

# Design Examples for Large-Span Culverts

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## Design 9.7 m Span by 3.7 m Rise Low Profile Arch, H = 1.0 m

### MathCad Units and Range Variables

$$\text{kN} := 224.8 \cdot \text{lbf} \quad \text{MPa} := 145.1379 \cdot \frac{\text{lbf}}{\text{in}^2} \quad i := 1, 2 \dots 4$$

Mathcad Terminology:

$:=$  defines a term  
 $=$  presents result of a calculation

$$\text{kPa} := \frac{\text{MPa}}{1000} \quad \text{GPa} := 1000 \cdot \text{MPa}$$

### Installation Conditions

$$\begin{aligned} \text{Depth of burial} & \quad H := 1.0 \cdot \text{m} \\ \text{Width of Structural Backfill} & \quad W := 4.85 \cdot \text{m} \\ \text{Live load} & \\ \text{Design Tandem} & \quad P := 222 \cdot \text{kN} \\ \text{Multiple presence factor} & \quad mp := 1.2 \\ \text{Tire length:} & \quad L_o := 250 \cdot \text{mm} \\ \text{Axle plus Wheel Width} & \quad W_T := 2300 \cdot \text{mm} \\ \text{Lane load} & \quad \text{Lane} := 9.3 \cdot \frac{\text{kN}}{\text{m}} \\ \text{Lane load width} & \quad \text{Lane}_W := 3 \cdot \text{m} \end{aligned}$$

### Culvert Geometry

$$\begin{aligned} \text{Span} & \quad S := 9.7 \cdot \text{m} \\ \text{Rise} & \quad R := 3.7 \cdot \text{m} \\ \text{Upper Rise} & \quad R_u := 3.2 \cdot \text{m} \\ \text{Top Radius} & \quad R_t := 6.3 \cdot \text{m} \\ \text{Top Angle} & \quad \theta_{\text{top}} := 80 \cdot \text{deg} \\ \text{Span/ Rise Ratio} & \quad \frac{S}{R} = 2.62 \\ \text{Topchord} & \quad := \sin\left(\frac{\theta_{\text{top}}}{2}\right) \cdot R_t \cdot 2 \\ \text{Topchord} & \quad = 8.10 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Impact} \quad I_{\text{mp}} & \quad := \text{if}\left(H < 2.44 \cdot \text{m}, 1.33 - 0.33 \cdot \frac{H}{2.44 \cdot \text{m}}, 1\right) \quad I_{\text{mp}} = 1.19 \\ \text{Live load distribution with depth of fill} & \quad \text{LLDF} := 1.15 \end{aligned}$$

### Culvert Material Properties

$$F_y := 227.6 \cdot \text{MPa} \quad E_p := 200 \cdot \text{GPa}$$

### Soil Properties:

Structural Backfill (Sn95): Ms selected from table in Specifications based on vertical pressure:

$$\begin{aligned} \text{Density} \quad \gamma_s & \quad := 18.84 \cdot \text{kN} \cdot \text{m}^{-3} & \text{Friction angle (loose)} & \quad \phi := 36 \cdot \text{deg} \\ & & \text{Poisson's ratio} & \quad \nu := 0.3 \\ p_{\text{crown}} & \quad := \gamma_s \cdot H \quad p_{\text{crown}} = 19 \text{ kPa} & M_{\text{sCrown}} & \quad := 15.5 \cdot \text{MPa} \\ p_{\text{side}} & \quad := \gamma_s \cdot \left(H + \frac{R}{2}\right) \quad p_{\text{side}} = 54 \text{ kPa} & M_{\text{sSide}} & \quad := 19.4 \cdot \text{MPa} \\ \text{Native soil: Soft Clay (See Table C2.3-1)} & & M_{\text{sBottom}} & \quad := 21 \cdot \text{MPa} \\ M_{\text{sN}} & \quad := 5 \cdot \text{MPa} \end{aligned}$$

**Design factors**

Load factors	Earth	Max	$\gamma_{EMax} := 1.3$	Resistance factors	Thrust	$\phi_c := 0.7$
		Min	$\gamma_{EMin} := 0.9$		Bending	$\phi_b := 0.9$
	Live		$\gamma_{LL} := 1.75$		Soil	$\phi_s := 0.9$
					Buckling	$\phi_{bck} := 0.7$

**Trial Section Properties**

Structural Plate = 150 mm by 50 mm by 6.324 mm,  
with circumferential stiffeners (2nd plate of same gage)

Basic plate:	$I_u := 2395 \cdot \frac{\text{mm}^4}{\text{mm}}$	$A := 7.74 \cdot \frac{\text{mm}^2}{\text{mm}}$	
Stiffened plate:	$I_p := 4790 \cdot \frac{\text{mm}^4}{\text{mm}}$	$M_p := 54.92 \cdot \frac{\text{kN} \cdot \text{m}}{\text{m}}$	$M_y := 38.21 \cdot \frac{\text{kN} \cdot \text{m}}{\text{m}}$

**MINIMUM STIFFNESS**

$$FF_{max} := 0.17 \cdot \frac{\text{mm}}{\text{N}}$$

$$FF := \frac{(2 \cdot R_t)^2 \cdot (1 - \sin(\phi))^3}{0.07 \cdot E_p \cdot I_p}$$

$$FF = 0.17 \frac{\text{mm}}{\text{N}}$$

$$\text{MinimumStiffness} := \text{if}(FF < FF_{max}, \text{"OK"}, \text{"Stiffeners Required"})$$

$$\text{MinimumStiffness} = \text{"OK"}$$

## THRUST CAPACITY

### Compute Vertical Arching Factor and Earth Load

**1.  $F_{WS}$**

$$K_{VAF_i} := \frac{1.9 - 1.15 \cdot \frac{W}{S}}{1.2}$$

$$K_{VAF} := \max(K_{VAF_i})$$

$$K_{VAF} = 1.33$$

$$\text{SoilRatio} := \text{if} \left( \frac{M_{sSide}}{M_{sN}} < 100, \frac{M_{sSide}}{M_{sN}}, 100 \right)$$

$$F_{WS} := 1.2 + 0.5 \cdot \log(\text{SoilRatio}) (K_{VAF} - 1.2)$$

$$F_{WS} = 1.24$$

**2.  $F_{S/R}$**

$$F_{sr_i} := \frac{1 - \frac{S}{R}}{0}$$

$$F_{SR} := \max(F_{sr_i})$$

$$F_{SR} = 0.00$$

**3.  $F_{H/S}$**

$$hs_{lim_i} := \frac{0.8 - 0.5 \cdot \frac{S}{R}}{0.3}$$

$$HS_{lim} := \max(hs_{lim_i})$$

$$HS_{lim} = 0.30$$

$$F_{hs_i} := \frac{2.5 \cdot \left( HS_{lim} - \frac{H}{S} \right)}{0}$$

$$F_{HS} := \max(F_{hs_i})$$

$$F_{HS} = 0.49$$

**4. VAF**

$$VAF := F_{WS} + F_{SR} + F_{HS}$$

$$VAF = 1.73$$

**5. Earth Load**

$$K_{sp} := \text{if} \left( 0.172 + 0.019 \cdot \frac{S}{R_u} < 0.5, 0.172 + 0.019 \cdot \frac{S}{R_u}, 0.5 \right) \quad K_{sp} = 0.23$$

$$W_{SP} := \gamma_s \cdot S \cdot (H + K_{sp} \cdot R_u) \quad W_{SP} = 317 \frac{\text{kN}}{\text{m}}$$

$$W_E := VAF \cdot W_{SP}$$

$$W_E = 548 \frac{\text{kN}}{\text{m}}$$

**6. Lane Load**

$$W_{Lane} := Lane \cdot \left( \frac{Lane_W}{Lane_W + LLDF \cdot H} \right) \quad W_{Lane} = 6.7 \frac{\text{kN}}{\text{m}}$$

**7. Live Load**

$$L_L := L_o + LLDF \cdot H \quad L_L = 1.40 \text{ m}$$

$$L_W := W_T + LLDF \cdot H \quad L_W = 3.45 \text{ m}$$

$$W_{LL} := \frac{0.7 \cdot mp \cdot I_{mp} \cdot P \cdot R_t}{L_L \cdot L_W} \quad W_{LL} = 291 \frac{\text{kN}}{\text{m}}$$

**Total Factored Thrust**

$$T_f := \frac{\gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + \gamma_{LL} \cdot W_{Lane}}{2} \quad T_f = 616 \frac{\text{kN}}{\text{m}}$$

**Check Capacity for Hoop Thrust**

$$\text{Factored Axial Resistance} \quad R_T := \phi_c \cdot F_y \cdot A \quad R_T = 1234 \frac{\text{kN}}{\text{m}}$$

$$\text{Status}_{Thrust} := \text{if}(R_T > T_f, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{Thrust} = \text{"OK"}$$

**Check Buckling Capacity**

Stiffened top arc with live load thrust

$$R_h := \frac{11.4}{\left(11 + \frac{S}{H}\right)}$$

$$R_h = 0.55$$

$$C_n := 0.55$$

$$K_b := \frac{(1 - 2 \cdot \nu)}{(1 - \nu)^2}$$

$$K_b = 0.82$$

$$R_b := \left[ 1.2 \cdot \phi_{bck} \cdot C_n \cdot (E_p \cdot I_p)^{\frac{1}{3}} \cdot \left( (\phi_s \cdot M_{sCrown} \cdot K_b) \right)^{\frac{2}{3}} \right] \cdot R_h$$

$$R_b = 1270 \frac{\text{kN}}{\text{m}}$$

$$T_f = 616 \frac{\text{kN}}{\text{m}}$$

$$\text{StatusBucklingtop} := \text{if}(R_b > T_f, \text{"OK"}, \text{"Redesign"})$$

$$\text{StatusBucklingtop} = \text{"OK"}$$

Unstiffened bottom arc without live load thrust

$$T_E := \frac{\gamma_{EMax} \cdot W_E}{2}$$

$$H_{bot} := H + 0.75 \cdot R$$

$$R_h := \frac{11.4}{\left(11 + \frac{S}{H_{bot}}\right)}$$

$$R_h = 0.84$$

$$C_n := 0.55$$

$$R_{bE} := \left[ 1.2 \cdot \phi_{bck} \cdot C_n \cdot (E_p \cdot I_u)^{\frac{1}{3}} \cdot \left( (\phi_s \cdot M_{sBottom} \cdot K_b) \right)^{\frac{2}{3}} \right] \cdot R_h$$

$$R_{bE} = 1883 \frac{\text{kN}}{\text{m}}$$

$$T_E = 356 \frac{\text{kN}}{\text{m}}$$

$$\text{StatusBucklingbottom} := \text{if}(R_{bE} > T_E, \text{"OK"}, \text{"Redesign"})$$

$$\text{StatusBucklingbottom} = \text{"OK"}$$



**FLEXURAL CAPACITY**

Bending stiffness factor

$$S_B := \frac{\phi_s \cdot M_{sSide} \cdot S^3}{E_p \cdot I_p} \quad S_B = 16634$$

**Earth Load Moment**

$$K_{e_i} := \frac{0.05 \cdot \left( 1 - \frac{S_B}{S_B + 400} \right)}{0.0025}$$

$$K_E := \max(K_{e_i})$$

$$K_E = 0.0025$$

$$M_E := \gamma_s \cdot S^2 \cdot H \cdot K_E \quad M_E = 4.43 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Live Load Moment**

$$K_{ll_i} := \frac{0.02 \cdot \left( 1.05 - \frac{S_B}{S_B + 800} \right)}{0.001}$$

$$K_{LL} := \max(K_{ll_i})$$

$$K_{LL} = 0.0019$$

$$M_{LL} := 2 \cdot W_{LL} \cdot R_t \cdot K_{LL} \quad M_{LL} = 7.02 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Lane Load Moment - Compute with same formula as earth load moment**

$$K_{Lane_i} := \frac{0.05 \cdot \left( 1 - \frac{S_B}{S_B + 400} \right)}{0.0025}$$

$$K_{Lane} := \max(K_{Lane_i})$$

$$K_{Lane} = 0.0025$$

$$M_{Lane} := W_{Lane} \cdot S \cdot K_{Lane} \quad M_{Lane} = 0.16 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Check if total live load moment is greater than 15% of plastic moment capacity**

$$\text{FlexureCheck} := \text{if} \left[ \gamma_{LL} \cdot (M_{LL} + M_{Lane}) > 0.15 \cdot (\phi_b \cdot M_p), \text{"Required"}, \text{"Not Required"} \right]$$

$$\text{FlexureCheck} = \text{"Required"}$$

**Construction Moment****Maximum allowed extension of top chord**

$$\text{Topchord}_{\max} := 1.0 \quad \text{times original top chord}$$

$$\text{Select } R_{\text{ntrial}} \text{ to give } \delta_{\text{chord}} = 0.0 \quad R_{\text{ntrial}} := 6.3 \cdot \text{m}$$

$$\text{CurvMin} := 0.005 \cdot \text{m}^{-1}$$

$$\delta_{\text{chord}} := 2 \cdot R_{\text{ntrial}} \cdot \sin\left(\frac{\theta_{\text{top}} \cdot R_{\text{t}}}{2 \cdot R_{\text{ntrial}}}\right) - \text{Topchord}_{\max} \cdot \text{Topchord} \quad \delta_{\text{chord}} = 0.0000 \text{ m}$$

$$\text{Curv} := \text{if}\left(\left|\frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right| < \text{CurvMin}, \text{CurvMin}, \frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right) \quad \text{Curv} = 0.005 \text{ m}^{-1}$$

$$M_{\text{sidemax}} := E_{\text{p}} \cdot I_{\text{p}} \cdot \text{Curv}$$

$$M_{\text{sidemax}} = 4.79 \text{ kN} \cdot \frac{\text{m}}{\text{m}}$$

**Minimum allowed top chord**

$$\text{Topchord}_{\min} := 0.98 \quad \text{times original top chord}$$

$$\text{Select } R_{\text{ntrial}} \text{ to give } \delta_{\text{chord}} = 0.0 \quad R_{\text{ntrial}} := 5.66 \cdot \text{m}$$

$$\delta_{\text{chord}} := 2 \cdot R_{\text{ntrial}} \cdot \sin\left(\frac{\theta_{\text{top}} \cdot R_{\text{t}}}{2 \cdot R_{\text{ntrial}}}\right) - \text{Topchord}_{\min} \cdot \text{Topchord} \quad \delta_{\text{chord}} = 0.00 \text{ m}$$

$$\text{Curv} := \text{if}\left(\left|\frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right| < \text{CurvMin}, \text{CurvMin}, \frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right) \quad \text{Curv} = -0.018 \text{ m}^{-1}$$

$$M_{\text{sidemin}} := E_{\text{p}} \cdot I_{\text{p}} \cdot \text{Curv}$$

$$M_{\text{sidemin}} = -17.21 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

$$\text{MaxConstructionM} := \text{if}\left(\left|M_{\text{sidemax}}\right| > \left|M_{\text{sidemin}}\right|, \left|M_{\text{sidemax}}\right|, \left|M_{\text{sidemin}}\right|\right)$$

$$\text{ConstructionControl} := \text{if}\left(\text{MaxConstructionM} < M_{\text{y}}, \text{"OK"}, \text{"Reduce Construction Moment"}\right)$$

$$\text{ConstructionControl} = \text{"OK"}$$

**Total Moment**

$$M_{u_i} :=$$

$\frac{\gamma_{EMax} \cdot -M_{sidemin} - \gamma_{EMin} \cdot M_E + \gamma_{LL} \cdot (M_{LL})}{\gamma_{EMin} \cdot M_{sidemax} + \gamma_{EMax} \cdot M_E + \gamma_{LL} \cdot (M_{LL} + M_{Lane})}$
---

$$M_u = \begin{pmatrix} 30.67 \\ 22.65 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{kN \cdot m}{m} \quad M_U := \max(M_u)$$

$$M_U = 30.67 \frac{kN \cdot m}{m}$$

**Check total moment against total capacity**

$$M_n := \phi_b \cdot M_p$$

$$M_n = 49.43 \frac{kN \cdot m}{m}$$

$$Status_{Flexure} := \text{if}(M_n > M_U, "OK", "Redesign")$$

$$Status_{Flexure} = "OK"$$

**COMBINED THRUST AND BENDING**

Reduce thrust to reflect that peak thrust and moment do not occur at same location

$$T_{fSh} := \frac{0.67 \cdot (\gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + \gamma_{LL} \cdot W_{Lane})}{2} \quad T_{fSh} = 413 \frac{\text{kN}}{\text{m}}$$

$$T_{fCr} := \frac{0.5 \cdot \gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + 0.5 \gamma_{LL} \cdot W_{Lane}}{2} \quad T_{fCr} = 435 \frac{\text{kN}}{\text{m}}$$

$$T_{fRed} := \text{if}(T_{fCr} > T_{fSh}, T_{fCr}, T_{fSh}) \quad T_{fRed} = 435 \frac{\text{kN}}{\text{m}}$$

$$\text{Combined}_1 := \begin{array}{c} \frac{T_{fRed}}{R_T} + \frac{8}{9} \cdot \frac{M_U}{M_n} \\ \frac{T_{fRed}}{2 \cdot R_T} + \frac{M_U}{M_n} \end{array} \quad \text{Combined} = \begin{pmatrix} 0.90 \\ 0.80 \\ 0.00 \\ 0.00 \end{pmatrix} \quad \frac{T_{fRed}}{R_T} = 0.35$$

$$\frac{M_U}{M_n} = 0.62$$

$$\text{Indx} := \text{if} \left( \frac{T_{fRed}}{R_T} \geq 0.2, 1, 2 \right) \quad \text{Indx} = 1.00$$

$$\text{Status}_{\text{Combined}} := \text{if}(\text{Combined}_{\text{Indx}} < 1, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{\text{Combined}} = \text{"OK"}$$

**DESIGN SUMMARY**

MinimumStiffness = "OK"

StatusThrust = "OK"

StatusBucklingtop = "OK"

StatusBucklingbottom = "OK"

FlexureCheck = "Required"

StatusFlexure = "OK"

StatusCombined = "OK"

Combined<sub>Indx</sub> = 0.90

$$A = 7.74 \frac{\text{mm}^2}{\text{mm}}$$

$$I_u = 2395 \frac{\text{mm}^4}{\text{mm}}$$

$$I_p = 4790 \frac{\text{mm}^4}{\text{mm}}$$

$$M_p = 54.92 \frac{\text{kN}\cdot\text{m}}{\text{m}}$$

$$T_f = 616 \frac{\text{kN}}{\text{m}}$$

$$M_U = 30.67 \frac{\text{kN}\cdot\text{m}}{\text{m}}$$

ConstructionControl = "OK"

Topchord<sub>max</sub> = 1.00Topchord<sub>min</sub> = 0.98**Notes**

- Circumferential stiffeners are used to meet minimum stiffness requirement
- Seam strength must also be checked and must be greater than  $T_f$ .

## Design 31.2 ft Span by 19 ft Rise Elliptical Culvert, H = 2 ft

### MathCad Units and Range Variables

$$\text{ksi} := 1000 \cdot \text{psi}$$

$$k := 1000 \cdot \text{lbf}$$

$$i := 1, 2 \dots 4$$

Mathcad Terminology:

$:=$  defines a term

$=$  presents result of a calculation

### Installation Conditions

$$\text{Depth of burial} \quad H := 2.0 \cdot \text{ft}$$

$$\text{Width of Structural Backfill} \quad W := 30 \cdot \text{ft}$$

Live load

$$\text{Design Tandem} \quad P := 50 \cdot k$$

$$\text{Multiple presence factor} \quad mp := 1.2$$

$$\text{Tire length:} \quad L_o := 10 \cdot \text{in}$$

$$\text{Axle plus Wheel Width} \quad W_T := 7.67 \cdot \text{ft}$$

$$\text{Lane load} \quad \text{Lane} := 640 \cdot \frac{\text{lbf}}{\text{ft}}$$

$$\text{Lane load width} \quad \text{Lane}_W := 10 \cdot \text{ft}$$

$$\text{Impact} \quad I_{mp} := \text{if} \left( H < 8 \cdot \text{ft}, 1.33 - 0.33 \cdot \frac{H}{8 \cdot \text{ft}}, 1 \right)$$

Live load distribution with depth of fill

### Culvert Geometry

$$\text{Span} \quad S := 31.2 \cdot \text{ft}$$

$$\text{Rise} \quad R := 19.0 \cdot \text{ft}$$

$$\text{Upper Rise} \quad R_u := 9.5 \cdot \text{ft}$$

$$\text{Top Radius} \quad R_t := 21.0 \cdot \text{ft}$$

$$\text{Top Angle} \quad \theta_{top} := 80 \cdot \text{deg}$$

$$\text{Span/ Rise Ratio} \quad \frac{S}{R} = 1.64$$

$$\text{Topchord} := \sin \left( \frac{\theta_{top}}{2} \right) \cdot R_t \cdot 2$$

$$\text{Topchord} = 27.00 \text{ ft}$$

$$I_{mp} = 1.25$$

$$\text{LLDF} := 1.15$$

### Culvert Material Properties

$$F_y := 33 \cdot \text{ksi}$$

$$E_p := 29000 \cdot \text{ksi}$$

### Soil Properties:

Structural Backfill (Sn95): Ms selected from table in Specifications based on vertical pressure:

$$\text{Density} \quad \gamma_s := 120 \cdot \text{lbf} \cdot \text{ft}^{-3}$$

$$\text{Friction angle (loose)} \quad \phi := 36 \cdot \text{deg}$$

$$\text{Poisson's ratio} \quad \nu := 0.3$$

$$p_{crown} := \gamma_s \cdot H \quad p_{crown} = 1.67 \text{ psi}$$

$$M_{sCrown} := 2100 \cdot \text{psi}$$

$$p_{side} := \gamma_s \cdot \left( H + \frac{R}{2} \right) \quad p_{side} = 9.58 \text{ psi}$$

$$M_{sSide} := 2950 \cdot \text{psi}$$

Native soil: Soft Clay (See Table C2.3-1)

$$M_{sBottom} := 3050 \cdot \text{psi}$$

$$M_{sN} := 750 \cdot \text{MPa}$$

**Design factors**

Load factors	Earth	Max	$\gamma_{EMax} := 1.3$	Resistance factors	Thrust	$\phi_c := 0.7$
		Min	$\gamma_{EMin} := 0.9$		Bending	$\phi_b := 0.9$
	Live		$\gamma_{LL} := 1.75$		Soil	$\phi_s := 0.9$
					Buckling	$\phi_{bck} := 0.7$

**Trial Section Properties**

Structural Plate = 6 in. by 2 in. by 0.276 in.,  
with circumferential stiffeners (2nd plate of same gage)

Basic plate:	$I_u := 0.166 \cdot \frac{\text{in}^4}{\text{in}}$	$A := 4.119 \cdot \frac{\text{in}^2}{\text{ft}}$	
Stiffened plate:	$I_p := 0.332 \cdot \frac{\text{in}^4}{\text{in}}$	$M_p := 167 \cdot \frac{\text{in} \cdot \text{k}}{\text{ft}}$	$M_y := 115 \cdot \frac{\text{in} \cdot \text{k}}{\text{ft}}$

**MINIMUM STIFFNESS**

$$FF_{\max} := 30 \cdot \frac{\text{in}}{\text{k}}$$

$$FF := \frac{(2 \cdot R_t)^2 \cdot (1 - \sin(\phi))^3}{0.07 \cdot E_p \cdot I_p}$$

$$FF = 26.40 \frac{\text{in}}{\text{k}}$$

$$\text{MinimumStiffness} := \text{if}(FF < FF_{\max}, \text{"OK"} , \text{"Stiffeners Required"})$$

$$\text{MinimumStiffness} = \text{"OK"}$$

## THRUST CAPACITY

### Compute Vertical Arching Factor and Earth Load

**1.  $F_{WS}$**

$$K_{VAF_i} := \frac{1.9 - 1.15 \cdot \frac{W}{S}}{1.2}$$

$$K_{VAF} := \max(K_{VAF_i})$$

$$K_{VAF} = 1.20$$

$$\text{SoilRatio} := \text{if} \left( \frac{M_{sSide}}{M_{sN}} < 100, \frac{M_{sSide}}{M_{sN}}, 100 \right)$$

$$F_{WS} := 1.2 + 0.5 \cdot \log(\text{SoilRatio}) (K_{VAF} - 1.2)$$

$$F_{WS} = 1.20$$

**2.  $F_{S/R}$**

$$F_{sr_i} := \frac{1 - \frac{S}{R}}{0}$$

$$F_{SR} := \max(F_{sr_i})$$

$$F_{SR} = 0.00$$

**3.  $F_{H/S}$**

$$hs_{lim_i} := \frac{0.8 - 0.5 \cdot \frac{S}{R}}{0.3}$$

$$HS_{lim} := \max(hs_{lim_i})$$

$$HS_{lim} = 0.30$$

$$F_{hs_i} := \frac{2.5 \cdot \left( HS_{lim} - \frac{H}{S} \right)}{0}$$

$$F_{HS} := \max(F_{hs_i})$$

$$F_{HS} = 0.59$$

**4. VAF**

$$VAF := F_{WS} + F_{SR} + F_{HS}$$

$$VAF = 1.79$$



**5. Earth Load**

$$K_{sp} := \text{if} \left( 0.172 + 0.019 \cdot \frac{S}{R_u} < 0.5, 0.172 + 0.019 \cdot \frac{S}{R_u}, 0.5 \right) \quad K_{sp} = 0.23$$

$$W_{SP} := \gamma_s \cdot S \cdot (H + K_{sp} \cdot R_u) \quad W_{SP} = 16 \frac{k}{ft}$$

$$W_E := VAF \cdot W_{SP}$$

$$W_E = 28 \frac{k}{ft}$$

**6. Lane Load**

$$W_{Lane} := Lane \cdot \left( \frac{LaneW}{LaneW + LLDF \cdot H} \right) \quad W_{Lane} = 0.5 \frac{k}{ft}$$

**7. Live Load**

$$L_L := L_o + LLDF \cdot H \quad L_L = 3.13 \text{ ft}$$

$$L_W := W_T + LLDF \cdot H \quad L_W = 9.97 \text{ ft}$$

$$W_{LL} := \frac{0.7 \cdot mp \cdot I_{mp} \cdot P \cdot R_t}{L_L \cdot L_W} \quad W_{LL} = 35 \frac{k}{ft}$$

**Total Factored Thrust**

$$T_f := \frac{\gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + \gamma_{LL} \cdot W_{Lane}}{2} \quad T_f = 50 \frac{k}{ft}$$

**Check Capacity for Hoop Thrust**

$$\text{Factored Axial Resistance} \quad R_T := \phi_c \cdot F_y \cdot A \quad R_T = 95 \frac{k}{ft}$$

$$\text{Status}_{Thrust} := \text{if}(R_T > T_f, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{Thrust} = \text{"OK"}$$

**Check Buckling Capacity**

Stiffened top arc with live load thrust

$$R_h := \frac{11.4}{\left(11 + \frac{S}{H}\right)}$$

$$R_h = 0.43$$

$$C_n := 0.55$$

$$K_b := \frac{(1 - 2 \cdot \nu)}{(1 - \nu)^2}$$

$$K_b = 0.82$$

$$R_b := \left[ 1.2 \cdot \phi_{bck} \cdot C_n \cdot (E_p \cdot I_p)^{\frac{1}{3}} \cdot \left( (\phi_s \cdot M_{sCrown} \cdot K_b) \right)^{\frac{2}{3}} \right] \cdot R_h$$

$$R_b = 67 \frac{k}{ft}$$

$$T_f = 50 \frac{k}{ft}$$

$$Status_{Bucklingtop} := \text{if}(R_b > T_f, "OK", "Redesign")$$

$$Status_{Bucklingtop} = "OK"$$

Unstiffened bottom arc without live load thrust

$$T_E := \frac{\gamma_{EMax} \cdot W_E}{2}$$

$$H_{bot} := H + 0.75 \cdot R$$

$$R_h := \frac{11.4}{\left(11 + \frac{S}{H_{bot}}\right)}$$

$$R_h = 0.88$$

$$C_n := 0.55$$

$$R_{bE} := \left[ 1.2 \cdot \phi_{bck} \cdot C_n \cdot (E_p \cdot I_u)^{\frac{1}{3}} \cdot \left( (\phi_s \cdot M_{sBottom} \cdot K_b) \right)^{\frac{2}{3}} \right] \cdot R_h$$

$$R_{bE} = 141 \frac{k}{ft}$$

$$T_E = 18 \frac{k}{ft}$$

$$Status_{Bucklingbottom} := \text{if}(R_{bE} > T_E, "OK", "Redesign")$$

$$Status_{Bucklingbottom} = "OK"$$

**FLEXURAL CAPACITY**

Bending stiffness factor

$$S_B := \frac{\phi_s \cdot M_{sSide} \cdot S^3}{E_p \cdot I_p} \quad S_B = 14472$$

**Earth Load Moment**

$$K_{e_i} := \frac{0.05 \cdot \left( 1 - \frac{S_B}{S_B + 400} \right)}{0.0025}$$

$$K_E := \max(K_{e_i})$$

$$K_E = 0.0025$$

$$M_E := \gamma_s \cdot S^2 \cdot H \cdot K_E \quad M_E = 7.01 \frac{\text{in} \cdot \text{k}}{\text{ft}}$$

**Live Load Moment**

$$K_{ll_i} := \frac{0.02 \cdot \left( 1.05 - \frac{S_B}{S_B + 800} \right)}{0.001}$$

$$K_{LL} := \max(K_{ll_i})$$

$$K_{LL} = 0.0020$$

$$M_{LL} := 2 \cdot W_{LL} \cdot R_t \cdot K_{LL} \quad M_{LL} = 36.35 \frac{\text{in} \cdot \text{k}}{\text{ft}}$$

**Lane Load Moment - Compute with same formula as earth load moment**

$$K_{Lane_i} := \frac{0.05 \cdot \left( 1 - \frac{S_B}{S_B + 400} \right)}{0.0025}$$

$$K_{Lane} := \max(K_{Lane_i})$$

$$K_{Lane} = 0.0025$$

$$M_{Lane} := W_{Lane} \cdot S \cdot K_{Lane} \quad M_{Lane} = 0.49 \frac{\text{in} \cdot \text{k}}{\text{ft}}$$

**Check if total live load moment is greater than 15% of plastic moment capacity**

$$\text{FlexureCheck} := \text{if} \left[ \gamma_{LL} \cdot (M_{LL} + M_{Lane}) > 0.15 \cdot (\phi_b \cdot M_p), \text{"Required"}, \text{"Not Required"} \right]$$

$$\text{FlexureCheck} = \text{"Required"}$$

**Construction Moment****Maximum allowed extension of top chord**

$$\text{Topchord}_{\max} := 1.002 \text{ times original top chord}$$

$$\text{Select } R_{\text{ntrial}} \text{ to give } \delta_{\text{chord}} = 0.0 \quad R_{\text{ntrial}} := 21.254 \cdot \text{ft}$$

$$\text{CurvMin} := 0.0015 \cdot \text{ft}^{-1}$$

$$\delta_{\text{chord}} := 2 \cdot R_{\text{ntrial}} \cdot \sin\left(\frac{\theta_{\text{top}} \cdot R_t}{2 \cdot R_{\text{ntrial}}}\right) - \text{Topchord}_{\max} \cdot \text{Topchord} \quad \delta_{\text{chord}} = -0.0001 \text{ ft}$$

$$\text{Curv} := \text{if}\left(\left|\frac{1}{R_t} - \frac{1}{R_{\text{ntrial}}}\right| < \text{CurvMin}, \text{CurvMin}, \frac{1}{R_t} - \frac{1}{R_{\text{ntrial}}}\right) \quad \text{Curv} = 0.0015 \text{ ft}^{-1}$$

$$M_{\text{sidemax}} := E_p \cdot I_p \cdot \text{Curv}$$

$$M_{\text{sidemax}} = 14.44 \frac{\text{in} \cdot \text{k}}{\text{ft}}$$

**Minimum allowed top chord**

$$\text{Topchord}_{\min} := 0.995 \text{ times original top chord}$$

$$\text{Select } R_{\text{ntrial}} \text{ to give } \delta_{\text{chord}} = 0.0 \quad R_{\text{ntrial}} := 20.40 \cdot \text{ft}$$

$$\delta_{\text{chord}} := 2 \cdot R_{\text{ntrial}} \cdot \sin\left(\frac{\theta_{\text{top}} \cdot R_t}{2 \cdot R_{\text{ntrial}}}\right) - \text{Topchord}_{\min} \cdot \text{Topchord} \quad \delta_{\text{chord}} = -0.0002 \text{ ft}$$

$$\text{Curv} := \text{if}\left(\left|\frac{1}{R_t} - \frac{1}{R_{\text{ntrial}}}\right| < \text{CurvMin}, \text{CurvMin}, \frac{1}{R_t} - \frac{1}{R_{\text{ntrial}}}\right) \quad \text{Curv} = 0.0015 \text{ ft}^{-1}$$

$$M_{\text{sidemin}} := E_p \cdot I_p \cdot \text{Curv}$$

$$M_{\text{sidemin}} = 14.44 \frac{\text{in} \cdot \text{k}}{\text{ft}}$$

$$\text{MaxConstructionM} := \text{if}\left(\left|M_{\text{sidemax}}\right| > \left|M_{\text{sidemin}}\right|, \left|M_{\text{sidemax}}\right|, \left|M_{\text{sidemin}}\right|\right)$$

$$\text{ConstructionControl} := \text{if}\left(\text{MaxConstructionM} < M_y, \text{"OK"}, \text{"Reduce Construction Moment"}\right)$$

$$\text{ConstructionControl} = \text{"OK"}$$

**Total Moment**

$$M_{u_i} :=$$

$\frac{\gamma_{EMax} \cdot -M_{sidemin} - \gamma_{EMin} \cdot M_E + \gamma_{LL} \cdot (M_{LL})}{\gamma_{EMin} \cdot M_{sidemax} + \gamma_{EMax} \cdot M_E + \gamma_{LL} \cdot (M_{LL} + M_{Lane})}$
---

$$M_u = \begin{pmatrix} 38.53 \\ 86.57 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{in} \cdot \text{k}}{\text{ft}} \quad M_U := \max(M_u)$$

$$M_U = 86.57 \frac{\text{in} \cdot \text{k}}{\text{ft}}$$

**Check total moment against total capacity**

$$M_n := \phi_b \cdot M_p$$

$$M_n = 150.30 \frac{\text{in} \cdot \text{k}}{\text{ft}}$$

$$\text{Status}_{\text{Flexure}} := \text{if}(M_n > M_U, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{\text{Flexure}} = \text{"OK"}$$

**COMBINED THRUST AND BENDING**

Reduce thrust to reflect that peak thrust and moment do not occur at same location

$$T_{fSh} := \frac{0.67 \cdot (\gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + \gamma_{LL} \cdot W_{Lane})}{2}$$

$$T_{fSh} = 33 \frac{k}{ft}$$

$$T_{fCr} := \frac{0.5 \cdot \gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + 0.5 \gamma_{LL} \cdot W_{Lane}}{2}$$

$$T_{fCr} = 40 \frac{k}{ft}$$

$$T_{fRed} := \text{if}(T_{fCr} > T_{fSh}, T_{fCr}, T_{fSh})$$

$$T_{fRed} = 40 \frac{k}{ft}$$

$$\text{Combined}_1 :=$$

$$\begin{array}{c} \frac{T_{fRed}}{R_T} + \frac{8}{9} \cdot \frac{M_U}{M_n} \\ \hline \frac{T_{fRed}}{2 \cdot R_T} + \frac{M_U}{M_n} \end{array}$$

$$\text{Combined} = \begin{pmatrix} 0.94 \\ 0.79 \\ 0.00 \\ 0.00 \end{pmatrix}$$

$$\frac{T_{fRed}}{R_T} = 0.42$$

$$\frac{M_U}{M_n} = 0.58$$

$$\text{Indx} := \text{if} \left( \frac{T_{fRed}}{R_T} \geq 0.2, 1, 2 \right) \quad \text{Indx} = 1.00$$

$$\text{Status}_{\text{Combined}} := \text{if}(\text{Combined}_{\text{Indx}} < 1, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{\text{Combined}} = \text{"OK"}$$

**DESIGN SUMMARY**

MinimumStiffness = "OK"

StatusThrust = "OK"

StatusBucklingtop = "OK"

StatusBucklingbottom = "OK"

FlexureCheck = "Required"

StatusFlexure = "OK"

StatusCombined = "OK"

Combined<sub>Indx</sub> = 0.94

$$A = 4.12 \frac{\text{in}^2}{\text{ft}}$$

$$T_f = 50 \frac{\text{k}}{\text{ft}}$$

$$I_u = 0.1660 \frac{\text{in}^4}{\text{in}}$$

$$M_U = 86.57 \frac{\text{in} \cdot \text{k}}{\text{ft}}$$

$$I_p = 0.3320 \frac{\text{in}^4}{\text{in}}$$

$$M_p = 167.00 \frac{\text{in} \cdot \text{k}}{\text{ft}}$$

ConstructionControl = "OK"

Topchord<sub>max</sub> = 1.00Topchord<sub>min</sub> = 1.00**Notes**

- Circumferential stiffeners are used to meet minimum stiffness requirement
- Seam strength must also be checked and must be greater than  $T_f$ .

## Design 14.3 m Span by 8.6 m Elliptical Culvert, H = 2 m

### MathCad Units and Range Variables

$$\text{kN} := 224.8 \cdot \text{lbf} \quad \text{MPa} := 145.1379 \cdot \frac{\text{lbf}}{\text{in}^2} \quad i := 1, 2 \dots 4$$

Mathcad Terminology:

$:=$  defines a term

$=$  presents result of a calculation

$$\text{kPa} := \frac{\text{MPa}}{1000} \quad \text{GPa} := 1000 \cdot \text{MPa}$$

### Installation Conditions

$$\begin{aligned} \text{Depth of burial} & \quad H := 2.0 \cdot \text{m} \\ \text{Width of Structural Backfill} & \quad W := 14.3 \cdot \text{m} \\ \text{Live load} & \\ \text{Design Tandem} & \quad P := 222 \cdot \text{kN} \\ \text{Multiple presence factor} & \quad mp := 1.2 \\ \text{Tire length:} & \quad L_o := 250 \cdot \text{mm} \\ \text{Axle + Wheel Width} & \quad W_T := 2300 \cdot \text{mm} \\ \text{Lane load} & \quad \text{Lane} := 9.3 \cdot \frac{\text{kN}}{\text{m}} \\ \text{Lane load width} & \quad \text{Lane}_W := 3 \cdot \text{m} \end{aligned}$$

### Culvert Geometry

$$\begin{aligned} \text{Span} & \quad S := 14.3 \cdot \text{m} \\ \text{Rise} & \quad R := 8.6 \cdot \text{m} \\ \text{Upper Rise} & \quad R_u := 4.3 \cdot \text{m} \\ \text{Top Radius} & \quad R_t := 9.6 \cdot \text{m} \\ \text{Top Angle} & \quad \theta_{\text{top}} := 80 \cdot \text{deg} \\ \text{Span/ Rise Ratio} & \quad \frac{S}{R} = 1.66 \\ \text{Topchord} & \quad := \sin\left(\frac{\theta_{\text{top}}}{2}\right) \cdot R_t \cdot 2 \\ \text{Topchord} & \quad = 12.34 \cdot \text{m} \end{aligned}$$

$$\text{Impact} \quad I_{\text{mp}} := \text{if}\left(H < 2.44 \cdot \text{m}, 1.33 - 0.33 \cdot \frac{H}{2.44 \cdot \text{m}}, 1\right) \quad I_{\text{mp}} = 1.06$$

$$\text{Live load distribution with depth of fill} \quad \text{LLDF} := 1.15$$

### Culvert Material Properties

$$F_y := 227.6 \cdot \text{MPa} \quad E_p := 200 \cdot \text{GPa}$$

### Soil Properties:

Structural Backfill (Sn95): Ms selected from table in Specifications based on vertical pressure:

$$\begin{aligned} \text{Density} \quad \gamma_s & := 18.84 \cdot \text{kN} \cdot \text{m}^{-3} & \text{Friction angle (loose)} & \quad \phi := 36 \cdot \text{deg} \\ & & \text{Poisson's ratio} & \quad \nu := 0.3 \end{aligned}$$

$$\begin{aligned} p_{\text{crown}} & := \gamma_s \cdot H & p_{\text{crown}} & = 38 \cdot \text{kPa} & M_{\text{sCrown}} & := 18.1 \cdot \text{MPa} \\ p_{\text{side}} & := \gamma_s \cdot \left(H + \frac{R}{2}\right) & p_{\text{side}} & = 119 \cdot \text{kPa} & M_{\text{sSide}} & := 23.1 \cdot \text{MPa} \end{aligned}$$

Native soil: Soft Clay (See Table C2.3-1)

$$M_{\text{sBottom}} := 24 \cdot \text{MPa}$$

$$M_{\text{sN}} := 5 \cdot \text{MPa}$$



**Design factors**

Load factors	Earth	Max	$\gamma_{EMax} := 1.3$	Resistance factors	Thrust	$\phi_c := 0.7$
		Min	$\gamma_{EMin} := 0.9$		Bending	$\phi_b := 0.9$
	Live		$\gamma_{LL} := 1.75$		Soil	$\phi_s := 0.9$
					Buckling	$\phi_{bck} := 0.7$

**Trial Section Properties**

Structural Plate = 150 mm by 50 mm by 7.112 mm,  
with circumferential stiffeners of same structural plate

Basic plate:	$I_u := 2717 \cdot \frac{\text{mm}^4}{\text{mm}}$	$A := 8.72 \cdot \frac{\text{mm}^2}{\text{mm}}$	
Stiffened plate:	$I_p := 5434 \cdot \frac{\text{mm}^4}{\text{mm}}$	$M_p := 62.08 \cdot \frac{\text{kN} \cdot \text{m}}{\text{m}}$	$M_y := 42.79 \cdot \frac{\text{kN} \cdot \text{m}}{\text{m}}$

**MINIMUM STIFFNESS**

$$FF_{\max} := 0.17 \cdot \frac{\text{mm}}{\text{N}}$$

$$FF := \frac{(2 \cdot R_t)^2 \cdot (1 - \sin(\phi))^3}{0.07 \cdot E_p \cdot I_p}$$

$$FF = 0.34 \frac{\text{mm}}{\text{N}}$$

MinimumStiffness := if( $FF < FF_{\max}$ , "OK" , "Stiffeners Required")

MinimumStiffness = "Stiffeners Required"

NOTE: Use longitudinal stiffeners in addition to circumferential stiffeners (Longitudinal stiffeners not designed in this example)

## THRUST CAPACITY

### Compute Vertical Arching Factor and Earth Load

**1.  $F_{WS}$**

$$K_{VAF_i} := \frac{1.9 - 1.15 \cdot \frac{W}{S}}{1.2}$$

$$K_{VAF} := \max(K_{VAF_i})$$

$$K_{VAF} = 1.20$$

$$\text{SoilRatio} := \text{if} \left( \frac{M_{sSide}}{M_{sN}} < 100, \frac{M_{sSide}}{M_{sN}}, 100 \right)$$

$$F_{WS} := 1.2 + 0.5 \cdot \log(\text{SoilRatio}) (K_{VAF} - 1.2)$$

$$F_{WS} = 1.20$$

**2.  $F_{S/R}$**

$$F_{sr_i} := \frac{1 - \frac{S}{R}}{0}$$

$$F_{SR} := \max(F_{sr_i})$$

$$F_{SR} = 0.00$$

**3.  $F_{H/S}$**

$$hs_{lim_i} := \frac{0.8 - 0.5 \cdot \frac{S}{R}}{0.3}$$

$$HS_{lim} := \max(hs_{lim_i})$$

$$HS_{lim} = 0.30$$

$$F_{hs_i} := \frac{2.5 \cdot \left( HS_{lim} - \frac{H}{S} \right)}{0}$$

$$F_{HS} := \max(F_{hs_i})$$

$$F_{HS} = 0.40$$

**4. VAF**

$$VAF := F_{WS} + F_{SR} + F_{HS}$$

$$VAF = 1.60$$

**5. Earth Load**

$$K_{sp} := \text{if} \left( 0.172 + 0.019 \cdot \frac{S}{R_u} < 0.5, 0.172 + 0.019 \cdot \frac{S}{R_u}, 0.5 \right) \quad K_{sp} = 0.24$$

$$W_{SP} := \gamma_s \cdot S \cdot (H + K_{sp} \cdot R_u) \quad W_{SP} = 811 \frac{\text{kN}}{\text{m}}$$

$$W_E := VAF \cdot W_{SP}$$

$$W_E = 1298 \frac{\text{kN}}{\text{m}}$$

**6. Lane Load**

$$W_{Lane} := Lane \cdot \left( \frac{Lane_W}{Lane_W + LLDF \cdot H} \right) \quad W_{Lane} = 5.3 \frac{\text{kN}}{\text{m}}$$

**7. Live Load**

$$L_L := L_o + LLDF \cdot H \quad L_L = 2.55 \text{ m}$$

$$L_W := W_T + LLDF \cdot H \quad L_W = 4.60 \text{ m}$$

$$W_{LL} := \frac{0.7 \cdot mp \cdot I_{mp} \cdot P \cdot R_t}{L_L \cdot L_W} \quad W_{LL} = 162 \frac{\text{kN}}{\text{m}}$$

**Total Factored Thrust**

$$T_f := \frac{\gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + \gamma_{LL} \cdot W_{Lane}}{2} \quad T_f = 990 \frac{\text{kN}}{\text{m}}$$

**Check Capacity for Hoop Thrust**

$$\text{Factored Axial Resistance} \quad R_T := \phi_c \cdot F_y \cdot A \quad R_T = 1390 \frac{\text{kN}}{\text{m}}$$

$$\text{Status}_{Thrust} := \text{if}(R_T > T_f, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{Thrust} = \text{"OK"}$$

**Check Buckling Capacity**

Stiffened top arc with live load thrust

$$R_h := \frac{11.4}{\left(11 + \frac{S}{H}\right)}$$

$$R_h = 0.63$$

$$C_n := 0.55$$

$$K_b := \frac{(1 - 2 \cdot \nu)}{(1 - \nu)^2}$$

$$K_b = 0.82$$

$$R_b := \left[ 1.2 \cdot \phi_{bck} \cdot C_n \cdot (E_p \cdot I_p)^{\frac{1}{3}} \cdot \left( (\phi_s \cdot M_{sCrown} \cdot K_b) \right)^{\frac{2}{3}} \right] \cdot R_h$$

$$R_b = 1676 \frac{\text{kN}}{\text{m}}$$

$$T_f = 990 \frac{\text{kN}}{\text{m}}$$

$$\text{StatusBucklingtop} := \text{if}(R_b > T_f, \text{"OK"}, \text{"Redesign"})$$

$$\text{StatusBucklingtop} = \text{"OK"}$$

Unstiffened bottom arc without live load thrust

$$T_E := \frac{\gamma_{EMax} \cdot W_E}{2}$$

$$H_{bot} := H + 0.75 \cdot R$$

$$R_h := \frac{11.4}{\left(11 + \frac{S}{H_{bot}}\right)}$$

$$R_h = 0.90$$

$$C_n := 0.55$$

$$R_{bE} := \left[ 1.2 \cdot \phi_{bck} \cdot C_n \cdot (E_p \cdot I_u)^{\frac{1}{3}} \cdot \left( (\phi_s \cdot M_{sBottom} \cdot K_b) \right)^{\frac{2}{3}} \right] \cdot R_h$$

$$R_{bE} = 2296 \frac{\text{kN}}{\text{m}}$$

$$T_E = 844 \frac{\text{kN}}{\text{m}}$$

$$\text{StatusBucklingbottom} := \text{if}(R_{bE} > T_E, \text{"OK"}, \text{"Redesign"})$$

$$\text{StatusBucklingbottom} = \text{"OK"}$$

**FLEXURAL CAPACITY**

Bending stiffness factor

$$S_B := \frac{\phi_s \cdot M_{sSide} \cdot S^3}{E_p \cdot I_p} \quad S_B = 55939$$

**Earth Load Moment**

$$K_{e_i} := \frac{0.05 \cdot \left( 1 - \frac{S_B}{S_B + 400} \right)}{0.0025}$$

$$K_E := \max(K_{e_i})$$

$$K_E = 0.0025$$

$$M_E := \gamma_s \cdot S^2 \cdot H \cdot K_E \quad M_E = 19.26 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Live Load Moment**

$$K_{ll_i} := \frac{0.02 \cdot \left( 1.05 - \frac{S_B}{S_B + 800} \right)}{0.001}$$

$$K_{LL} := \max(K_{ll_i})$$

$$K_{LL} = 0.0013$$

$$M_{LL} := 2 \cdot W_{LL} \cdot R_t \cdot K_{LL} \quad M_{LL} = 3.98 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Lane Load Moment - Compute with same formula as earth load moment**

$$K_{Lane_i} := \frac{0.05 \cdot \left( 1 - \frac{S_B}{S_B + 400} \right)}{0.0025}$$

$$K_{Lane} := \max(K_{Lane_i})$$

$$K_{Lane} = 0.0025$$

$$M_{Lane} := W_{Lane} \cdot S \cdot K_{Lane} \quad M_{Lane} = 0.19 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Check if total live load moment is greater than 15% of plastic moment capacity**

$$\text{FlexureCheck} := \text{if} \left[ \gamma_{LL} \cdot (M_{LL} + M_{Lane}) > 0.15 \cdot (\phi_b \cdot M_p), \text{"Required"}, \text{"Not Required"} \right]$$

$$\text{FlexureCheck} = \text{"Not Required"}$$

**Construction Moment****Maximum allowed extension of top chord**

$$\text{Topchord}_{\max} := 1.005 \text{ times original top chord}$$

$$\text{Select } R_{\text{ntrial}} \text{ to give } \delta_{\text{chord}} = 0.0 \quad R_{\text{ntrial}} := 9.9 \cdot \text{m}$$

$$\text{CurvMin} := 0.005 \cdot \text{m}^{-1}$$

$$\delta_{\text{chord}} := 2 \cdot R_{\text{ntrial}} \cdot \sin\left(\frac{\theta_{\text{top}} \cdot R_{\text{t}}}{2 \cdot R_{\text{ntrial}}}\right) - \text{Topchord}_{\max} \cdot \text{Topchord} \quad \delta_{\text{chord}} = 0.0003 \text{ m}$$

$$\text{Curv} := \text{if}\left(\left|\frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right| < \text{CurvMin}, \text{CurvMin}, \frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right) \quad \text{Curv} = 0.005 \text{ m}^{-1}$$

$$M_{\text{sidemax}} := E_{\text{p}} \cdot I_{\text{p}} \cdot \text{Curv}$$

$$M_{\text{sidemax}} = 5.44 \text{ kN} \cdot \frac{\text{m}}{\text{m}}$$

**Minimum allowed top chord**

$$\text{Topchord}_{\min} := 1.00 \text{ times original top chord}$$

$$\text{Select } R_{\text{ntrial}} \text{ to give } \delta_{\text{chord}} = 0.0 \quad R_{\text{ntrial}} := 9.6 \cdot \text{m}$$

$$\delta_{\text{chord}} := 2 \cdot R_{\text{ntrial}} \cdot \sin\left(\frac{\theta_{\text{top}} \cdot R_{\text{t}}}{2 \cdot R_{\text{ntrial}}}\right) - \text{Topchord}_{\min} \cdot \text{Topchord} \quad \delta_{\text{chord}} = 0.00 \text{ m}$$

$$\text{Curv} := \text{if}\left(\left|\frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right| < \text{CurvMin}, \text{CurvMin}, \frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right) \quad \text{Curv} = 0.005 \text{ m}^{-1}$$

$$M_{\text{sidemin}} := E_{\text{p}} \cdot I_{\text{p}} \cdot \text{Curv}$$

$$M_{\text{sidemin}} = 5.44 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

$$\text{MaxConstructionM} := \text{if}\left(\left|M_{\text{sidemax}}\right| > \left|M_{\text{sidemin}}\right|, \left|M_{\text{sidemax}}\right|, \left|M_{\text{sidemin}}\right|\right)$$

$$\text{ConstructionControl} := \text{if}\left(\text{MaxConstructionM} < M_{\text{y}}, \text{"OK"}, \text{"Reduce Construction Moment"}\right)$$

$$\text{ConstructionControl} = \text{"OK"}$$

**Total Moment**

$$M_{u_1} :=$$

$\frac{\gamma_{EMax} \cdot -M_{sidemin} - \gamma_{EMin} \cdot M_E + \gamma_{LL} \cdot (M_{LL})}{\gamma_{EMin} \cdot M_{sidemax} + \gamma_{EMax} \cdot M_E + \gamma_{LL} \cdot (M_{LL} + M_{Lane})}$
---

$$M_u = \begin{pmatrix} -17.44 \\ 37.23 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{kN} \cdot \text{m}}{\text{m}} \quad M_U := \max(M_u)$$

$$M_U = 37.23 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Check total moment against total capacity**

$$M_n := \phi_b \cdot M_p$$

$$M_n = 55.87 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

$$\text{Status}_{\text{Flexure}} := \text{if}(M_n > M_U, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{\text{Flexure}} = \text{"OK"}$$

**COMBINED THRUST AND BENDING**

Reduce thrust to reflect that peak thrust and moment do not occur at same location

$$T_{fSh} := \frac{0.67 \cdot (\gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + \gamma_{LL} \cdot W_{Lane})}{2}$$

$$T_{fSh} = 663 \frac{\text{kN}}{\text{m}}$$

$$T_{fCr} := \frac{0.5 \cdot \gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + 0.5 \gamma_{LL} \cdot W_{Lane}}{2}$$

$$T_{fCr} = 566 \frac{\text{kN}}{\text{m}}$$

$$T_{fRed} := \text{if}(T_{fCr} > T_{fSh}, T_{fCr}, T_{fSh})$$

$$T_{fRed} = 663 \frac{\text{kN}}{\text{m}}$$

$$\text{Combined}_1 :=$$

$$\begin{array}{c} \frac{T_{fRed}}{R_T} + \frac{8}{9} \cdot \frac{M_U}{M_n} \\ \hline \frac{T_{fRed}}{2 \cdot R_T} + \frac{M_U}{M_n} \end{array}$$

$$\text{Combined} = \begin{pmatrix} 1.07 \\ 0.90 \\ 0.00 \\ 0.00 \end{pmatrix}$$

$$\frac{T_{fRed}}{R_T} = 0.48$$

$$\frac{M_U}{M_n} = 0.67$$

$$\text{Indx} := \text{if} \left( \frac{T_{fRed}}{R_T} \geq 0.2, 1, 2 \right) \quad \text{Indx} = 1.00$$

$$\text{Status}_{\text{Combined}} := \text{if}(\text{Combined}_{\text{Indx}} < 1, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{\text{Combined}} = \text{"Redesign"}$$



**DESIGN SUMMARY**

MinimumStiffness = "Stiffeners Required"	$A = 8.72 \frac{\text{mm}^2}{\text{mm}}$	$T_f = 990 \frac{\text{kN}}{\text{m}}$
StatusThrust = "OK"		
StatusBucklingtop = "OK"	$I_u = 2717 \frac{\text{mm}^4}{\text{mm}}$	$M_U = 37.23 \frac{\text{kN}\cdot\text{m}}{\text{m}}$
StatusBucklingbottom = "OK"		
FlexureCheck = "Not Required"	$I_p = 5434 \frac{\text{mm}^4}{\text{mm}}$	
StatusFlexure = "OK"	$M_p = 62.08 \frac{\text{kN}\cdot\text{m}}{\text{m}}$	
StatusCombined = "Redesign"		
Combined <sub>Indx</sub> = 1.07		
ConstructionControl = "OK"		
Topchord <sub>max</sub> = 1.00		
Topchord <sub>min</sub> = 1.00		

**Notes**

- Circumferential stiffeners are used to meet minimum stiffness requirement
- Seam strength must also be checked and must be greater than  $T_f$ .

## Design 9.2 m Span by 9.0 m Rise Pear Shaped Culvert, H = 8 m

### MathCad Units and Range Variables

$$\text{kN} := 224.8 \cdot \text{lbf} \quad \text{MPa} := 145.1379 \cdot \frac{\text{lbf}}{\text{in}^2} \quad i := 1, 2 \dots 4$$

Mathcad Terminology:

$:=$  defines a term

$=$  presents result of a calculation

$$\text{kPa} := \frac{\text{MPa}}{1000} \quad \text{GPa} := 1000 \cdot \text{MPa}$$

### Installation Conditions

$$\begin{aligned} \text{Depth of burial} & \quad H := 8.0 \cdot \text{m} \\ \text{Width of Structural Backfill} & \quad W := 4.6 \cdot \text{m} \\ \text{Live load} & \\ \text{Design Tandem} & \quad P := 222 \cdot \text{kN} \\ \text{Multiple presence factor} & \quad mp := 1.2 \\ \text{Tire width:} & \quad W_o := 500 \cdot \text{mm} \\ \text{Tire length:} & \quad L_o := 250 \cdot \text{mm} \\ \text{Wheel Spacing} & \quad W_T := 1800 \cdot \text{mm} \\ \text{Lane load} & \quad \text{Lane} := 9.3 \cdot \frac{\text{kN}}{\text{m}} \\ \text{Lane load width} & \quad \text{Lane}_W := 3 \cdot \text{m} \end{aligned}$$

$$\text{Impact} \quad I_{mp} := \text{if} \left( H < 2.44 \cdot \text{m}, 1.33 - 0.33 \cdot \frac{H}{2.44 \cdot \text{m}}, 1 \right)$$

Live load distribution with depth of fill

### Culvert Geometry

$$\begin{aligned} \text{Span} & \quad S := 9.2 \cdot \text{m} \\ \text{Rise} & \quad R := 9.0 \cdot \text{m} \\ \text{Upper Rise} & \quad R_u := 3.0 \cdot \text{m} \\ \text{Top Radius} & \quad R_t := 6.7 \cdot \text{m} \\ \text{Top Angle} & \quad \theta_{top} := 67 \cdot \text{deg} \\ \text{Span/ Rise Ratio} & \quad \frac{S}{R} = 1.02 \\ \text{Side Radius} & \quad R_{side} := 7.4 \cdot \text{m} \end{aligned}$$

$$\text{Topchord} := \sin \left( \frac{\theta_{top}}{2} \right) \cdot R_t \cdot 2$$

$$\begin{aligned} \text{Topchord} & = 7.40 \text{ m} \\ I_{mp} & = 1.00 \end{aligned}$$

$$\text{LLDF} := 1.15$$

### Culvert Material Properties

$$F_y := 227.6 \cdot \text{MPa} \quad E_p := 200 \cdot \text{GPa}$$

### Soil Properties:

Structural Backfill (Sn95): Ms selected from table in Specifications based on vertical pressure:

$$\text{Density} \quad \gamma_s := 18.84 \cdot \text{kN} \cdot \text{m}^{-3}$$

$$\text{Friction angle (loose)} \quad \phi := 36 \cdot \text{deg}$$

$$\text{Poisson's ratio} \quad \nu := 0.3$$

$$p_{crown} := \gamma_s \cdot H \quad p_{crown} = 151 \text{ kPa}$$

$$M_{sCrown} := 24.6 \cdot \text{MPa}$$

$$p_{side} := \gamma_s \cdot \left( H + \frac{R}{2} \right) \quad p_{side} = 235 \text{ kPa}$$

$$M_{sSide} := 28.8 \cdot \text{MPa}$$

Native soil: Soft Clay (See Table C2.3-1)

$$M_{sBottom} := 30 \cdot \text{MPa}$$

$$M_{sN} := 5 \cdot \text{MPa}$$

**Design factors**

Load factors	Earth	Max	$\gamma_{EMax} := 1.3$	Resistance factors	Thrust	$\phi_c := 0.7$
		Min	$\gamma_{EMin} := 0.9$		Bending	$\phi_b := 0.9$
	Live		$\gamma_{LL} := 1.75$		Soil	$\phi_s := 0.9$
					Buckling	$\phi_{bck} := 0.7$

**Trial Section Properties**

Structural Plate = 150 mm by 50 mm by 6.324 mm,  
without circumferential stiffeners

$$\begin{aligned} \text{Basic plate:} \quad I_u &:= 2395 \cdot \frac{\text{mm}^4}{\text{mm}} & A &:= 7.74 \cdot \frac{\text{mm}^2}{\text{mm}} \\ \text{Stiffened plate:} \quad I_p &:= 2395 \cdot \frac{\text{mm}^4}{\text{mm}} & M_p &:= 27.46 \cdot \frac{\text{kN} \cdot \text{m}}{\text{m}} & M_y &:= 19.11 \cdot \frac{\text{kN} \cdot \text{m}}{\text{m}} \end{aligned}$$

**MINIMUM STIFFNESS****Top Plates**

$$\begin{aligned} FF_{\max} &:= 0.17 \cdot \frac{\text{mm}}{\text{N}} \\ FF &:= \frac{(2 \cdot R_t)^2 \cdot (1 - \sin(\phi))^3}{0.07 \cdot E_p \cdot I_p} & FF &= 0.37 \frac{\text{mm}}{\text{N}} \end{aligned}$$

$$\text{MinimumStiffness} := \text{if}(FF < FF_{\max}, \text{"OK"}, \text{"Stiffeners Required"})$$

$$\text{MinimumStiffness} = \text{"Stiffeners Required"}$$

**Side Plates**

$$\begin{aligned} FF_{\text{side}} &:= \frac{(2 \cdot R_{\text{side}})^2 \cdot (1 - \sin(\phi))^3}{0.07 \cdot E_p \cdot I_p} & FF_{\max} &:= 0.17 \cdot \frac{\text{mm}}{\text{N}} \\ & & FF_{\text{side}} &= 0.46 \frac{\text{mm}}{\text{N}} \end{aligned}$$

$$\text{MinimumStiffness}_{\text{side}} := \text{if}(FF < FF_{\max}, \text{"OK"}, \text{"Increase Side Stiffness"})$$

$$\text{MinimumStiffness}_{\text{side}} = \text{"Increase Side Stiffness"}$$

NOTE: Use heavier gage for side plates  
or adjust flexibility requirement, or increase control  
of side compaction

## THRUST CAPACITY

### Compute Vertical Arching Factor and Earth Load

**1.  $F_{W/S}$**

$$K_{ws_i} := \frac{1.9 - 1.15 \cdot \frac{W}{S}}{1.2}$$

$$K_{WS} := \max(K_{ws_i})$$

$$K_{WS} = 1.33$$

$$\text{SoilRatio} := \text{if} \left( \frac{M_{sSide}}{M_{sN}} < 100, \frac{M_{sSide}}{M_{sN}}, 100 \right)$$

$$F_{WS} := 1.2 + 0.5 \cdot \log(\text{SoilRatio}) (K_{WS} - 1.2)$$

$$F_{WS} = 1.25$$

**2.  $F_{S/R}$**

$$F_{sr_i} := \frac{1 - \frac{S}{R}}{0}$$

$$F_{SR} := \max(F_{sr_i})$$

$$F_{SR} = 0.00$$

**3.  $F_{H/S}$**

$$hs_{lim_i} := \frac{0.8 - 0.5 \cdot \frac{S}{R}}{0.3}$$

$$HS_{lim} := \max(hs_{lim_i})$$

$$HS_{lim} = 0.30$$

$$F_{hs_i} := \frac{2.5 \cdot \left( HS_{lim} - \frac{H}{S} \right)}{0}$$

$$F_{HS} := \max(F_{hs_i})$$

$$F_{HS} = 0.00$$

**4. VAF**

$$VAF := F_{WS} + F_{SR} + F_{HS}$$

$$VAF = 1.25$$

**5. Earth Load**

$$K_{sp} := \text{if} \left( 0.172 + 0.019 \cdot \frac{S}{R_u} < 0.5, 0.172 + 0.019 \cdot \frac{S}{R_u}, 0.5 \right) \quad K_{sp} = 0.23$$

$$W_{SP} := \gamma_s \cdot S \cdot (H + K_{sp} \cdot R_u) \quad W_{SP} = 1506 \frac{\text{kN}}{\text{m}}$$

$$W_E := VAF \cdot W_{SP}$$

$$W_E = 1879 \frac{\text{kN}}{\text{m}}$$

**6. Lane Load**

$$W_{Lane} := Lane \cdot \left( \frac{Lane_W}{Lane_W + LLDF \cdot H} \right) \quad W_{Lane} = 2.3 \frac{\text{kN}}{\text{m}}$$

**7. Live Load**

$$L_L := L_o + LLDF \cdot H \quad L_L = 9.45 \text{ m}$$

$$L_W := W_o + W_T + LLDF \cdot H \quad L_W = 11.50 \text{ m}$$

$$W_{LL} := \frac{0.7 \cdot mp \cdot I_{mp} \cdot P \cdot R_t}{L_L \cdot L_W} \quad W_{LL} = 11 \frac{\text{kN}}{\text{m}}$$

**Total Factored Thrust**

$$T_f := \frac{\gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + \gamma_{LL} \cdot W_{Lane}}{2} \quad T_f = 1234 \frac{\text{kN}}{\text{m}}$$

**Check Capacity for Hoop Thrust**

$$\text{Factored Axial Resistance} \quad R_T := \phi_c \cdot F_y \cdot A \quad R_T = 1234 \frac{\text{kN}}{\text{m}}$$

$$\text{Status}_{Thrust} := \text{if}(R_T > T_f, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{Thrust} = \text{"OK"}$$

**Check Buckling Capacity**

Stiffened top arc with live load thrust

$$R_h := \frac{11.4}{\left(11 + \frac{S}{H}\right)} \quad R_h = 0.94$$

$$C_n := 0.55 \quad K_b := \frac{(1 - 2 \cdot \nu)}{(1 - \nu)^2} \quad K_b = 0.82$$

$$R_b := \left[ 1.2 \cdot \phi_{bck} \cdot C_n \cdot (E_p \cdot I_p)^{\frac{1}{3}} \cdot \left( (\phi_s \cdot M_{sCrown} \cdot K_b) \right)^{\frac{2}{3}} \right] \cdot R_h \quad R_b = 2338 \frac{\text{kN}}{\text{m}}$$

$$T_f = 1234 \frac{\text{kN}}{\text{m}}$$

$$\text{StatusBucklingtop} := \text{if}(R_b > T_f, \text{"OK"}, \text{"Redesign"})$$

$$\text{StatusBucklingtop} = \text{"OK"}$$

Unstiffened bottom arc without live load thrust

$$T_E := \frac{\gamma_{EMax} \cdot W_E}{2}$$

$$H_{bot} := H + 0.75 \cdot R$$

$$R_h := \frac{11.4}{\left(11 + \frac{S}{H_{bot}}\right)} \quad R_h = 0.98$$

$$C_n := 0.55$$

$$R_{bE} := \left[ 1.2 \cdot \phi_{bck} \cdot C_n \cdot (E_p \cdot I_u)^{\frac{1}{3}} \cdot \left( (\phi_s \cdot M_{sBottom} \cdot K_b) \right)^{\frac{2}{3}} \right] \cdot R_h \quad R_{bE} = 2789 \frac{\text{kN}}{\text{m}}$$

$$T_E = 1221 \frac{\text{kN}}{\text{m}}$$

$$\text{StatusBucklingbottom} := \text{if}(R_{bE} > T_E, \text{"OK"}, \text{"Redesign"})$$

$$\text{StatusBucklingbottom} = \text{"OK"}$$

**FLEXURAL CAPACITY**

Bending stiffness factor

$$S_B := \frac{\phi_s \cdot M_{sSide} \cdot S^3}{E_p \cdot I_p} \quad S_B = 42137$$

**Earth Load Moment**

$$K_{e_i} := \frac{0.05 \cdot \left( 1 - \frac{S_B}{S_B + 400} \right)}{0.0025}$$

$$K_E := \max(K_{e_i})$$

$$K_E = 0.0025$$

$$M_E := \gamma_s \cdot S^2 \cdot H \cdot K_E \quad M_E = 31.89 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Live Load Moment**

$$K_{ll_i} := \frac{0.02 \cdot \left( 1.05 - \frac{S_B}{S_B + 800} \right)}{0.001}$$

$$K_{LL} := \max(K_{ll_i})$$

$$K_{LL} = 0.0014$$

$$M_{LL} := 2 \cdot W_{LL} \cdot R_t \cdot K_{LL} \quad M_{LL} = 0.21 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Lane Load Moment - Compute with same formula as earth load moment**

$$K_{Lane_i} := \frac{0.05 \cdot \left( 1 - \frac{S_B}{S_B + 400} \right)}{0.0025}$$

$$K_{Lane} := \max(K_{Lane_i})$$

$$K_{Lane} = 0.0025$$

$$M_{Lane} := W_{Lane} \cdot S \cdot K_{Lane} \quad M_{Lane} = 0.05 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Check if total live load moment is greater than 15% of plastic moment capacity**

$$\text{FlexureCheck} := \text{if} \left[ \gamma_{LL} \cdot (M_{LL} + M_{Lane}) > 0.15 \cdot (\phi_b \cdot M_p), \text{"Required"}, \text{"Not Required"} \right]$$

$$\text{FlexureCheck} = \text{"Not Required"}$$

**Construction Moment****Maximum allowed extension of top chord**

$$\text{Topchord}_{\max} := 1.0 \quad \text{times original top chord}$$

$$\text{Select } R_{\text{ntrial}} \text{ to give } \delta_{\text{chord}} = 0.0 \quad R_{\text{ntrial}} := 6.7 \cdot \text{m}$$

$$\text{CurvMin} := 0.005 \cdot \text{m}^{-1}$$

$$\delta_{\text{chord}} := 2 \cdot R_{\text{ntrial}} \cdot \sin\left(\frac{\theta_{\text{top}} \cdot R_{\text{t}}}{2 \cdot R_{\text{ntrial}}}\right) - \text{Topchord}_{\max} \cdot \text{Topchord} \quad \delta_{\text{chord}} = 0.0000 \text{ m}$$

$$\text{Curv} := \text{if}\left(\left|\frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right| < \text{CurvMin}, \text{CurvMin}, \frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right) \quad \text{Curv} = 0.005 \text{ m}^{-1}$$

$$M_{\text{sidemax}} := E_{\text{p}} \cdot I_{\text{p}} \cdot \text{Curv}$$

$$M_{\text{sidemax}} = 2.40 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Minimum allowed top chord**

$$\text{Topchord}_{\min} := 0.98 \quad \text{times original top chord}$$

$$\text{Select } R_{\text{ntrial}} \text{ to give } \delta_{\text{chord}} = 0.0 \quad R_{\text{ntrial}} := 5.778 \cdot \text{m}$$

$$\delta_{\text{chord}} := 2 \cdot R_{\text{ntrial}} \cdot \sin\left(\frac{\theta_{\text{top}} \cdot R_{\text{t}}}{2 \cdot R_{\text{ntrial}}}\right) - \text{Topchord}_{\min} \cdot \text{Topchord} \quad \delta_{\text{chord}} = 0.00 \text{ m}$$

$$\text{Curv} := \text{if}\left(\left|\frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right| < \text{CurvMin}, \text{CurvMin}, \frac{1}{R_{\text{t}}} - \frac{1}{R_{\text{ntrial}}}\right) \quad \text{Curv} = -0.024 \text{ m}^{-1}$$

$$M_{\text{sidemin}} := E_{\text{p}} \cdot I_{\text{p}} \cdot \text{Curv}$$

$$M_{\text{sidemin}} = -11.42 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

$$\text{MaxConstructionM} := \text{if}\left(\left|M_{\text{sidemax}}\right| > \left|M_{\text{sidemin}}\right|, \left|M_{\text{sidemax}}\right|, \left|M_{\text{sidemin}}\right|\right)$$

$$\text{ConstructionControl} := \text{if}\left(\text{MaxConstructionM} < M_{\text{y}}, \text{"OK"}, \text{"Reduce Construction Moment"}\right)$$

$$\text{ConstructionControl} = \text{"OK"}$$



**Total Moment**

$$M_{u_1} :=$$

$\frac{\gamma_{EMax} \cdot -M_{sidemin} - \gamma_{EMin} \cdot M_E + \gamma_{LL} \cdot (M_{LL})}{\gamma_{EMin} \cdot M_{sidemax} + \gamma_{EMax} \cdot M_E + \gamma_{LL} \cdot (M_{LL} + M_{Lane})}$
---

$$M_u = \begin{pmatrix} -13.49 \\ 44.08 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{kN} \cdot \text{m}}{\text{m}} \quad M_U := \max(M_u)$$

$$M_U = 44.08 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

**Check total moment against total capacity**

$$M_n := \phi_b \cdot M_p$$

$$M_n = 24.71 \frac{\text{kN} \cdot \text{m}}{\text{m}}$$

$$\text{Status}_{\text{Flexure}} := \text{if}(M_n > M_U, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{\text{Flexure}} = \text{"Redesign"}$$

**COMBINED THRUST AND BENDING**

Reduce thrust to reflect that peak thrust and moment do not occur at same location

$$T_{fSh} := \frac{0.67 \cdot (\gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + \gamma_{LL} \cdot W_{Lane})}{2}$$

$$T_{fSh} = 826 \frac{\text{kN}}{\text{m}}$$

$$T_{fCr} := \frac{0.5 \cdot \gamma_{EMax} \cdot W_E + \gamma_{LL} \cdot W_{LL} + 0.5 \gamma_{LL} \cdot W_{Lane}}{2}$$

$$T_{fCr} = 622 \frac{\text{kN}}{\text{m}}$$

$$T_{fRed} := \text{if}(T_{fCr} > T_{fSh}, T_{fCr}, T_{fSh})$$

$$T_{fRed} = 826 \frac{\text{kN}}{\text{m}}$$

$$\text{Combined}_1 :=$$

$$\begin{array}{c} \frac{T_{fRed}}{R_T} + \frac{8}{9} \cdot \frac{M_U}{M_n} \\ \frac{T_{fRed}}{2 \cdot R_T} + \frac{M_U}{M_n} \end{array}$$

$$\text{Combined} = \begin{pmatrix} 2.26 \\ 2.12 \\ 0.00 \\ 0.00 \end{pmatrix}$$

$$\frac{T_{fRed}}{R_T} = 0.67$$

$$\frac{M_U}{M_n} = 1.78$$

$$\text{Indx} := \text{if} \left( \frac{T_{fRed}}{R_T} \geq 0.2, 1, 2 \right) \quad \text{Indx} = 1.00$$

$$\text{Status}_{\text{Combined}} := \text{if}(\text{Combined}_{\text{Indx}} < 1, \text{"OK"}, \text{"Redesign"})$$

$$\text{Status}_{\text{Combined}} = \text{"Redesign"}$$

**DESIGN SUMMARY**

MinimumStiffness = "Stiffeners Required"	$A = 7.74 \frac{\text{mm}^2}{\text{mm}}$	$T_f = 1234 \frac{\text{kN}}{\text{m}}$
MinimumStiffness <sub>side</sub> = "Increase Side Stiffness"		
Status <sub>Thrust</sub> = "OK"	$I_u = 2395 \frac{\text{mm}^4}{\text{mm}}$	$M_U = 44.08 \frac{\text{kN} \cdot \text{m}}{\text{m}}$
Status <sub>Bucklingtop</sub> = "OK"		
Status <sub>Bucklingbottom</sub> = "OK"	$I_p = 2395 \frac{\text{mm}^4}{\text{mm}}$	
FlexureCheck = "Not Required"	$M_p = 27.46 \frac{\text{kN} \cdot \text{m}}{\text{m}}$	
Status <sub>Flexure</sub> = "Redesign"		
Status <sub>Combined</sub> = "Redesign"		
Combined <sub>Indx</sub> = 2.26		
ConstructionControl = "OK"		
Topchord <sub>max</sub> = 1.00		
Topchord <sub>min</sub> = 0.98		

**Notes**

- Circumferential stiffeners are used to meet minimum stiffness requirement
- Seam strength must also be checked and must be greater than  $T_f$ .

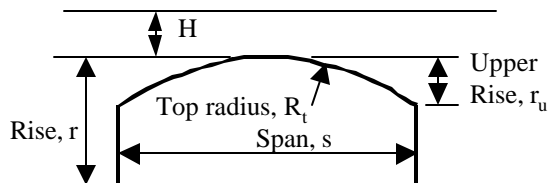
## Long Span Concrete Culvert - Load Calculations

### English Units

#### Input

##### Geometry

Outside span	$s_o := 460 \cdot \text{in}$
Rise	$r := 138 \cdot \text{in}$
Upper rise	$r_u := 54.18 \cdot \text{in}$
Top Radius	$R_t := 486 \cdot \text{in}$
Thick.	$\text{Side}_t := 14 \cdot \text{in}$ $\text{Arch}_t := 12 \cdot \text{in}$



##### Soil & Backfill

		Initial Values:	$\text{Lat}_1 := 0$	$\text{Lat}_2 := 0.3$
Backfill type:	$\text{Soil} := \text{"Sn95"}$	$\text{Lat}_1 := \text{if}(\text{Soil} = \text{"Sn95"}, 0.05, 0)$		
Soil unit weight	$\gamma_s := 120 \cdot \frac{\text{lbf}}{\text{ft}^3}$	$\text{Lat}_1 := \text{if}(\text{Soil} = \text{"Sn90"} \vee \text{Soil} = \text{"Si95"}, .025, \text{Lat}_1)$		
		$\text{Lat}_2 := \text{if}(\text{Soil} = \text{"Sn95"} \vee \text{Soil} = \text{"Sn90"} \vee \text{Soil} = \text{"Si95"}, 0.4, \text{Lat}_2)$		
Depth of fill	$H := 2.0 \cdot \text{ft}$	$\text{Lat}_2 := \text{if}(\text{Soil} = \text{"Sn85"} \vee \text{Soil} = \text{"Si90"} \vee \text{Soil} = \text{"Cl95"}, 0.4, \text{Lat}_2)$		
Lateral pressure coefficient		Final Values	$\text{Lat}_1 = 0.050$	$\text{Lat}_2 = 0.40$

$$K_H := \text{Lat}_2 + \text{Lat}_1 \cdot \frac{H}{3.28 \cdot \text{ft}} \quad K_H := \text{if}(K_H < 0.6, K_H, 0.6) \quad K_H = 0.43$$

- No groundwater above footings of culvert
- Native soil: dense granular

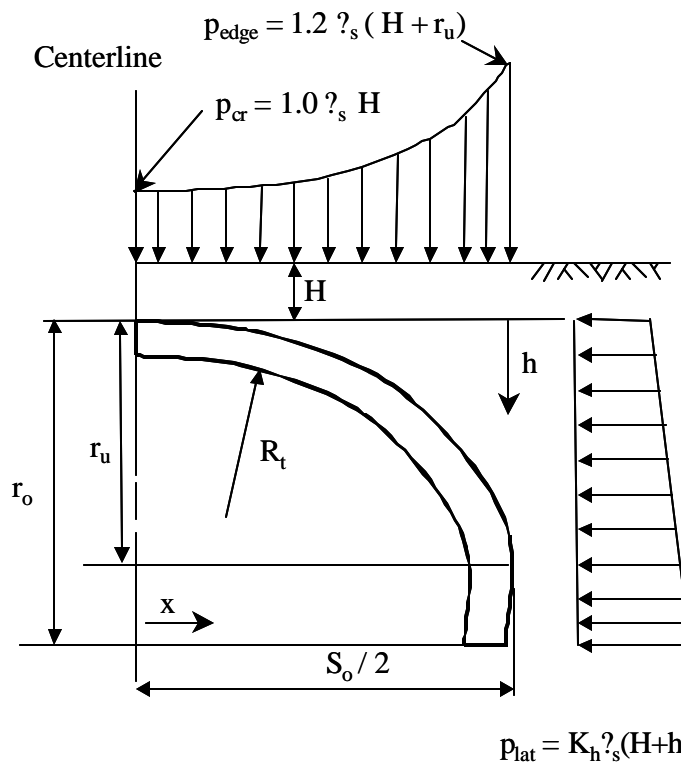
##### Live load

Design Tandem, Load per axle	$P := 25000 \cdot \text{lbf}$
Multiple presence factor	$mp := 1.2$
Impact Factor	$I_0 := 1 + 0.33 \cdot \left( \frac{8 - \frac{H}{\text{ft}}}{8} \right)$
	$I_1 := 1$
	$IM := \max(I) \quad IM = 1.25$

Earth Loads

Vertical Pressure

Soil prism load



$$K_{VAF} := 0.172 + 0.019 \cdot \frac{S_o}{r_u} \quad K_{VAF} = 0.33$$

$$W_{sp} := \gamma_s \cdot S_o \cdot (H + K_{VAF} \cdot r_u) \quad W_{sp} = 16123 \frac{\text{lbf}}{\text{ft}}$$

Soil pressure over crown

$$p_{cr} := \gamma_s \cdot H \quad p_{cr} = 240 \frac{\text{lbf}}{\text{ft}^2} \quad p_{cr} = 20.00 \frac{\text{lbf}}{\text{in} \cdot \text{ft}}$$

Soil pressure at edge

$$p_{edge} := 1.2 \cdot \gamma_s \cdot (H + r_u) \quad p_{edge} = 938 \frac{\text{lbf}}{\text{ft}^2} \quad p_{edge} = 78.18 \frac{\text{lbf}}{\text{in} \cdot \text{ft}}$$

Lateral Pressure

Top

$$P_{lattop} := K_H \cdot \gamma_s \cdot H \quad P_{lattop} = 8.61 \frac{\text{lbf}}{\text{in} \cdot \text{ft}}$$

Bottom

$$P_{latbot} := K_H \cdot \gamma_s \cdot (H + r) \quad P_{latbot} = 58.12 \frac{\text{lbf}}{\text{in} \cdot \text{ft}}$$

Total lateral load

$$P_{Lat} := \frac{(P_{lattop} + P_{latbot})}{2} \cdot r \quad P_{Lat} = 4604 \frac{\text{lbf}}{\text{ft}}$$

## Example Calculations for Loads on Concrete Culverts

Comm. 96232  
Date: 11/30/2001

**Note:** Frame model used in analysis will be based on centerline dimensions. To assure that all load on culvert is placed on the model the pressures need to be scaled up by the ratio of the outside dimensions to the centerline dimensions.

Vertical Pressures:

$$Scv := \frac{s_o}{s_o - Side_t} \quad Scv = 1.03 \quad p_{cr} := p_{cr} \cdot Scv \quad p_{cr} = 20.63 \frac{\text{lbf}}{\text{in} \cdot \text{ft}}$$

$$p_{edge} := p_{edge} \cdot Scv \quad p_{edge} = 80.63 \frac{\text{lbf}}{\text{in} \cdot \text{ft}}$$

Lateral Pressures

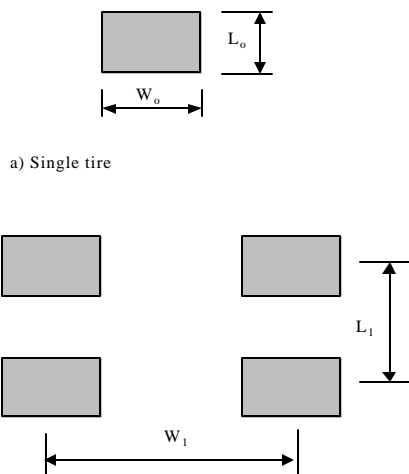
$$Scl := \frac{r}{r - 0.5 \cdot Arch_t} \quad Scl = 1.05 \quad p_{laptop} := p_{laptop} \cdot Scl \quad p_{laptop} = 9.00 \frac{\text{lbf}}{\text{in} \cdot \text{ft}}$$

$$p_{latbot} := p_{latbot} \cdot Scl \quad p_{latbot} = 60.76 \frac{\text{lbf}}{\text{in} \cdot \text{ft}}$$

## Example Calculations for Loads on Concrete Culverts

Comm. 96232  
Date: 11/30/2001

### Live Load



Live load distribution factor:	$LLDF := 1.15$
Wheel length:	$L_o := 10 \cdot \text{in}$
Wheel width	$W_o := 20 \cdot \text{in}$
Wheel spacing	$W_1 := 72 \cdot \text{in}$
Axle spacing (width)	$L_1 := 48 \cdot \text{in}$
Distribution width	$D_w := 40 \cdot \text{in}$
Single wheel load*	$P_w := \frac{P \cdot \text{mp} \cdot \text{IM}}{2}$
*includes multiple presence and impact factors	$P_w = 18713 \text{ lbf}$

At depth of structure:

Patch width for single wheel:	$WS_0 := W_o + LLDF \cdot H + D_w$	$WS_0 = 87.60 \text{ in}$
Patch width for multiple wheels: i.e. wheels interact.	$WS_1 := (W_o + W_1) + LLDF \cdot H + D_w$	$WS_1 = 159.60 \text{ in}$
Depth of interaction:	$H_{\text{wint}} := \frac{(W_1 - W_o - D_w)}{LLDF}$	$H_{\text{wint}} = 0.87 \text{ ft}$
Effective patch width:	$W_s := \text{if}(H < H_{\text{wint}}, WS_0, WS_1)$	$W_s = 159.60 \text{ in}$
	$P_{wa} := \text{if}(H < H_{\text{wint}}, P_w, 2P_w)$	$P_{wa} = 37425 \text{ lbf}$
Patch length for single wheel:	$LS_0 := (L_o + LLDF \cdot H)$	$LS_0 = 37.60 \text{ in}$
Patch length for multiple wheels: i.e. wheel pressures interact.	$LS_1 := [(L_o + L_1) + LLDF \cdot H]$	$LS_1 = 85.60 \text{ in}$
Depth of interaction:	$H_{\text{lint}} := \frac{(L_1 - L_o)}{LLDF}$	$H_{\text{lint}} = 2.754 \text{ ft}$
Effective patch length:	$L_s := \text{if}(H < H_{\text{lint}}, LS_0, LS_1)$	$L_s = 37.60 \text{ in}$
	$P_{wb} := \text{if}(H < H_{\text{lint}}, P_{wa}, 2P_{wa})$	$P_{wb} = 37425 \text{ lbf}$

## Example Calculations for Loads on Concrete Culverts

Comm. 96232  
Date: 11/30/2001

Effective live load pressure on structure:

$$p := \frac{P_{wb}}{W_s \cdot L_s}$$

$$p = 74.84 \frac{\text{lbf}}{\text{in} \cdot \text{ft}}$$

Length of effective pressure

$$L_s := \text{if}(L_s < s_o, L_s, s_o)$$

$$L_s = 37.60 \text{ in}$$

$$\text{Total live load } P := L_s \cdot p$$

$$P = 2814 \frac{\text{lbf}}{\text{ft}}$$

- Notes: 1. This analysis assumes that the length dimension is parallel to the direction of travel and that the direction of travel is across the culvert span.
2. If  $H$  is less than  $H_{\text{lint}}$ , then the structure must be loaded with two loads of magnitude,  $P$ , and length  $L_s$ , spaced a distance  $L_1$ .



## **CALCULATIONS TO EVALUATE REINFORCING REQUIREMENTS FOR CONSPAN REINFORCED CONCRETE CULVERT - Depth of Fill = 2.0 ft (0.6 m)**

### **DIMENSIONS AND MATERIAL STRENGTHS**

Horizontal span of culvert.....	$S_i := 36\text{-ft}$
Crown radius.....	$r_t := 72\text{-ft}$
Clear cover over reinforcement.....	$t_b := 1.5\text{-in}$
Width of section being designed.....	$b := 12\text{-in}\cdot\text{ft}^{-1}$
Design compressive strength of concrete.....	$f_{cp} := 6\text{-ksi}$
Yield strength of steel reinforcement.....	$f_y := 65\text{-ksi}$
Maximum developable stirrup material strength (not greater than $f_y$ or anchorage strength)....	$f_v := 60\text{-ksi}$
Spacing of circumferential reinforcement.....	$s := 2\text{-in}$
Circumferential reinforcementT provided in one (n=1) or multiple (n=2) layers.....	$n := 1$
Reinforcement Type.....	$R_{type} := 2$
1 = smooth wire or plain bars	
2 = welded smooth wire fabric with 8 in. maximum spacing of longitudinals	
3 = welded deformed wire fabric, deformed wire, deformed bars or any reinforcement with stirrups	
Load factor for selfweight.....	$\gamma_{SW} := 1.35$
Load factor for earth pressure.....	$\gamma_E := 0.9, 1.35$
Load factor for live load.....	$\gamma_L := 1.35$
Resistance factor for flexure.....	$\phi_f := 0.95$
Resistance factor for radial tension.....	$\phi_r := 0.90$
Resistance factor for diagonal tension.....	$\phi_v := 0.90$
Radial tension process factor.....	$F_{rp} := 1.0$
Diagonal tesion process factor.....	$F_{vp} := 1.0$
Crack control factor.....	$F_{cr} := 0.9$

Design Forces From Frame Analysis:

$M_{uactual} :=$	$N_u :=$	$M_s :=$	$N_s :=$	$V_{vuactual} :=$
$\begin{pmatrix} 0.00 \\ -208.05 \\ -431.82 \\ -669.29 \\ -796.93 \\ -927.41 \\ -1060.47 \\ -804.51 \\ -580.65 \\ -381.99 \\ -203.63 \\ -41.14 \\ 108.06 \\ 246.50 \\ 376.38 \\ 473.09 \\ 497.71 \end{pmatrix}$	$\begin{pmatrix} 22.33 \\ 21.98 \\ 21.63 \\ 21.28 \\ 21.09 \\ 20.87 \\ 21.67 \\ 20.21 \\ 19.05 \\ 18.13 \\ 17.36 \\ 16.74 \\ 16.22 \\ 15.78 \\ 15.42 \\ 15.05 \\ 14.98 \end{pmatrix}$	$\begin{pmatrix} 0.00 \\ -129.55 \\ -276.57 \\ -438.81 \\ -528.03 \\ -620.41 \\ -715.65 \\ -532.10 \\ -375.01 \\ -238.77 \\ -119.33 \\ -13.13 \\ 82.00 \\ 168.13 \\ 247.01 \\ 304.89 \\ 319.81 \end{pmatrix}$	$\begin{pmatrix} 16.21 \\ 15.95 \\ 15.68 \\ 15.41 \\ 15.26 \\ 15.09 \\ 15.61 \\ 14.60 \\ 13.79 \\ 13.16 \\ 12.65 \\ 12.24 \\ 11.90 \\ 11.62 \\ 11.39 \\ 11.17 \\ 11.12 \end{pmatrix}$	$\begin{pmatrix} -10.84 \\ -11.75 \\ -12.54 \\ -13.23 \\ -13.55 \\ -13.83 \\ 12.34 \\ 10.77 \\ 9.53 \\ 8.53 \\ 7.75 \\ 7.11 \\ 6.58 \\ 6.16 \\ 5.80 \\ 2.96 \\ 0.36 \end{pmatrix}$

### Forces from frame analysis:

Moments are in.-k/ft

Thrusts and shears are k/ft

$M_{uactual}$	= factored moment with proper sign + for tension on inside, - for tension on outside
$M_u$	= moment with all signs positive.
$N_u$	= factored thrust, + is compression
$V_{vuactual}$	= factored shear with proper sign
$V_{vu}$	= factored shear with all signs positive
$M_s$	= service load moment
$N_s$	= service load thrust

Add Units to arrays and convert moments and shears to all positive signs:

$$M_u := M_{uactual} \cdot (\text{in} \cdot \text{k}) \cdot \text{ft}^{-1}$$

$$M_{u_i} := \text{if}(M_{u_i} < 0, -M_{u_i}, M_{u_i})$$

$$N_u := N_u \cdot \text{k} \cdot \text{ft}^{-1}$$

$$M_s := M_s \cdot \text{in} \cdot \text{k} \cdot \text{ft}^{-1}$$

$$M_{s_i} := \text{if}(M_{s_i} < 0, -M_{s_i}, M_{s_i})$$

$$N_s := N_s \cdot \text{k} \cdot \text{ft}^{-1}$$

$$V_{vu} := V_{vuactual} \cdot \text{k} \cdot \text{ft}^{-1}$$

$$V_{vu_i} := \text{if}(V_{vu_i} < 0, -V_{vu_i}, V_{vu_i})$$

$$M_{vu_i} := M_{u_i}$$

$$N_{vu_i} := N_{u_i}$$

Note: Structure and all loads are symmetric, only Nodes 1 to 17 are presented in analysis.

Node 1 is base of leg

Node 7 is corner of segment

Node 17 is crown

# Wall Thickness and Depth to Centroid from Compression Face

i =

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

	( 14.00 )	( 12.02 )
	14.00	12.02
	14.00	12.02
	14.31	12.33
	15.91	13.93
	20.04	18.06
	23.24	21.26
	20.84	18.96
h :=	16.14	d := 14.16
	13.30	11.32
	12.21	10.23
	12.0	10.02
	12.00	10.02
	12.00	10.02
	12.00	10.02
	12.00	10.02
	12.00	10.02
	( 12.00 )	( 10.02 )

Add units:

h := h·in

d := d·in

## 1.1 Reinforcement for Flexural Strength

Define the compressive strength per inch of thickness as.....

$$g := 0.85 \cdot b \cdot f_{cp}$$

$$g = 5.10 \text{ ksi}$$

Required area of flexural steel

$$A_{sf_i} := \frac{1}{f_y} \cdot \left[ g \cdot \phi_f \cdot d_i - N_{u_i} - \sqrt{g \cdot \left( g \cdot (\phi_f \cdot d_i)^2 - N_{u_i} \cdot (2 \cdot \phi_f \cdot d_i - h_i) - 2 \cdot M_{u_i} \right)} \right]$$

	1
1	-0.210
2	0.081
3	0.403
4	0.730
5	0.780
6	0.675
7	0.641
8	0.528
9	0.514
10	0.397
11	0.165
12	-0.094
13	0.021
14	0.259
15	0.488
16	0.663
17	0.708

$A_{sf} = \frac{\text{in}^2}{\text{ft}}$

## 1.2 Minimum Flexural Reinforcement

Minimum reinforcement area....

$$A_{smin_i} := \text{for } j \in i$$

$$\left( \text{if}(i < 7, 0.002 \cdot 12 \cdot 14, .002 \cdot 12 \cdot 12) \cdot \text{in}^2 \cdot \text{ft}^{-1} \right)$$

Governing Reinforcement.....  $A_{sf_i} := \text{if}(A_{smin_i} > A_{sf_i}, A_{smin_i}, A_{sf_i})$

	1
1	0.336
2	0.336
3	0.403
4	0.730
5	0.780
6	0.675
7	0.641
8	0.528
9	0.514
10	0.397
11	0.288
12	0.288
13	0.288
14	0.288
15	0.488
16	0.663
17	0.708

$A_{sf} = \frac{\text{in}^2}{\text{ft}}$

### 1.3 Maximum Flexural Reinforcement without Stirrups (Radial Tension)

Radius of the inside layer of reinforcement.....  $r_s := r_t + t_b$  .....  $r_s = 865.50 \text{ in}$

Size factor for radial tension, fixed value for large span culverts.....  $F_{rt} := 0.8$

Radial tension index:

$$M_{\max} := -\min(M_{u\text{actual}}) \cdot \text{in} \cdot \text{k} \cdot \text{ft}^{-1}$$

$$a_i := \text{for } j \in i$$

$$\text{if} \left( \frac{-M_{\max}}{\text{in} \cdot \text{k} \cdot \text{ft}^{-1}} \neq M_{u\text{actual}_i}, 0, i \right) \quad n_{\text{ndx}} := \max(a) \quad n_{\text{ndx}} = 7.00$$

Parameters at Critical Radial Tension Section

$$M_{\max} = 1.06 \times 10^3 \text{ in} \cdot \text{k} \cdot \text{ft}^{-1} N_{u_{n_{\text{ndx}}}} = 21.67 \text{ k} \cdot \text{ft}^{-1} \quad d_{n_{\text{ndx}}} = 21.26 \text{ in}$$

$$R_{rt} := \frac{M_{\max} - 0.45 \cdot N_{u_{n_{\text{ndx}}}} \cdot d_{n_{\text{ndx}}}}{1.2 \cdot b \cdot d_{n_{\text{ndx}}} \cdot \phi_f \cdot r_s \cdot \sqrt{f_{cp}} \cdot \text{psi} \cdot F_{rt} \cdot F_{rp}} \quad R_{rt} = 0.06$$

Maximum Reinforcement Area

$$\beta_1 := \text{if} \left[ f_{cp} < 4 \cdot \text{ksi}, 0.85, \text{if} \left[ f_{cp} > 8 \cdot \text{ksi}, 0.65, 0.85 - 0.05 \cdot \left( \frac{f_{cp}}{\text{ksi}} - 4 \right) \right] \right] \beta_1 = 0.75$$

$$A_{s\max} := \frac{1}{f_y} \cdot \left( \frac{55 \cdot \phi_f \cdot \beta_1 \cdot d \cdot f_{cp}}{87 + f_y \cdot \text{ksi}^{-1}} - 0.75 \cdot N_u \right)$$

	1
1	3.17
2	3.18
3	3.18
4	3.28
5	3.73
6	4.92
7	5.82
$A_{s\max} =$	8 5.18
	9 3.82
	10 3.02
	11 2.72
	12 2.67
	13 2.67
	14 2.68
	15 2.68
	16 2.69

$\frac{\text{in}^2}{\text{ft}}$

Evaluate limits on Maximum Reinforcement

RadialTension := if (R<sub>rt</sub> < 1, "ok", "Stirrups Required")

MaxCompression<sub>i</sub> := if (A<sub>sf<sub>i</sub></sub> < A<sub>smax<sub>i</sub></sub>, "ok", "NotOK")

Note: If maximum compression is "NotOK" then the options for redesign include:

- increase concrete strength
- increase depth of section
- design section as a compression member with ties

RadialTensi

MaxCompression =

	1
1	"ok"
2	"ok"
3	"ok"
4	"ok"
5	"ok"
6	"ok"
7	"ok"
8	"ok"
9	"ok"
10	"ok"
11	"ok"
12	"ok"
13	"ok"
14	"ok"
15	"ok"
16	"ok"

## 1.4 Flexural Reinforcement Requirements for Crack Width Control

Crack control coefficients.....  $B_1 := \left( \frac{t_b \cdot s}{2 \cdot n \cdot \text{in}^2} \right)^{0.333}$  .....  $B_1 = 1.145$

$C_1 := \text{if}(R_{\text{type}} \geq 3, 1.9, \text{if}(R_{\text{type}} \geq 2, 1.5, 1.0))$ .....  $C_1 = 1.50$

Service level load eccentricity.....  $e_i := \frac{M_{s_i}}{N_{s_i}} + d_i - \frac{h_i}{2}$

Lower bound enforced for  $e_i/d_i$  ratio.....  $\text{edratio}_i := \text{if}\left(\frac{e_i}{d_i} > 1.15, \frac{e_i}{d_i}, 1.15\right)$

Flexural design parameter.....  $j_i := \text{if}\left(0.9 < 0.74 + \frac{\text{edratio}_i}{10}, 0.9, 0.74 + \frac{\text{edratio}_i}{10}\right)$

Flexural design parameter.....  $i_{cr_i} := \left(1 - \frac{j_i}{\text{edratio}_i}\right)^{-1}$

Moment-thrust contribution factor.....  $K_i := \frac{1}{i_{cr_i} \cdot j_i} \cdot \left[ M_{s_i} + N_{s_i} \cdot \left(d_i - \frac{h_i}{2}\right) \right]$

Required area of flexural reinforcement steel for crack width control at service load design based upon **Equation B.7**.....  $A_{scr_i} := \frac{B_1 \cdot \text{psi}^{-1}}{30000 \cdot \phi_f \cdot d_i \cdot F_{cr}} \cdot \left[ K_i - C_1 \cdot (h_i)^2 \cdot \sqrt{f_{cp} \cdot \text{psi}} \right]$

$A_{scr_i} := \text{if}\left[\left(A_{scr_i}\right) < 0, 0, A_{scr_i}\right]$

$e_i =$		$\text{edratio}_i =$	$j_i =$	$i_{cr_i} =$	$A_{scr_i} =$
5.02	in	1.15	0.86	3.90	0.00 in <sup>2</sup> ·ft <sup>-1</sup>
13.14		1.15	0.86	3.90	0.00
22.66		1.89	0.90	1.91	0.00
33.65		2.73	0.90	1.49	0.36
40.58		2.91	0.90	1.45	0.39
49.15		2.72	0.90	1.49	0.00
55.49		2.61	0.90	1.53	0.00
44.99		2.37	0.90	1.61	0.00
33.28		2.35	0.90	1.62	0.00
22.81		2.02	0.90	1.81	0.00
13.56		1.33	0.87	2.93	0.00
5.09		1.15	0.86	3.90	0.00
10.91		1.15	0.86	3.90	0.00
18.49		1.85	0.90	1.95	0.00
25.71		2.57	0.90	1.54	0.05
31.32		3.13	0.90	1.40	0.34
32.78		3.27	0.90	1.38	0.41



Select limiting area based on cracking & flexure:..  $A_{s_i} := \text{if} \left( A_{scr_i} > A_{sf_i}, A_{scr_i}, A_{sf_i} \right)$

$$i2 := 1, 2 \dots 2$$

$$M_{uactual_1} = 0.00$$

Governing positive reinforcement

$$A_{smpos_i} := \text{if} \left( M_{uactual_i} > 0, A_{s_i}, 0 \cdot \frac{\text{in}^2}{\text{ft}} \right)$$

$$A_{sg_1} := \max(A_{smpos})$$

$$A_{sg_1} = 0.71 \frac{\text{in}^2}{\text{ft}}$$

Governing Negative reinforcement

$$A_{smneg_i} := \text{if} \left( M_{uactual_i} < 0, A_{s_i}, 0 \cdot \frac{\text{in}^2}{\text{ft}} \right)$$

$$A_{sg_2} := \max(A_{smneg})$$

$$A_{sg_2} = 0.78 \frac{\text{in}^2}{\text{ft}}$$

Depth to tension reinforcement at governing locations:

User input is required

$$d_{g_1} := d_{17}$$

$$d_{g_2} := d_3$$

	1
1	0.34
2	0.34
3	0.40
4	0.73
5	0.78
6	0.67
7	0.64
8	0.53
9	0.51
10	0.40
11	0.29
12	0.29
13	0.29
14	0.29
15	0.49
16	0.66
17	0.71

$$A_s = \frac{\text{in}^2}{\text{ft}}$$

$$A_{sg} = \left( \frac{0.71}{0.78} \right) \frac{\text{in}^2}{\text{ft}}$$

$$d_g = \left( \frac{10.02}{12.02} \right) \text{in}$$

## 1.5 Shear Strength Calculations

Moment for  $M/V_d$  ratio .....  $M_{nu_i} := M_{vu_i} - N_{vu_i} \cdot \left( \frac{4 \cdot h_i - d_i}{8} \right)$   $M_{nu_i} := \text{if} (M_{nu_i} < 0, 0, M_{nu_i})$

Reinforcement ratio.....  $\rho_{i2} := \text{if} \left( 0.02 < \frac{A_{sg_{i2}}}{b \cdot d_{g_{i2}}}, 0.02, \frac{A_{sg_{i2}}}{b \cdot d_{g_{i2}}} \right)$   $\rho_{i2} = \frac{0.0059}{0.0054}$

$\rho_{g_i} := \text{if} (M_{uactual_i} > 0, \rho_2, \rho_1)$

Factor for depth of section .....  $F_{d_i} := 0.8 + \frac{1.6 \text{ in}}{d_i}$   $F_{d_{i2}} := \text{if} (F_{d_{i2}} > 1.3, 1.3, F_{d_{i2}})$

Thrust factor .....  $F_{n_i} := 1 + \frac{N_{vu_i} \cdot \text{psi}^{-1}}{24000 \cdot h_i}$

$F_c$  factor for shear capacity calculation.....  $F_{c_i} := 1 + \frac{d_i}{r_t + 0.5 \cdot h_i}$   $F_{c_i} := \text{if} (i < 7, 1, \text{if} (i > 26, 1, F_{c_i}))$

$M_{nu} =$

	1
1	0
2	87
3	313
4	550
5	666
6	765
7	866
8	642
9	461
10	287
11	120
12	0
13	31
14	172
15	303
16	402
17	427

$\frac{\text{in} \cdot \text{k}}{\text{ft}}$

$\rho_g =$

	1
1	$5.89 \cdot 10^{-3}$
2	$5.89 \cdot 10^{-3}$
3	$5.89 \cdot 10^{-3}$
4	$5.89 \cdot 10^{-3}$
5	$5.89 \cdot 10^{-3}$
6	$5.89 \cdot 10^{-3}$
7	$5.89 \cdot 10^{-3}$
8	$5.89 \cdot 10^{-3}$
9	$5.89 \cdot 10^{-3}$
10	$5.89 \cdot 10^{-3}$
11	$5.89 \cdot 10^{-3}$
12	$5.89 \cdot 10^{-3}$
13	$5.41 \cdot 10^{-3}$
14	$5.41 \cdot 10^{-3}$
15	$5.41 \cdot 10^{-3}$
16	$5.41 \cdot 10^{-3}$
17	$5.41 \cdot 10^{-3}$

$F_d =$

	1
1	0.93
2	0.93
3	0.93
4	0.93
5	0.91
6	0.89
7	0.88
8	0.88
9	0.91
10	0.94
11	0.96
12	0.96
13	0.96
14	0.96
15	0.96
16	0.96
17	0.96

$F_n =$

	1
1	1.01
2	1.01
3	1.01
4	1.01
5	1.00
6	1.00
7	1.00
8	1.00
9	1.00
10	1.00
11	1.00
12	1.00
13	1.00
14	1.00
15	1.00
16	1.00
17	1.00

$F_c =$

	1
1	1.00
2	1.00
3	1.00
4	1.00
5	1.00
6	1.00
7	1.02
8	1.02
9	1.02
10	1.01
11	1.01
12	1.01
13	1.01
14	1.01
15	1.01
16	1.01
17	1.01

Shear capacity at critical section.....  $V_{b_i} := \phi_v \cdot b \cdot d_i \cdot F_{vp} \cdot \sqrt{f_{cp} \cdot \psi_i} \cdot \left( 1.1 + 63 \cdot \rho_{g_i} \right) \cdot \frac{F_{d_i} \cdot F_{n_i}}{F_{c_i}}$

$$MVD_i := \text{if} \left( \frac{M_{nu_i}}{V_{vu_i} \cdot d_i} > 3, 3, \frac{M_{nu_i}}{V_{vu_i} \cdot d_i} \right)$$

$$V_{c_i} := \text{if} \left( V_{b_i} \cdot \frac{4}{MVD_i + 1} > 4.5 \cdot \sqrt{f_{cp} \cdot \psi_i} \cdot d_i, 4.5 \cdot \sqrt{f_{cp} \cdot \psi_i} \cdot d_i, \frac{V_{b_i} \cdot 4}{MVD_i + 1} \right)$$

Diagonal tension index.....  $R_{dt_i} := \frac{V_{vu_i}}{V_{c_i}}$

Evaluate Diagonal Tension Strength  $DT_{strength_i} := \text{if} (R_{dt_i} > 1, \text{"Strength Exceeded"} , \text{"OK"})$

		$MVD_i =$		$V_{c_i} =$		$R_{dt_i} =$	
				$\frac{k}{ft}$			
$V_b =$	1	13.88	0.00	50.28	0.22	1	"OK"
	2	13.88	0.62	34.32	0.34	2	"OK"
	3	13.88	2.08	18.05	0.69	3	"OK"
	4	14.18	3.00	14.18	0.93	4	"OK"
	5	15.76	3.00	15.76	0.86	5	"OK"
	6	19.82	3.00	19.82	0.70	6	"OK"
	7	22.43	3.00	22.43	0.55	7	"OK"
	8	20.27	3.00	20.27	0.53	8	"OK"
	9	15.72	3.00	15.72	0.61	9	"OK"
	10	13.01	2.97	13.10	0.65	10	"OK"
	11	11.96	1.51	19.05	0.41	11	"OK"
	12	11.76	0.00	41.91	0.17	12	"OK"
	13	11.51	0.47	31.31	0.21	13	"OK"
	14	11.51	2.78	12.18	0.51	14	"OK"
	15	11.51	3.00	11.51	0.50	15	"OK"
	16	11.51	3.00	11.51	0.26	16	"OK"
	17	11.51	3.00	11.51	0.03	17	"OK"

### Check if Circumferential Reinforcement Can Be Increased to Improve Shear Strength

$$A_{sinc_i} := \text{if} \left[ R_{dt_i} > 1, \frac{0.01587 \cdot V_{vu_i}}{\phi_v \cdot F_{vp} \cdot \sqrt{f_{cp} \cdot \psi_i}} \cdot \left( \frac{F_{c_i}}{F_{d_i} \cdot F_{n_i}} \right) - ((0.01746)) \cdot d_i, A_{s_i} \right]$$

$$\rho_{inc_i} := \frac{A_{sinc_i}}{b \cdot d_i} \quad A_{sinc_i} := \text{if} \left( \rho_{inc_i} > 0.02, 10^5 \cdot \text{in}^2 \cdot \text{ft}^{-1}, A_{sinc_i} \right)$$

$$DT_{inc_i} := \text{if} \left( \rho_{inc_i} > 0.02, \text{"Stirrups Requ'd"}, \text{"OK"} \right)$$

If increased reinforcement ratio is greater than 2% than stirrups must be used

**Governing Design**

$$A_{si_i} := \text{if} \left( M_{uactual_i} > 0, A_{sinc_i}, 0 \cdot \frac{\text{in}^2}{\text{ft}} \right) \quad A_{sinside} := \max(A_{si_i})$$

$$A_{so_i} := \text{if} \left( M_{uactual_i} < 0, A_{sinc_i}, 0 \cdot \frac{\text{in}^2}{\text{ft}} \right) \quad A_{sooutside} := \max(A_{so_i})$$

		$A_{si_i} =$		$A_{so_i} =$		$\rho_{inc_i} =$	
		$\frac{\text{in}^2}{\text{ft}}$		$\frac{\text{in}^2}{\text{ft}}$			
$A_{sinc} =$	1	0.336	0.00	0.00	0.002	$DT_{inc} =$	1
	2	0.336	0.00	0.34	0.002		2
	3	0.403	0.00	0.40	0.003		3
	4	0.730	0.00	0.73	0.005		4
	5	0.780	0.00	0.78	0.005		5
	6	0.675	0.00	0.67	0.003		6
	7	0.641	0.00	0.64	0.003		7
	8	0.528	0.00	0.53	0.002		8
	9	0.514	0.00	0.51	0.003		9
	10	0.397	0.00	0.40	0.003		10
	11	0.288	0.00	0.29	0.002		11
	12	0.288	0.00	0.29	0.002		12
	13	0.288	0.29	0.00	0.002		13
	14	0.288	0.29	0.00	0.002		14
	15	0.488	0.49	0.00	0.004		15
	16	0.663	0.66	0.00	0.006		16
	17	0.708	0.71	0.00	0.006		17

### Design Summary:

Flexural  
Criteria  
Only

$$A_{sg} = \left( \frac{0.71}{0.78} \right) \frac{\text{in}^2}{\text{ft}}$$

Flexure, crack,  
and diagonal  
tension criteria

$$A_{sinside} = 0.71 \frac{\text{in}}{\text{ft}}$$

$$A_{sooutside} = 0.78 \frac{\text{in}}{\text{ft}}$$

Note - If  $A_s$  is listed as  $10^5 \text{ in}^2/\text{ft}$ , the shear strength cannot be adequately increased by increasing the circumferential reinforcement. Stirrup reinforcement, a thicker section or increased concrete strength are possible adjustments to the design.

1	"OK"
2	"OK"
3	"OK"
4	"OK"
5	"OK"
6	"OK"
7	"OK"
8	"OK"
9	"OK"
10	"OK"
11	"OK"
12	"OK"
13	"OK"
14	"OK"
15	"OK"
16	"OK"
17	"OK"