beta_1 \cdot c}{2}\right) + T_{f3} \cdot (d_{p2} - c) + T_{f4} \cdot (d_{p3} - c) + T_{f5} \cdot (d_{p4} - c) + T_{f6} \cdot (d_{p5} - c) + T_{f7}$$

$\phi := 0.75$ [for CFRP prestressed beams] [AASHTO-CFRP Art. 1.5.3.2]

$\phi \cdot M_n = \left(4.78 \cdot 10^3\right)\ ft \cdot kip$ $M_u = \left(4.56 \cdot 10^3\right)\ ft \cdot kip$ $> M_u$ [OK]

Shear Design

Transverse shear reinforcement will be provided where [AASHTO Eq. 5.7.2.3-1]

$V_u > 0.5\ \phi \cdot \left(V_c + V_p\right)$

Where,
V_u=factored shear force (kips)
V_c=nominal shear resistance provided by tensile stresses in the concrete (kips)
V_p=component of prestressing in the direction of shear force (kips)
ϕ=0.90=resistance factor for shear [AASHTO Art. 5.5.4.2]

Critical Section for Shear [AASHTO Art. 5.7.3.2]

The location of the critical section for shear shall be taken as d_v from the internal face of the support.

Where, d_v= effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultant of the tensile and compressive force due to flexure. It need not to be taken less than the greater of $0.9\,d_e$ or $0.72\,h$ (in).

$= d_e - \dfrac{a}{2}$ [AASHTO Art. 5.7.2.8]

Where,
a = depth of compression block
h = overall depth of composite section
d_e= Effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (For harped and draped configuration, this values varies along the length) For CFRP prestressed beams, this value can be taken as the centroid of prestressing CFRP at that location (substituting f_{ps} by f_{pu} and A_s@3#in AASHTO Eq. 5.8.2.9-2)

$y_{bse} := x_p = 15\ in$

$d_e := h_g + h_d - y_{bse} = 46.5\ in$ $a := \beta_1 \cdot c = 3.1\ in$

Effective Shear Depth $d_v := d_e - \dfrac{a}{2} = 44.95\ in$

$0.9 \cdot d_e = 41.85\ in$ $0.72 \cdot \left(h_g + h_d\right) = 44.28\ in$

$d_1 := \max\left(0.9 \cdot d_e, 0.72 \cdot \left(h_g + h_d\right)\right) = 44.28\ in$

$$d_v := \max\left(d_1, d_v\right) = 44.95\ in$$

The bearing width is yet to be determined. It is conservatively assumed zero and the critical section for shear is locatied at the distance of

$$x_c := d_v = 44.95\ in \qquad \frac{x_c}{L_{design}} = 0.04$$

(0.04*L*) from the centerline of the bearing, where *L* is the design span length.
The value of d_e is calculated at the girder end, which can be refined based on the critical section location. However, it is conservative not to refine the value of d_e based on the critical section 0.04*L*. The value, if refined, will have a small difference (PCI 2017).

Shear Stress

Shear stress in the concrete (v_u) is given as:

$$v_u = \frac{V_u - \phi V_p}{\phi \cdot b_v \cdot d_v} \qquad \text{[AASHTO Eq. 5.7.2.8-1]}$$

Where,
β = A factor indicating the ability of diagonally cracked concrete to transmit tension
ϕ = resistance factor for shear
$\phi := 0.9$
b_v = effective web width (in)
$b_v := b_w - 3\ in = 5\ in$ [Duct size used is 2 inches. Subtract 2x1.5 in.=3 in.
$d_v = 44.95\ in$
V_u=factored shear force at specified section at Strength Limit I state

Using the equation to calculate shear force due to the design truck $x := x_c = 44.95\ in$

$$V_{truck1}(x) := P_1 \cdot \frac{\left(L_{design} - x\right)}{L_{design}} + P_2 \cdot \frac{\left(L_{design} - x - 14\ ft\right)}{L_{design}} + P_3 \cdot \frac{\left(L_{design} - x - 28\ ft\right)}{L_{design}}$$

$$V_{truck2}(x) := P_1 \cdot \frac{-(x - 14\ ft)}{L_{design}} + P_2 \cdot \frac{\left(L_{design} - x\right)}{L_{design}} + P_3 \cdot \frac{\left(L_{design} - x - 14\ ft\right)}{L_{design}}$$

$$V_{truck1}(x) = 61.54\ kip$$

$$V_{truck2}(x) = 40.74\ kip$$

$$V := \max\left(V_{truck1}(x), V_{truck2}(x)\right) = 61.54\ kip$$

Distributed bending shear due to truck load including dynamic load allowance (V_{LT}) is calculated as follows:

V_{LT} = (Moment per lane due to truck load)(*DFS*)(1+*IM*/100)

$$IM := 33$$

$$D_{S.Interior} = 0.56$$

$$V_{LT} := V \cdot D_{S.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 45.84\ kip$$

The maximum shear force (V_L) due to a uniformly distributed lane load of 0.64 klf are calculated using the following formulas given by the *PCI Design Manual* (PCI 2017).

Maximum bending moment, $V_x\ =\ 0.64\ \dfrac{kip}{ft}\ \dfrac{\left(L_{design} - x\right)^2}{2\ L_{design}}$

where:
x = Distance from centerline of bearing to section at which the shear force is calculated, ft.
L = Design span length

$$V_L := 0.64\ \frac{kip}{ft}\ \frac{\left(L_{design} - x\right)^2}{2\ L_{design}} = 26.45\ kip$$

$$V_{LL} := D_{S.Interior} \cdot V_L = 14.82\ kip$$

$$V_{ws} := w_{ws} \cdot \left(0.5 \cdot L_{design} - x\right) = 6.19\ kip$$

$$V_u := 1.25\ \left(V_g + V_s + V_b\right) + 1.5 \cdot V_{ws} + 1.75 \cdot \left(V_{LT} + V_{LL}\right) = 193.15\ kip$$

V_p = Component of the effective prestressing force in the direction of the applied shear, kips
= (force per strand)(number of harped strands) $\left(sin\left(\Psi\right)\right)$

$$V_p := 0\ kip$$

Therefore,

$$v_u := \frac{V_u - \phi \cdot V_p}{\phi \cdot b_v \cdot d_v} = 0.95\ ksi$$

Contribution of Concrete to Nominal Shear Resistance [AASHTO Art. 5.7.3.3]

The contribution of the concrete to the nominal shear resistance is given as:
[AASHTO Eq. 5.7.3.3-3]

$$V_c = 0.0316\ \beta \cdot \sqrt{f'_{cGirder}} \cdot b_v \cdot d_v$$

where:
β = A factor indicating the ability of diagonally cracked concrete to transmit tension
$f'_{cGirder}$ = Compressive strength of concrete at service
b_v = Effective web width taken as the minimum web width within the depth d_v,
d_v = Effective shear depth

Strain in Flexural Tension Reinforcement

The θ and β values are determined based on the strain in the flexural tension reinforcement. The strain in the reinforcement, ε_f, is determined assuming that the section contains at least the minimum transverse reinforcement as specified in AASHTO-CFRP Eq. 1.8.3.2-1

$$\varepsilon_f = \frac{\dfrac{M_u}{d_v} + 0.5 \cdot N_u + 0.5 \cdot \left(V_u - V_p\right) - A_{pf} \cdot f_{po}}{E_p \cdot A_{pf}}$$

M_u =Applied factored bending moment at specified section.

$$M_{wsv} := w_{ws} \cdot x \cdot \left(0.5 \cdot L_{design} - x\right) = 23.18\ ft \cdot kip$$

$$M := \max\left(M_{truck1}(x), M_{truck2}(x)\right) = 230.52\ ft \cdot kip$$

$$M_L := 0.5 \cdot 0.64\ \frac{kip}{ft} \cdot (x)\ \left(L_{design} - x\right) = 103.4\ ft \cdot kip$$

$$M_{LLv} := D_{M.Interior} \cdot M_L = 57.91\ ft \cdot kip$$

$$M_{LTv} := D_{M.Interior} \cdot \left(1 + \frac{IM}{100}\right) \cdot M = 171.72\ ft \cdot kip$$

$$M_{uv} := 1.25\ \left(M_{gv} + M_{sv} + M_{bv}\right) + 1.50 \cdot M_{wsv} + 1.75 \cdot \left(M_{LTv} + M_{LLv}\right) = 797.7\ ft \cdot kip$$

$$M_u := \max\left(M_{uv}, V_u \cdot d_v\right) = 797.7\ ft \cdot kip$$

N_u = Applied factored normal force at the specified section, 0.04L = 0 kips

$$N_u := 0 \qquad n_h := 4 \qquad \Psi := \frac{2 \cdot 25.98 \cdot in \cdot 2}{L_{span}}\ rad = 0.1$$

f_{po} = Parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi). For pretensioned members, AASHTO Art. C5.7.3.4.2 indicates that f_{po} can be taken as the stress in strands when the concrete is cast around them, which is jacking stress f_{pj}, or f_{pu}.

$$f_{po} := f_{pi} = 254.81\ ksi \qquad A_s := 0\ in^2 \qquad E_f = \left(2.27 \cdot 10^7\right)\ psi \qquad A_{pf} = 0.29\ in^2$$

V_p = Component of the effective prestressing force in the direction of the applied shear, kips
= (force per strand)(number of harped strands) $(sin(\Psi))$

$$V_p := p_e \cdot n_h \cdot \sin(\Psi) = 23.16\ kip$$

$$\varepsilon_f := \frac{\frac{M_u}{d_v} + 0.5\ kip \cdot N_u + (V_u - V_p) - (n_p - n_h) \cdot A_{pf} \cdot f_{po}}{E_f \cdot (n_p - n_h) \cdot A_{pf}} = -6.36 \cdot 10^{-3}$$

Since this value is negative, ε_s should be recalculated using AASHTO Eq. 5.7.3.4.2-4 replacing the denominator by $(E_c \cdot A_{ct} + E_f \cdot A_{pft})$

A_{ct} = Area of the concrete on the flexural tension side below $h/2$

$$A_{ct} := \left(\frac{h_g + h_d}{2} - h_{btf} - h_{brf}\right) \cdot b_w + h_{btf} \cdot b_{btf} + h_{brf} \cdot b_{brf} = 399\ in^2$$

$$\varepsilon_f := \frac{\frac{M_u}{d_v} + 0.5\ kip \cdot N_u + (V_u - V_p) - (n_p - n_h) \cdot A_{pf} \cdot f_{po}}{(E_c \cdot A_{ct} + E_f \cdot (n_p - n_h) \cdot A_{pf})} = -2.22 \cdot 10^{-4}$$

$$\varepsilon_f := \max(\varepsilon_f, -0.40 \cdot 10^{-3}) = -2.22 \cdot 10^{-4}$$

Therefore, β, factor indicating the ability of diagonally cracked concrete to transmit tension and shear can be calculated as:

$$\beta := \frac{4.8}{1 + 750 \cdot \varepsilon_f} = 5.76$$ [AASHTO Eq. 5.7.3.4.2-1]

And, θ, angle of inclination of diagonal compressive stress can be calculated as:

$$\theta := 29 + 3500 \cdot \varepsilon_f = 28.22$$ [AASHTO Eq. 5.7.3.4.2-3]

$$\theta := 28.23\ deg$$

Computation of Concrete Contribution

The contribution of the concrete to the nominal shear resistance is given as:

$$V_c := 0.0316\ \beta \cdot \sqrt{\frac{f'_c}{ksi}} \cdot b_v \cdot d_v \cdot ksi = 122.74\ kip$$

Contribution of Reinforcement to Nominal Shear Resistance

if $V_u < \phi \cdot \frac{(V_c + V_p)}{2}$ [AASHTO Eq. 5.7.2.3-1] = "Transverse shear reinforcement will be provided"

"Transverse reinforcement shall not be provided"

else

"Transverse shear reinforcement will be provided"

Required Area of Shear Reinforcement

The required area of transverse shear reinforcement is:

$$\frac{V_u}{\phi} \le V_n \qquad V_n = V_c + V_p + V_s$$

Where,

V_s =Shear force carried by transverse reinforcement

$$V_s := \frac{V_u}{\phi} - V_c - V_p = 68.71\ kip$$ [Minimum Shear Reinforcement shall be provided]

Determine Spacing of Reinforcement [AASHTO Art. 5.7.2.6]

Check for maximum spacing of transverse reinforcement

check if $v_u < 0.125\ f'_c$

or $v_u \ge 0.125\ f'_c$

$$s_{max} := \begin{cases} \min(0.8 \cdot d_v, 24\ in) & \text{if } v_u < 0.125\ f'_c \\ \min(0.4 \cdot d_v, 12\ in) & \text{else} \end{cases} = 24\ in$$

Use $s := 22\ in$

$$\begin{aligned}&\text{if } s < s_{max}\\&\quad \| \text{“transverse reinforcement spacing OK”}\\&\text{else}\\&\quad \| \text{“transverse reinforcement spacing NOT OK”}\end{aligned} = \text{“transverse reinforcement spacing OK”}$$

Minimum Reinforcement Requirement [AASHTO Eq. 5.7.2.5-1]

The area of transverse reinforcement should not be less than:

$$0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot \frac{b_v \cdot s}{f_y}$$

$$A_{vmin} := 0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot \frac{b_v \cdot s}{f_y}\ ksi = 0.17\ in^2$$

Use #4 bar double-legged stirrups at 12 in. c/c,

$$A_{vprov} := 2 \cdot \left(0.20\ in^2\right) = 0.4\ in^2$$

$$V_s := \frac{A_{vprov} \cdot f_y \cdot d_v \cdot \cot(\theta)}{s} = 91.34\ kip \qquad V_{sprov} := V_s$$

$$\begin{aligned}&\text{if } A_{vprov} > A_{vmin}\\&\quad \| \text{“Minimum shear reinforcement criteria met”}\\&\text{else}\\&\quad \| \text{“Minimum shear reinforcement criteria not met”}\end{aligned} = \text{“Minimum shear reinforcement criteria met”}$$

Therefore, #4 stirrups with 2 legs shall be provided at 22 in spacing

Maximum Nominal Shear Resistance

In order to ensure that the concrete in the web of the girder will not crush prior to yielding of the transverse reinforcement, the AASHTO Specifications give an upper limit for V_n as follows:

$$V_n = 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p \qquad \text{[AASHTO Eq. 5.7.3.3-2]}$$

Comparing the above equation with AASHTO Eq. 5.8.3.3-1

$$V_c + V_s \le 0.25 \cdot f'_c \cdot b_v \cdot d_v = 1$$

$$V_c + V_s = 214.08 \ kip$$

$$0.25 \cdot f'_c \cdot b_v \cdot d_v = 505.71 \ kip$$

This is a sample calculation for determining the transverse reinforcement requirement at the critical section. This procedure can be followed to find the transverse reinforcement requirement at increments along the length of the girder.

Interface Shear Transfer [AASHTO Art. 5.7.4]

Factored Interface Shear

To calculate the factored interface shear between the girder and slab, the procedure in the commentary of AASHTO Art. 5.7.4.5 will be used. This procedure calculates the factored interface shear force per unit length.

At the Strength I Limit State, the factored interface shear force, V_{hi}, at a section on a per unit basis is:

$$V_{hi} = \frac{V_1}{d_v}$$ [AASHTO Eq. C5.7.4.5-7]

where: V_1 = factored shear force at specified section due to total load (noncomposite and composite loads)

The AASHTO Specifications does not identify the location of the critical section. For convenience, it will be assumed here to be the same location as the critical section for vertical shear, at point 0.04L.

$$V_u = 193.15 \ kip$$

$$V_1 := V_u = 193.15 \ kip$$

$$V_{hi} := \frac{V_1}{d_v} = 4.3 \ \frac{kip}{in}$$

Required Nominal Interface Shear Resistance

The required nominal interface shear resistance (per unit length) is:

$$V_{ni} = \frac{V_{ri}}{\phi}$$ [AASHTO Eq. 5.7.4.3-1]

where: $V_{ri} \ge V_{ui}$ [AASHTO Eq. 5.7.4.3-2]

where, $V_{ui} := V_{hi} = 4.3 \ \frac{kip}{in}$

Therefore, $V_{ni} = \frac{V_{ui}}{\phi}$

$$V_{ni} := \frac{V_{ui}}{\phi} = 4.77 \ \frac{kip}{in}$$

Required Interface Shear Reinforcement

The nominal shear resistance of the interface surface (per unit length) is:

$V_{ni} \ = c_1 \, A_{cv} \, + \, \mu \, (A_{vf} \ f_y \, + \, P_c)$ [AASHTO Eq. 5.7.4.3-3]

where:

c_1 = Cohesion factor [AASHTO Art. 5.7.4.4]
μ = Friction factor [AASHTO Art. 5.7.4.4]
A_{cv} = Area of concrete engaged in shear transfer, in.2
A_{vf} = Area of shear reinforcement crossing the shear plane, in.2
P_c = Permanent net compressive force normal to the shear plane, kips
f_y = Shear reinforcement yield strength, ksi

For concrete normal-weight concrete placed against a clean concrete surface, free of laitance, with surface intentionally roughened to an amplitude of 0.25 in: [AASHTO Art. 5.7.4.4]

$c_1 := 0.28 \ ksi$
$\mu := 1$

The actual contact width, *bv*, between the slab and the girder is 20 in.

$A_{cv} := b_{trf} = 240 \ \frac{in^2}{ft}$ $d_v = 44.95 \ in$

$P_c := 0 \ kip$

The AASHTO Eq. 5.8.4.1-3 can be solved for A_{vf} as follows:

$$A_{vf} := \frac{V_{ni} - c_1 \cdot A_{cv} - \frac{\mu \cdot P_c}{in}}{\mu \cdot f_y} = -0.17 \ \frac{in^2}{ft}$$

The provided vertical shear reinforcement $\frac{A_{vprov}}{s} = 0.22 \ \frac{in^2}{ft}$

Since, $\frac{A_{vprov}}{s} > A_{vf}$,

The provided reinforcement for vertical shear is sufficient to resist interface shear.

$$A_{vfprov} := \frac{A_{vprov}}{s} = 0.22\ \frac{in^2}{ft}$$

Minimum Interface Shear Reinforcement

The cross-sectional area of the interface shear reinforcement, A_{vf}, croossing the interface are, A_{cv}, shall satisfy

$$\text{Minimum } A_{vf} \geq \frac{0.05 \cdot A_{cv}}{f_y}$$ [AASHTO Eq. 5.7.4.2-1]

$$A_{vf1} := \frac{0.05 \cdot A_{cv}}{\frac{f_y}{ksi}} = 0.2\ \frac{in^2}{ft}$$

The minimum interface shear reinforcement, A_{vf}, need not exceed the lesser of the amount determined using Eq. 5.7.4.2-1 and the amount needed to resist $1.33 \frac{V_{ui}}{\phi}$ as determined using Eq. 5.8.4.1-3.

$$A_{vf2} := \frac{1.33\ \frac{V_{ui}}{\phi} - c_1 \cdot A_{cv} - \frac{\mu \cdot P_c}{in}}{\mu \cdot f_y} = 0.15\ \frac{in^2}{ft}$$

Therefore, minimum amount of shear reinforcement

$$A_{vfmin} := min\left(A_{vf1}, A_{vf2}\right) = 0.15\ \frac{in^2}{ft}$$

if $A_{vfprov} > A_{vfmin}$
  "Minm. Interface shear reinforcement OK"
else
  "Minm. Interfaceshear reinforcement NOT OK"

= "Minm. Interface shear reinforcement OK"

Maximum Nominal Shear Resistance

$$V_{niprov} := c_1 \cdot A_{cv} + \mu \cdot A_{vf} \cdot f_y = 57.29\ \frac{kip}{ft}$$ [AASHTO Eq. 5.7.4.3-3]

The nominal shear resistance, V_{ni}, used in the design shall not be greater than the lesser of

$V_{ni} \le k_1 \cdot f'_c \cdot A_{cv}$ [AASHTO Eq. 5.7.4.3-4]

$V_{ni} \le k_2 \cdot A_{cv}$ [AASHTO Eq. 5.7.4.3-5]

Where: For a cast-in-place concrete slab on clean concrete girder surfaces, free of laitance with surface roughened to an amplitude of 0.25 in.

$k_1 := 0.30 \quad k_2 := 1.8\ ksi$

$$k_1 \cdot f'_c \cdot A_{cv} = 648\ \frac{1}{ft} \cdot kip$$

$$k_2 \cdot A_{cv} = 432\ \frac{1}{ft} \cdot kip$$

$V_{niprov} < k_1 \cdot f'_c \cdot A_{cv} = 1$ [1=OK]

$V_{niprov} < k_2 \cdot A_{cv} = 1$ [1=OK]

Minimum Longitudinal Reinforcement Requirement [AASHTO Art. 1.8.3.3]

Longitudinal reinforcement should be proportioned so that at each section the following equation is satisfied:

$$\sum_{x=1}^{n} A_{pf} \cdot f_{pu} \ge \frac{M_u}{d_v \cdot \phi_f} + 0.5 \cdot \frac{N_u}{\phi_n} + \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right) cot(\theta) \quad \text{[AASHTO-CFRP Eq. 1.8.3.3-1]}$$

where: $n_p \cdot A_{pf}$ = area of prestressing steel on the flexural tension side of the member at section under consideration (in)

f_{pu} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi) conservatively taken as effective prestress

M_u = factored bending moment at the section corresponding to the factored shear force (kip-ft)

V_u = factored shear force at section under consideration (kip)

V_p = component of the effective prestressing force in direction of the applied shear (kip) = 0

V_s = shear resistance provided by the transverse reinforcement at the section under investigation as given by Eq. 5.7.3.3-4, except that Vs shall not be taken as greater than Vu /φ (kip)

ϕ_f = resistance factor for flexure

ϕ_n = resistance factor for axial resistance

ϕ_v = resistance factor for shear

θ = angle of inclination of diagonal compressive stresses used in determining the nominal shear resistance of the section under investigation as determined by AASHTO Eq. 5.7.3.4.2-3 (degrees)

Required Reinforcement at Face of Bearing

Width of the bearing is assumed to be zero. This assumption is more conservative for these calculations. Thus, the failure crack assumed for this analysis radiates from the centerline of the bearing, 6 in. from the end of the beam.

As 6 in. is very close to the end of the beam, shear and moment values at the end of the beam are used

Inputs

$excel_{\text{"F1"}} := 3.28084\ (L_{design})$ $\quad excel_{\text{"A13"}} := 0.5 \cdot 3.28084\ (L_{design})$

$excel_{\text{"B1"}} := 0.0000685217\ (w_g)$

$excel_{\text{"D1"}} := 0.0000685217\ (w_d + w_h)$

$excel_{\text{"B2"}} := 0.0000685217\ (w_b)$

w_g	0.822		0.61042		90.000		L	
w_{SD}	0.109							

Distance (x)	Section (x/L)	Dead Load						Total D
		Girder Weight		Slab Weight		Barrier weight		
		Shear	Moment	Shear	Moment	Shear	Moment	Shear
ft.		k	k-ft	k	k-ft	k	k-ft	k
0	0.000	36.990	0.00	27.469	0.00	4.890	0.00	69.35
0.291	0.003	36.751	10.73	27.291	7.97	4.858	1.42	68.90
10.858	0.121	28.065	353.18	20.841	262.27	3.710	46.69	52.62
21.717	0.241	19.139	609.47	14.212	452.59	2.530	80.57	35.88
32.575	0.362	10.213	768.82	7.584	570.93	1.350	101.64	19.15
43.433	0.483	1.288	831.26	0.957	617.30	0.170	109.89	2.41
45.000	0.500	0.000	832.27	0.000	618.05	0.000	110.02	0.00

Outputs

$V_g := excel_{\text{"C7"}} \cdot kip \quad V_s := excel_{\text{"E7"}} \cdot kip \quad V_b := excel_{\text{"G7"}} \cdot kip$

$M_{gv} := excel_{\text{"D7"}} \cdot ft \cdot kip \quad M_{sv} := excel_{\text{"F7"}} \cdot ft \cdot kip \quad M_{bv} := excel_{\text{"H7"}} \cdot ft \cdot kip$

$x := 0\ ft$

$V_{truck1}(x) = 64.53\ kip \qquad V_{truck2}(x) = 43.73\ kip$

$V := \max\left(V_{truck1}(x), V_{truck2}(x)\right) = 64.53\ kip$

$$V_{LT} := V \cdot D_{S.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 48.07\ kip$$

$$V_L := 0.64\ \frac{kip}{ft}\ \frac{\left(L_{design} - x\right)^2}{2\ L_{design}} = 28.8\ kip$$

$$V_{LL} := D_{S.Interior} \cdot V_L = 16.13\ kip$$

$$V_{ws} := w_{ws} \cdot \left(0.5 \cdot L_{design} - x\right) = 6.75\ kip$$

$$V_u := 1.25\ \left(V_g + V_s + V_b\right) + 1.5 \cdot V_{ws} + 1.75 \cdot \left(V_{LT} + V_{LL}\right) = 209.17\ kip$$

$$M_{wsv} := w_{ws} \cdot x \cdot \left(0.5 \cdot L_{design} - x\right) = 0\ ft \cdot kip$$

$$M := \max\left(M_{truck1}(x), M_{truck2}(x)\right) = 0\ ft \cdot kip$$

$$M_L := 0.5 \cdot 0.64\ \frac{kip}{ft} \cdot (x)\ \left(L_{design} - x\right) = 0\ ft \cdot kip$$

$$M_{LLv} := D_{M.Interior} \cdot M_L = 0\ ft \cdot kip$$

$$M_{LTv} := D_{M.Interior} \cdot \left(1 + \frac{IM}{100}\right) \cdot M = 0\ ft \cdot kip$$

$$V_u := 1.25\ \left(V_g + V_s + V_b\right) + 1.5 \cdot V_{ws} + 1.75 \cdot \left(V_{LT} + V_{LL}\right) = 209.17\ kip$$

$$M_{uv} := 1.25\ \left(M_{gv} + M_{sv} + M_{bv}\right) + 1.50 \cdot M_{wsv} + 1.75 \cdot \left(M_{LTv} + M_{LLv}\right) = 0\ ft \cdot kip$$

$\phi_f := 0.75$ $\qquad$ $\phi_n := 1$ $\qquad$ $\phi_v := 0.9$ $\qquad$ $V_s := V_{sprov}$

$$\frac{M_{uv}}{d_v \cdot \phi_f} + 0.5 \cdot \frac{N_u \cdot kip}{\phi_n} + \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right)\ \cot(\theta) = 304.68\ kip$$

The crack plane crosses the centroid of the 12 straight strands at a distance of

$$x_c := 6 + 3.659 \cdot \cot(\theta) = 12.82$$

in. from the girder end. Because the transfer length is 24 in., the available prestress from 12 straight strands is a fraction of the effective prestress, f_{pe}, in these strands. The 4 draped strands do not contribute to the tensile capacity since they are not on the flexural tension side of the member.

$$\left(n_p - n_h\right) \cdot A_{pf} \cdot f_{pe} \cdot \frac{x_c \cdot in}{24\ in} = 390.52\ kip \qquad \text{[AASHTO-CFRP Eq. 1.8.3.3-1]}$$

$$\left(n_p - n_h\right) \cdot A_{pf} \cdot f_{pe} \cdot \frac{x_c \cdot in}{24\ in} \geq \frac{M_u}{d_v \cdot \phi_f} + 0.5 \cdot \frac{N_u \cdot kip}{\phi_n} + \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right)\ \cot(\theta)$$

Therefore, additional longitudinal reinforcement is not required

<u>Deflection and Camber</u> [Upward deflection is negative]

Deflection Due to Prestressing Force at Transfer

$P_{t1} := n_p \cdot p = \left(1.16 \cdot 10^3\right)\ kip$

$e' := e_c - e_{ce} = 8.5\ in$ difference between the eccentricity of the prestressing CFRP at midspan and at the end of the beam

$a := \dfrac{L_{design}}{2} = 45\ ft$

$$\Delta_{pt} := \frac{-P_{t1}}{E_{ci} \cdot I_g} \left(\frac{e_c \cdot \left(L_{design}\right)^2}{8} - \frac{e' \cdot a^2}{6} \right) = -2.09\ in$$

Deflection Due to Beam Self-Weight

$$\Delta_g = \frac{5 \cdot w_g \cdot \left(L_{girder}\right)^4}{384 \cdot E_{ci} \cdot I_g}$$

Deflection due to beam self-weight at transfer:

$$\Delta_{gt} := \frac{5 \cdot w_g \cdot \left(L_{span}\right)^4}{384 \cdot E_{ci} \cdot I_g} = 1.02\ in$$

Deflection due to beam self-weight used to compute deflection at erection:

$$\Delta_{ge} := \frac{5 \cdot w_g \cdot \left(L_{design}\right)^4}{384 \cdot E_{ci} \cdot I_g} = 0.98\ in$$

Deflection Due to Slab and Haunch Weights

$$\Delta_{gd} := \frac{5 \cdot w_d \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I_{comp}} = 0.26\ in$$

Deflection Due to Rail/Barrier and Future Wearing Surface (Overlay)

$$\Delta_{bws} := \frac{5 \cdot \left(w_b + w_{ws}\right) \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I_{comp}} = 0.11\ in$$

$C_a := \Psi_b\left(t_d, t_i\right) = 0.96$ [From previous calculation of the creep of concrete]

Camber at transfer $\Delta_t := \Delta_{pt} + \Delta_{gt} = -1.07\ in$

Total deflection before deck placement

$$\varDelta_{d1} := \left(\varDelta_{pt} + \varDelta_{gt}\right)\ \left(1 + C_a\right) = -2.09\ in$$

Total deflection after deck placement

$$\varDelta_{d2} := \left(\varDelta_{pt} + \varDelta_{gt}\right)\ \left(1 + C_a\right) + \varDelta_{gd} = -1.83\ in$$

Total deflection on composite section

$$\varDelta := \left(\varDelta_{pt} + \varDelta_{gt}\right)\ \left(1 + C_a\right) + \varDelta_{gd} + \varDelta_{bws} = -1.71\ in$$

The deflection criteria in S2.5.2.6.2 (live load deflection check) is considered optional. The bridge owner may select to invoke this criteria if desired.

Deflection Due to Live Load and Impact

Live load deflection limit (optional) = Span / 800 [AASHTO Art. 2.5.2.6.2]

$$\varDelta_{Ll} := \frac{L_{design}}{800} = 1.35\ in$$

If the owner invokes the optional live load deflection criteria specified in AASHTO Article 2.5.2.6.2, the deflection is the greater of:

- That resulting from the design truck alone, or[AASHTO Art. 3.6.1.3.2]
- That resulting from 25% of the design truck taken together with the design lane load.

Therefore, the distribution factor for deflection, DFD, is calculated as follows:

$$DFD := \frac{4}{N_{beams}} = 0.67$$

However, it is more conservative to use the distribution factor for moment

Deflection due to Lane Load:

Design lane load, $w_{LL} := 0.64 \cdot \frac{kip}{ft} \cdot DFD = 0.43\ \frac{kip}{ft}$

$$\varDelta_{LL} := \frac{5 \cdot w_{LL} \cdot \left(L_{design}\right)^4}{384 \cdot E_c \cdot I_{comp}} = 0.19\ in$$

Deflection due to Design Truck Load and Impact:

To obtain maximum moment and deflection at midspan due to the truck load, set the spacing between the rear axles to 14 ft, and let the centerline of the beam coincide with the middle point of the distance between the inner 32-kip axle and the resultant of the truck load, as shown in Figure 15.6-1.

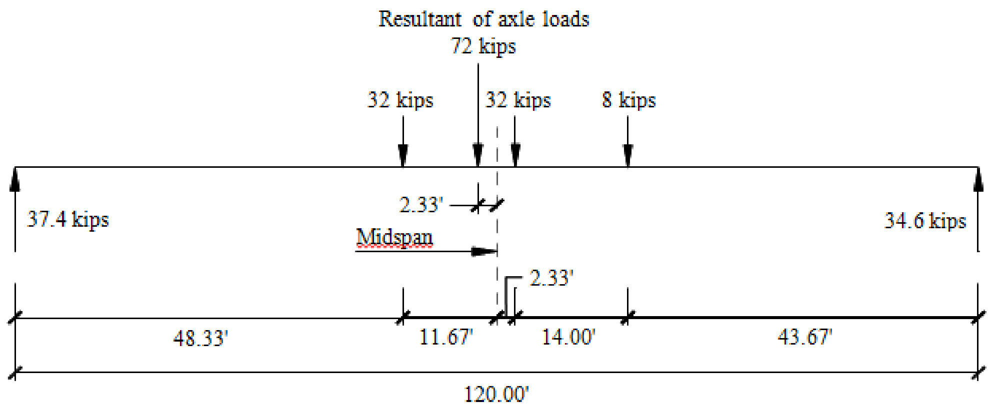


Figure 15.6-1: Design Truck Axle Load Position for Maximum Bending Moment

The deflection at point x due to a point load at point a is given by the following equations:

$$\Delta = \frac{P \cdot b \cdot x}{6 \cdot E_c \cdot I_{comp} \cdot L} \left(L^2 - b^2 - x^2\right) \qquad \text{for } x<a$$

$$\Delta = \frac{P \cdot b}{6 \cdot E_c \cdot I_{comp} \cdot L} \left((x-a)^3 \cdot \frac{L}{b} + \left(L^2 - b\right)^2 \cdot x - x^3 \right) \qquad \text{for } x>a$$

where: P = point load

L = span length

x = location at which deflection is to determined

b = $L - a$

E_c = modulus of elasticity of precast beam at service loads

I_{comp} = gross moment of inertia of the composite section

$$E_c = \left(5.45 \cdot 10^3\right) \ ksi$$

Inputs

$excel_{\text{"D1"}} := 3.28084 \left(L_{design}\right)$ $excel_{\text{"B2"}} := \left(I_{comp}\right) \cdot \left(39.3701^4\right)$

$excel_{\text{"B1"}} := 1.45038 \cdot 10^{-7}\ E_c$

E_c	5448.345	L_{design}	90			
I_{comp}	617603.9					
Axle load	a	b	x	Δ		
P(kips)	(ft)	(ft)	(ft)	in.		
32	33.33	56.6	45	0.222564		
32	47.33	42.67	45	0.248557		
8	61.33	28.67	45	0.048579		

Outputs

$\delta_1 := excel_{\text{"E5"}} \cdot in$ $\delta_2 := excel_{\text{"E6"}} \cdot in$ $\delta_3 := excel_{\text{"E7"}} \cdot in$

The total deflection =

$$\Delta_{LT} := \delta_1 + \delta_2 + \delta_3 = 0.52\ in$$

Including impact and the distribution factor, the deflection at midspan due to the design truck load is:

$$\Delta_{LT} := \Delta_{LT} \cdot D_{M.Interior} \cdot \left(1 + \frac{IM}{100}\right) = 0.39\ in$$

Therefore, the live load deflection is the greater of:

$$\Delta_L := \begin{array}{l} \text{if } \Delta_{LT} > 0.25 \cdot \Delta_{LT} + \Delta_{LL} \\ \quad \| \Delta_{LT} \\ \text{else} \\ \quad \| 0.25 \cdot \Delta_{LT} + \Delta_{LL} \end{array} = 0.39\ in$$

$$\begin{array}{l} \text{if } \Delta_{Ll} > \Delta_L \\ \quad \| \text{"Deflection Limit Satisfied"} \\ \text{else} \\ \quad \| \text{"Deflection Limit Not Satisfied"} \end{array} = \text{"Deflection Limit Satisfied"}$$